POPULATION DENSITY, OPTIMAL INFRASTRUCTURE AND ECONOMIC GROWTH

Sumit S. Deole
Aix-Marseille School of Economics

Abstract

This paper develops an OLG model with endogenous fertility and education decisions to analyze the impact of transportation infrastructure on economic growth. This study argues that there is an optimal distribution of infrastructure in the region to boost the transition of an economy from economic stagnation towards the developing phase. The study assumes that the population induced productivity improvements and provision of optimal infrastructure in the region will form the two necessary conditions for this transition. In line with what is actually seen during the process of development, if the transportation costs as a fraction of labor income are assumed to be decreasing, results obtained under section 2 imply a paradox. At a macro-economic level, the model proposes simpler micro-foundations of the geographical interpretations of economic growth in order to study the effect of population density on growth. The higher population density enables the set-up costs of additional infrastructure in that region to be created, opening the possibility of enhanced welfare. But lagging behind on additional investments in infrastructure could prolong the transition. Using time series data for years 1960-2010 on India, the study examines the role of transportation infrastructure on economic growth. Empirical evidences, with the help of OLS estimations, suggest that there is a negative impact of infrastructure costs on the fertility decisions of the parents and similarly, a positive impact of population density on economic growth. With the help of Granger causality test, study confirms the unidirectional causality between population density and economic growth. The study also reflects the government's emphasis on building new roads than building new railway tracks considering the costs involved. (JEL: J13, O40, R28)

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Notes

[1]Email address: sumitdeole@gmail.com
[2]This study was carried out as a part of master thesis project work in Aix-Marseille School of Economics.The author successfully defended his work on 25th June 2012.

1 Introduction

Contribution of different growth enhancing policies in the aggregate development of the economy has been a central question in the literature of economic growth. Accordingly, public education, foreign investments, domestic financial markets, international trade have been considered important engines of economic growth of the country. Domestic human and physical capital development do form the initial conditions for the above mentioned growth engines and hence literacy and infrastructure improvements are usually set as primary goals. The measurement of the impact of human capital development on growth could easily be quantified from the different empirical findings, where in the absence of proper physical capital (available infrastructure etc) in the country, one sees an outward migration of educated people towards the developed world. On the other hand, the impact of infrastructure development alone is seen as inequality enhancing for different regions of the same country. The fact that infrastructure services are often provided by the public sector and as a result they are often not priced at all, or are rationed, one finds it difficult to estimate the private productivity of infrastructure capital (Canning, 1999). This study investigates the impact of infrastructure availability on the fertility and education decisions of individuals and then at macro level, on the economic growth of the country.

The main focus of this paper is going to be the socio-economic changes in India in the post-independence era, i.e. mainly from the year 1960-2010. In India, the region around river Ganga, mainly, consisting of states like Bihar and Uttar Pradesh, accounts for almost 35% of the population of India but its GDP is not in that proportion with regards to the national GDP. In post independence times, this region faced a lot of problems which were similar to other regions in the country, for example, inequality, poverty, illiteracy, etc, but unlike other regions, poor infrastructure development led it into the poverty trap and severity of these problems magnified. As shown in the figure 1, population density pattern for these regions diverged from that of aggregate Indian population density. One of the reasons behind this divergence could be the asymmetric effect of international or inter-regional trade of this region with the other parts of the country and with other countries resulting into the channeling of the trade gains into the population growth (Galor and Mountford, 2008). Out-migration, and relatively high and sluggish

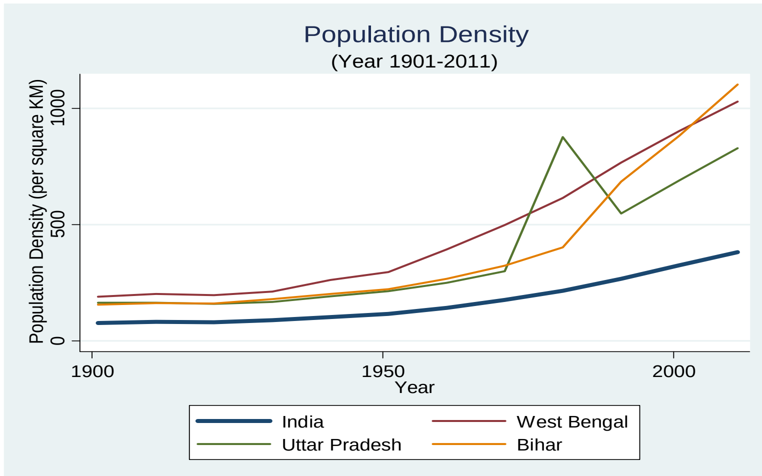

Figure 1: Population Density 1901-2011

total fertility rates are some of the indicators of this.

While studying the role of physical capital in the country, the crucial role played by Indian railways in the total transportation infrastructure can not be ignored. The ticket prices of rail network in India are mostly under the control of central government and there is a relatively frequent change in the price structure. Unlike city and interstate bus system, rail system lacks proper ticket checking on stations and moreover, a separate compartment for poor makes it more reliable source of transportation. The consecutive selection of minister of railways from this poor region, made a political statement to get the increased share of the infrastructure towards these states which connected them with other growing parts of the country so that individuals could move freely to receive better health facilities, and job opportunities. If rail transport is assumed to be the only important transport channel for poor population or small businesses to move then one should see a positive effect both on education and business development in the highly connected region. As a result, pure investment in railway infrastructure will bring in positive impact on economic activity and also on the process of human capital accumulation in the region. Studies of growth enhancing effects of infrastructure provision go back to Rosenstein Rodan's (1943) view of Big Push, which recommended huge infrastructure investments in order to alleviate poverty and experience growth. Agénor (2010) further studied the theory of development based on public infrastructure as the main engine of growth. According to him, the positive effect of infrastructure investments on GDP is not restricted to the capital stock creation via economies of scale but its mere existence is competition enhancing for the economy via network externalities. As shown in figure 2 below, since the time of the independence of the country, there has been a slow increase in the number of stations and also the number of passengers in the country. But there has been a large increase in passenger kilometers traveled, indicating the preference of passengers to travel large distances. One of the main reasons of this shift could be the geographic set up of the country in post independence era, where different states were formed on the basis of different languages. Population traveled long distances keeping in mind the economic gains of doing so or for education purposes explaining the part of the out-migration which resulted into dense metros. Physical capital development complementing to the investment in human capital development may encourage economic activities in the region.

Does the infrastructure investment causes more dense population? The answer is yes. Beyzatlar and Kuştepeli (2011) in their empirical study on Turkish annual data from 1950-2004, pointed out that there is a short as well as long run positive relationship between the length of railway infrastructure and population density. It is already known that higher population density can lower the set up costs facilitating further creation of new infrastructure (Fujita, M., 2002). Glover and Simon (1975) also showed that the denser population brings in more infrastructures per worker in the region concluding the empirical analysis with a result that population density is a significant cause of higher road density.

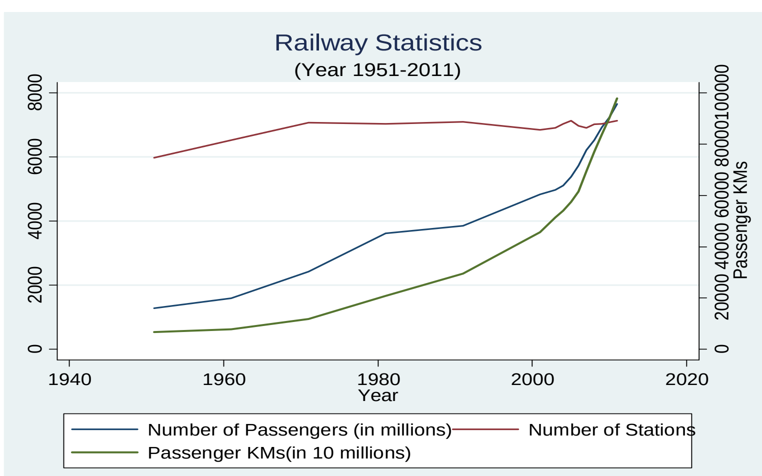

Figure 2: Railway Statistics

As argued by Becker, Glaeser, and Murphy (1999), larger population encourages greater specialization in the form of assimilation of ideas. On the similar lines, Galor and Weil (2000) discussed the positive effect of population density on the technological progress. But such an effect only shows up through the increase in the profitability of the infrastructure providing facility with increase in population density (Boucekkine, R. et al., 2007). Failing to provide the optimum level of infrastructure, results into economic contraction or under-performance of the economy.

Following from the description above, one requires two necessary conditions on policy making of the economic development to take the economy from stagnation towards the economic development and these are: a) population induced productivity improvements, and b) provision of optimal infrastructure in the region. While modeling the population induced productivity improvements to study the optimal infrastructure allocation, Boucekkine, R. et al. (2007) has important shortcoming, i.e. exogenous demographics. Endogenizing fertility in the model a la Boucekkine, R. et al.(2007) is very complicated and hence, OLG model a la Croix-Doepke (2003) is introduced. The micro-foundations of geographical interpretation of economic growth is very well explained in Boucekkine, R. et al. (2007) which are originally absent in Croix-Doepke (2003) as well as Galor (2005). The computations involving micro-foundations of geographical interpretations in presence of OLG model with endogenous fertility are complicated, so they are incorporated in a very simple manner in this article. That is to say, the computations for optimal number of public facilities, i.e. number of stations are omitted from this study.

The study commences with an overlapping generation (OLG) model allowing the fertility decisions of the parents endogenous for given infrastructure costs in the section 2. This section is further divided in subsections studying different scenarios in which infrastructure costs affect parental decisions of educating their children as well as how many children to reproduce. In the fourth subsection, the central authority problem of station location or the decision to build new public facility is studied with the help of simpler micro-foundations of geographical interpretations of economic growth. Mathematical properties of the profit function of the public facility are studied deeply in this subsection. Third section consists of the empirical analysis investigating important theoretical relationships proved in section 2 via simple OLS regressions as well as Granger causality techniques.

2 Theoretical Exploration

The model economy is populated with overlapping generations of people. All the decisions made by the individuals are in their adulthood. Infrastructure is provided by the central authority. The term infrastructure can broadly mean any public facility provided by the central authority, but for specificity, transportation infrastructure is considered in this setting. Parental decision making includes decision related to the number of children they should have, i.e. the fertility decision, decision to educate their progenies, and also decision related to the location of their residence, i.e. where to reside: (I) Near workplace or (II) Near school or (III) Reside far from both, workplace as well as school?

2.1 Residing near Workplace

In this setting, parents decide to reside near their workplace and school location for their children is secondary decision on their part. As a result, there are no infrastructure costs on their part but only children incur the transportation costs for their daily commute to the school.

