Breakdown of the market for developable land around a stochastic city with public goods

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Abstract

The option value on developable land in the vicinity of a city depends on its public investment path. We investigate the implied relationship between urban public goods provision and the market for developable land. First best investment in public goods and land conversion maximizes the total value of all urban and developable land. It will be implemented by a city that has the exclusive right to develop. Landowners may freeride on urban public investment if development rights are universal. Hence, a competitive market for developable land breaks down. Moreover, the core of a bargaining game between landowners is empty, even if their number is comparably small, so market power is unlikely to promote efficient trade. A breakdown will lead to underprovision of public goods and suboptimal urban growth.

1 Introduction

Owners of land in the vicinity of a city benefit from its success, as expectations about the value of developable land in future use capitalize into its current price. Investment in public goods and services that make a city attractive to its residents are an important constituent of success. Expectations about the future public investment path thus contribute to the option value of developable land. Our paper explores the close relationship between the provision of urban public goods and the market for developable land that is implied by this capitalization effect.

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We analyze a monocentric city that attracts residents through provision of a local public good, like infrastructure, cultural facilities or an appealing cityscape, and through a wage or amenity component that evolves according to a stochastic process. Investment in the public good and conversion of agricultural land to urban use are irreversible. On the first-best investment path, land should be developed when its value in urban use exceeds agricultural land value by conversion costs plus an insurance premium, while public goods should be provided until the rise in value of developed land offsets costs at the margin. The option value of future public investments is shown to capitalize in the value of developable land, since increments the value of land that is already developed are used to cover the investment costs. The first-best investment path optimizes the total value of all urban and developable land net of costs. We investigate the outcome that obtains when the urban developer does not own any developable land under alternative institutional arrangements.

The assignment of development rights turns out to play a crucial role. Whereas control of land use is typically a local affair in the US, higher levels of government coordinate planning in most European countries (Fischel (2001)). We capture this dichotomy by distinguishing exclusive and universal development rights. Development rights are universal in our analysis if each landowner has the right to convert agricultural land to urban use. This reflects a situation in which neither the central city municipality nor any planning authority at the metropolitan, state or federal level has the power to constrain development of land by newly incorporated or existing suburban municipalities. The case in which the urban developer has an exclusive development right reflects coordination of land use planning at least at the metropolitan level.

We find that an urban developer with exclusive development rights will implement the first best investment strategy, since he can acquire land at its agricultural value. However, universal development rights are bound to induce a breakdown of the market for developable land. Dispersed landowners are then able to freeride on public investment by developing their own parcel. Land value thus exceeds what an urban developer can afford to pay, since he has to bear the costs of future public investments. As a result, no owner sells her land to the developer and in turn, being unable to appropriate the benefits, the developer loses the incentive to invest in public goods. The city will grow at a suboptimal pace if public goods are undersupplied.

In order to investigate whether market power can overcome the freerider problem, we consider a bargaining game between the urban developer and a small number of landowners. In particular, we ask whether the gains from cooperation suffice to dissuade each individual owner from freeriding. It turns out that this is the case only if the concentration of ownership is very strong, although the core of the bargaining game will never be empty if there are only one or two landowners. However, any deal between the developer and a even moderate number of landowners is likely to be unstable. Market power does not inhibit efficient trade if development rights are exclusive, because the possibility to freeride on public goods provision is effectively ruled out.

Our analysis thus suggests the exclusive assignment of development rights in metropolitan areas as an instrument to enhance efficient urban public goods provision. It may enable cities to expand through annexation or incorporation, because land value does not reflect the possibility to freeride in the outside option. Alternatively, a metropolitan planning authority may use its zoning power to extract surplus from suburban developments and invest it in public goods in the city centre. The wide availability and use of this instrument in Europe may contribute to explaining higher levels of urban public investment relative to the typical US city.

2 Background

Capozza and Helsley (1990) study optimal conversion decisions and the value of land in a city in which labour demand shocks evolve according to a geometric Brownian motion. Our analysis extends this framework with urban public investment and yields comparable results if the stock of public goods is held constant. Several other papers analyze the theory of real option value and optimal conversion of land, see for instance Titman (1985), Capozza and Li (1994) and Capozza and Li (2002). These studies endogenize capital intensity, which is held constant in Capozza and Helsley (1990) and in our present paper, yet they ignore urban spatial structure as well as public goods. One important implication of the introduction of public goods is that, depending on the institutional context, decentralized investment decisions may cease to be optimal.

Suburban exploitation of central city public goods is not a new theme in the fiscal federalism literature, see Bradford and Oates (1974) for an early discussion. More generally, the existence of local public expenditure spillovers is well acknowledged, see Oates (1999) for an overview and Sole-Olle (2006) for some recent empirical evidence from Spanish municipalities. While the tax and transfer system offers one way to deal with such externalities, our analysis points to the organization of the market for developable land as an alternative avenue.

