

Common Drifting Volatility in Large Bayesian VARs*

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Abstract

The estimation of large Vector Autoregressions with stochastic volatility using standard methods is computationally very demanding. In this paper we propose to model conditional volatilities as driven by a single common (stationary or integrated) unobserved factor. This is justified by the observation that the pattern of estimated volatilities in empirical analyses is often very similar across variables. Using a combination of a standard natural conjugate prior for the VAR coefficients, and an independent prior on a common stochastic volatility factor, we derive the posterior densities for the parameters of the resulting BVAR with common stochastic volatility (BVAR-CSV). Under the chosen prior the conditional posterior of the VAR coefficients features a Kroneker structure that allows for fast estimation, even in a large system. Using US and UK data, we show that, compared to a model with constant volatilities, our proposed common volatility model significantly improves model fit and forecast accuracy. The gains are comparable to or as great as the gains achieved with a conventional stochastic volatility specification that allows independent volatility processes for each variable. But our common volatility specification greatly speeds computations.

Keywords: Bayesian VARs, stochastic volatility, forecasting, prior specification.

J.E.L. Classification: C11, C13, C33, C53.

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1 Introduction

Several recent papers have shown that two key ingredients for the empirical success of Vector Autoregressions are the use of a rather large information set and the inclusion of drifting volatilities in the model. Banbura, Giannone, and Reichlin (2010), Carriero, Kapetanios, and Marcellino (2011), and Koop (2012) show that a system of 15-20 variables performs better than smaller systems in point forecasting and structural analysis. With small models, studies such as Clark (2011), Cogley and Sargent (2005), and Primiceri (2005) show how the inclusion of drifting volatility is key for understanding the dynamics of macroeconomic variables and for density forecasting. Koop and Korobilis (2012) show that a computational shortcut for allowing time-varying volatility (roughly speaking, using a form of exponential smoothing of volatility) improves the accuracy of point and density forecasts from larger VARs.

However, introducing stochastic volatility within a Vector Autoregressions poses serious computational burdens, and typically all the empirical implementations of such models have been limited to a handful of variables (3 to 5). The computational burden is driven by the use of Markov Chain Monte Carlo (MCMC) estimation methods needed to accommodate stochastic volatility (the same applies to Bayesian estimation of other models of time-varying volatilities, including Markov Switching and GARCH). In particular, as noted in such studies as Sims and Zha (1998), the challenge with larger VAR models is that drawing the VAR coefficients from the conditional posterior involves computing a (variance) matrix with the number of rows and columns equal to the number of variables squared times the number of lags (plus one if a constant is included). The size of this matrix increases with the square of the number of variables in the model, making CPU time requirements highly nonlinear in the number of variables. An alternative to MCMC sampling is the local scale model proposed by Uhlig (1997), which can be estimated using nonlinear filtering and importance sampling. However, the implementation of nonlinear filtering, the calibration of the importance sampling density via a maximization involving all the VAR coefficients, and the high risk of inefficiency inherent in the importance sampling procedure make difficult the application of this method to a large VAR.

In this paper we propose a computationally effective way to model stochastic volatility, to greatly speed up computations for smaller VAR models and make estimation tractable for larger models. The proposed method hinges on the observation that the pattern of estimated volatilities in empirical analyses is often very similar across variables. We propose to model conditional volatilities as driven by a single common (stationary or integrated)

unobserved factor. Our volatility model corresponds to the stochastic discount factor model described in Jacquier, Polson, and Rossi (1995). While Jacquier, Polson, and Rossi (1995) had in mind using the model in an asset return context, we incorporate the volatility model in a VAR. Using a combination of (1) a standard natural conjugate prior for the VAR coefficients and (2) an independent prior on a common stochastic volatility factor, we derive the posterior densities for the parameters of the resulting BVAR with common stochastic volatility (BVAR-CSV). Under the chosen prior the conditional posterior of the VAR coefficients features a Kroneker structure that allows for fast estimation. Hence, the BVAR-CSV can be also estimated with a larger set of endogenous variables.

Our proposed volatility model treats the commonality as multiplicative. We need both the single factor and the multiplicative structure in order to be able to define a prior and factor out volatility in such a way as to exploit the Kroneker structure that is needed to speed up the VAR computations. Prior work by Pajor (2006) considered the same basic model of volatility for the errors of a VAR(1) process, in just a few variables, without the VAR prior we incorporate to speed up computations. Still other work in such studies as Osiewalski and Pajor (2009) and references therein has considered common volatility within GARCH-type specifications. Some other papers introduce the commonality in volatility as additive. For example, in an asset return context, Chib, Nardari, and Shephard (2002, 2006) and Jacquier, Polson and Rossi (1995) employ a factor structure multivariate stochastic volatility model. In a macro context, in a setup similar to that used in some finance research, Del Negro and Otrok (2008), Liu, Mumtaz and Theophilopoulou (2011), and Mumtaz and Surico (2012) develop a factor model with stochastic volatility. Viewed this way, the factor structure multivariate stochastic volatility model or factor model with stochastic volatility is somewhat different from the one proposed here: in the BVAR-CSV we have a VAR that captures cross-variable correlations in conditional means and captures a common factor in just volatility; in these other models, the factor captures both cross-variable correlations in conditional means and drives commonality in volatility.

To establish the value of our proposed model, we compare CPU time requirements, measures of in-sample fit, volatility estimates, and forecast accuracy (both point and density) across VAR models of different sizes and specifications. The model specifications include: a VAR with constant volatilities; a VAR with stochastic volatility that treats the volatilities of each variable as independent, as pioneered in Cogley and Sargent (2005) and Primiceri (2005); and our proposed VAR with common stochastic volatility, where the latter can be either stationary or integrated. More specifically, using VARs for US data, we first document the computational efficiency and fit gains associated with imposing common volatility. We

then compare alternative estimates of volatility, for both 4-variable and 8-variable systems, and show that there is substantial evidence of common volatility. We then proceed to examine real-time forecasts from 4-variable and 8-variable macroeconomic models for the US, finding that the imposition of common stochastic volatility consistently improves the accuracy of real-time point forecasts (RMSEs) and, in particular, density forecasts (log predictive scores). We also compare final-vintage forecasts from 15-variable models for the US data and again find that common stochastic volatility in general improves forecast accuracy. In terms of volatility specification, the integrated and stationary (but highly persistent) volatility models produce similar in-sample and forecast results.

Finally, as a robustness check, we repeat much of the analysis using UK data, obtaining broadly similar results. Most notably, despite evidence of more heterogeneity in the volatility patterns across variables for the UK than for the US, we find the BVAR with common stochastic volatility significantly improves the accuracy of forecasts.¹ Actually, the gains are comparable to those for the US when using a BVAR as the benchmark, and are even larger with a simple AR model for each variable as the benchmark. Furthermore, the gains apply to both point and density forecasts.

We interpret these results as evidence that the BVAR-CSV model efficiently summarizes the information in a rather large dataset and successfully accounts for changing volatility, in a way that is much more computationally efficient than in the conventional approach that treats the volatility of each variable as independent.

The structure of the paper is as follows. Section 2 presents the model, discusses the priors, derives the posteriors (with additional details in the Appendix), and briefly describes the other BVAR models to which we compare the results from our proposed BVAR-CSV model. Section 3 discusses the MCMC implementation. Section 4 presents our US-based evidence, including computational time for the estimates of alternative models and full-sample volatility estimates and presents the forecasting exercise for the 4-, 8- and 15-variable BVAR-CSV. Section 5 examines the robustness of our key findings using data for the UK. Section 6 summarizes the main results and concludes.

¹As detailed below, in light of the more limited availability of real-time data for the UK than the US, our UK results are based on final vintage data, not real-time data.

2 The BVAR-CSV model

2.1 Model Specification

Let y_t denote the $n \times 1$ vector of model variables and p the number of lags. Define the following: Π_0 = an $n \times 1$ vector of intercepts; $\Pi(L) = \Pi_1 - \Pi_2 L - \dots - \Pi_p L^{p-1}$; A = a lower triangular matrix with ones on the diagonal and coefficients a_{ij} in row i and column j (for $i = 2, \dots, n, j = 1, \dots, i-1$), where $a_i, i = 2, \dots, n$ denotes the vector of coefficients in row i ; and $S = \text{diag}(1, s_2, \dots, s_n)$.

The VAR(p) with common stochastic volatility takes the form:

$$y_t = \Pi_0 + \Pi(L)y_{t-1} + v_t, \quad (1)$$

$$v_t = \lambda_t^{0.5} A^{-1} S^{1/2} \epsilon_t, \quad \epsilon_t \sim N(0, I_n), \quad (2)$$

$$\log(\lambda_t) = \log(\lambda_{t-1}) + \nu_t, \quad \nu_t \sim \text{iid } N(0, \phi). \quad (3)$$

As is standard in macroeconomic VARs with stochastic volatility, the log variance λ_t follows a random walk process, with innovations having a variance of ϕ . Here, there is a single volatility process that is common to all variables, and drives the time variation in the entire variance covariance matrix of the VAR errors. As we will see, empirically this assumption yields sizable forecasting gains with respect to a specification with constant volatility. Moreover, it leads to major computational gains with respect to a model with n independent stochastic volatilities, with in general no major losses and often gains in forecasting accuracy. The scaling matrix S allows the variances of each variable to differ by a factor that is constant over time. The setup of S reflects an identifying normalization that the first variable's loading on common volatility is 1. Similarly, the matrix A rescales the covariances.

Under the above specification, the residual variance-covariance matrix for period t is $\text{var}(v_t) = \Sigma_t \equiv \lambda_t A^{-1} S A^{-1'}$. To simplify some notation, we define $\tilde{A} = S^{-1/2} A$. Then the inverse of the reduced-form variance-covariance matrix simplifies to $\Sigma_t^{-1} = \frac{1}{\lambda_t} \tilde{A}' \tilde{A}$.

2.2 Priors

The parameters of the model consist of the following: The parameters in the matrices Π_0 and $\Pi(L)$, which we collect in a $k \times n$ (where $k = 1 + np$) matrix $\Pi = (\Pi_0, \Pi_1, \dots, \Pi_p)'$; the lower A (non-zero and non-unit elements), composed of vectors $a_i, i = 2, \dots, n$; the diagonal elements of S , composed of $s_i, i = 2, \dots, n$; ϕ ; and the initial volatility λ_0 . The model also includes the latent states $\lambda_t, t = 1, \dots, T$. Below, we use Λ to refer to the history of variances from 1 to T .

We use $N(a, b)$ to denote a normal distribution (either univariate or multivariate) with mean a and variance b . We use $IG(a, b)$ to denote an inverse gamma distribution with scale term a and degrees of freedom b . We specify priors for the parameter blocks of the model, as follows (implementation details are given below).

$$\text{vec}(\Pi) | A, S \sim N(\text{vec}(\underline{\mu}_\Pi), \underline{\Omega}_\Pi) \quad (4)$$

$$a_i \sim N(\underline{\mu}_{a,i}, \underline{\Omega}_{a,i}), \quad i = 2, \dots, n \quad (5)$$

$$s_i \sim IG(d_s \cdot \underline{s}_i, d_s), \quad i = 2, \dots, n \quad (6)$$

$$\phi \sim IG(d_\phi \cdot \underline{\phi}, d_\phi) \quad (7)$$

$$\log \lambda_0 \sim N(\underline{\mu}_\lambda, \underline{\Omega}_\lambda) \quad (8)$$

To make estimation with large models tractable, the prior variance for $\text{vec}(\Pi)$ needs to be specified with a factorization that permits a Kroneker structure. To be able to exploit a Kroneker structure and achieve computational gains, we need not only a single common, multiplicative volatility factor but also a prior that permits factorization. Specifically, we use a prior conditional on $\tilde{A} = S^{-1/2}A$, of the following form:

$$\underline{\Omega}_\Pi = (\tilde{A}'\tilde{A})^{-1} \otimes \underline{\Omega}_0, \quad (9)$$

where $\underline{\Omega}_0$ incorporates the kind of symmetric coefficient shrinkage typical of the natural conjugate Normal-Wishart prior. Under the usual Minnesota-style specification of the Normal-Wishart prior for $\underline{\Omega}_0$, the prior variance takes account of volatility (and relative volatilities of different variables) by using variance estimates from some training sample. Note that the use of a prior for the coefficients conditional on volatility is in alignment with the natural conjugate Normal-Wishart prior, but it does depart from the setup of Clark (2011) and Clark and Davig (2011), in which, for a VAR with independent stochastic volatilities, the coefficient prior was unconditional.

The prior used here, combined with the assumption of a single volatility factor, implies that the posterior distribution of the VAR coefficients, conditional on \tilde{A} and Λ , will have a variance featuring a Kroneker structure. As a result the computations required to draw from such a distribution via MC sampling are of order $n^3 + k^3$ rather than of order n^3k^3 . Details on how these assumptions yield computational improvements are given in the Appendix. While such computational advantages can be considered minor with a small system, they become crucial in estimating larger VARs.

2.3 Coefficient posteriors

The parameters Π , a_i , s_i , and ϕ have closed form conditional posterior distributions which we present here. Draws from these conditionals will constitute Gibbs sampling steps in our MCMC algorithm. Drawing from the process λ_t instead will involve a Metropolis step and is discussed below. We define some additional notation incorporated in the computation of certain moments. In the Appendix we show that the conditional posterior distributions of Π , a_i , s_i , and ϕ take the following forms:

$$\text{vec}(\Pi)|A, S, \phi, \Lambda, y \sim N(\text{vec}(\bar{\mu}_\Pi), \bar{\Omega}_\Pi) \quad (10)$$

$$a_i|\Pi, S, \phi, \Lambda, y \sim N(\bar{\mu}_{a,i}, \bar{\Omega}_{a,i}), \quad i = 2, \dots, n \quad (11)$$

$$s_i|\Pi, A, \phi, \Lambda, y \sim IG\left(d_s \cdot \underline{s}_i + \sum_{t=1}^T (\tilde{v}_{i,t}^2 / \lambda_t), d_s + T\right), \quad i = 2, \dots, n \quad (12)$$

$$\phi|\Pi, A, S, \Lambda, y \sim IG\left(d_\phi \cdot \underline{\phi} + \sum_{t=1}^T \nu_t^2, d_\phi + T\right), \quad (13)$$

where y is a nT -dimensional vector containing all the data and where $\tilde{v}_t = Av_t$.

The mean and variance of the conditional posterior normal distribution for $\text{vec}(\Pi)$ take the following forms:

$$\text{vec}(\bar{\mu}_\Pi) = \bar{\Omega}_\Pi \left\{ \text{vec} \left(\sum_{t=1}^T X_t y_t' \Sigma_t^{-1} \right) + \underline{\Omega}_\Pi^{-1} \text{vec}(\underline{\mu}_\Pi) \right\} \quad (14)$$

$$\bar{\Omega}_\Pi = \left(\tilde{A}' \tilde{A} \right)^{-1} \otimes \left(\underline{\Omega}_0^{-1} + \sum_{t=1}^T \left(\frac{1}{\lambda_t} X_t X_t' \right) \right)^{-1}. \quad (15)$$

Again, the key to the computational advantage of this model is the Kroneker structure of the conditional posterior variance. Achieving this Kroneker structure requires both a single, multiplicative volatility factor and the conditional prior described above.

In practice, the posterior mean of the coefficient matrix can be written in an equivalent form that may often be more computationally efficient. This equivalent form is obtained by defining data vectors normalized by the standard deviation of volatility, to permit rewriting the VAR in terms of conditionally homoskedastic variables: specifically, let $\tilde{y}_t = \lambda_t^{-0.5} y_t$ and $\tilde{X}_t = \lambda_t^{-0.5} X_t$. Then, the posterior mean of the matrix of coefficients can be equivalently written as

$$\bar{\mu}_\Pi = \left(\sum_{t=1}^T \tilde{X}_t \tilde{X}_t' + \underline{\Omega}_0^{-1} \right)^{-1} \left(\underline{\Omega}_0^{-1} \underline{\mu}_\Pi + \sum_{t=1}^T \tilde{y}_t \tilde{X}_t' \right), \quad (16)$$

or, using full-data matrices,

$$\bar{\mu}_{\Pi} = \left(\tilde{X}'\tilde{X} + \underline{\Omega}_0^{-1} \right)^{-1} \left(\underline{\Omega}_0^{-1} \underline{\mu}_{\Pi} + \tilde{X}'\tilde{y} \right). \quad (17)$$

As detailed in Cogley and Sargent (2005), the mean and variance of the posterior normal distribution for the rows of A are obtained from moments associated with regressions, for $i = 2, \dots, n$, of $v_{i,t}/(s_i\lambda_t)^{0.5}$, on $v_{j,t}/(s_i\lambda_t)^{0.5}$, where $j = 1, \dots, i-1$. Treating each equation i separately, let $Z_i'Z_i$ denote the second moment matrix of the variables on the right-hand side of the regression, and $Z_i'z_i$ denote the product of the right-hand side with the dependent variable. Then, for each i , the posterior mean and variance of the normal distribution are as follows:

$$\bar{\mu}_{a,i} = \bar{\Omega}_{a,i}(Z_i'z_i + \underline{\Omega}_{a,i}^{-1}\underline{\mu}_{a,i}) \quad (18)$$

$$\bar{\Omega}_{a,i} = (Z_i'Z_i + \underline{\Omega}_{a,i}^{-1})^{-1}. \quad (19)$$

2.4 Volatility

Our treatment of volatility follows the approach of Cogley and Sargent (2005) and Jacquier, Polson, and Rossi (1994). We define:

$$w_t = S^{-1/2}\tilde{v}_t = S^{-1/2}Av_t \quad (20)$$

where in the second equality we used $\tilde{v}_t = Av_t$. Given the assumptions of the model w_t are i.i.d. Gaussian processes. We will refer to w_t as the orthogonalized errors of the system. In the Appendix we show that the conditional posterior kernel for the full history of λ_t , summarised in Λ , is given by:

$$\pi(\Lambda|\Pi, A, S, \phi, y) = \prod_{t=1}^T \pi(\lambda_t|\lambda_{t-1}, \lambda_{t+1}, \phi, w_t), \quad (21)$$

where the disappearance of all values beyond 1 lead and lag is a consequence of the Markov property of the process assumed for λ_t . The generic term in (21) is proportional to:

$$\pi(\lambda_t|\lambda_{t-1}, \lambda_{t+1}, \phi, w_t) \propto \lambda_t^{-n \times 1.5} \prod_{i=1}^n \exp(-0.5w_{it}^2/\lambda_t) \quad (22)$$

$$\cdot \exp(-0.5(\ln \lambda_t - \mu_t)^2/\sigma_c^2), \quad (23)$$

where the parameters μ_t and σ_c^2 are the conditional mean and variance of $\log \lambda_t$ given λ_{t-1} and λ_{t+1} . The kernel in (23) can be used as a basis to construct a sequence of Metropolis steps in which draws from λ_t are obtained starting from $t = 1$ and ending in $t = T$. More details on the derivation and implementation of this Metropolis step are given in the Appendix.

In the empirical application we also consider a case in which λ_t features stationary dynamics, and in particular we replace (3) with an AR(1) process:

$$\log(\lambda_t) = \psi_0 + \psi_1 \log(\lambda_{t-1}) + \nu_t, \quad \nu_t \sim \text{iid } N(0, \phi). \quad (24)$$

Estimation for this case is very similar to that for the random walk specification, as the Markov property of λ_t is still preserved under (24). The implied kernel will be similar to (23), the only difference being conditioning and computing moments taking into account the presence of the parameters ψ_0 and ψ_1 .

2.5 Other models for comparison

To establish the merits of our proposed model, we will consider estimates from a VAR with independent stochastic volatilities for each variable (denoted BVAR-SV) and a VAR with constant volatilities (denoted BVAR).

The BVAR-SV model takes the form

$$y_t = \Pi_0 + \Pi(L)y_{t-1} + v_t, \quad (25)$$

$$v_t = A^{-1}\Lambda_t^{0.5}\epsilon_t, \quad \epsilon_t \sim N(0, I_n), \quad \Lambda_t = \text{diag}(\lambda_{1,t}, \dots, \lambda_{n,t}), \quad (26)$$

$$\log(\lambda_{i,t}) = \log(\lambda_{i,t-1}) + \nu_{i,t}, \quad \nu_{i,t} \sim N(0, \phi_i), \quad i = 1, n,$$

With this model, the residual variance-covariance matrix for period t is $\text{var}(v_t) \equiv \Sigma_t = A^{-1}\Lambda_t A^{-1'}$.

In the interest of brevity, we don't spell out all of the priors and posteriors for the model. However, as detailed in Clark (2011) and Clark and Davig (2011), the prior for the VAR coefficients is unconditional, rather than conditional as in the BVAR-CSV. From a computational perspective, the key difference between the BVAR-SV and BVAR-CSV models is that the posterior variance for the (VAR) coefficients of the BVAR-SV model does not have the overall Kroneker structure of the posterior variance for the coefficients of the BVAR-CSV model (given in equation (15)). For the BVAR-SV specification, the posterior

mean (the vector of coefficients) and variance are:

$$\text{vec}(\bar{\mu}_{\Pi}) = \bar{\Omega}_{\Pi} \left\{ \text{vec} \left(\sum_{t=1}^T X_t y_t' \Sigma_t^{-1} \right) + \underline{\Omega}_{\Pi}^{-1} \text{vec}(\underline{\mu}_{\Pi}) \right\} \quad (27)$$

$$\bar{\Omega}_{\Pi}^{-1} = \underline{\Omega}_{\Pi}^{-1} + \sum_{t=1}^T (\Sigma_t^{-1} \otimes X_t X_t'). \quad (28)$$

The BVAR takes the form

$$y_t = \Pi_0 + \Pi(L)y_{t-1} + v_t, \quad v_t \sim N(0, \Sigma).$$

For this model, we use the Normal-inverted Wishart prior and posterior detailed in such studies as Kadiyala and Karlsson (1997) and Banbura, Giannone, and Reichlin (2010).

3 Implementation

After detailing the priors used in estimating the BVAR-CSV model, this section provides some evidence on the behavior of our proposed algorithm in Monte Carlo-generated data. The section then details the priors used in estimating the BVAR-SV and BVAR models.

3.1 Specifics on priors: BVAR-CSV model

For our proposed BVAR-CSV model, we set the prior moments of the VAR coefficients along the lines of the common Minnesota prior, without cross-variable shrinkage:

$$\underline{\mu}_{\Pi} = 0, \quad \text{such that } E[\Pi_l^{(ij)}] = 0 \quad \forall i, j, l \quad (29)$$

$$\underline{\Omega}_0 \text{ such that the entry corresponding to } \Pi_l^{(ij)} = \begin{cases} \frac{\theta^2}{l^2} \frac{\sigma_1^2}{\sigma_j^2} & \text{for } l > 0 \\ \varepsilon^2 \sigma_1^2 & \text{for } l = 0 \end{cases}. \quad (30)$$

With all of the variables of our VAR models transformed for stationarity (in particular, we use growth rates of GDP, the price level, etc.), we set the prior mean of all the VAR coefficients to 0.² The variance matrix $\underline{\Omega}_0$ is defined to be consistent with the usual Minnesota prior variance, which is a diagonal matrix. Note that σ_1^2 , the prior variance associated with innovations to equation 1, enters as it does to reflect the normalization of S , in which all

²Our proposed BVAR-CSV specification can also be directly applied to models in levels with unit root priors, with the appropriate modification of the prior means on the coefficients. Including priors on sums of coefficients and initial observations as in such studies as Sims and Zha (1998) is also possible, subject to appropriate adjustment for the conditional heteroskedasticity of y_t and X_t .

variances are normalized by σ_1^2 . With a bit of algebra, omitted for brevity, by plugging in $A = I_n$ and $S_{ii} = \sigma_i^2/\sigma_1^2$, the prior $\underline{\Omega}_\Pi = (\tilde{A}'\tilde{A})^{-1} \otimes \underline{\Omega}_0$ can be shown to equal the conventional Minnesota prior given below for the BVAR-SV model. The shrinkage parameter θ measures the tightness of the prior: when $\theta \rightarrow 0$ the prior is imposed exactly and the data do not influence the estimates, while as $\theta \rightarrow \infty$ the prior becomes loose and results will approach standard *GLS* estimates. We set $\theta = 0.2$ and $\varepsilon = 1000$. The term $1/l^2$ determines the rate at which the prior variance decreases with increasing lag length. To set the scale parameters σ_i^2 we follow common practice (see e.g. Litterman, 1986; Sims and Zha, 1998) and fix them to the variance of the residuals from a univariate AR(4) model for the variables, computed for the estimation sample.

Following Cogley and Sargent (2005), we use an uninformative prior for the elements in the matrix A :

$$\underline{\mu}_{a,i} = 0, \quad \underline{\Omega}_{a,i} = 1000^2 \cdot I_{i-1}. \quad (31)$$

In line with other studies such as Cogley and Sargent (2005), we make the priors on the volatility-related parameters loosely informative. Specifically, the prior scale and shape parameters for the elements \underline{s}_i in S and for ϕ are:

$$\underline{s}_i = \hat{s}_{i,OLS}, \quad d_s = 3, \quad (32)$$

$$\underline{\phi} = 0.01, \quad d_\phi = 10. \quad (33)$$

Our choices for the prior degrees of freedom and mean for ϕ are based on the results of some Monte Carlo experiments described in the next subsection. We have verified that using somewhat higher or lower values for both the prior mean and degrees of freedom doesn't much change the overall model fit as measured by the log predictive likelihood.

Finally, the prior moments for the initial value of the volatility process are:

$$\underline{\mu}_\lambda = \log \hat{\lambda}_{0,OLS}, \quad \underline{\Omega}_\lambda = 4. \quad (34)$$

In the prior for S , the mean $\hat{s}_{i,OLS}$ is set on the basis of residual variances obtained from AR models fit with the estimation sample (in line with common practice). For each variable, we estimate an AR(4) model. For each $j = 2, \dots, n$, we regress the residual from the AR model for j on the residuals associated with variables 1 through $j-1$ and compute the error variance (this step serves to filter out covariance as reflected in the A matrix). Letting $\hat{\sigma}_{i,0}^2$ denote these error variances, we set the prior mean on the relative volatilities at $\hat{s}_{i,OLS} = \hat{\sigma}_{i,0}^2/\hat{\sigma}_{1,0}^2$ for $i = 2, \dots, n$. In the prior for log volatility in period 0, we use just the first variable of

the VAR, following the same steps in obtaining residual variances $\hat{\sigma}_{1,0}^2$, but with data from a training sample of the 40 observations preceding the estimation sample.³ We set the prior mean of log volatility in period 0 at $\log \hat{\lambda}_{0,OLS} = \log \hat{\sigma}_{1,0}^2$.

In light of the potential for a unit root in volatility to adversely affect the accuracy of longer-horizon forecasts, we also consider below some results for a version of the BVAR-CSV model with stationary volatility, as given in (24). We use a prior very tightly centered on a persistent but stationary volatility process, setting the prior mean and variance of the slope coefficient ψ_1 at, respectively, 0.95 and 0.00001. The prior mean and variance of the intercept ψ_0 are set to 0.0 and 0.5, respectively.

3.2 Monte Carlo evidence on properties of BVAR-CSV estimates

To check the behavior of our algorithm for estimating the BVAR-CSV model and explore the effects of different settings for the prior on the volatility parameter ϕ , we estimated the model in 100 artificial datasets generated from an 8-variable VAR(2) with common stochastic volatility. The parameters of the DGP were roughly set to correspond to actual estimates of the BVAR-CSV in US data. In these experiments, we consider both the random walk and stationary versions of the volatility process. In the model with random walk volatility, we used a ϕ of 0.025 in the data-generating process. In the model with an AR(1) process for volatility, we set the AR(1) coefficient at 0.95 and ϕ at 0.05. In the artificial datasets, we estimate the models with a larger sample of 186 observations corresponding to our full sample for the US and a smaller sample of 104 observations corresponding to the data that would be available for estimation a few years into our forecast sample. For both models and both samples, we consider estimates obtained under different combinations of the settings of the prior degrees of freedom d_ϕ and the prior mean $\underline{\phi}$.

In the interest of brevity, we will focus on the behavior of the estimates of ϕ (we have verified that the estimated time series of volatilities reasonably match up to the time series of true volatilities underlying each artificial dataset). The table below reports the median estimate of ϕ in 100 Monte Carlo datasets, where the estimate of ϕ for each dataset is the posterior mean. Consider the first panel, which provides results from estimating the BVAR-CSV model in artificial data generated from a VAR with common stochastic volatility in which the true ϕ is equal to 0.025. As might be expected, the posterior mean estimates of ϕ are a little sensitive to the prior, more so in the smaller sample than in the larger sample. In

³In the real-time forecasting analysis, for the vintages in which a training sample of 40 observations is not available, the prior is set using the training sample estimates available from the most recent vintage with 40 training sample observations.

general, using a very low prior mean for ϕ consistently delivers an estimate well below the true value, while using a slightly high prior mean consistently delivers an posterior mean that is a little above the true value. With this random walk volatility specification, using a prior mean of either 0.01 or 0.035 may be seen as delivering a posterior mean that is reasonably close to the true value of ϕ , without much sensitivity to whether the degrees of freedom is set at 5 or 10.

Now consider the second panel, which provides results from estimating the stationary version of the BVAR-CSV model in artificial data generated from a VAR with common stochastic volatility that follows an AR(1) process with a true ϕ equal to 0.05. For this model specification, the posterior mean estimates of ϕ generally exceed the true value, more so in the smaller sample than the larger. For this specification, too, the posterior mean estimates of ϕ are a little sensitive to the prior, particularly the degrees of freedom for ϕ . Setting the degrees of freedom at 10 delivers a posterior mean estimate of ϕ closer to the true value than does setting the degrees of freedom of 5.

Based on this evidence, to try to strike a balance across different models while using a single prior for simplicity, in all our estimates of BVARs with common stochastic volatility we set the prior degrees of freedom d_ϕ at 10 and the prior mean $\underline{\phi}$ at 0.01. The Monte Carlo evidence suggests that, were the true model to have common stochastic volatility that follows a random walk, this prior would deliver posterior estimates reasonably close to truth, where truth includes a fair amount of movement over time in volatility. The evidence also suggests that were the true model to include stationary common volatility, this prior delivers estimates of ϕ as close as it may be possible to get without making the prior very tight.

**Medians of Posterior Mean Estimates of ϕ
in 100 Artificial Datasets**

Estimation sample of 186 obs. Estimation sample of 104 obs.

BVAR-CSV, ϕ in DGP = 0.025

| | $\underline{\phi} = 0.001$ | $\underline{\phi} = 0.01$ | $\underline{\phi} = 0.035$ | $\underline{\phi} = 0.001$ | $\underline{\phi} = 0.01$ | $\underline{\phi} = 0.035$ |
|---------------|----------------------------|---------------------------|----------------------------|----------------------------|---------------------------|----------------------------|
| $d_\phi = 5$ | 0.019 | 0.022 | 0.030 | 0.013 | 0.021 | 0.033 |
| $d_\phi = 10$ | 0.013 | 0.019 | 0.031 | 0.006 | 0.017 | 0.034 |

BVAR-stationary CSV, ϕ in DGP = 0.05

| | $\underline{\phi} = 0.001$ | $\underline{\phi} = 0.01$ | $\underline{\phi} = 0.035$ | $\underline{\phi} = 0.001$ | $\underline{\phi} = 0.01$ | $\underline{\phi} = 0.035$ |
|---------------|----------------------------|---------------------------|----------------------------|----------------------------|---------------------------|----------------------------|
| $d_\phi = 5$ | 0.078 | 0.080 | 0.083 | 0.093 | 0.096 | 0.103 |
| $d_\phi = 10$ | 0.067 | 0.069 | 0.077 | 0.075 | 0.079 | 0.089 |

3.3 Specifics on priors: BVAR-SV and BVAR models

For the BVAR-SV model, we use a conventional Minnesota prior, without cross-variable shrinkage:

$$\bar{\mu}_\Pi \text{ such that } E[\Pi_l^{(ij)}] = 0 \quad \forall i, j, l \quad (35)$$

$$\underline{\Omega}_\Pi \text{ such that } V[\Pi_l^{(ij)}] = \begin{cases} \frac{\theta^2}{l^2} \sigma_i^2 \sigma_j^2 & \text{for } l > 0 \\ \varepsilon^2 \sigma_i^2 & \text{for } l = 0 \end{cases} \quad (36)$$

Consistent with our prior for the BVAR-CSV model, we set $\theta = 0.2$ and $\varepsilon = 1000$, and we set the scale parameters σ_i^2 at estimates of residual variances from AR(4) models from the estimation sample.

In the prior for the volatility-related components of the model, we follow an approach similar to that for the BVAR-CSV model. Broadly, our approach to setting volatility-related priors is similar to that used in such studies as Clark (2011), Cogley and Sargent (2005), and Primiceri (2005). The prior for A is uninformative, as described above. For the prior on each ϕ_i , we follow Clark (2011) and use a mean of 0.035 and 3 degrees of freedom. For the initial value of the volatility of each equation i , we use

$$\underline{\mu}_{\lambda,i} = \log \hat{\lambda}_{i,0,OLS}, \quad \underline{\Omega}_\lambda = 4. \quad (37)$$

To obtain $\log \hat{\lambda}_{i,0,OLS}$, we use a training sample of 40 observations preceding the estimation

sample to fit AR(4) models for each variable and, for each $j = 2, \dots, n$, we regress the residual from the AR model for j on the residuals associated with variables 1 through $j - 1$ and compute the error variance (this step serves to filter out covariance as reflected in the A matrix). Letting $\hat{\sigma}_{i,0}^2$ denote these error variances, we set the prior mean of log volatility in period 0 at $\log \hat{\lambda}_{i,0,OLS} = \log \hat{\sigma}_{i,0}^2$.⁴

3.4 MCMC Algorithm

We estimate the BVAR-CSV model with a five-step Metropolis-within-Gibbs MCMC algorithm.⁵ The Metropolis step is used for volatility estimation, following Cogley and Sargent (2005), among others. The other steps rely on Gibbs samplers. In order to facilitate the description of some of the steps, we rewrite the VAR as in Cogley and Sargent (2005) and Primiceri (2005):

$$A(y_t - \Pi_0 - \Pi(L)y_{t-1}) \equiv \tilde{v}_t = \lambda_t^{0.5} S^{1/2} \epsilon_t. \quad (38)$$

Step 1: Draw the matrix of VAR coefficients Π conditional on A , S , ϕ , and Λ , using the conditional (normal) distribution for the posterior given in equation (10).