2.1.1 Model Specifications

The model considers an economy consisting of individuals who live for three periods: childhood, adulthood and old age. Adults make all the decisions. The utility of a household depends on consumption (c_t), future consumption (d_{t+1}), number of children (n_t) and the human capital of their children (h_{t+1}).

$$U = \max \left\{ \ln(c_t) + \beta \ln(d_{t+1}) + \gamma \ln(n_t h_{t+1}) \right\}$$

In above equation, β represents the preference of the adult for future and it represents his motivation for savings and hence capital formation. γ represents the altruism factor. Following on De la Croix and Doepke (2003), the study assumes that, human capital of teachers equals the average human capital of the population. Another assumption entails that there is an intra-family transmission of human capital suggesting that educated parents create positive externality on children's human capital level. This transmission is called an externality as the level of human capital of parents in first place is not their decision but is considered as already given. Following on the discussion of location selection problem of parents, the adult could choose to reside near work or near school depending on the costs involved in doing so. The study will commence with the first case, i.e. case (i) on the budget constraint where parents have decided to locate near workplace and the budget constraint is as follows:

$$c_t + s_t + (e_t + g_t)w_t\overline{h}_t n_t = w_t h_t(1 - \varphi n_t) \tag{1}$$

where w_t represents wage per unit of human capital,and s_t represents his ability to save. e_t represents the schooling time per children, $\overline{h}_t$ represents average human capital, $w_t\overline{h}_t$ represents the labor income in efficiency units and hence education cost per child is given by $e_t w_t \overline{h}_t$. Factor g_t is the fraction of the labor income devoted to pay the usage of public services and is introduced in order to anticipate for transportation spending of parents on the tickets of their children to travel from home to the school. So the part $g_t w_t \overline{h}_t$ represents the payment for the usage of public facilities (e.g. tickets). The study also takes into account the fact that up to certain age, children have to be accompanied by an adult to be able to take the bus. So even if the nominal price of the ticket is zero, because the adult is constrained to accompany his child, the real price of the ticket is not zero. Hence it is assumed that g_t is never zero in this setting. On the other hand, adult uses his/her time endowment in two respects: total time available hours (assumed it as 1), and time for child rearing per children, (φ). Person's future consumption (d_{t+1}) is dependent on his saving rate (s_t) in present and the rate of returns to his savings, i.e. R_{t+1}.

$$d_{t+1} = R_{t+1}s_t, \tag{2}$$

$$h_{t+1} = B_t(\theta + e_t)^{\eta}(h_t)^{\tau}(\overline{h}_t)^{\kappa} \tag{3}$$

While explaining the development of human capital of the children, the following things should be considered. The parameter θ captures the positive human capital of the children, in the circumstances when he/she is not being educated. The parameter τ represents the inter-generation transmission of human capital within the household. Parameter κ represents the quality of education system or schooling. Parameters B, θ satisfy the positivity condition, i.e. $B, \theta > 0$. The efficiency parameter, B_t is defined as,

$$B_t = (1 + \rho)^{(1-\tau-\kappa)t} \tag{4}$$

As assumed in Rangazas (2000), to introduce endogenous growth in the model, a condition on $\kappa = 1 - \tau$ is made compatible. The human capital of the parents is distributed as given by the function $F_t(h_t)$. With fertility decision in mind, the evolution of the total population is given by,

$$P_{t+1} = P_t \int_0^{\infty} n_t dF_t(h_t) \tag{5}$$

where, P_{t+1} is the total population at time $(t+1)$ and n_t is the fertility rate. The distribution of human capital function evolves according to

$$F_{t+1}(h) = \frac{P_t}{P_{t+1}} \int_0^\infty n_t I(h_{t+1} \leq h) dF_t(h_t) \tag{6}$$

where, $I(.)$ is a typical indicator function.

Similarly, the average human capital is given by,

$$\overline{h}_t = \int_0^\infty h_t dF_t(h_t) \tag{7}$$

where, $\overline{h}_t$ is the average human capital.

There is an unique consumption good and it is produced by a single representative firm. The production technology used by this firm is given by,

$$Y_t = AL_t \tag{8}$$

where, L_t is the aggregate labor supply. There is no capital in the equation of the firm because it is being assumed that firms don't need capital and infrastructure provided by the government is used by the children and not the firms. This assumption also helps to remain consistent with prime motive of this analysis to study the accumulation of human capital. The profit function of the firm is given by, $Y_t - w_t L_t$.

Definition 1 *For given initial distribution of human capital $F_0(h_0)$ and for an initial population size P_0, an equilibrium consists of wages $\{w_t\}$, aggregate quantities $\{L_t, \overline{h}_t, P_{t+1}\}$, distributions $\{F_{t+1}(h_{t+1})\}$ and decision rules $\{c_t, d_{t+1}, s_t, n_t, e_t, g_t, h_{t+1}\}$*

1) The decision rules of the household maximize utility subject to the constraints explained in equations 1, 2, and 3.

2) Firms producing unique consumer good maximize profits by hiring labor inputs.

3) The wages of labor inputs, w_t clear markets

4) Human capital distribution follows equation 6

Now, solving the households' decision rules $c_t, e_t, s_t, n_t, h_{t+1}, d_{t+1}$ in order to maximize the utility subject to the constraints (1),(2) and (3);

Substituting constraints in the lagrange as follows,

$$\begin{aligned} L = & \ln(w_t h_t (1 - \varphi n_t) - s_t - (e_t + g_t) n_t w_t \overline{h}_t) + \beta \ln(R_{t+1} s_t) \\ & + \gamma(\ln n_t B_t + \eta \ln(\theta + e_t) + \tau \ln h_t + \kappa \ln \overline{h}_t) \end{aligned} \tag{9}$$

For computational simplicity and sectional analysis of the population with distributed human capital, different parameters of the programme are made to form a condition that, parents with higher human capital level are identified by the condition: $x_t > \frac{\theta - g_t\eta}{\varphi\eta}$ and those with lower levels of human capital are identified by the condition $x_t \leq \frac{\theta - g_t\eta}{\varphi\eta}$. Where x_t is relative human capital and is given by $x_t = \frac{h_t}{\overline{h}_t}$. At equilibrium, these conditions become $(\varphi + g_t)\eta > \theta$ for parents with higher level of human capital and $(\varphi + g_t)\eta \leq \theta$ for parents with lower level of human capital.

There is an interior solution for the optimal level of education and is obtained by solving the optimization problem for the household with higher human capital ($x_t > \frac{\theta - g_t\eta}{\varphi\eta}$) as shown in the appendix A1,

Now solving for the equation for e_t,

$$e_t = \frac{\varphi h_t \eta + g_t \eta \overline{h}_t - \overline{h}_t \theta}{\overline{h}_t(1-\eta)}$$

Taking $\overline{h}_t$ common from numerator and denominator, the above equation in terms of x_t, i.e. relative human capital of a household could be obtained.

$$e_t = \frac{\varphi x_t \eta + g_t \eta - \theta}{(1-\eta)} \tag{10}$$

The above equation shows that the infrastructure costs as a fraction of labor income work as complementary to the schooling time in this setting. In the process of development, it is seen that the transportation costs as a fraction of labor income are decreasing over time and with this assumption, result obtained in the above equation gives paradoxical results.

Similarly, the equation for number of children is obtained as follows,

$$n_t = \frac{\gamma h_t(1-\eta)}{(1+\beta+\gamma)(-\overline{h}_t\theta + g_t\overline{h}_t + \varphi h_t)}$$

n_t when expressed in terms of x_t is,

$$n_t = \frac{\gamma x_t(1-\eta)}{(1+\beta+\gamma)(\varphi x_t - \theta + g_t)} \tag{11}$$

From the equation derived above for n_t, it is clear that the number of children or the fertility rate of the parent is non-increasing with the transportation costs as a fraction of labor income, they have to pay in order to educate their children. As mentioned earlier, during the process of development, it is seen that the transportation costs as a fraction of labor income decreases over time and with this assumption, result obtained in the above equation gives paradoxical results.

And the equation for s_t,

$$s_t = \frac{\beta w_t h_t}{(1+\beta+\gamma)}$$

A constant saving rate is obtained which is in line with Croix-Doepke (2003).

The labor market condition required for equilibrium analysis is given by,

$$L_t = P_t(\int_0^\infty h_t(1-\varphi n_t)dF_t(h_t) - \int_0^\infty n_t e_t \overline{h}_t dF_t(h_t)) \tag{12}$$

On the right side of the above equation there is aggregate labor demand and on the right side, there is labor supplied by households in efficiency units. At equilibrium, human capital of individuals is equal to the average human capital of the society and the labor market condition becomes,

$$L_t = P_t(\int_0^\infty \overline{h}_t(1-\varphi n_t - n_t e_t)dF_t(\overline{h}_t)) \tag{13}$$

In other words, above equation entails that the time devoted to teaching is not available for working for the parents. After defining all the conditions above, the equilibrium of the economy can be defined as shown in section 2.1.2.

Proposition 2 *Quantity-Quality Trade-off: Skilled people invest more in the quality of their children than their quantity, i.e. investment in education of the progeny increases with the relative human capital of the parents and number of children given birth to decreases with the relative human capital of the parents.*

In other words, above proposition means that the education costs on the children of parents with higher relative human capital are increasing and at the same time, the number of children given birth to, is decreasing. To prove this well documented fact, simple derivatives of equation 10 and 11 with respect to x_t are calculated as shown in appendix A2. Another wordy explanation of the Quality-Quantity trade-off is that rich people end up improving the quality of their children and poor people end up reproducing more children ($\frac{\partial e_t}{\partial x_t} > 0$ and $\frac{\partial n_t}{\partial x_t} < 0$). The reason behind this trade-off could be that the cost of education is fixed, while the time cost of raising many children increases with income.

Proposition 3 *Transportation costs negatively impact fertility decisions of the parents.*

To prove this proposition, equation 11 is used. The fertility decisions obtained in equation 11, are derived with respect to transportation costs as shown in the appendix A3. From the appendix A3, it is clear that there is a negative impact of transportation costs on the number of children parents give birth to.

2.1.2 Balanced Growth Path

The Balanced Growth Path (BGP) is being carried out in order to study the dynamic behavior of the economy and the equilibrium conditions are written in terms of the variables that are kept constant. The growth rate of average human capital, the population growth rate N_t, distribution of relative human capital levels, and deflated level of average human capital $\widetilde{h}$ are given as,

$k_t = \frac{\overline{h}_{t+1}}{\overline{h}_t}, N_t = \frac{P_{t+1}}{P_t}, G_t(x_t) = F_t(x_t\overline{h}_t), \widetilde{h} = \frac{\overline{h}}{(1+\sigma)^t}$

Using these variables, let's rewrite equations 5, 6, and 7 as follows,

$$N_t = \int_0^{\infty} n_t dG_t(x_t) \tag{14}$$

$$G_{t+1}(x) = \frac{1}{N_t}\int_0^{\infty} n_t I(x_{t+1} \leq x) dG_t(x_t) \tag{15}$$

$$1 = \int_0^{\infty} x_t dG_t(x_t) \tag{16}$$

At equilibrium, at firm level, marginal costs equalize marginal productivity and from equation 8,

$$w_t = A \tag{17}$$

Now, lets try to find the range of fertility rate. Equation explained in 11, denotes the fertility rate of parents with higher human capital level. Parents with low human capital levels, end up producing more number of children instead of concentrating on the quality of the children. The fertility rate of the parents with low human capital level is given by,

$$n_t = \frac{\gamma}{\varphi(1+\beta+\gamma)}$$

The range of fertility rates in terms of x_t is,

$$n_t = Min[\frac{(1-\eta)\gamma x_t}{(\varphi x_t + g_t - \theta)(1+\beta+\gamma)}, \frac{\gamma}{\varphi(1+\beta+\gamma)}] \tag{18}$$

From equation 3 and Proposition 2, the equation of human capital of the children becomes,

$$x_{t+1} = \frac{Bx_t^{\tau}}{k_t}(\theta + Max[0, \frac{\eta(\varphi x_t + g_t) - \theta}{(1-\eta)}])^{\eta}(\widetilde{h})^{\tau+\kappa-1} \tag{19}$$

And also, the labor marker clearing condition as explained in 12, becomes

$$\frac{L_t}{P_t\overline{h}_t} = \int_0^{\frac{\theta - g_t\eta}{\eta\varphi}} \frac{(1+\beta)x_t}{1+\beta+\gamma} dG_t(x_t) + \int_{\frac{\theta - g_t\eta}{\eta\varphi}}^{\infty} (1-\gamma\frac{\varphi(1-\eta)x_t + (\eta\varphi x_t + g_t - \theta)}{(\varphi x_t + g_t - \theta)(1+\beta+\gamma)})x_t dG_t(x_t) \tag{20}$$

Now, lets study the Balanced Growth Path analysis to study the long run behavior of the economy.