Our breakdown result in the case of a limited number of landowners fits in with a recent literature on bargaining in the presence of externalities, which challenges the conventional 'Coase theorem logic' that parties will negotiate efficiently in the absence of transaction costs. In particular, Maskin (2003) argues that formation of the 'grand coalition' is often implausible in situations in which coalitions generate positive externalities. This applies to our setting, since the public investment on which outsiders can freeride depends on the amount of land that is owned by members in the coalition. The core plays an important role in his analysis, since nonemptiness is a sufficient condition for the grand coalition to form. This motivates our focus on the core as a solution concept. Segal (1999) reviews a related literature on contracting between one principle and multiple agents in the presence of externalities, much like the urban developer and multiple landowners, which generally predicts inefficient outcomes as well. The private provision of public goods example, see also Bergstrom, Blume, and Varian (1986), seems particularly relevant in the context of our analysis.

3 The model

We consider an open city that is located on a one-dimensional linear space. Residents of the city derive utility from the consumption of numeraire goods c, the public good $g(X)$ and a random component f, according to the function

$$U(X,c,f) = f + g(X) + c,$$

where $g(X)$ is increasing and concave and where X is the stock of public goods. Each resident occupies one unit of land. The public good is located at the Central Consumer Area (CCA) in the city centre. We assume that the random component f follows a geometric Brownian motion with drift

$$df = \mu f dt + \sigma f dz,$$

where z is a Wiener process. Since f and c enter additively in the utility function, this random process may reflect income shocks as in Capozza and Helsley (1990) or stochastic variation in the (taste for) amenities on offer in the city.

Let ρ be the discount rate. In standard condition in this type of model is that the expected growth rate of f is smaller than the discount rate $\mu < \rho$. In our case, the boundedness of the value of the city imposes a further restriction on the drift parameter

$$\mu + \frac{\sigma^2}{2} < \frac{\rho}{2}. \tag{1}$$

It will be convenient to use a specific functional form for $g(X)$,

$$g(X) = \sqrt{\rho\chi X},$$

so that the marginal benefit is proportional to the inverse of the total benefit: $g'(X) = \frac{\rho\chi}{2} g(X)^{-1}$. It must be assumed that $\chi < \tau$, so that the optimal city size is finite.

Utility at any location in the city must be equal to the level that is on offer elsewhere, which is normalized to zero. Residents spend $\omega - \tau s - R(s)$ on the consumption of the numeraire good c, where ω is their exogenous income, s measures the distance to the CCA, τ is the per unit travel cost and $R(s)$ denotes the land rent at a distance s from the CCA. Hence, in a spatial equilibrium, land rents must equal

$$R(s) = f + \omega + g(X) - \tau s.$$

The total differential land rent obtains as

$$\begin{aligned} TDR(X,S,f) &\equiv 2\int_0^S R(s)\,ds \\ &= 2(f+\omega+g(X))S - \tau S^2, \end{aligned}$$

where S is the distance from the CCA to the edge of the city.The present expected value of all future rental flows of a city with fixed size S and public good X is given by

$$\begin{aligned} TDP(X,S,f) &\equiv E\left[\int_0^\infty TDR(X,S,f_t)\,e^{-\rho t}dt \middle| f_0 = f\right] \\ &= \frac{2Sf}{\rho-\mu} + \frac{2(\omega+g(X))S - \tau S^2}{\rho}. \end{aligned}$$

The expected rise in value from a marginal expansion of the city equals $TDP_S(X,S,f)$.

Conversion of land from agricultural to residential use requires an investment ξ per unit of land. Both land conversion and expansion of X are irreversible investments. The productivity of land in agricultural use is normalized to zero for the sake of simplicity.

4 First best

4.1 Optimal investment

The problem of surplus optimization coincides with the profit maximization problem of an urban planner who owns the city and all surrounding land. Let $V(X,S,f)$ denote the value of the city, which is defined as the present expected value of future rental flows on all land inside and outside of the current boundary, net of future investment and conversion costs. The problem is to find paths of X and S that maximize this value, while taking account of the irreversibility of land conversion and public investment.

There are values of f for which it is optimal to invest in the extension of neither X nor S. For these values of f, $V(X,S,f)$ satisfies

$$V(X,S,f) = TDR(X,S,f)\,dt + e^{-\rho dt}E\left[V(X,S,f+df)\right].$$

By expanding this equation and applying Ito's Lemma, we obtain the Bellman equation

$$\frac{1}{2}\sigma^2 f^2 V_{ff}(X,S,f) + \mu f V_f(X,S,f) - \rho V(X,S,f) + TDR(X,S,f) = 0.$$

Admissible solutions of this equation have the form

$$V(X,S,f) = B(X,S) f^{\beta} + TDP(X,S,f), \tag{2}$$

where $B(X,S)$ is a constant in f that is determined by suitable boundary conditions and where β is the positive root of the fundamental quadratic

$$\frac{1}{2}\sigma^2 \beta(\beta-1) + \mu\beta - \rho = 0. \tag{3}$$

Solutions with the negative root of this quadratic are not admissible, since the value of owning the city should be bounded for arbitrary small f. Comparative statics of β with regard to σ, μ and ρ are discussed in Dixit and Pindyck (1994).

