Downtown Parking and Traffic Congestion

A Diagrammatic Exposition*

Richard Arnott†

University California, Riverside

Eren Inci‡

Sabanci University

John Rowse§

University of Calgary

15 February 2012

Abstract

Through an extended numerical example, this paper develops a diagrammatic analysis of steady-state parking and traffic congestion in an isotropic downtown and provides systematic policy analysis. Unlike our previous work, the model incorporates curbside parking, garage parking, and price-sensitive travel demand in a unified setting. We examine the deadweight loss associated with underpriced curbside parking, as well as first- and second-best curbside parking capacities. We also explore the transient dynamics and stability of various downtown traffic equilibria.

*Arnott would like to acknowledge financial support through the US Department of Transportation (grant 1DTRT07-G-9000) and the California Department of Transportation (grant 65A0216), both through the University of California, Transportation Center (UCTC) at UC Berkeley, and Inci would like to acknowledge financial support from the Scientific and Technological Research Council of Turkey (TUBITAK Career Grant 111K051). The authors thank Ozan Bakis for helpful comments. Any remaining errors are authors' responsibility.

†*Address*: Department of Economics, University of California, Riverside, 4106 Sproul Hall, Riverside, CA 92521-0427 USA. *E-mail address*: richard.arnott@ucr.edu

‡*Tel.*: 90-216-483-9340; *fax*: 90-216-483-9250. *Address*: Sabanci University - FASS, Orhanli / Tuzla 34956 Istanbul TURKEY. *E-mail address*: ereninci@sabanciuniv.edu.

§*Address*: Department of Economics, University of Calgary Calgary, AB T2N 1N4 CANADA. *E-mail address*: rowse@ucalgary.ca

Keywords: cruising for parking, curbside parking, parking garages, parking policy, traffic congestion

JEL Classification: D04; L91; R41; R48

1 Introduction

For many years, urban transportation economists have analyzed downtown traffic congestion by applying economic and engineering tools developed in the 1960's and 1970's for the study of freeway congestion. Over the last decade, however, there has been increasing recognition that downtown traffic congestion differs in important ways from freeway congestion. One of the most important is parking. Parking is a major user of land downtown, curbside parking reduces street capacity, and cars cruising for parking contribute to the congestion on city streets. Another is that downtown travel takes place on a denser network, which increases the importance of nodal (intersection) congestion relative to classical link flow congestion. Yet another is that downtown traffic congestion is more heterogeneous in character, entailing queuing at intersections, spillbacks, traffic jams, car-pedestrian interaction, double parking, and congestion caused by cars entering and exiting curbside parking spaces and cruising for parking, in addition to classical link flow congestion.

William Vickrey (1994) was the first urban transportation economist to develop a model customized for the study of downtown traffic congestion. The central departure from the standard models is that, to avoid treating the complexity of downtown traffic, travel speed is assumed simply to be related to the density of traffic per unit area. Vickrey conceived of Manhattan as a bathtub. The height of water in the bathtub corresponds to traffic density per unit area; traffic entering Manhattan, as well as trips initiated within Manhattan, correspond to water flowing into the bathtub; and traffic leaving Manhattan, as well as trips terminating in Manhattan correspond to water flowing out of the bathtub. Traffic speed is negatively related to traffic density; traffic flow is the product of traffic speed and traffic density; and trip termination is proportional to traffic flow. Only recently have transportation engineers started to use traffic sensors to study the empirical properties of downtown traffic congestion. The early results (Geroliminis and Daganzo, 2008; Daganzo, Gayah, and Gonzales, 2011) provide strong support for Vickrey's conception, documenting the existence of a stable macroscopic fundamental diagram (relating average flow to average density) at the level of downtown neighborhoods.

In a series of papers (Arnott and Inci, 2006, 2010; Arnott and Rowse, 2009, 2011), we have been developing a sequence of models that build on Vickrey's conception. However, our models differ from Vickrey's conception in that they put parking and the interaction between downtown parking and downtown traffic congestion at center stage. Arnott and Inci (2006, 2010) examines steady-state equilibria in a bathtub model of downtown traffic congestion with curbside parking and price-sensitive demand, but without garage parking. Arnott and Rowse (2009, 2011) extends that model to allow for garage as well as curbside parking but to keep it analytically manageable assumes demand to be inelastic.

Building on our previous work, in this paper we develop an extended numerical example of a synthesized steady-state model that incorporates curbside parking, garage parking, and price-sensitive demand, presenting the results through a series of diagrams. Working through an extended numerical example, with diagrams, circumvents the technical complexity of the earlier papers and puts the economic intuition into sharper relief. We use the diagrammatic exposition to examine the deadweight loss associated with the underpricing of curbside parking (which is typical of US cities), as well as first-best and second-best (with the underpricing of curbside parking and of traffic congestion being the distortions) curbside parking capacity, and to explore the multiplicity and stability of equilibria.

Our principal results are as follows. First, in both the first and second best, it is efficient: at low levels of demand intensity to have all parking curbside and as demand intensity increases to expand curbside parking capacity; at intermediate levels of demand intensity, to provide garage parking, as well as curbside parking and as demand intensity increases to expand garage parking and to contract curbside parking; and at high levels of demand intensity, to provide all parking in garages and none curbside. Second, at those demand intensities where it is second-best efficient to provide no garage parking, curbside parking should be provided to the point where cruising for parking is just eliminated, and the second-best level of curbside parking is higher (lower) than the first-best level when curbside parking is underpriced (overpriced). And third, when parking is underpriced, the range of demand intensities over which it is efficient to have both garage and curbside parking is narrower in the second best than in the first best. The intelligent design of downtown parking policy needs to take into account intra-day dynamics. We hope that the steady-state analysis of this paper will provide a foundation for the construction of models that do so.

The paper is organized as follows. Section 2 develops the base model, setting the stage by adapting Walters' (1961) landmark model of highway congestion to downtown congestion without parking. Section 3 adds curbside parking to the base model, considering the first best and then the second best, in the short run and then in the long run. Section 4 extends the analysis by incorporating both curbside and garage parking. Section 5 investigates the stability of the various equilibria of the previous sections. Section 6 discusses directions for future research.

2 Downtown Traffic Congestion with No Parking

To set the base for further analysis, we start by adapting the familiar diagrammatic analysis of congested traffic equilibrium with price-sensitive demand due to Walters (1961) to downtown traffic. For the moment, we ignore downtown parking, essentially assuming that parking is costless. We assume that downtown is isotropic; one can imagine a boundless Manhattan network of one-way streets. We also assume that drivers are identical and that the demand for trips initiated per unit area-time is stationary and is a function of the full price of a trip, F:

$$D = D(F) \quad . \tag{1}$$

For simplicity we ignore the money costs of travel. Therefore, the user cost of a trip, UC, equals the travel time cost of a trip, which equals the trip length, m, times travel time per mile, t, times the value of time, ρ:

$$UC = \rho m t \quad . \tag{2}$$

Travel time per mile is an increasing, convex function of the density of traffic per unit area, V: $t(V)$, with $t' > 0$, $t'' > 0$, and with $t(0) > 0$ being free-flow travel time. In order to distinguish between the full trip price and user cost, we assume that a toll of size τ is applied, so that the full price of a trip equals user cost plus the toll:

$$F = UC + \tau \quad . \tag{3}$$

In steady state, the number of trips initiated per unit area-time equals the number of trips terminated per unit area-time. We refer to this as *the steady-state condition*, and the steady-state number of trips per unit area time as *throughput*,[1] and denote it by r. The steady-state number of trips initiated per unit area-time is given by the demand function. The steady-state number of trips terminated per unit area-time equals traffic density divided by the length of time each car spends in traffic, mt. Thus, the steady-state condition is

$$D(\rho m t(V) + \tau) = \frac{V}{m t(V)} \quad , \tag{4}$$

[1] Throughput has units of cars per unit area-time. In steady state, throughput is the same as the entry flow and exit flow per unit area. We avoid the term *flow* to avoid confusion. The fundamental identity of traffic flow states that flow, f, equals density times velocity. Applying that identity in the current context gives $f = V/t(V)$. Flow, therefore, equals throughput times trip length. Then, throughput measures the exit rate (= entry rate) from the flow of traffic, which equals flow divided by trip length.

which gives steady-state equilibrium traffic density.

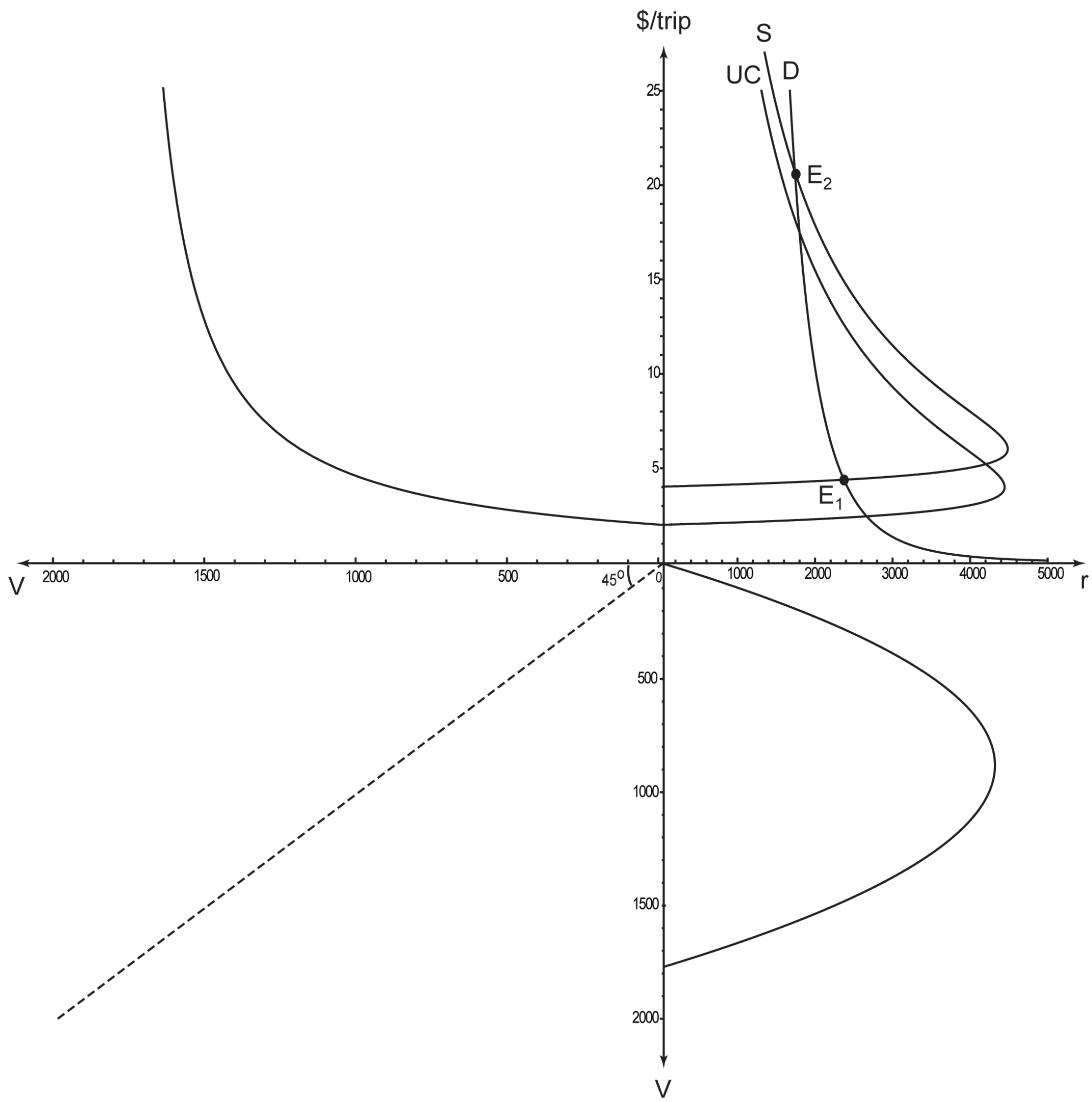

Figure 1: The Fundamental Traffic Diagram applied to downtown traffic

This equilibrium can be derived geometrically using the four-quadrant diagram of Figure 1. Quadrant II plots the relationship between user cost and traffic density ($UC = \rho m t(V)$, which combines (2) and $t = t(V)$). Quadrant III shows the 45-degree line. Quadrant IV depicts the steady-state relationship between traffic throughput and density,

$$r = \frac{V}{mt(V)} \quad . \tag{5}$$

The user cost curve in Quadrant I, marked as UC, relates user cost to throughput.[2] The supply curve relates the full price of a trip to throughput, and is labeled S in the figure. It is obtained as a vertical shift of the user cost curve by τ. The inverse demand function provides the demand relation between the full trip price and throughput, and equilibrium is given by the point of intersection of the demand and supply curves.

Figure 1 is plotted for specific functional forms and parameter values. The following are maintained throughout the paper:

$$D(F) = D_0 F^{-a} \tag{6}$$

$$t(T) = \frac{t_0}{1 - \frac{V}{V_j}} \tag{7}$$

with parameter values

$$a = 0.2, \quad t_0 = 0.05, \quad V_j = 1778.17, \quad m = 2.0, \quad \rho = 20.0 \quad . \tag{8}$$

The parameters chosen are the same as those assumed in Arnott and Inci (2006, 2010), and the basis for their choice is given in Arnott and Inci (2006). Demand is assumed to be iso-elastic, with demand elasticity equal to 0.2. The demand intensity parameter, D_0, is allowed to vary, in order to examine how equilibrium changes with demand. Travel congestion is described by Greenshields' Relation, which specifies a negative linear relationship between velocity and density,[3] and hence the form of the relationship between travel time and density depicted in Quadrant II. Free-flow travel time per mile, t_0, is 0.05 hrs, which corresponds to 20 mph. Jam density, V_j, is 1778.17 cars/ml^2. Trip distance is 2.0 mls and the value of time is \$20/hr. Figure 1 is drawn with the base case demand intensity of $D_0 = 3190.94$.

Following Vickrey, travel on the upward-sloping portion of the user cost curve is termed

[2] From (2) and $t = t(V)$, $V = t^{-1}(UC/(\rho m))$. Substituting this into (5) gives $r = t^{-1}(UC/(\rho m))/(UC/\rho)$.

[3] Greenshields' Relation has the property that travel is hypercongested if velocity is less than one-half free-flow, or equivalently if travel time per ml is more than double free-flow travel time per ml.

congested travel, and travel on the backward-bending portion is termed hypercongested travel. With congested travel, travel time and user cost increase with throughput. With hypercongested travel, travel time and user cost decrease with throughput. Congested travel corresponds to normal travel, and hypercongested travel to traffic jam situations.

Figure 1 shows two equilibria. At E_1 traffic flow is congested, at E_2 traffic flow is hypercongested. There is also an equilibrium, E_3, that cannot be shown in the diagram, corresponding to gridlock – zero flow and an infinite full trip price. It is generally accepted that E_1 is a stable equilibrium. The stability of equilibria on the backward-bending portion of the supply curve has been a matter of considerable dispute. Arnott and Inci (2010) examined the issue for a somewhat different model[4] that included curbside parking.

If the Arnott and Inci (2010) model had excluded curbside parking, the stability analysis would have proceeded as follows. Stability is defined with reference to a particular adjustment dynamic. The natural adjustment dynamic in this context is that the change in the density of cars equals the demand inflow, $D(F)$, minus the outflow, $T/(mt(T))$. With demand based on either myopic foresight (when a driver is deciding whether to take a trip, he bases his expectation of the full price on current traffic conditions) or perfect foresight, the analog of E_1 is indeed stable, while E_2 is unstable[5] and E_3 stable. Which of the two stable equilibria the traffic network attains depends on the density at the time when the demand function first became stationary.

Figure 2 focuses on the upward-sloping portion of the user cost curve. Aggregate user cost can be calculated as a function of throughput. On this portion of the user cost curve, marginal social cost is defined as the derivative of aggregate user cost with respect to throughput, and at a particular throughput equals the corresponding user cost plus the congestion externality cost (the cost to inframarginal users due to the increase in throughput slowing them down).

[4] Visit length is assumed to be Poisson distributed with mean length m.

[5] A steady-state equilibrium is unstable if the measure of initial traffic conditions achieving this equilibrium is zero. One may call E_2 *saddle-path stable* because it can be reached from initial traffic conditions on one of the arms of the steady state, which is a curve and thus of measure zero.

