Fat-Tailed Wishart Autoregressive Processes for Multivariate Stochastic Volatility with Jumps

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Short Summary:

In times of macroeconomic or financial turmoil volatility typically jumps suddenly to extreme values. We develop a distribution for volatility that allows for fat tails and that generalizes the Wishart Autoregressive Process. We show that this specification allows for a simple and efficient MCMC algorithm. By conditioning on some auxiliary variables, all volatility matrices can be sampled jointly from the joint conditional posterior. Because of its simplicity and efficiency, the MCMC algorithm can be used to tackle relatively large dimensions. In applications to real data we show that the extension to fat tails is empirically relevant.

JEL Classification: C10, C11, C58

Extended Abstract:

We are concerned with the following model:

$$y_t = \mu_t + \Omega_t^{1/2}\varepsilon_t \qquad t = 1, ..., T \tag{1}$$

where y_t is an observed $r \times 1$ vector and Ω_t is a time-varying $r \times r$ variance-covariance matrix. One possible way to model time-varying covariance matrices is to assume that Ω_t^{-1} follows the Wishart Autoregressive process proposed by Gourieroux, Jasiak and Sufana (2009, GJS henceforth). This can be defined by writing $\Omega_t^{-1} = \beta_t'\beta_t$, where β_t is a $n \times r$ matrix distributed according to:

$$\begin{aligned} \beta_t &= \beta_{t-1}\rho + \eta_t \\ vec(\eta_t) &\sim N(0, I_n \otimes \Sigma) \end{aligned} \tag{2}$$

where Σ is a $r \times r$ matrix, I_n is the identity matrix of order n and ρ is a diagonal matrix of dimension $r \times r$.

Here we propose an extension of the Wishart Autoregressive process defined by (2) that allows for more flexible tails in the distribution of Ω_t. To define this fat-tail Wishart Autoregressive process, instead of specifying a normal distribution for $\beta_t|\beta_{t-1}$ we assume instead the following:

$$\pi(\beta_t|\beta_{t-1}) \propto \left(\beta_t'\beta_t\right)^{-\nu/2} \exp\left(-\frac{1}{2}tr\left[\Sigma^{-1/2}\left(\beta_t - \beta_{t-1}\rho\right)'\left(\beta_t - \beta_{t-1}\rho\right)\right]\right) \qquad \nu \geq 0$$

The parameter ν controls the tail of the distribution. When $\nu = 0$ we have the Wishart Autoregressive process of GJS, but for $\nu > 0$ we have a distribution that gives higher weight to values of Ω_t^{-1} near 0, and therefore higher weight to large values of Ω_t. Firstly we analyze the statistical properties of the fat-tail Wishart process. Then we provide an MCMC algorithm that makes use of the introduction of T auxiliary variables. By conditioning on these auxiliary variables, all volatility matrices can be sampled jointly from the joint conditional posterior (given $\mu_1, ..., \mu_T$). Because of its simplicity and efficiency, the MCMC algorithm can be used to tackle relatively large dimensions. In applications to real data we show that the extension to fat tails is empirically relevant.

References:

Gourieroux, C., Jasiak, J. and Sufana, R. (2009) "The Wishart Autoregressive process of multivariate stochastic volatility" *Journal of Econometrics*, 150, 167-181