In order to determine $B(X,S)$ and the optimal investment paths for X and S, we impose value matching and smooth pasting of $V(X,S,f)$ at the boundary of the region where it is optimal not to invest. Public goods should be installed until the marginal contribution to the value of the city equals its cost. The value matching condition obtains from the requirement that this condition should also hold at the boundary of the region where it is not desirable to invest

$$V_X(X,S,f) = B_X(X,S) f^{\beta} + \frac{2g\prime(X)S}{\rho} = 1, \tag{4}$$

where the second equality follows from the substitution of equation (2). Optimality requires that this value matching condition is met in a smooth way with regard to f, or

$$V_{Xf}(X,S,f) = \beta B_X(X,S) f^{\beta-1} = 0. \tag{5}$$

This implies that once new land has been converted to urban use, there is no option value to delaying public investment. Investment in X should take place simultaneously with extension of S. Substitution of this smooth pasting condition into (4) yields Paul Samuelson's rule $2g\prime(X)S = \rho$: invest in the public good occurs until the total willingness to pay for addition of a marginal unit equals the cost of invesment. Using $g(X) = \sqrt{\rho\chi X}$, we obtain

$$\begin{aligned} X(S) &= \frac{\chi}{\rho}S^2, \\ g(X) &= \chi S. \end{aligned} \tag{6}$$

Land should be converted to urban use until the contribution of the marginal unit to the value of the city equals the cost of conversion. By making use of (2), we thus obtain the value matching condition

$$V_S(X,S,f) = B_S(X,S) f^{\beta} + TDP_S(X,S,f) = 2\xi. \tag{7}$$

In a deterministic world, the planner would extend the city until $TDP_S(X,S,f) = 2\xi$, the marginal revenue of further extension equals the conversion cost. Equation (7) shows that uncertainty and irreversibility drive a wedge $B_S(X,S) f^{\beta}$ between the present expected value of the land rents that flow from a marginal extension of the city and the cost of conversion. This wedge may be interpreted as the (foregone) value of holding the option to make this investment at some future point in time or as an insurance premium. Furthermore, optimality requires smooth pasting, so

$$V_{Sf}(X,S,f) = \beta B_S(X,S) f^{\beta-1} + \frac{2}{\rho-\mu} = 0. \tag{8}$$

We are now ready to derive $f(X,S)$, the boundary of the region in which investment is not profitable. Solving (8) for $B_S(X,S)$ and substituting the result into (7), we may write

$$TDP_S(X,S,f) - 2\xi = \frac{2}{\beta(\rho-\mu)} f(X,S). \tag{9}$$

By writing out $TDP_S(X,S,f)$ and substituting (6), we may characterize $f(X,S)$ as

$$\widehat{f}(S) = \Delta\left[\xi - \frac{\omega-(\tau-\chi)S}{\rho}\right]. \tag{10}$$

where $\Delta \equiv \frac{\beta}{\beta-1}(\rho-\mu)$. A hat on a function implies that we have substituted the argument X for $X(S)$. From this equation it can be seen why we need to assume that $\chi < \tau$. Otherwise, we would have $\widehat{f}_S(S) < 0$, so places further out would be developed earlier, implying an infinite optimal city size.

Substitution of (10) into (7) yields

$$\widehat{B}\prime(S) = -2\frac{\Delta^{-\beta}}{\beta-1}\left[\xi - \frac{\omega-(\tau-\chi)S}{\rho}\right]^{1-\beta}. \tag{11}$$

The total value of the city may then be written as the net discounted value of the rental income of the city at its current size S plus the option value $\widehat{B}(S) f^{\beta}$ of future extensions of the city and the public good,

$$\begin{aligned} \widehat{V}(S,f) &= \widehat{TDP}(S,f) + \widehat{B}(S) f^{\beta}, \\ \widehat{B}(S) &= -\int_{S}^{\infty} \widehat{B}\prime(s)\, ds \\ &= \frac{1}{\tau-\chi}\frac{2\rho}{\beta-2}\frac{\Delta^{-\beta}}{\beta-1}\left[\xi - \frac{\omega-(\tau-\chi)S}{\rho}\right]^{2-\beta}, \end{aligned} \tag{12}$$

where we have used that $\beta > 2$ in the evaluation of $\widehat{B}(S)$, which follows from assumption (1). The option value would be infinite otherwise. Note that although public investment will rise with f, the increase in rental values within the urban boundary is exactly offset by investment costs following (6). This explains why the expected rise in inframarginal rents due to public investment does not contribute to the value of the city.

4.2 The value of land

In order to complete our characterization of the first-best outcome, we derive the value of a unit of land gross of conversion and public investment costs as the expected present discounted value of all future rental flows on a unit of land at distance s from the CCA. We write $P^c(s,f,S)$ for a piece of land in the city, i.e. $s \leq S$, and $P^a(s,f,S)$ for a piece of developable land.