Step 2: Draw the coefficients in A conditional on Π , S , ϕ , and Λ , using the conditional (normal) distribution for the posterior given in (11). This step follows the approach detailed in Cogley and Sargent (2005), except that, in our model, the VAR coefficients Π are constant over time.

Step 3: Draw the elements of S conditional on Π , A , ϕ , and Λ , using the conditional (IG) distribution for the posterior given above in (12)

Using equation (38), for each equation $i = 2, \dots, n$, we have that $\tilde{v}_{i,t}/\lambda_t^{0.5} = s_i^{1/2} \epsilon_{i,t}$. We can then draw s_i using a posterior distribution that incorporates information from the sample variance of $\tilde{v}_{i,t}/\lambda_t^{0.5}$.

Step 4: Draw the time series of volatility λ_t conditional on Π , A , S , and ϕ , using a Metropolis step.

Step 5: Draw the variance ϕ , conditional on Π , A , S , and Λ , using the conditional (IG) distribution for the posterior given in (13)

We estimate the BVAR-SV model with a similar algorithm, modified to drop the step for sampling S and to draw time series of volatilities of all variables, not just common volatility.

⁴In the real-time forecasting analysis, for the vintages in which a training sample of 40 observations is not available, the prior is set using the training sample estimates available from the most recent vintage with 40 training sample observations.

⁵While not detailed in the interest of brevity, we follow Cogley and Sargent (2005) in including in the algorithm checks for explosive autoregressive draws, rejecting explosive draws and re-drawing to achieve a stable draw.

We estimate the BVAR by sampling from Normal-inverted Wishart conjugate posteriors for the error variance matrix and VAR coefficients.

In all cases, we obtain forecast distributions by sampling as appropriate from the posterior distribution. For example, in the case of the BVAR-CSV model, for each set of draws of parameters, we: (1) simulate volatility time paths over the forecast interval using the random walk structure of log volatility; (2) draw shocks to each variable over the forecast interval with variances equal to the draw of Σ_{t+h} ; and (3) use the VAR structure of the model to obtain paths of each variable. We form point forecasts as means of the draws of simulated forecasts and density forecasts from the simulated distribution of forecasts. Conditional on the model, the posterior distribution reflects all sources of uncertainty (latent states, parameters, and shocks over forecast interval).

4 Empirical results with US data

4.1 Data and design of the forecast exercise

In most of our analysis, we consider models of a maximum of eight variables, at the quarterly frequency: growth of output, growth of personal consumption expenditures (PCE), growth of business fixed investment (in equipment, software, and structures, denoted BFI), growth of payroll employment, the unemployment rate, inflation, the 10-year Treasury bond yield, and the federal funds rate. This particular set of variables was chosen in part on the basis of the availability of real-time data for forecast evaluation. Consistent with such studies as Clark (2011), we also consider a four-variable model, in output, unemployment, inflation, and the funds rate. We also examine forecasts from a 15-variable model, similar to the medium-sized model of Koop (2012), using his data.

For the 4- and 8-variable models, we consider both full-sample estimates and real-time estimates. Our full-sample estimates are based on current vintage data taken from the FAME database of the Federal Reserve Board. The quarterly data on unemployment and the interest rates are constructed as simple within-quarter averages of the source monthly data (in keeping with the practice of, e.g., Blue Chip and the Federal Reserve). Growth and inflation rates are measured as annualized log changes (from $t - 1$ to t).

For the 15-variable model, we report only forecasts based on current vintage data, using data from Koop (2012). The set of variables is listed in Tables 9 and 10 (please see Koop’s paper for additional details). The data are transformed as detailed in Koop (2012).

The forecast evaluation period runs from 1985:Q1 through 2010:Q4 (2008:Q4 for the 15-

variable analysis), and the forecasting models are estimated using data starting in 1965:Q1. We report results for forecasts at horizons of 1, 2, 4, 8, and 12 quarters ahead.

In the real-time forecast analysis of models with 4 or 8 variables, output is measured as GDP or GNP, depending on data vintage. Inflation is measured with the GDP or GNP deflator or price index. Quarterly real-time data on GDP or GNP, PCE, BFI, payroll employment, and the GDP or GNP price series are taken from the Federal Reserve Bank of Philadelphia’s Real-Time Data Set for Macroeconomists (RTDSM). For simplicity, hereafter “GDP” and “GDP price index” refer to the output and price series, even though the measures are based on GNP and a fixed weight deflator for much of the sample. In the case of unemployment, the Treasury yield, and the fed funds rate, for which real-time revisions are small to essentially non-existent, we simply abstract from real-time aspects of the data, and we use current vintage data.

Our analysis of real-time forecasts uses real-time data vintages from 1985:Q1 through 2011:Q2. As described in Croushore and Stark (2001), the vintages of the RTDSM are dated to reflect the information available around the middle of each quarter. Normally, in a given vintage t , the available NIPA data run through period $t - 1$. For each forecast origin t starting with 1985:Q1, we use the real-time data vintage t to estimate the forecast models and construct forecasts for periods t and beyond.⁶

As discussed in such sources as Croushore (2005), Romer and Romer (2000), and Sims (2002), evaluating the accuracy of real-time forecasts requires a difficult decision on what to take as the actual data in calculating forecast errors. The GDP data available today for, say, 1985, represent the best available estimates of output in 1985. However, output as defined and measured today is quite different from output as defined and measured in 1970. For example, today we have available chain-weighted GDP; in the 1980s, output was measured with fixed-weight GNP. Forecasters in 1985 could not have foreseen such changes and the potential impact on measured output. Accordingly, we follow studies such as Clark (2011), Faust and Wright (2009), and Romer and Romer (2000) and use the second available estimates of GDP/GNP, PCE, BFI, payroll employment, and the GDP/GNP deflator as actuals in evaluating forecast accuracy. In the case of h -step ahead forecast made for period $t + h - 1$ with vintage t data ending in period $t - 1$, the second available estimate is normally taken from the vintage $t + h + 1$ data set. In light of the abstraction from real-time revisions in unemployment and the interest rates, for these series the real-time data correspond to the final vintage data.

Finally, note that, throughout our analysis, we include four lags in all of our models.

⁶The starting point of the model estimation sample is always 1965:Q1.

With Bayesian methods that naturally provide shrinkage, many prior studies have used the same approach of setting the lag length at the data frequency (e.g., Banbura, Giannone, and Reichlin (2010), Clark (2011), Del Negro and Schorfheide (2004), Koop (2012), and Sims (1993)).

4.2 Results on MCMC convergence properties and CPU time requirements

We begin with documenting the convergence properties of our MCMC algorithm for the BVAR-CSV model compared to existing algorithms for the BVAR-SV and BVAR models and with comparing CPU time requirements for each type of model.

Table 1 reports summary statistics for the distributions of inefficiency factors (IF) for the posterior estimates of all groups of model parameters. We consider 4-variable and 8-variable BVARs with independent and common volatility (CSV and stationary CSV), using skip intervals of 5, 10, or 20 draws, intended to yield reasonable mixing properties (sufficiently low IF's). As noted above, all BVARs have four lags. The IF is the inverse of the relative numerical efficiency measure of Geweke (1992), and can be written as $1 + 2 \sum_{k=1}^{\infty} \rho_k$, where ρ_k is the k -th order autocorrelation of the chain of retained draws. To estimate the IF we use the Newey and West (1987) kernel and a bandwidth of 4 percent of the sample of draws.

These convergence measures reveal three broad patterns: first, the IF's tend to rise as the model size increases from 4 to 8 variables; second, the IF's are lower for the BVAR-CSV than for the BVAR-SV, so that the skip interval can be lower for BVAR-CSV than for BVAR-SV; third, in general the IF's are lower for stationary CSV than for CSV, but the differences are small. More specifically, the table indicates that for the 4-variable BVAR-SV, the highest IF is for the set of parameters ϕ_i , $i = 1, \dots, n$, the innovation variances in the random walk models for the log variances $\lambda_{i,t}$. For the 4-variable BVAR-CSV, the highest IF is instead for the scaling matrix S (which allows the variances of each variable to differ by a factor that is constant over time). For the BVAR-SV the IFs are substantially reduced when the skip interval increases from 10 to 20, from 5 to 10 for the BVAR-CSV, and all the values are anyway lower than 20, which is typically regarded as satisfactory (see e.g. Primiceri (2005)). For the 8-variable BVAR-CSV the IF is again highest for S . In this case a skip interval of 10 would still produce IFs much lower than 20, and even lower than 10 for stationary CSV.

Based on this evidence, in all subsequent analysis in the paper, our results are based on 5000 retained draws, obtained from a larger sample of draws in which we set the skip interval

as follows: BVAR-CSV, skip interval of 5 for both stationary and integrated volatility; BVAR-SV, skip interval of 10. This is sufficient to have good mixing properties. In all cases, we initialize the MCMC chain with 5000 draws, which are discarded.

As to the CPU time requirements for the different models, Table 2 shows that they increase substantially when increasing the number of variables and/or adding stochastic volatility to the BVAR.⁷ As noted above, a key determinant of the CPU time requirements is the size of the posterior variance matrix that must be computed for sampling the VAR coefficients; the size of the matrix is a function of the square of the number of variables in the model. The CPU time for models with independent stochastic volatilities can be considerable. For our quarterly data sample of 1965:Q1-2011:Q2, it takes about 84 minutes to estimate the 4-variable BVAR-SV and 880 minutes (14.7 hours) to estimate the 8-variable BVAR-SV. The time requirement for the 8-variable BVAR-SV makes it infeasible to consider the model in a real-time forecast evaluation. Moreover, this time requirement likely deters other researchers and practitioners from using the independent stochastic volatility specification with models of more than a few variables (a deterrence evident in the fact that existing studies have not considered more than a handful of variables).

Introducing stochastic volatility through our common volatility specification yields significant computational gains relative to the independent volatility specification. With 4 variables, the BVAR-CSV estimation takes about 19 minutes, compared to almost 84 for the BVAR-SV. With 8 variables, the BVAR-CSV estimation takes nearly 47 minutes, compared to 879.5 minutes (14.7 hours) for the BVAR-SV, and only slightly more (50 minutes) for the stationary CSV.

It is worthwhile mentioning that, for all specifications, the CPU time results in Table 2 are based on a total of 105,000 draws. However, as mentioned, because the CSV estimation has better mixing properties than SV, CSV has an even stronger advantage than what the Table suggests.

As noted earlier in the paper, these computational gains stem from the Kroneker structure of the coefficient variance matrix that results from having a single multiplicative volatility factor and the coefficient prior developed above. With these computational gains, we can readily consider stochastic volatility in the form of common volatility in our real-time forecasting analysis, for models of 4, 8, or 15 variables.⁸

⁷We estimated the models with 2.93 GHZ processors, using the RATS software package.

⁸While we don't include the result in the table because the estimation sample isn't the same, estimating the BVAR-CSV with 15 variables takes about 162 minutes.

4.3 Full-sample results

Having established the computational advantages of our proposed common volatility specification, we turn now to a comparison of volatility estimates from the common volatility model (BVAR-CSV) versus a model that allows independent volatility processes for each variable (BVAR-SV). We consider both 4-variable and 8-variable models.

Figure 1 reports the volatility estimates for the 4-variable BVAR-SV, which, despite the independence across variables, are fairly similar in shape across variables, with higher volatility in the 1970s, a marked decrease starting in the early 1980s, in line with the literature on the Great Moderation, and a new increase with the onset of the financial crisis. Figures 2 and 3 show the same estimates for, respectively, the BVAR with CSV and stationary CSV, which are of course equal across variables apart from a scaling factor. These common volatility estimates follow paths quite similar to those obtained from the BVAR-SV model, with some more variability over time for the stationary CSV.

Figures 4-6 present corresponding estimates for the 8-variable specifications. The shape of Figure 4's volatility estimates from the BVAR-SV model that allows independent volatilities across variables are again similar across variables. The similarity is reflected in high correlations (ranging from 0.58 to 0.97) of each volatility estimate with the first principal component computed from the posterior median volatility estimates of each variable (the principal component is reported in Figure 7).⁹ The common volatility estimates from the BVAR-CSV and stationary CSV models follow paths similar to the BVAR-SV estimates. Figure 7 shows that the common volatility estimate closely resembles the first principal component computed from the posterior median volatility estimates obtained with the BVAR-SV model. There are just some differences in the first part of the sample, where the stationary CSV estimate in particular is more volatile. The overall correlation between the common volatility estimates and the principal component is 0.94 and 0.90, for the CSV and stationary CSV estimates respectively.

Based on these results, it seems that, in applications to at least standard macroeconomic VARs in US data, our common stochastic volatility specification can effectively capture time variation in conditional volatilities. Of course, in real time, reliable estimation of volatility may prove to be more challenging, in part because, at the end of the sample, only one-sided filtering is possible, and in part because of data revisions. Accordingly, in Figures 8-12 we compare time series of volatility estimates from five different real-time data vintages. In the

⁹To compute the principal component, we take the posterior median estimates of volatility from the BVAR-SV model, standardize them, and compute the principal component as described in such studies as Stock and Watson (2002).

4-variable case, we consider estimates from the BVAR-CSV, stationary CSV and BVAR-SV models. In the 8-variable case, in light of the computational burden of the BVAR-SV model, we only consider results for the BVAR-CSV and stationary CSV specifications.

Three main messages emerge from the real-time estimates in Figures 8-12. First, the commonality in the volatility estimates for the four variables in the BVAR-SV is confirmed for each vintage (Figure 8). For GDP growth and inflation, data revisions can shift the estimated volatility path but typically have little effect on the contours of the volatility estimate. The larger shifts in the volatility paths tend to be associated with benchmark or large annual revisions of the NIPA accounts. In the case of the unemployment and federal funds rates, volatility estimates tend to change less across vintages, presumably because the underlying data are not revised. Second, the BVAR-CSV and stationary CSV volatility estimates for the 4-variable model are also quite similar across vintages (Figures 9-10), even more than in the SV case. Finally, applied to the 8-variable model, the BVAR-CSV and stationary CSV specifications yield volatility estimates that follow very similar patterns across vintages (Figures 11-12). Again, contours are very similar across vintages, although data revisions can move the levels of volatility across data vintages, less so in the stationary CSV case.

To assess more generally how the competing models fit the full sample of data, we follow studies such as Geweke and Amisano (2010) in using 1-step ahead predictive likelihoods. The predictive likelihood is closely related to the marginal likelihood: the marginal likelihood can be expressed as the product of a sequence of 1-step ahead predictive likelihoods. In our model setting, the predictive likelihood has the advantage of being simple to compute. For model M_i , the log predictive likelihood is defined as

$$\text{LPL}(M_i) = \sum_{t=t_0}^T \log p(y_t^o | y^{(t-1)}, M_i), \quad (39)$$

where y_t^o denotes the observed outcome for the data vector y in period t , $y^{(t-1)}$ denotes the history of data up to period $t-1$, and the predictive density is multivariate normal. Finally, in computing the log predictive likelihood, we sum the log values over the period 1980:Q1 through 2011:Q2.

The log predictive likelihood (LPL) estimates reported in Table 3 show that our proposed common volatility specification significantly improves the fit of a BVAR. In the four-variable case, the LPL of the BVAR-CSV model is about 108 points higher than the LPL of the constant volatility BVAR (in log units, a difference of just a few points would imply a

meaningful difference in fit and, in turn, model probabilities). In the eight-variable case, the BVAR-CSV also fits the data much better than the BVAR, with a LPL difference of about 82 points. In the four-variable case, extending the volatility specification to permit independent volatilities for each variable offers comparable improvements in fit: the LPL of the BVAR-SV is about 2 points higher than the BVAR-CSV, but 2 points lower than the stationary CSV specification. In the eight-variable BVAR, the CSV specification is instead slightly better than the stationary CSV specification, but the difference is 1 point only.¹⁰

Hence, in addition to sizable gains in terms of CPU time with respect to SV, our proposed BVAR-CSV specifications also yield much of the gain in model fit to be achieved by allowing stochastic volatility.

4.4 Real-time forecast results

In this subsection we compare the relative performance of the 4-variable BVAR-SV model and the 4- and 8- variable BVAR-CSV models, starting with point forecasts and moving next to density forecasts. We also include univariate AR(2) models in the context, since they are known to be tough benchmarks, but our main focus is on the relative performance of BVARs with constant volatility versus (stationary or integrated) common and independent stochastic volatility.¹¹ As mentioned above, the evaluation sample is 1985Q1-2010Q4, we consider four forecast horizons, and the exercise is conducted in a real time manner, using recursive estimation with real time data vintages.

Table 4 reports the root mean squared error (RMSE) of each model relative to that of the BVAR, and the absolute RMSE for the BVAR for the 4-variable case (including GDP growth, unemployment, GDP inflation and the Fed funds rate). Hence, entries less than 1 indicate that the indicated model has a lower RMSE than the BVAR. Table A4 in the Appendix contains the same results but using AR(2) models as benchmarks. To provide a rough gauge of whether the RMSE ratios are significantly different from 1, we use the Diebold and Mariano (1995) t-statistic for equal MSE, applied to the forecast of each model relative to the benchmark. Our use of the Diebold-Mariano test with forecasts that are, in many cases, nested is a deliberate choice. Monte Carlo evidence in Clark and McCracken (2011a,b) indicates that, with nested models, the Diebold-Mariano test compared against normal critical values can be viewed as a somewhat conservative (conservative in the sense of

¹⁰We don't report LPL results for the 8-variable BVAR-SV specification because the CPU time requirements for the model effectively rule out using it for forecast evaluation and for computing the LPL.

¹¹The AR(2) models are estimated with the same Normal-inverted Wishart approach we have described for the BVAR. However, to correspond to common use of least-squares estimated AR models as benchmarks, we make the prior uninformative, by setting the shrinkage hyperparameter θ set at 1000.

tending to have size modestly below nominal size) test for equal accuracy in the finite sample. As most of the alternative models can be seen as nesting the benchmark, we treat the tests as one-sided, and only reject the benchmark in favor of the null (i.e., we don't consider rejections of the alternative model in favor of the benchmark). Differences in accuracy that are statistically different from zero are denoted by one, two, or three asterisks, corresponding to significance levels of 10%, 5%, and 1%, respectively. The underlying p-values are based on t-statistics computed with a serial correlation-robust variance, using a rectangular kernel, $h - 1$ lags, and the small-sample adjustment of Harvey, Leybourne, and Newbold (1997).

Four main comments can be made based on the figures in Table 4 (and A4). First, including independent stochastic volatility in the BVAR model systematically improves the forecasts, and in general the gains are statistically significant. Second, constraining the volatility to be common in general further improves the forecasts. The BVAR-CSV produces lower RMSEs than the BVAR-SV in 13 out of 20 cases, with the BVAR-SV doing slightly better only for the interest rate. While the advantages of the BVAR-CSV model over the BVAR-SV specification are small or modest, they are consistent. Third, the performance of integrated and stationary CSV is quite similar. Fourth, the AR benchmark produces the lowest RMSEs for GDP growth and inflation. Instead, the BVARs with volatility are better for inflation and the Fed funds rate, and the gains with respect to the AR are statistically significant.

Table 5 provides corresponding results for the 8-variable case, adding consumption, investment, employment, and the Treasury yield to the variable set. Table 5 shows two main results. First, the larger BVAR is better than the 4-variable BVAR in 13 out of 20 cases. This is in line with several findings in the literature showing that a larger information set generally yields more accurate forecasts — see, e.g., Banbura, Giannone, and Reichlin (2010), Carriero, Clark and Marcellino (2011), Carriero, Kapetanios, and Marcellino (2011), and Koop (2012). More precisely, compared to the small model, the large system consistently yields more accurate point forecasts of GDP growth and unemployment, except at the 12 quarter horizon, while the large model is beaten at short horizons for GDP inflation and the Fed funds rate. Second, all the main patterns found using the 4-variable specifications are confirmed. In particular, the introduction of stochastic volatility improves the point forecasts, especially at short horizons, and often with significant gains. Compared to the BVAR, our proposed BVAR-CSV model lowers the RMSE in at least 65% of the cases (26 out of 40, rising to 29 out of 40 for the stationary CSV), and in many cases the gains are statistically significant. Constraining the volatility to be common again further improves the forecasts. The BVAR-CSV produces lower RMSEs than the BVAR-SV in 27 out of 40

cases, including the short term forecasts of the federal funds rate (which were better forecast by the BVAR in the 4-variable case). Also in this case the performance of integrated and stationary CSV is similar. The BVAR with stationary CSV fares rather well also against the AR benchmark, beating the AR models in about 60% of the cases, slightly less for the CSV, even though BVARs generally have a difficult time beating AR models in data since 1985.

The RMSE, while informative and commonly used for forecast comparisons, is based on the point forecasts only and therefore ignores the rest of the forecast density. Of course the introduction of drifting volatility in a VAR makes it particularly well suited for density forecasting; for a 4-variable model, Clark (2011) shows that adding independent stochastic volatilities to a VAR significantly improves density forecasts. The overall calibration of the density forecasts can be measured with log predictive density scores, motivated and described in such sources as Geweke and Amisano (2010). At each forecast origin, we compute the log predictive score using the quadratic approximation of Adolfson, Linde, and Villani. (2007).¹² Specifically, we compute the log score with:

$$s_t(y_{t+h}^o) = -0.5 \left(n \log(2\pi) + \log |V_{t+h|t}| + (y_{t+h}^o - \bar{y}_{t+h|t})' V_{t+h|t}^{-1} (y_{t+h}^o - \bar{y}_{t+h|t}) \right), \quad (40)$$

where y_{t+h}^o denotes the observed outcome, $\bar{y}_{t+h|t}$ denotes the posterior mean of the forecast distribution, and $V_{t+h|t}$ denotes the posterior variance of the forecast distribution.

Table 6 reports differences in log scores with respect to the BVAR for the 4-variable case, such that entries greater than 0 indicate that the model has a better average log score (better density forecast) than the benchmark BVAR model. Table A6 in the Appendix contains the same results but using the AR as benchmark. To provide a rough gauge of the statistical significance of differences in average log scores, we use the Amisano and Giacomini (2007) t-test of equal means, applied to the log score for each model relative to the benchmark BVAR forecast. We view the tests as a rough gauge because, with nested models, the asymptotic validity of the Amisano and Giacomini (2007) test requires that, as forecasting moves forward in time, the models be estimated with a rolling, rather than expanding, sample of data. As most of the alternative models can be seen as nesting the benchmark, we treat the tests as one-sided, and only reject the benchmark in favor of the null (i.e., we don't consider rejections of the alternative model in favor of the benchmark). Differences in average scores that are statistically different from zero are denoted by one,

¹²In some limited checks, we obtained qualitatively similar results with some other approaches to computing the predictive score. In our application, though, the quadratic approximation is easier to use.

two, or three asterisks, corresponding to significance levels of 10%, 5%, and 1%, respectively. The underlying p-values are based on t-statistics computed with a serial correlation-robust variance, using a rectangular kernel, $h - 1$ lags, and the small-sample adjustment of Harvey, Leybourne, and Newbold (1997).

The results in Table 6 yield four main conclusions. First, consistent with Clark (2011), compared to a BVAR with constant volatilities, adding independent stochastic volatility to a BVAR model almost always improves the density forecasts, with some exceptions for 2- and 3-year ahead forecasts. Second, as in the RMSE comparison, constraining the volatility to be common in general further improves the forecasts, in particular when using the stationary CSV specification. The BVAR-CSV beats the BVAR-SV model in 14 out of 20 cases, increasing to 16 out of 20 cases for the stationary CSV, with the BVAR-SV model doing slightly better only for short term forecasts for inflation and the interest rate. Third, the relative performance of the AR model deteriorates with respect to the RMSE comparison. It is now better than the BVAR-CSV in only 6 out of 20 cases (5 out of 20 for the stationary CSV), and better than the BVAR-SV in only 7 out of 20 cases. Fourth, the system scores are systematically better for the BVAR with stationary CSV, except at the 3-year horizon where the BVAR is better.

Table 7 provides corresponding results for the 8-variable case. Focusing first on the original four variables (GDP growth, unemployment, GDP inflation and the Fed funds rate), the 8-variable model often improves on the density forecast accuracy of the 4-variable model. As in the RMSE comparison, the larger BVAR is generally better for growth and unemployment, worse for inflation, and better for the Fed funds rate but only at medium and long horizons. Compared to the BVAR benchmark, our proposed model with common stochastic volatility generally improves the accuracy of density forecasts, although more so at short horizons than long horizons. In particular, the BVAR-CSV is clearly better than the simple BVAR for $h = 1, 2$, improving average scores in 14 out of 16 cases (15 out of 16 for the stationary CSV). But at the longer horizons of 1- up to 3- year ahead, log scores from the BVAR-CSV model are worse than those from the BVAR in 18 out of 24 cases (same for the stationary CSV). Constraining the volatility to be common does not lead to consistent losses, nor gains. The BVAR-CSV model outperforms the BVAR-SV in 18 cases out of 40 (rising to 20 out of 40 for the stationary CSV). With respect to the results obtained for point forecasts, these results show a relatively worse performance of the BVAR-CSV model, but there are still not significant nor systematic losses in using the BVAR-CSV rather than the BVAR-SV. The BVAR with stationary CSV performs consistently well for some selected variables such as Consumption, BFI, Employment and Unemployment. Finally, by the

density metric, the BVAR-CSV specification beats the AR models in more than 50% of the cases (22 out of 40, same for the stationary CSV, and often concentrated for $h=1,2$), and in most of these cases the gains are statistically significant.

The log predictive density scores measure the overall calibration of the density forecasts, but sometimes the tail behavior can be of specific interest, for example to assess whether relatively large or small values of the target variable can be also accurately predicted. Hence, for the 4- and 8-variable BVARs, we have also computed two-sided and one-sided coverage rates, and in the Appendix we report tables with either the probability of outcome outside the 70% interval (Appendix Tables A16 and A19), or the probability of outcome in the 15% left or right tails (Appendix Tables A17, A18, A20, A21). The main insights from the results are the following. First, the coverage rates are better for the BVAR than for the univariate AR models. Second, adding SV to the BVAR helps substantially. Third, it is unclear whether using CSV is better than SV, but generally it is not worse. Fourth, and perhaps not surprisingly, the worst performance in terms of coverage is for the Federal Funds rate and for unemployment. Fifth, when looking separately at the performance in the right and left tails, the positive role of CSV emerges again, but even with CSV the empirical coverages are often different from the theoretical 15% values.

In light of the seemingly high volatility of the period surrounding the recent (2007-2009) financial crisis and recession, it is natural to ask how the relative performance of our competing models is affected by this period. Hence, we have computed the average log predictive scores for the 4- and 8-variable VARs over the period 1985Q1-2008Q2, dropping the worst part of the recession. In the interest of brevity, we simply summarize results here, with detailed figures provided in the Appendix Tables A13 and A14. Three main comments can be made based on the 4-variable BVARs. First, the absolute forecast density performance of the BVAR systematically improves when the most problematic crisis quarters are excluded. Second, the BVAR-SV remains preferable to the BVAR, and its relative performance improves also at longer horizons. Admittedly, this means that including stochastic volatility in our BVARs tended to harm density forecast accuracy during the crisis. The reason is that, reflecting the Great Moderation, the forecast distribution was narrower for BVARs with time-varying volatility than BVARs with constant volatility. In turn, some of the outcomes of the recession were further in the tails of forecast distributions of models with time-varying volatility than of models with constant volatilities. Third, the BVAR with stationary CSV is even better than the BVAR-SV. Finally, in the 8-variable BVARs, CSV is even more helpful than in the smaller models, while the relative performance of the stationary and integrated CSV is less clear-cut.

In summary, the estimation analysis confirms that estimated volatilities are often similar across variables when estimated unrestrictedly, and therefore provides support for our common volatility specification. The forecast results are also favourable, in the sense that the typical ranking according to both the RMSE and the log predictive score is BVAR-CSV, BVAR-SV and BVAR, with the stationary CSV overall performing slightly better than CSV though in general the differences are minor. The ranking is clear-cut in the 4-variable case and when using RMSE, while the performance is more mixed in the 8-variable case and when using the log predictive score as the evaluation criterion. However, even in these cases the BVAR-CSV is preferred in the majority of cases, and in particular for shorter forecast horizons. The relative performance of the BVAR-CSV further improves when the sample ends in 2008Q2.

4.5 Current-vintage forecasting results for 15-variable model

In light of recent evidence that medium-scale BVARs often yield forecasts more accurate than small BVARs (e.g., Banbura, Giannone, and Reichlin (2010) and Koop (2012)), we also consider forecasts from 15-variable models. In this exercise, we use Koop’s data (2012) and variable transformations, for the variables listed in Tables 8 and 9. In the interest of brevity, we compare just a BVAR-CSV (with integrated or stationary volatility) and simple BVAR specification.¹³ Tables 8 and 9 provide the results on RMSEs and average log scores, respectively.

Consistent with our real time results for smaller models, the RMSE results in Table 8 show that adding common stochastic volatility to the 15-variable model yields fairly consistent, small improvements in the accuracy of point forecasts, concentrated at shorter horizons. More specifically, at forecast horizons of 1 or 2 quarters, the RMSE ratios are below 1 for almost all variables. In contrast, at a forecast horizon of 8 or 12 quarters, the RMSE ratios are slightly above 1 for most variables, though the differences with the BVAR are small.

Similarly, the average score results in Table 9 show that including common stochastic volatility in a model typically improves forecast accuracy at shorter forecast horizons. At the 1-quarter horizon, the BVAR-CSV model yields a better score for all variables (with rough statistical significance in 1/2 of the cases). The BVAR-CSV also yields better scores for most variables at the 2-quarter horizon. But at the 8- and 12-quarter horizon, the BVAR yields slightly better scores than the BVAR-CSV for about 50% of the variables.

¹³The run-time on the BVAR-CSV with 15 variables is about 162 minutes, based on the same number of draws we use for the figures given for the other models in Table 2. For the simple BVAR, we generate forecasts with a simple Normal-Wishart prior and posterior, simulating 5000 forecast draws.

The differences between the stationary and integrated CSV specifications are even smaller than in the models for fewer variables.

5 Additional evidence: the case of the UK

The results we have obtained are favourable to the BVAR-CSV but one may wonder whether they are country specific. To provide additional evidence on the robustness of the good performance of the BVAR-CSV, in this section we repeat the key components of the US-based analysis of the previous section with data for the UK: the comparison of BVAR-CSV and BVAR-SV volatility estimates and forecast performance, for an 8-variable macroeconomic model.

5.1 Data and design of the forecast exercise

As for the US, we consider models of eight variables, at the quarterly frequency, with variables selected to match the US case as closely as possible. Specifically, the variables under analysis include growth of real GDP, growth of real household consumption expenditure, growth of real gross fixed investment, growth of employment, the unemployment rate, inflation as measured by the GDP deflator, a yield on bellwether (10-year when issued) government bonds, and the 3-month interbank rate.¹⁴ In light of the sharp mean shifts that occurred in the nominal variables (inflation and the interest rates) between the start of the sample and the early 1990s, we include these nominal variables in the model as differences (i.e., as the change in inflation and the change in each interest rate). After forming forecasts of the changes of these variables, we cumulate to obtain forecasts of the levels of inflation and the interest rate.

In light of the more limited availability of real-time data for the UK, all of our results use current vintage data taken from the FAME database of the Federal Reserve Board. The quarterly data on the interest rates are constructed as simple within-quarter averages of the source monthly data. Growth and inflation rates are measured as annualized log changes (from $t - 1$ to t).

The full forecast evaluation period runs from 1985:Q1 through 2011:Q2. For each forecast origin t starting with 1985:Q1, we use data through quarter $t - 1$ to estimate the forecast models and construct forecasts for periods t and beyond. The starting point of the model estimation sample is always 1972:Q3, the earliest possible for our data on the included variables.

¹⁴We constructed the GDP deflator as the ratio of nominal to real (chain-weight) GDP.

The results on forecast accuracy cover forecast horizons of 1, 2, 4, 8, and 12 quarters ahead. As in the case of the US, for all variables, we forecast the future growth rates, inflation rates, or levels for each quarterly horizon.

5.2 Full sample results

Figures 13-15 present volatility estimates from the 8-variable BVAR-SV and BVAR-CSV models. Figure 13's volatility estimates from the BVAR-SV model that allows independent volatilities across variables are broadly similar across variables. However, compared to the US estimates (Figure 4), the UK estimates show somewhat more heterogeneity. Volatility generally trends down for inflation and interest rates, trends up for unemployment and employment, and shows the familiar volatility moderation in the period 1985-2008 for the growth rates of GDP, investment and consumption. Despite some heterogeneity, each volatility estimate is significantly correlated (with correlations ranging from 0.40 to 0.96 for all variables except employment, for which the correlation is -0.30) with the first principal component computed from the posterior median volatility estimates of each variable (the principal component is reported in Figure 16).¹⁵

The common volatility estimates from the BVAR-CSV model shown in Figure 14 follow paths broadly similar to the BVAR-SV estimates, for most variables. The same holds for the estimates for the stationary CSV, reported in Figure 15, though the latter are a bit more volatile. Figure 16 shows that the common volatility estimate closely resembles the first principal component computed from the posterior median volatility estimates obtained with the BVAR-SV model; the correlation between the common volatility estimate and the principal component is 0.99. Both the common volatility estimate and the principal component of the individual volatility estimates from the BVAR-SV model decreases rather monotonically from the early 1970s till about 2005, and increases mildly after that.

While our proposed model seems to reasonably capture variation over time in conditional volatilities, how much does that matter for the full sample model fit? The log predictive likelihood (LPL) estimates reported in Table 10 show that our proposed common volatility specification significantly improves the fit of the 8-variable BVAR. The LPL of the BVAR-CSV model is about 50 points higher than the LPL of the constant volatility BVAR, with the stationary CSV performing only one point worse.

Overall, the UK results, like the US results, suggest our proposed common stochastic

¹⁵To compute the principal component, we take the posterior median estimates of volatility from the BVAR-SV model, standardize them, and compute the principal component as described in such studies as Stock and Watson (2002).

volatility specification can effectively capture time variation in conditional volatilities. However, in the UK evidence, there is more heterogeneity across variables. Accordingly, it could be that the forecasting gains from our proposed BVAR-CSV model could be more limited than for the US. We will assess whether this is the case in the next subsection.