Proposition 4 *In the long run, in the case where, for the given g, every individual in the economy has the same level of human capital ($dG(1) = 1$) and also $\eta(\varphi+g) > \theta$, then there is a balanced growth path and growth factor of output growth and human capital is given by $k^* = B(\frac{\eta(\varphi+g-\theta)}{1-\eta})^{\eta}$.*

As per mentioned in the conditions of the proposition, every individual is endowed with same level of human capital, i.e. $h_t = \overline{h}_t$ and hence relative human capital of the individual becomes, $x_t = \frac{\overline{h}_t}{\overline{h}_t} = 1$. Also, every individuals will be able to attend the school and hence maximum schooling effect. From the condition that the model studies endogenous growth ($\kappa = 1 - \tau$) and from equations 14, 15, 16, 18,and 19,

$$\begin{aligned}
1 &= \frac{B}{k^*}\frac{1}{}(\theta + \frac{\eta(\varphi x_t + g) - \theta}{(1-\eta)})^{\eta}(\widetilde{h})^{0} \\
k^* &= B\ (\theta + \frac{\eta(\varphi + g) - \theta}{(1-\eta)})^{\eta} \\
k^* &= B\ (\frac{\theta(1-\eta) + \eta(\varphi + g) - \theta}{(1-\eta)})^{\eta} \\
k^* &= B\ (\frac{\theta(1-\eta) + \eta(\varphi + g) - \theta}{(1-\eta)})^{\eta}
\end{aligned}$$

$$k^* = B(\frac{\eta(\varphi + g - \theta)}{(1-\eta)})^{\eta} \tag{21}$$

Hence QED. But in the process of development, it is seen that the transportation costs as a fraction of labor income decreases over time and with this assumption in hand, the result obtained in the above equation gives paradoxical results. As in the assumption of the proposition, at $\eta(\varphi + g) > \theta$ at equilibrium, there is a positive growth in this setting.

Corollary 5 *Steady state growth rate is positively impacted by the transportation costs.*

Following on the derived formula for growth factor of output growth in proposition 4, $k^* = B(\frac{\eta(\varphi+g-\theta)}{(1-\eta)})^\eta$. Now deriving k^* with respect to g, gives

$$\frac{\partial k^*}{\partial g} = \frac{\partial B(\frac{\eta(\varphi+g-\theta)}{(1-\eta)})^\eta}{\partial g}$$
$$\frac{\partial k^*}{\partial g} = B\eta(\frac{\eta(\varphi+g-\theta)}{(1-\eta)})^{\eta-1}\frac{\eta}{(1-\eta)}$$
$$\frac{\partial k^*}{\partial g} > 0$$

Hence, steady state growth is positively impacted by transportation costs and QED.

2.2 Residing in a School District

In this subsection, the study investigates the impact of parental choice to locate near school on their decision to educate children, fertility decisions and hence on the economic growth of the economy. In this scenario, parents pay transportation costs to reach their workplace but for the children going to school, no transportation costs are incurred. The real time example of this scenario is Belgium, where children are exempted from transportation ticket costs. There is no change in the optimizing utility equation or else the constraints except in the budget constraint, which becomes,

$$c_t + s_t + g_t\overline{h}_t w_t + e_t\overline{h}_t n_t w_t = w_t h_t(1 - \varphi n_t - q)$$

where, q represents, a constant representing time taken by the individual to reach his/her workplace. This is a novelty introduced in this subsection. Lagrangian and effective equations for education and number of children are as shown in appendix A4.

$$e_t = \frac{\varphi h_t \eta - \overline{h}_t \theta}{\overline{h}_t(1-\eta)} \tag{22}$$

From the above equation for education costs, it is clear that education costs are independent of the transportation costs incurred for parents and are different from the one obtained in previous section.

$$n_t = \frac{\gamma(h_t(1-q) - g_t\overline{h}_t)(1-\eta)}{(\varphi h_t - \overline{h}_t\theta)(1+\beta+\gamma)} \tag{23}$$

From the above mentioned equation for number of children parents should give birth to, it is clear that transportation costs incurred for parents and time spent by parents in traveling impact the number of children negatively. But in the process of development, it is seen that the transportation costs as a fraction of labor income decreases over time and with this assumption, result obtained in the above equation gives paradoxical results.

Proposition 6 *In the long run, in the case where, for the given* g*, every individual in the economy has the same level of human capital* ($dG(1) = 1$) *and also* $\eta\varphi > \theta$*, then there is a balanced growth path and growth factor of output growth and human capital is given by* $k^* = B(\frac{\eta(\varphi-\theta)}{1-\eta})^\eta$.

In order to derive the proof of this proposition, the method undertaken to prove the proof of proposition 4 is undertaken. With the similar logic and with the help of equation 22, the equation 19 becomes,

$$x_{t+1} = \frac{Bx_t^\tau}{k_t}(\theta + Max[0, \frac{\varphi x_t\eta - \theta}{(1-\eta)}])^\eta(\widetilde{h})^{\tau+\kappa-1} \tag{24}$$

Hence, at steady state, when $x_t = 1, k_t = k^*$and for endogenous growth setting, i.e. $\kappa = 1 - \tau$, equation 24 becomes,

$$\begin{aligned}
1 &= \frac{B}{k^*}(\theta + \frac{\varphi\eta - \theta}{(1-\eta)})^\eta(\widetilde{h})^0 \\
k^* &= B(\theta + \frac{\varphi\eta - \theta}{(1-\eta)})^\eta \\
k^* &= B(\frac{\varphi\eta - \theta + \theta(1-\eta)}{(1-\eta)})^\eta \\
k^* &= B(\frac{\varphi\eta - \theta\eta}{(1-\eta)})^\eta
\end{aligned}$$

$$k^* = B(\eta(\frac{\varphi - \theta}{(1-\eta)}))^\eta \tag{25}$$

Hence QED. As in the assumption of the proposition, at $\varphi > \theta$, there is a positive growth in this setting. On contrary to results obtained in the section 2.1, the economic growth is independent of infrastructure costs in this setting. The reason behind this lies in the assumption that parents decide to reside near school and schooling decision is independent of ticket payments of parents.

2.3 Both, parents as well as children incur transportation costs

In this case, parents reside away from their destination, that is, away from their workplace as well as from the school. Hence, parents need to pay for infrastructure costs incurred by them as well as their children. In this case, same as the case before, there is no change in the optimizing utility equation or else the constraints except on the budget constraint, which becomes,

$$c_t + s_t + g_t \overline{h}_t w_t + (e_t + g_t) n_t \overline{h}_t w_t = w_t h_t (1 - \varphi n_t - q)$$

Lagrangian and effective equations for education and number of children comes as following. Complete calculations are done in appendix A5.

Now solving the optimization problem as shown in the appendix A5, the equation of e_t is given as,

$$e_t = \frac{\eta(h_t \varphi + g_t \overline{h}_t) - \overline{h}_t \theta}{\overline{h}_t (1 - \eta)} \tag{26}$$

From the above equation for education costs, it is clear that education costs are dependent on the transportation costs incurred by parents but strictly because of the commute of the children towards school and hence there is an impact on the education related decisions. In the process of development, it is seen that the transportation costs as a fraction of labor income decreases over time and with this assumption, result obtained in the above equation gives paradoxical results.

$$n_t = \frac{\gamma(1 - \eta)(h_t - q h_t - g_t \overline{h}_t)}{(-\theta \overline{h}_t + h_t \varphi + g_t \overline{h}_t)(1 + \beta + \gamma)} \tag{27}$$

From the above mentioned equation for number of children parents should give birth to, it is clear that transportation costs incurred for parents and time spent by parents in traveling impact the number of children negatively. But in the process of development, it is seen that the transportation costs as a fraction of labor income decreases over time and with this assumption, result obtained in the above equation gives paradoxical results.

Proposition 7 *In the long run, in this case where, for the given g, every individual in the economy has the same level of human capital ($dG(1) = 1$) and also $\eta(\varphi + g) > \theta$, then there is a balanced growth path and growth factor of output growth and human capital is given by $k^* = B(\frac{\eta(\varphi+g-\theta)}{1-\eta})^\eta$.*

In order to derive the proof of this proposition, the method undertaken to prove the proof of proposition 4 is undertaken. With the similar logic and with the help of equation 26, the equation 19 becomes,

$$x_{t+1} = \frac{Bx_t^\tau}{k_t}(\theta + Max[0, \frac{\eta(x_t\varphi + g_t) - \theta}{(1-\eta)}])^\eta(\widetilde{h})^{\tau+\kappa-1} \tag{28}$$

Hence, at steady state, when $x_t = 1, k_t = k^*$and for endogenous growth setting, i.e. $\kappa = 1 - \tau$, equation 28 becomes,

$$\begin{aligned} 1 &= \frac{B}{k^*}(\theta + \frac{\eta(\varphi + g) - \theta}{(1-\eta)})^\eta(\widetilde{h})^0 \\ k^* &= B(\theta + \frac{\eta(\varphi + g) - \theta}{(1-\eta)})^\eta \\ k^* &= B(\frac{\eta(\varphi + g - \theta)}{(1-\eta)})^\eta \end{aligned}$$

$$k^* = B(\frac{\eta(\varphi + g - \theta)}{(1-\eta)})^\eta \tag{29}$$

Hence QED.

As already explained in equation 21 of section 2.1, in the process of development, it is seen that the transportation costs as a fraction of labor income decreases over time and with this assumption, result obtained in the above equation gives paradoxical results. The difference in the assumption of this subsection from that of 2.1, puts pressure on the savings rate of parents but decision to educate their children incorporates the actual growth in this setting.

2.4 Station Location Policy

For this subsection studying the infrastructure allocation policy, the location problem under consideration is in continuous space. Individuals born at the same date are located at different location uniformly. The individuals in the cohort have different human capital h, distributed as given by the distribution $F_t(h_t)$. The migration of households is only acceptable considering people are optimizing the distance for commuting to near school or near work.

In this scenario, the study considers the existence of a central authority that determines the optimal number of transport stations to be built in the area. The set-up cost for implementing a new transport facility (train station) is C. If no station is created, the transportation costs are infinite. The first assumption comes from the logical realization that building cost of the new station is inversely proportional to total population present in the area. At each date, a number E of stations created to serve the population. The framework follows the one explained in Bos (1965) and Boucekkine et al (2007). The second assumption supposes that the heterogenous population (with respect to human capital level) is distributed uniformly in the region and also equally among all the stations. It is considered that the transportation costs are the function of distance and as the distance from the destination is reduced, the transportation costs will also decrease.

Considering that the population is evenly spaced, optimal setting is when stations are equidistantly and uniformly distributed. To avoid indeterminacy, it is assumed that there is a station located at 0 and their further location is given by $(1-j)/E$, where $j = 1, .., E$. The potential catchment area of the station is given by below figure. The figure represents a circular segment $[-1/(2E), 1/(2E)]$.

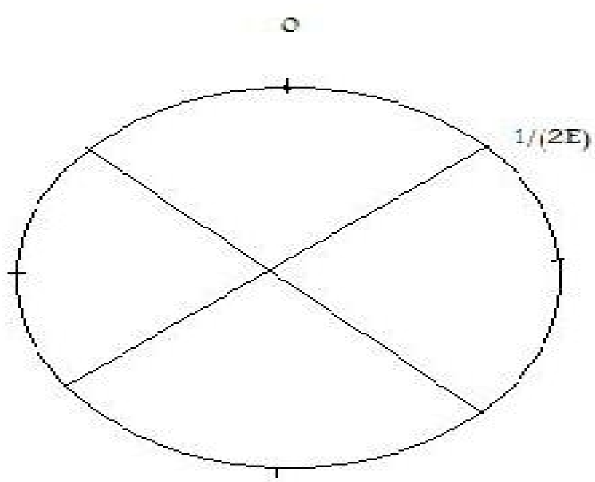

Figure 3: Station Location

The assumption on human capital helps in stating that population with higher level of human capital will definitely use the station facility. Similar logic applies for the distance of individual from the station, as the distance increases, less people (the ones with higher level of human capital) will use the facility. To keep focus clear and calculations simpler, homogeneity condition on population's human capital level is implemented wherever feasible.