Since $g(X) = \chi S$, $P^c(s,f,S)$ evolves according to the Bellman equation

$$\frac{1}{2}\sigma^2 f^2 P^c_{ff}(s,f,S) + \mu f P^c_f(s,f,S) - \rho P^c(s,f,S) + f + \omega + \chi S - \tau s = 0.$$

The general solution of this equation is

$$P^c(s,f,S) = C^c(s,S) f^{\beta} + \frac{f}{\rho-\mu} + \frac{\omega+\chi S-\tau s}{\rho}. \tag{13}$$

In order to determine $C^c(s,S)$, we impose the condition that on the boundary $\widehat{f}(S)$, a marginal change in S leaves $P^c(s,f,S)$ unaffected. This boundary condition follows from ruling out arbitrage opportunities. Suppose on the contrary that $P^c_S\left(s,\widehat{f}(S),S\right) > 0$. In this case, growth of $P^c(s,f,S)$ would be in the same order of magnitude as growth in f once it hit $\widehat{f}(S)$. Due to the volatile behaviour of f on short time intervals, capital gains on holding the unit of land would dominate rental income and opportunity costs, thus creating an arbitrage opportunity. Hence, when f approaches $\widehat{f}(S)$, the extension of S must be fully anticipated in $P^c(s,f,S)$. Once the actual investment takes place, the rise in payoff due to an increased level of public goods is exactly offset by the loss in this capitalized expectation of future investment.

It is implied that

$$C_S^c(s,S) = -\frac{\chi}{\rho}\Delta^{-\beta}\left[\xi - \frac{\omega-(\tau-\chi)S}{\rho}\right]^{-\beta},$$

where $\widehat{f}(S)$ from (10) has been substituted for f. Note that $C_S^c(s,S)$ does not depend on s, so henceforth we will write $C_S^c(S)$. Integration with regard to S yields

$$\begin{aligned} C^c(S) &= \frac{\chi}{\tau-\chi}\frac{\Delta^{-\beta}}{\beta-1}\left[\xi - \frac{\omega-(\tau-\chi)S}{\rho}\right]^{1-\beta} + I^c \\ &= -\frac{1}{2}\frac{\chi}{\tau-\chi}\widehat{B}\prime(S) + I^c, \end{aligned}$$

where I^c is a constant of integration and where the second step follows from substitution of (11). By applying a limit argument, it can be seen that the constant of integration I^c must equal zero: for small χ, when people hardly value the public good, the term $C^c(s,S)f^\beta$ must vanish in expression (13). Hence, the present expected value of all future rents on a piece of urban land is given by

$$P^c(s,f,S) = \frac{f}{\rho-\mu} + \frac{\omega+\chi S-\tau s}{\rho} - \frac{1}{2}\frac{\chi}{\tau-\chi}\widehat{B}\prime(S)f^\beta. \tag{14}$$

The first two terms in this expression equal the present value of all future rental flows at the current level of public investment. The third term reflects the present value of increments in land rents due to future public investment. Since the planner applies Samuelson's rule, this part of land value will pay for future investments in the public good, which explains why it does not contribute to $\widehat{V}(S,f)$.

The present expected value of all future rents on a unit of undeveloped land $P^a(s,f)$, $s>S$, evolves according to the Bellman equation

$$\frac{1}{2}\sigma^2 f^2 P_{ff}^a(s,f) + \mu f P_f^a(s,f) - \rho P^a(s,f) = 0, \tag{15}$$

which has the general solution

$$P^a(s,f) = C^a(s) f^\beta. \tag{16}$$

When f hits $\widehat{f}(s)$, value matching implies that $P^a(s,f)$ must equal $P^c(s,f,s)$. Hence, we have

$$C^a(s)\widehat{f}(s)^\beta = \frac{\widehat{f}(s)}{\rho-\mu} + \frac{\omega-(\tau-\chi)s}{\rho} - \frac{1}{2}\frac{\chi}{\tau-\chi}\widehat{B}\prime(s)\widehat{f}(s)^\beta.$$

Making use of the value matching condition with regard to S in (7), we rewrite this expression as

$$
\begin{aligned}
C^{a}(s)\,\widehat{f}(s)^{\beta} &= \xi+\frac{\chi}{\chi-\tau}\left[\xi-\frac{\widehat{f}(s)}{\rho-\mu}-\frac{\omega-(\tau-\chi)\,s}{\rho}\right] \\
&= \xi+\frac{1}{2}\frac{\chi}{\chi-\tau}\widehat{B\prime}(s)\,\widehat{f}(s)^{\beta}.
\end{aligned}
$$

Subsitution into (16) yields

$$
P^{a}(s,f)=\xi\left(\frac{f}{\widehat{f}(s)}\right)^{\beta}-\frac{1}{2}\frac{\chi}{\tau-\chi}\widehat{B\prime}(s)\,f^{\beta}-\frac{1}{2}\widehat{B\prime}(s)\,f^{\beta}. \tag{17}
$$

The first term of this expression pays for the conversion cost once f hits $\widehat{f}(s)$. The second term pays for future investments in the public good. After netting out conversion and investment costs, the third remaining term is the part of land value that contributes to surplus of the planner $\widehat{V}(S,f)$.