5.3 Forecast results

In this subsection we compare the relative performance of 8-variable BVARs with or without common stochastic volatility (all with four lags), starting with point forecasts and moving next to density forecasts. As for the US, we also include univariate AR(2) models in the comparison. The model priors are the ones described in section 3. As mentioned above, the evaluation sample is 1985Q1-2011Q2, we consider four forecast horizons, and the exercise is conducted in a pseudo-real time manner, using recursive estimation but a single data vintage (the most recent available).

Table 11 reports the root mean squared error (RMSE) of each model relative to that of the BVAR, and the RMSE level for the BVAR. Hence, entries less than 1 indicate that the model has a lower RMSE than the BVAR. Table A11 in the Appendix contains the same results but using the AR model as benchmark.¹⁶

Our point forecast results for the UK suggest that the AR is often outperformed by the constant volatility BVAR, contrary to the US results. The former has a lower RMSE than the latter in only 12 out of 40 cases (8 variables and 5 forecast horizons), beating the BVAR only for inflation and bond yields. The differences are however rarely statistically significant, except for inflation and some 1-step ahead forecasts (see also Appendix Table A11, in which the AR model is the benchmark).

Adding common stochastic volatility to the BVAR model improves the forecasts in 23 out of 40 cases with a BVAR benchmark (Table 11), and in 25 out of 40 cases with the AR benchmark (Appendix Table A11). Similar figures are obtained for the stationary CSV specification: improvements in 21 out of 40 cases with the BVAR benchmark and 23 out of 40 with the AR benchmark. Against the BVAR benchmark, the BVAR-CSV's payoff in forecast accuracy is almost uniform at horizons up to two quarters, with mixed results for four-quarter forecasts, and generally worse outcomes at longer horizon, except for inflation

¹⁶ As for the US, we use the Diebold-Mariano t-statistic for equal MSE, applied to the forecast of each model relative to the benchmark, to provide a rough gauge of whether the RMSE ratios are significantly different from 1. The tests are one-sided, only rejecting the alternative model in favor of the benchmark. Differences in accuracy that are statistically different from zero are denoted by one, two, or three asterisks, corresponding to significance levels of 10%, 5%, and 1%, respectively. The underlying p-values are based on t-statistics computed with a serial correlation-robust variance, using a rectangular kernel, $h - 1$ lags, and the small-sample adjustment of Harvey, Leybourne, and Newbold (1997).

and interest rates. The comparison with the BVAR-SV shows mixed evidence that any of the two models outperforms the other, with both models being better in 19 occasions, and equal in 2. Against the AR benchmark, the BVAR-CSV model consistently improves the accuracy of forecasts of real variables, but not nominal variables. In general, the larger improvements in RMSEs are often statistically significant, though less so than for the US (Table 5).

Moving now to the evaluation of the density forecasts, Table 12 reports differences in log scores with respect to the BVAR, such that entries greater than 0 indicate that the BVAR-CSV has a better average log score (better density forecast) than the benchmark BVAR model. Table A12 in the Appendix presents comparable results using the AR model as benchmark.¹⁷ Both tables provide the levels of average log scores for the benchmark model.

Against either the constant volatility BVAR or AR benchmark, our proposed common stochastic volatility BVAR yields significant gains in density forecast accuracy, especially at shorter horizons. Compared to the BVAR, the BVAR-CSV model yields a better score in 25 out of 40 cases, decreasing to 19 out of 40 for the stationary CSV. These figures are only slightly lower than those for the US and, as for the US, the gains are larger and concentrated at the one- and two-quarter horizons, when the BVAR-CSV is better for all variables except Investment, Employment and Unemployment. Not surprisingly, these are the variables whose BVAR-SV estimates of volatility (estimates obtained by treating the estimates of volatility as independent) are more different from the common stochastic volatility estimate. The density forecast performance deteriorates at longer horizons, but not so much for the nominal variables. The comparison with the BVAR-SV shows that the latter is better at short horizons, while the BVAR-CSV produces consistently good forecasts at the very long horizons. On balance, however, the evidence is that the two models perform similarly. Compared to AR model forecasts, the BVAR-CSV model yields a better score in 29 out of 40 cases, 20 out of 40 for the stationary CSV specification, again with gains that are concentrated at shorter forecast horizons. However, when all 8 variables are considered jointly, the BVAR-CSV model (significantly) beats both the AR and the BVAR models at all horizons, presumably because the BVAR forecasts better account for the covariances

¹⁷ As for the US, to provide a rough gauge of the statistical significance of differences in average log scores, we use the Amisano-Giacomini (2007) t-test of equal means, applied to the log score for each model relative to the benchmark BVAR forecast. Again, we treat the tests as one-sided. Differences in average scores that are statistically different from zero are denoted by one, two, or three asterisks, corresponding to significance levels of 10%, 5%, and 1%, respectively. The underlying p-values are based on t-statistics computed with a serial correlation-robust variance, using a rectangular kernel, $h - 1$ lags, and the small-sample adjustment of Harvey, Leybourne, and Newbold (1997).

among variables.

In terms of tail coverage, the results are qualitatively similar to those for the US, though quantitatively slightly worse (see Appendix Tables 22-24). In particular, BVAR-CSV has better coverage than BVAR, except for Investment, Employment and Unemployment, since as mentioned for these variables the volatility pattern is rather different from the estimated CSV.

If we then repeat the density forecast evaluation ending the sample in 2008Q2 to exclude the worst quarters of the crisis, the results are again qualitatively similar to those for the US. In particular, the gains from CSV are even slightly larger, though the differences with respect to the full sample results are generally smaller than for the US.

In summary, notwithstanding the higher heterogeneity in independent estimates of volatility for each variable, the forecasting gains from the BVAR-CSV are generally confirmed for the UK. Broadly, the gains for the UK are just slightly lower than those for the US, for both point and density forecasts, and are present even when excluding the crisis period.

6 Conclusions

In this paper we propose to model conditional volatilities as driven by a single (stationary or integrated) common unobserved factor. Using a combination of a standard natural conjugate prior for the VAR coefficients, and an independent prior on a common stochastic volatility factor, we derive the posterior densities for the parameters of the resulting BVAR with common stochastic volatility (BVAR-CSV). Under the chosen prior the conditional posterior of the VAR coefficients features a Kroneker structure that allows for fast estimation.

Empirically, we start with systems composed of 4 and 8 US variables, and we show that there is substantial evidence of common volatility. We then examine the accuracy of real-time forecasts from VARs with constant volatility, independent stochastic volatilities, and our proposed common stochastic volatility. We find that compared to a model with constant volatilities, our proposed common volatility model significantly improves model fit and forecast accuracy, with little differences from treating the volatility factor as integrated or stationary but persistent. The gains are comparable to or as great as the gains achieved with a conventional stochastic volatility specification that allows independent volatility processes for each variable. But our common volatility specification greatly speeds computations.

As a robustness check, we repeat the volatility and forecasting analysis using comparable UK data. Notwithstanding slightly higher heterogeneity in the estimated volatility across variables than for the US, the BVAR-CSV still delivers improved accuracy of both point

and density forecasts, in particular at short horizons.

We interpret these results as evidence that the BVAR-CSV efficiently summarizes the information in a possibly large dataset and accounts for changing volatility, while helping to significantly reduce computation costs relative to a model with independent stochastic volatilities. For these reasons this class of models should have a wide range of applicability for forecasting and possibly also for policy simulation exercises.

7 Appendix: Derivation of conditional posterior distribution for MCMC sampler

In this Appendix we derive the conditional posterior distributions used in the MCMC scheme. For ease of exposition, we recall the model:

$$y_t = \Pi_0 + \Pi(L)y_{t-1} + v_t, \quad (41)$$

$$v_t = \lambda_t^{0.5} \epsilon_t, \quad \epsilon_t \sim N(0, S), \quad (42)$$

$$\log(\lambda_t) = \log(\lambda_{t-1}) + \nu_t, \quad \nu_t \sim \text{iid } N(0, \phi). \quad (43)$$

7.1 Likelihood of the VAR

We start by stating the likelihood function of the VAR. An alternative representation for the VAR in (41) is:

$$y_t = \Pi_0 + \Pi_1 y_{t-1} + \dots + \Pi_p y_{t-p} + v_t \quad (44)$$

By defining the $k \times n$ (where $k = 1 + np$) matrix $\Pi = (\Pi_0, \Pi_1, \dots, \Pi_p)'$ and collecting the intercept and lagged variables in the $k \times 1$ vector $x_t = (1, y'_{t-1}, y'_{t-2}, \dots, y'_{t-p})'$ we have:

$$y_t = \Pi' x_t + v_t. \quad (45)$$

Now consider the T equations (45) for each observations $t = 1, \dots, T$. By stacking them by columns and then transposing the system we get:

$$Y = X\Pi + v, \quad (46)$$

where Y is a $T \times n$ data-matrix with rows y'_t , X is a $T \times k$ data-matrix with rows $x'_t = (1, y'_{t-1}, y'_{t-2}, \dots, y'_{t-p})$ and v is a $T \times n$ matrix of disturbances with rows v'_t . Consider now

the T equations involving the generic dependent variable j in the system above. We have:

$$\begin{matrix} Y_j &= & X \Pi_j &+ & v_j, \\ T \times 1 & & k \times 1 & & T \times 1 \end{matrix}, \quad (47)$$

where Y_j is the j -th column of Y , and Π_j the j -th column of Π . By stacking these equations by column for $j = 1, \dots, n$, and defining $y = \text{vec}(Y)$, $Z = (I \otimes X)$, $v = \text{vec}(v)$ we can write:

$$y = Z \text{vec}(\Pi) + v. \quad (48)$$

Under our specification, the residual variance-covariance matrix for period t is $\text{Var}(v_t) = \Sigma_t \equiv \lambda_t A^{-1} S A^{-1'} = \lambda_t (\tilde{A}' \tilde{A})^{-1}$. Define a diagonal matrix having the whole history of λ in the main diagonal:

$$\Lambda = \text{diag}(\lambda_1, \dots, \lambda_T). \quad (49)$$

The variance of v is given by¹⁸:

$$Q = (\tilde{A}' \tilde{A})^{-1} \otimes \Lambda. \quad (50)$$

¹⁸This can be derived as follows:

$$Q = E[vv'] = \begin{bmatrix} E[\mathbf{v}_1 \mathbf{v}_1'] & \dots & E[\mathbf{v}_N \mathbf{v}_1'] \\ \dots & E[\mathbf{v}_i \mathbf{v}_j']_{T \times T} & \dots \\ E[\mathbf{v}_1 \mathbf{v}_N'] & \dots & E[\mathbf{v}_N \mathbf{v}_N'] \end{bmatrix}$$

The generic term $E[\mathbf{v}_i \mathbf{v}_j']$ in the above matrix is equal to:

$$\begin{aligned} E[\mathbf{v}_i \mathbf{v}_j'] &= \begin{bmatrix} E[v_1^{(i)} v_1^{(j)'}] & \dots & E[v_T^{(i)} v_1^{(j)'}] \\ E[v_1^{(i)} v_T^{(j)'}] & \dots & E[v_T^{(i)} v_T^{(j)'}] \end{bmatrix} = \begin{bmatrix} [\Sigma_1]_{ij} & \dots & \\ & \dots & [\Sigma_T]_{ij} \end{bmatrix} \\ &= \begin{bmatrix} \lambda_1 \left[(\tilde{A}' \tilde{A})^{-1} \right]_{ij} & \dots & \\ & \dots & \lambda_T \left[(\tilde{A}' \tilde{A})^{-1} \right]_{ij} \end{bmatrix} = \left[(\tilde{A}' \tilde{A})^{-1} \right]_{ij} \cdot \Lambda \end{aligned}$$

Therefore we have:

$$Q = \begin{bmatrix} \left[(\tilde{A}' \tilde{A})^{-1} \right]_{11} \cdot \Lambda & \dots & \left[(\tilde{A}' \tilde{A})^{-1} \right]_{N1} \cdot \Lambda \\ \dots & \left[(\tilde{A}' \tilde{A})^{-1} \right]_{ij} \cdot \Lambda & \dots \\ \left[(\tilde{A}' \tilde{A})^{-1} \right]_{1N} \cdot \Lambda & \dots & \left[(\tilde{A}' \tilde{A})^{-1} \right]_{NN} \cdot \Lambda \end{bmatrix} = (\tilde{A}' \tilde{A})^{-1} \otimes \Lambda$$

The error term in (48) has the following conditional distribution:

$$v|\Pi, A, S, \phi, \Lambda, Z \sim N(0, Q). \quad (51)$$

It follows that the likelihood of (48) is:

$$p(y|\Pi, A, S, \phi, \Lambda, Z) = 2\pi^{-\frac{Tn}{2}} |Q|^{-\frac{1}{2}} \exp(-(y - Z\text{vec}(\Pi))' Q^{-1} (y - Z\text{vec}(\Pi))) / 2 \quad (52)$$

7.2 Drawing $\Pi|A, S, \phi, \Lambda, y$

Conditionally on the other parameters of the model, the posterior of Π can be obtained by using standard results for the $N - IW$ natural conjugate prior. The assumed prior distribution is:

$$\text{vec}(\Pi)|A, S \sim N(\text{vec}(\underline{\mu}_\Pi), \underline{\Omega}_\Pi) \quad (53)$$

Given the likelihood in (52) this is a conditionally natural conjugate prior. Indeed, by combining (53) with (52), the conditional posterior kernel of $\text{vec}(\Pi)$ is given by:

$$p(\text{vec}(\Pi)|A, S, \phi, \Lambda, y) \propto p(y|\text{vec}(\Pi), A, S, \phi, \Lambda) p(\text{vec}(\Pi)|A, S), \quad (54)$$

which is the normal kernel typically found when combining a normal likelihood with a normal prior. As shown e.g. in Geweke (2005), the conditional posterior will be multivariate normal with moments:

$$\text{vec}(\bar{\mu}_\Pi) = \bar{\Omega}_\Pi(\underline{\Omega}_\Pi^{-1} \text{vec}(\underline{\mu}_\Pi) + Z' Q^{-1} y) \quad (55)$$

$$\bar{\Omega}_\Pi = (\underline{\Omega}_\Pi^{-1} + Z' Q^{-1} Z)^{-1}. \quad (56)$$

A draw Π^i from the conditional posterior can be obtained as:

$$\text{vec}(\Pi^i) = \text{vec}(\bar{\mu}_\Pi) + \text{chol}(\bar{\Omega}_\Pi) \times u \quad (57)$$

where u is a $kn \times 1$ standard Gaussian vector process. If one were to extract many draws using (57), the bottleneck would be the computation of the matrices Q^{-1} and $\bar{\Omega}_\Pi$, both necessary to get $\bar{\mu}_\Pi$, and the Cholesky decomposition of $\bar{\Omega}_\Pi$, necessary to perform the draw. All these computations require $(kn)^3$ elementary operations. However, thanks to the assumptions made on (i) the structure of the prior variance and (ii) the common nature of

the volatility, the production of a draw Π^i can be dramatically simplified.¹⁹

To start with, the fact that Q features a Kronecker structure, $Q = (\tilde{A}'\tilde{A}^{-1} \otimes \Lambda)$, which is a consequence of (ii), means that the moments $Z'Q^{-1}y$ and $Z'Q^{-1}Z$ can be written as follows:

$$Z'Q^{-1}Z = (I \otimes X') (\tilde{A}'\tilde{A} \otimes \Lambda^{-1}) (I \otimes X) = \tilde{A}'\tilde{A} \otimes X'\Lambda^{-1}X, \quad (58)$$

$$Z'Q^{-1}y = (I \otimes X')(\tilde{A}'\tilde{A} \otimes \Lambda^{-1})y = (\tilde{A}'\tilde{A} \otimes X'\Lambda^{-1})y. \quad (59)$$

Using these moments, as well as the fact that $\underline{\Omega}_{\Pi}^{-1}$ has a Kronecker structure (assumption (i)), one can get the following expressions for posterior variance:

$$\begin{aligned} \bar{\Omega}_{\Pi} &= (\tilde{A}'\tilde{A} \otimes \underline{\Omega}_0^{-1} + \tilde{A}'\tilde{A} \otimes X'\Lambda^{-1}X)^{-1} \\ &= (\tilde{A}'\tilde{A})^{-1} \otimes (\underline{\Omega}_0^{-1} + X'\Lambda^{-1}X)^{-1} \end{aligned} \quad (60)$$

We will see shortly how the fact that $\bar{\Omega}_{\Pi}$ features a Kronecker structure improves the computations. The posterior mean can be written as:

$$\begin{aligned} \text{vec}(\bar{\mu}_{\Pi}) &= \bar{\Omega}_{\Pi}(\underline{\Omega}_{\Pi}^{-1}\text{vec}(\underline{\mu}_{\Pi}) + Z'Q^{-1}y) \\ &= \bar{\Omega}_{\Pi}[(\tilde{A}'\tilde{A}) \otimes \underline{\Omega}_0^{-1}]\text{vec}(\underline{\mu}_{\Pi}) + [\tilde{A}'\tilde{A} \otimes X'\Lambda^{-1}]y \\ &= [(\tilde{A}'\tilde{A})^{-1} \otimes (\underline{\Omega}_0^{-1} + X'\Lambda^{-1}X)^{-1}][(\tilde{A}'\tilde{A}) \otimes \underline{\Omega}_0^{-1}]\text{vec}(\underline{\mu}_{\Pi}) + \\ &\quad [(\tilde{A}'\tilde{A})^{-1} \otimes (\underline{\Omega}_0^{-1} + X'\Lambda^{-1}X)^{-1}][\tilde{A}'\tilde{A} \otimes X'\Lambda^{-1}]y + \\ &= [I \otimes (\underline{\Omega}_0^{-1} + X'\Lambda^{-1}X)^{-1}\underline{\Omega}_0^{-1}]\text{vec}(\underline{\mu}_{\Pi}) + \\ &\quad [I \otimes (\underline{\Omega}_0^{-1} + X'\Lambda^{-1}X)^{-1}X'\Lambda^{-1}]\text{vec}(Y) \end{aligned} \quad (61)$$

By using $\text{vec}(ABC) = (C' \otimes A)\text{vec}(B)$ we have:

$$\begin{aligned} \text{vec}(\bar{\mu}_{\Pi}) &= \text{vec}([(\underline{\Omega}_0^{-1} + X'\Lambda^{-1}X)^{-1}\underline{\Omega}_0^{-1}]\underline{\mu}_{\Pi}) + \text{vec}([(\underline{\Omega}_0^{-1} + X'\Lambda^{-1}X)^{-1}X'\Lambda^{-1}]Y) \\ \Rightarrow \bar{\mu}_{\Pi} &= (\underline{\Omega}_0^{-1} + X'\Lambda^{-1}X)^{-1}(\underline{\Omega}_0^{-1}\underline{\mu}_{\Pi} + X'\Lambda^{-1}Y) \end{aligned} \quad (62)$$

¹⁹These two assumptions imply that both the variance of the error term and the variance of the prior are symmetric across equations. This means that both the likelihood and the prior of the model (as well as the resulting conjugate posterior) will have a matricvariate normal distribution, as in Kadiyala and Karlsson (1997). If a random variable Z has a matricvariate normal distribution, then its vectorisation $z = \text{vec}(Z)$ will be multivariate normal, but the converse is not necessarily true. The matricvariate form allows to write down the likelihood and priors using the representation in (46), i.e. a multivariate regression, which simplifies computations. This result is related to what happens in the case of a SUR model featuring the same regressors in each equation. In such case, due to the symmetry in the likelihood, the SUR can be written in the form of multivariate regression. Clearly in the case at hand not only the likelihood but also the prior needs to feature such a symmetry in order for the posterior to feature a matricvariate distribution.

Similarly, the term $chol(\bar{\Omega}_{\Pi}) \times u$ can be written in a computationally more convenient way. Define a $k \times n$ matrix U such that $u = \text{vec}(U)$, then we have:

$$\begin{aligned} chol(\bar{\Omega}_{\Pi}) \times u &= chol((\tilde{A}'\tilde{A})^{-1} \otimes (\underline{\Omega}_0^{-1} + X'\Lambda^{-1}X)^{-1}) \times \text{vec}(U) \\ &= \text{vec}(chol[(\underline{\Omega}_0^{-1} + X'\Lambda^{-1}X)^{-1}] \times U \times chol[(\tilde{A}'\tilde{A})^{-1}]') \end{aligned} \quad (63)$$

where we have exploited the fact that $\bar{\Omega}_{\Pi}$ has the Kronecker structure shown in (60). Therefore, using (62) and (63) in (57) and removing the vectorization operator from all terms we get:

$$\Pi^i = \bar{\mu}_{\Pi} + (chol((\underline{\Omega}_0^{-1} + X'\Lambda^{-1}X)^{-1})) \times U \times chol((\tilde{A}'\tilde{A})^{-1})' \quad (64)$$

This considerably speeds up the computations, because the two Choleski decompositions above, as well as the computation of $\bar{\mu}_{\Pi}$ require only operations of at most order $k^3 + n^3$. Finally we note that due to the fact that Λ is diagonal, $\bar{\mu}_{\Pi}$ and $X'\Lambda^{-1}X$ can be computed as follows, which may speed up the computations depending on the type of software package used:

$$X'\Lambda^{-1}X = \sum_{t=1}^T \frac{1}{\lambda_t} X_t X_t' \quad (65)$$

$$\bar{\mu}_{\Pi} = \left(\underline{\Omega}_0^{-1} + \sum_{t=1}^T \frac{1}{\lambda_t} X_t X_t' \right)^{-1} \left(\underline{\Omega}_0^{-1} \underline{\mu}_{\Pi} + \sum_{t=1}^T \frac{1}{\lambda_t} X_t y_t' \right) \quad (66)$$

The expression in (66) also shows that the posterior mean of the coefficient matrix can be obtained by defining data vectors normalized by the standard deviation of volatility, to permit rewriting the VAR in terms of conditionally homoskedastic variables: specifically, let $\tilde{y}_t = \lambda_t^{-0.5} y_t$ and $\tilde{X}_t = \lambda_t^{-0.5} X_t$. Then, the posterior mean of the matrix of coefficients can be equivalently written as:

$$\bar{\mu}_{\Pi} = \left(\sum_{t=1}^T \tilde{X}_t \tilde{X}_t' + \underline{\Omega}_0^{-1} \right)^{-1} \left(\underline{\Omega}_0^{-1} \underline{\mu}_{\Pi} + \sum_{t=1}^T \tilde{y}_t \tilde{X}_t' \right), \quad (67)$$

which is of course equivalent to (66).

7.3 Drawing $a_i | \Pi, S, \phi, \Lambda, y$

The kernel of the posterior distribution of the covariance elements a_i , conditional on the remaining coefficients, has been derived in Cogley and Sargent's (2005) Appendix B.2.4, in

particular equations 76 through 78. The resulting conditional posterior is:

$$a_i | \Pi, S, \phi, \Lambda, y \sim N(\bar{\mu}_{a,i}, \bar{\Omega}_{a,i}), \quad i = 2, \dots, n. \quad (68)$$

7.4 Drawing $s_i | \Pi, A, \phi, \Lambda, y$

Recall the model in (48), which has the likelihood given in (52). The matrix Q^{-1} can be written as:

$$\begin{aligned} Q^{-1} &= (\tilde{A}'\tilde{A}) \otimes \Lambda^{-1} \\ &= (A'S^{-1}A) \otimes \Lambda^{-1} \\ &= (A' \otimes I_T)(I_n \otimes \Lambda^{-1/2})(S^{-1} \otimes I_T)(I_n \otimes \Lambda^{-1/2})(A \otimes I_T) \\ &= A^{*'}\Lambda^*(S^{-1} \otimes I_T)\Lambda^*A^*, \end{aligned} \quad (69)$$

where we defined $\Lambda^{*-1/2} = I_n \otimes \Lambda^{-1/2}$ and $A^* = A \otimes I$. Define the rescaled residuals $v^* = \Lambda^{*-1/2}A^*(y - \text{Zvec}(\Pi))$. The cross-product term in (52) can be written as:

$$\begin{aligned} &(y - \text{Zvec}(\Pi))'Q^{-1}(y - \text{Zvec}(\Pi)) \\ &= (y - \text{Zvec}(\Pi))'A^{*'}\Lambda^{*-1/2}(S^{-1} \otimes I_T)\Lambda^{*-1/2}A^*(y - \text{Zvec}(\Pi)) \\ &= v^{*'}(S^{-1} \otimes I_T)v^* \\ &= \text{tr}(V^{*'}V^*S^{-1}) \\ &= \text{tr}(RS^{-1}), \end{aligned} \quad (70)$$

where we used $\text{tr}(A'BCD') = \text{vec}(A)'(D \otimes B)\text{vec}(C)$ and we defined $\text{vec}(V^*) = v^*$ and $V^{*'}V^* = R$. Note that $V^* = Y - X\Pi$ so the matrix R is the scatter matrix of rescaled residuals. Using (70), the likelihood in (52) can be written as:

$$\begin{aligned} p(y | \Pi, A, S, \phi, \Lambda, Z) &= 2\pi^{-\frac{Tn}{2}} |A^{*'}\Lambda^{*-1/2}(S^{-1} \otimes I_T)\Lambda^{*-1/2}A^*|^{-\frac{1}{2}} \cdot \exp(-\text{tr}(RS^{-1})/2) \\ &\propto |S|^{-\frac{T}{2}} \exp(-\text{tr}(RS^{-1})/2) \end{aligned} \quad (71)$$

which is the kernel of a Wishart distribution for S^{-1} , or of an Inverse Wishart distribution for S . The conjugate prior for this distribution is:

$$S^{-1} \sim W(R_0^{-1}, d_s), \quad (72)$$

with prior density:

$$\begin{aligned}
p(S^{-1}) &= 2^{-d_s n/2} \cdot \pi^{-(n-1)n/4} |R_0^{-1}|^{d_s/2} \cdot \left(\prod_{i=1}^p \Gamma[(d_s + i - 1)/2] \right)^{-1} \\
&\quad \cdot |S^{-1}|^{(d_s-1-n)/2} \cdot \exp(-\text{tr}(R_0 S^{-1})/2).
\end{aligned} \tag{73}$$

The prior used in the paper is an inverse gamma for s_i , which can be interpreted as a special case of $S \sim IW(R_0, d_s)$ where S is diagonal, in particular we set $R_0 = \text{diag}(d_s \cdot \underline{s}_i)$. The posterior kernel is therefore:

$$\begin{aligned}
p(S^{-1}|y, \Pi, A, \phi, \Lambda) &\propto p(y|\Pi, A, S, \phi, \Lambda) * p(S^{-1}) \\
&\propto \left[2\pi^{-\frac{Tn}{2}} |S|^{-\frac{T}{2}} \exp(-\text{tr}(RS^{-1})/2) \right] \cdot \\
&\quad \left[2^{-d_s n/2} \cdot \pi^{-(n-1)n/4} |R_0^{-1}|^{d_s/2} \cdot \left(\prod_{i=1}^p \Gamma[(d_s + i - 1)/2] \right)^{-1} \right. \\
&\quad \left. \cdot |S^{-1}|^{(d_s-1-p)/2} \cdot \exp(-\text{tr}(R_0^{-1} S^{-1})/2) \right] \\
&\propto |S|^{-\frac{T}{2}} |S^{-1}|^{(d_s-1-p)/2} \cdot \exp(-\text{tr}(RS^{-1})/2) \cdot \exp(-\text{tr}(R_0 S^{-1})/2) \\
&\propto |S^{-1}|^{\frac{T+d_s-1-n}{2}} \cdot \exp(-\text{tr}(R + R_0)S^{-1})/2),
\end{aligned} \tag{74}$$

which is the kernel of $S^{-1}|\Pi, A, \phi, \Lambda, y \sim W((R + R_0)^{-1}, T + d_s)$, which implies that $S|\Pi, A, \phi, \Lambda, y \sim IW(R + R_0, T + d_s)$. Recalling that the generic element of R is $r_{ij} = v_i^{*'} v_j^*$, the sum of squares of the rescaled residuals, and that we are imposing diagonality on R , we see that $R = \sum_{t=1}^T (\hat{v}_{i,t}^2 / \lambda_t)$ and the posterior scale matrix is given by $\text{diag}(d_s \cdot \underline{s}_i) + \sum_{t=1}^T (\hat{v}_{i,t}^2 / \lambda_t)$.

7.5 Drawing $\lambda_t|\Pi, A, S, \phi, y$

To draw from the conditional distribution of λ_t we use a modification of the algorithm proposed by Cogley and Sargent's (2005), which uses a sequence of Metropolis steps. The kernel of this distribution is given by:

$$\pi(\Lambda|\Pi, A, S, \phi, y) = \prod_{t=1}^T \pi(\lambda_t|\lambda_{t-1}, \lambda_{t+1}, \phi, w_t) \tag{75}$$

where $w_t = (w_{1t}, \dots, w_{nt})$ is a vector of orthogonalized residuals and where the disappearance of all values beyond 1 lead and lag is a consequence of the Markov property of the process assumed for λ_t . As $w_t = S^{-1/2} \hat{v}_t = S^{-1/2} A v_t$ and $v_t = y_t - \Pi_0 - \Pi(L)y_{t-1}$, the rescaled residuals w_t contain all the information given by Π, A, S, y so we have substituted condi-

tioning with respect to these variables with conditioning with respect to w_t to simplify the notation. The generic element $\pi(\lambda_t|\lambda_{t-1}, \lambda_{t+1}, \phi, w_t)$ in the products (75) can be factorised as:

$$\pi(\lambda_t|\lambda_{t-1}, \lambda_{t+1}, \phi, w_t) \propto p(w_t|\lambda_t, \phi)p(\lambda_{t-1}|\lambda_t, \phi)p(\lambda_t|\lambda_{t+1}, \phi) \quad (76)$$

The distribution $p(w_t|\lambda_t, \phi)$ is a normal, while $p(\lambda_{t-1}|\lambda_t, \phi)$ and $p(\lambda_t|\lambda_{t+1}, \phi)$ are log-normal. Writing them down we have:

$$\begin{aligned} p(w_t|\lambda_t, \phi) &\propto \lambda_t^{-0.5} \exp(-0.5w_{1t}^2/\lambda_t) \times \lambda_t^{-0.5} \exp(-0.5w_{2t}^2/\lambda_t) \\ &\quad \times \dots \times \lambda_t^{-0.5} \exp(-0.5w_{nt}^2/\lambda_t) \end{aligned} \quad (77)$$

$$p(\lambda_{t-1}|\lambda_t, \phi)p(\lambda_t|\lambda_{t+1}, \phi) = \lambda_t^{-1} \exp(-0.5(\ln \lambda_t - \mu_t)^2/\sigma_c^2) \quad (78)$$

In (77) there are no cross terms because the orthogonalised residuals are by construction independent. Equation (78) comes from the product of the two lognormals, and it is slightly different at the beginning and end of the sample. The parameters μ_t and σ_c^2 are the conditional mean and variance of $\log \lambda_t$ given λ_{t-1} and λ_{t+1} . With the random walk process, for periods 2 through $T-1$, the conditional mean and variance are $\mu_t = (\log \lambda_{t-1} + \log \lambda_{t+1})/2$ and $\sigma_c^2 = \phi/2$, respectively (the conditional mean and variance are a bit different for periods 1 and T and can be found in Cogley and Sargent (2005)).

Draws from λ_t are obtained sequentially starting from $t = 1$ and ending in $t = T$. In each period, a candidate draw λ_t^* is extracted from a proposal distribution, and then it is accepted with probability a . By choosing as proposal distribution $q(\theta) \propto p(\lambda_{t-1}|\lambda_t, \phi)p(\lambda_t|\lambda_{t+1}, \phi)$ the acceptance probability will simplify to:

$$a = \min \left(\frac{p(w_t|\lambda_t^*, \phi)}{p(w_t|\lambda_t, \phi)}, 1 \right); \quad (79)$$

where:

$$\frac{p(w_t|\lambda_t^*, \phi)}{p(w_t|\lambda_t, \phi)} = \frac{\lambda_t^{*-n \times 0.5} \prod_{i=1}^n \exp(-0.5w_{it}^2/\lambda_t^*)}{\lambda_t^{-n \times 0.5} \prod_{i=1}^n \exp(-0.5w_{it}^2/\lambda_t)} \quad (80)$$

7.6 Drawing $\phi|\Pi, A, S, \Lambda, y$

Finally, the assumed process for volatility is:

$$\log(\lambda_t) = \log(\lambda_{t-1}) + \nu_t, \quad \nu_t \sim \text{iid } N(0, \phi). \quad (81)$$

Given an Inverse gamma prior, deriving the posterior for the parameter ϕ is standard:²⁰

$$\phi|\Pi, A, S, \Lambda, y \sim IG(d_\phi \cdot \underline{\phi} + \sum_{t=1}^T \nu_t^2, d_\phi + T). \quad (82)$$

In some of the empirical applications, we modify equation (81) to allow for mean reversion in the stochastic volatility. In particular, we consider an AR(1) specification:

$$\log(\lambda_t) = \psi_0 + \psi_1 \log(\lambda_{t-1}) + \nu_t, \quad \nu_t \sim \text{iid } N(0, \phi). \quad (83)$$

Again, under Gaussian prior for ψ_0 and ψ_1 , the resulting conditional posteriors will be Gaussian with moments given by the combination of prior and data moments. The remaining steps of the algorithm described above do not change, apart from having to condition also on ψ_0 and ψ_1 whenever one conditions on ϕ .

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²⁰See e.g. Cogley and Sargent's (2005) equation 69.