Now, lets study the important factors/conditions to be considered for the erection of single station in the region following aforementioned simple geographical interpretation.

2.4.1 Population Dynamics

In this case, the rate of growth of population is equal to the fertility rate and with the help of which, the study undertakes fertility decisions of parents forward. As a result, total population becomes endogenous in this model. Hence the equation for total population from equation 5 is,

$$P_{t+1} = P_t \int_0^\infty n_t dF_t(h_t) \tag{30}$$

where, P_{t+1} is the total population present at time $t+1$, P_t is the population present at time t and n_t is total fertility rate at time t.

2.4.2 The Incentive for Central Authority to Build the infrastructure

In line with the assumptions made in the previous subsection, every individual in the population uses the transportation facility. Since, there is a heterogeneity in the human capital of the population, their spread is very important for the central authority to determine the profitability of the facility. To make calculations simpler, it is supposed that central authority foresees the demography and then decide on the transportation costs to be paid by the individuals and if more number of stations are needed to be built in the region. There is a perfect foresight in the model for central authority on demography and hence population of $t+1$ is considered more important while deciding transportation costs at time t. The nominal benefits brought about by the station are thus given by,

$\int_0^\infty g_t A h_t P_{t+1} dF_t(h_t)$

where P_{t+1} is the population at time $t+1, h_t$ is the human capital at time t, A is introduced in order to control for the technological progress. And hence the profit function of the facility becomes,

$$\Pi(g_t) = \int_0^\infty g_t A h_t P_{t+1} dF_t(h_t) - C \tag{31}$$

substituting equation 5,

$$\Pi(g_t) = \int_0^\infty g_t A h_t P_t \int_0^\infty n_t dF_t(h_t) dF_t(h_t) - C \tag{32}$$

Proposition 8 *For the homogeneity of human capital in the population, no stations will be built in the region for the condition that* $P_{t+1} <$

$\frac{C}{\int_0^\infty g_t A\overline{h_t} dF_t(\overline{h_t})}$, *i.e. the costs involved in building stations are not covered by the aggregate benefits, then no stations are built.*

As mentioned in the proposition, the population is homogenous in human capital (all the individuals have same human capital level, i.e. $\overline{h_t}$), and above derived equation for profitability of the facility becomes,

$\Pi(g_t) = \int_0^\infty g_t A\overline{h_t} P_{t+1} dF_t(\overline{h_t}) - C$

For this facility, being a loss making entity, above equation becomes,

$\int_0^\infty g_t A\overline{h_t} P_{t+1} dF_t(\overline{h_t}) - C < 0$

The equation after rearranging for P_{t+1},

$$P_{t+1} < \frac{C}{\int_0^\infty g_t A\overline{h_t} dF_t(\overline{h_t})} \tag{33}$$

Hence, QED. This proposition can also be interpreted in a way to determine the threshold level on population, needed to be present in order to make the station facility profitable.

Lemma 9 *At $g_t \cong 0$, the facility, with given benefit function, turns into the loss making entity.*

As explained earlier, benefit function is a linear function of transportation cost as a fraction of labor income, i.e. g_t as shown below.

$$\Pi(g_t) = \int_0^\infty g_t A h_t P_{t+1} dF_t(h_t) - C$$

At $g_t \cong 0$, the equation becomes,

$$\begin{aligned}\Pi(g_t) &= [0 - C] \\ &= -C\end{aligned}$$

Hence, the given lemma is proved.

Lemma 10 *Properties of* $\Pi(g)$

1) It is obviously a continuous function.

2) Limits on profit function when g_t grows infinitely.

$\lim_{g_t w_t \overline{h}_t \to 0} \Pi(g_t) = -C$

$\lim_{g_t w_t \overline{h}_t \to \infty} \Pi(g_t) = indefinite$ ($-C$ or ∞)

Lemma 11 *The optimization problem of* $\Pi(g_t)$ *at* $E > 0$ *admits a solution* G*, such that for all* $g > G$*, the profit of the entity enters into a loss,* $\Pi(g_t) < 0$*.*

This assumed level on transportation cost as a fraction of labor income, i.e. G does also indicate the affordability of the population to use the availed facility. For the ticket set at $g > G$, nobody uses the facility and the facility turns into a loss making entity. This level of affordability of ticket on population is very well explained in Boucekkine et al. (2007) and is useful to mention here.

It can also be said that the optimization problem admits a maximum at $g\epsilon[0; G]$.

$$\frac{\partial \Pi(g)}{\partial g_t}(0) = \int_0^\infty A h_t P_{t+1} dF_t(h_t)$$
$$\frac{\partial \Pi(g)}{\partial g_t}(0) > 0$$

This is also a global maximum because of the condition already obtained: $\Pi(g_t = 0) = -CE$ for all $g > G$.

Proposition 12 *In the case, when* $\overline{g} = \overline{g}(E)$*, an unique and global optimal solution for* (g) *exists and the maximized profit is necessarily non-negative.*

In this case, the optimal transportation cost as a fraction of labor income $\overline{g}$ is independent of the fixed cost incurred, i.e. C and is a function of the number of stations available in the region, i.e. E. The condition of positive profitability of the facility suggests that the condition, $\int_0^\infty \overline{g}(E) A h_t P_{t+1} dF_t(h_t) \geq C$, should prevail. As it could be derived from implicit theorem, $\overline{g}(E)$ is a differential function, and as a result of which, $\xi(E) = \int_0^\infty \overline{g}(E) A h_t P_{t+1} dF_t(h_t)$ is also a continuous and a differential function. But in this case, at $E \to \infty$, the rate at which population uses the facility, goes to zero, because $\overline{g}(E) < \overline{g}$, where $\overline{g}$ is the optimized transportation costs for maximum profit. Hence, the study entails a boundary condition on the number of stations to be built in order to achieve positive or zero profit condition, i.e. non negative profits. Moreover, the non-zero condition on the number of station facilities will still prevail because positive profitability condition will not hold for $E = 0$. Hence, mathematically, the optimization problem with respect to (E) is given as,

$$S = \{(g)\epsilon R^2 : 0 \leq E; \xi(E) \geqslant C; 0 \leq g \leq \overline{g}\}$$

The set S is a compact subset of R^2 and because $\Pi(g)$ is continuous in (g) it must reach a maximum in S. The non-negativity of the profit comes immediately from the above mentioned definitions of $\overline{g}$ and $\xi(E)$.

2.4.3 Alternative Specifications

After studying optimal behavior of the central authority for static profits in subsection 2.4.2, this study turns towards two alternative specifications, which can be studied here.

Case I (inter-temporal profits): The profit function explained above in equation 32, is a problem of static optimization but the case of inter-temporal profits for public infrastructure facility can also be studied by considering all the sequences of transportation costs and the profit function becomes,

$$\Pi = \int_0^\infty \delta^t \Pi(g_t) dt$$

$$\Pi = \int_0^\infty \delta^t (\int_0^\infty g_t A h_t P_t \int_0^\infty n_t dF_t(h_t) dF_t(h_t) - C) dt \tag{34}$$

where, Π is the inter-temporal profits of the public facility, and δ is the discount factor.

Case II (zero profits) : Although, it is not a reality, zero profit condition on central authority is welfare enhancing. The mathematical specification of the model at zero profits thus becomes

$$\Pi(g_t) = 0$$

$$\int_0^\infty g_t A \overline{h_t} P_{t+1} dF_t(h_t) - C = 0 \tag{35}$$

which implies that, $\int_0^\infty g_t A \overline{h_t} P_{t+1} dF_t(h_t) = C$.

Perhaps, it is not possible to crave for welfare enhancing zero profit conditions on central authority in the fastly growing countries like India, where investment funds for infrastructure are considered much important, but it is a possible scenario.

3 Empirical Evidences

This study now turns to an evaluation of the inter-relationship between the stock of public capital, human capital, and output per capita. This empirical evaluation offers a link to the aforementioned theoretical model as it allows to map the coefficient estimates for different variables like population density, total fertility rate, transportation costs, infrastructure availability in terms of number of stations and total track length of railway constructed and per capita GDP.

The partial aim of this study is to theoretically investigate the transmission mechanism of the impact of transportation infrastructure on the household decision making of reproduction and then collectively on the economic growth of the country. While doing so, in the first part, the study verifies the impact of transportation infrastructure on the total fertility rate. At macro level, infrastructure with relatively inexpensive transportation costs should increase the usage of available infrastructure, resulting into allocation of resources towards their efficient use. Urbanization and increased economic activity are few of the consequences of this. Hence in the second part, study investigates the impact of infrastructure and resulted population density on the GDP per capita of the country.

To verify these above mentioned relations, simple method of ordinary least square (OLS) is used. In the following subsection, previously obtained OLS results are verified using Granger's causality.

3.1 The Data

The data under observation mainly focuses on India for the period from 1960-2010. This data has some limitations for some variables indicating infrastructure availability, for example, in terms of data collection for the first few decades when decadal data is readily available instead of annual data. To be more specific, data on number of railway stations in India is decadal for first few decades and since then annual data is present. The aggregate data on per capita GDP, total fertility rate (TFR), total length of railway lines and population density could be found on World Bank database (http://databank.worldbank.org) and is available for all the years under observations. The aggregate data on variables relevant to Indian railways is taken from the statistical summary pages of Indian railways (http://www.indianrail.gov.in), Planning commission of India's web-site and several reports on the performance of Indian railways, for example, India: Appraisal of Thirteenth Railway Project by World Bank, report on rate of dividend for 2006-07 and other Ancillary Matters presented in Fourteenth Loksabha in the year of 2006. Before starting to employ the econometric tests, it is deemed valuable to graph the variables in question and provide descriptive statistics about them.

Lets commence the descriptive statistics with the data collected on the variables describing the state of Indian railways. Following on figure 2, it is clear that the total number of railway stations in the country did not change with the state of economic activity. This number kept fluctuating in between its lowest value in the year 1961 of 6523 stations to 7146 stations in the year 2005. In the year 2010, there were 7083 railway stations in the country, the value which is not that far away from its all time mean, i.e. 6976 stations. Similarly, for the total length of railway track during the observed time period. This length varied from 56247 kilometers in the year 1961 to 63506 kilometers in the year 1998. Continuing with the consolidation and making more use of available infrastructure, Indian railways put more emphasis on increasing the passenger and freight rates. Average rate charged per passenger kilometer was 1.71 paise in the year of 1961, which reached the maximum of 25.9 paise in the year 2010. The rates were increased slowly during the early observed period but significantly increased in late 1980s and for the period from there on till early 2000s. Similar trend could be seen in terms of rates charged for freight transportation. As a result, these trends in rates were reflected in net revenues earned by Indian railways. Periodic mean net revenue of Indian railways was INR 2.13e+10. Net revenue reached its periodic minimum of 5.54e+08 in the year of 1974 and maximum of INR 1.83e+11 in the year 2008 following massive privatization of railway property.

As shown in figure 4, it can easily be understood that the share of railway transportation in total transport sector of the country has declined continuously since the independence. This suggests the overemphasis on railway transportation for the intra-country movement of the passengers and freights and hence increasing economic or migration activity is caught by the government's concentration on road transport. Due to improved role of private transportation in road transportation and the role of road transportation in total transportation sector of India, the study should consider both, road as well as rail infrastructure while continuing with the empirical exploration.