5 Universal development rights

From inspection of (14) and (17) it is seen that $\widehat{V}(S,f)$ equals the total land value net of conversion and public investment costs. Hence, a developer who owns all urban and developable land will implement the first best strategy. We now turn to the question whether the first best allocation can be decentralized when the city or urban developer owns all land up to S only. This section considers the case in which owners of developable land or farmers have the right to develop it themselves, whereas the case of exclusive development rights is dealt with in the next section.

5.1 Dispersed ownership

When ownership of developable land is dispersed, an outcome in which the city invests in public goods cannot be a Nash equilibrium. This is easily seen for the case of optimal public investment, by inspecting the decomposition of developable land value in (17). The second term, which captures benefits of future public investment once the land is converted, cancels out against investment costs for the urban developer, but not for a farmer who develops the land herself. Since she does not contribute to public investment, this term reflects the gains of freeriding. The land would thus be worth more to her as a freerider than to the city, so she would not sell.

Now consider the urban developer's public investment strategy, provided that he obtains only a strictly positive share of all land. Value matching and smooth pasting conditions imply that public goods are installed until at the margin, investment costs offset the benefits as capitalized in the land he owns. The developer can thus appropriate only a share of the benefits from investment if he does not own all land, which implies that he invests less. However, while public investment does not obey Samuelson's rule, it is still positive and it continues to be financed from land revenue. Hence, the maximum he can afford to pay is the land value net of conversion costs holding public investment constant. This is less than farmers who sold their land could have obtained by developing the land themselves, i.e. by freeriding on public investment by the developer. It follows that an outcome in which a strictly positive share of land is sold to the city cannot be a Nash equilibrium.

If no owner sells his land, then the planner cannot reap any of the benefits of further public investment, so the level of public goods remains constant at $\overline{X} = X(S)$. Since urban expansion is not associated with any external effects, the conversion decision of landowners coincides with the first best conversion decision. By simultaneously solving (7) and (8), while holding X constant at $\overline{X}$, we obtain

$$f\left(\overline{X}, s\right) = \Delta\left(\xi - \frac{\omega + g\left(\overline{X}\right) - \tau s}{\rho}\right),$$

$$B_S\left(\overline{X}, s\right) = -2\frac{\Delta^{-\beta}}{\beta - 1}\left(\xi - \frac{\omega + g\left(\overline{X}\right) - \tau s}{\rho}\right)^{1-\beta}.$$

Since $g(X)$ and $f(X, s)$ are increasing in X, we have $f(X, s) > \widehat{f}(s)$ for $s > S$: land beyond the urban boundary gets developed at a later moment than under the optimal public investment scheme. The present value of future rents on a unit of agricultural land equals

$$P^a\left(s, \overline{X}, f\right) = \xi\left(\frac{f}{f\left(\overline{X}, s\right)}\right)^{\beta} - \frac{1}{2}B_S\left(\overline{X}, s\right)f^{\beta}.$$

Hence, the gap between land value to the farmer and the urban developer has disappeared. While this suggests that farmers would be indifferent to selling their land to the urban, such a strategy would not be subgame perfect. They know that once the developer owns the land, he will find it optimal to invest in public goods and create an opportunity for freeriding.

The gains from first best investment relative to this breakdowm outcome read

$$\begin{aligned} \Gamma &= -f^{\beta}\int_{S}^{\infty}\left[\widehat{B}\prime(s) - B_s\left(\overline{X}, s\right)\right]ds \\ &= \frac{\chi}{\tau}f^{\beta}\widehat{B}(S), \end{aligned} \tag{18}$$

where for $\overline{X}$ we subtituted the optimal value of X at S, using $g(X) = \sqrt{\rho\chi X}$. Thus, a fraction χ/τ of total surplus in the first best allocation is due to the implementation of an optimal public investment path. It constitutes the gains from trade that can be divided between parties if the coordination problem between the developer and the farmers can be overcome.

5.2 Market power

Concentration of land ownership may overcome the coordination problem and prevent a breakdown, provided that it is possible to distribute the gains from trade in such a way that no party or coalition of parties has an incentive to opt out and freeride on the investment of others. In other words, the core of the bargaining game must be nonempty. We consider the case in which all developable land is concentrated in the hands of one monopolist, as well as the case in which there are multiple landowners. In order keep the analysis tractible, we assume symmetry, i.e. each party owns the same share of land at each distance from the CCA. In the case of two landowners, one may think of one party owning all land to the left of the city and the other owning all land to the right. In the case of more than two landowners, developable land may be throught of as being divided into strips of equal width that are owned by different parties.

In the monopoly case, the core consists of all distribution rules that grant the urban planner at least the present value of all land within the urban boundary S and the monopolist at least the present value rents on all land further out, if the public good remains at $\overline{X}$. Hence, any distribution of the gains from trade in (18) is in the core.