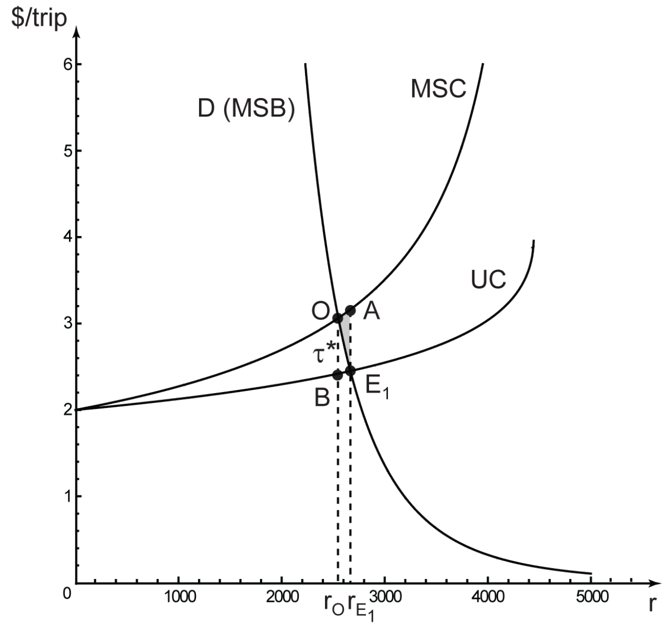

Figure 2: Equilibrium and social optimum with no parking

Figure 2 displays the user cost curve and the marginal social cost curve, labeled MSC. In social surplus analysis, the demand curve is interpreted as the marginal social benefit curve, labeled MSB, so that optimal throughput occurs at the point of intersection of the marginal social cost and demand curves, O.

The optimal throughput can be achieved by setting an optimal congestion toll equal to the congestion externality cost, evaluated at the social optimum, τ^*. In the no-toll equilibrium, E_1, too many cars travel on the road since travel is underpriced due to drivers not paying for slowing other drivers down. The deadweight loss associated with having no toll is given by the shaded area AE_1O, and equals the loss in social surplus from travel at throughput r_{E_1} compared to throughput r_O. These results are, of course, broadly familiar, but we have been careful to derive them precisely in the context of steady-state traffic congestion in an isotropic downtown area, since we shall build on them in the sections that follow, which add parking.[6]

[6] We could extend the analysis to solve for optimal road capacity. But, here and throughout the paper we take road capacity as fixed, although the portion of it allocated for curbside parking may vary.

3 Downtown Traffic Congestion with Only Curbside Parking

We now modify the model to take into account that drivers must park. In this section, we rule out garage parking and consider only curbside parking. Curbside parking affects the analysis in four ways. First, increasing the amount of curbside allocated to parking reduces the road space available for traffic flow, which reduces jam density.[7] Second, the amount of curbside parking constrains the throughput of the downtown traffic network to be no more than the curbside turnover rate, which we term *curbside parking capacity constraint* (CPC); if there are P curbside parking spaces per unit area and if the visit duration is l, then curbside parking capacity is P/l; it is the maximum throughput that curbside parking can accommodate. Third, if there is insufficient curbside parking to ration the demand, given the curbside parking fee, cruising for parking occurs, with travel time costs, including cruising-for-parking time costs, adjusting to clear the market. And fourth, drivers pay a curbside parking fee per unit time (meter rate)[8], f.

To simplify, we provide a crude treatment of parking search. We assume that each driver travels to his destination block. If a space is available, he takes it, and if it is not he drives around the destination block until a space opens up. Thus, curbside parking involves no walking. Furthermore, we ignore the random variation that occurs due to the small number of parking spaces on each block, and assume that curbside parking is either *saturated* (fully occupied) everywhere, or *unsaturated* everywhere.[9]

We shall first consider optimal curbside parking pricing. We shall then examine optimal

[7] We assume that curbside allocated to parking reduces jam density by the same amount whatever the occupancy rate of the curbside parking. The rationale is that, under at least moderately congested conditions, even if only one curbside parking space is occupied on one side of the block, traffic flow is effectively excluded from that lane for the entire block.

[8] To keep the analysis simple, we consider only linear curbside parking payment schedules.

[9] Realistically, at the level of the downtown area, there is a gradual transition between unsaturated and saturated parking (Martens, Benenson, and Levy, 2010). As the demand for curbside parking increases, curbside parking becomes saturated on an increasingly high proportion of blocks.

curbside parking capacity, conditional on curbside parking being efficiently priced (first-best capacity) and inefficiently priced (second-best capacity). In all our analysis, we assume that no congestion tolling is employed. Because the distance traveled and the visit duration are fixed, the first best can be achieved just by efficiently pricing curbside parking, even though congestion tolling is not employed; the efficient parking fee includes the optimal congestion toll. This is why we refer to the optimal capacity with efficient curbside parking pricing as first-best.

We have already distinguished between throughput and flow. Steady-state throughput is the rate at which trips are initiated and terminated per unit area-time. Steady-state flow is the number of car-miles traveled per unit area-time. When cruising for parking occurs, there is a further distinction between throughput and flow – flow includes cars that are cruising for parking.

Two adjustments need to be made to the specification of the congestion technology to accommodate curbside parking. First, it is necessary to account for the reduction in road capacity due to curbside parking. We assume that *effective jam density* is related to the amount of street space allocated to traffic flow. In particular, where Ω is the jam density with no curbside parking, effective jam density, V_j, equals jam density times the proportion of street space allocated to traffic flow, $1-P/P_{\max}$, where P is the density of curbside parking spaces per unit area and $P_{\max}$ its maximum value. Thus,

$$V_j = \Omega(1 - \frac{P}{P_{\max}}) \quad . \tag{9}$$

Second, the specification of the congestion technology needs to account for the congestion interaction between cars in transit and cars cruising for parking. We make the simple assumption that a car cruising for parking generates θ times as much congestion as a car in transit. Thus, where C is the density of cars per unit area that are cruising for parking, the travel time function is

$$t(T, C, P) = \frac{t_0}{1 - \frac{T+\theta C}{V_j}} \quad . \tag{10}$$

We maintain the following parameters for the rest of the paper:[10]

$$\theta = 1.5, \quad \Omega = 2667.36, \quad P_{\max} = 11136 \quad . \tag{11}$$

For the base case, we also assume that the curbside parking fee is \$1/hr, so that the parking fee for the trip is \$2, and that curbside parking is permitted on one side of the street everywhere, so that $P = 3712$ and $P/l = 1856$. In the analysis that follows, we start with the short run, where the level of curbside parking is fixed, and then move to the long run, where the level of curbside parking is a policy choice variable.

3.1 The short run with only curbside parking

We start with the first-best planning problem and its decentralization.

3.1.1 First-best optimum in the short run with only curbside parking

Consider a benevolent social planner who has direct control of the transportation system and its users. She would never choose to have cruising for parking because the same throughput (and hence the same social benefit) can be achieved at lower cost without it. Since the amount of curbside parking is fixed, she chooses throughput to maximize social surplus.

[10] The parameters are drawn from Arnott and Inci (2006) and were chosen to be broadly consistent with observation. A city block is assumed to be 1/8 ml long, the one-way streets to have three lanes, and roads to be 33 ft wide. Then, each side of a block is 627 ft long. If parking is on one side of the street, so that two sides of every city block have curbside parking, the maximum length of curbside around each city block that could be devoted to parking is 1254 ft. But some of this curbside is used for crosswalks. On two sides of a city block, there are four crosswalks. We assume that each crosswalk is 9 ft wide, so that the amount of curbside around each city block allocated to parking is 1218 ft. With 21 ft devoted to each curbside parking space, the number of curbside parking spaces on each block is 58. And since there are 64 blocks per ml^2, the number of curbside parking spaces per ml^2 is 3712.

Resource cost per unit time is ρT. Thus, where $X(r)$ is the social benefit from throughput r (which equals the area under the inverse demand curve up to throughput level r), she faces the maximization problem

$$\max_{r,T} X(r) - \rho T \tag{12}$$

$$s.t.$$

$$r = \frac{T}{mt(T, 0, P)}$$

$$r \leq \frac{P}{l} \quad .$$

The first constraint is the steady-state condition and the second is the curbside parking capacity constraint. Figure 3 displays the solutions with the functional forms for the congestion and demand functions, as well as the base case parameters, specified earlier. Two demand functions are considered, D_1 and D_2, which correspond to the demand function given in (6) with different levels of demand intensity. The curbside parking capacity constraint is labeled CPC. We define the (unconstrained) *short-run marginal social cost of throughput* as $\rho[dT/dr]$, where dT/dr is calculated from the steady-state condition, holding curbside parking capacity fixed and ignoring the curbside parking capacity constraint, and label the corresponding locus as $SRMSC(r; P)$. It equals the user cost plus the congestion externality cost.

With demand level D_1, the curbside parking constraint does not bind, and the first-best optimum, O_1, is at the point of intersection of the demand and $SRMSC$ curves. Since we have assumed that the reduction in roadside capacity caused by curbside parking depends on the amount of curbside that is allocated to curbside parking, independent of its occupancy rate, the marginal driver generates no parking externality. Short-run marginal social cost therefore equals user cost plus the congestion externality cost, so that the social optimum can be decentralized by setting the parking fee equal to the congestion externality cost, which is

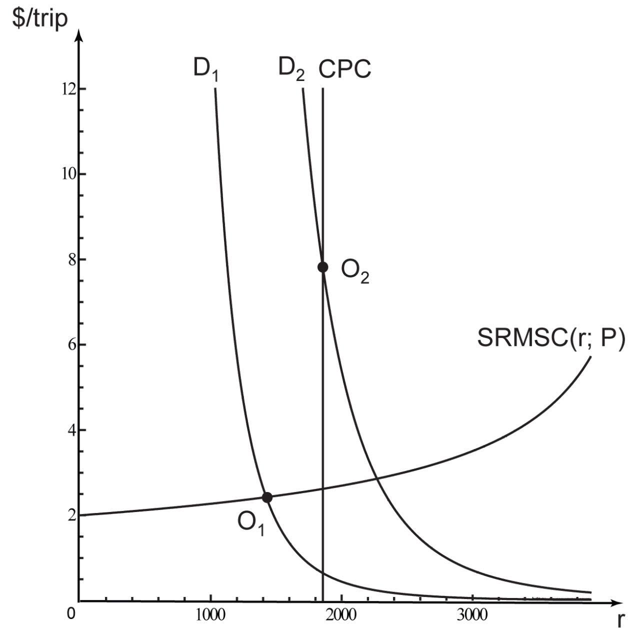

Figure 3: First-best optimum in the short run with only curbside parking

calculated per Figure 2.

With demand level D_2, the curbside parking capacity constraint binds, and the first-best optimum, O_2, is at the point of intersection of the demand curve and the curbside parking capacity constraint. The short-run marginal social cost now equals the user cost plus the congestion externality cost plus a parking scarcity rent.[11] The social optimum can then be decentralized by charging a congestion toll equal to the congestion externality cost, evaluated at the social optimum, plus a parking scarcity rent. Since in the model each trip has a fixed length and visit duration, the social optimum can also be decentralized without an explicit congestion toll by charging a parking fee equal to the congestion externality cost and the parking scarcity rent.

We may simplify the analysis by defining the *capacity-constrained short-run marginal*

[11] Let λ be the Lagrange multiplier on the steady-state condition and μ the multiplier on the curbside parking capacity constraint. The first-order condition with respect to r is $X'(r) - \lambda - \mu = 0$. The first-order condition with respect to T is $-\rho + \lambda(1/(mt) - Tt_T/(mt^2)) = 0$, so that $\lambda = \rho mt/(1 - Tt_T/t)$. Here, ρmt is the travel time cost of the marginal traveler and $\lambda - \rho mt$ is the congestion externality cost imposed by the marginal traveler on inframarginal travelers. Thus, λ equals $SRMSC(P/l; P)$ and μ, the parking scarcity rent, equals $MSB - SRMSC(P/l; P)$.

social cost curve as the short-run marginal social curve up to the capacity constraint, combined with that portion of the capacity constraint above its point of intersection with the short-run marginal social cost curve. The short-run, first-best optimum then lies at the point of intersection of the demand curve and the capacity-constrained short-run marginal cost curve.

3.1.2 Second-best optimum in the short run with only curbside parking

There are two distortions in the second-best problem. No congestion toll can be charged, and the parking fee is set suboptimally low. The second-best optimization problem in the short run is degenerate in that the constraints determine the solution. The second-best optimum is therefore the equilibrium that generates the highest social surplus.

An equilibrium may entail unsaturated or saturated parking. Consider first equilibria with unsaturated parking. Since parking is unsaturated, there is no cruising for parking. The user cost is $UC = \rho mt(T, 0, P)$ and the full price is $F = UC + fl$, where T satisfies the steady-state condition. From these results the *unsaturated user cost curve* for the exogenous level of P, $UC(r; P)$ can be derived, which is completely analogous to the user cost curve derived in the previous section, except that curbside parking reduces road capacity. At levels of throughput where the curbside parking capacity constraint does not bind the supply curve is obtained as the unsaturated user cost curve shifted up by the curbside parking fee, and any point of intersection of the demand curve and this portion of the supply curve is an unsaturated equilibrium.

Now consider equilibrium with saturated parking. Parking is saturated because the curbside parking capacity constraint binds, and except in the situation where it just binds there is cruising for parking. Equilibrium therefore entails two density variables, the density of cars in transit and the density of cars cruising for parking. They are determined by two equilibrium conditions. The first is the familiar steady-state condition but here modified to take into account cruising for parking:

$$D(F) = \frac{T}{mt(T,C,P)} \quad , \tag{13}$$

where the full price equals the cost of in-transit time, plus the expected cost of cruising-for-parking time, plus the parking fee:

$$F = \rho mt(T,C,P) + \frac{\rho Cl}{P} + fl \quad . \tag{14}$$

Since C cars are cruising for parking and since the turnover rate of parking spaces is P/l, the probability that a car cruising for parking gets a space per unit time is $P/(Cl)$, so that expected cruising-for-parking time is Cl/P. The second equilibrium condition, the *cruising-for-parking equilibrium condition*, is that the rate at which cars enter cruising for parking, which equals the rate at which they exit the in-transit pool, equals the rate at which cars exit cruising for parking, which equals the parking turnover rate:

$$\frac{T}{mt(T,C,P)} = \frac{P}{l} \quad . \tag{15}$$

The steady-state condition and the cruising-for-parking equilibrium condition provide two non-linear equations in two unknowns, T and C. Their analysis is complex. Arnott and Inci (2006) derive the conditions under which the two curves intersect in T-C space, and for which therefore there exists a saturated equilibrium. Furthermore, they prove that, if a saturated equilibrium exists, it is unique. Here, we derive the properties we need for our diagrammatic analysis through heuristic argument. We ask: What are the minimum and maximum full prices consistent with saturated parking, and therefore with (15) being satisfied?[12] The minimum full price involves congested travel with no cruising for parking, as intuition suggests. For a given level of P, (15) has two roots for T, and the smaller (for which

[12] This entails minimizing and maximizing, respectively, (14) subject to (15).

travel is congested) corresponds to the minimum full price. Less obviously, the maximum full price corresponds to the larger root (for which travel is hypercongested). A higher price corresponds to a worse traffic jam, which is inconsistent with the level of throughput P/l.

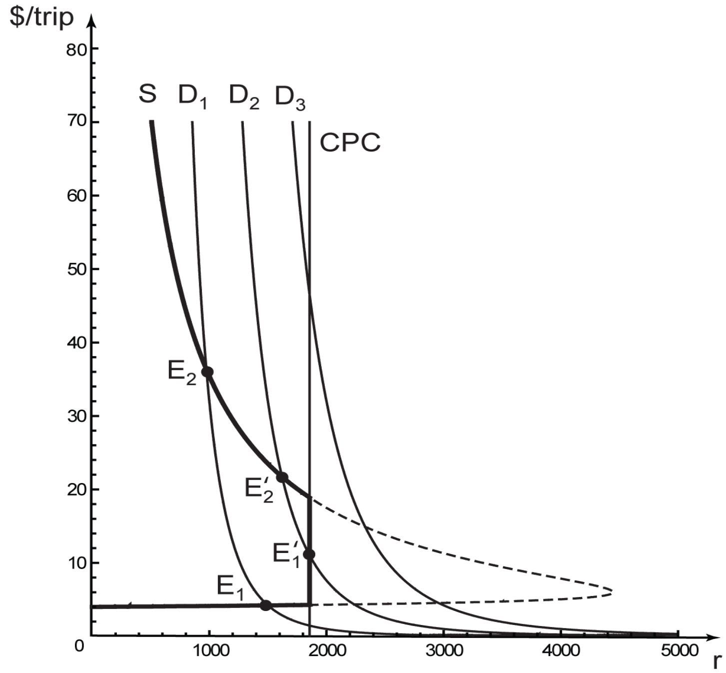

Figure 4: Equilibria in the short run with only curbside parking

Turn to Figure 4. First, plot the unsaturated user cost curve for the level of P corresponding to the curbside parking capacity constraint. Second, shifting this up by fl gives the corresponding *unsaturated full price curve*. Third, draw in the curbside parking capacity constraint, which constrains throughput to be no greater than P/l. The portions of the unsaturated full price curve to the left of the curbside parking capacity constraint – where it does not bind – form part of the supply curve. The portion to the right of the curbside parking capacity constraint is drawn as a dashed line since it is not relevant to the analysis.