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Volatility estimate: BVAR(4), full stochastic volatility (standard deviation)

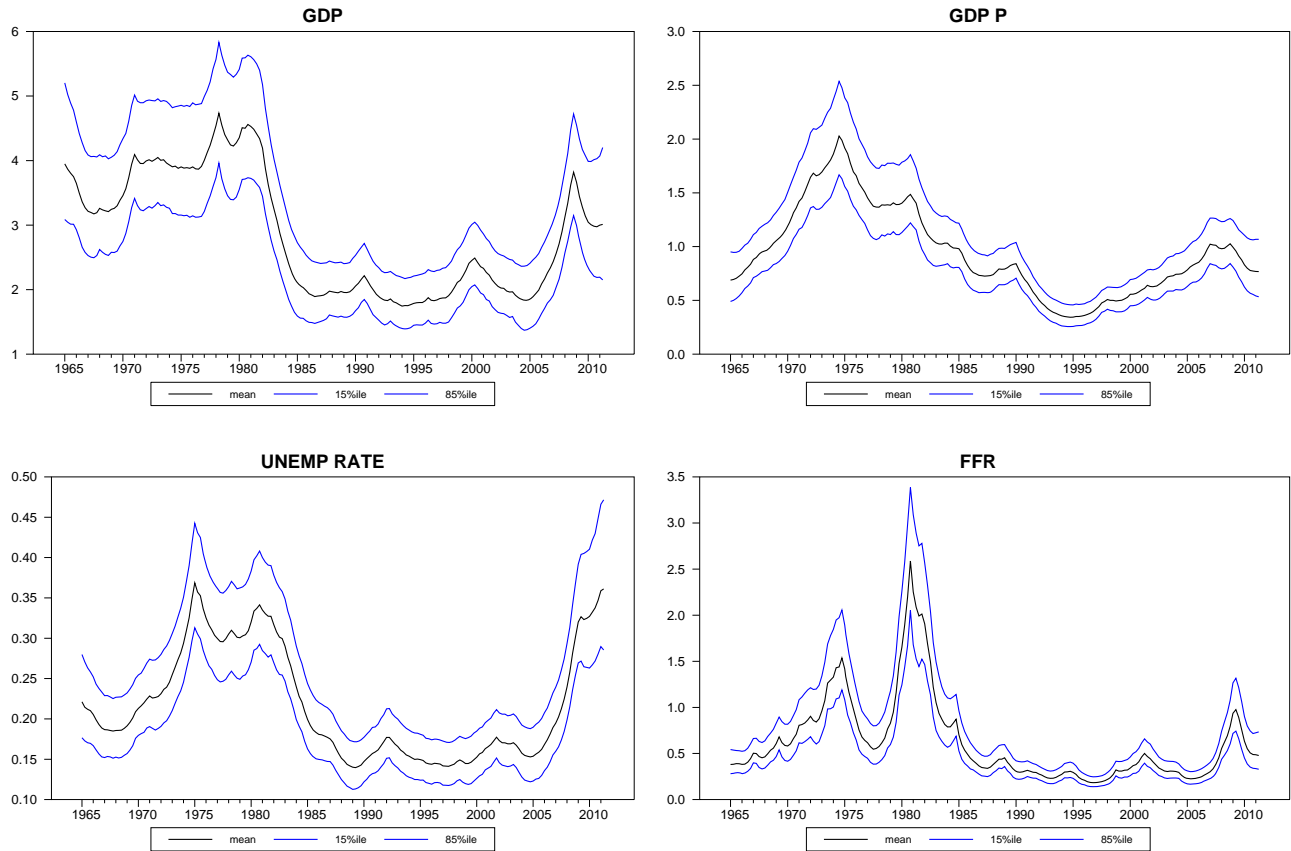


Figure 1: Volatility estimates from 4-variable BVAR-SV, final vintage data

Volatility estimate: BVAR(4), common stochastic volatility (standard deviation)

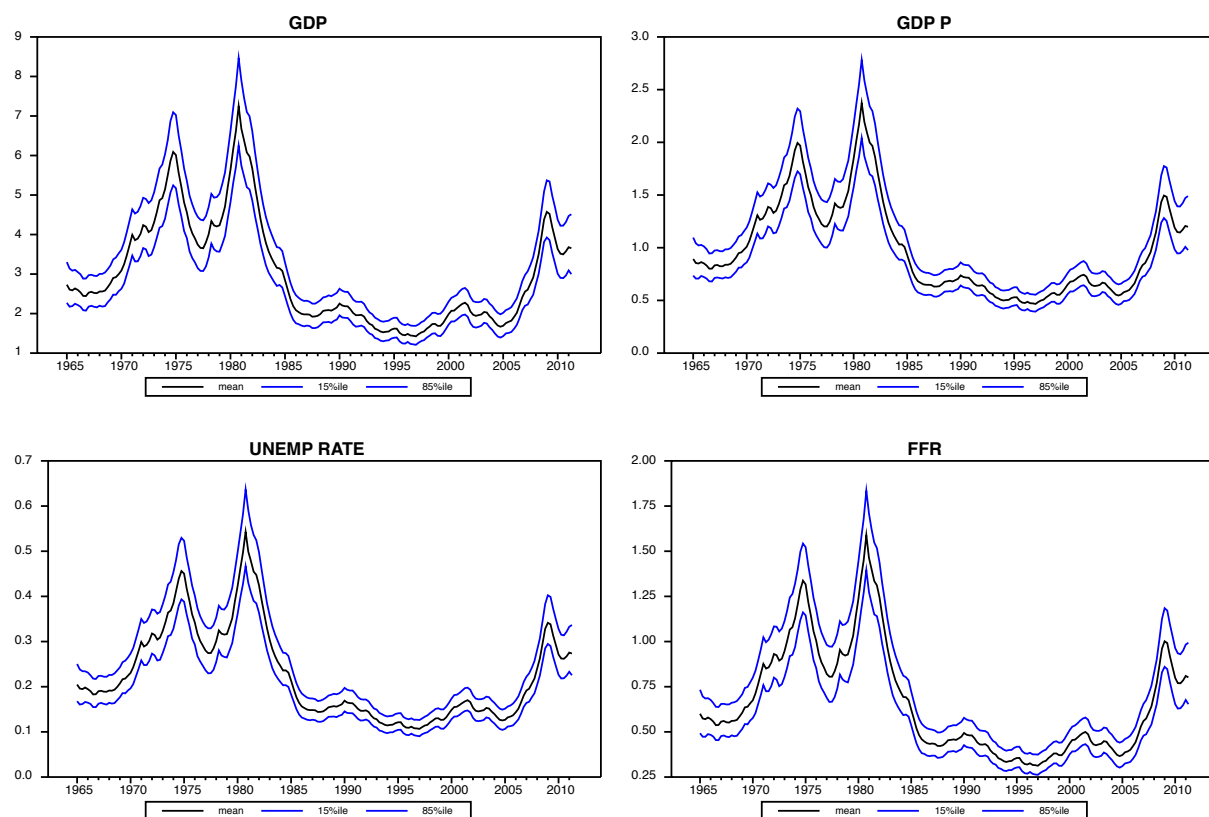


Figure 2: Volatility estimates from 4-variable BVAR-CSV, final vintage data

Volatility estimate: BVAR(4), common stochastic volatility (standard deviation)

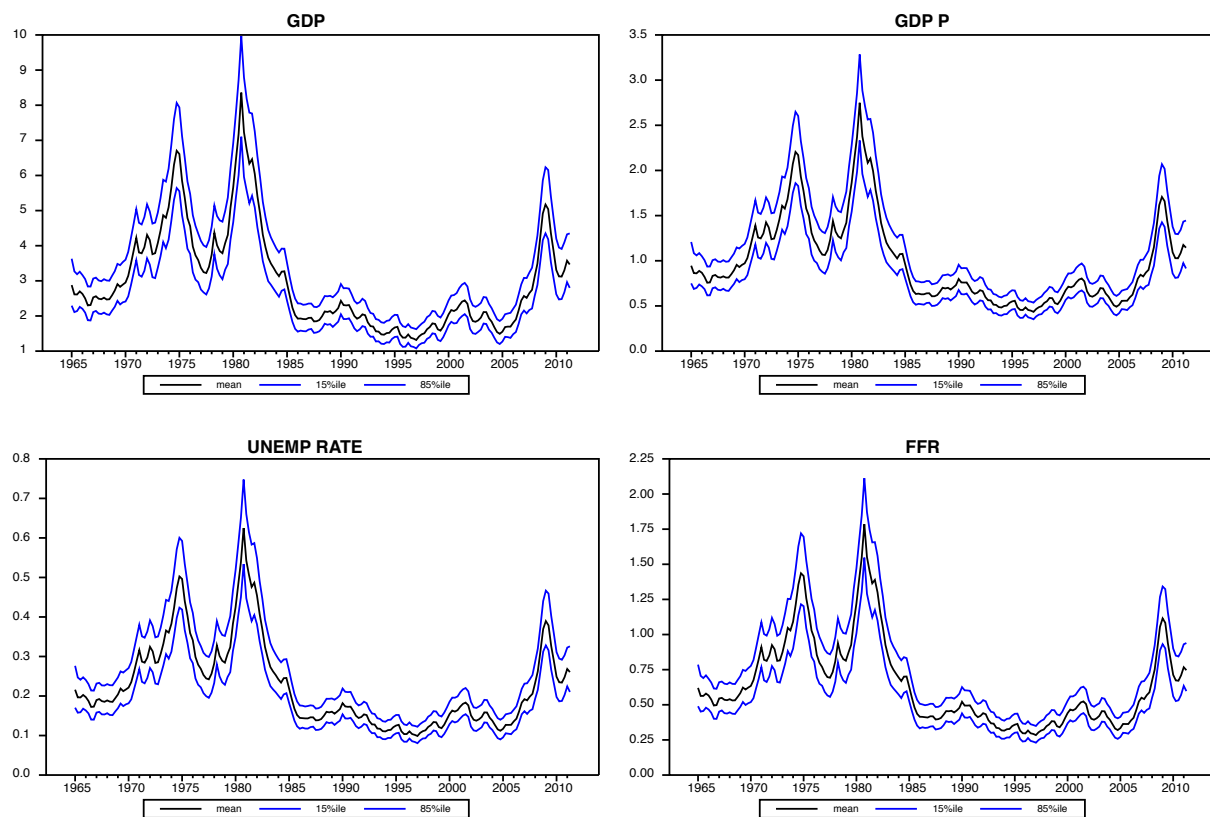


Figure 3: Volatility estimates from 4-variable BVAR-stationary CSV, final vintage data

Volatility estimate: BVAR(4), full stochastic volatility (standard deviation)

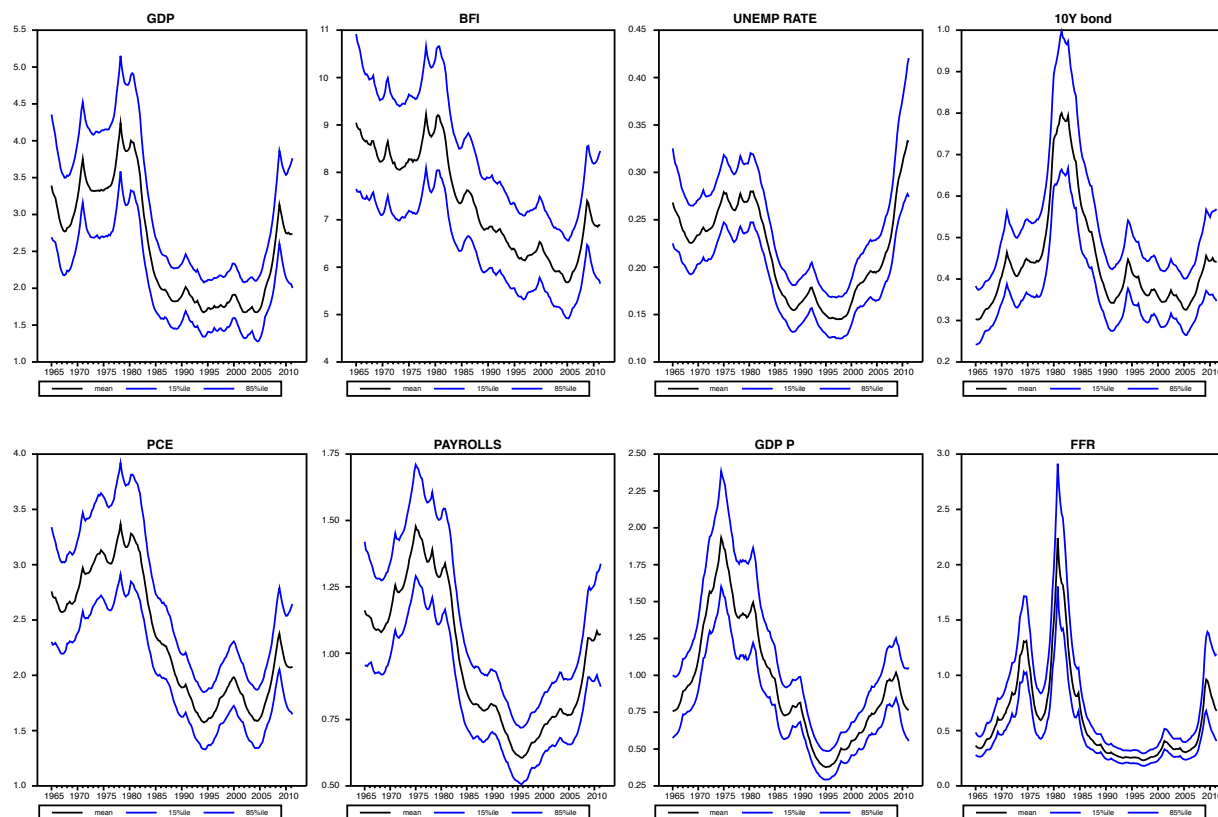


Figure 4: Volatility estimates from 8-variable BVAR-SV, final vintage data

Volatility estimate: BVAR(4), common stochastic volatility (standard deviation)

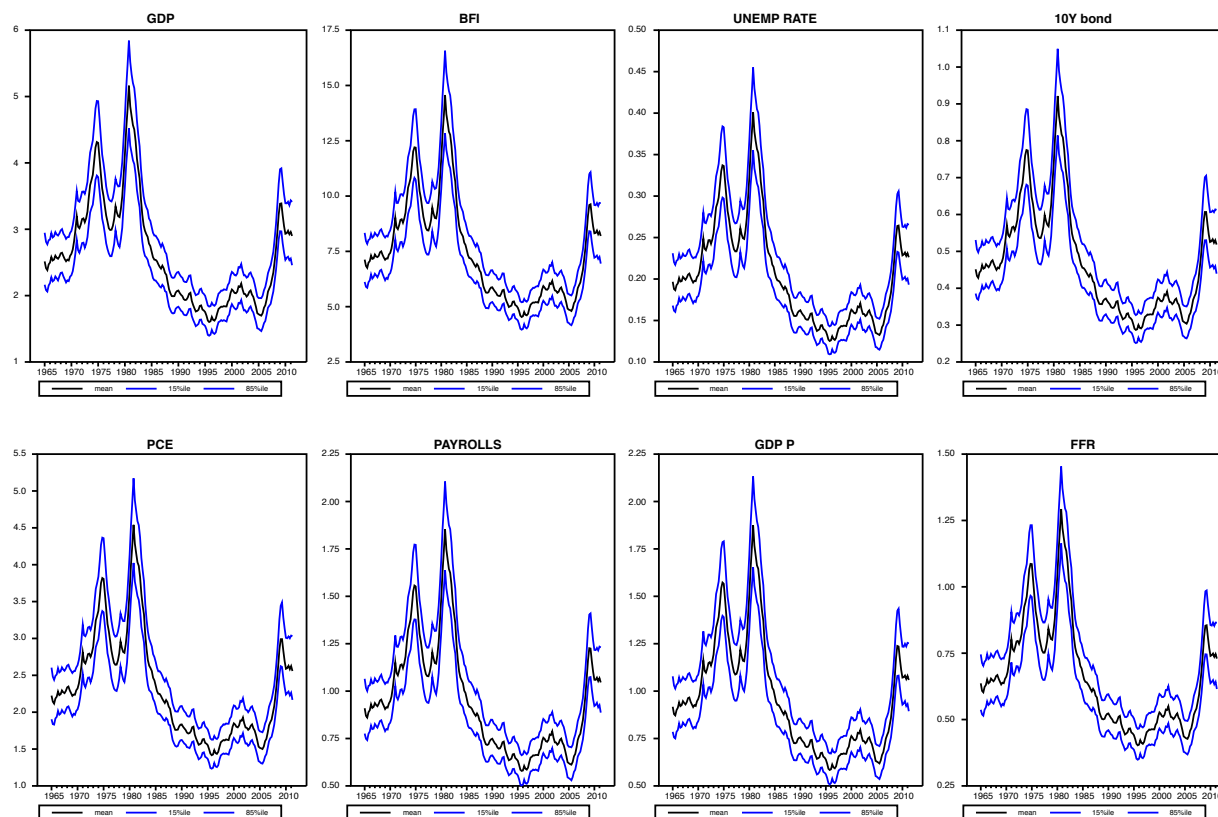


Figure 5: Volatility estimates from 8-variable BVAR-CSV, final vintage data

Volatility estimate: BVAR(4), common stochastic volatility (standard deviation)

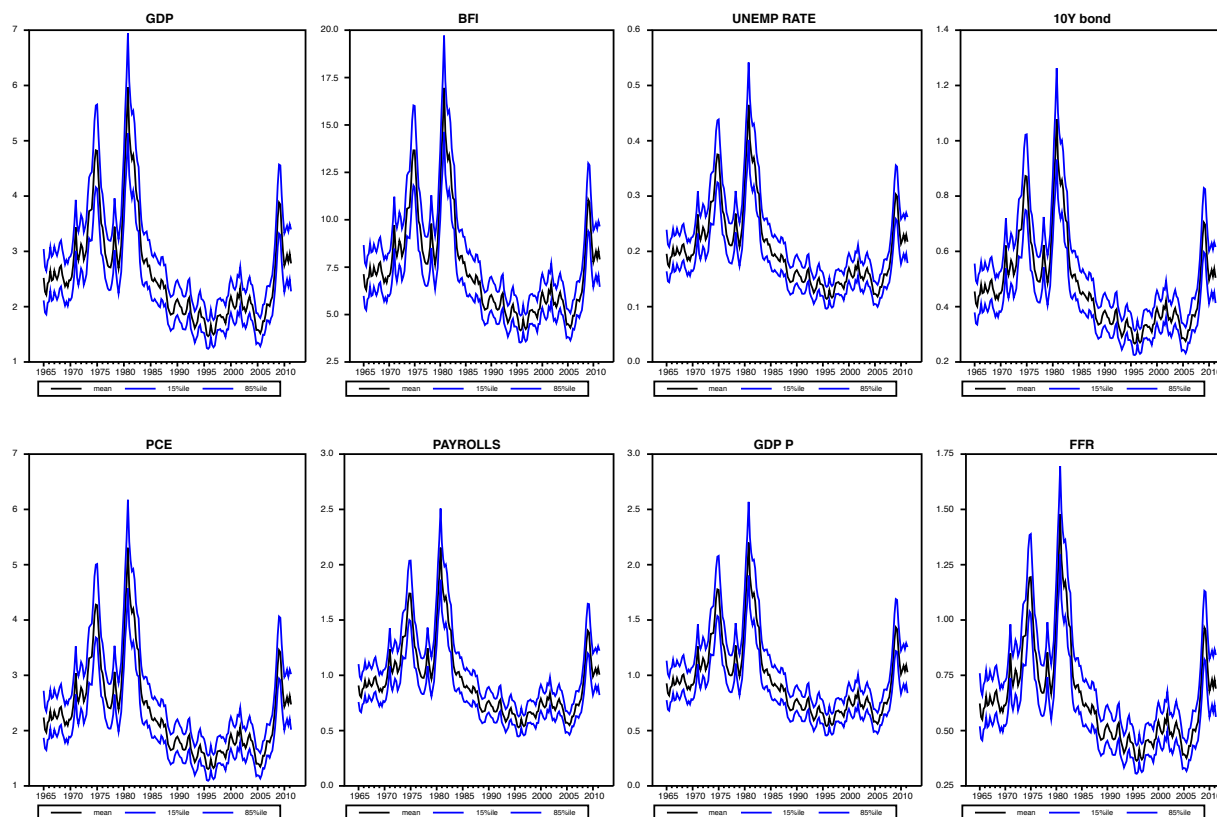


Figure 6: Volatility estimates from 8-variable BVAR-stationary CSV, final vintage data

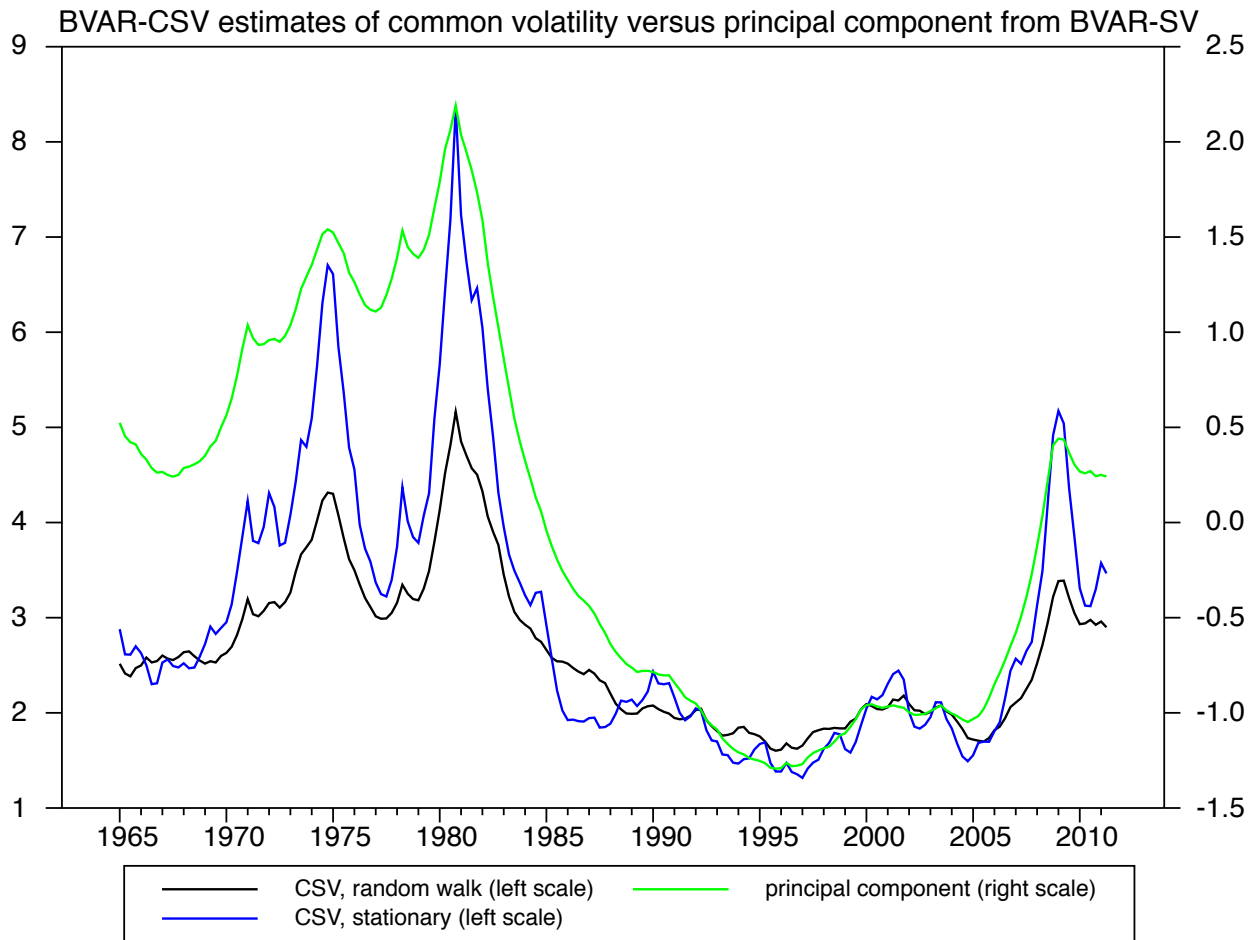


Figure 7: Principal component of BVAR-SV estimates of volatility versus common factor of volatility in 8-variable BVAR-CSV, final vintage data

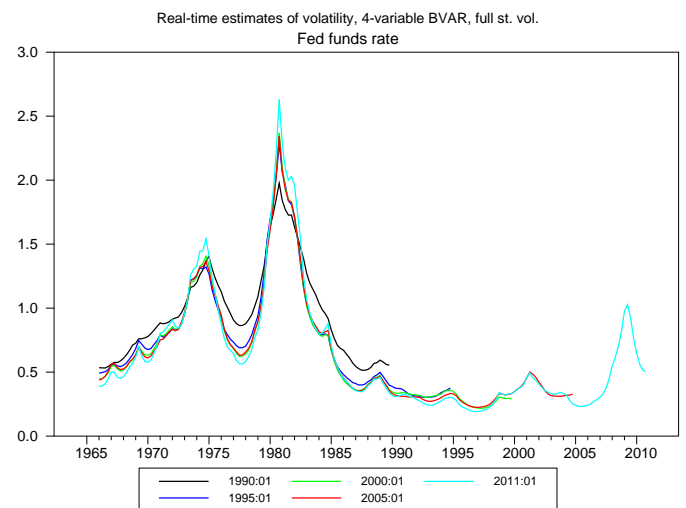
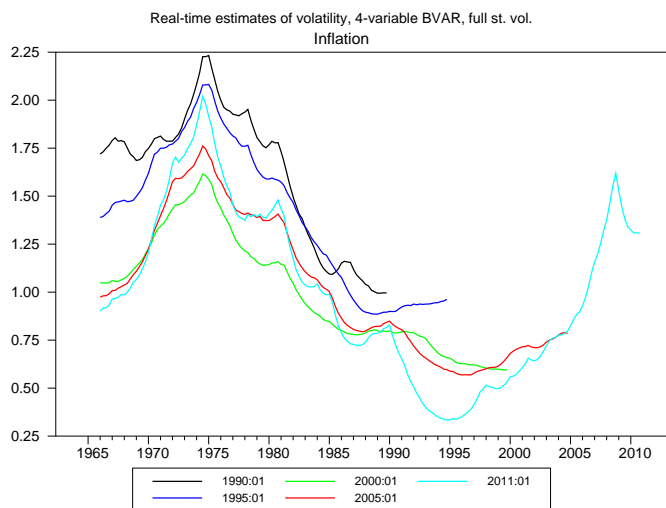
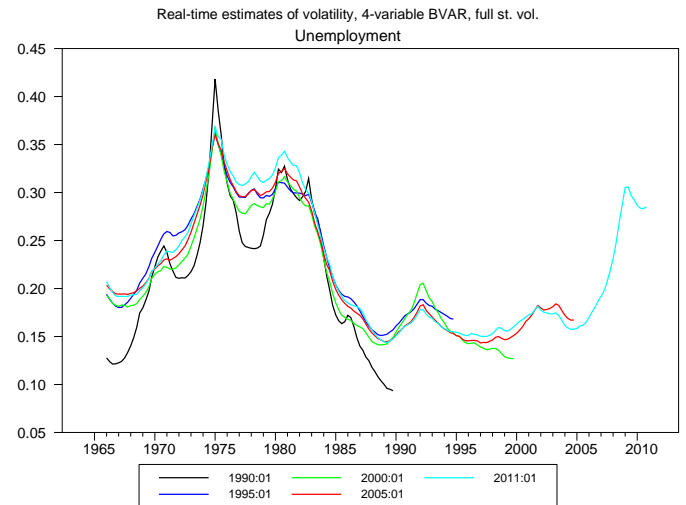
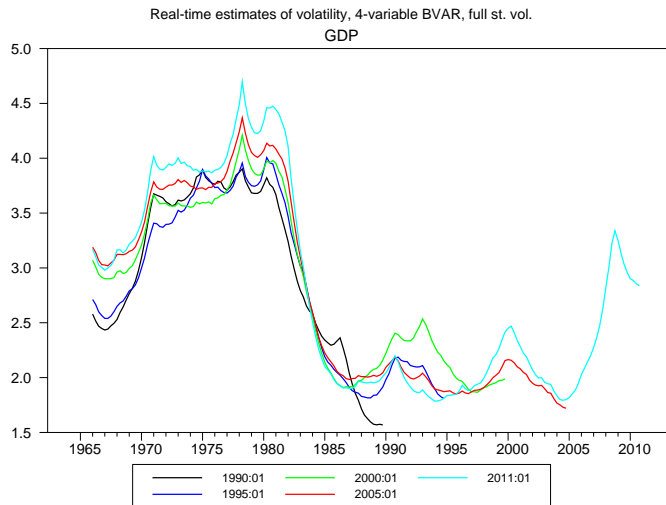


Figure 8: Volatility estimates from 4-variable BVAR-SV, real-time data

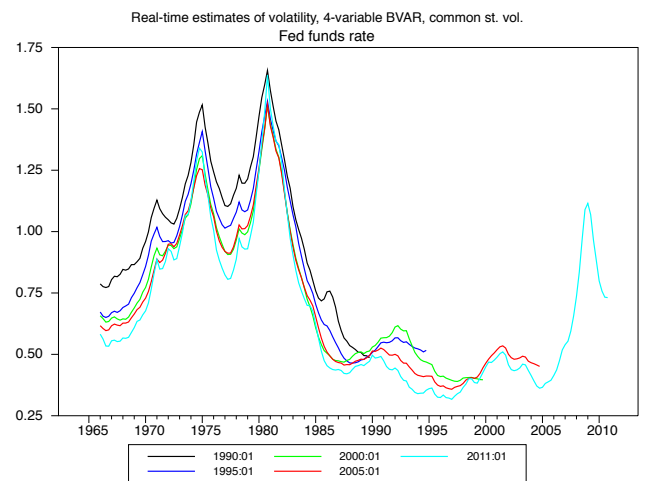
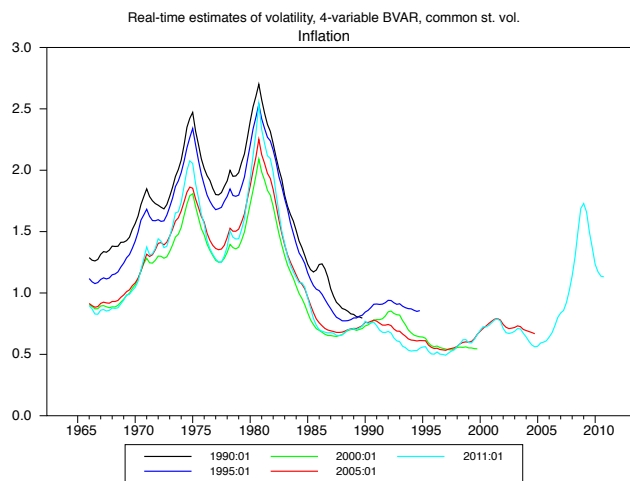
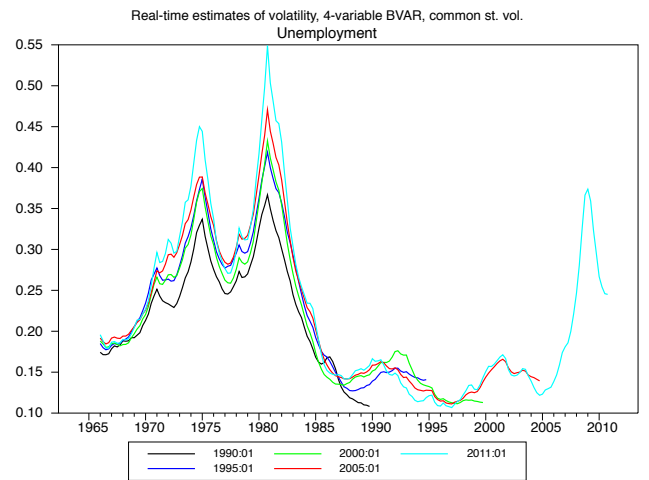
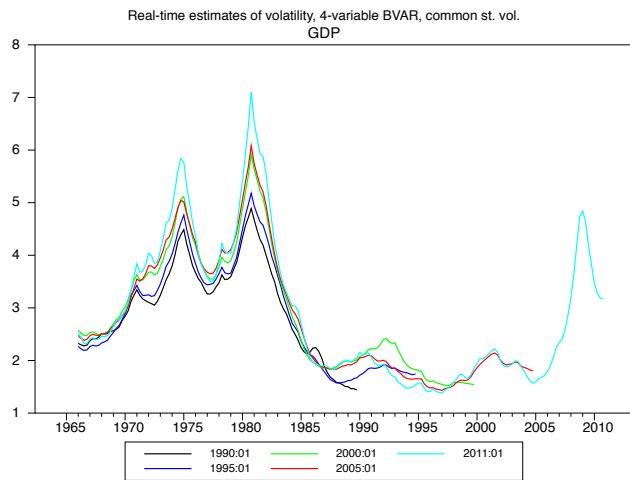


Figure 9: Volatility estimates from 4-variable BVAR-CSV, real-time data

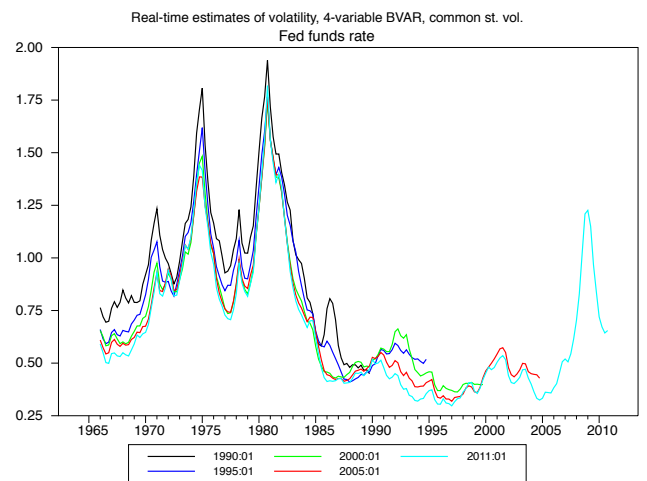
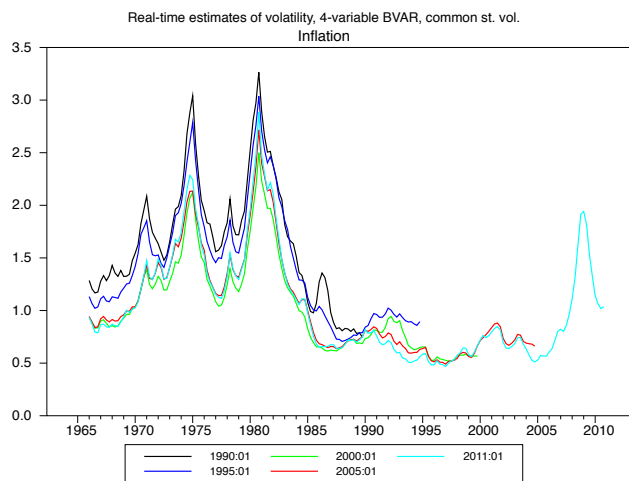
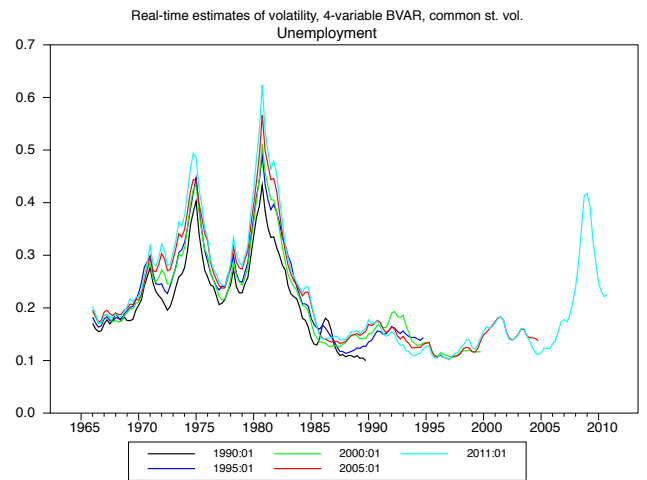
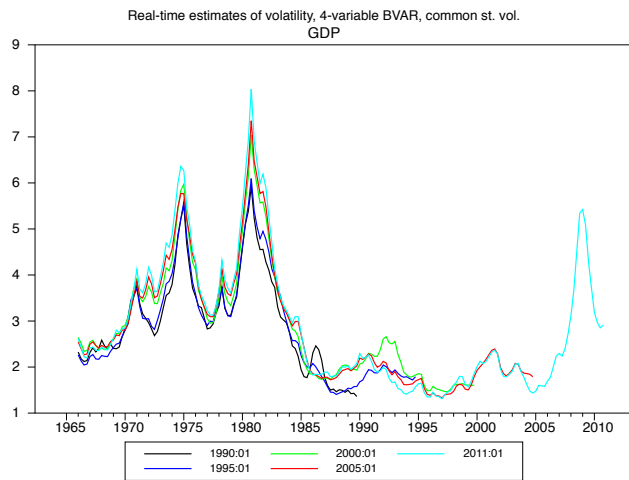