Now consider a situation in which there are $n \geq 2$ landowners, who all own the same share of land $1/n$ at each distance $s > S$ from the CCA. The grand coalition G that will implement the first-best policy consists of the urban planner and the full set of n landowers. If this grand coalition G forms, then the total surplus $\widehat{V}^G(S, f)$ that has to be divided amongst coalition partners is given in (12). For the core to be nonempty, a division must exist such that no coalition can gain a higher surplus by defecting. In particular, this means that each landowner i must get at least $\widehat{V}^i(S, f)$, the surplus that she would obtain by freeriding on the investment by all other members of the coalition. By symmetry, each landowner must get the same share of surplus, so it must hold that

$$\widehat{TDP}(S, f) + n\widehat{V}^i(S, f) \leq \widehat{V}^G(S, f), \tag{19}$$

where $\widehat{TDP}(S, f)$ equals the value of defection for the urban developer C. Freeriding becomes less beneficial if it is done by a larger number of landowners, because the smaller remaining coalition will invest less in public goods. Hence, condition (19) is also sufficient for the core to be nonempty.

5.2.1 Investment strategy by $G\backslash i$

In order to determine $\widehat{V}^{i}(S,f)$, we have to derive the investment strategy of the remaining coalition $G\backslash i$. Suppose that at some future point in time, the city will have reached $T>S$ on the land of the coalition. At this point, the flow of rents accruing to $G\backslash i$ equals

$$\begin{aligned} TDR^{G\backslash i}(X,S,T,f) &= 2\int_0^S R(s)\,ds + 2\frac{n-1}{n}\int_S^T R(s)\,ds \\ &= 2(f+\omega+g(X))\left(\frac{1}{n}S+\frac{n-1}{n}T\right) - \tau\left(\frac{1}{n}S^2+\frac{n-1}{n}T^2\right). \end{aligned}$$

Holding X constant, the present expected value of all future rental flows will equal

$$TDP^{G\backslash i}(X,S,T,f) = 2\left(\frac{f}{\rho-\mu}+\frac{\omega+g(X)}{\rho}\right)\left(\frac{1}{n}S+\frac{n-1}{n}T\right) - \frac{\tau}{\rho}\left(\frac{1}{n}S^2+\frac{n-1}{n}T^2\right).$$

The value of the coalition has the form

$$V^{G\backslash i}(X,S,T,f) = TDP^{G\backslash i}(X,S,T,f) + D(X,S,T)\,f^{\beta}.$$

Optimal investment in X follows from value matching and smooth pasting conditions

$$\begin{aligned} D_X(X,S,T)\,f^{\beta} + 2\frac{g\prime(X)}{\rho}\left(\frac{1}{n}S+\frac{n-1}{n}T\right) &= 1, \\ \beta D_X(X,S,T)\,f^{\beta-1} &= 0. \end{aligned}$$

Hence, it must be the case that $D_X(X,S,T)=0$ and therefore the public investment path $X^{G\backslash i}(S,T)$ solves $2g\prime(X)\left(\frac{1}{n}S+\frac{n-1}{n}T\right)=\rho$. This implies

$$\begin{aligned} X^{G\backslash i}(S,T) &= \frac{\chi}{\rho}\left(\frac{1}{n}S+\frac{n-1}{n}T\right)^2, \\ g\left[X^{G\backslash i}(S,T)\right] &= \chi\left(\frac{1}{n}S+\frac{n-1}{n}T\right), \end{aligned}$$

which replaces condition (6). Value matching and smooth pasting with regard to T yield

$$\begin{aligned} \widehat{D}_T(S,T)\,f^{\beta} + 2\frac{n-1}{n}\left[\frac{f}{\rho-\mu}+\frac{\omega+\chi\left(\frac{1}{n}S+\frac{n-1}{n}T\right)-\tau T}{\rho}\right] &= \xi, \\ \beta\widehat{D}_T(S,T)\,f^{\beta-1} + 2\frac{n-1}{n}\frac{1}{\rho-\mu} &= 0. \end{aligned}$$

By solving these two equations simultaneously, we obtain the boundary $\widehat{f}^{G\backslash i}(S,T)$ as

$$\widehat{f}^{G\backslash i}(S,T)=\Delta\left[\xi-\frac{\omega+\chi\left(\frac{1}{n}S+\frac{n-1}{n}T\right)-\tau T}{\rho}\right].$$

Hence

$$\widehat{D}_T(S,T)=-2\frac{n-1}{n}\frac{\Delta^{-\beta}}{\beta-1}\left[\xi-\frac{\omega+\chi\left(\frac{1}{n}S+\frac{n-1}{n}T\right)-\tau T}{\rho}\right]^{1-\beta}.$$

The value of this coalition follows as

$$V^{G\backslash i}(X,S,T,f)=TDP^{G\backslash i}(X,S,T,f)+\int_T^\infty\left[-\widehat{D}_T(S,t)\,f^\beta\right]dt,$$

evaluated at $X=X^{G\backslash i}(S,T)$. For $T=S$, we obtain

$$\widehat{V}^{G\backslash i}(S,f)=\widehat{TDP}(S,f)+\frac{n-1}{n}\frac{\tau-\chi}{\tau-\frac{n-1}{n}\chi}\widehat{B}(S)\,f^\beta. \tag{20}$$

This expression illustrates that by losing out landowner n, the remaining members of G lose a fraction $1/n$ of the option value on all developable land and this option value is further eroded by implementing a suboptimal public investment path.