The minimum full price at which a saturated equilibrium can exist is the lower point of intersection of the unsaturated full price curve and the curbside parking capacity constraint, and the maximum full price is the upper point of intersection. Thus, a saturated equilibrium must lie on the portion of the curbside parking capacity constraint above the upward-sloping portion of the unsaturated full price curve and below the backward-bending portion. This is the third piece of the supply curve.

We may alternatively define the *capacity-constrained user cost curve*, in the same way that we defined the capacity-constrained short-run marginal social cost curve in the previous section, except that it contains only that portion of the curbside parking capacity constraint between the lower and upper portions of the unsaturated user cost curve. We can then derive the supply curve as the capacity-constrained user cost curve shifted up by the amount of the parking fee.

Figure 4 shows three demand curves, each corresponding to a different demand intensity. While not obvious from the diagram, for all three demand curves, gridlock is an equilibrium. The steady-state condition is satisfied since the entry flow and the exit flow are both zero. Since throughput is zero, the curbside parking capacity constraint does not bind, so that the gridlock equilibrium is unsaturated. In section 5, we shall argue that, throughout the paper, the gridlock equilibria are stable.

With low demand intensity (in the figure, D_1, with demand intensity equal to 2000), there are three equilibria: E_1, which is unsaturated, congested, and stable; E_2, which is unsaturated, hypercongested and unstable; and the gridlock equilibrium. With medium demand intensity (in the figure, D_2, with demand intensity equal to 3000), there are again three equilibria: E_1', which is saturated and stable, and may be either congested or hypercongested); E_2', which is unsaturated, hypercongested, and unstable; and the gridlock equilibrium. With high demand intensity (in the figure, D_3, with demand intensity equal to 4000), the equilibria corresponding to E_1 and E_2 disappear, with only the gridlock equilibrium remaining. Later, we shall display the various equilibria, as a function of demand intensity, in a bifurcation diagram.

Let us consider the equilibrium E_1' in more detail. In this saturated equilibrium, the stocks of cars cruising for parking and in-transit adjust to clear the market,[13] such that the full price is at the point of intersection of the demand curve and the curbside parking capacity constraint. The equilibrium values of T and C are 444.28 and 394.02, so that travel time is 0.1197 hrs per ml, which implies a velocity of 8.36 mph and hence hypercongested travel. The full trip price equals \$11.03, of which \$4.78 is in-transit travel time cost, \$4.24 is expected cruising-for-parking time cost, and \$2.00 is the parking fee.

We now consider the deadweight loss associated with inefficient pricing in the equilibrium E_1'. The deadweight loss equals social surplus at the optimum minus social surplus in the equilibrium. In the example, the social optimum too is at E_1'. Since there is no cruising for parking in the social optimum, the socially optimal level of T is the smaller root solving $T/(mt(T,0,P)) = P/l$, which is 210.74, so that travel time is 0.0567 hrs per ml, which corresponds to a velocity of 17.64 mph and an in-transit travel cost of \$2.27. Thus, the deadweight loss due to inefficient pricing is $\$12,528$ per ml^2-hr, corresponding to \$6.75 per driver. The social optimum could be decentralized by charging each driver \$8.75 for curbside parking for the two hours.

Since in the model both trip length and visit duration are fixed, it makes no difference whether this charge takes the form of a congestion toll or a parking fee. But generally it does. Determining the second-best optimal toll when the meter rate is set inefficiently or the second-best optimal meter rate when the congestion toll is set inefficiently requires enriching the model, making either trip length or visit duration a choice variable. While the equilibrium full price can be decomposed into three shadow prices, the decomposition is not particularly insightful.[14]

Figure 5 portrays the same space as Figure 4, but has a different focus. It plots the

[13] Without cruising for parking, the throughput demanded would exceed the throughput supplied (curbside parking capacity). Cruising for parking serves as a dissipative rationing mechanism. Since individuals are identical, it is consistent to assume that the aggregate demand is the sum of their identical demand curves. Thus, there is no issue of whether the rationed commodity goes to those who value it the most.

[14] Even though the second-best optimum is degenerate, its shadow prices are nonetheless informative. We may write the second-best optimum problem as

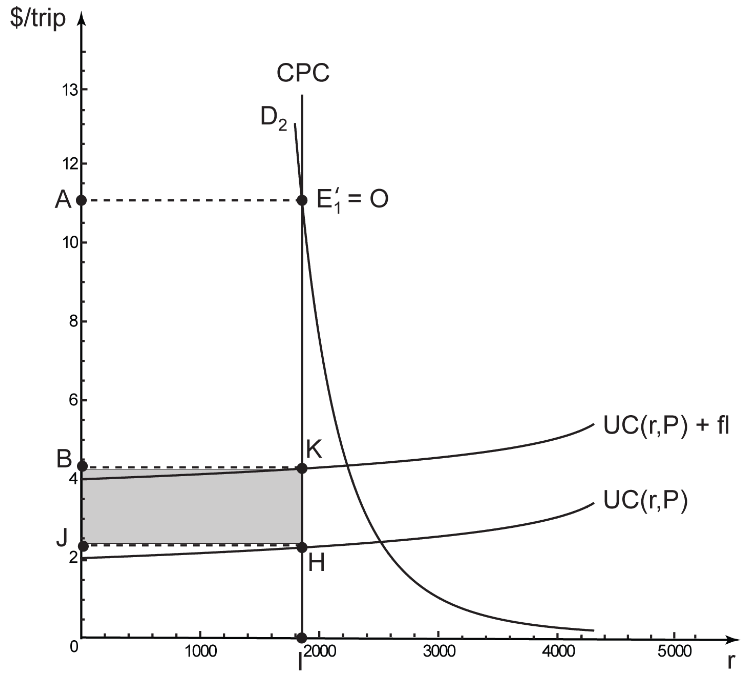

Figure 5: Equilibrium and social optimum in the short run with only curbside parking

unsaturated user cost curve, $UC(r;P)$ and the demand function D_2. The social surplus at the optimum equals social benefit, the area below the demand curve and to left of the curbside parking capacity constraint, minus aggregate user cost, $0JHI$. It, therefore, equals the area $JAE_1'H$ plus the area above AE_1' and below the demand curve. The former area is the shadow curbside parking rent, which would accrue to the government if it were to decentralize the social optimum by setting the parking fee at its first-best level, and the second term is consumer surplus. The social surplus at the equilibrium with underpriced

$$\begin{aligned} &\max_{r,C,T} X(r) - \rho(T+C) && \\ s.t. \quad & r = T/(mt(T,C,P) && \lambda \\ & r \leq P/l && \mu \\ & r = D(\rho mt(T,C,P) + \rho Cl/P + fl). && \phi \end{aligned}$$

This is the same as the first-best optimization problem except for the addition of the third constraint. The first-order conditions are $r : X'(r) - \lambda - \mu - \rho = 0$; $T : -\rho + \lambda(1/(mt) - Tt_T/(mt^2)) + \phi D' \rho mt_T) = 0$; $C : -\rho - \lambda Tt_C/(mt^2) + \phi D'(\rho mt_C + \rho l/P) = 0$. The first-order condition with respect to T indicates that the marginal social cost can be decomposed into the sum of three shadow prices. But, the first-order conditions with respect to T and C suggest that the decomposition is not particularly insightful. If the parking fee is a choice variable, ϕ is zero, and, as noted in footnote 11, the optimal parking fee can be decomposed into a congestion externality cost and a parking scarcity rent.

curbside parking (the degenerate second best optimum) equals consumer surplus plus parking fee revenue, $JBKH$. Thus, the deadweight loss due to the underpricing of curbside parking is $BAE_1'K$. Raising the meter rate does not alter consumer surplus but converts deadweight loss dollar for dollar into tax revenue. Thus, the extra revenue is raised not just with no excess burden but also with no burden at all. An obvious question is therefore why local governments choose to forgo such an efficient source of revenue.

3.2 The long run with only curbside parking

We now turn to the determination of first-best and second-best curbside parking capacity.

3.2.1 First-best curbside parking capacity in the long run with only curbside parking

When curbside parking is saturated, increasing curbside parking capacity by a small amount has two effects, one positive and one negative. The positive effect is to raise throughput and hence the social benefit from travel, the area under the demand curve and to the left of the curbside parking capacity constraint. The negative effect is to reduce the amount of road space available to traffic flow, which causes the unsaturated user cost curve to rise. These effects are displayed in Figure 6. Increasing curbside parking capacity from P to P' causes the curbside parking capacity constraint to shift to the right, which generates the surplus to marginal drivers of $HALJ$, equal to the benefit they receive minus the user cost they incur. But it also causes the unsaturated user cost curve to shift up, increasing the costs of inframarginal drivers by $KBHG$, and reducing their surplus by the same amount. With first-best optimal capacity the two areas are equal.

It will be instructive to determine first-best optimal capacity through an alternative geometric construct. Figure 7 plots the short-run marginal social cost of throughput with

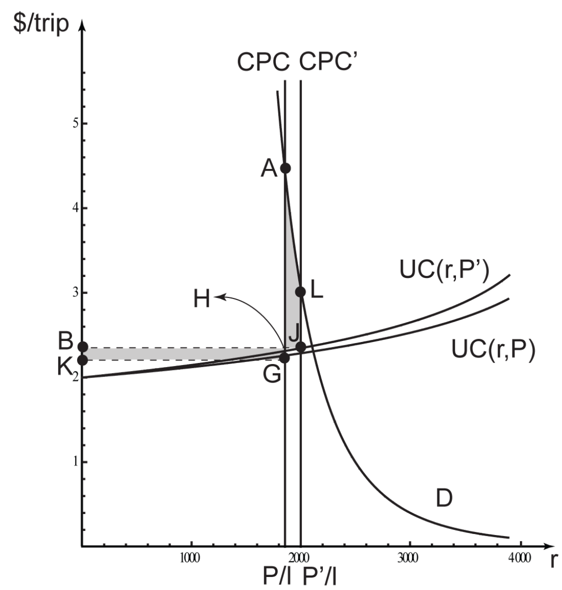

Figure 6: The first-best optimal curbside parking capacity in the long run with only curbside parking

parking capacity of 3712, which constrains throughput to be less that 1856. M indicates the point of intersection of $SRMSC(r; P)$ and the curbside parking capacity constraint, $r = P/l$. It, therefore, corresponds to the point $SRMSC(P/l; P)$. It is also at the kink point of the capacity-constrained short-run marginal social cost curve with a curbside parking capacity of P. If parking capacity is reduced slightly, the throughput of 1856 cannot be achieved. If parking capacity is increased slightly, the throughput of 1856 can be achieved but at a higher short-run marginal cost. Thus, M gives the minimum marginal social cost associated with the throughput of 1856.

There is a point corresponding to M for every level of throughput, up to some maximum. Joining these points gives the long-run marginal social cost curve, labeled $LRMSC$. This curve is defined up to the throughput, $r_{\max}$, at which the curbside parking capacity constraint is tangent to the corresponding unsaturated user cost curve. $r_{\max}$ is the maximum level of throughput that can be accommodated on downtown streets, which is attained when $P = lr_{\max}$. Note that $LRMSC$ is the locus of the kink points of the capacity-constrained short-run marginal social cost curves, and also the lower envelope of the capacity-constrained

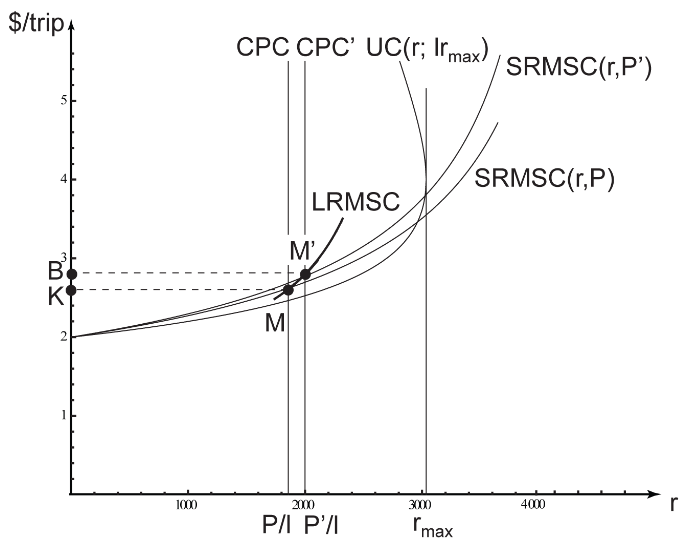

Figure 7: The short-run marginal social cost of throughput with only curbside parking

marginal social cost curves.

Long-run marginal social cost equals $\rho(dT/dr)$, where dT/dr is the change in T induced by a change in r such that: i) the steady-state condition is satisfied; and ii) parking capacity is increased along with r such that the curbside parking capacity constraint just binds. The expression for dT/dr is obtained by totally differentiating $r = T/(mt(T, 0, rl)$. Thus,

$$LRMSC(r) = \rho\frac{dT}{dr} = \rho\frac{\partial T}{\partial r} + \rho\frac{\partial T}{\partial P}\frac{dP}{dr} = \rho\frac{\partial T}{\partial r} + \rho l\frac{\partial T}{\partial P} \quad . \tag{16}$$

Turn now to Figure 8. The long-run social optimum occurs where the demand curve intersects the $LRMSC$ curve,[15] with throughput r^*, at point O ($r^* = 2007.65$ and $LRMSC(r^*) = 2.9941$ with $D_0 = 2500$). By construction, the parking capacity constraint "just binds" so that $P^* = lr^*$; parking is saturated but has no scarcity rent. $UC(r; lr^*)$ is the unsaturated user cost associated with throughput r^* and curbside parking capacity lr^*, and $LRMSC(r)$ the corresponding long-run marginal social cost.

Decentralization of the social optimum entails charging a parking fee equal to the differ-

[15]If the demand curve lies everywhere below the long-run marginal social cost curve, optimal throughput is zero.

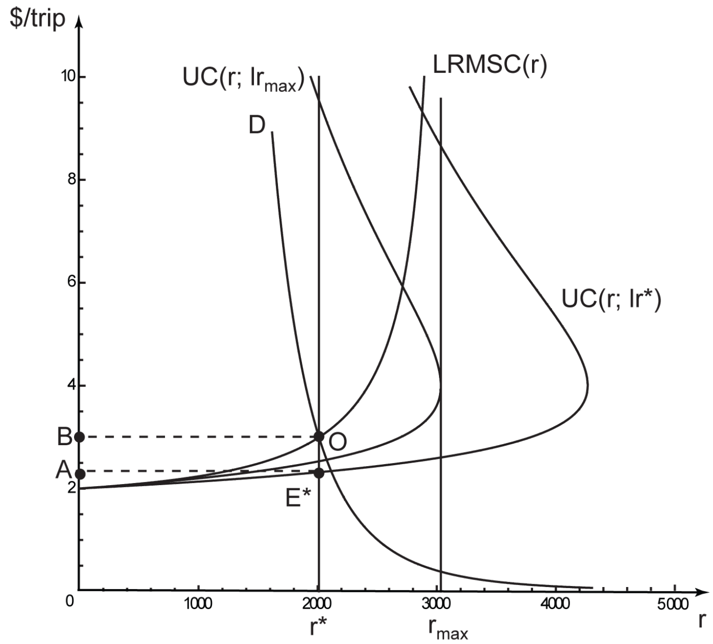

Figure 8: First-best optimal curbside parking capacity in the long run with only curbside parking

ence between long-run marginal social cost and user cost.[16] Thus, the first-best parking fee solves

$$f^*(r)l = LRMSC(r) - UC(r; rl) = \left[\rho \frac{\partial T}{\partial r} - UC(r; rl)\right] + \left\{\rho l \frac{\partial T}{\partial P}\right\} \quad . \tag{17}$$

It was established earlier that the term in square brackets equals the congestion externality cost. We label the term in curly brackets the *parking externality cost*. When throughput is increased by one unit, the number of curbside parking spaces is increased by l units. This reduces the road space available for travel and hence increases congestion. The parking externality cost is the increase in in-transit travel time costs associated with this increased congestion.