Figure 10: Volatility estimates from 4-variable BVAR-stationary CSV, real-time data

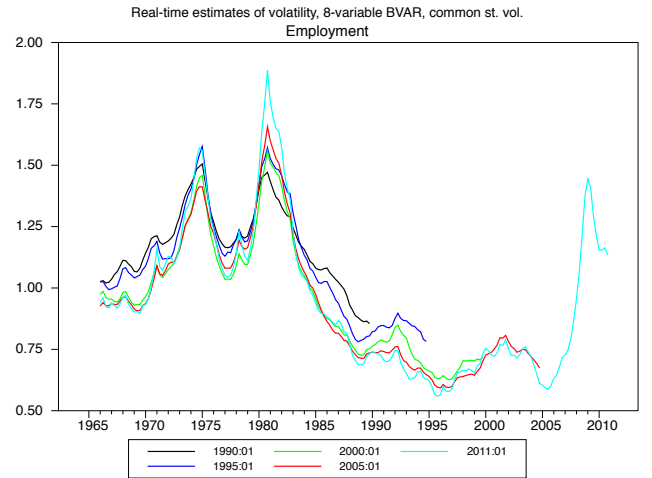
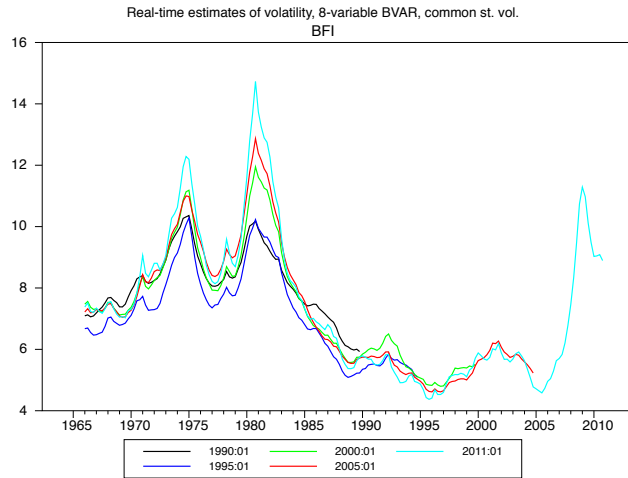
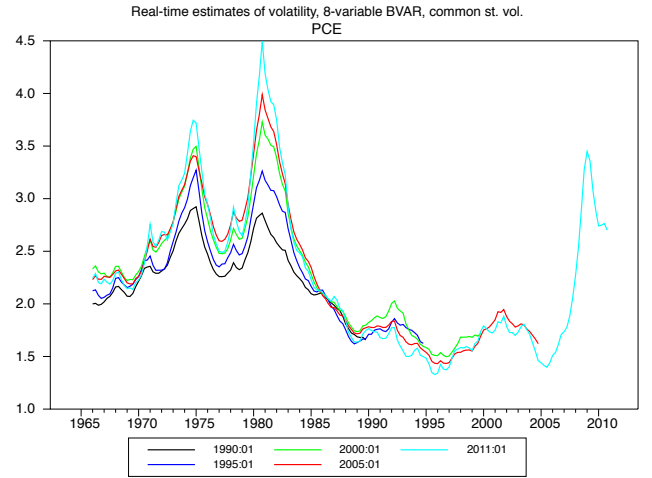
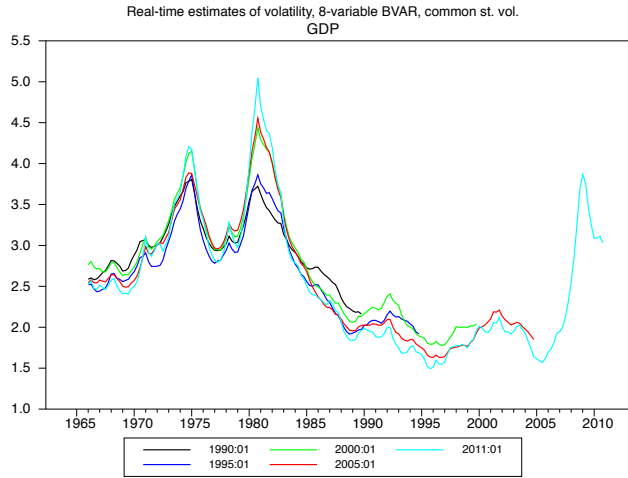


Figure 11: Volatility estimates from 8-variable BVAR-CSV, real-time data

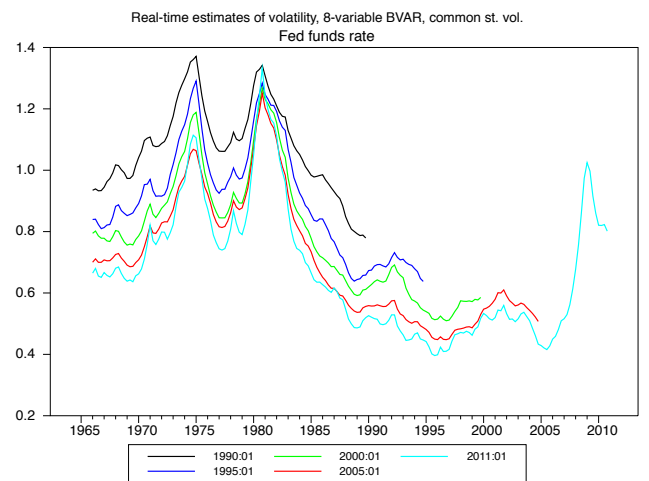
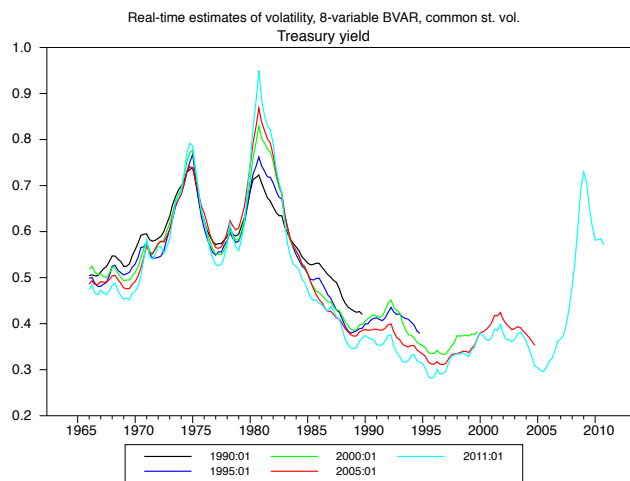
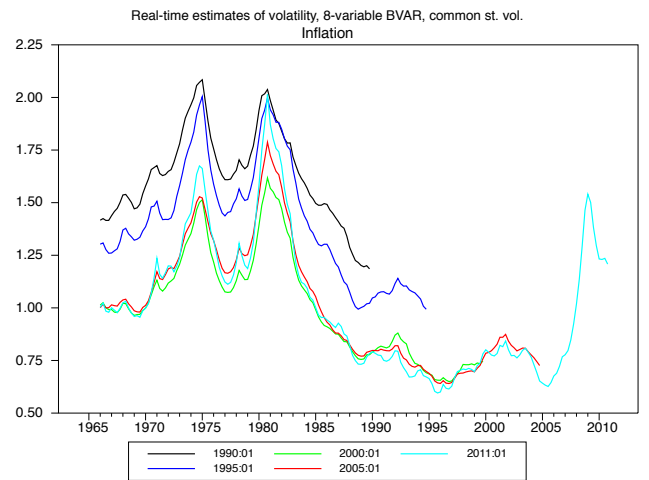
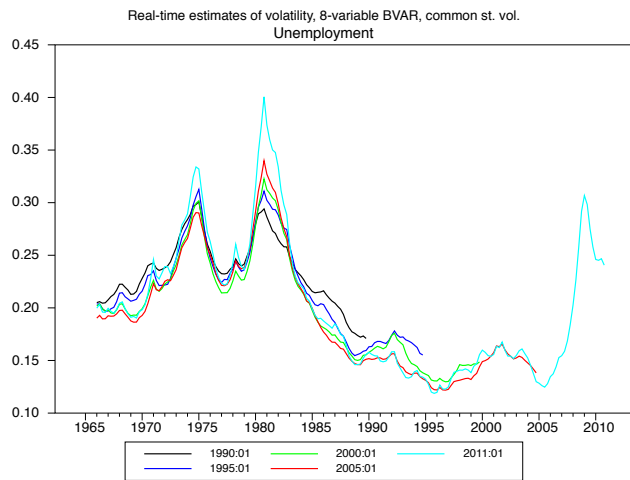


Figure 11: Volatility estimates from 8-variable BVAR-CSV, real-time data, continued

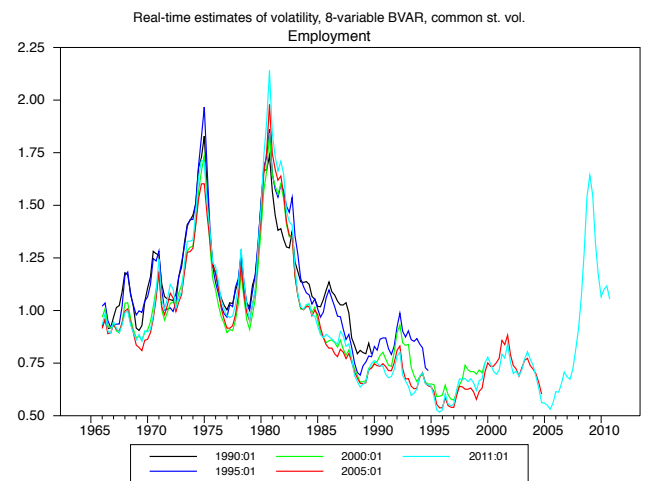
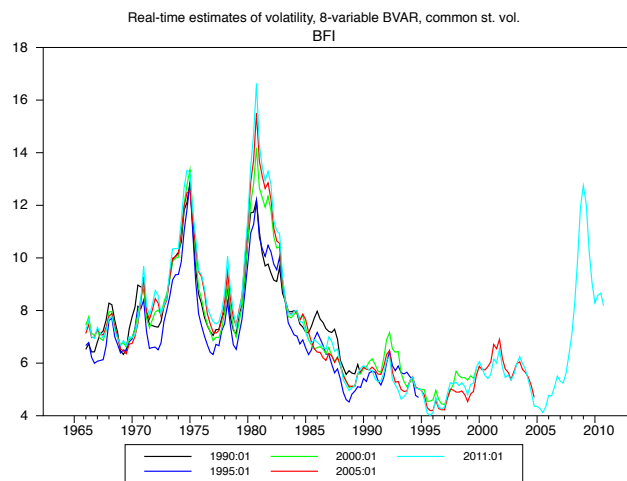
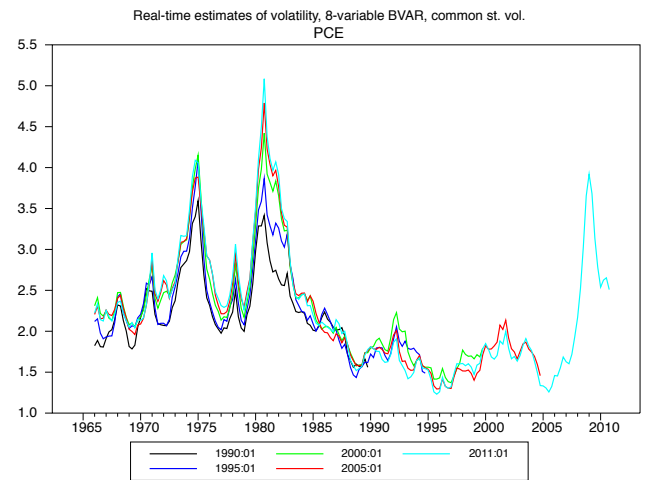
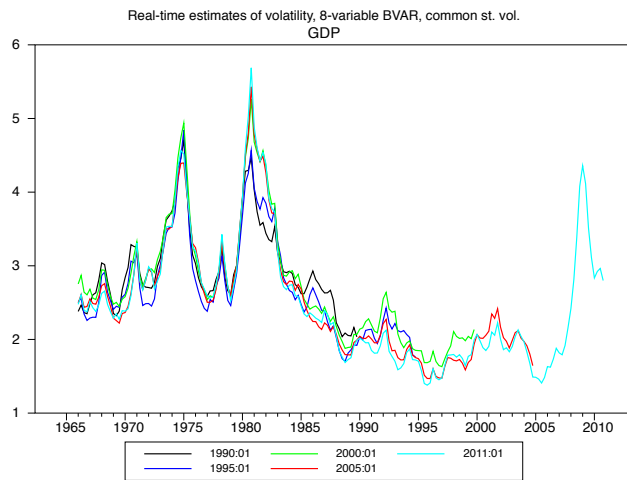


Figure 12: Volatility estimates from 8-variable BVAR-stationary CSV, real-time data

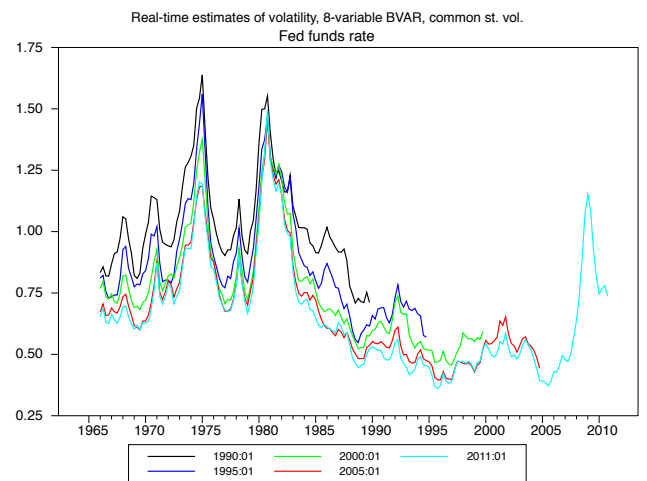
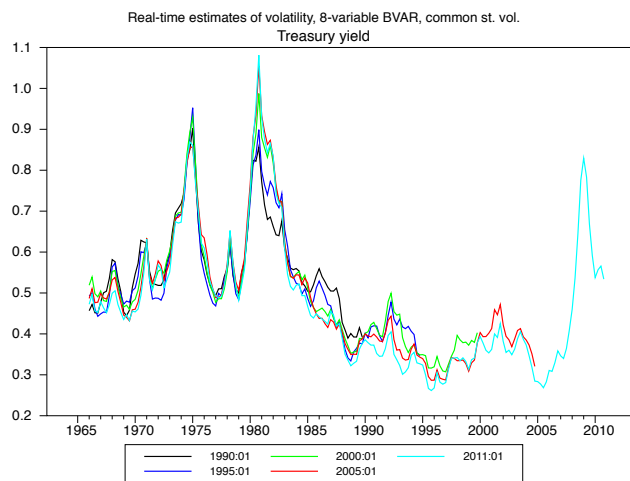
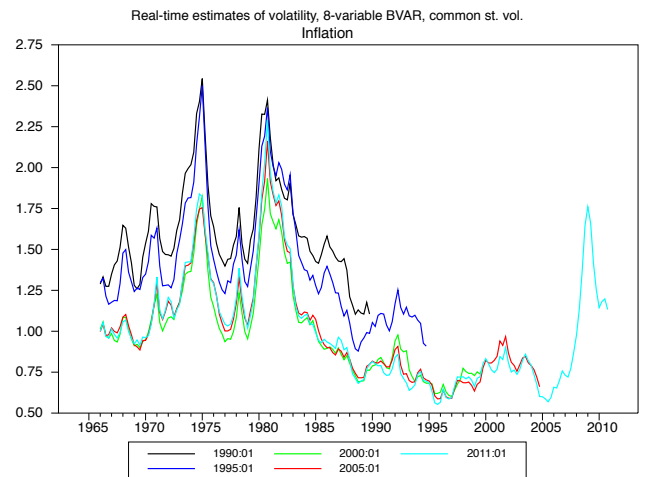
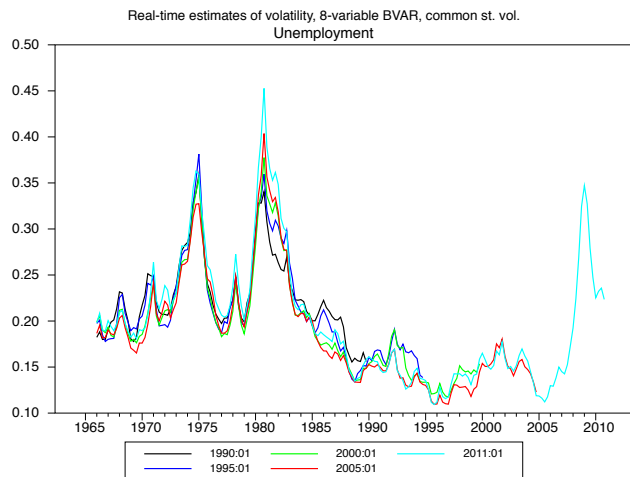


Figure 12: Volatility estimates from 8-variable BVAR-stationary CSV, real-time data, continued

Volatility estimate: BVAR(4), full stochastic volatility (standard deviation)

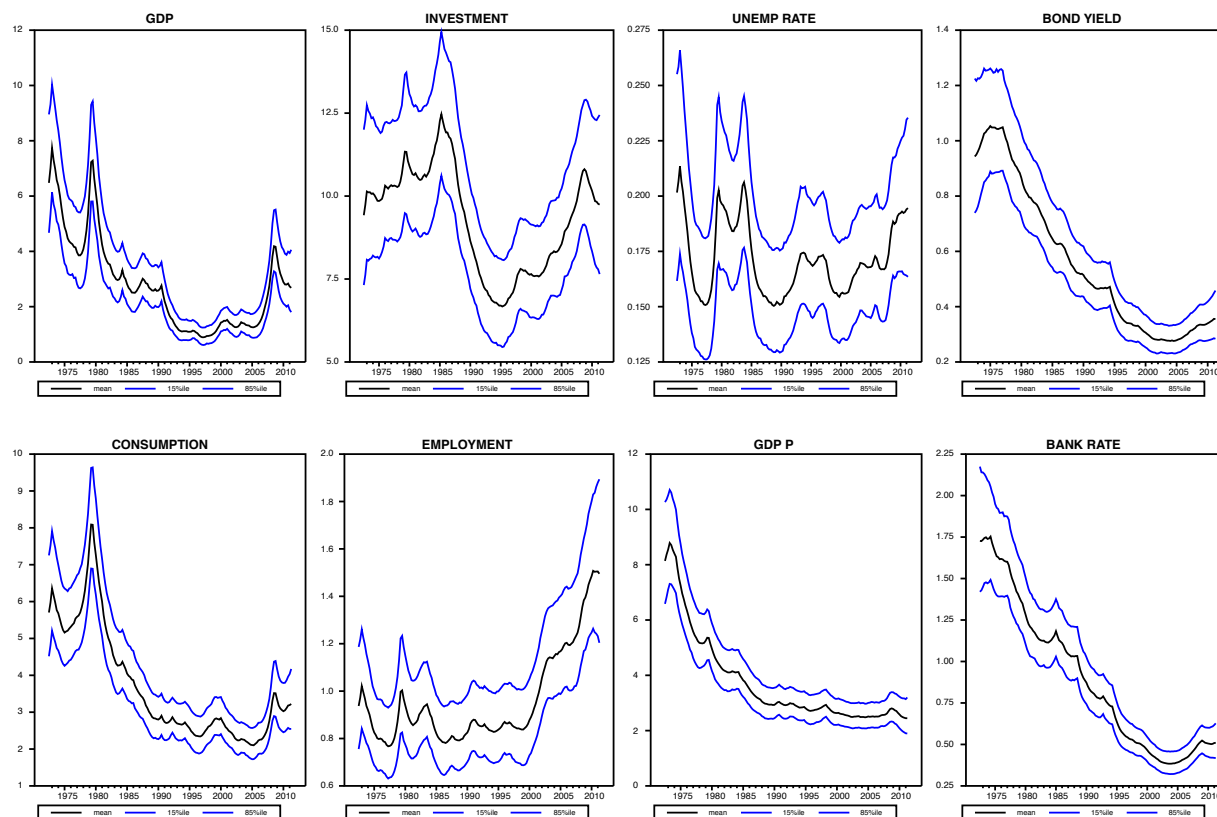


Figure 13: Volatility estimates from 8-variable BVAR-SV, UK data

Volatility estimate: BVAR(4), common stochastic volatility (standard deviation)

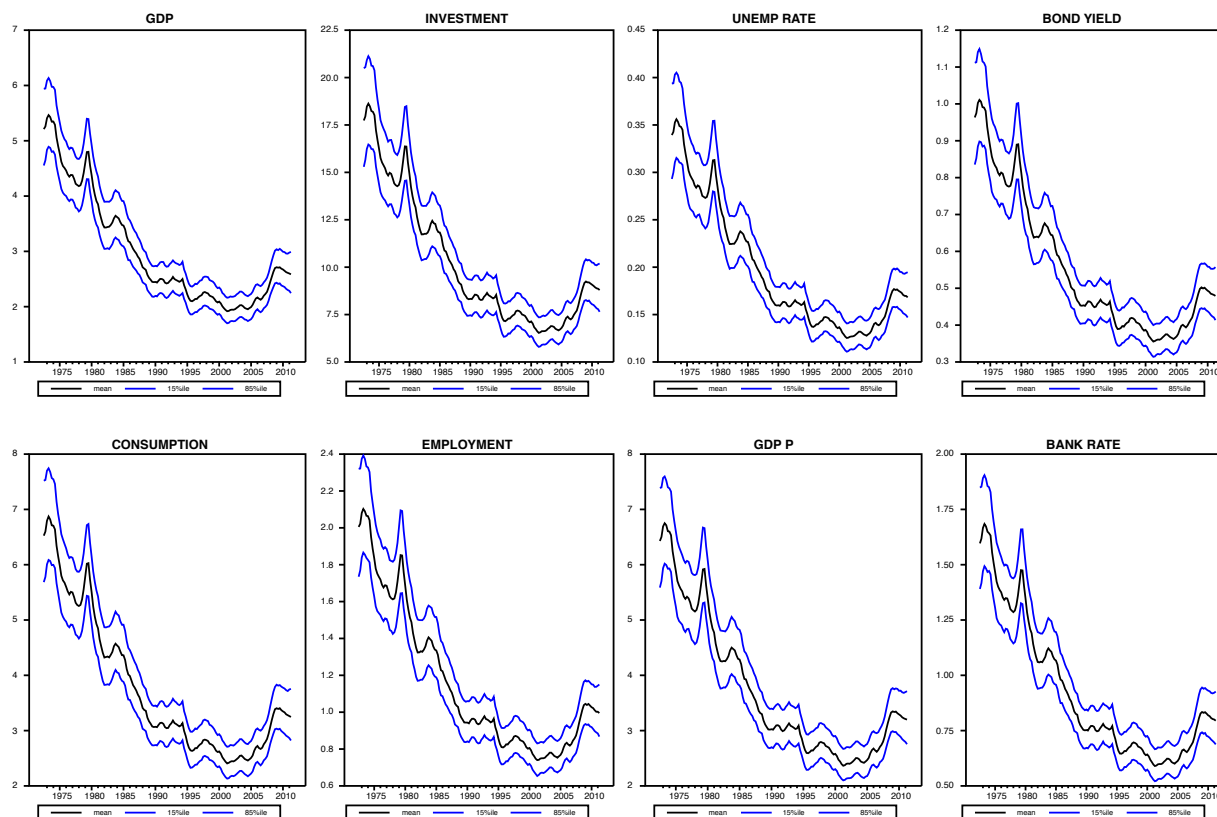


Figure 14: Volatility estimates from 8-variable BVAR-CSV, UK data

Volatility estimate: BVAR(4), common stochastic volatility (standard deviation)

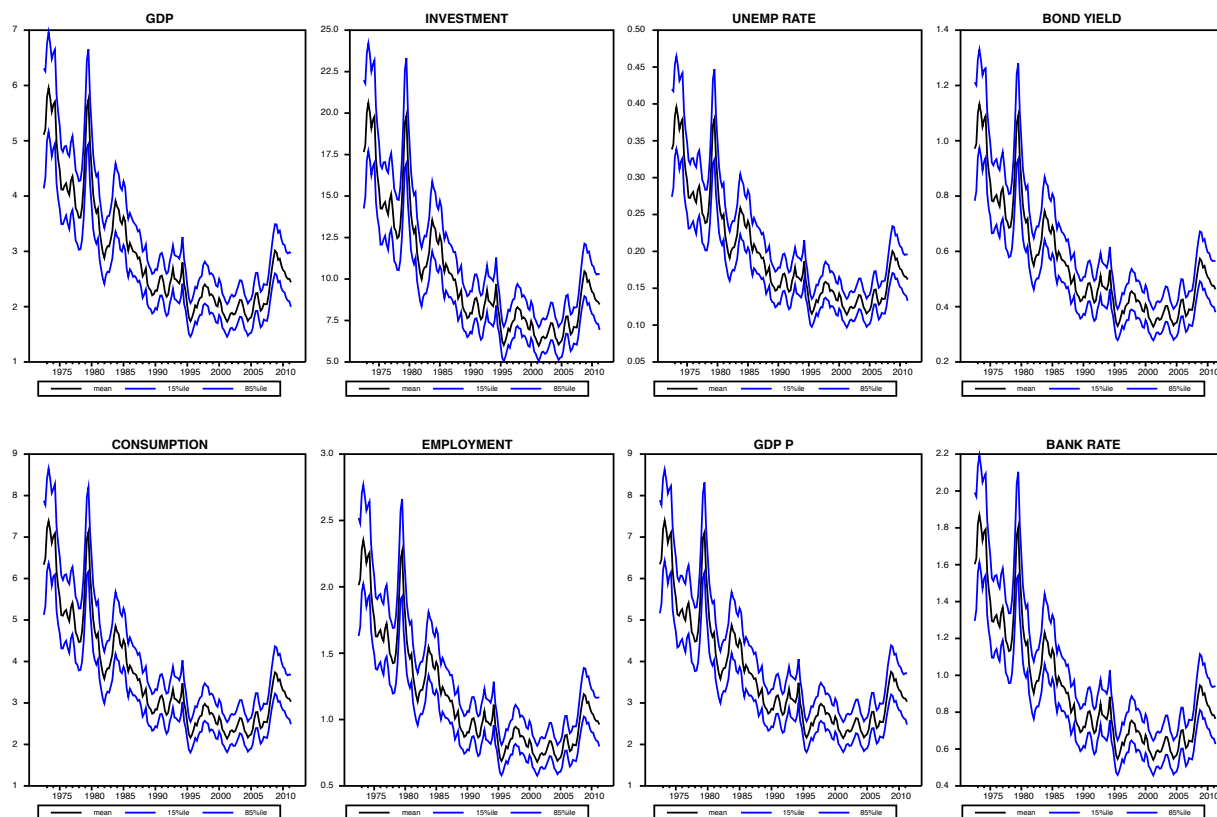


Figure 15: Volatility estimates from 8-variable BVAR-stationary CSV, UK data

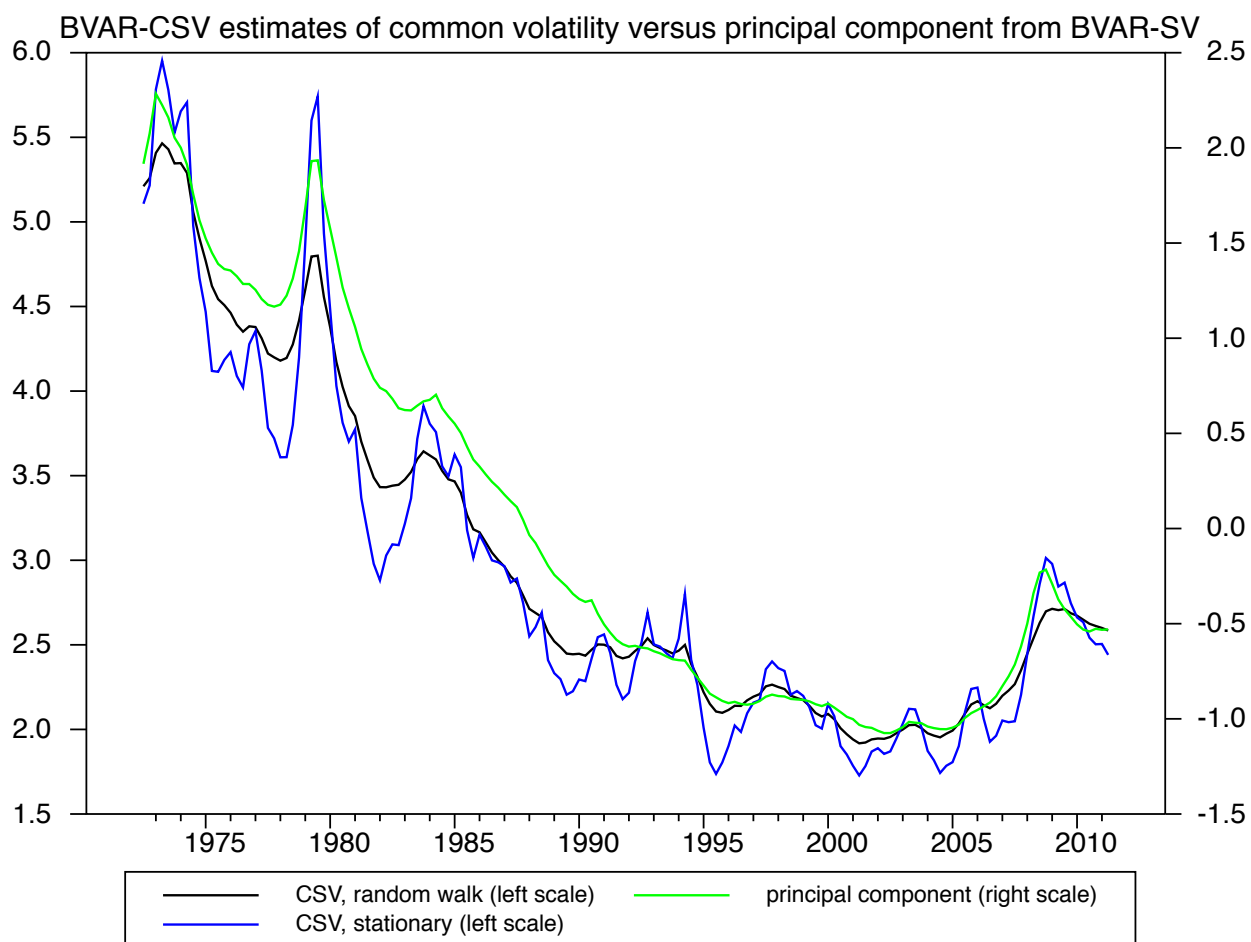


Figure 16: Principal component of BVAR-SV estimates of volatility versus common factor of volatility in 8-variable BVAR-CSV, UK data

**Table 1. Summary of Inefficiency Factors
for Various Model Specifications**

| parameter block | # parameters | median | mean | min | max |
|---|--------------|--------|-------|------|-------|
| <i>4 variables, independent st. vol.: skip interval of 10</i> | | | | | |
| Π | 68 | 1.40 | 1.47 | 0.79 | 2.74 |
| A | 6 | 1.65 | 1.87 | 1.09 | 3.00 |
| ϕ | 4 | 11.54 | 11.13 | 7.80 | 13.65 |
| Λ | 744 | 4.81 | 4.96 | 0.80 | 14.93 |
| <i>4 variables, independent st. vol.: skip interval of 20</i> | | | | | |
| Π | 68 | 1.09 | 1.19 | 0.54 | 2.03 |
| A | 6 | 1.04 | 1.30 | 0.77 | 2.32 |
| ϕ | 4 | 8.12 | 7.92 | 4.12 | 11.34 |
| Λ | 744 | 2.82 | 2.92 | 0.68 | 8.39 |
| <i>4 variables, common st. vol.: skip interval of 5</i> | | | | | |
| Π | 68 | 1.08 | 1.08 | 0.64 | 2.32 |
| A | 6 | 1.18 | 1.16 | 0.88 | 1.44 |
| S | 3 | 10.19 | 10.20 | 9.77 | 10.62 |
| ϕ | 1 | 8.25 | 8.25 | 8.25 | 8.25 |
| Λ | 186 | 5.89 | 6.10 | 3.11 | 10.22 |
| <i>4 variables, common st. vol.: skip interval of 10</i> | | | | | |
| Π | 68 | 0.99 | 0.99 | 0.55 | 1.65 |
| A | 6 | 1.04 | 1.06 | 0.82 | 1.24 |
| S | 3 | 4.58 | 4.50 | 3.95 | 4.97 |
| ϕ | 1 | 3.30 | 3.30 | 3.30 | 3.30 |
| Λ | 186 | 3.08 | 3.17 | 1.80 | 5.77 |
| <i>4 variables, stationary common st. vol.: skip interval of 5</i> | | | | | |
| Π | 68 | 1.06 | 1.08 | 0.67 | 2.03 |
| A | 6 | 1.23 | 1.23 | 0.94 | 1.41 |
| S | 3 | 7.33 | 6.70 | 5.06 | 7.70 |
| ϕ | 1 | 5.18 | 5.18 | 5.18 | 5.18 |
| Λ | 186 | 3.22 | 3.28 | 1.72 | 6.12 |
| <i>4 variables, stationary common st. vol.: skip interval of 10</i> | | | | | |
| Π | 68 | 1.05 | 1.04 | 0.60 | 1.51 |
| A | 6 | 1.09 | 1.09 | 0.79 | 1.37 |
| S | 3 | 3.75 | 3.93 | 3.23 | 4.81 |
| ϕ | 1 | 2.66 | 2.66 | 2.66 | 2.66 |
| Λ | 186 | 1.74 | 1.84 | 0.82 | 3.29 |

Notes:

1. For each individual parameter, the inefficiency factor is estimated as $1 + 2 \sum_{k=1}^{\infty} \rho_k$, where ρ_k is the k -th order autocorrelation of the chain of retained draws. The estimates use the Newey-West kernel and a bandwidth of 4 percent of the sample of draws.
2. The table provides summary statistics for the inefficiency factors computed for groups of model parameters.