In this paragraph, lets concentrate on demographic characteristics of the country. Population density of the country significantly increased from 149 people per square kilometer in the year of 1961 to 393.88 people per square kilometer in the year 2010. Main reasons behind this could be increased urbanization in the country. In the year of 1971, only 20.22% of population used to live in urban areas and this percentage increased by almost half till the year of 2011, where 31.16% of total population lives in urban areas. Here point to be noted that, urbanization in highly dense states of India is still very low. In the state of Bihar, urbanization is only 11.29% and in the state of Uttar Pradesh, it is only 22.28%. Total fertility rate of the country gradually decreased over the same period. TFR in the year of 1961 was 5.87, which gradually decreased to 2.663 in 2009. Reasons favoring this reduction in TFR could be such as literacy improvements, increased investment in public education, various awareness programmes etc. On the other hand, following on figure 5, GDP per capita increased over the period under observation.

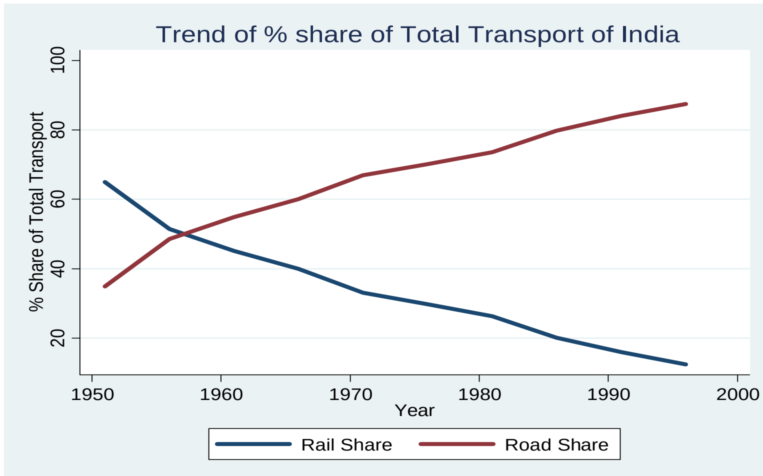

Figure 4: Railway Statistics

GDP per capita in 1960 was INR 691.0811 at its all time minimum level which gradually increased to INR 37621.52 in 2010, which is its all time maximum level. The mean GDP for the time under observation is INR 9441.106.

From figure 6, it is clear that life expectancy at birth has increased significantly since the observed year 1960. The improvement is of more than 50% from the age in years of 42.5 years in 1960 to 64.77 years in 2010. This could also be explained with the help of reduction in crude death rate of the country during the observed time period. Crude death rate (CDR) of India reduced to 34.85% in 2009 of what it was in the year of 1960. The crude birth rate (CBR), is another effective indicator nicely suggesting quality-quantity hypothesis with literacy improvements, also reduced to 54.52% for the given period. From the figure 6, it can be seen that the gap between CBR and CDR increased during 1970s due to improved health conditions, hence resulted in improved population growth rate of the country.

3.2 Regression Analysis

Regression analysis consists of two regression tables. All the variables used are log-linearized. Lag of all the independent variables are taken so as to check for possible endogeneity among regressors. Regression 1 tries to investigate the impact of infrastructure availability parameters (i.e. average rate charged per passenger kilometer by Indian railway, length of railway lines, total number of stations, and road length), and quality

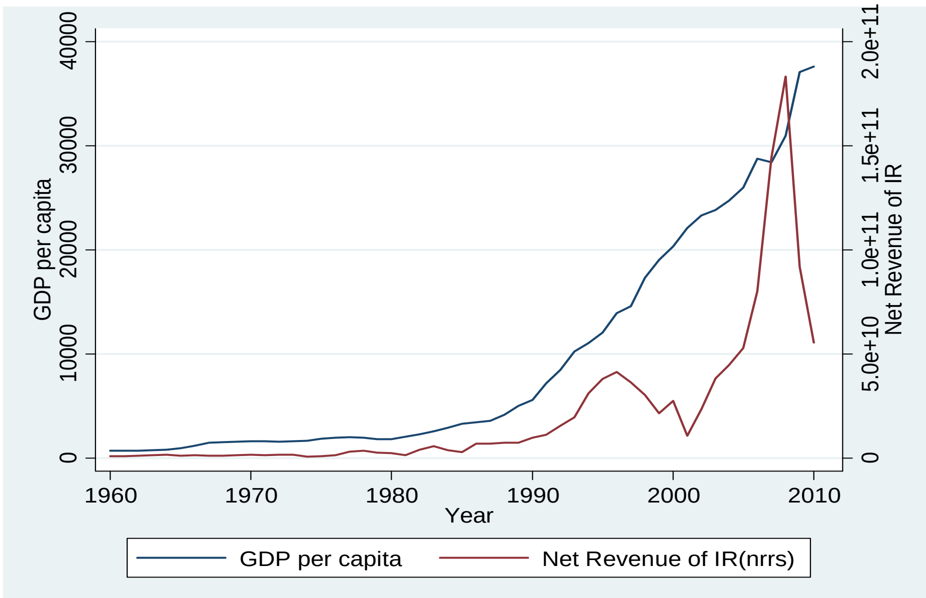

Figure 5: GDP per capita and Net Revenue of Indian Railways

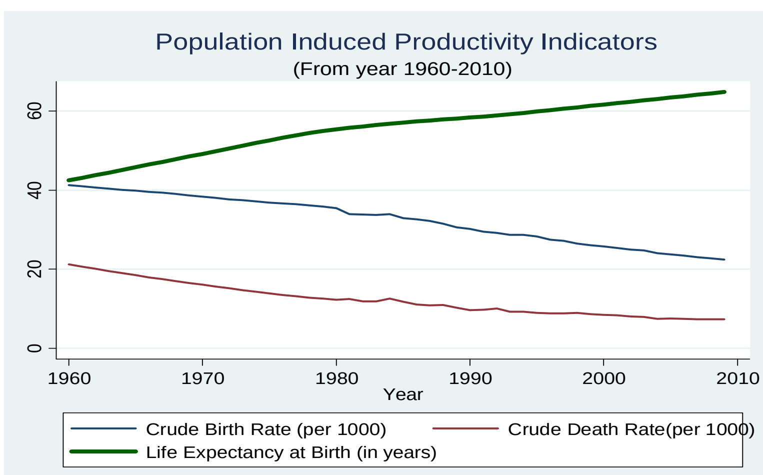

Figure 6: Demographic Indicators

of life parameters (i.e. infant mortality, and life expectancy at birth) on the total fertility rate (TFR) of the country. Ultimate aim of regression 1 is to study the impact of infrastructure costs on TFR of the country supporting theoretical result derived in equation 11. Similarly, regression 2 investigates the impact of infrastructure availability parameters (i.e. length of railway lines, and road length), and quality of life parameters (i.e. infant mortality, life expectancy at birth, and population density) on per capita GDP of the country. Ultimate aim of regression 2 is to study the impact of population density on per capita GDP of the country. The regression models under consideration are as following:

$$\begin{aligned} Log(TFR_t) = {} & B + B_1 Log(RateperPassKm_{t-1}) + B_2 Log(RailRoute_{t-1}) \\ & + B_3 Lag.\log(RoadLength_{t-1}) + B_4 Log(Nu\,\mathrm{ms}tat_{t-1}) + \\ & B_5 Log(IM_{t-1}) + B_6 Log(LE_{t-1}) + U_t \end{aligned}$$

$$\begin{aligned} y_t = {} & c + c_1 Log(PopDens_{t-1}) + c_2 Log(IM_{t-1}) + c_3 Log(LE_{t-1}) \\ & + c_4 Log(RoadLength_{t-1}) + c_4 Log(RailRoute_{t-1}) + e_t \end{aligned}$$

Where, subscript t represents time series in between 1960 and 2010, 50 samples in total. y represents per capita GDP, $RoadLength$ represents the length of total roads, $RailRoute$ represents the length of railway route, IM represents the infant mortality of the country, LE represents the life expectancy at birth of the country, TFR represents the total fertility rate of the country, $PopDens$ represents the population density of the country and $RateperPassKm$ represents the average rate charged by Indian railways per passenger KM

Referring to table 1, study proves the negative and significant relationship between the infrastructure costs (ticket) and the total fertility rate of the country. The sign and significance of this relationship prevails even if we control for relevant factors emphasizing on railway and road transport infrastructure, then for quality of life determinants, for example, life expectancy at birth and infant mortality. As explained in data section, data on number of stations is not available for all the years under consideration, hence the results of this regression can not be said to be credible but they do give some basic idea about the intended analysis in section 2.

From table 2, it is clear that there is a positive and significant impact of population density on the GDP per capita of the country. This significance and positive impact prevails even if the parameters indicating transportation level as well as quality of life determinants are controlled.

Table 1: OLS Estimates-I

Variables:	log(TFR)	log(TFR)	log(TFR)	log(TFR)
L1.log(Average Rate per Passenger	-0.26720***	-0.20427**	-0.09703**	-0.10025*
	0.00822	0.07177	0.04226	0.04437
L1.log(Length of Railway Lines)		-3.19195	-0.98192	-0.89653
		3.07441	1.66995	0.59695
L1.log(Number of Stations)		0.41663	0.00787	0.21535
		1.06721	0.63054	0.14187
L1.log(Road Lengths)			-0.2811***	-0.04901
			0.04812	0.02894
L1.log(Infant Mortality)				0.83532***
				0.10293
L1.log(Life Expectancy)				-0.59636
				0.64486
L1.log(Population Density)				0.78562
				0.53926
Constant	0.69305***	32.37777	15.94600	4.11972
	0.02220	33.00301	18.00265	6.00932
R-squared	0.97509	0.94072	0.98639	0.99956

Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

Table 2: OLS Estimates-II, 1960-2010

Variables:	log(GDP)	log(GDP)	log(GDP)
L1.log(population density)	5.11601***	2.95937**	3.76000***
	0.10116	0.45945	0.75122
L1.log(Life expectancy)		-3.98903***	-4.06088***
		0.50720	0.86195
L1.log(Infant mortality)		-3.39936***	-3.16447**
		0.26717	0.40538
L1.log(length of roads)			-0.17865*
			0.10101
L1.Log length railway lines			-4.09571
			2.71604
Constant	-19.86541***	24.45793***	65.87836**
	0.55997	2.41191	26.50742
Observations	49	49	34
R-squared	0.98195	0.99873	0.99862

Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

During the sampled time period, the phenomenon of urbanization was seen as increasing very rapidly in the form of increased population density of the country. Urbanized population mainly worked in technologically advanced modern sector, being displaced from less productive (usual assumption as none) agriculture sector and perhaps this explains the larger value of coefficient representing impact of population density on per capita GDP growth.

Empirical analysis suggested that infrastructure costs actually impacted negatively on TFR of the country, which is in line with the theoretical model explained in section 2. At macro levels, shifting of population towards urban areas, i.e. aforementioned urbanization definitely played an important role in the growth of Indian per capita GDP.

3.3 Testing the Causality

Causality testing is introduced here in order to neutralize the insignificance of having less number of observations in the regression analysis. Another important reason behind this inclusion is to cross verify the results obtained via OLS estimates above.

3.3.1 Test Methodology

The relationships to be tested are the ones in between GDP per capita, total fertility rate (TFR), net revenue earned by Indian railways, and the population density of India. Considering the type of data available, i.e. Time series data, the study commences with the two step process to test for co-integration, originally introduced by Engle and Granger in 1987. This process consists of an unit root test and simple OLS regression test to test the co-integration involved between different variables used. At first, an OLS regression is run for the variables under test and then residuals are obtained in order to test for unit roots with Augmented Dickey-Fuller test. Co-integration test is done in order to verify the existence of long run equilibrium. With the help of co-integration, we can separate short and long run relationship among variables. It can be also used in order to improve long run forecasting accuracy.

Causality relationship analysis comes after unit root testing and co-integration analysis. An error correction term of co-integrating vectors is included in the causality testing if the variables are found to be co-integrating. A variable is said to Granger cause another variable if the past values (i.e. lags) of the first variable are significantly explaining the current values of the second variable. The relationship verification done is bidirectional.