[16] Charging this fee supports the socially optimal allocation. Whether its application results in the social optimum being attained depends on the initial allocation and the adjustment dynamics. Charging this fee would not, for example, unlock a gridlock equilibrium.

3.2.2 Second-best optimal curbside parking capacity in the long run with only curbside parking

We now turn to the determination of optimal second-best capacity, where the distortion is underpriced curbside parking. Start at a saturated equilibrium. Increasing curbside parking capacity a small amount unambiguously increases social surplus. With underpriced curbside parking, social surplus equals consumer surplus plus parking fee revenue. Increasing curbside parking causes the equilibrium to move down the demand curve, which causes both consumer surplus and parking fee revenue to increase. Now, start at an unsaturated, stable equilibrium. Reducing parking capacity a small amount unambiguously increases social surplus. There are two cases to consider, that where the initial equilibrium lies on the upward-sloping portion of the supply curve, and that where it lies on the backward-bending portion.

Consider first the case where the initial equilibrium lies on the upward-sloping portion of the supply curve. Reducing parking capacity lowers this portion of the supply curve, which increases social surplus. Consider next the case where the initial equilibrium lies on the backward-bending portion of the supply curve. Stability of the equilibrium requires that the demand curve be flatter than the supply curve. Reducing parking capacity raises this portion of the supply curve, which causes the equilibrium to move down the demand curve, which again increases social surplus. Thus, a second-best optimum entails the capacity constraint just binding.

This line of reasoning points to a method for determining second-best optimal throughput and capacity. Plot the $UC(r;rl)$ curve, along which the capacity constraint just binds. Shifting the curve up by the amount of the parking fee generates the long-run supply curve, which we label $LRS(r)$ in Figure 9. The second-best optimal throughput, r^{**}, corresponds to that point of intersection[17] of the demand curve and the long-run supply curve with the highest level of throughput[18] (and hence the highest level of social surplus), which occurs at

[17] If the demand curve lies everywhere below the long-run supply curve, second-best capacity is zero.

[18] The second-best optimum is supported by the level of parking capacity $P^{**} = lr^{**}$. Provision of P^{**}

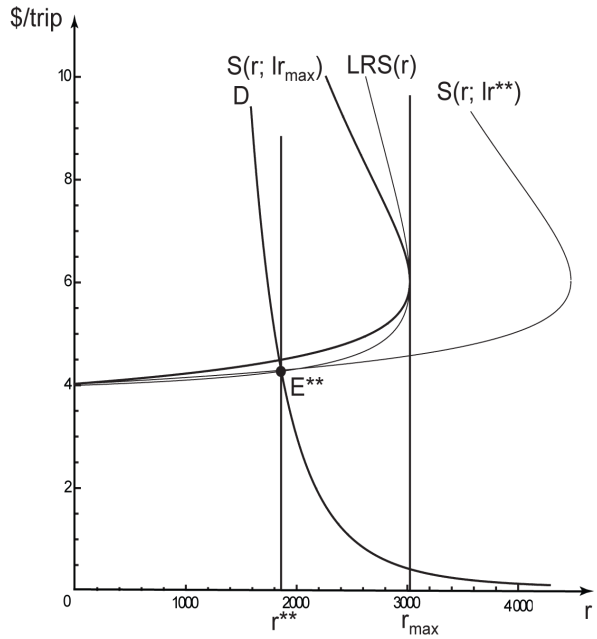

Figure 9: Second-best optimal curbside parking capacity in the long run with only curbside parking

$r^{**} = 1869.83$ and $LRS(r^{*}) = 4.2726$ when $D_0 = 2500$.

The relationship between first- and second-best optimal capacities is shown in Figure 10, which plots the long-run marginal social cost curve, the long-run supply curve, and the demand curve. With both first- and second-best optimal capacities the curbside parking capacity constraint just binds. Thus, parking capacity equals throughput times visit length, so that the analysis can be conducted in terms of throughput. The first-best optimum is at the point of intersection of the long-run marginal social cost curve and the demand curve. The second-best optimum lies at that point of intersection of the demand curve and the long-run supply curve with the highest level of throughput.

Consider first the case where demand is "low", so that the demand curve intersects the long-run supply curve on its upward-sloping portion. There are two sub-cases. In the first, which corresponds to D_1, E_1, and O_1 in the figure, at a price of \$1/hr curbside parking is

curbside parking spaces does not however guarantee that the second-best optimum will be achieved. For example, if the economy starts in the gridlock equilibrium, adjusting the amount of parking provided will not unlock the gridlock.

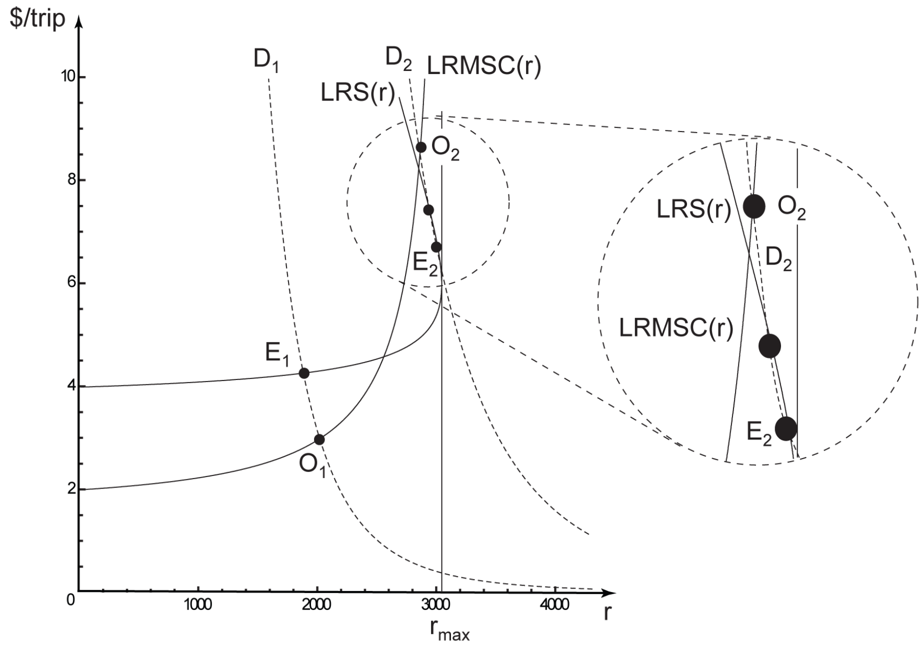

Figure 10: The relationship between first- and second-best optimal parking capacity in the long run with only curbside parking

over priced, so that the long-run supply curve lies above $LRMSC$. The first-best optimum is given by O_1, for which the amount of parking is P^*. With this amount of parking capacity, equilibrium would entail excess curbside parking capacity. The second-best optimum occurs at E_1. Thus, $P^{**} < P^*$ and $r^{**} < r^*$. In the second sub-case, which is not displayed in the Figure, demand is "moderate". At the level of throughput at which the demand curve intersects $LRMSC$, the long-run supply curve lies below $LRMSC$, so that at P^* parking is underpriced and there is cruising for parking. The second best therefore entails a higher level of parking than P^*; $P^{**} > P^*$ and $r^{**} > r^*$.

Consider next the case where demand is "high", so that the demand curve intersects the long-run supply curve but not on the upward-sloping portion of the LRS curve. This case corresponds to D_2 in the figure. In addition to the gridlock equilibrium, which entails zero parking capacity, there are two equilibria on the backward-bending portion of the long-run supply curve. The upper equilibrium (shown as an unlabeled dot) is unstable. The lower equilibrium marked as E_2 is the second-best optimum. At this equilibrium, the long-run supply curve may lie above or below the $LRMSC$ curve. If it lies below the $LRMSC$ curve (the situation shown in the diagram) at the price of \$1/hr, curbside parking is underpriced and second-best optimal capacity exceeds first-best optimal capacity. If it lies above the $LRMSC$ curve, second-best capacity falls short of first-best optimal capacity. Note that over the range of demand intensities for which this case applies, as demand intensity increases, second-best parking capacity falls.

At “very high” levels of demand intensity, the demand curve intersects the long-run supply curve only with gridlock, and the second-best level of curbside parking capacity is zero.

4 Dowtown Traffic Congestion with Both Curbside and Garage Parking

In the downtowns of small towns, the suburbs of small cities, and the residential neighborhoods of medium-sized cities, there is typically enough parking space curbside to accommodate demand without severely impeding traffic flow. But in most locations where traffic congestion is a serious problem, curbside parking needs to be supplemented by off-street parking, whether in a parking lot or garage.

We shall treat off-street parking — which we shall refer to generically as garage parking – in the simplest possible way, by assuming that it is provided continuously over space by the private sector at a constant cost of $c = \$2.5/\text{hr}$. In fact, in the downtowns of major metropolitan areas, because of economies of scale in garage construction,[19] there is typically

[19] There are other reasons. Because of the weight of cars, garage parking above the ground floor of a general purpose building typically does not conform to code, and garage parking on the ground floor is typically uneconomical because that space has its highest use in retail. Also, because of the cost of vertically transporting cars, high-rise parking structures are not observed, which makes them economical on back streets. Finally, parking structures are viewed by many planners as unsightly, and so are often zoned off major streets. Parking lots, meanwhile, are typically irregularly spaced because they are a temporary land use, while a developer is waiting for a favorable time to build on a vacant lot.

an irregular grid of parking garages, some public, some private, which engage in spatial competition with one another. Arnott and Rowse (2009) model this spatial competition, taking into account the technology of garage construction. But here we provide a simpler treatment[20,21] in order to focus on the interaction between curbside and garage parking. As we shall see, analysis of even this simplest model is complex.

4.1 First-best optimal parking capacity with both curbside and garage parking

In the first best, efficient supply can be analyzed independently of the level of demand. In the first stage, for each level of throughput, the planner decides on the combination of curbside and garage parking that minimizes total cost. And in the second, she decides on the surplus-maximizing level of throughput. The third step in the analysis is to derive alternative mechanisms whereby the first-best optimum can be decentralized. The full first-best problem is to maximize social surplus subject to the steady-state condition. Social surplus equals social benefit minus social cost. Social benefit equals the area under the demand curve up to the chosen level of throughput and social cost equals total in-transit travel time cost and total garage parking costs.[22] The efficient supply problem, which we

[20] We could alternatively assume that there is an upward-sloping supply schedule, as was done in Arnott, Rave, and Schöb (2005), chapter 2. If the schedule starts at the origin, then some garage parking is always provided, and the dichotomy between low-demand situations where garage parking is not provided and higher-demand situations where it is disappears.

[21] Our modeling also ignores minimum parking requirements, whereby property owners are required to provide a minimum amount of off-street parking depending on the land use and floor area. Minimum parking requirements were originally introduced in the downtown areas of US cities in the 1950's, in response to a widespread perception that the market did not provide enough private parking downtown, and their use rapidly spread to suburban areas (Jakle and Sculle, 2004). The literature provides no economic analysis of the validity of this perception. Curbside parking was at the time provided free, and observers may have incorrectly attributed the excess demand this generated (in the form of cruising for parking) to a market failure on the supply side. Currently the mainstream view is that the minimum parking requirements introduced at that time were excessive, encouraging travel by car rather than mass transit, and some cities, notably San Francisco and Boston, have replaced their minimum parking requirements with maximum parking constraints on new developments.

[22] The planning problem is $\max_{P,r,T} X(r) - \rho T - (rl - P)c$ subject to $r = T/(mt(T, 0, P))$ and $0 \leq P \leq rl$. Each of the garage parking spaces is provided at a cost of c. The first constraint is the steady-state condition. The second one imposes the constraints that neither the number of garage spaces nor the number of curbside

consider first, is to minimize social cost, holding throughput fixed.

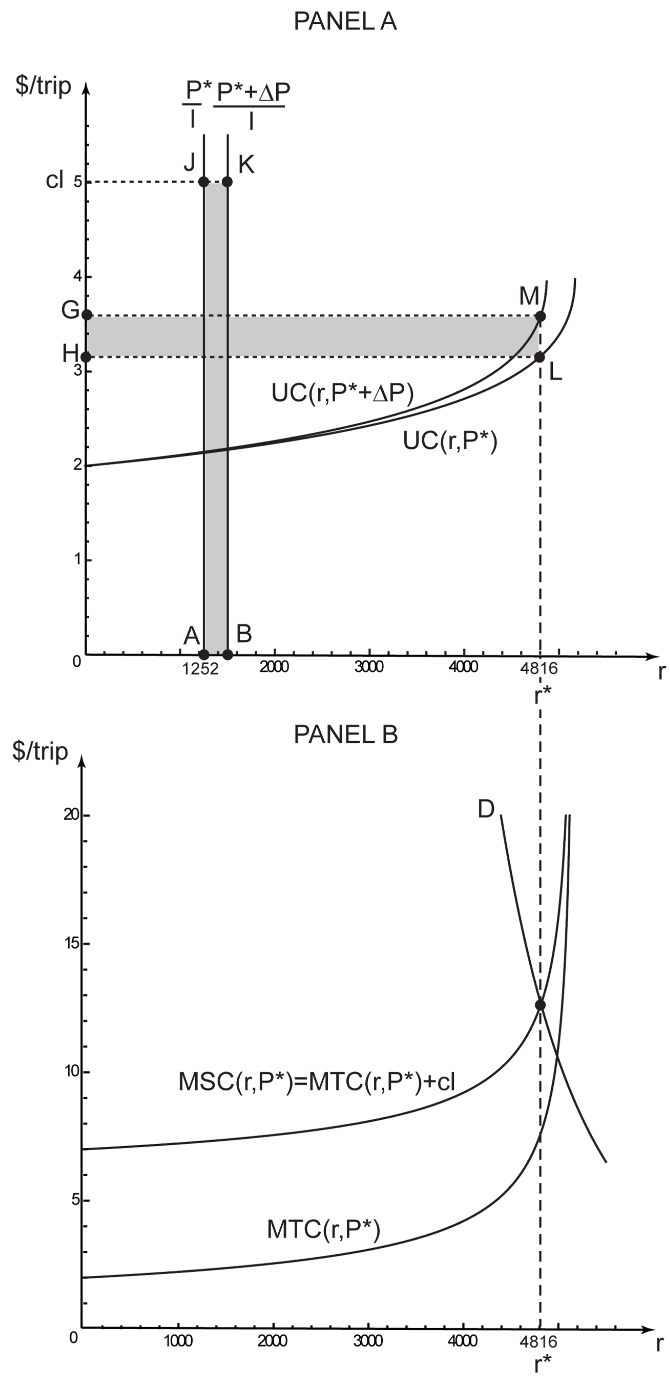

Figure 11: User cost for a given level of curbside parking capacity (Panel A) and the full first-best optimum (Panel B) with both curbside and garage parking

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parking spaces can be negative. Holding r fixed, the planning problem is $\min_{P,T} \rho T + (rl - P)c$ subject to the same pair of constraints.

Panel A of Figure 11 plots user cost for the example's first-best level of curbside parking capacity (which entails both curbside and garage parking). Total resource costs equal total user costs plus garage parking costs. Consider increasing curbside parking capacity a small amount, holding throughput fixed. Doing so has two effects. The first is to decrease effective jam density, causing the user cost curve to shift up, from $UC(r, P^*)$ to $UC(r, P^* + \Delta P)$ in the figure, and total user costs to increase by the area HGML. The second is to decrease the number of garage parking spaces that need to be provided, which is given by the area $AJKB$, which equals $(\Delta P/l)cl = c\Delta P$.

For the given number of travelers, with an interior (both curbside and garage parking are used) optimum, first-best optimal capacity is such that a unit increase in curbside parking capacity causes total user cost to increase to increase by cl, the saving in garage parking costs. Corner optima are possible too. At low levels of throughput, when everyone parks curbside, the decrease in total user cost from reducing the number of curbside parking spaces by one unit is less than c. At high levels of throughput, when everyone parks in a garage, the increase in total user cost from reducing the number of curbside parking spaces is greater than c.

We now determine optimal throughput. At an interior first-best optimum, the change in social surplus from an extra traveler is the same whether he is accommodated by increasing the amount of curbside parking or the amount of garage parking. Assume the latter. The marginal social cost of the added traveler, MSC, is then the marginal travel cost, $MTC(r; P)$, plus the garage cost, cl. And the optimal number of travelers is such that the marginal social cost of an added traveler equals the marginal social benefit. Panel B of Figure 11 displays the full first-best optimum. With the base case parameters, first-best optimal curbside parking capacity is $P^*/l = 1252$, and first-best optimal throughput is $r^* = 4816$. When throughput is sufficiently low that providing only curbside parking is cost minimizing, marginal social cost equals $MTC(r; rl)$. When throughput is sufficiently high that providing only garage parking is cost minimizing, marginal social cost is $MTC(r; 0) + cl$.