**Table 1, Continued. Summary of Inefficiency Factors
for Various Model Specifications**

| parameter block | # parameters | median | mean | min | max |
|---|--------------|--------|-------|-------|-------|
| <i>8 variables, independent st. vol.: skip interval of 10</i> | | | | | |
| Π | 264 | 1.47 | 1.56 | 0.65 | 3.94 |
| A | 28 | 2.00 | 2.31 | 1.17 | 5.36 |
| ϕ | 8 | 14.01 | 13.39 | 5.42 | 21.01 |
| Λ | 1488 | 8.74 | 9.61 | 1.42 | 27.89 |
| <i>8 variables, independent st. vol.: skip interval of 20</i> | | | | | |
| Π | 264 | 1.18 | 1.23 | 0.63 | 2.62 |
| A | 28 | 1.44 | 1.62 | 0.87 | 3.42 |
| ϕ | 8 | 7.85 | 8.13 | 4.91 | 12.85 |
| Λ | 1488 | 4.71 | 4.93 | 0.77 | 12.55 |
| <i>8 variables, common st. vol.: skip interval of 5</i> | | | | | |
| Π | 264 | 1.07 | 1.12 | 0.47 | 4.40 |
| A | 28 | 0.97 | 1.07 | 0.74 | 1.48 |
| S | 7 | 26.33 | 26.09 | 22.45 | 27.68 |
| ϕ | 1 | 9.16 | 9.16 | 9.16 | 9.16 |
| Λ | 186 | 13.99 | 14.40 | 7.82 | 21.89 |
| <i>8 variables, common st. vol.: skip interval of 10</i> | | | | | |
| Π | 264 | 0.97 | 1.01 | 0.47 | 2.62 |
| A | 28 | 0.96 | 0.98 | 0.57 | 1.37 |
| S | 7 | 11.34 | 10.63 | 7.86 | 11.78 |
| ϕ | 1 | 5.52 | 5.52 | 5.52 | 5.52 |
| Λ | 186 | 6.18 | 6.18 | 3.91 | 9.82 |
| <i>8 variables, common st. vol.: skip interval of 20</i> | | | | | |
| Π | 264 | 0.95 | 0.98 | 0.41 | 2.05 |
| A | 28 | 0.96 | 0.96 | 0.62 | 1.32 |
| S | 7 | 5.99 | 5.88 | 5.07 | 6.46 |
| ϕ | 1 | 3.45 | 3.45 | 3.45 | 3.45 |
| Λ | 186 | 3.47 | 3.57 | 2.33 | 5.42 |
| <i>8 variables, stationary common st. vol.: skip interval of 5</i> | | | | | |
| Π | 264 | 1.01 | 1.02 | 0.51 | 1.93 |
| A | 28 | 1.01 | 1.03 | 0.57 | 1.70 |
| S | 7 | 7.51 | 7.48 | 6.45 | 8.00 |
| ϕ | 1 | 4.89 | 4.89 | 4.89 | 4.89 |
| Λ | 186 | 3.65 | 3.63 | 1.49 | 6.77 |
| <i>8 variables, stationary common st. vol.: skip interval of 10</i> | | | | | |
| Π | 264 | 0.93 | 0.96 | 0.46 | 1.63 |
| A | 28 | 0.96 | 0.97 | 0.50 | 1.51 |
| S | 7 | 4.72 | 4.85 | 4.14 | 5.52 |
| ϕ | 1 | 2.33 | 2.33 | 2.33 | 2.33 |
| Λ | 186 | 2.38 | 2.39 | 0.89 | 4.11 |
| <i>8 variables, stationary common st. vol.: skip interval of 20</i> | | | | | |
| Π | 264 | 0.92 | 0.94 | 0.36 | 1.60 |
| A | 28 | 0.93 | 0.95 | 0.57 | 1.57 |
| S | 7 | 2.92 | 2.87 | 2.08 | 3.87 |
| ϕ | 1 | 1.28 | 1.28 | 1.28 | 1.28 |
| Λ | 186 | 1.46 | 1.49 | 0.58 | 2.49 |

Notes:

1. For each individual parameter, the inefficiency factor is estimated as $1 + 2 \sum_{k=1}^{\infty} \rho_k$, where ρ_k is the k -th order autocorrelation of the chain of retained draws. The estimates use the Newey-West kernel and a bandwidth of 4 percent of the sample of draws.

2. The table provides summary statistics for the inefficiency factors computed for groups of model parameters.

Table 2. CPU time requirements for different models
(105,000 draws)

| model | CPU time (minutes) |
|---|-----------------------|
| BVAR(4), 4 variables, independent stochastic volatility | 83.6 |
| BVAR(4), 4 variables, common stochastic volatility | 19.0 |
| BVAR(4), 4 variables, stationary common stochastic volatility | 19.8 |
| BVAR(4), 8 variables, independent stochastic volatility | 879.5 |
| BVAR(4), 8 variables, common stochastic volatility | 47.1 |
| BVAR(4), 8 variables, stationary common stochastic volatility | 50.4 |

Note: The reported CPU run times are averages across 10 different sets of model estimates, based on different MCMC chains.

Table 3. Log predictive likelihoods, 1980:Q1-2011:Q2

| model | log PL |
|---|-----------|
| BVAR(4), 4 variables | -661.880 |
| BVAR(4), 4 variables, independent stochastic volatility | -551.284 |
| BVAR(4), 4 variables, common stochastic volatility | -553.176 |
| BVAR(4), 4 variables, stationary common stochastic volatility | -549.630 |
| BVAR(4), 8 variables | -1541.749 |
| BVAR(4), 8 variables, common stochastic volatility | -1459.193 |
| BVAR(4), 8 variables, stationary common stochastic volatility | -1458.193 |

Notes: The table reports log predictive likelihoods, formed as the sum of 1-step ahead likelihoods, over the period 1980:Q1 through 2011:Q2. The estimates are based on final vintage data, not real-time data.

**Table 4. Real-Time Forecast RMSEs, 4-variable BVARs,
1985:Q1-2010:Q4**

(RMSEs for BVAR benchmark, RMSE ratios in all others)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| GDP growth | 0.816 *** | 0.803 *** | 0.871 ** | 1.035 | 1.025 |
| Unemployment | 0.779 ** | 0.751 * | 0.756 * | 0.866 | 1.066 |
| GDP inflation | 1.024 | 1.025 | 0.964 | 0.896 *** | 0.856 *** |
| Fed funds rate | 0.974 | 1.114 | 1.230 | 1.180 | 1.143 |
| BVAR | | | | | |
| GDP growth | 2.533 | 2.742 | 2.651 | 2.281 | 2.320 |
| Unemployment | 0.247 | 0.499 | 1.011 | 1.580 | 1.507 |
| GDP inflation | 1.114 | 1.220 | 1.480 | 2.107 | 2.557 |
| Fed funds rate | 0.506 | 0.868 | 1.447 | 2.405 | 2.936 |
| BVAR-SV | | | | | |
| GDP growth | 0.953 *** | 0.935 ** | 0.948 * | 1.002 | 1.011 |
| Unemployment | 0.991 | 0.971 * | 0.956 * | 0.981 | 1.038 |
| GDP inflation | 0.945 *** | 0.926 *** | 0.862 *** | 0.826 *** | 0.795 *** |
| Fed funds rate | 0.885 *** | 0.932 ** | 0.973 | 0.969 | 0.939 ** |
| BVAR-CSV | | | | | |
| GDP growth | 0.917 *** | 0.899 *** | 0.937 * | 1.020 | 1.019 |
| Unemployment | 0.956 ** | 0.933 ** | 0.922 * | 0.973 | 1.052 |
| GDP inflation | 0.936 *** | 0.902 *** | 0.834 *** | 0.785 *** | 0.742 *** |
| Fed funds rate | 0.947 *** | 0.971 ** | 0.984 | 0.971 | 0.934 *** |
| BVAR-stationary CSV | | | | | |
| GDP growth | 0.919 *** | 0.900 *** | 0.944 | 1.023 | 1.020 |
| Unemployment | 0.973 * | 0.948 ** | 0.935 * | 0.984 | 1.066 |
| GDP inflation | 0.927 *** | 0.894 *** | 0.821 *** | 0.776 *** | 0.734 *** |
| Fed funds rate | 0.946 *** | 0.974 * | 0.999 | 0.999 | 0.970 ** |

Notes: For the forecasts from AR models, the BVAR with independent stochastic volatilities, and the BVAR with common stochastic volatility, entries less than 1 indicate the model has a lower RMSE than the benchmark. To provide a rough gauge of whether the RMSE ratios are significantly different from 1, we use the Diebold-Mariano t -statistic for equal MSE, applied to the forecast of each model relative to the benchmark. Differences in accuracy that are statistically different from zero are denoted by one, two, or three asterisks, corresponding to significance levels of 10%, 5%, and 1%, respectively. The underlying p -values are based on t -statistics computed with a serial correlation-robust variance, using a rectangular kernel, $h - 1$ lags, and the small-sample adjustment of Harvey, Leybourne, and Newbold (1997).

Table 5. Real-Time Forecast RMSEs, 8-variable BVARs, 1985:Q1-2010:Q4

(RMSEs for BVAR benchmark, RMSE ratios in all others)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| GDP growth | 0.912 * | 0.873 ** | 0.922 * | 1.014 | 1.012 |
| Consumption | 0.877 *** | 0.899 *** | 0.902 *** | 0.990 | 1.002 |
| BFI | 1.085 | 1.054 | 1.066 | 1.101 | 1.016 |
| Employment | 0.889 ** | 0.828 *** | 0.822 ** | 0.926 | 0.993 |
| Unemployment | 0.852 * | 0.827 | 0.834 | 0.892 | 1.041 |
| GDP inflation | 0.991 | 0.993 | 0.984 | 0.944 | 0.897 ** |
| Treasury yield | 0.930 | 1.007 | 1.210 | 1.398 | 1.410 |
| Fed funds rate | 0.809 * | 0.993 | 1.274 | 1.319 | 1.259 |
| BVAR | | | | | |
| GDP growth | 2.266 | 2.523 | 2.506 | 2.327 | 2.350 |
| Consumption | 2.480 | 2.446 | 2.518 | 2.342 | 2.273 |
| BFI | 8.566 | 9.427 | 9.963 | 9.639 | 10.210 |
| Employment | 1.074 | 1.657 | 2.173 | 2.177 | 2.101 |
| Unemployment | 0.225 | 0.453 | 0.916 | 1.535 | 1.542 |
| GDP inflation | 1.150 | 1.259 | 1.450 | 1.999 | 2.440 |
| Treasury yield | 0.456 | 0.704 | 0.928 | 1.085 | 1.295 |
| Fed funds rate | 0.609 | 0.974 | 1.397 | 2.150 | 2.666 |
| BVAR-SV | | | | | |
| GDP growth | 1.007 | 0.998 | 0.997 | 1.000 | 1.014 |
| Consumption | 0.968 *** | 0.982 * | 1.013 | 1.023 | 1.025 |
| BFI | 0.999 | 0.997 | 0.996 | 1.001 | 1.008 |
| Employment | 0.985 | 0.987 | 0.973 | 0.954 ** | 0.973 * |
| Unemployment | 1.051 | 1.024 | 1.005 | 1.005 | 1.028 |
| GDP inflation | 0.957 *** | 0.938 *** | 0.898 *** | 0.874 *** | 0.848 *** |
| Treasury yield | 0.966 ** | 0.963 ** | 0.987 | 0.926 * | 0.868 ** |
| Fed funds rate | 0.956 | 1.008 | 1.004 | 0.948 * | 0.895 ** |
| BVAR-CSV | | | | | |
| GDP growth | 0.986 | 0.959 ** | 0.981 | 1.001 | 1.002 |
| Consumption | 0.960 *** | 0.960 *** | 0.986 | 1.021 | 1.019 |
| BFI | 1.003 | 1.007 | 1.016 | 1.008 | 0.997 |
| Employment | 0.927 *** | 0.918 *** | 0.921 ** | 0.937 ** | 0.956 |
| Unemployment | 1.052 | 1.018 | 0.985 | 0.980 | 1.017 |
| GDP inflation | 0.950 *** | 0.912 *** | 0.866 *** | 0.821 *** | 0.789 *** |
| Treasury yield | 0.973 * | 1.001 | 1.023 | 1.005 | 0.956 * |
| Fed funds rate | 0.935 ** | 0.947 ** | 0.971 | 0.988 | 0.953 *** |
| BVAR-stationary CSV | | | | | |
| GDP growth | 0.979 | 0.948 ** | 0.980 | 0.999 | 0.997 |
| Consumption | 0.961 *** | 0.961 *** | 0.985 | 1.025 | 1.020 |
| BFI | 1.006 | 1.004 | 1.018 | 1.009 | 0.993 |
| Employment | 0.936 *** | 0.921 *** | 0.922 ** | 0.936 * | 0.952 |
| Unemployment | 1.073 | 1.032 | 0.995 | 0.983 | 1.020 |
| GDP inflation | 0.942 *** | 0.897 *** | 0.849 *** | 0.800 *** | 0.768 *** |
| Treasury yield | 0.974 * | 0.999 | 1.024 | 1.017 | 0.967 |
| Fed funds rate | 0.924 ** | 0.936 ** | 0.967 | 0.991 | 0.961 ** |

Note: See the notes to Table 4.

Table 6. Average log predictive scores, 4-variable BVARs, 1985:Q1-2010:Q4

(avg. score for benchmark BVAR, differences in scores in all others)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| GDP growth | 0.033 | 0.055 | 0.014 | -0.037 | -0.028 |
| Unemployment | 0.178 *** | 0.230 | 0.530 | 0.888 | 0.252 |
| GDP inflation | -0.019 | -0.016 | 0.002 | 0.077 *** | 0.146 *** |
| Fed funds rate | 0.026 | -0.074 | -0.137 | -0.139 | -0.127 |
| BVAR | | | | | |
| All variables | -4.923 | -6.250 | -8.014 | -9.406 | -9.273 |
| GDP growth | -2.391 | -2.461 | -2.460 | -2.422 | -2.438 |
| Unemployment | -0.056 | -0.769 | -1.761 | -2.687 | -2.205 |
| GDP inflation | -1.549 | -1.662 | -1.856 | -2.170 | -2.365 |
| Fed funds rate | -1.162 | -1.557 | -1.960 | -2.362 | -2.542 |
| BVAR-SV | | | | | |
| All variables | 0.818 *** | 0.560 ** | 0.284 | -0.699 | -1.060 |
| GDP growth | 0.099 *** | 0.026 | 0.036 | -0.001 | -0.024 |
| Unemployment | 0.161 *** | 0.055 | -0.355 | -0.906 | -1.133 |
| GDP inflation | 0.090 *** | 0.107 *** | 0.172 *** | 0.155 *** | 0.194 *** |
| Fed funds rate | 0.537 *** | 0.297 *** | 0.025 | -0.119 | -0.146 |
| BVAR-CSV | | | | | |
| All variables | 0.783 *** | 0.668 *** | 0.286 | -0.807 | -1.482 |
| GDP growth | 0.176 *** | 0.133 *** | 0.058 | 0.003 | -0.010 |
| Unemployment | 0.199 *** | 0.083 | -0.227 | -0.813 | -1.469 |
| GDP inflation | 0.070 * | 0.098 ** | 0.181 *** | 0.165 *** | 0.210 *** |
| Fed funds rate | 0.433 *** | 0.301 *** | 0.082 | -0.153 | -0.303 |
| BVAR-stationary CSV | | | | | |
| All variables | 0.872 *** | 0.854 *** | 0.697 *** | 0.200 | -0.303 |
| GDP growth | 0.192 *** | 0.170 *** | 0.093 ** | 0.012 | -0.099 |
| Unemployment | 0.247 *** | 0.197 * | 0.049 | -0.111 | -0.587 |
| GDP inflation | 0.071 ** | 0.099 ** | 0.175 *** | 0.194 *** | 0.249 *** |
| Fed funds rate | 0.450 *** | 0.314 *** | 0.101 | -0.037 | -0.097 |

Notes: For the forecasts from AR models, the BVAR with independent stochastic volatilities, and the BVAR with common stochastic volatility, entries greater than 0 indicate the model has a better average log score (better density forecast) than the benchmark model. To provide a rough gauge of the statistical significance of differences in average log scores, we use the Amisano-Giacomini t -test of equal means, applied to the log score for each model relative to the benchmark forecast. Differences in average scores that are statistically different from zero are denoted by one, two, or three asterisks, corresponding to significance levels of 10%, 5%, and 1%, respectively. The underlying p -values are based on t -statistics computed with a serial correlation-robust variance, using a rectangular kernel, $h - 1$ lags, and the small-sample adjustment of Harvey, Leybourne, and Newbold (1997).

Table 7. Average log predictive scores, 8-variable BVARs, 1985:Q1-2010:Q4

(avg. score for benchmark BVAR, differences in scores in all others)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| All variables | -0.633 | -1.006 | -0.859 | -0.681 | -1.065 |
| GDP growth | -0.046 | -0.006 | -0.016 | -0.020 | -0.015 |
| Consumption | 0.090 ** | 0.064 * | 0.056 * | 0.000 | -0.007 |
| BFI | -0.092 | -0.033 | -0.026 | -0.080 | -0.001 |
| Employment | 0.018 | 0.105 | 0.216 | 0.083 | 0.024 |
| Unemployment | 0.101 * | 0.148 | 0.360 | 0.645 | 0.236 |
| GDP inflation | 0.001 | 0.000 | -0.014 | 0.031 | 0.096 * |
| Treasury yield | 0.054 * | -0.037 | -0.183 | -0.237 | -0.221 |
| Fed funds rate | 0.028 | -0.055 | -0.163 | -0.209 | -0.198 |
| BVAR | | | | | |
| All variables | -12.335 | -14.217 | -16.504 | -18.346 | -18.563 |
| GDP growth | -2.312 | -2.400 | -2.430 | -2.439 | -2.451 |
| Consumption | -2.333 | -2.327 | -2.355 | -2.311 | -2.292 |
| BFI | -3.578 | -3.724 | -3.788 | -3.732 | -3.784 |
| Employment | -1.520 | -1.955 | -2.303 | -2.270 | -2.228 |
| Unemployment | 0.021 | -0.686 | -1.590 | -2.444 | -2.188 |
| GDP inflation | -1.570 | -1.678 | -1.840 | -2.125 | -2.316 |
| Treasury yield | -0.664 | -1.071 | -1.367 | -1.626 | -1.828 |
| Fed funds rate | -1.165 | -1.577 | -1.934 | -2.292 | -2.471 |
| BVAR-SV | | | | | |
| All variables | 0.687 *** | 0.369 | 0.134 | -0.505 | -0.630 |
| GDP growth | 0.048 | 0.011 | 0.011 | 0.004 | -0.020 |
| Consumption | 0.032 | -0.013 | -0.103 | -0.102 | -0.094 |
| BFI | -0.057 | -0.149 | -0.168 | -0.171 | -0.243 |
| Employment | 0.049 | -0.076 | -0.199 | -0.226 | -0.137 |
| Unemployment | 0.065 ** | -0.037 | -0.391 | -0.848 | -0.777 |
| GDP inflation | 0.058 ** | 0.079 *** | 0.116 *** | 0.125 *** | 0.155 ** |
| Treasury yield | 0.057 *** | 0.041 * | 0.044 | 0.085 ** | 0.090 * |
| Fed funds rate | 0.422 *** | 0.267 *** | 0.133 ** | 0.064 | 0.054 |
| BVAR-CSV | | | | | |
| All variables | 0.603 *** | 0.478 ** | -0.279 | -1.006 | -1.354 |
| GDP growth | 0.055 ** | 0.046 | -0.016 | -0.011 | -0.053 |
| Consumption | 0.007 | -0.016 | -0.073 | -0.075 | -0.076 |
| BFI | 0.031 | 0.005 | -0.222 | -0.194 | -0.317 |
| Employment | 0.112 *** | 0.060 | -0.090 | -0.170 | -0.252 |
| Unemployment | 0.103 ** | -0.013 | -0.336 | -0.707 | -0.873 |
| GDP inflation | 0.042 | 0.072 ** | 0.101 *** | 0.141 *** | 0.192 *** |
| Treasury yield | 0.078 *** | 0.033 | 0.014 | 0.007 | -0.017 |
| Fed funds rate | 0.274 *** | 0.203 *** | 0.083 | -0.010 | -0.066 |
| BVAR-stationary CSV | | | | | |
| All variables | 0.668 *** | 0.612 *** | 0.157 | -0.135 | -0.255 |
| GDP growth | 0.051 ** | 0.049 ** | -0.022 | -0.059 | -0.160 |
| Consumption | 0.018 | -0.002 | -0.034 | -0.081 | -0.151 |
| BFI | 0.066 | 0.058 | -0.098 | -0.064 | -0.137 |
| Employment | 0.102 *** | 0.093 * | 0.006 | -0.021 | -0.101 |
| Unemployment | 0.118 *** | 0.032 | -0.190 | -0.278 | -0.389 |
| GDP inflation | 0.052 ** | 0.074 ** | 0.093 *** | 0.117 ** | 0.144 * |
| Treasury yield | 0.056 *** | 0.019 | 0.002 | -0.022 | -0.067 |
| Fed funds rate | 0.266 *** | 0.195 *** | 0.070 | -0.014 | -0.067 |

Note: See the notes to Table 6.

Table 8. Forecast RMSEs, 15-variable BVARs, 1985:Q1-2008:Q4
(RMSEs for BVAR benchmark, RMSE ratios for BVAR-CSV)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|------------------------------------|-----------|----------|----------|----------|-----------|
| BVAR | | | | | |
| GDP | 1.923 | 2.253 | 2.355 | 2.323 | 2.325 |
| CPI | 2.014 | 2.197 | 2.282 | 2.274 | 2.312 |
| Fed funds rate | 0.426 | 0.506 | 0.503 | 0.516 | 0.511 |
| Consumption | 1.890 | 2.129 | 2.203 | 2.172 | 2.134 |
| Industrial production | 3.132 | 4.023 | 4.276 | 4.267 | 4.180 |
| Capacity Utilization | 0.853 | 1.561 | 2.544 | 3.477 | 3.740 |
| Unemployment rate | 0.156 | 0.198 | 0.233 | 0.231 | 0.226 |
| Housing starts | 30.354 | 44.957 | 64.460 | 80.796 | 89.056 |
| PPI for finished goods | 4.362 | 4.705 | 4.815 | 4.826 | 4.929 |
| PCE price index | 1.442 | 1.541 | 1.596 | 1.593 | 1.611 |
| Real average hourly earnings | 1.727 | 1.785 | 1.877 | 1.870 | 1.819 |
| S&P stock price index, industrials | 27.497 | 28.915 | 28.918 | 27.775 | 27.439 |
| 10-year Treasury bond yield | 0.442 | 0.454 | 0.437 | 0.402 | 0.394 |
| Effective exchange rate | 13.185 | 13.280 | 13.013 | 12.552 | 12.581 |
| Payroll employment | 0.908 | 1.422 | 1.916 | 2.038 | 1.999 |
| BVAR-CSV | | | | | |
| GDP | 0.995 | 0.983 | 1.011 | 1.021 | 1.017 |
| CPI | 1.001 | 1.000 | 0.997 | 1.000 | 1.002 |
| Fed funds rate | 0.932 *** | 0.970 * | 1.012 | 1.020 | 1.032 |
| Consumption | 1.009 | 0.982 | 1.007 | 1.017 | 1.020 |
| Industrial production | 0.991 | 0.999 | 0.999 | 1.013 | 1.024 |
| Capacity Utilization | 0.960 ** | 0.970 * | 0.974 | 0.988 | 1.031 |
| Unemployment rate | 0.990 | 0.992 | 0.995 | 1.020 | 1.017 |
| Housing starts | 1.005 | 0.996 | 1.028 | 1.052 | 1.037 |
| PPI for finished goods | 0.992 * | 0.991 ** | 0.992 ** | 1.000 | 1.001 |
| PCE price index | 0.984 ** | 0.997 | 0.997 | 0.999 | 1.002 |
| Real average hourly earnings | 0.990 | 1.016 | 1.015 | 1.021 | 1.029 |
| S&P stock price index, industrials | 0.982 * | 0.988 | 1.000 | 1.017 | 1.009 |
| 10-year Treasury bond yield | 0.987 | 1.005 | 1.003 | 0.997 | 1.001 |
| Effective exchange rate | 0.996 | 1.003 | 0.993 * | 1.004 | 0.999 |
| Payroll employment | 0.965 * | 0.964 * | 0.991 | 1.036 | 1.047 |
| BVAR-stationary CSV | | | | | |
| GDP | 1.000 | 0.982 * | 1.010 | 1.018 | 1.017 |
| CPI | 1.001 | 0.998 | 0.998 | 1.003 | 1.001 |
| Fed funds rate | 0.928 *** | 0.963 * | 1.016 | 1.026 | 1.038 |
| Consumption | 1.013 | 0.982 | 1.011 | 1.013 | 1.019 |
| Industrial production | 0.995 | 0.997 | 1.000 | 1.007 | 1.025 |
| Capacity Utilization | 0.963 ** | 0.969 * | 0.974 | 0.992 | 1.037 |
| Unemployment rate | 0.994 | 0.994 | 1.000 | 1.013 | 1.027 |
| Housing starts | 1.008 | 0.997 | 1.030 | 1.047 | 1.033 |
| PPI for finished goods | 0.991 * | 0.991 ** | 0.995 * | 1.002 | 1.000 |
| PCE price index | 0.980 ** | 0.998 | 0.997 | 1.002 | 0.999 |
| Real average hourly earnings | 0.988 | 1.015 | 1.015 | 1.024 | 1.024 |
| S&P stock price index, industrials | 0.981 * | 0.988 | 1.002 | 1.016 | 1.010 |
| 10-year Treasury bond yield | 0.989 | 1.002 | 1.006 | 1.000 | 1.000 |
| Effective exchange rate | 0.998 | 1.003 | 0.995 | 1.002 | 0.996 |
| Payroll employment | 0.981 | 0.972 | 1.002 | 1.041 | 1.057 |

Note: See the notes to Table 4.

Table 9. Average log predictive scores, 15-variable BVARs, 1985:Q1-2008:Q4
(*avg. score for benchmark BVAR, differences in scores for BVAR-CSV*)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|------------------------------------|-----------|-----------|-----------|-----------|-----------|
| BVAR | | | | | |
| All variables | -30.861 | -32.504 | -33.810 | -34.563 | -34.912 |
| GDP | -2.251 | -2.361 | -2.415 | -2.419 | -2.425 |
| CPI | -2.186 | -2.309 | -2.335 | -2.349 | -2.390 |
| Fed funds rate | -0.985 | -1.096 | -1.142 | -1.173 | -1.176 |
| Consumption | -2.119 | -2.229 | -2.265 | -2.267 | -2.262 |
| Industrial production | -2.725 | -2.910 | -2.992 | -3.004 | -2.997 |
| Capacity Utilization | -1.438 | -1.932 | -2.402 | -2.734 | -2.812 |
| Unemployment rate | 0.272 | 0.081 | -0.057 | -0.068 | -0.065 |
| Housing starts | -4.938 | -5.299 | -5.653 | -5.867 | -5.948 |
| PPI for finished goods | -2.971 | -3.092 | -3.093 | -3.122 | -3.170 |
| PCE price index | -1.823 | -1.916 | -1.953 | -1.968 | -1.994 |
| Real average hourly earnings | -1.978 | -2.018 | -2.060 | -2.060 | -2.045 |
| S&P stock price index, industrials | -4.764 | -4.821 | -4.831 | -4.780 | -4.756 |
| 10-year Treasury bond yield | -0.626 | -0.658 | -0.645 | -0.615 | -0.610 |
| Effective exchange rate | -4.089 | -4.074 | -4.045 | -3.972 | -3.981 |
| Payroll employment | -1.548 | -1.871 | -2.129 | -2.192 | -2.172 |
| BVAR-CSV | | | | | |
| All variables | 1.402 *** | 0.951 *** | 0.703 * | 0.158 | -0.143 |
| GDP | 0.089 *** | 0.073 *** | 0.044 | 0.049 | 0.025 |
| CPI | 0.009 | -0.131 | -0.116 | -0.045 | -0.208 |
| Fed funds rate | 0.298 *** | 0.264 *** | 0.216 *** | 0.179 *** | 0.165 *** |
| Consumption | 0.094 *** | 0.091 *** | 0.026 | -0.011 | 0.011 |
| Industrial production | 0.131 *** | 0.083 ** | 0.017 | 0.008 | 0.016 |
| Capacity Utilization | 0.232 *** | 0.100 | -0.032 | -0.203 | -0.295 |
| Unemployment rate | 0.106 *** | 0.094 ** | 0.033 | -0.021 | 0.036 |
| Housing starts | 0.147 *** | 0.037 | -0.120 | -0.257 | -0.187 |
| PPI for finished goods | 0.116 | -0.030 | -0.024 | 0.040 | -0.065 |
| PCE price index | 0.061 | -0.076 | -0.048 | 0.018 | -0.103 |
| Real average hourly earnings | 0.062 | -0.001 | 0.014 | 0.009 | -0.032 |
| S&P stock price index, industrials | 0.072 | 0.022 | -0.007 | -0.035 | -0.064 |
| 10-year Treasury bond yield | 0.031 | 0.011 | 0.026 * | 0.019 | 0.002 |
| Effective exchange rate | 0.071 *** | 0.044 ** | 0.055 ** | 0.032 * | 0.012 |
| Payroll employment | 0.165 *** | 0.130 *** | 0.008 | -0.107 | -0.063 |
| BVAR-stationary CSV | | | | | |
| All variables | 1.457 *** | 0.973 *** | 0.731 *** | -0.108 | -1.093 |
| GDP | 0.081 *** | 0.062 *** | 0.017 | -0.042 | -0.145 |
| CPI | 0.049 | -0.102 | 0.014 | 0.066 | 0.004 |
| Fed funds rate | 0.287 *** | 0.245 *** | 0.167 *** | 0.058 | -0.051 |
| Consumption | 0.084 *** | 0.083 *** | 0.010 | -0.064 | -0.124 |
| Industrial production | 0.127 *** | 0.085 ** | 0.015 | -0.034 | -0.113 |
| Capacity Utilization | 0.230 *** | 0.116 ** | -0.005 | -0.131 | -0.179 |
| Unemployment rate | 0.098 *** | 0.093 *** | 0.029 | -0.044 | -0.090 |
| Housing starts | 0.145 *** | 0.052 | -0.081 | -0.126 | -0.063 |
| PPI for finished goods | 0.160 | 0.024 | 0.055 | 0.120 | 0.059 |
| PCE price index | 0.076 | -0.052 | 0.041 | 0.068 | 0.000 |
| Real average hourly earnings | 0.063 | 0.003 | 0.017 | -0.025 | -0.122 |
| S&P stock price index, industrials | 0.082 | 0.061 * | 0.012 | -0.005 | -0.087 |
| 10-year Treasury bond yield | 0.025 | -0.006 | -0.015 | -0.096 | -0.204 |
| Effective exchange rate | 0.070 ** | 0.055 ** | 0.064 ** | 0.010 | -0.059 |
| Payroll employment | 0.153 *** | 0.124 *** | 0.016 | -0.085 | -0.111 |

Note: See the notes to Table 6.

Table 10. Log predictive likelihoods for UK models, 1985:Q1-2011:Q2

| model | log PL |
|---|-----------|
| BVAR(4), 8 variables | -1508.500 |
| BVAR(4), 8 variables, common stochastic volatility | -1458.368 |
| BVAR(4), 8 variables, stationary common stochastic volatility | -1459.564 |

Notes: The table reports log predictive likelihoods, formed as the sum of 1-step ahead likelihoods, over the period 1985:Q1 through 2011:Q2.