Main hypothetical relationships to be tested are as follows:

1) The relationship between GDP per capita in Indian rupees and the

total fertility rate of the country for the period under observation, i.e. from 1960-2010. With the help of this hypothesis, the study obtains the quality-quantity trade-off, suggesting a shift of the population towards quality of the future generation than giving rise to the basic fertility. The transmission mechanism could also flow through improved literacy, health conditions, infrastructure increase and its result on reduced total fertility rate.

$$GDP(t) = \sum_{j=1}^{p} A_{1j} TFR(t-j) + \sum_{j=1}^{p} A_{2j} GDP(t-j) + u_i$$
$$TFR(t) = \sum_{j=1}^{p} B_{1j} GDP(t-j) + \sum_{j=1}^{p} B_{2j} TFR(t-j) + v_i$$

The null hypothesis to be tested are:

1) $H_1 : A_{1j} = 0, j = 1, ..., p$ which means TFR does not Granger cause GDP per capita.

2) $H_1 : B_{1j} = 0, j = 1, ..., p$ which means GDP per capita does not Granger cause TFR.

If both the hypothesis are not rejected, then it can safely be said that GDP per capita does not Granger cause TFR and TFR does not Granger cause GDP per capita which at the end indicates that these two variables are independent of each other. Rejection of first hypothesis suggests that TFR does Granger cause GDP per capita and rejection of second hypothesis suggests that GDP per capita does Granger cause TFR. Rejection of both the hypothesis suggest that the causality is bidirectional.

2) With similar null hypothesis construction as above in relationship 1, the relationship between GDP per capita and the population density of the country is to be tested.

3) With similar setting as above in relationship 1, the relationship between net revenue earned by Indian railways and the population density of the country is to be tested. With the help of this hypothesis, the study verifies the above discussed impact of population density on the profitability of the transportation providing facility, i.e. net revenue earned by Indian railways, the biggest infrastructure providing institution in India.

3.3.2 Results

Above discussed co-integration tests and Granger's Causality tests are calculated using STATA software. Here, study uses logarithmic values of every variable under test. EG-ADF test is used to test the co-integration here, which is two step process proposed by Engle and Granger in 1987. If the variables under consideration are integrated of the same order, i.e.

if they have the same number of unit roots, then only the co-integration analysis is feasible. In the table below for co-integration strategy, EG-ADF test is performed and in both the scenarios under test, i.e. log(GDP per capita in Indian rupees) and log(total fertility rate of India), and log(population density of India), the study finds the evidences of the long run co-integration relationship.

Table 3: Cointegration Strategy

Variables Under Test	**Test Statistics**	**P-value**
l(GDP) and l(TFR)	-4.631	0.0001*
l(GDP) and l(Population Density)	-3.232	0.0182**
l(Net Revenue) and l(Population Density)	-2.922	0.0428**

Note: *, ** and *** denotes the unit root existence at 1%, 5% and 10% significance levels respectively. These critical values are based on critical values are based on MacKinnon (1991)

In Table 3, it is clear that there exists unit root in all the three relationships performed. Hence, now Granger's causality tests can be performed. Granger causality test, as can be seen in the table (4), takes into account the co-integration relationships between the variables discussed in the co-integration strategy and tests the causality of this long run in addition to the short run causality by determined lag lengths.

Table 4: Granger's Causality Test Strategy

Null Hypothesis	F-Stats	#(lags)	Pr>F
l(NR) doesnt cause l(PopDens)	0.52	2	0.5997
l(PopDens) doesnt cause l(NR)	6.98	2	0.0024*
l(GDP) doesnt cause l(PopDens)	2.13	2	0.1313
l(PopDens) doesnt cause l(GDP)	2.51	2	0.0933***
l(TFR) doesnt cause l(GDP)	2.41	2	0.1017
l(GDP) doesnt cause l(TFR)	10.31	2	0.0002*
l(GDP) doesnt cause l(NR)	6.17	2	0.0044*
l(NR) doesnt cause l(GDP)	2.88	2	0.0666**

Note: *, ** and *** denotes indicate the rejection of the null hypothesis at 1, 5 and 10% significance levels respectively

According to the results posted in table 4, the relationship between net revenue and population density is only one way significant. The direction goes from population density towards net revenue of Indian Railways. On the other hand, population density Granger causes GDP per capita in rupees in one way only, i.e. from population density to GDP per capita. It is also evident that TFR and population density both have significant impact on GDP per capita of the country.

4 Conclusion

The purpose of this paper has been to study the impact of infrastructure availability or transportation costs on individual fertility decisions and decisions related to the education of the progeny and as a result, on the economic growth of the region. The first part of this paper explored a theoretical setting in which, the parental fertility decisions were made endogenous and the impact of infrastructure costs on education and fertility decisions is verified. The second part of this model investigated the profitability condition on the institutions investing in infrastructure and its welfare enhancing impact. In section 3 of the paper, the empirical study cross-verified the results obtained in the model, applying simple OLS estimations and Granger's causality techniques on Indian data from year 1960-2010.

The Croix-Doepke(2003) model was selected for theoretical exploration in order to have fertility decisions endogenous, which are absent in Boucekkine et al. (2007). In the process of development, the infrastructure costs as a fraction of labor income should decrease over time, schooling time of children and economic growth should increase over time, and the fertility rate should decrease in time, but in the theoretical results derived in section 2.1, the study obtained contradictory results for this assumption.

The first contribution of this paper is to introduce infrastructure costs as a decision variable for human capital accumulation of the children in the standard model of differential fertility and economic growth. Results prove the process of making the fertility decisions endogenous for the varying transportation costs. Secondly, the model studied three different scenarios on the basis where parents choose the location of their residence and computed the growth factor of the output growth at steady state. Third, the model proposes the micro-foundations for the effect of population density on growth. The location of transport facility is chosen optimally either to maximize the profit of the whole transport system or to increase the welfare of the society. In this set-up, higher population density should increase the number of facilities, opening the possibility for individuals to achieve higher education and initiating the development process in the economically stagnated economy.

Via empirical evidences, the study suggests that there is a significantly negative impact of average rate charged per passenger kilometer, i.e. transportation cost on the total fertility rate of the country. The study also investigated the significant positive impact of population density on the GDP per capita. From Granger causality testing, it is clear that population density unidirectionally Granger causes increase in the per capita GDP as well as the net revenue of the Indian railways, i.e. the infrastructure providing facility of the country. It is also verified that there is an unidirectional and significant impact of income growth on the total fertility rate of the country.

During the sampled time period, the phenomenon of urbanization was seen as increasing very rapidly in the form of increased population density of the country. This increased population density put a lot of pressure on available infrastructure. Government's emphasis on building new roads than new railway lines and also the positive profitability condition forced Indian railways to keep on increasing the average rate charged per passenger kilometer, i.e. the passenger tickets. Improved profitability of railways should bring in more investment funds and make them less dependent on government aid but continued dependency of Indian railways on increasing tariffs was not truly welfare enhancing. Hence, the study also reflects the government's emphasis on building new roads than building new railway tracks considering the large fixed costs involved.

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A Appendix

A.1 Individual problem when resided near Workplace

The individual optimization problem mentioned in the section 2.1 is solved in this section following on the Lagrangian mentioned in equation 10. First order conditions with respect to variables to be calculated are carried as following.

$$\begin{aligned} L = & \ln(w_t h_t(1-\varphi n_t) - s_t - (e_t + g_t) n_t w_t \overline{h}_t) + \beta \ln(R_{t+1} s_t) \\ & + \gamma(\ln n_t B_t + \eta \ln(\theta + e_t) + \tau \ln h_t + \kappa \ln \overline{h}_t) \end{aligned} \tag{36}$$

Now taking the first order condition of the lagrangian with respect to s_t,

$$\frac{\partial L}{\partial s_t} = 0$$

$$\frac{-1}{c_t} + \frac{\beta R_{t+1}}{R_{t+1} s_t} = 0$$

$$s_t = \beta c_t$$

$$s_t = \beta(w_t h_t(1-\varphi n_t) - s_t - (e_t + g_t) n_t w_t \overline{h}_t)$$

$$s_t(1+\beta) = \beta(w_t h_t(1-\varphi n_t) - (e_t + g_t) n_t w_t \overline{h}_t)$$

$$s_t = \frac{\beta(w_t h_t(1-\varphi n_t) - (e_t + g_t) n_t w_t \overline{h}_t)}{(1+\beta)}$$

Taking first order condition of lagrangian with respect to e_t gives,

$$\frac{\partial L}{\partial e_t} = 0$$

$$\frac{-\overline{h}_t n_t w_t}{w_t h_t(1-\varphi n_t) - s_t - (e_t+g_t)n_t w_t \overline{h}_t} + \frac{\eta\gamma}{\theta + e_t} = 0$$

$$\overline{h}_t n_t w_t(\theta + e_t) = \eta\gamma(w_t h_t(1-\varphi n_t) - s_t - (e_t+g_t)n_t w_t \overline{h}_t)$$

substituting value of saving rate s_t calculated in the above equation,

$$\overline{h}_t n_t w_t(\theta + e_t) = \eta\gamma(w_t h_t(1-\varphi n_t) - \tfrac{\beta(w_t h_t(1-\varphi n_t)-(e_t+g_t)n_t w_t \overline{h}_t)}{(1+\beta)} - (e_t+g_t)n_t w_t \overline{h}_t)$$

$$\begin{aligned}\overline{h}_t n_t(\theta + e_t)(1+\beta) = \eta\gamma(h_t(1-\varphi n_t)(1+\beta) - \beta(h_t(1-\varphi n_t) \\ -(e_t+g_t)n_t \overline{h}_t)) - (1+\beta)(e_t+g_t)n_t \overline{h}_t)\end{aligned}$$

$$\overline{h}_t n_t w_t(\theta + e_t)(1+\beta) = \eta\gamma(w_t h_t(1-\varphi n_t) - (e_t+g_t)n_t w_t \overline{h}_t)$$

Now, first order condition of lagrangian with respect to n_t gives,

$$\frac{\partial L}{\partial n_t} = 0$$

$$\frac{-\varphi w_t h_t - (e_t+g_t)w_t \overline{h}_t}{w_t h_t(1-\varphi n_t) - s_t - (e_t+g_t)n_t w_t \overline{h}_t} + \frac{\gamma}{n_t} = 0$$

$$\begin{aligned}&\gamma(w_t h_t(1-\varphi n_t) - \frac{\beta(w_t h_t(1-\varphi n_t) - (e_t+g_t)w_t n_t \overline{h}_t)}{(1+\beta)} \\ &-(e_t+g_t)n_t w_t \overline{h}_t) \\ &= n_t(w_t \varphi h_t + (e_t+g_t)w_t \overline{h}_t)\end{aligned}$$

$$n_t(\varphi w_t h_t + (e_t+g)\overline{h}_t w_t) = \frac{\gamma}{(1+\beta)}(w_t h_t(1-\varphi n_t) - (e_t+g)\overline{h}_t w_t n_t)$$

$$\varphi w_t h_t n_t + (e_t+g)\overline{h}_t w_t n_t = \frac{\gamma}{(1+\beta)}(w_t h_t(1-\varphi n_t) - (e_t+g)\overline{h}_t w_t n_t)$$

Using equation calculated just above this first order condition,

$$\varphi h_t + (e_t + g)\overline{h}_t = \frac{\overline{h}_t(\theta + e_t)(1+\beta)}{\eta}$$

$$\varphi h_t + (e_t + g)\overline{h}_t = \frac{\overline{h}_t(\theta + e_t)}{\eta}$$

$$\varphi h_t \eta + \eta(e_t + g)\overline{h}_t = \overline{h}_t(\theta + e_t)$$

$$\begin{aligned} e_t(\overline{h}_t - \eta\overline{h}_t) &= \varphi h_t \eta + g\eta\overline{h}_t - \overline{h}_t\theta \\ e_t &= \frac{\varphi h_t \eta + g\eta\overline{h}_t - \overline{h}_t\theta}{\overline{h}_t(1-\eta)} \end{aligned}$$

$$e_t = \frac{\varphi h_t \eta + g\eta\overline{h}_t - \overline{h}_t\theta}{\overline{h}_t(1-\eta)}$$

This is the solution for the equation of education costs per children referred in equation 11.