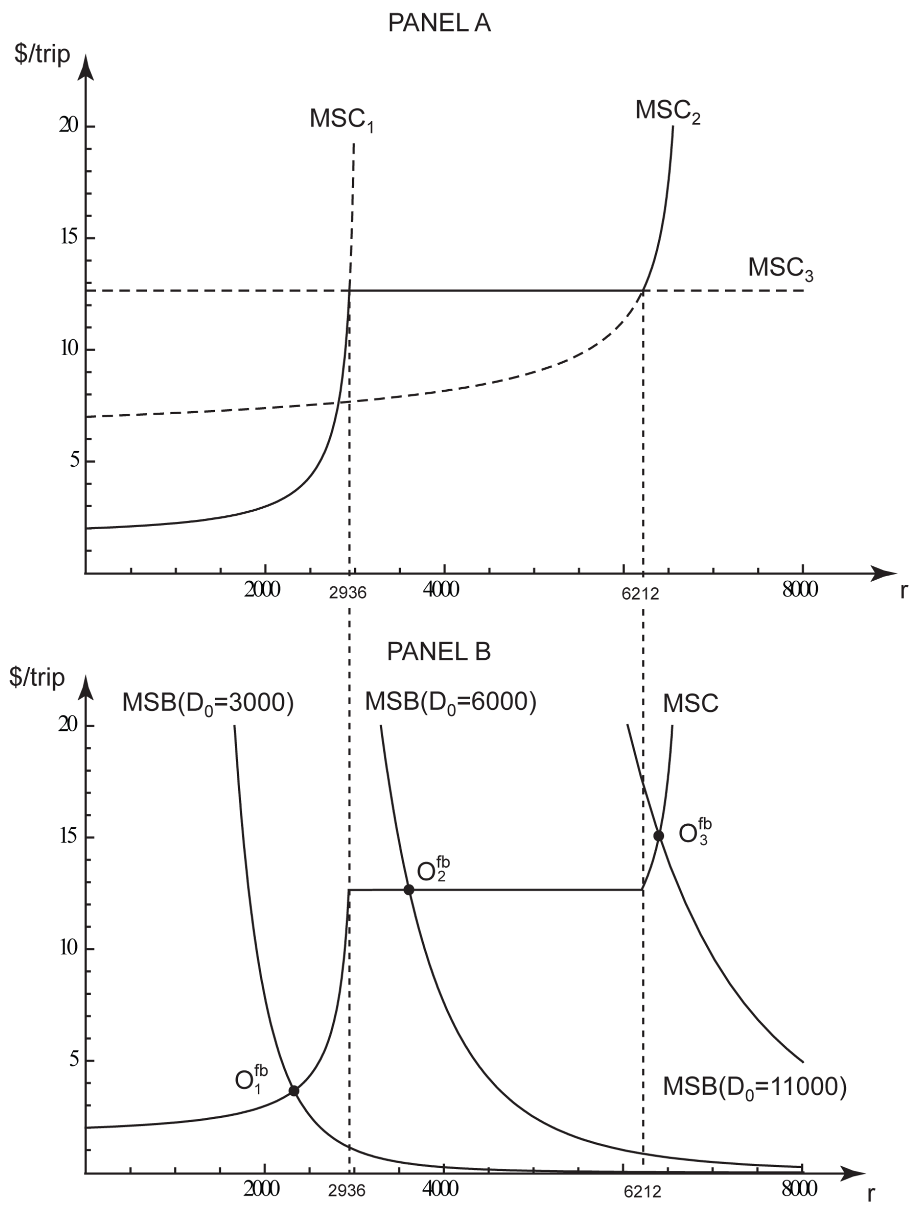


Figure 12: Regimes in the first-best optimum with both curbside and garage parking

Figure 12, Panel A displays these results diagrammatically, using the base-case supply-side parameters. Three marginal social cost curves are drawn: MSC_1 is the marginal social cost for régime 1, where only curbside parking is provided; MSC_2 is the marginal social cost for régime 2, where both curbside and garage parking are provided, with the amount of each chosen to minimize total cost; and MSC_3 is the marginal social cost for régime 3, where only garage parking is provided. MSC_2 is defined only for those levels of throughput at which it is efficient to provide both curbside and garage parking. The marginal social cost curve is then the lower envelope of MSC_1, MSC_2, and MSC_3.

For low levels of throughput, below $r' = 2936$, providing only curbside parking is efficient; for high levels of throughput, above $r'' = 6213$, providing only garage parking is efficient; and in between it is efficient to provide both curbside and garage parking. Resource costs are $RC = \rho T + c(rl - P)$, so that the marginal social cost of throughput is $\rho dT/dr + c(l - dP/dr)$. In the case of accommodating the marginal throughput via curbside parking, $dP/dr = l$; in the case via curbside parking, $dP/dr = 0$; and in the case where both curbside and garage parking is provided, dP/dr depends on the details of the congestion technology (with the congestion technology we assume, dP/dr is a negative constant).

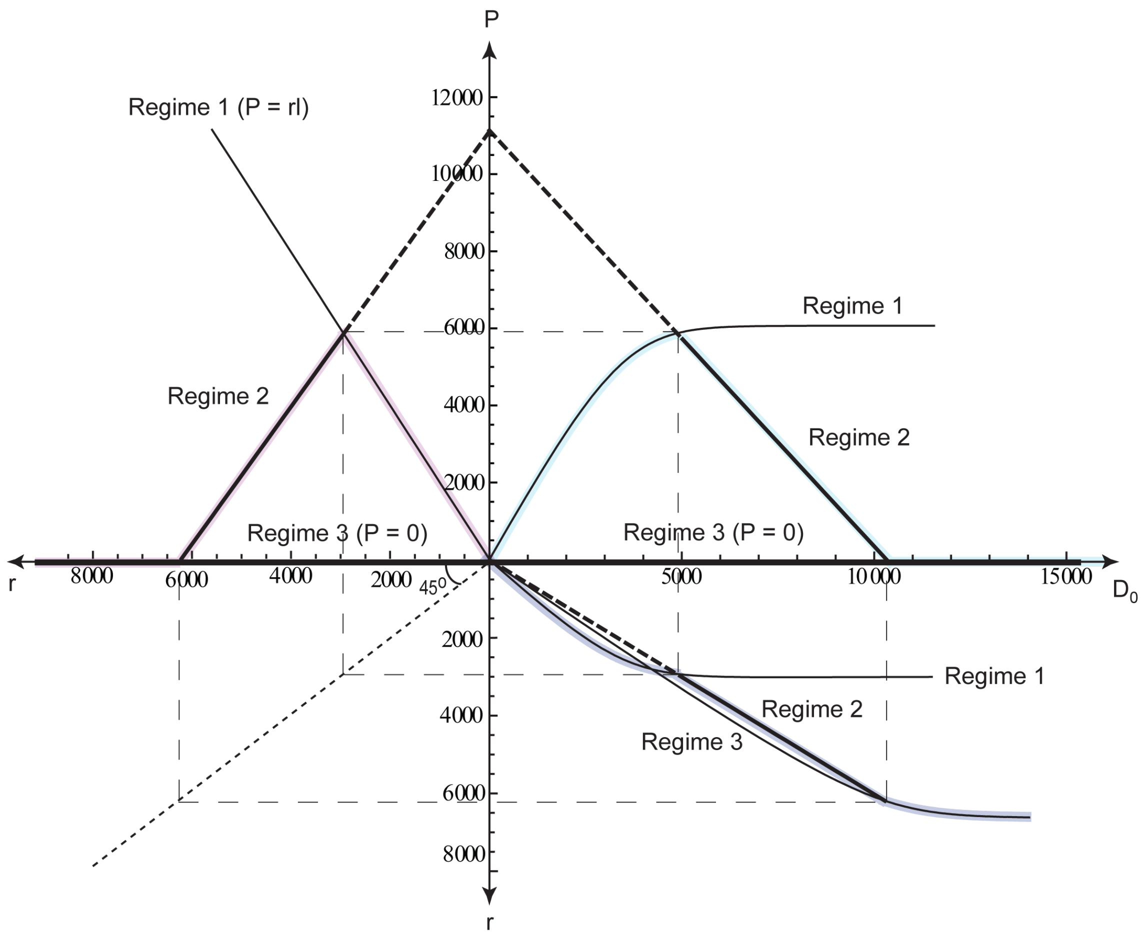
It is noteworthy that MSC_2 is constant over the interval of throughput where it is defined. The result is due to the form of the congestion function assumed.[23] When throughput is increased by one unit, the number of curbside parking spaces is reduced by an amount such that travel time remains fixed, which implies from the steady-state condition that T increases in the same proportion as r. Furthermore, the reduction in the number of curbside parking spaces associated with a unit increase in throughput, remains constant, Thus, the marginal social cost equals the marginal travel time cost, which remains constant, plus the cost due to the increase in the number of garage spaces needed, which is also constant.[24]

Figure 12, Panel B displays the social optimum at three different levels of demand in-

[23] In this region, the minimization problem reduces to $\min_{T,P} \rho T + c(rl - P)$ subject to $r = T/(mt(T, 0, P))$. The first-order condition for T is $\rho - \lambda r(1/T - t_T/t) = 0$ and for P is $-c + \lambda r t_P/t = 0$ where λ is the Lagrange multiplier on the constraint. Recall that $t = t_0/(1 - T/V_j) = t_0 V_j/(V_j - T)$ where $V_j = \Omega(1 - P/P_{\max})$. The steady-state condition and the first-order conditions remain satisfied, if, as r increases, T and V_j increase in the same proportion. t remains the same; the steady-state condition remains satisfied; and $t/t_T = (V_j - T)$ and $t/t_P = P_{\max} V_j (V_j - T)/(T\Omega)$ increase in the same proportion as T and V_j, so that the first-order conditions remain satisfied. Now, V_j increasing in the same proportion as r is equivalent to the condition that $dV_j/V_j = dr/r$, so that $dV_j/dr = V_j/r$. $dV_j/dr = (dV_j/dP)(dP/dr) = -(\Omega/P_{\max})dP/dr$. Hence, $dP/dr = -V_j P_{\max}/(r\Omega)$, which is a constant.

[24] For the example, when $P = 0$, $r'' = 6213$, and $V_j = \Omega$. Thus, $dP/dr = -V_j P_{\max}/(r''\Omega) = -P_{\max}/r'' = 1.79$, which implies that in régime 2 a unit increase in throughput is accompanied by a reduction in the number of curbside parking spaces of 1.79, and hence an increase in the number of garage spaces of 3.79. When the stock of curbside parking spaces is so adjusted, the marginal social cost of the marginal traveler equals his travel cost (\$3.17) plus the cost of the extra garage spaces that need to be supplied (\$9.48).

tensity, a low demand intensity social optimum where only curbside parking is optimal, O_1^{fb}, a medium demand intensity social optimum where a mix of curbside and garage parking is optimal, O_2^{fb}, and a high demand intensity social optimum where only garage parking is optimal, O_3^{fb}. Figure 13 displays how the social optimum changes as a function of demand intensity in D_0-P, P-r, and D_0-r space.



Note: Highlighted paths denote optimal regimes.

Figure 13: Regimes in the first-best optimum with both curbside and garage parking

We now turn to decentralization of the social optimum. We assume that garage parking is provided by the private sector at cost and that there is no congestion tolling in place. Decentralization of the optimum then requires that the curbside parking fee be set equal to the difference between the marginal social cost of a trip and the user cost (which equals the conventional congestion externality cost, as well as the curbside parking externality cost), and that the garage parking fee be set equal to the difference between the marginal social cost of a trip and the user cost, inclusive of the garage parking fee, which equals the congestion externality cost.

4.2 The second best with both curbside and garage parking

Analysis of second-best pricing and capacity when there is both curbside and garage parking is complicated by cruising for parking.

4.2.1 The short run with both curbside and garage parking

We consider the short-run situation where no congestion tolling is applied, where curbside parking is priced below the unit cost of garage parking (which is generally the case in US cities though apparently not in all of Western Europe), and where the government does not subsidize garage parking.[25] Again, since the government has no policy instruments at its disposal, the optimization problem is degenerate, and the optimum coincides with the equilibrium.

When both curbside and garage parking are available, drivers will choose whichever is cheaper. Thus, the stock of cars cruising for parking adjusts so that the full prices of curbside and garage parking are equalized: $fl + \rho Cl/P = cl$. We term this the *full (parking) price equalization condition*. Rearranging, we have that

$$C = \hat{C} \equiv (c - f)\frac{P}{\rho} \quad ; \tag{18}$$

thus, when both curbside and garage parking are provided in equilibrium, the stock of cars

[25] Subsidizing garage parking would reduce cruising for parking when curbside parking is provided but would result in excessive travel. The second best would entail lowering the garage fee to the point where the marginal decrease in cruising-for-parking costs were exactly offset by the marginal deadweight due to excessive travel.

cruising for parking increases in proportion to the differential between curbside and garage parking rates and to the amount of curbside parking. This yields the obvious but important result that cruising for parking can be eliminated by providing no curbside parking.

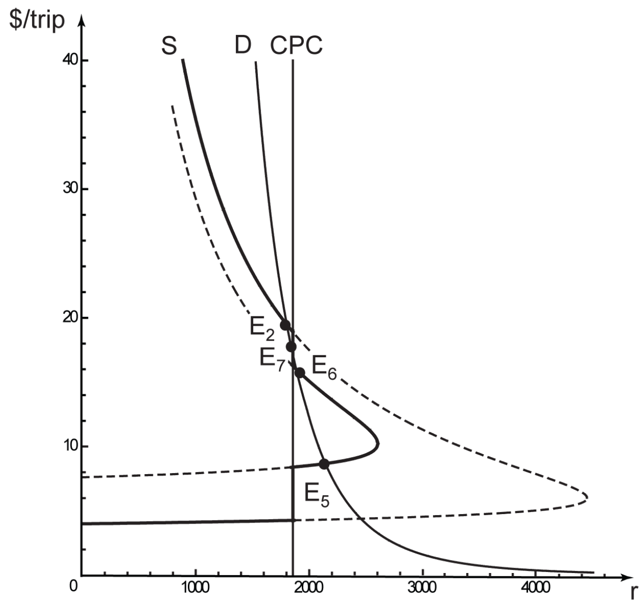


Figure 14: Equilibria with both curbside and garage parking

Figure 14 is like Figure 4 but adds garage parking. We start by defining two different short-run full price curves for the same level of curbside parking, P. The first corresponds to the situation where a driver pays the curbside parking fee but experiences no cruising for parking, so that the full price of travel is $F_1 = \rho mt(T, 0, P) + fl$. The second corresponds to the situation where curbside parking is saturated and garage parking occurs, so that there is cruising for parking with the stock of cars cruising for parking given by (18), so that the full price of travel is $F_2 = \rho mt(T, \hat{C}, P) + cl$ $(= \rho mt(T, \hat{C}, P) + \rho Cl/P + fl)$.

Since the stock of cars cruising for parking reduces effective jam density, the second full price line has a lower $r_{\max}$. And since garage parking is more expensive than curbside parking, the second full price line has a higher y-intercept. In the example, the second full price line lies inside and to the left of the first, but this need not be the case. Now add the curbside parking capacity constraint of $P/l = 1856$. To the left of the constraint, parking is unsaturated and the stock of cars cruising for parking is zero, so that the first full price line applies. To the right of the constraint, garage parking is provided, so that the (18) holds and the second full price line applies. There is also an intermediate régime where curbside parking is saturated but the stock of cars cruising for parking that clears the market without garage parking is not sufficiently large to make the provision of garage parking profitable.

The supply curve, shown as the bold line S in the Figure, contains five portions: (i) the portion of the second full price line to the right of the parking constraint; (ii) the two portions of the first full price line to the left of the parking constraint; and (iii) two segments of the curbside parking capacity constraint, each joining the first and second full price lines.[26] On (i), the scarcity rent on curbside parking is zero and there is no cruising for parking; on (ii) the scarcity rent on curbside parking is cl, and this rent is dissipated by cruising for parking, given by (18); and on (iii) the scarcity rent on curbside parking is between 0 and cl, and this rent is dissipated by cruising for parking, with the stock of cars cruising for parking varying between 0 and $\hat{C}$.