5.2.2 The value of defection for i

Next, we have to solve for the optimization problem of i, taking the path of X as given. Let T_i denote the city size on the strip of land that belongs to i. We distinguish T_i from T in order to allow for the possibility that this strip grows at a different rate than the part belonging to the coalition $G\backslash i$. The flow of rents to i equals

$$\begin{aligned}TDR^i(T_i,T,f)&=\frac{2}{n}\int_S^{T_i}R(s)\,ds\\&=\frac{2}{n}\left[f+\omega+g\left(X^{G\backslash i}(T)\right)\right](T_i-S)-\frac{1}{n}\tau\left(T_i^2-S^2\right).\end{aligned}$$

Holding $X^{G\backslash i}(T)$ constant, the present expected value of all future rental flows will equal

$$TDP^i(T_i,T,f)=\frac{2}{n}\frac{(T_i-S)\,f}{\rho-\mu}+\frac{2}{n}\frac{\left(\omega+g\left(X^{G\backslash i}(T)\right)\right)(T_i-S)-\frac{1}{2}\tau\left(T_i^2-S^2\right)}{\rho}.$$

The value of defection for i has the form

$$V^i(T_i,T,f)=E(T_i,T)\,f^\beta+TDP^i(T_i,T,f).$$

On the boundary $\widehat{f}^{G\backslash i}(T)$, we impose the condition that $V_T^i(T_i, T, f) = 0$. This condition states that investment in X must be anticipated in land prices, in order to rule out arbitrage opportunities. It follows that

$$E_T(T_i, T)\left(\widehat{f}^{G\backslash i}(T)\right)^\beta + \frac{2\chi}{n\rho}\frac{n-1}{n}(T_i - S) = 0,$$

where we have substituted $g(X) = \sqrt{\chi\rho X}$. Substitution of $\widehat{f}^{G\backslash i}(T)$ yields

$$E_{T_{G\backslash i}}(T_i, T) = -\frac{2\chi}{n\rho}\frac{n-1}{n}(T_i - S)\,\Delta^{-\beta}\left[\xi - \frac{1}{\rho}\left(\omega + \chi\left(\frac{1}{n}S + \frac{n-1}{n}T\right) - \tau T\right)\right]^{-\beta}.$$

Integration with regard to T yields

$$\begin{aligned} E(T_i, T) &= (T_i - S)\frac{2}{n}\frac{\frac{n-1}{n}\chi}{\tau - \frac{n-1}{n}\chi}\frac{\Delta^{-\beta}}{\beta - 1}\left[\xi - \frac{1}{\rho}\left(\omega + \chi\left(\frac{1}{n}S + \frac{n-1}{n}T\right) - \tau T\right)\right]^{1-\beta} + F(T_i) \\ &= -(T_i - S)\frac{1}{n-1}\frac{\frac{n-1}{n}\chi}{\tau - \frac{n-1}{n}\chi}\widehat{D}\prime(T) + F(T_i), \end{aligned}$$

where $F(T_i)$ is a constant of integration that may depend on T_i but not on T.

The value matching condition with regard to T_i now reads

$$\frac{2}{n}\left(\frac{f}{\rho - \mu} + \frac{\omega + g\left(X^{G\backslash i}(T)\right) - \tau T_i}{\rho}\right) + \left(F\prime(T_i) - \frac{1}{n-1}\frac{\frac{n-1}{n}\chi}{\tau - \frac{n-1}{n}\chi}\widehat{D}\prime(T)\right)f^\beta = \xi.$$

Note that there is an additional term on the benefit side of urban expansion, which reflects expected gains through future extensions of the public good. We obtain the smooth pasting condition

$$\frac{2}{n}\frac{1}{\rho - \mu} + \beta\left(F\prime(T_i) - \frac{1}{n-1}\frac{\frac{n-1}{n}\chi}{\tau - \frac{n-1}{n}\chi}\widehat{D}\prime(T)\right)f^{\beta - 1} = 0.$$

Solving for $f^i(T_i, T)$ yields

$$f^i(T_i, T) = \Delta\left[\xi - \frac{1}{\rho}\left(\omega + \chi\left(\frac{1}{n}S + \frac{n-1}{n}T\right) - \tau T_i\right)\right].$$

It follows that for $T_i = T$, we have $f^i(T_i, T) = \widehat{f}^{G\backslash i}(T)$, i.e. T_i follows exactly the same path as T and we may substitute $T_i = T$. Hence, substitution into the smooth pasting condition yields

$$F\prime(T) - \frac{1}{n-1}\frac{\frac{n-1}{n}\chi}{\tau - \frac{n-1}{n}\chi}\widehat{D}\prime(T) = -\frac{2}{n}\frac{\Delta^{-\beta}}{\beta - 1}\left[\xi - \frac{1}{\rho}\left(\omega + \chi\left(\frac{1}{n}S + \frac{n-1}{n}T\right) - \tau T\right)\right]^{1-\beta},$$

or

$$F\prime\left(T\right)=\frac{1}{n-1}\left(1+\frac{\frac{n-1}{n}\chi}{\tau-\frac{n-1}{n}\chi}\right)\widehat{D}\prime\left(T\right).$$