Now, lets substitute calculated values of e_t in the equation, to calculate the value of n_t,

$$\begin{aligned} \varphi h_t n_t + (\frac{\varphi h_t \eta + g\eta\overline{h}_t - \overline{h}_t\theta}{\overline{h}_t(1-\eta)} + g_t)\overline{h}_t n_t &= \frac{\gamma}{(1+\beta)}(h_t(1-\varphi n_t) \\ &\quad -(\frac{\varphi h_t \eta + g\eta\overline{h}_t - \overline{h}_t\theta}{\overline{h}_t(1-\eta)} + g_t)\overline{h}_t n_t \end{aligned}$$

$$\begin{aligned} \varphi h_t n_t + (\frac{\varphi h_t \eta + g\overline{h}_t\eta - \overline{h}_t\theta}{\overline{h}_t(1-\eta)} + g)\overline{h}_t n_t &= \frac{\gamma}{(1+\beta)}(h_t(1-\varphi n_t) \\ &\quad -(\frac{\varphi h_t \eta + g\overline{h}_t\eta - \overline{h}_t\theta}{\overline{h}_t(1-\eta)} + g)\overline{h}_t n_t) \end{aligned}$$

$$\begin{aligned} &\varphi h_t n_t \overline{h}_t(1-\eta)(1+\beta) \\ &+(\varphi h_t \eta + g\overline{h}_t\eta - \overline{h}_t\theta)n_t\overline{h}_t(1+\beta) \\ +g n_t \overline{h}_t \overline{h}_t(1-\eta)(1+\beta) = &\gamma(\overline{h}_t h_t(1-\varphi n_t)(1-\eta) \\ &-(\varphi h_t \eta + g\overline{h}_t\eta - \overline{h}_t\theta)\overline{h}_t n_t - g\overline{h}_t n_t \overline{h}_t(1-\eta) \end{aligned}$$

$$
\begin{aligned}
&\varphi h_t n_t \overline{h}_t(1-\eta)(1+\beta+\gamma)\\
&+(\varphi h_t\eta + g\overline{h}_t\eta - \overline{h}_t\theta)n_t\overline{h}_t(1+\beta+\gamma)\\
+gn_t\overline{h}_t\overline{h}_t(1-\eta)(1+\beta+\gamma) = &\gamma\overline{h}_t h_t(1-\eta))
\end{aligned}
$$

$$
n_t(1+\beta+\gamma)(\varphi h_t\eta + g\overline{h}_t\eta - \overline{h}_t\theta + g\overline{h}_t(1-\eta) + \varphi h_t(1-\eta)) = \gamma h_t(1-\eta)
$$

$$
n_t = \frac{\gamma h_t(1-\eta)}{(1+\beta+\gamma)(\varphi h_t\eta + g\overline{h}_t\eta - \overline{h}_t\theta + g\overline{h}_t(1-\eta) + \varphi h_t(1-\eta))}
$$

$$
n_t = \frac{\gamma h_t(1-\eta)}{(1+\beta+\gamma)(-\overline{h}_t\theta + g\overline{h}_t + \varphi h_t)}
$$

$$
n_t = \frac{\gamma h_t(1-\eta)}{(1+\beta+\gamma)(-\overline{h}_t\theta + g\overline{h}_t + \varphi h_t)}
$$

$$
n_t = \frac{\gamma x_t(1-\eta)}{(1+\beta+\gamma)(\varphi x_t - \theta + g)}
$$

After the substitution, the equation for number of children as referred in equation 12 is calculated above.

Now, to calculate saving rate s_t, we will substitute above calculated values of n_t and e_t and we get,

$$
s_t = \frac{\beta(w_t h_t(1-\varphi n_t) - (e_t+g_t)n_t w_t\overline{h}_t)}{(1+\beta)}
$$

$$
s_t = \frac{\beta(w_t h_t(1-\varphi\frac{\gamma h_t(1-\eta)}{(1+\beta+\gamma)(-\overline{h}_t\theta+g\overline{h}_t+\varphi h_t)}) - (\frac{\varphi h_t\eta+g\eta\overline{h}_t-\overline{h}_t\theta}{\overline{h}_t(1-\eta)} + g_t)\frac{\gamma h_t(1-\eta)w_t\overline{h}_t}{(1+\beta+\gamma)(-\overline{h}_t\theta+g\overline{h}_t+\varphi h_t)})}{(1+\beta)}
$$

$$
s_t = \frac{\beta(w_t h_t(1-\varphi\frac{\gamma h_t(1-\eta)}{(1+\beta+\gamma)(-\overline{h}_t\theta+g\overline{h}_t+\varphi h_t)}) - (\frac{\varphi h_t\eta+g\eta\overline{h}_t-\overline{h}_t\theta}{\overline{h}_t(1-\eta)} + g_t)\frac{\gamma h_t(1-\eta)w_t\overline{h}_t}{(1+\beta+\gamma)(-\overline{h}_t\theta+g\overline{h}_t+\varphi h_t)})}{(1+\beta)}
$$

$$
\begin{aligned}
s_t = &\frac{\beta}{(1+\beta)(1+\beta+\gamma)(-\overline{h}_t\theta + g\overline{h}_t + \varphi h_t)}[w_t h_t((1+\beta+\gamma)(-\overline{h}_t\theta + g_t\overline{h}_t + \varphi h_t)\\
&-h_t\varphi\gamma(1-\eta)) - (\varphi h_t\eta + g\overline{h}_t\eta - \overline{h}_t\theta)\gamma h_t w_t - (1-\eta)g_t\gamma h_t w_t\overline{h}_t]
\end{aligned}
$$

$$s_t = \frac{\beta w_t h_t}{(1+\beta)(1+\beta+\gamma)(-\overline{h}_t\theta + g\overline{h}_t + \varphi h_t)}[(1+\beta+\gamma)(-\overline{h}_t\theta + g\overline{h}_t + \varphi h_t) - h_t\varphi\gamma)$$
$$+(\overline{h}_t\theta)\gamma - g_t\gamma\overline{h}_t]$$

$$s_t = \frac{w_t h_t \beta}{(1+\beta)(1+\beta+\gamma)(-\overline{h}_t\theta + g_t\overline{h}_t + \varphi h_t)}[(1+\beta+\gamma)(-\overline{h}_t\theta + g_t\overline{h}_t + \varphi h_t)$$
$$-\gamma(h_t\varphi - \overline{h}_t\theta + g_t\overline{h}_t)]$$

$$s_t = \frac{\beta w_t h_t}{(1+\beta)}[1 - \frac{\gamma}{(1+\beta+\gamma)}]$$
$$s_t = \frac{\beta w_t h_t}{(1+\beta+\gamma)}$$

This is the equation for saving rate of parents mentioned in equation 13 in section 2.

A.2 Proposition 1

Following on proposition 1, let's calculate the derivative of the equation of education costs e_t with respect to relative human capital of parents, i.e. x_t.

As proved in equation 11, we have,

$$e_t = \frac{\varphi h_t\eta + g\eta\overline{h}_t - \overline{h}_t\theta}{\overline{h}_t(1-\eta)}$$
$$e_t = \frac{\varphi x_t\eta + g\eta - \theta}{(1-\eta)}$$

$$\frac{\partial e_t}{\partial x_t} = \frac{\partial(\frac{\varphi x_t\eta + g\eta - \theta}{(1-\eta)})}{\partial x_t}$$
$$= \frac{\varphi\eta}{(1-\eta)}$$
$$\rightarrow \text{Sign is positive}$$

Similarly, lets calculated the derive equation 11 with respect to x_t.

$$
\begin{aligned}
\frac{\partial n_t}{\partial x_t} &= \frac{\partial(\frac{\gamma x_t(1-\eta)}{(1+\beta+\gamma)(\varphi x_t-\theta+g)})}{\partial x_t} \\
&= \frac{(1+\beta+\gamma)(\varphi x_t-\theta+g)\gamma(1-\eta)-\gamma x_t(1-\eta)(1+\beta+\gamma)\varphi}{((1+\beta+\gamma)(\varphi x_t-\theta+g))^2} \\
&= \frac{\gamma(1-\eta)(-\theta+g)}{(1+\beta+\gamma)(\varphi x_t-\theta+g)^2} \\
&\rightarrow \text{ Sign is negative if } (\theta > g)
\end{aligned}
$$

A.3 Proposition 2

From equation 10 involving education decisions,

$$e_t = \frac{\varphi h_t \eta + g\eta \overline{h}_t - \overline{h}_t \theta}{\overline{h}_t(1-\eta)}$$

After deriving e_t with respect to g_t,we have,

$$\frac{\partial e_t}{\partial g_t} = \frac{\eta}{(1-\eta)}$$
$$\rightarrow \text{ Sign is positive}$$

From equation 11 for fertility decisions, we have

$$n_t = \frac{\gamma h_t(1-\eta)}{(1+\beta+\gamma)(-\overline{h}_t\theta + g\overline{h}_t + \varphi h_t)}$$

After taking derivative with respect to g_t, we have,

$$\frac{\partial n_t}{\partial g_t} = \frac{-\gamma h_t \overline{h}_t(1-\eta)}{(1+\beta+\gamma)(-\overline{h}_t\theta + g\overline{h}_t + \varphi h_t)^2}$$
$$\rightarrow \text{Sign is negative}$$

Hence, proposition 4 is proved.

A.4 Individual problem when resided Near School

In line with the changes in the budget constraint as mentioned in the section 2.2 of the article, we have the changed Lagrangian as following.

$$
\begin{aligned}
L = & \ln(w_t h_t(1-\varphi n_t - q) - s_t - g_t w_t \overline{h}_t - e_t \overline{h}_t n_t w_t) + \beta \ln(R_{t+1} s_t) \\
& + \gamma(\ln n_t B_t + \eta \ln(\theta + e_t) + \tau \ln h_t + \kappa \ln \overline{h}_t) \qquad (37)
\end{aligned}
$$

First order conditions with respect to variables to be calculated are calculated in order to get the aforementioned formulae.