The demand curve in Figure 14 is drawn for $D_0 = 3300$. In this example, there are five equilibria, one of which, the gridlock equilibrium, cannot be displayed on the diagram. As expected, these equilibria alternate between stable and unstable. There are three stable equilibria. One is the gridlock equilibrium, which as before we label E_3; the second, E_7, is a hypercongested equilibrium with saturated curbside parking, cruising for parking, and no garage parking; and the third, E_5, is a congested equilibrium with saturated curbside parking, cruising for parking, and garage parking. How the set of equilibria changes as demand intensity increases will be considered later.

Figure 15, which is analogous to Figure 5, displays the deadweight losses associated with the two stable equilibria (other than the gridlock equilibrium). The short-run marginal social cost curve with the curbside parking capacity constraint $P/l = 1856$, $MSC(r; P)$ is given

[26] If the garage parking fee is increased, first the top segment and then the bottom segment disappear.

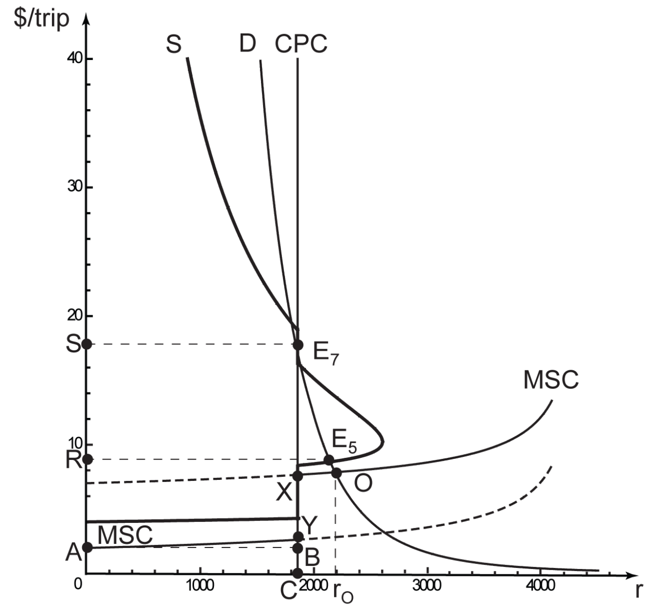


Figure 15: Deadweight losses associated with the two stable equilibria

by the locus $AYXO$, where O is the short-run social optimum. Notice the discontinuous increase in the locus at curbside parking capacity, which indicates that the level of curbside parking is suboptimal. The social surplus at the optimum is given by the area between the demand and marginal social cost curves up to the first-best optimal level of throughput, r_O.

The social surplus at the equilibrium E_5 equals consumer surplus plus the curbside parking fee revenue, $0ABC$. Thus, the deadweight loss associated with the equilibrium E_5 is given by the area $ARE_5OXY - 0ABC$, which is \$8488/hr. The deadweight loss at the equilibrium E_7 is determined analogously and is given by the area $ASE_7OXY - 0ABC$, which is \$26474/hr. The Figure reinforces a point made in discussing Figure 5, that the deadweight loss due to underpricing curbside parking can be substantial. It also illustrates another source of possible deadweight loss. If, when demand is first stationary, traffic is hypercongested, downtown traffic may end up at an inferior stable equilibrium. In the example, ending up at equilibrium E_7 rather than E_5 generates additional deadweight loss of RSE_7E_5, which in the example equals \$17986.

Figure 16 examines the social benefit from increasing the curbside meter rate such that

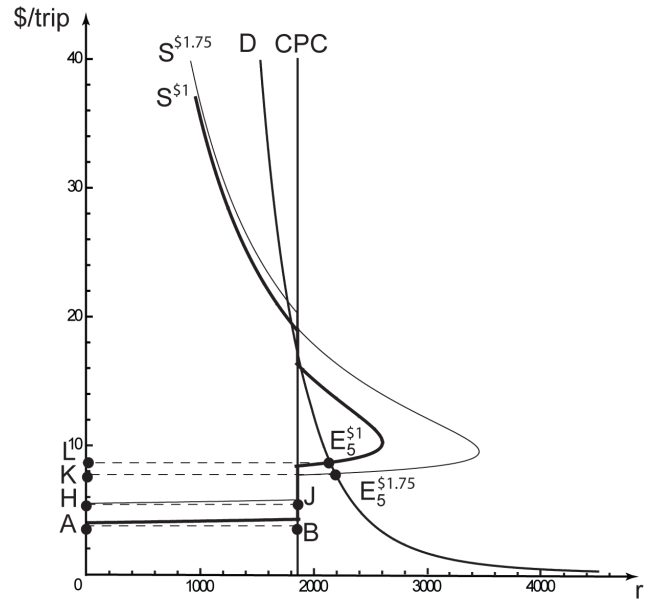


Figure 16: Social benefit from increasing the curbside meter rate

the price differential between garage and curbside parking is halved, in the example raising the parking fee from \$1.00 to \$1.75/hr. Doing so changes the value of $\hat{C}$. Denote the corresponding supply curves by $S^{\$1}$ and $S^{\$1.75}$, and the corresponding type-5 equilibria by $E_5^{\$1}$ and $E_5^{\$1.75}$. The gain in parking meter revenue is given by the area $ABJH$, while the gain in consumer surplus equals the area $LE_5^{\$1}E_5^{\$1.75}K$. Thus, in contrast to the previous section where the gain in social surplus from increasing the meter rate equals the increase in meter revenue, here the gain in social surplus may be several times the increase in meter revenue. The marginal burden of curbside parking fee revenue is then negative. In the previous section, the increase in the meter rate simply converted travel costs dollar for dollar into meter revenue. Here, the increase in the meter rate converts cruising-for-parking time costs dollar for dollar into dollar into meter revenue, with the added gain that the decrease in the stock of cars cruising for parking reduces traffic congestion, benefiting everyone.

Figure 17 displays a bifurcation diagram, indicating the equilibrium throughputs at each level of demand intensity with the base-case parameter values. Equilibria of type 4 are congested, stable, and unsaturated, and correspond to equilibria of type 1 in Figure 4.

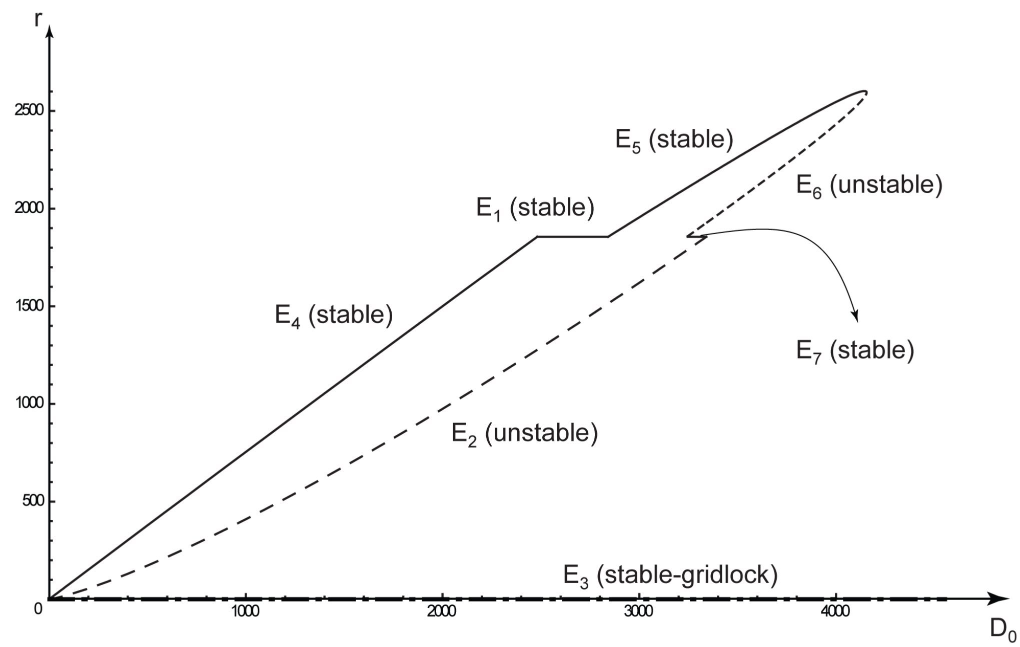


Figure 17: Bifurcation diagram

Equilibria of type 1 occur if the demand curve intersects the curbside parking capacity constraint below the portion of the supply corresponding to the full price line with garage parking, and correspond to equilibria of type 1' in Figure 4. With both these equilibrium types, there is no garage parking. The other equilibrium types are illustrated in Figure 14.

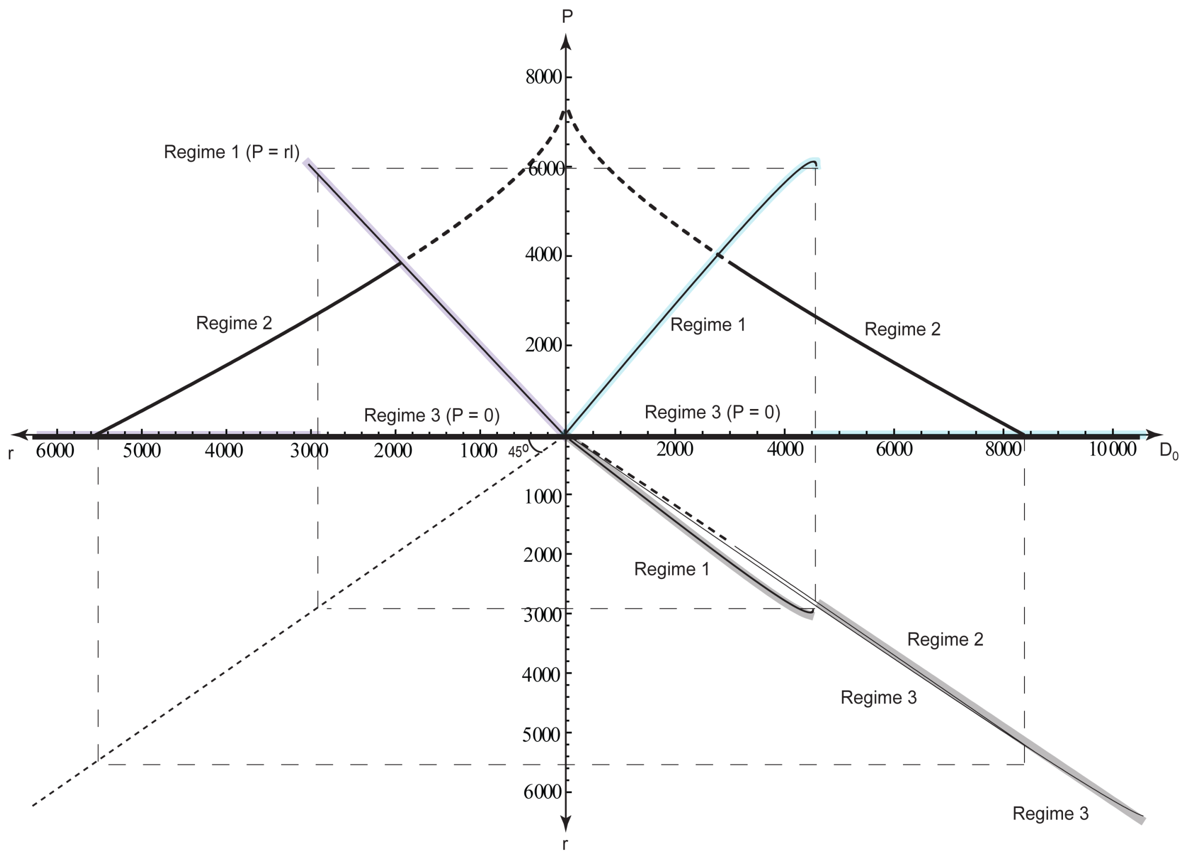
Start at low levels of demand intensity. There is more than enough curbside parking to accommodate the demand. All three equilibria are unsaturated. E_4 is the most efficient of the equilibria and is congested; E_2 is hypercongested and unstable; and E_3 is the gridlock equilibrium, which is an equilibrium for all levels of demand intensity. As demand intensity increases, a level is reached at which the efficient equilibrium becomes saturated, switching from a type-4 equilibrium to a type-1 equilibrium. There is cruising for parking but not enough to make garage parking economically viable. For an interval of higher demand intensities, the type 1 equilibrium coexists with equilibria of types 2 and 3.

As demand intensity increases even further, another level of demand intensity is reached at which the scarcity rent on curbside parking becomes sufficiently high to make garage parking profitable, and the type 1 equilibrium switches to a type 5 equilibrium, which was described above, and coexists with equilibria of types 2 and 3. As the level of demand intensity increases further, another level of demand intensity is reached at which two new types of equilibria emerge, types 6 and 7. A type 6 equilibrium has both saturated curbside parking and no garage parking, and is hypercongested and stable. As demand intensity increases further, another critical level of demand intensity is reached at which the type 2 and type 7 equilibria disappear, leaving only equilibria of types 3, 5, and 6. Finally, the demand intensity becomes so high that downtown street use can be rationed only with gridlock.

4.2.2 Second-best optimal capacity with both curbside and garage parking

As in the previous subsection, we work with three régimes. In régime 1 there is only curbside parking, in régime 2 there is both curbside and garage parking, and in régime 3 there is only garage parking. As we shall see, the second-best analysis is considerably more complex than the first-best analysis. The way we shall proceed is, for each D_0, to first determine second-best optimal curbside parking capacity in each of the three régime and then to determine which of the three régimes is optimal. Subscripts 1, 2, and 3 denote the three régimes.

Start with the analysis of régime 1. We have already solved for second-best optimal curbside parking capacity when there is only curbside parking, in section 3.2.2. It is second-best optimal to choose curbside parking capacity such that it is saturated without generating cruising for parking. Thus, $P_1^{**} = r_1^{**}l$ and $D(UC(r_1^{**}, r_1^{**}l) + fl; D_0) = r_1^{**}$, which gives an implicit function relating r_1^{**} (and hence P_1^{**}) to D_0. For the numerical example, these functions are displayed in Figure 18. Two points are worthy of note. First, for $D_0 > 4341$, the only equilibrium is the gridlock equilibrium; with only curbside parking, such high levels of demand require a very high price to ration the demand, which can be achieved only via gridlock. Second, for a small interval of demand intensities below $D_0 = 4341$, second-best throughput and curbside parking capacity falls as demand intensity rises; this corresponds to the stable, hypercongested equilibrium identified in section 3.2.2.

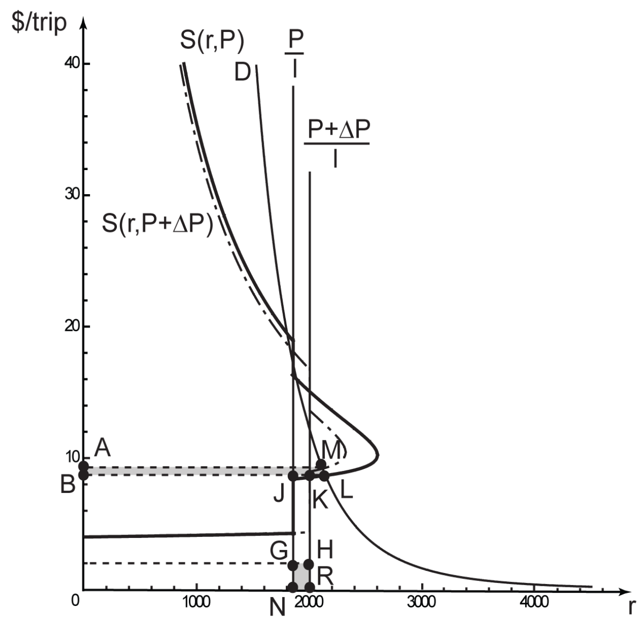


Note: Highlighted paths denote optimal regimes.

Figure 18: Regimes in the second-best optimum

Turn to régime 2. Now, the second-best curbside capacity analysis of the previous section must be substantially modified. Expanding curbside parking has four effects. First, curbside parking revenue increases; second, fewer garage spaces need to be constructed; third, *per* (18) the stock of cars cruising for parking increases; and fourth, the amount of road space for traffic circulation is reduced. We shall provide two different analyses, the first which is more intuitive, the second more technical.

Here is the first analysis. Social surplus can be decomposed into consumer surplus, government surplus, and producer surplus. Since garage parking is priced at cost, there is no producer surplus, and government surplus is simply the curbside meter revenue. Thus, second-best curbside parking capacity is that which maximizes the sum of consumer surplus and curbside meter revenue.



Note: Please note that the bottom part of the supply curve corresponding to $P + \Delta P$ becomes indistinguishable from the bottom part of the supply curve corresponding to P in this figure.