**Table 11. Forecast RMSEs, 8-variable BVARs,
UK data, 1985:Q1-2011:Q2**
(RMSEs for BVAR benchmark, RMSE ratios in all others)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|----------|----------|-----------|-----------|
| AR | | | | | |
| GDP growth | 1.026 | 1.027 | 1.055 | 1.032 | 1.016 |
| Consumption | 1.060 | 1.036 | 1.063 | 1.010 | 1.023 |
| Investment | 1.077 | 1.015 | 1.023 | 1.025 | 1.023 |
| Employment | 1.095 | 1.068 | 1.061 | 1.048 | 1.042 |
| Unemployment | 1.062 | 1.071 | 1.040 | 1.016 | 0.980 |
| GDP inflation | 0.922 * | 0.932 | 0.820 ** | 0.757 *** | 0.756 ** |
| Bond yield | 0.956 | 0.973 | 0.954 | 0.878 | 0.796 *** |
| Bank rate | 1.019 | 1.039 | 1.047 | 1.014 | 0.958 |
| BVAR | | | | | |
| GDP growth | 2.470 | 2.574 | 2.783 | 2.807 | 2.786 |
| Consumption | 3.167 | 3.142 | 3.471 | 3.419 | 3.423 |
| Investment | 10.406 | 10.597 | 10.153 | 10.204 | 9.304 |
| Employment | 1.085 | 1.167 | 1.474 | 1.681 | 1.686 |
| Unemployment | 0.182 | 0.323 | 0.684 | 1.380 | 1.897 |
| GDP inflation | 2.886 | 2.874 | 3.252 | 3.964 | 4.218 |
| Bond yield | 0.453 | 0.704 | 0.973 | 1.505 | 2.067 |
| Bank rate | 0.762 | 1.200 | 1.844 | 3.054 | 3.949 |
| BVAR-SV | | | | | |
| GDP growth | 0.993 | 1.030 | 1.012 | 1.026 | 1.040 |
| Consumption | 0.982 | 1.015 | 1.002 | 1.028 | 1.072 |
| Investment | 0.990 | 0.989 | 0.993 | 1.001 | 1.011 |
| Employment | 1.024 | 1.027 | 1.014 | 0.992 | 1.014 |
| Unemployment | 1.084 | 1.089 | 1.072 | 1.038 | 1.037 |
| GDP inflation | 0.997 | 0.996 | 0.945 ** | 0.893 ** | 0.881 |
| Bond yield | 0.993 | 0.991 | 1.003 | 1.021 | 1.050 |
| Bank rate | 0.982 | 0.981 | 0.990 | 1.007 | 1.024 |
| BVAR-CSV | | | | | |
| GDP growth | 0.955 *** | 0.995 | 1.026 | 1.030 | 1.032 |
| Consumption | 0.984 | 1.009 | 1.025 | 1.038 | 1.058 |
| Investment | 0.996 | 1.010 | 1.021 | 1.017 | 1.019 |
| Employment | 0.976 ** | 0.991 | 1.013 | 1.049 | 1.056 |
| Unemployment | 0.995 | 0.996 | 0.999 | 1.015 | 1.038 |
| GDP inflation | 0.997 | 1.013 | 0.972 | 0.933 * | 0.954 |
| Bond yield | 0.984 | 0.974 * | 0.937 ** | 0.889 *** | 0.884 ** |
| Bank rate | 0.992 | 0.981 | 0.963 | 0.941 | 0.940 |
| BVAR-stationary CSV | | | | | |
| GDP growth | 0.959 ** | 0.994 | 1.026 | 1.033 | 1.035 |
| Consumption | 0.957 ** | 0.998 | 1.023 | 1.042 | 1.064 |
| Investment | 1.005 | 1.007 | 1.017 | 1.019 | 1.007 |
| Employment | 0.988 | 1.006 | 1.022 | 1.068 | 1.076 |
| Unemployment | 1.000 | 0.986 | 0.989 | 1.010 | 1.038 |
| GDP inflation | 0.986 | 0.987 | 0.976 | 0.940 | 0.936 |
| Bond yield | 0.991 | 0.990 | 0.952 * | 0.923 * | 0.933 |
| Bank rate | 1.002 | 0.992 | 0.964 | 0.951 | 0.965 |

Note: See the notes to Table 4.

**Table 12. Average log predictive scores, 8-variable BVARs,
UK data, 1985:Q1-2011:Q2**

(avg. score for benchmark BVAR, differences in scores in all others)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| All variables | -1.082 | -1.024 | -1.204 | -0.912 | -0.625 |
| GDP growth | -0.018 | -0.008 | -0.028 | -0.018 | -0.020 |
| Consumption | -0.022 | -0.003 | -0.026 | -0.016 | -0.029 |
| Investment | -0.081 | -0.021 | -0.029 | -0.030 | -0.036 |
| Employment | -0.102 | -0.065 | -0.050 | -0.019 | -0.008 |
| Unemployment | -0.064 | -0.070 | -0.041 | -0.015 | 0.013 |
| GDP inflation | 0.052 *** | 0.115 *** | 0.197 *** | 0.277 *** | 0.309 *** |
| Bond yield | 0.001 | -0.016 | -0.003 | 0.044 | 0.092 *** |
| Bank rate | -0.044 | -0.063 | -0.070 | -0.002 | 0.048 |
| BVAR | | | | | |
| All variables | -14.268 | -15.855 | -17.764 | -19.924 | -21.097 |
| GDP growth | -2.531 | -2.574 | -2.631 | -2.640 | -2.638 |
| Consumption | -2.725 | -2.737 | -2.805 | -2.803 | -2.807 |
| Investment | -3.755 | -3.785 | -3.750 | -3.762 | -3.700 |
| Employment | -1.545 | -1.584 | -1.840 | -1.990 | -1.979 |
| Unemployment | 0.246 | -0.306 | -1.049 | -1.754 | -2.064 |
| GDP inflation | -2.705 | -2.828 | -3.096 | -3.441 | -3.669 |
| Bond yield | -0.770 | -1.200 | -1.576 | -2.009 | -2.296 |
| Bank rate | -1.283 | -1.740 | -2.172 | -2.657 | -2.929 |
| BVAR-SV | | | | | |
| All variables | 0.787 *** | 0.512 ** | 0.221 | -0.081 | -0.190 |
| GDP growth | 0.273 *** | 0.126 | -0.116 | -0.211 | -0.184 |
| Consumption | 0.139 *** | 0.101 * | 0.033 | 0.013 | -0.150 |
| Investment | 0.012 | 0.021 | 0.015 | -0.021 | -0.142 |
| Employment | 0.029 | -0.062 | -0.078 | -0.035 | -0.004 |
| Unemployment | -0.057 | -0.110 | -0.206 | -0.126 | -0.120 |
| GDP inflation | 0.144 *** | 0.195 *** | 0.242 *** | 0.220 *** | 0.095 |
| Bond yield | 0.168 *** | 0.132 *** | 0.132 ** | 0.118 | 0.052 |
| Bank rate | 0.227 *** | 0.171 *** | 0.081 | 0.010 | -0.080 |
| BVAR-CSV | | | | | |
| All variables | 0.397 *** | 0.408 *** | 0.321 * | 0.150 | 0.096 |
| GDP growth | 0.136 *** | 0.099 * | 0.029 | -0.026 | 0.040 |
| Consumption | 0.122 *** | 0.091 ** | 0.042 | 0.040 | 0.024 |
| Investment | -0.007 | -0.027 | -0.041 | -0.022 | 0.002 |
| Employment | -0.052 | -0.032 | -0.131 | -0.277 | -0.298 |
| Unemployment | -0.010 | -0.031 | -0.115 | -0.208 | -0.232 |
| GDP inflation | 0.150 *** | 0.171 *** | 0.202 *** | 0.228 *** | 0.219 *** |
| Bond yield | 0.128 *** | 0.110 *** | 0.120 *** | 0.136 *** | 0.152 ** |
| Bank rate | 0.131 *** | 0.124 *** | 0.102 *** | 0.076 ** | 0.088 ** |
| BVAR-stationary CSV | | | | | |
| All variables | 0.473 *** | 0.358 *** | 0.181 | -0.198 | -0.598 |
| GDP growth | 0.174 *** | 0.098 | -0.042 | -0.084 | -0.120 |
| Consumption | 0.130 *** | 0.100 ** | 0.020 | -0.048 | -0.126 |
| Investment | -0.014 | -0.029 | -0.053 | -0.066 | -0.167 |
| Employment | -0.044 | -0.047 | -0.113 | -0.150 | -0.124 |
| Unemployment | 0.024 | 0.010 | -0.077 | -0.175 | -0.183 |
| GDP inflation | 0.150 *** | 0.161 *** | 0.177 *** | 0.142 *** | 0.082 ** |
| Bond yield | 0.090 *** | 0.061 ** | 0.065 ** | 0.044 | 0.015 |
| Bank rate | 0.079 *** | 0.059 ** | 0.043 | 0.013 | -0.016 |

Note: See the notes to Table 6.

Appendix Table A4. Real-Time Forecast RMSEs, 4-variable BVARs, 1985:Q1-2010:Q4

(RMSEs for benchmark AR, RMSE ratios in all others)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| GDP growth | 2.067 | 2.202 | 2.310 | 2.360 | 2.378 |
| Unemployment | 0.192 | 0.375 | 0.764 | 1.369 | 1.605 |
| GDP inflation | 1.140 | 1.250 | 1.427 | 1.888 | 2.188 |
| Fed funds rate | 0.493 | 0.967 | 1.780 | 2.837 | 3.357 |
| BVAR | | | | | |
| GDP growth | 1.226 | 1.245 | 1.148 | 0.967 ** | 0.975 |
| Unemployment | 1.284 | 1.332 | 1.323 | 1.154 | 0.938 |
| GDP inflation | 0.977 | 0.976 | 1.037 | 1.116 | 1.169 |
| Fed funds rate | 1.027 | 0.897 | 0.813 *** | 0.848 *** | 0.875 *** |
| BVAR-SV | | | | | |
| GDP growth | 1.168 | 1.164 | 1.088 | 0.969 | 0.986 |
| Unemployment | 1.273 | 1.293 | 1.264 | 1.132 | 0.974 |
| GDP inflation | 0.923 *** | 0.904 *** | 0.894 *** | 0.921 *** | 0.929 *** |
| Fed funds rate | 0.909 | 0.836 ** | 0.791 *** | 0.822 *** | 0.822 *** |
| BVAR-CSV | | | | | |
| GDP growth | 1.124 | 1.120 | 1.075 | 0.986 | 0.993 |
| Unemployment | 1.228 | 1.243 | 1.220 | 1.123 | 0.988 |
| GDP inflation | 0.914 *** | 0.880 *** | 0.865 *** | 0.876 *** | 0.867 *** |
| Fed funds rate | 0.973 | 0.871 | 0.800 ** | 0.823 *** | 0.817 *** |
| BVAR-stationary CSV | | | | | |
| GDP growth | 1.127 | 1.121 | 1.083 | 0.989 | 0.995 |
| Unemployment | 1.249 | 1.263 | 1.237 | 1.136 | 1.000 |
| GDP inflation | 0.906 *** | 0.872 *** | 0.851 *** | 0.867 *** | 0.857 *** |
| Fed funds rate | 0.972 | 0.874 | 0.812 ** | 0.847 ** | 0.848 ** |

Notes: For the forecasts from BVAR models, entries less than 1 indicate the model has a lower RMSE than the benchmark. To provide a rough gauge of whether the RMSE ratios are significantly different from 1, we use the Diebold-Mariano t -statistic for equal MSE, applied to the forecast of each model relative to the benchmark. Differences in accuracy that are statistically different from zero are denoted by one, two, or three asterisks, corresponding to significance levels of 10%, 5%, and 1%, respectively. The underlying p -values are based on t -statistics computed with a serial correlation-robust variance, using a rectangular kernel, $h - 1$ lags, and the small-sample adjustment of Harvey, Leybourne, and Newbold (1997).

Appendix Table A5. Real-Time Forecast RMSEs, 8-variable BVARs, 1985:Q1-2010:Q4

(RMSEs for benchmark AR, RMSE ratios in all others)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| GDP growth | 2.067 | 2.202 | 2.310 | 2.360 | 2.378 |
| Consumption | 2.174 | 2.200 | 2.272 | 2.320 | 2.277 |
| BFI | 9.293 | 9.936 | 10.619 | 10.616 | 10.369 |
| Employment | 0.955 | 1.372 | 1.787 | 2.015 | 2.085 |
| Unemployment | 0.192 | 0.375 | 0.764 | 1.369 | 1.605 |
| GDP inflation | 1.140 | 1.250 | 1.427 | 1.888 | 2.188 |
| Treasury yield | 0.424 | 0.709 | 1.123 | 1.518 | 1.826 |
| Fed funds rate | 0.493 | 0.967 | 1.780 | 2.837 | 3.357 |
| BVAR | | | | | |
| GDP growth | 1.096 | 1.146 | 1.085 | 0.986 | 0.988 |
| Consumption | 1.141 | 1.112 | 1.108 | 1.010 | 0.998 |
| BFI | 0.922 *** | 0.949 * | 0.938 * | 0.908 *** | 0.985 *** |
| Employment | 1.125 | 1.208 | 1.216 | 1.080 | 1.007 |
| Unemployment | 1.174 | 1.209 | 1.199 | 1.122 | 0.960 |
| GDP inflation | 1.009 | 1.007 | 1.016 | 1.059 | 1.115 |
| Treasury yield | 1.075 | 0.993 | 0.826 ** | 0.715 *** | 0.709 *** |
| Fed funds rate | 1.237 | 1.007 | 0.785 ** | 0.758 ** | 0.794 ** |
| BVAR-SV | | | | | |
| GDP growth | 1.104 | 1.143 | 1.082 | 0.986 | 1.002 |
| Consumption | 1.105 | 1.092 | 1.122 | 1.033 | 1.023 |
| BFI | 0.921 *** | 0.946 * | 0.935 * | 0.909 ** | 0.993 |
| Employment | 1.108 | 1.193 | 1.183 | 1.030 | 0.980 |
| Unemployment | 1.234 | 1.239 | 1.206 | 1.127 | 0.987 |
| GDP inflation | 0.965 | 0.945 * | 0.913 * | 0.926 | 0.946 |
| Treasury yield | 1.038 | 0.956 | 0.815 ** | 0.662 *** | 0.616 *** |
| Fed funds rate | 1.182 | 1.015 | 0.788 * | 0.718 ** | 0.711 ** |
| BVAR-CSV | | | | | |
| GDP growth | 1.081 | 1.098 | 1.064 | 0.988 | 0.990 |
| Consumption | 1.095 | 1.068 | 1.093 | 1.031 | 1.017 |
| BFI | 0.925 ** | 0.955 * | 0.954 | 0.915 ** | 0.982 *** |
| Employment | 1.043 | 1.109 | 1.120 | 1.012 | 0.963 |
| Unemployment | 1.235 | 1.231 | 1.182 | 1.100 | 0.976 |
| GDP inflation | 0.958 | 0.919 ** | 0.880 *** | 0.869 *** | 0.880 *** |
| Treasury yield | 1.045 | 0.994 | 0.846 * | 0.719 *** | 0.678 *** |
| Fed funds rate | 1.156 | 0.953 | 0.762 ** | 0.749 ** | 0.757 *** |
| BVAR-stationary CSV | | | | | |
| GDP growth | 1.073 | 1.087 | 1.064 | 0.985 | 0.985 |
| Consumption | 1.096 | 1.069 | 1.092 | 1.035 | 1.018 |
| BFI | 0.927 ** | 0.952 * | 0.955 | 0.916 ** | 0.977 *** |
| Employment | 1.053 | 1.112 | 1.121 | 1.011 | 0.959 |
| Unemployment | 1.260 | 1.248 | 1.194 | 1.102 | 0.980 |
| GDP inflation | 0.950 * | 0.903 *** | 0.862 *** | 0.847 *** | 0.856 *** |
| Treasury yield | 1.047 | 0.992 | 0.846 * | 0.727 *** | 0.686 *** |
| Fed funds rate | 1.142 | 0.942 | 0.759 ** | 0.751 ** | 0.763 *** |

Note: See the notes to Appendix Table A4.

Appendix Table A6. Average log predictive scores, 4-variable BVARs, 1985:Q1-2010:Q4

(avg. score for benchmark AR, differences in scores in all others)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| GDP growth | -2.358 | -2.406 | -2.446 | -2.459 | -2.466 |
| Unemployment | 0.122 | -0.539 | -1.231 | -1.799 | -1.953 |
| GDP inflation | -1.568 | -1.678 | -1.853 | -2.094 | -2.220 |
| Fed funds rate | -1.136 | -1.631 | -2.097 | -2.501 | -2.669 |
| BVAR | | | | | |
| GDP growth | -0.033 | -0.055 | -0.014 | 0.037 *** | 0.028 *** |
| Unemployment | -0.178 | -0.230 | -0.530 | -0.888 | -0.252 |
| GDP inflation | 0.019 | 0.016 | -0.002 | -0.077 | -0.146 |
| Fed funds rate | -0.026 | 0.074 ** | 0.137 *** | 0.139 *** | 0.127 *** |
| BVAR-SV | | | | | |
| GDP growth | 0.065 | -0.029 | 0.022 | 0.036 | 0.004 |
| Unemployment | -0.018 | -0.175 | -0.885 | -1.794 | -1.385 |
| GDP inflation | 0.109 *** | 0.122 *** | 0.169 *** | 0.079 ** | 0.048 |
| Fed funds rate | 0.511 *** | 0.372 *** | 0.162 | 0.020 | -0.019 |
| BVAR-CSV | | | | | |
| GDP growth | 0.143 *** | 0.078 | 0.043 | 0.040 | 0.018 |
| Unemployment | 0.020 | -0.147 | -0.757 | -1.701 | -1.721 |
| GDP inflation | 0.089 ** | 0.113 ** | 0.179 *** | 0.088 ** | 0.064 |
| Fed funds rate | 0.407 *** | 0.375 *** | 0.219 * | -0.014 | -0.177 |
| BVAR-stationary CSV | | | | | |
| GDP growth | 0.159 *** | 0.115 ** | 0.079 | 0.049 * | -0.071 |
| Unemployment | 0.069 | -0.033 | -0.481 | -0.999 | -0.839 |
| GDP inflation | 0.091 ** | 0.115 ** | 0.173 *** | 0.117 *** | 0.104 *** |
| Fed funds rate | 0.424 *** | 0.388 *** | 0.238 ** | 0.102 | 0.029 |

Notes: For the forecasts from BVAR models, entries greater than 0 indicate the model has a better average log score (better density forecast) than the benchmark model. To provide a rough gauge of the statistical significance of differences in average log scores, we use the Amisano-Giacomini t -test of equal means, applied to the log score for each model relative to the benchmark. Differences in average scores that are statistically different from zero are denoted by one, two, or three asterisks, corresponding to significance levels of 10%, 5%, and 1%, respectively. The underlying p -values are based on t -statistics computed with a serial correlation-robust variance, using a rectangular kernel, $h - 1$ lags, and the small-sample adjustment of Harvey, Leybourne, and Newbold (1997).

Appendix Table A7. Average log predictive scores, 8-variable BVARs, 1985:Q1-2010:Q4

(avg. score for benchmark AR, differences in scores in all others)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| All variables | -12.968 | -15.223 | -17.364 | -19.027 | -19.627 |
| GDP growth | -2.358 | -2.406 | -2.446 | -2.459 | -2.466 |
| Consumption | -2.243 | -2.264 | -2.299 | -2.310 | -2.300 |
| BFI | -3.671 | -3.757 | -3.815 | -3.812 | -3.784 |
| Employment | -1.502 | -1.850 | -2.087 | -2.187 | -2.203 |
| Unemployment | 0.122 | -0.539 | -1.231 | -1.799 | -1.953 |
| GDP inflation | -1.568 | -1.678 | -1.853 | -2.094 | -2.220 |
| Treasury yield | -0.610 | -1.108 | -1.550 | -1.863 | -2.048 |
| Fed funds rate | -1.136 | -1.631 | -2.097 | -2.501 | -2.669 |
| BVAR | | | | | |
| All variables | 0.633 *** | 1.006 ** | 0.859 | 0.681 | 1.065 |
| GDP growth | 0.046 ** | 0.006 | 0.016 | 0.020 *** | 0.015 *** |
| Consumption | -0.090 | -0.064 | -0.056 | -0.000 | 0.007 * |
| BFI | 0.092 *** | 0.033 | 0.026 | 0.080 * | 0.001 |
| Employment | -0.018 | -0.105 | -0.216 | -0.083 | -0.024 |
| Unemployment | -0.101 | -0.148 | -0.360 | -0.645 | -0.236 |
| GDP inflation | -0.001 | -0.000 | 0.014 | -0.031 | -0.096 |
| Treasury yield | -0.054 | 0.037 | 0.183 *** | 0.237 *** | 0.221 *** |
| Fed funds rate | -0.028 | 0.055 | 0.163 *** | 0.209 *** | 0.198 ** |
| BVAR-SV | | | | | |
| All variables | 1.320 *** | 1.374 *** | 0.993 | 0.176 | 0.435 |
| GDP growth | 0.095 * | 0.017 | 0.027 | 0.024 | -0.005 |
| Consumption | -0.057 | -0.077 | -0.159 | -0.103 | -0.087 |
| BFI | 0.036 | -0.116 | -0.141 | -0.091 | -0.242 |
| Employment | 0.031 | -0.181 | -0.415 | -0.310 | -0.161 |
| Unemployment | -0.035 | -0.185 | -0.750 | -1.493 | -1.013 |
| GDP inflation | 0.056 * | 0.079 *** | 0.130 *** | 0.094 * | 0.058 |
| Treasury yield | 0.003 | 0.078 | 0.227 ** | 0.322 *** | 0.311 ** |
| Fed funds rate | 0.394 *** | 0.322 *** | 0.297 *** | 0.272 *** | 0.252 ** |
| BVAR-CSV | | | | | |
| All variables | 1.236 *** | 1.483 *** | 0.580 | -0.324 | -0.290 |
| GDP growth | 0.101 *** | 0.052 | -0.000 | 0.009 | -0.038 |
| Consumption | -0.082 | -0.080 | -0.129 | -0.075 | -0.068 |
| BFI | 0.124 * | 0.038 | -0.195 | -0.114 | -0.317 |
| Employment | 0.094 ** | -0.045 | -0.306 | -0.253 | -0.276 |
| Unemployment | 0.003 | -0.161 | -0.696 | -1.352 | -1.109 |
| GDP inflation | 0.040 | 0.072 * | 0.115 *** | 0.110 *** | 0.096 * |
| Treasury yield | 0.024 | 0.070 | 0.197 ** | 0.244 * | 0.203 |
| Fed funds rate | 0.246 *** | 0.257 *** | 0.246 *** | 0.199 *** | 0.132 |
| BVAR-stationary CSV | | | | | |
| All variables | 1.301 *** | 1.617 *** | 1.016 | 0.547 | 0.810 |
| GDP growth | 0.097 *** | 0.055 | -0.005 | -0.039 | -0.145 |
| Consumption | -0.072 | -0.066 | -0.090 | -0.081 | -0.143 |
| BFI | 0.159 ** | 0.091 ** | -0.072 | 0.016 | -0.137 |
| Employment | 0.084 * | -0.012 | -0.210 | -0.104 | -0.125 |
| Unemployment | 0.018 | -0.116 | -0.550 | -0.923 | -0.625 |
| GDP inflation | 0.050 * | 0.073 ** | 0.106 *** | 0.086 ** | 0.048 |
| Treasury yield | 0.002 | 0.056 | 0.185 ** | 0.215 ** | 0.154 |
| Fed funds rate | 0.238 *** | 0.249 *** | 0.233 *** | 0.194 *** | 0.131 |

Note: See the notes to Appendix Table A6.

**Appendix Table A11. Forecast RMSEs, 8-variable BVARs,
UK data, 1985:Q1-2011:Q2**

(RMSEs for benchmark AR, RMSE ratios in all others)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|----------|----------|----------|-----------|
| AR | | | | | |
| GDP growth | 2.535 | 2.644 | 2.937 | 2.897 | 2.829 |
| Consumption | 3.359 | 3.256 | 3.692 | 3.452 | 3.503 |
| Investment | 11.207 | 10.755 | 10.390 | 10.464 | 9.517 |
| Employment | 1.188 | 1.246 | 1.563 | 1.763 | 1.757 |
| Unemployment | 0.193 | 0.346 | 0.711 | 1.402 | 1.860 |
| GDP inflation | 2.660 | 2.680 | 2.666 | 3.001 | 3.190 |
| Bond yield | 0.433 | 0.685 | 0.928 | 1.322 | 1.646 |
| Bank rate | 0.776 | 1.247 | 1.931 | 3.096 | 3.782 |
| BVAR | | | | | |
| GDP growth | 0.975 | 0.973 | 0.948 | 0.969 | 0.985 |
| Consumption | 0.943 * | 0.965 | 0.940 | 0.990 | 0.977 |
| Investment | 0.929 * | 0.985 | 0.977 | 0.975 | 0.978 |
| Employment | 0.913 *** | 0.937 ** | 0.943 * | 0.954 | 0.960 |
| Unemployment | 0.942 * | 0.933 | 0.962 | 0.985 | 1.020 |
| GDP inflation | 1.085 | 1.073 | 1.220 | 1.321 | 1.322 |
| Bond yield | 1.046 | 1.027 | 1.048 | 1.139 | 1.256 |
| Bank rate | 0.981 | 0.962 | 0.955 | 0.987 | 1.044 |
| BVAR-SV | | | | | |
| GDP growth | 0.968 * | 1.003 | 0.959 | 0.994 | 1.024 |
| Consumption | 0.926 ** | 0.980 | 0.943 | 1.019 | 1.047 |
| Investment | 0.919 ** | 0.975 | 0.970 | 0.976 | 0.988 |
| Employment | 0.935 ** | 0.962 | 0.956 * | 0.947 | 0.973 |
| Unemployment | 1.021 | 1.017 | 1.030 | 1.022 | 1.058 |
| GDP inflation | 1.082 | 1.068 | 1.153 | 1.180 | 1.164 |
| Bond yield | 1.038 | 1.018 | 1.051 | 1.163 | 1.318 |
| Bank rate | 0.964 | 0.944 | 0.945 | 0.993 | 1.068 |
| BVAR-CSV | | | | | |
| GDP growth | 0.931 ** | 0.969 | 0.973 | 0.998 | 1.016 |
| Consumption | 0.927 * | 0.973 | 0.963 | 1.028 | 1.034 |
| Investment | 0.925 * | 0.995 | 0.998 | 0.992 | 0.996 |
| Employment | 0.891 *** | 0.928 ** | 0.955 | 1.001 | 1.014 |
| Unemployment | 0.938 * | 0.930 | 0.960 | 0.999 | 1.058 |
| GDP inflation | 1.082 | 1.087 | 1.186 | 1.232 | 1.261 |
| Bond yield | 1.029 | 1.000 | 0.982 | 1.013 | 1.109 |
| Bank rate | 0.974 | 0.944 | 0.919 | 0.928 | 0.981 |
| BVAR-stationary CSV | | | | | |
| GDP growth | 0.935 * | 0.968 | 0.973 | 1.001 | 1.019 |
| Consumption | 0.903 ** | 0.963 | 0.962 | 1.032 | 1.040 |
| Investment | 0.933 * | 0.992 | 0.994 | 0.994 | 0.984 |
| Employment | 0.902 *** | 0.942 ** | 0.963 | 1.019 | 1.033 |
| Unemployment | 0.942 * | 0.920 | 0.951 | 0.995 | 1.059 |
| GDP inflation | 1.070 | 1.058 | 1.191 | 1.242 | 1.237 |
| Bond yield | 1.036 | 1.017 | 0.998 | 1.052 | 1.172 |
| Bank rate | 0.983 | 0.954 | 0.921 | 0.939 | 1.008 |

Note: See the notes to Appendix Table A4.

**Appendix Table A12. Average log predictive scores, 8-variable BVARs,
UK data, 1985:Q1-2011:Q2**

(avg. score for benchmark AR, differences in scores in all others)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| All variables | -15.350 | -16.879 | -18.969 | -20.835 | -21.722 |
| GDP growth | -2.548 | -2.581 | -2.659 | -2.658 | -2.658 |
| Consumption | -2.747 | -2.740 | -2.831 | -2.819 | -2.836 |
| Investment | -3.835 | -3.805 | -3.779 | -3.792 | -3.736 |
| Employment | -1.647 | -1.649 | -1.891 | -2.009 | -1.987 |
| Unemployment | 0.182 | -0.377 | -1.090 | -1.769 | -2.051 |
| GDP inflation | -2.653 | -2.713 | -2.900 | -3.164 | -3.360 |
| Bond yield | -0.768 | -1.216 | -1.579 | -1.966 | -2.204 |
| Bank rate | -1.327 | -1.802 | -2.242 | -2.659 | -2.881 |
| BVAR | | | | | |
| All variables | 1.082 *** | 1.024 *** | 1.204 *** | 0.912 *** | 0.625 *** |
| GDP growth | 0.018 | 0.008 | 0.028 * | 0.018 | 0.020 |
| Consumption | 0.022 | 0.003 | 0.026 | 0.016 | 0.029 * |
| Investment | 0.081 * | 0.021 | 0.029 ** | 0.030 | 0.036 ** |
| Employment | 0.102 ** | 0.065 * | 0.050 | 0.019 | 0.008 |
| Unemployment | 0.064 | 0.070 | 0.041 | 0.015 | -0.013 |
| GDP inflation | -0.052 | -0.115 | -0.197 | -0.277 | -0.309 |
| Bond yield | -0.001 | 0.016 | 0.003 | -0.044 | -0.092 |
| Bank rate | 0.044 *** | 0.063 ** | 0.070 * | 0.002 | -0.048 |
| BVAR-SV | | | | | |
| All variables | 1.869 *** | 1.536 *** | 1.426 *** | 0.830 ** | 0.435 |
| GDP growth | 0.290 *** | 0.133 | -0.088 | -0.193 | -0.164 |
| Consumption | 0.161 *** | 0.104 * | 0.059 | 0.029 | -0.121 |
| Investment | 0.093 ** | 0.042 | 0.044 * | 0.009 | -0.105 |
| Employment | 0.132 ** | 0.002 | -0.028 | -0.016 | 0.004 |
| Unemployment | 0.008 | -0.039 | -0.165 | -0.111 | -0.133 |
| GDP inflation | 0.092 *** | 0.079 *** | 0.045 | -0.056 | -0.214 |
| Bond yield | 0.166 *** | 0.148 *** | 0.135 ** | 0.074 | -0.040 |
| Bank rate | 0.271 *** | 0.234 *** | 0.151 ** | 0.013 | -0.128 |
| BVAR-CSV | | | | | |
| All variables | 1.479 *** | 1.432 *** | 1.525 *** | 1.062 *** | 0.721 |
| GDP growth | 0.153 *** | 0.107 * | 0.057 | -0.008 | 0.060 |
| Consumption | 0.145 *** | 0.094 ** | 0.068 | 0.056 | 0.053 |
| Investment | 0.073 | -0.006 | -0.013 | 0.008 | 0.038 * |
| Employment | 0.050 | 0.033 | -0.080 | -0.258 | -0.290 |
| Unemployment | 0.054 | 0.039 | -0.075 | -0.192 | -0.245 |
| GDP inflation | 0.097 *** | 0.055 ** | 0.005 | -0.049 | -0.090 |
| Bond yield | 0.127 *** | 0.126 *** | 0.123 ** | 0.092 * | 0.060 |
| Bank rate | 0.175 *** | 0.186 *** | 0.171 *** | 0.078 | 0.040 |
| BVAR-stationary CSV | | | | | |
| All variables | 1.555 *** | 1.382 *** | 1.385 *** | 0.714 ** | 0.028 |
| GDP growth | 0.192 *** | 0.105 | -0.014 | -0.066 | -0.101 |
| Consumption | 0.153 *** | 0.103 ** | 0.046 | -0.033 | -0.097 |
| Investment | 0.067 | -0.008 | -0.025 | -0.037 | -0.131 |
| Employment | 0.059 | 0.017 | -0.063 | -0.131 | -0.116 |
| Unemployment | 0.088 * | 0.080 | -0.036 | -0.160 | -0.196 |
| GDP inflation | 0.098 *** | 0.046 * | -0.020 | -0.135 | -0.227 |
| Bond yield | 0.088 *** | 0.077 ** | 0.068 * | 0.001 | -0.076 |
| Bank rate | 0.124 *** | 0.121 *** | 0.113 *** | 0.015 | -0.064 |

Note: See the notes to Appendix Table A6.

Appendix Table A13. Average log predictive scores, 4-variable BVARs, 1985:Q1-2008:Q2

(avg. score for benchmark BVAR, differences in scores in all others)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| GDP growth | -0.010 | -0.023 | -0.049 | -0.042 | -0.024 |
| Unemployment | 0.064 ** | -0.038 | -0.083 | -0.230 | -0.395 |
| GDP inflation | -0.016 | -0.013 | 0.005 | 0.085 *** | 0.143 *** |
| Fed funds rate | -0.000 | -0.100 | -0.136 | -0.121 | -0.120 |
| BVAR | | | | | |
| All variables | -4.536 | -5.547 | -6.667 | -7.769 | -8.194 |
| GDP growth | -2.291 | -2.308 | -2.323 | -2.340 | -2.362 |
| Unemployment | 0.139 | -0.369 | -0.932 | -1.280 | -1.303 |
| GDP inflation | -1.512 | -1.665 | -1.844 | -2.170 | -2.348 |
| Fed funds rate | -1.142 | -1.531 | -1.926 | -2.287 | -2.440 |
| BVAR-SV | | | | | |
| All variables | 0.869 *** | 0.543 *** | 0.434 ** | 0.203 | 0.172 |
| GDP growth | 0.130 *** | 0.081 | 0.138 *** | 0.139 *** | 0.095 *** |
| Unemployment | 0.167 *** | 0.094 | -0.064 | -0.205 | -0.109 |
| GDP inflation | 0.096 *** | 0.123 *** | 0.184 *** | 0.156 *** | 0.193 *** |
| Fed funds rate | 0.591 *** | 0.329 *** | 0.105 | 0.039 | 0.020 |
| BVAR-CSV | | | | | |
| All variables | 0.839 *** | 0.675 *** | 0.553 ** | 0.131 | 0.053 |
| GDP growth | 0.177 *** | 0.144 *** | 0.147 *** | 0.148 *** | 0.164 *** |
| Unemployment | 0.219 *** | 0.111 | -0.087 | -0.324 | -0.219 |
| GDP inflation | 0.094 ** | 0.126 ** | 0.213 *** | 0.181 *** | 0.249 *** |
| Fed funds rate | 0.467 *** | 0.331 *** | 0.146 | -0.051 | -0.132 |
| BVAR-stationary CSV | | | | | |
| All variables | 0.863 *** | 0.745 *** | 0.608 *** | 0.346 * | 0.178 |
| GDP growth | 0.177 *** | 0.153 *** | 0.132 *** | 0.050 *** | -0.054 |
| Unemployment | 0.237 *** | 0.157 * | 0.016 | -0.111 | -0.123 |
| GDP inflation | 0.086 ** | 0.129 *** | 0.202 *** | 0.201 *** | 0.255 *** |
| Fed funds rate | 0.483 *** | 0.342 *** | 0.158 * | 0.027 | 0.002 |

Note: See the notes to Table 6.