Evaluated at $T=S$, we have

$$\begin{aligned}\widehat{V}^{i}\left(S,f\right) &= \frac{1}{n-1}\left(1+\frac{\frac{n-1}{n}\chi}{\tau-\frac{n-1}{n}\chi}\right)\int_{S}^{\infty}\left[-\widehat{D}\prime\left(s\right)f^{\beta}\right]ds \\ &= \frac{1}{n}\left(1+\frac{\frac{n-1}{n}\chi}{\tau-\frac{n-1}{n}\chi}\right)\frac{\tau-\chi}{\tau-\frac{n-1}{n}\chi}\widehat{B}\left(S\right)f^{\beta}. \qquad (21)\end{aligned}$$

By comparison with (20), developable land is seen to be more valuable to the freerider than to the remaining coalition.

5.2.3 Evaluation of the core condition

We define Σ as the share of gains from trade that is available for the city after paying each landowner enough to prevent her from freeriding, or

$$\Sigma\equiv\frac{\widehat{V}^{G}\left(S,f\right)-\widehat{TDP}\left(S,f\right)-n\widehat{V}^{i}\left(S,f\right)}{\Gamma}.$$

This share is positive if and only if condition (19) is met. Substitution of (12), (18) and (21) yields

$$\begin{aligned}\Sigma &= \frac{\tau}{\chi}\left[1-\left(1+\frac{\frac{n-1}{n}\chi}{\tau-\frac{n-1}{n}\chi}\right)\frac{\tau-\chi}{\tau-\frac{n-1}{n}\chi}\right] \\ &= \left(\widehat{\chi}+\frac{1}{\left(n-1\right)^{2}}-1\right)\left(\frac{n-1}{n-\left(n-1\right)\widehat{\chi}}\right)^{2},\end{aligned}$$

where $\widehat{\chi}\equiv\chi/\tau$. Hence, the core nonempty if and only if

$$\frac{\chi}{\tau}\geq1-\frac{1}{\left(n-1\right)^{2}}.$$

This inequality is satisfied for $n=2$, but it will not hold if there are more landowners and $\widehat{\chi}$ is not too large. Moreover, for any $\widehat{\chi}<1$, the core will be empty for n sufficiently large. We have

$$\lim_{n\to\infty}\Sigma=-\frac{1}{1-\widehat{\chi}}<0.$$

In summary, if landowners have the right to develop, then bargaining may be relied upon to attain an efficient outcome only if their number is small. If there are many landowners, then the money needed to prevent each party to refrain from freeriding exceeds the gains from trade.

6 Exclusive development rights for the city

When ownership of agricultural land is dispersed, the first best allocation may still obtain if land use is regulated in such a way that the right to develop rests exclusively with the city. In that case, for an owner of agricultural land, the outside option to selling is to continue farming. Competition amongst many small sellers of land ensures that the urban developer needs to pay no more than this outside option. Hence, the price of developable land equals its oppportunity cost in agricultural use, normalized to zero in our model, just as in the case in which the developer owns all developable land outright.

This result extends to the case of bargaining with a finite number of landowners. In particular, the assignment that grants all surplus to the city and nothing to the landowners is always in the core. Any coalition that includes the city thus attains the same surplus as in the grand coalition and any coalition that doesn't cannot realise any surplus, since it does not have the right to develop land.

Even if the assignment that grants no surplus to landowners is always in the core, owners with a large portfolio of land may be able to secure a share of the surplus in the bargaining process. Consider the extreme case of monopoly. In the outside option, no development takes place at all. Hence, the gains from trade that will be shared by the two parties consist of the option value on all developable land under optimal public investment. The core consists of all distributition rules that grant the city at least the present value of all land within the urban boundary S if the public good remains at $\overline{X}$. Small farmers may have an incentive to sell their land to middlemen who hold large land portfolios and are thus able to secure a positive share of the surplus.

7 Conclusions and discussion

The exclusive assignment of development rights at the metropolitan level avoids a breakdown of the market for developable land and undersupply of public goods. Since planning is often coordinated at higher levels of government in European cities, our analysis thus predicts more generous provision of urban public goods than in the typical US city. This prediction is consistent with the stylized observation in Brueckner, Thisse, and Zenou (1999) that centres of European cities offer a high level of amenities and attract high income households, whereas the rich have fled to the suburbs in US cities. While history may explain part of the amenity differential, its exploitation and maintenance require substantial public investment. Higher levels of investment in public transportation within European cities (Nechyba and Walsh (2004)) are equally consistent with the predictions of our model. On the other hand, it should be pointed out that more centralized tax and transfer systems may also explain higher public investment levels in European cities. A full discussion of the advantages and disadvantages of these alternative instruments to internalize external benefits of public investment is beyond the scope of this paper.

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