Figure 19: The second-best optimal curbside parking capacity with curbside and garage parking

Figure 19 displays two short-run supply curves, one with curbside parking capacity P, $S(r, P)$, the other with curbside parking capacity $P + \Delta$, $S(r, P + \Delta P)$. The shape of the supply curves is similar to that shown in Figure 14. The expansion of curbside parking capacity causes the curbside parking capacity constraint to shift to the right, and the portion of the supply curve to the right of the curbside parking capacity constraint to shift to the left. The latter causes equilibrium throughput to fall from r_L to r_M, resulting in a loss of consumer surplus of $BAML$. The increase in curbside parking revenue is given by $NGHR$. At an interior optimum of curbside parking capacity, the two areas are equal.

One result follows immediately. There should be no curbside parking when it is provided free; providing curbside parking reduces consumer surplus with no offsetting increase in curbside parking revenue. Another result is that, with reasonable restrictions on demand[27] and when the second-best level of curbside parking is interior, the second-best level of curbside parking is decreasing in demand intensity. The social benefit from raising curbside parking capacity by one unit equals the increase in parking fee revenue,[28] lf, independent of demand. The social cost from raising curbside capacity by one unit equals the loss in consumer surplus, which is[29] $lr\rho m(dt/dP)$. As demand intensity rises, equilibrium traffic congestion worsens. Due to the convexity of the congestion technology, the loss in travel time from the unit increase in curbside parking capacity increases with congestion, and hence with the level of demand intensity. Normally, an increase in demand should be associated with a higher marginal social cost of curbside parking capacity locus and hence a lower second-best optimal level of curbside parking. This can be seen from the diagram. As the demand curve shifts out, the vertical distance between $S(r, P+\Delta P)$ and $S(r, P)$ increases due to increased congestion, which will normally cause the area $BAML$ to increase. A related result is that above some level of demand intensity, no curbside should be allocated to parking.

[27] We have proved the result with constant demand elasticity less than 4.0.

[28] Raising curbside parking capacity (P/l) by one unit increases P by l units.

[29] Consumer surplus is $CS = \int_0^r MWP(r', D_0)dr' - MWP(r, D_0)r$, where r is the equilibrium throughput and MWP is the marginal willingness to pay. The change in consumer surplus from increasing parking capacity by one unit is $dCS/d(P/l) = l(dCS/dP) = -lr(dMWP(r, D_0)/dP)$. Now, at equilibrium, the marginal willingness to pay equals full price,$F = \rho mt + cl$. Thus, the marginal social cost from raising curbside parking capacity by one unit equals $lr\rho m(dt/dP)$.

Our second analysis builds on the second-best optimization problem for regime 2[30]

$$\max_{r,T,C,P} X(r; D0) - \rho(T + C) - c(rl - P) \tag{19}$$

$$s.t.$$

$$r = \frac{T}{mt(T, C, P)}$$

$$r = D(\rho mt(T, C, P) + cl)$$

$$C = \frac{(c - f)P}{\rho}$$

The maximand is social surplus, which equals social benefit, $X(r; D0)$, minus social cost, which equals the total cost of time in transit, ρT, plus the total cost of cruising for parking, ρC, plus garage parking costs, $c(rl - P)$. The first constraint is the steady-state condition; the second constraint is that throughput equal the demand for travel when the full price equals the trip cost of a garage parker; and the third constraint is the full price equalization condition, that the parking cost of a garage parker equal that for a curbside parker (which includes cruising for parking), or equivalently that the trip price for a garage parker is the same as that for a curbside parker.

A solution to this maximization with $P < 0$ or with $P > rl$ are not economically sensible. To treat this, we say that régime 2 occurs only over that interval of demand intensities for which $rl > P > 0$.

Note first that, for each r, the second constraint determines a unique trip time, which allows the first constraint to be solved for a unique value of T. Furthermore, an increase in r requires that t fall, which can be shown to imply that P fall (holding trip time fixed, since travel is congested, throughput is increased by decreasing the amount of curbside parking, and hence the stock of cars cruising for parking as well). In this way, we can determine the

[30] We have defined régime 2 to exist over only the range of demand intensities for which it is second-best optimal to have both curbside and garage parking. We could alternatively add a fourth constraint to the second-best maximization problem, that $0 \leq P \leq rl$, and say that régime 2 is defined for that range of demand intensities for which this constraint does not bind.

equilibrium values of T, C, and P as functions of r, $T_2(r; D_0)$, $C_2(r; D_0)$, and $P_2(r; D_0)$, from which we obtain resource costs as a function of r and D_0, $RC_2(r; D_0)$. $dRC_2(r; D_0)/dr$ is the régime 2 long-run marginal social cost curve. The régime 2 second-best optimum occurs at the point of intersection of the régime 2 long-run marginal social curve and the demand curve, over that range of demand intensities for which $P_2^{**}(D_0)$ satisfies $r_2^{**}(D_0)l > P_2^{**}(D_0) > 0$.

Figure 18 displays régime 2 second-best optimum as a function of D_0. Two points bear note. First, régime 2 is defined only for an interval of demand, from $D_0 = 2980$ to $D_0 = 8369$. Below this interval, the social welfare maximization solution gives $P > rl$, which corresponds to its being second-best optimal to provide no garage parking; and above this interval, the social welfare maximization solution gives $P < 0$, which corresponds to its being second-best optimal to provide only garage parking.

Finaly consider régime 3. In this regime, $P = 0$, and $D(UC(r_3^{**}, 0) + cl; D_0) = r_3^{**}$ gives the implicit function $r_3^{*}(D_0)$ relating r_3^{**} to D_0. These results are displayed in Figure 18.

We shall now put the pieces together. The most straightforward way to determine the full second-best optimum is to solve for social surplus as a function of D_0 for each of the three régimes (taking into account that régime 2 is defined over only an interval of D_0), and then for each level of D_0 to choose that régime which maximizes social surplus. Figure 18 displays the results, by highlighting the optimal régime for each level of demand intensity.

One result is particularly striking. With the parameters of the numerical example, the full second-best optimum never entails having both curbside and garage parking; that is, it is never second-best optimal to operate in régime 2. The intuition is straightforward. Cruising for parking occurs only in régime 2. The price differential between garage and curbside parking is sufficiently large that it generates so much cruising for parking that it is more efficient to have either only curbside parking or only garage parking. The second-best optimum entails only curbside parking (régime 1) up to demand intensity $D_0 = 4395$, and only garage parking (régime 3) above this level of demand intensity.

This result illustrates that the underpricing of curbside parking can generate a nonconvexity that results in a discontinuity in second-best optimal parking policy. As demand intensity crosses the critical level of demand intensity, it becomes optimal to switch from providing as much curbside parking as the street system can sustain (viz., as long as the demand can be rationed without gridlock occurring) to providing none at all.

The second major result is that, with the parameters of the numerical example, curbside parking is second-best optimal as long as the street system can sustain it. This result occurs because of the large cost differential between curbside and garage parking.

The third major result, which is again specific to the parameters of the numerical example, is that there is no level of demand intensity for which it is second-best optimal to have parking on both sides of the street (which corresponds to $P = 7424$). The reason is that with so much curbside parking the street system cannot accommodate the traffic flow that would be needed to keep the curbside parking saturated, and gridlock becomes the only means to ration the demand.

We have assumed that garage parking is provided by the private sector at cost. The second-best optimum would be the same if garage parking were provided by the public sector at cost. But it would be different if garage parking were subsidized. Start with the situation where garage parking fee is set equal to the curbside meter rate. The second-best optimal amount of curbside parking would be such that social surplus would be unchanged if one more curbside parking space were provided. This would result in more congested travel, hence a higher trip price, and hence reduced throughput, but also a reduction in the number of garage spaces that would need to be provided. From a different perspective, the fall in consumer surplus would be exactly offset by the increase in government revenue – a rise in the revenue from curbside fees and a fall in the subsidy to garage parking. The underpricing of parking would give rise to excessive travel, associated with which would be a deadweight loss. Reducing the garage parking subsidy would reduce the size of this source of deadweight loss, but would introduce another source of deadweight loss – cruising for parking. The second best would involve minimizing the sum of these two deadweight losses.

4.3 Comparison of first- and second-best optimal capacities

In the previous section, we showed that, when there is only curbside parking, second-best capacity exceeds first-best capacity when curbside parking is underpriced, and falls short of it when parking is overpriced. Furthermore, the second best never entails curbside parking being saturated without cruising for parking. When both curbside and garage parking may be provided, the results are considerably more complex. Since obtaining general results appears difficult, we shall focus on the numerical example.

Figure 20 plots first- and second-best curbside parking capacity, as a function of demand intensity. All three régimes are present in the first best. For low levels of demand intensity, up to $D_0 = 4878$, it is efficient to provide only curbside parking; for intermediate levels of demand intensity, between $D_0 = 4878$ and $D_0 = 10321$, it is efficient to have both curbside and garage parking, with the amount of curbside parking declining monotonically with demand intensity; and for high levels of demand intensity, above $D_0 = 10321$, it is efficient to have only garage parking. In the second best, in contrast, régime 2, with both curbside and garage parking, is second-best efficient for no interval of demand intensity. Cruising for parking occurs only in régime 2, and, with the parameters of the numerical example, generates cruising-for-parking costs that make operation in régime 2 too costly for it to be second-best efficient. Only curbside parking is provided for demand intensities up to $D_0 = 4395$, and above that no curbside parking is provided.

Several other points bear note. First, the curbside parking fee of \$1/hr results in the overpricing of curbside parking for demand intensities below 3454, so that over this interval second-best curbside parking capacity falls short of the first-best level; while for levels of demand intensity between 3454 and 4395, curbside parking is underpriced, so that second-

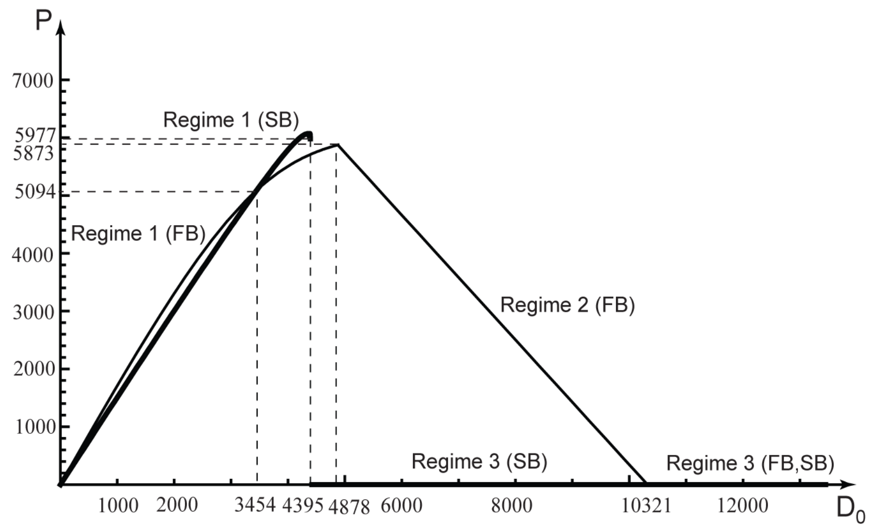


Figure 20: Comparison of the first-best and the second-best optima

best curbside parking capacity exceeds the first-best level. Second, in the interval of demand intensities between 4395 and 4878, only curbside parking is provided in the first best and only garage parking in the second best. The reason is that it is efficient for the planner to respond to the underpricing of curbside parking by expanding curbside parking capacity so that no cruising for parking occurs, which reduces throughput. Third, it is efficient to eliminate curbside parking for a larger interval of demand intensities in the second best than in the first best.

Figure xxxx displays the deadweight loss per ml^2-hr from underpricing curbside parking, as a function of demand intensity. It starts at zero with zero demand intensity, rises to a maximum per unit throughput of xxxx at a demand intensity of xxxx, and to an aggregate maximum of xxxx at a demand intensity of xxxx, and then declines to zero for high demand intensities for which only garage parking is provided.

It would be imprudent to generalize from a specific numerical example. Nevertheless, the numerical example does illustrate some general policy insights. First, for high levels of demand intensity, it is inefficient to provide curbside parking. The reason is that the social value of road space is higher for traffic flow than for curbside parking. This important yet rather obvious insight is applied in practice. In congested cities, curbside parking is virtually never provided along major arterial roads during peak periods. Second, underpricing curbside parking can introduce considerable distortion, even when the amount of curbside parking is optimized. In our numerical example, the maximum distortion was $xxxx per unit throughput. Furthermore, holding fixed demand intensity, deadweight loss is a convex function of the extent of underpricing. From an alternative perspective, raising curbside meter rates may generate efficiency gains that are several times the increased curbside meter revenue generated.

Underpricing curbside parking makes it considerably more difficult to determine optimal curbside parking capacity. Our model abstracted from many important aspects of curbside parking – variation in demand over time and space, and user heterogeneity in trip distance, value of time, parking duration, curbside parking time limits, and stochasticity – and provided a very simplified treatment of garage parking. Yet even in our model the determination of second-best policy was challenging. In our model, underpricing curbside parking is always dysfunctional in the sense of lowering social surplus, and often dysfunctional in the sense of lowering consumer surplus as well. In our analysis, we have deliberately ignored a potentially valuable, second-best policy tool – the subsidization of garage parking by the public authority. In practice, much garage parking is subsidized privately, through employer-provided subsidized or free parking, and parking validation for shopping.

5 Stability Analysis

Arnott and Inci (2010) provided a thorough stability analysis of equilibria in a variant of the model presented above with only curbside parking. In this section, we extend their analysis to investigate the stability of equilibria when both curbside and garage parking are present, and when only garage parking is present. Stability analysis of traffic congestion has proved difficult since it requires solving for the out-of-equilibrium dynamics of traffic flow over time and space. The treatment of downtown as isotropic simplifies the analysis considerably since at any point in time traffic flow is the same throughout the downtown area; the analysis then entails solving ordinary rather than partial differential equations. Arnott and Inci further simplified the problem by making some special assumptions[31] that render the differential equation system autonomous (time does not enter the analysis explicitly), which permits phase-plane/state-space analysis. The arrows give the direction of motion, under the assumption that drivers decide whether to travel based on myopic expectations (more precisely, the entry rate at time t is assumed to depend on the *perceived* full price of a trip, which depends only on traffic conditions at time t.

We first introduce a new piece of notation to facilitate geometric presentation of the stability analyis in 2D space. We define

$$R = \begin{cases} C & \text{for } R \geq 0 \\ Q - P & \text{for } R \leq 0 \end{cases} , \tag{20}$$

where Q is the stock of occupied curbside parking spaces. In words, when R is positive, which corresponds to saturated curbside parking, it equals the stock of cars cruising for parking, and when R is negative, which corresponds to unsaturated curbside parking, it equals minus the stock of unoccupied curbside parking spaces. This allows us to depict the transition between saturated and unsaturated parking in a single phase plane. As R increases from being negative to being positive, the stock of unoccupied curbside parking spaces shrinks, until at $R = 0$ parking is saturated with no cruising for parking, and then remains saturated with the stock of cars cruising for parking increasing.

Figure 21 displays the stability analysis with only curbside parking for the base case

[31] They assume that trip lengths are negative exponentially distributed, which implies that the exit rate from the in-transit pool at time t depends only on the stock of cars in transit and cruising for parking at that point in time, and not on the history of congestion. They also assume that visit durations are negative exponentially distributed, which implies that the exit rate from curbside parking depends only on the amount of curbside parking.

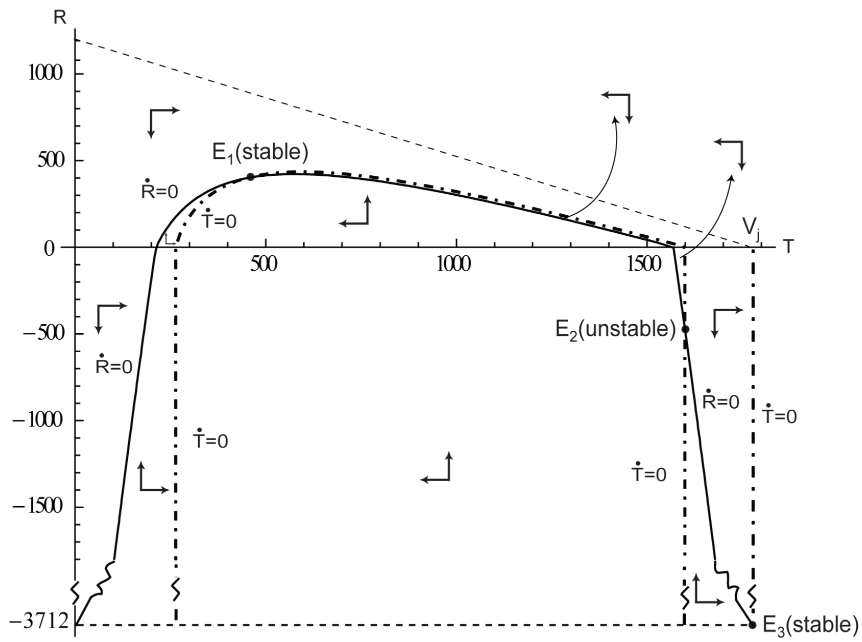


Figure 21: Transient dynamics of downtown traffic when there is only curbside parking

level of curbside parking, $P = 3712$, and for the demand intensity indicated by $D_2 = 3000$ in Figure 4. The state of the system is characterized by T and R, with T on the horizontal axis and R on the vertical axis. Above $R = 0$, parking is saturated and there is cruising for parking, and below $R = 0$ parking is unsaturated and there are unoccupied curbside parking spaces. The arrows indicate the direction of motion of the two state variables. Three loci are displayed in T-R space. The first, the dashed line, is the jam density line; combinations of T and R to the right of the line are infeasible.