Lets take the first order condition with respect to the saving rate(s_t),

$$\frac{\partial L}{\partial s_t} = 0$$

$$\frac{-1}{c_t} + \frac{\beta R_{t+1}}{R_{t+1} s_t} = 0$$

$$s_t = \beta c_t$$

$$s_t = \beta(w_t h_t(1 - \varphi n_t - q) - s_t - g w_t \overline{h}_t - e_t \overline{h}_t n_t w_t)$$

$$s_t(1 + \beta) = \beta(w_t h_t(1 - \varphi n_t - q) - g_t w_t \overline{h}_t - e_t \overline{h}_t n_t w_t)$$

$$s_t = \frac{\beta(w_t h_t(1 - \varphi n_t - q) - g_t w_t \overline{h}_t - e_t \overline{h}_t n_t w_t)}{(1 + \beta)}$$

Lets take the first order condition with respect to e_t,

$$\frac{\partial L}{\partial e_t} = 0$$

$$\frac{-\overline{h}_t n_t w_t}{w_t h_t(1 - \varphi n_t - q) - s_t - g_t w_t \overline{h}_t - e_t \overline{h}_t n_t w_t} + \frac{\eta\gamma}{\theta + e_t} = 0$$

$$\overline{h}_t n_t w_t(\theta + e_t) = \eta\gamma(w_t h_t(1 - \varphi n_t - q) - s_t - g_t w_t \overline{h}_t - e_t \overline{h}_t n_t w_t)$$

substituting value of saving rate s_t calculated in the above equation,

$$\begin{aligned}\overline{h}_t n_t w_t(\theta + e_t) = \eta\gamma(&w_t h_t(1 - \varphi n_t - q) \\ &- \frac{\beta(w_t h_t(1 - \varphi n_t - q) - g_t w_t \overline{h}_t - e_t \overline{h}_t n_t w_t)}{(1 + \beta)} \\ &- g_t w_t \overline{h}_t - e_t \overline{h}_t n_t w_t)\end{aligned}$$

$$\begin{aligned}\overline{h}_t n_t w_t(\theta + e_t)(1 + \beta) = \eta\gamma(&w_t h_t(1 - \varphi n_t - q)(1 + \beta) \\ &- \beta(w_t h_t(1 - \varphi n_t - q) - g_t w_t \overline{h}_t - e_t \overline{h}_t n_t w_t) \\ &- (1 + \beta)(g_t w_t \overline{h}_t + e_t \overline{h}_t n_t w_t)\end{aligned}$$

$$\overline{h}_t n_t w_t(\theta + e_t)(1+\beta) = \eta\gamma(w_t h_t(1-\varphi n_t - q) - g_t w_t \overline{h}_t - e_t \overline{h}_t n_t w_t)$$

Now, lets take the first order condition with respect to n_t gives,

$$\frac{\partial L}{\partial n_t} = 0$$

$$\frac{-\varphi w_t h_t - e_t \overline{h}_t w_t}{w_t h_t(1-\varphi n_t - q) - s_t - g_t w_t \overline{h}_t - e_t \overline{h}_t n_t w_t} + \frac{\gamma}{n_t} = 0$$

$$n_t(\varphi w_t h_t + e_t \overline{h}_t w_t) = \gamma(w_t h_t(1-\varphi n_t - q) - \frac{\beta(w_t h_t(1-\varphi n_t - q) - g_t w_t \overline{h}_t - e_t \overline{h}_t n_t w_t)}{(1+\beta)}$$
$$- g_t w_t \overline{h}_t - e_t \overline{h}_t n_t w_t)$$
$$n_t(\varphi w_t h_t + e_t \overline{h}_t w_t) = \frac{\gamma}{(1+\beta)}(w_t h_t(1-\varphi n_t - q) - g_t w_t \overline{h}_t - e_t \overline{h}_t n_t w_t)$$

Using equation calculated just above this first order condition,

$$n_t(\varphi w_t h_t + e_t \overline{h}_t w_t) = \frac{\overline{h}_t n_t w_t(\theta + e_t)}{\eta}$$

$$\varphi h_t \eta + \eta(e_t \overline{h}_t) = \overline{h}_t(\theta + e_t)$$

$$e_t(\overline{h}_t - \eta \overline{h}_t) = \varphi h_t \eta - \overline{h}_t \theta$$

$$e_t = \frac{\varphi h_t \eta - \overline{h}_t \theta}{\overline{h}_t(1-\eta)}$$

This is the solution for the equation of education costs per children referred in equation 25.

Now, lets substitute calculated values of e_t in the equation, to calculate the value of n_t,

$$\overline{h}_t n_t w_t(\theta + \frac{\varphi h_t \eta - \overline{h}_t \theta}{\overline{h}_t(1-\eta)})(1+\beta) = \eta\gamma(w_t h_t(1-\varphi n_t - q) - g w_t \overline{h}_t - \frac{(\varphi h_t \eta - \overline{h}_t \theta)}{(1-\eta)} n_t w_t)$$

$$n_t(\varphi h_t - \overline{h}_t \theta)(1+\beta+\gamma) = \gamma(h_t(1-q)(1-\eta) - (1-\eta) g \overline{h}_t)$$
$$n_t = \frac{\gamma(h_t(1-q) - g\overline{h}_t)(1-\eta)}{(\varphi h_t - \overline{h}_t \theta)(1+\beta+\gamma)}$$

After the substitution, the equation for number of children as referred in equation 26 is calculated above.

Hence, Individual Optimization problem when he decides to reside near School is solved as above.

A.5 Both, parents as well as children pay transportation costs

In line with the changes in the budget constraint as mentioned in the section 2.3 of the article, the Lagrangian has been changed as following.

$$\begin{aligned} L = &\ln(w_t h_t(1-\varphi n_t - q) - s_t - g_t w_t \overline{h}_t - (e_t+g_t)\overline{h}_t n_t w_t) + \beta \ln(R_{t+1} s_t) \\ &+\gamma(\ln n_t B_t + \eta \ln(\theta+e_t) + \tau \ln h_t + \kappa \ln \overline{h}_t) \end{aligned} \tag{38}$$

First order conditions with respect to variables to be calculated are calculated in order to get the aforementioned formulae.

Lets take the first order condition with respect to the saving rate(s_t),

$$\frac{\partial L}{\partial s_t} = 0$$

$$\frac{-1}{c_t} + \frac{\beta R_{t+1}}{R_{t+1} s_t} = 0$$

$$\begin{aligned} s_t &= \beta c_t \\ s_t &= \beta(w_t h_t(1-\varphi n_t - q) - s_t - g_t w_t \overline{h}_t - (e_t+g_t)\overline{h}_t n_t w_t) \\ s_t(1+\beta) &= \beta(w_t h_t(1-\varphi n_t - q) - g_t w_t \overline{h}_t - (e_t+g_t)\overline{h}_t n_t w_t) \\ s_t &= \frac{\beta(w_t h_t(1-\varphi n_t - q) - g_t w_t \overline{h}_t - (e_t+g_t)\overline{h}_t n_t w_t)}{(1+\beta)} \end{aligned}$$

Lets take the first order condition with respect to e_t,

$$\frac{\partial L}{\partial e_t} = 0$$

$$\begin{aligned} \frac{-\overline{h}_t n_t w_t}{w_t h_t(1-\varphi n_t - q) - s_t - g_t w_t \overline{h}_t - (e_t+g_t)\overline{h}_t n_t w_t} + \frac{\eta\gamma}{\theta+e_t} = 0 \\ \eta\gamma(w_t h_t(1-\varphi n_t - q) - s_t - g_t w_t \overline{h}_t - (e_t+g_t)\overline{h}_t n_t w_t) = \overline{h}_t n_t w_t(\theta+e_t) \end{aligned}$$

substituting the value of s_t calculated as above, we get,

$$
\begin{aligned}
\overline{h}_t n_t w_t(\theta + e_t) &= \eta\gamma(w_t h_t(1 - \varphi n_t - q) - \frac{\beta(w_t h_t(1 - \varphi n_t - q) - g_t w_t \overline{h}_t - (e_t + g_t)\overline{h}_t n_t w_t)}{(1+\beta)} \\
&\quad - g_t w_t \overline{h}_t - (e_t + g_t)\overline{h}_t n_t w_t) \\
\overline{h}_t n_t w_t(\theta + e_t)(1+\beta) &= \eta\gamma(w_t h_t(1 - \varphi n_t - q) - g_t w_t \overline{h}_t - (e_t + g_t)\overline{h}_t n_t w_t)
\end{aligned}
$$

Now, lets take the first order condition with respect to n_t gives,

$$
\frac{\partial L}{\partial n_t} = 0
$$

$$
\begin{aligned}
\frac{-w_t h_t \varphi - e_t \overline{h}_t w_t - g_t \overline{h}_t w_t}{w_t h_t(1 - \varphi n_t - q) - s_t - g_t w_t \overline{h}_t - (e_t + g_t)\overline{h}_t n_t w_t} + \frac{\gamma}{n_t} &= 0 \\
(w_t h_t(1 - \varphi n_t - q) - s_t - g_t w_t \overline{h}_t - (e_t + g_t)\overline{h}_t n_t w_t)\gamma &= n_t(w_t h_t \varphi + e_t \overline{h}_t w_t + g_t \overline{h}_t w_t)
\end{aligned}
$$

$$
\begin{aligned}
n_t(w_t h_t \varphi + e_t \overline{h}_t w_t + g_t \overline{h}_t w_t) &= (w_t h_t(1 - \varphi n_t - q) - g_t - e_t \overline{h}_t n_t w_t - g n_t)\gamma \\
&\quad - \frac{\beta(w_t h_t(1 - \varphi n_t - q) - g_t - e_t \overline{h}_t n_t w_t - g n_t}{(1+\beta)} \\
(w_t h_t(1 - \varphi n_t - q) - g_t \overline{h}_t w_t - e_t \overline{h}_t n_t w_t - g_t \overline{h}_t w_t n_t)\gamma &= n_t(w_t h_t \varphi + e_t \overline{h}_t w_t + g_t \overline{h}_t w_t)(1+\beta)
\end{aligned}
$$

using equation derived above,

$$
\begin{aligned}
\frac{\overline{h}_t n_t w_t(\theta + e_t)(1+\beta)}{\eta} &= n_t(w_t h_t \varphi + e_t \overline{h}_t w_t + g_t \overline{h}_t w_t)(1+\beta) \\
\overline{h}_t w_t(\theta + e_t) &= \eta(w_t h_t \varphi + e_t \overline{h}_t w_t + g_t \overline{h}_t w_t) \\
e_t \overline{h}_t w_t(1 - \eta) &= \eta(w_t h_t \varphi + g_t \overline{h}_t w_t) - \overline{h}_t w_t \theta
\end{aligned}
$$

$$
e_t = \frac{\eta(w_t h_t \varphi + g_t \overline{h}_t w_t) - w_t \overline{h}_t \theta}{w_t \overline{h}_t(1 - \eta)}
$$

$$
e_t = \frac{\eta(h_t \varphi + g_t \overline{h}_t) - \overline{h}_t \theta}{\overline{h}_t(1 - \eta)} \tag{39}
$$

$$
e_t = \frac{\eta(x_t \varphi + g_t) - \theta}{(1 - \eta)} \tag{40}
$$

Putting this equation in above derived equation obtained when FOC for n_t is used.

$$
\begin{aligned}
\overline{h}_t n_t w_t(\theta + \frac{\eta(h_t\varphi + g_t\overline{h}_t) - \overline{h}_t\theta}{\overline{h}_t(1-\eta)})(1+\beta) &= \eta\gamma(w_t h_t(1-\varphi n_t - q) - g_t w_t \overline{h}_t \\
&\quad -(\frac{\eta(h_t\varphi + g_t\overline{h}_t) - \overline{h}_t\theta}{\overline{h}_t(1-\eta)} + g_t)\overline{h}_t n_t w_t) \\
n_t(-\theta\overline{h}_t + h_t\varphi + g_t\overline{h}_t)(1+\beta) &= \gamma(h_t(1-\varphi n_t - q)(1-\eta) \\
&\quad -g_t\overline{h}_t + g_t\overline{h}_t\eta - (\eta(h_t\varphi) - \overline{h}_t\theta + g_t\overline{h}_t)n_t)) \\
n_t(-\theta\overline{h}_t + h_t\varphi + g_t\overline{h}_t)(1+\beta) &= \gamma(h_t(1-\varphi n_t - q)(1-\eta) \\
&\quad -g_t\overline{h}_t + g_t\overline{h}_t\eta - (\eta(h_t\varphi) - \overline{h}_t\theta + g_t\overline{h}_t)n_t)) \\
n_t(-\theta\overline{h}_t + h_t\varphi + g_t\overline{h}_t)(1+\beta+\gamma) &= \gamma(((h_t - qh_t) - \eta(h_t - qh_t)) \\
&\quad -g_t\overline{h}_t + g_t\overline{h}_t\eta)) \\
n_t &= \frac{\gamma(1-\eta)(h_t - qh_t - g_t\overline{h}_t)}{(-\theta\overline{h}_t + h_t\varphi + g_t\overline{h}_t)(1+\beta+\gamma)} \\
n_t &= \frac{\gamma(1-\eta)(x_t - qx_t - g_t)}{(-\theta + x_t\varphi + g_t)(1+\beta+\gamma)}
\end{aligned}
$$

Hence, derived.