The second locus is the $\dot{R} = 0$ locus. For $R \geq 0$, the locus corresponds to the cruising-for-parking equilibrium condition $\dot{C} = 0 = T/(mt(T, C, P)) - P/l$, along which the stock of cars cruising for parking remains unchanged; below this locus, the stock of cars cruising for parking is increasing, and above it the stock is decreasing. For $R \leq 0$, the locus corresponds to $\dot{Q} = 0 = T/(mt(T, C, P)) - Q/l$; below this locus, the stock of occupied curbside parking spaces is increasing, and above it the stock is decreasing.

The third locus is the steady-state condition that $\dot{T} = 0 = D(F) - T/(mt(T, C, P)) = D\left(\rho mt(T, C, P)\right) + \rho Cl/P + fl) - T/(mt(T, C, P))$. When curbside parking is saturated, the $\dot{T} = 0$ locus is a curve in T-C space, above which the stock of cars in-transit is increasing and below which it is decreasing. When curbside parking is unsaturated, $C = 0$, the $\dot{T} = 0$ corresponds to those levels of T for which the stock of cars in transit remains unchanged. There are three such levels of T, all corresponding to points of intersection of the unsaturated user cost curve, shifted up by the curbside parking fee, and the demand curve. The one furthest to the left corresponds to the the upward-sloping portion of the user cost curve, the middle one to the backward-bending portion of the curve, and the one on the right to gridlock. The stock of cars in transit is increasing for T lower than the T furthest to the left and between the middle T and the gridlock T, and is decreasing between the T furthest to the left and the middle T.

Consistent with Figure 4, there are three equilibria. The equilibrium E_1 in Figure 21 corresponds to the equilibrium E_1' in Figure 4, and is saturated, stable, and congested. The equilibrium E_2 in Figure 21 corresponds to the equilibrium E_2' in Figure 4, and is unsaturated, stable, and hypercongested. The equilibrium E_3 in Figure 21 corresponds to the gridlock equilibrium, which cannot be displayed in Figure 4.

In the remainder of the section we show the stability analysis can be adapted to the situation with both curbside and garage parking, and then apply the adapted stability analysis to determine the stability of the equilibria analyzed in section 4.

In the analysis of section 4, since the curbside parking fee is lower than the garage parking fee, garage parking occurs only when curbside parking is saturated. Thus, allowing for garage parking does not affect the stability analysis when curbside parking is unsaturated, and hence the portion of the phase plane with negative R. The addition of garage parking adds the full price equalization condition that $R \leq \hat{C} = (c - f)P/\rho$. When $R < \hat{C}$, curbside parking is cheaper than garage parking so that no one parks in a garage, and the stability analysis of Figure 21 continues to apply. When $R > \hat{C}$, however, the stability analysis of Figure 21 needs to be modified. If $R > \hat{C}$, garage parking is cheaper than curbside parking. We assume that when this occurs the number of cars cruising for parking falls instantaneously such that $R = \hat{C}$ is satisfied. Thus, above $C = \hat{C}$, the direction of motion is vertically downward. Otherwise, the direction of motion in the phase plane is unchanged.

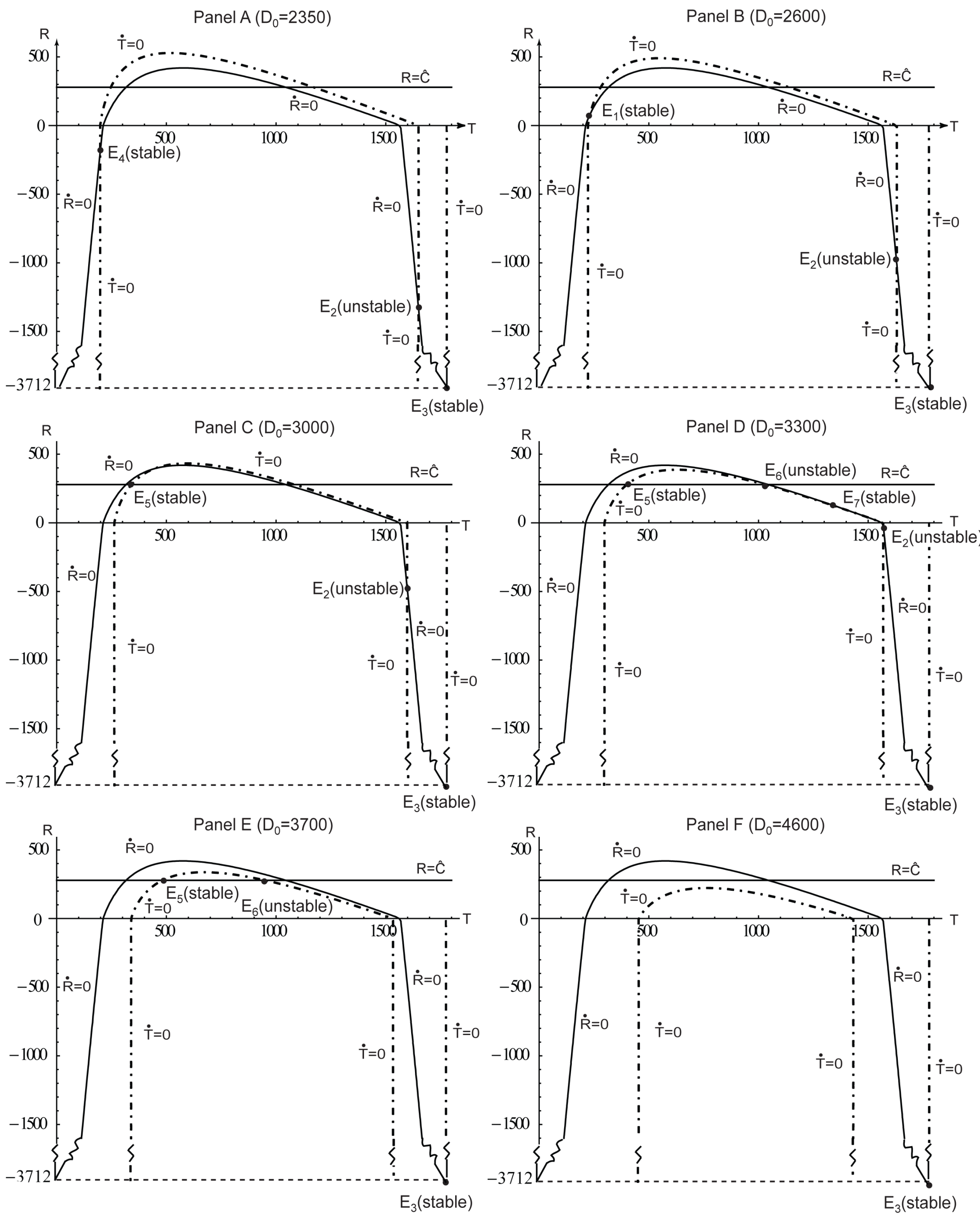


Figure 22: Transient dynamics of downtown traffic when there are curbside and garage parking

Figure 22 portrays the phase plane for six different levels of demand intensity. Recall that an increase in demand intensity has no effect on the $\dot{R} = 0$ but causes the $\dot{T} = 0$ locus to shift downward. Start with Panel A, which has the lowest level of demand intensity. Qualitatively, this corresponds to the situation shown in Figure 4 with demand intensity D_1. All three curbside parking equlbria are unsaturated, so that there is no demand for garage parking. Turn next to Panel B, with the next lowest level of demand intensity. Qualitatively, this panel corresponds to the situation shown in Figure 4 with demand intensity D_2. The equilibrium corresponding to E_1' in that figure is saturated. Cruising for parking occurs, but the stock of cars cruising for parking is not sufficient to make the provision of garage parking profitable. In Panel B, this corresponds to the equilibrium E_1 lying below the $R = \hat{C}$ locus.

Now turn to Panel C. The demand intensity is the same as that used in the construction of Figure 21. Thus, comparison of Panel C, Figure 22, and Figure 21 shows how admitting garage parking alters the equilibria of Figure 21. Now the stock of cars cruising for parking in equilibrium E_1' in Figure 4 is sufficiently high to make garage parking profitable. Garage parking is provided, and the equilibrium E_1' in Figure 4 is replaced by the equilibrium with the same qualitative properties as E_5 in Figure 14, which is saturated, stable, and congested. The other two equilibria remain unsaturated.

In Panel D, demand intensity is close to that for the demand curve drawn in Figure 14 so that the equilibria are qualitatively the same. There are now five equilibria. In the stable, congested equilibrium E_5, garage parking is provided and curbside parking is saturated. In the unstable, hypercongested equilibrium E_6, garage parking is provided and curbside parking is saturated. In the stable, hypercongested equilibrium E_7, curbside parking is saturated but the stock of cars cruising for parking is insufficient for garage parking to be profitable. In the unstable, hypercongested equilibrium E_2, curbside parking is unsaturated. Finally, there is the gridlock equilibrium. Panel E corresponds to Figure 14 but with a higher level of demand intensity such that the equilibria E_7 and E_2 disappear. Panel F corresponds to Figure 14 with an even higher level of demand intensity such that only the gridlock equilibrium remains. Thus, the stability analysis of Figure 22 confirms the stability properties of the various equilibria asserted in the bifurcation diagram of Figure 17.

The above discussion has been mechanical. It will be useful to provide some intuition, which can be done by describing the process of adjustment along three sample trajectories in Panel D. Let us start with a situation in which downtown is empty. Then the demand is turned on at the demand intensity $D_0 = 3330$, and remains at that level forever. Cars start entering the city streets, traffic density builds, and an increasing number of curbside parking spaces become occupied.[32] With unsaturated parking, the trajectory lies between the $\dot{R} = 0$ and $\dot{T} = 0$ loci. In due course, parking becomes saturated and cruising for parking commences. The stock of cars in-transit and cruising for parking continue to increase, which corresponds to the trajectory continuing to lie between the $\dot{R} = 0$ and $\dot{T} = 0$ loci, but now with saturated parking and cruising for parking. In due course, the stock of cars stock of cars cruising for parking becomes sufficiently large that it becomes profitable for garage parking to be provided. The stock of cars in transit continues to increase and the stock of garage parking spaces to be expanded until the equilibrium E_5 is reached.[33]

Consider the unstable equilibrium E_6. Since the equilibrium is saddlepoint stable, its stable arms are the boundary between E_5 and E_7's zones of attraction. Start slightly to the left of E_6 on $R = \hat{C}$. There is both curbside and garage parking, and cruising for parking satisfies the full price equalization condition, which continues to be satisfied throughout the adjustment process. The stock of cars in transit is slightly lower than at E_6. Turn to Figure 14, which describes the same situation as Panel D, but in another space. On the demand side, a stock of cars in transit slightly below that at E_6 results in the trip price being somewhat lower than at E_6 and the entry rate therefore being somewhat higher.

[32] Recall that the adjustment process assumes, first, that trip lengths are negative exponentially distributed with mean m, so that parking spaces start becoming occupied as soon as there is traffic on the road, and, second, that the entry rate at time t depends upon the stock of cars in transit and cruising for parking at that point in time.

[33] This adjustment process ignores the durability of garage parking spaces.

On the supply side, because traffic is hypercongested, the lower stock of cars in transit implies a higher exit rate (throughput). Because the demand curve is steeper than the supply curve at E_6, the quantity of trips supplied is higher than at E_6 by more than the quantity of trips demanded, which results in a fall in the stock of cars in transit. In due course, the reduction in the stock of cars in transit becomes sufficiently large that travel becomes congested, and continued reductions in the stock of cars in transit causes throughput to fall, while the quantity of trips demanded continues to rise. This eventually results in achievement of the stable equilibrium at E_5.

The story is similar if the starting point is slightly to the right of E_6 in Figure 22, Panel D. The initial stock of cars in transit is slightly higher than at E_6. On the demand side, a stock of cars in transit slightly above that at E_6 results in the trip price being somewhat higher than at E_6, and the entry rate therefore being somewhat lower. On the supply side, because traffic is hypercongested, the higher stock of cars in transit implies a lower exit rate. Because the demand curve is steeper than the supply curve, the quantity of trips supplied is lower than at E_6 by more than the quantity of trips demanded, which results in an increase in the stock of cars in transit, and traffic become increasingly hypercongested. The reduced throughput causes a reduced demand for garage parking and eventually zero demand.

6 Concluding Remarks

This paper provides a diagrammatic analysis of downtown parking and traffic congestion policy. Diagrammatic analysis is insightful since it draws on geometric intuition, but it can only go so far. The above analysis omits a number of important considerations, which cannot be easily handled via diagrammatic analysis. First, households are assumed to be identical, but of course driver heterogeneity is important. Arnott and Rowse (2011) explore how drivers who differ in their visit durations and values of the time sort themselves between curbside and garage parking, and how, when drivers differ, curbside parking time limits can be used to reduce cruising for parking.

Second, only steady-state equilibria are explored but the demand for parking spaces varies systematically over the course of the day. Ideally the meter rate would be adjusted over the day to clear the market for curbside parking. However, most cities apply single-step curbside parking fees, in which the curbside parking fee is fixed over the business day and free at other times, with the result that cruising for parking occurs during peak hours. Third, mass transit is ignored. Via the Envelope Theorem, our analysis carries through if mass transit is organized efficiently, treated implicitly in the demand function. But if mass transit is not organized efficiently, welfare analysis should take into account how parking policy affects the deadweight losses in the mass transit market.

Fourth, our analysis assumes that garage parking is supplied and priced at constant cost. Extending the analysis to treat an upward-sloping supply curve for garage parking is straightforward, but extending it to treat spatial competition between garage parking operators is not. The spatial competition model presented in Arnott (2006) and Arnott and Rowse (2009) is coherent but its behavior is likely unrealistic since it ignores capacity constraints. The market power exercised by garage parking operators is likely sufficiently important that it should be explicitly treated in the analysis of downtown parking policy. Fifth, the analysis assumes downtown to be isotropic, but of course spatial variation in parking policy reflecting spatial variation in traffic is important; resident parking regulation in residential neighborhoods is one example.[34]

Sixth, our analysis pays no attention to land use, except for the allocation of exogenous road space to parking. This may be a reasonable short-run assumption in the context of downtown traffic congestion, but over longer periods the allocation of downtown space to roads is an important aspect of downtown traffic policy, and the effects of downtown parking policy on land use both inside and outside the downtown area may be significant.

[34]For reasons explained earlier, treating spatial variation analytically is likely to prove intractable. In policy analysis it can be treated by employing a downtown traffic and parking microsimlutor, such as VISSIM.

Seventh, our models ignore two important aspects of downtown parking, heavily subsidized employer-provided parking[35] and minimum parking requirements. These may be treated as exogenous in a model of downtown parking, and better yet should be derived as properties of equilibrium. Hasker and Inci (2011) obtains minimum parking requirements as a property of an equilibrium in the context of shopping mall parking.

Eighth, our analysis assumes that downtown traffic congestion is appropriately modeled using classic traffic flow theory, which was developed from freeway traffic. One alternative is to model congestion using intersection queuing theory. Another is to employ a traffic and parking microsimulator. Ninth, much of our second-best analysis takes the underpricing of curbside parking to be an exogenous distortion. This can reasonably be challenged since typically the downtown parking authority determines both meter rates and the allocation of curbside to parking. Also, there seems to widespread agreement that downtown curbside parking is underpriced due to lobbying by downtown merchant associations, who argue that it is needed to compete with free suburban shopping center parking and to keep downtown vital. If this is correct, then parking policy should be evaluated either taking these objectives into account or taking them into account via political economy constraints.

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