Appendix Table A14. Average log predictive scores, 8-variable BVARs, 1985:Q1-2008:Q2

(avg. score for benchmark BVAR, differences in scores in all others)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| All variables | -0.800 | -1.465 | -1.828 | -1.950 | -1.980 |
| GDP growth | -0.050 | -0.047 | -0.051 | -0.027 | -0.017 |
| Consumption | 0.047 | 0.028 | 0.030 | -0.004 | -0.007 |
| BFI | -0.071 | -0.060 | -0.094 | -0.116 | -0.024 |
| Employment | 0.012 | 0.023 | 0.096 | 0.032 | -0.007 |
| Unemployment | -0.028 | -0.119 | -0.190 | -0.231 | -0.323 |
| GDP inflation | -0.003 | -0.006 | -0.010 | 0.036 | 0.097 * |
| Treasury yield | 0.036 | -0.074 | -0.220 | -0.268 | -0.264 |
| Fed funds rate | -0.021 | -0.098 | -0.165 | -0.164 | -0.166 |
| BVAR | | | | | |
| All variables | -11.680 | -13.154 | -14.759 | -16.124 | -16.692 |
| GDP growth | -2.252 | -2.284 | -2.321 | -2.355 | -2.370 |
| Consumption | -2.258 | -2.244 | -2.264 | -2.236 | -2.220 |
| BFI | -3.405 | -3.445 | -3.481 | -3.489 | -3.539 |
| Employment | -1.428 | -1.765 | -2.024 | -2.032 | -2.008 |
| Unemployment | 0.231 | -0.288 | -0.825 | -1.279 | -1.374 |
| GDP inflation | -1.525 | -1.672 | -1.829 | -2.121 | -2.302 |
| Treasury yield | -0.647 | -1.034 | -1.343 | -1.579 | -1.750 |
| Fed funds rate | -1.122 | -1.533 | -1.896 | -2.244 | -2.394 |
| BVAR-SV | | | | | |
| All variables | 0.832 *** | 0.586 *** | 0.570 *** | 0.457 * | 0.156 |
| GDP growth | 0.087 ** | 0.081 * | 0.129 *** | 0.108 *** | 0.067 ** |
| Consumption | 0.049 ** | 0.067 *** | 0.026 | 0.015 | -0.007 |
| BFI | 0.000 | 0.002 | -0.030 | -0.038 | -0.060 |
| Employment | 0.098 ** | 0.037 | 0.057 | 0.030 | -0.006 |
| Unemployment | 0.092 *** | 0.055 | -0.038 | -0.129 | -0.086 |
| GDP inflation | 0.063 *** | 0.088 *** | 0.124 *** | 0.121 ** | 0.154 ** |
| Treasury yield | 0.056 *** | 0.036 | 0.050 | 0.119 *** | 0.112 ** |
| Fed funds rate | 0.488 *** | 0.309 *** | 0.179 *** | 0.105 * | 0.067 |
| BVAR-CSV | | | | | |
| All variables | 0.639 *** | 0.582 *** | 0.469 *** | 0.418 ** | 0.399 ** |
| GDP growth | 0.073 *** | 0.094 *** | 0.096 *** | 0.078 ** | 0.082 ** |
| Consumption | 0.043 * | 0.063 ** | 0.029 | 0.019 | 0.040 * |
| BFI | -0.022 | -0.004 | -0.029 | -0.038 | -0.018 |
| Employment | 0.123 *** | 0.105 ** | 0.084 | 0.034 | 0.051 |
| Unemployment | 0.157 *** | 0.107 *** | 0.048 | -0.010 | -0.049 |
| GDP inflation | 0.061 ** | 0.091 *** | 0.127 *** | 0.154 *** | 0.219 *** |
| Treasury yield | 0.100 *** | 0.045 * | 0.041 | 0.070 * | 0.085 ** |
| Fed funds rate | 0.280 *** | 0.201 *** | 0.121 ** | 0.080 | 0.045 |
| BVAR-stationary CSV | | | | | |
| All variables | 0.655 *** | 0.599 *** | 0.515 *** | 0.346 * | -0.049 |
| GDP growth | 0.059 *** | 0.070 *** | 0.044 * | -0.053 | -0.150 |
| Consumption | 0.040 * | 0.052 ** | 0.017 | -0.065 | -0.145 |
| BFI | -0.006 | 0.002 | -0.017 | -0.074 | -0.117 |
| Employment | 0.107 *** | 0.110 *** | 0.111 * | 0.031 | -0.036 |
| Unemployment | 0.150 *** | 0.105 *** | 0.052 | 0.026 | -0.054 |
| GDP inflation | 0.063 *** | 0.092 *** | 0.111 *** | 0.116 ** | 0.138 |
| Treasury yield | 0.079 *** | 0.032 | 0.023 | 0.001 | -0.044 |
| Fed funds rate | 0.267 *** | 0.189 *** | 0.099 ** | 0.035 | -0.028 |

Note: See the notes to Table 6.

**Appendix Table A15. Average log predictive scores, 8-variable BVARs,
UK data, 1985:Q1-2008:Q2**

(avg. score for benchmark BVAR, differences in scores in all others)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| All variables | -0.911 | -0.831 | -0.956 | -0.828 | -0.490 |
| GDP growth | -0.025 | -0.015 | -0.017 | -0.011 | -0.007 |
| Consumption | -0.040 | -0.020 | -0.030 | -0.012 | -0.017 |
| Investment | -0.063 | -0.007 | -0.025 | -0.023 | -0.025 |
| Employment | -0.083 | -0.044 | -0.052 | -0.017 | 0.001 |
| Unemployment | -0.017 | -0.053 | -0.056 | -0.035 | 0.019 |
| GDP inflation | 0.053 *** | 0.115 *** | 0.193 *** | 0.273 *** | 0.324 *** |
| Bond yield | -0.009 | -0.021 | -0.014 | 0.034 | 0.092 *** |
| Bank rate | -0.036 | -0.046 | -0.066 | -0.022 | 0.048 |
| BVAR | | | | | |
| All variables | -14.144 | -15.743 | -17.632 | -19.794 | -21.102 |
| GDP growth | -2.479 | -2.501 | -2.541 | -2.561 | -2.563 |
| Consumption | -2.693 | -2.705 | -2.757 | -2.769 | -2.774 |
| Investment | -3.729 | -3.749 | -3.688 | -3.708 | -3.641 |
| Employment | -1.433 | -1.491 | -1.737 | -1.960 | -1.970 |
| Unemployment | 0.307 | -0.225 | -0.933 | -1.697 | -2.101 |
| GDP inflation | -2.721 | -2.845 | -3.111 | -3.468 | -3.715 |
| Bond yield | -0.780 | -1.215 | -1.593 | -2.029 | -2.332 |
| Bank rate | -1.302 | -1.753 | -2.159 | -2.617 | -2.926 |
| BVAR-SV | | | | | |
| All variables | 0.788 *** | 0.625 *** | 0.585 ** | 0.227 | -0.154 |
| GDP growth | 0.313 *** | 0.252 ** | 0.261 *** | 0.176 ** | 0.052 |
| Consumption | 0.153 *** | 0.149 *** | 0.177 *** | 0.095 * | -0.100 |
| Investment | 0.008 | 0.019 | 0.029 | -0.004 | -0.154 |
| Employment | -0.013 | -0.097 | -0.115 | -0.039 | -0.007 |
| Unemployment | -0.056 | -0.115 | -0.199 | -0.107 | -0.105 |
| GDP inflation | 0.158 *** | 0.200 *** | 0.256 *** | 0.205 *** | 0.054 |
| Bond yield | 0.170 *** | 0.129 ** | 0.115 ** | 0.080 | -0.010 |
| Bank rate | 0.241 *** | 0.203 *** | 0.129 ** | 0.029 | -0.120 |
| BVAR-CSV | | | | | |
| All variables | 0.410 *** | 0.481 *** | 0.458 ** | 0.351 | 0.098 |
| GDP growth | 0.162 *** | 0.171 *** | 0.132 *** | 0.137 *** | 0.119 *** |
| Consumption | 0.132 *** | 0.124 *** | 0.119 *** | 0.098 ** | 0.063 |
| Investment | -0.016 | -0.023 | -0.002 | 0.014 | 0.018 |
| Employment | -0.081 | -0.039 | -0.114 | -0.285 | -0.329 |
| Unemployment | -0.011 | -0.018 | -0.053 | -0.155 | -0.242 |
| GDP inflation | 0.162 *** | 0.174 *** | 0.217 *** | 0.219 *** | 0.201 *** |
| Bond yield | 0.131 *** | 0.114 *** | 0.115 *** | 0.116 ** | 0.120 ** |
| Bank rate | 0.140 *** | 0.141 *** | 0.120 *** | 0.096 ** | 0.075 * |
| BVAR-stationary CSV | | | | | |
| All variables | 0.475 *** | 0.428 *** | 0.373 ** | -0.120 | -0.738 |
| GDP growth | 0.194 *** | 0.157 *** | 0.131 *** | 0.014 | -0.124 |
| Consumption | 0.137 *** | 0.128 *** | 0.097 *** | -0.019 | -0.140 |
| Investment | -0.033 | -0.038 | -0.037 | -0.057 | -0.205 |
| Employment | -0.078 | -0.058 | -0.118 | -0.160 | -0.146 |
| Unemployment | 0.025 | 0.006 | -0.046 | -0.140 | -0.182 |
| GDP inflation | 0.164 *** | 0.170 *** | 0.189 *** | 0.133 *** | 0.062 * |
| Bond yield | 0.099 *** | 0.068 ** | 0.064 * | 0.025 | -0.014 |
| Bank rate | 0.089 *** | 0.071 *** | 0.046 | 0.006 | -0.041 |

Note: See the notes to Table 6.

Appendix Table A16. Real-Time Coverage Rates, 2-sided, 4-variable BVARs, 1985:Q1-2010:Q4

(probability of outcome outside 70 percent interval)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| GDP growth | 0.067 *** | 0.078 *** | 0.099 *** | 0.103 *** | 0.108 *** |
| Unemployment | 0.135 *** | 0.087 *** | 0.109 *** | 0.144 ** | 0.194 |
| GDP inflation | 0.202 ** | 0.194 *** | 0.139 *** | 0.258 | 0.312 |
| Fed funds rate | 0.048 *** | 0.097 *** | 0.178 | 0.309 | 0.366 |
| BVAR | | | | | |
| GDP growth | 0.192 *** | 0.194 ** | 0.158 *** | 0.082 *** | 0.097 *** |
| Unemployment | 0.173 *** | 0.184 ** | 0.337 | 0.381 | 0.247 |
| GDP inflation | 0.212 ** | 0.204 ** | 0.238 | 0.371 | 0.527 * |
| Fed funds rate | 0.048 *** | 0.068 *** | 0.119 *** | 0.278 | 0.333 |
| BVAR-SV | | | | | |
| GDP growth | 0.317 | 0.320 | 0.277 | 0.227 | 0.194 ** |
| Unemployment | 0.269 | 0.340 | 0.416 | 0.464 | 0.559 *** |
| GDP inflation | 0.327 | 0.282 | 0.366 | 0.412 | 0.538 * |
| Fed funds rate | 0.192 ** | 0.301 | 0.307 | 0.392 | 0.398 |
| BVAR-CSV | | | | | |
| GDP growth | 0.298 | 0.272 | 0.307 | 0.278 | 0.237 |
| Unemployment | 0.308 | 0.340 | 0.406 | 0.515 ** | 0.548 *** |
| GDP inflation | 0.317 | 0.272 | 0.356 | 0.412 | 0.527 * |
| Fed funds rate | 0.135 *** | 0.223 | 0.277 | 0.392 | 0.376 |
| BVAR-stationary CSV | | | | | |
| GDP growth | 0.288 | 0.282 | 0.287 | 0.227 | 0.183 *** |
| Unemployment | 0.317 | 0.311 | 0.406 | 0.464 | 0.527 ** |
| GDP inflation | 0.356 | 0.320 | 0.307 | 0.392 | 0.441 |
| Fed funds rate | 0.144 *** | 0.204 * | 0.277 | 0.402 | 0.387 |

Appendix Table A17. Real-Time Coverage Rates, left tail, 4-variable BVARs, 1985:Q1-2010:Q4

(probability of outcome in 15 percent left tail)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| GDP growth | 0.038 *** | 0.039 *** | 0.069 ** | 0.072 ** | 0.075 ** |
| Unemployment | 0.048 *** | 0.019 *** | 0.030 *** | 0.031 *** | 0.108 |
| GDP inflation | 0.135 | 0.165 | 0.129 | 0.258 ** | 0.312 * |
| Fed funds rate | 0.048 *** | 0.097 | 0.178 | 0.309 | 0.366 ** |
| BVAR | | | | | |
| GDP growth | 0.163 | 0.175 | 0.149 | 0.062 ** | 0.065 *** |
| Unemployment | 0.000 | 0.000 | 0.010 *** | 0.000 | 0.032 *** |
| GDP inflation | 0.163 | 0.175 | 0.218 | 0.371 *** | 0.527 *** |
| Fed funds rate | 0.029 *** | 0.039 *** | 0.099 | 0.278 | 0.333 ** |
| BVAR-SV | | | | | |
| GDP growth | 0.231 * | 0.233 | 0.198 | 0.134 | 0.108 |
| Unemployment | 0.058 *** | 0.058 *** | 0.050 *** | 0.052 ** | 0.108 |
| GDP inflation | 0.221 * | 0.233 * | 0.337 *** | 0.412 *** | 0.538 *** |
| Fed funds rate | 0.106 | 0.184 | 0.228 | 0.361 * | 0.387 ** |
| BVAR-CSV | | | | | |
| GDP growth | 0.202 | 0.184 | 0.218 | 0.155 | 0.118 |
| Unemployment | 0.067 *** | 0.068 ** | 0.069 ** | 0.093 | 0.118 |
| GDP inflation | 0.202 | 0.214 | 0.317 *** | 0.402 *** | 0.527 *** |
| Fed funds rate | 0.067 *** | 0.107 | 0.218 | 0.361 * | 0.376 ** |
| BVAR-stationary CSV | | | | | |
| GDP growth | 0.192 | 0.204 | 0.218 | 0.134 | 0.097 |
| Unemployment | 0.077 *** | 0.049 *** | 0.050 *** | 0.041 *** | 0.097 |
| GDP inflation | 0.231 ** | 0.243 ** | 0.267 * | 0.392 *** | 0.441 ** |
| Fed funds rate | 0.067 *** | 0.107 | 0.208 | 0.351 * | 0.376 ** |

Appendix Table A18. Real-Time Coverage Rates, right tail, 4-variable BVARs, 1985:Q1-2010:Q4

(probability of outcome in 15 percent right tail)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| GDP growth | 0.029 *** | 0.039 *** | 0.030 *** | 0.031 *** | 0.032 *** |
| Unemployment | 0.087 ** | 0.068 ** | 0.079 | 0.113 | 0.086 |
| GDP inflation | 0.067 *** | 0.029 *** | 0.010 *** | 0.000 | 0.000 |
| Fed funds rate | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| BVAR | | | | | |
| GDP growth | 0.029 *** | 0.019 *** | 0.010 *** | 0.021 *** | 0.032 *** |
| Unemployment | 0.173 | 0.184 | 0.327 * | 0.381 ** | 0.215 |
| GDP inflation | 0.048 *** | 0.029 *** | 0.020 *** | 0.000 | 0.000 |
| Fed funds rate | 0.019 *** | 0.029 *** | 0.020 *** | 0.000 | 0.000 |
| BVAR-SV | | | | | |
| GDP growth | 0.087 ** | 0.087 ** | 0.079 ** | 0.093 | 0.086 * |
| Unemployment | 0.212 | 0.282 ** | 0.366 ** | 0.412 ** | 0.452 *** |
| GDP inflation | 0.106 | 0.049 *** | 0.030 *** | 0.000 | 0.000 |
| Fed funds rate | 0.087 ** | 0.117 | 0.079 * | 0.031 *** | 0.011 *** |
| BVAR-CSV | | | | | |
| GDP growth | 0.096 * | 0.087 ** | 0.089 * | 0.124 | 0.118 |
| Unemployment | 0.240 * | 0.272 * | 0.337 * | 0.423 ** | 0.430 *** |
| GDP inflation | 0.115 | 0.058 *** | 0.040 *** | 0.010 *** | 0.000 |
| Fed funds rate | 0.067 *** | 0.117 | 0.059 *** | 0.031 *** | 0.000 |
| BVAR-stationary CSV | | | | | |
| GDP growth | 0.096 * | 0.078 ** | 0.069 ** | 0.093 | 0.086 * |
| Unemployment | 0.240 * | 0.262 * | 0.356 ** | 0.423 ** | 0.430 *** |
| GDP inflation | 0.125 | 0.078 ** | 0.040 *** | 0.000 | 0.000 |
| Fed funds rate | 0.077 ** | 0.097 | 0.069 ** | 0.052 ** | 0.011 *** |

Appendix Table A19. Real-Time Coverage Rates, 2-sided, 8-variable BVARs, 1985:Q1-2010:Q4

(probability of outcome outside 70 percent interval)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| GDP growth | 0.067 *** | 0.078 *** | 0.099 *** | 0.103 *** | 0.108 *** |
| Consumption | 0.144 *** | 0.126 *** | 0.109 *** | 0.144 *** | 0.129 *** |
| BFI | 0.308 | 0.282 | 0.267 | 0.247 | 0.215 ** |
| Employment | 0.125 *** | 0.126 *** | 0.149 ** | 0.206 | 0.226 |
| Unemployment | 0.135 *** | 0.087 *** | 0.109 *** | 0.144 ** | 0.194 |
| GDP inflation | 0.202 ** | 0.194 *** | 0.139 *** | 0.258 | 0.312 |
| Treasury yield | 0.221 ** | 0.194 *** | 0.238 | 0.299 | 0.301 |
| Fed funds rate | 0.048 *** | 0.097 *** | 0.178 | 0.309 | 0.366 |
| BVAR | | | | | |
| GDP growth | 0.163 *** | 0.165 *** | 0.119 *** | 0.093 *** | 0.108 *** |
| Consumption | 0.269 | 0.252 | 0.248 | 0.155 *** | 0.118 *** |
| BFI | 0.308 | 0.243 | 0.257 | 0.216 ** | 0.247 |
| Employment | 0.183 *** | 0.291 | 0.366 | 0.340 | 0.247 |
| Unemployment | 0.135 *** | 0.184 ** | 0.277 | 0.381 | 0.290 |
| GDP inflation | 0.231 | 0.223 ** | 0.228 | 0.309 | 0.473 |
| Treasury yield | 0.154 *** | 0.214 * | 0.198 * | 0.196 | 0.204 |
| Fed funds rate | 0.058 *** | 0.068 *** | 0.079 *** | 0.216 | 0.312 |
| BVAR-SV | | | | | |
| GDP growth | 0.308 | 0.301 | 0.248 | 0.216 | 0.183 *** |
| Consumption | 0.317 | 0.330 | 0.366 | 0.268 | 0.237 |
| BFI | 0.269 | 0.282 | 0.248 | 0.258 | 0.269 |
| Employment | 0.221 * | 0.350 | 0.436 | 0.433 | 0.312 |
| Unemployment | 0.240 | 0.291 | 0.386 | 0.474 * | 0.527 ** |
| GDP inflation | 0.288 | 0.243 | 0.307 | 0.361 | 0.441 |
| Treasury yield | 0.212 ** | 0.262 | 0.297 | 0.227 | 0.258 |
| Fed funds rate | 0.183 *** | 0.214 * | 0.257 | 0.299 | 0.366 |
| BVAR-CSV | | | | | |
| GDP growth | 0.231 | 0.214 ** | 0.257 | 0.196 * | 0.140 *** |
| Consumption | 0.298 | 0.301 | 0.327 | 0.309 | 0.226 ** |
| BFI | 0.337 | 0.301 | 0.277 | 0.278 | 0.312 |
| Employment | 0.240 | 0.311 | 0.406 | 0.392 | 0.301 |
| Unemployment | 0.250 | 0.282 | 0.347 | 0.454 | 0.495 * |
| GDP inflation | 0.250 | 0.233 | 0.257 | 0.309 | 0.409 |
| Treasury yield | 0.240 | 0.301 | 0.297 | 0.237 | 0.258 |
| Fed funds rate | 0.125 *** | 0.175 ** | 0.208 | 0.268 | 0.355 |
| BVAR-stationary CSV | | | | | |
| GDP growth | 0.231 | 0.184 *** | 0.198 ** | 0.165 ** | 0.108 *** |
| Consumption | 0.298 | 0.311 | 0.287 | 0.278 | 0.215 ** |
| BFI | 0.337 | 0.291 | 0.267 | 0.247 | 0.258 |
| Employment | 0.260 | 0.291 | 0.406 | 0.381 | 0.280 |
| Unemployment | 0.269 | 0.311 | 0.337 | 0.412 | 0.495 * |
| GDP inflation | 0.240 | 0.223 * | 0.198 ** | 0.258 | 0.269 |
| Treasury yield | 0.221 * | 0.291 | 0.287 | 0.227 | 0.226 |
| Fed funds rate | 0.135 *** | 0.175 ** | 0.218 | 0.247 | 0.344 |

Appendix Table A20. Real-Time Coverage Rates, left tail, 8-variable BVARs, 1985:Q1-2010:Q4

(probability of outcome in 15 percent left tail)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| GDP growth | 0.038 *** | 0.039 *** | 0.069 ** | 0.072 ** | 0.075 ** |
| Consumption | 0.106 | 0.078 ** | 0.079 ** | 0.113 | 0.097 |
| BFI | 0.125 | 0.087 * | 0.109 | 0.103 | 0.108 |
| Employment | 0.096 * | 0.117 | 0.139 | 0.206 | 0.226 |
| Unemployment | 0.048 *** | 0.019 *** | 0.030 *** | 0.031 *** | 0.108 |
| GDP inflation | 0.135 | 0.165 | 0.129 | 0.258 ** | 0.312 * |
| Treasury yield | 0.154 | 0.175 | 0.218 | 0.299 ** | 0.301 *** |
| Fed funds rate | 0.048 *** | 0.097 | 0.178 | 0.309 | 0.366 ** |
| BVAR | | | | | |
| GDP growth | 0.115 | 0.136 | 0.109 | 0.072 ** | 0.075 ** |
| Consumption | 0.202 | 0.194 | 0.208 | 0.134 | 0.086 |
| BFI | 0.125 | 0.107 | 0.129 | 0.113 | 0.097 |
| Employment | 0.173 | 0.291 ** | 0.356 ** | 0.340 ** | 0.247 |
| Unemployment | 0.019 *** | 0.019 *** | 0.030 *** | 0.021 *** | 0.043 *** |
| GDP inflation | 0.163 | 0.204 | 0.208 | 0.309 ** | 0.473 ** |
| Treasury yield | 0.077 ** | 0.117 | 0.129 | 0.186 | 0.204 |
| Fed funds rate | 0.019 *** | 0.039 *** | 0.059 ** | 0.216 | 0.312 * |
| BVAR-SV | | | | | |
| GDP growth | 0.202 | 0.214 | 0.178 | 0.134 | 0.097 |
| Consumption | 0.212 | 0.223 | 0.257 ** | 0.155 | 0.140 |
| BFI | 0.125 | 0.146 | 0.139 | 0.134 | 0.129 |
| Employment | 0.202 | 0.311 ** | 0.406 *** | 0.433 *** | 0.312 ** |
| Unemployment | 0.058 *** | 0.078 ** | 0.089 * | 0.103 | 0.118 |
| GDP inflation | 0.212 * | 0.194 | 0.257 ** | 0.361 ** | 0.441 ** |
| Treasury yield | 0.115 | 0.165 | 0.198 | 0.216 | 0.258 |
| Fed funds rate | 0.038 *** | 0.058 *** | 0.139 | 0.278 | 0.366 ** |
| BVAR-CSV | | | | | |
| GDP growth | 0.144 | 0.146 | 0.178 | 0.124 | 0.065 ** |
| Consumption | 0.192 | 0.194 | 0.238 | 0.206 | 0.140 |
| BFI | 0.144 | 0.126 | 0.139 | 0.134 | 0.151 |
| Employment | 0.212 | 0.291 ** | 0.386 ** | 0.392 ** | 0.301 ** |
| Unemployment | 0.077 *** | 0.078 ** | 0.089 | 0.093 | 0.118 |
| GDP inflation | 0.173 | 0.184 | 0.208 | 0.299 ** | 0.409 ** |
| Treasury yield | 0.135 | 0.184 | 0.208 | 0.216 | 0.258 |
| Fed funds rate | 0.048 *** | 0.068 ** | 0.129 | 0.268 | 0.355 ** |
| BVAR-stationary CSV | | | | | |
| GDP growth | 0.154 | 0.126 | 0.129 | 0.093 | 0.065 ** |
| Consumption | 0.192 | 0.204 | 0.218 | 0.186 | 0.129 |
| BFI | 0.135 | 0.126 | 0.139 | 0.134 | 0.118 |
| Employment | 0.221 * | 0.282 ** | 0.386 ** | 0.381 ** | 0.280 * |
| Unemployment | 0.087 ** | 0.078 ** | 0.089 | 0.072 | 0.108 |
| GDP inflation | 0.163 | 0.165 | 0.168 | 0.247 | 0.269 * |
| Treasury yield | 0.125 | 0.175 | 0.198 | 0.206 | 0.226 |
| Fed funds rate | 0.038 *** | 0.068 ** | 0.119 | 0.247 | 0.344 ** |

Appendix Table A21. Real-Time Coverage Rates, right tail, 8-variable BVARs, 1985:Q1-2010:Q4

(probability of outcome in 15 percent right tail)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| GDP growth | 0.029 *** | 0.039 *** | 0.030 *** | 0.031 *** | 0.032 *** |
| Consumption | 0.038 *** | 0.049 *** | 0.030 *** | 0.031 *** | 0.032 *** |
| BFI | 0.183 | 0.194 | 0.158 | 0.144 | 0.108 |
| Employment | 0.029 *** | 0.010 *** | 0.010 *** | 0.000 | 0.000 |
| Unemployment | 0.087 ** | 0.068 ** | 0.079 | 0.113 | 0.086 |
| GDP inflation | 0.067 *** | 0.029 *** | 0.010 *** | 0.000 | 0.000 |
| Treasury yield | 0.067 *** | 0.019 *** | 0.020 *** | 0.000 | 0.000 |
| Fed funds rate | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| BVAR | | | | | |
| GDP growth | 0.048 *** | 0.029 *** | 0.010 *** | 0.021 *** | 0.032 *** |
| Consumption | 0.067 *** | 0.058 *** | 0.040 *** | 0.021 *** | 0.032 *** |
| BFI | 0.183 | 0.136 | 0.129 | 0.103 | 0.151 |
| Employment | 0.010 *** | 0.000 | 0.010 *** | 0.000 | 0.000 |
| Unemployment | 0.115 | 0.165 | 0.248 | 0.361 * | 0.247 |
| GDP inflation | 0.067 *** | 0.019 *** | 0.020 *** | 0.000 | 0.000 |
| Treasury yield | 0.077 *** | 0.097 | 0.069 ** | 0.010 *** | 0.000 |
| Fed funds rate | 0.038 *** | 0.029 *** | 0.020 *** | 0.000 | 0.000 |
| BVAR-SV | | | | | |
| GDP growth | 0.106 | 0.087 ** | 0.069 ** | 0.082 | 0.086 * |
| Consumption | 0.106 | 0.107 | 0.109 | 0.113 | 0.097 |
| BFI | 0.144 | 0.136 | 0.109 | 0.124 | 0.140 |
| Employment | 0.019 *** | 0.039 *** | 0.030 *** | -0.000 | -0.000 |
| Unemployment | 0.183 | 0.214 | 0.297 | 0.371 * | 0.409 ** |
| GDP inflation | 0.077 *** | 0.049 *** | 0.050 *** | -0.000 | -0.000 |
| Treasury yield | 0.096 * | 0.097 | 0.099 | 0.010 *** | -0.000 |
| Fed funds rate | 0.144 | 0.155 | 0.119 | 0.021 *** | -0.000 |
| BVAR-CSV | | | | | |
| GDP growth | 0.087 ** | 0.068 *** | 0.079 ** | 0.072 ** | 0.075 ** |
| Consumption | 0.106 | 0.107 | 0.089 | 0.103 | 0.086 |
| BFI | 0.192 | 0.175 | 0.139 | 0.144 | 0.161 |
| Employment | 0.029 *** | 0.019 *** | 0.020 *** | 0.000 | 0.000 |
| Unemployment | 0.173 | 0.204 | 0.257 | 0.361 * | 0.376 ** |
| GDP inflation | 0.077 *** | 0.049 *** | 0.050 *** | 0.010 *** | 0.000 |
| Treasury yield | 0.106 | 0.117 | 0.089 | 0.021 *** | 0.000 |
| Fed funds rate | 0.077 ** | 0.107 | 0.079 * | 0.000 | 0.000 |
| BVAR-stationary CSV | | | | | |
| GDP growth | 0.077 *** | 0.058 *** | 0.069 ** | 0.072 ** | 0.043 *** |
| Consumption | 0.106 | 0.107 | 0.069 * | 0.093 | 0.086 |
| BFI | 0.202 | 0.165 | 0.129 | 0.113 | 0.140 |
| Employment | 0.038 *** | 0.010 *** | 0.020 *** | 0.000 | 0.000 |
| Unemployment | 0.183 | 0.233 | 0.248 | 0.340 * | 0.387 ** |
| GDP inflation | 0.077 *** | 0.058 *** | 0.030 *** | 0.010 *** | 0.000 |
| Treasury yield | 0.096 * | 0.117 | 0.089 | 0.021 *** | 0.000 |
| Fed funds rate | 0.096 | 0.107 | 0.099 | 0.000 | 0.000 |

**Appendix Table A22. Coverage Rates, 2-sided, 8-variable BVARs,
UK data, 1985:Q1-2011:Q2**
(probability of outcome outside 70 percent interval)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| GDP growth | 0.066 *** | 0.076 *** | 0.068 *** | 0.061 *** | 0.063 *** |
| Consumption | 0.104 *** | 0.105 *** | 0.165 *** | 0.121 *** | 0.126 *** |
| Investment | 0.264 | 0.248 | 0.214 | 0.232 | 0.211 |
| Employment | 0.481 *** | 0.314 | 0.243 | 0.263 | 0.242 |
| Unemployment | 0.396 * | 0.314 | 0.252 | 0.283 | 0.232 |
| GDP inflation | 0.057 *** | 0.038 *** | 0.010 *** | 0.010 *** | 0.000 |
| Bond yield | 0.057 *** | 0.095 *** | 0.039 *** | 0.081 *** | 0.063 *** |
| Bank rate | 0.066 *** | 0.095 *** | 0.117 *** | 0.212 | 0.126 *** |
| BVAR | | | | | |
| GDP growth | 0.085 *** | 0.057 *** | 0.058 *** | 0.061 *** | 0.063 *** |
| Consumption | 0.132 *** | 0.095 *** | 0.136 *** | 0.121 *** | 0.116 *** |
| Investment | 0.349 | 0.295 | 0.262 | 0.212 | 0.211 |
| Employment | 0.462 *** | 0.314 | 0.282 | 0.374 | 0.347 |
| Unemployment | 0.283 | 0.248 | 0.243 | 0.293 | 0.295 |
| GDP inflation | 0.075 *** | 0.019 *** | 0.019 *** | 0.000 | 0.000 |
| Bond yield | 0.085 *** | 0.095 *** | 0.126 *** | 0.081 *** | 0.095 *** |
| Bank rate | 0.066 *** | 0.076 *** | 0.126 *** | 0.141 *** | 0.137 *** |
| BVAR-SV | | | | | |
| GDP growth | 0.274 | 0.286 | 0.282 | 0.253 | 0.211 * |
| Consumption | 0.274 | 0.248 | 0.282 | 0.263 | 0.305 |
| Investment | 0.283 | 0.248 | 0.243 | 0.253 | 0.242 |
| Employment | 0.425 ** | 0.276 | 0.282 | 0.354 | 0.358 |
| Unemployment | 0.377 | 0.352 | 0.340 | 0.374 | 0.484 |
| GDP inflation | 0.189 *** | 0.124 *** | 0.049 *** | 0.030 *** | -0.000 |
| Bond yield | 0.245 | 0.210 * | 0.155 *** | 0.172 ** | 0.168 * |
| Bank rate | 0.113 *** | 0.114 *** | 0.175 ** | 0.202 | 0.158 ** |
| BVAR-CSV | | | | | |
| GDP growth | 0.151 *** | 0.152 *** | 0.136 *** | 0.141 *** | 0.147 *** |
| Consumption | 0.189 *** | 0.181 *** | 0.214 * | 0.232 | 0.242 |
| Investment | 0.377 | 0.324 | 0.350 | 0.293 | 0.295 |
| Employment | 0.500 *** | 0.390 * | 0.330 | 0.455 ** | 0.495 ** |
| Unemployment | 0.377 | 0.362 | 0.330 | 0.343 | 0.474 ** |
| GDP inflation | 0.189 *** | 0.133 *** | 0.049 *** | 0.020 *** | 0.000 |
| Bond yield | 0.123 *** | 0.162 *** | 0.136 *** | 0.141 *** | 0.116 *** |
| Bank rate | 0.094 *** | 0.095 *** | 0.155 *** | 0.162 ** | 0.147 ** |
| BVAR-stationary CSV | | | | | |
| GDP growth | 0.160 *** | 0.162 *** | 0.165 *** | 0.202 * | 0.179 ** |
| Consumption | 0.208 ** | 0.190 *** | 0.243 | 0.232 | 0.253 |
| Investment | 0.349 | 0.314 | 0.350 | 0.283 | 0.242 |
| Employment | 0.509 *** | 0.371 | 0.340 | 0.424 * | 0.442 * |
| Unemployment | 0.349 | 0.362 | 0.320 | 0.354 | 0.442 |
| GDP inflation | 0.189 *** | 0.124 *** | 0.039 *** | 0.010 *** | 0.000 |
| Bond yield | 0.123 *** | 0.124 *** | 0.136 *** | 0.091 *** | 0.105 *** |
| Bank rate | 0.085 *** | 0.076 *** | 0.107 *** | 0.141 ** | 0.158 ** |

**Appendix Table A23. Coverage Rates, left tail, 8-variable BVARs,
UK data, 1985:Q1-2011:Q2**
(probability of outcome in 15 percent left tail)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| GDP growth | 0.057 *** | 0.067 ** | 0.058 *** | 0.051 *** | 0.063 ** |
| Consumption | 0.047 *** | 0.048 *** | 0.097 | 0.071 | 0.084 |
| Investment | 0.132 | 0.133 | 0.097 | 0.111 | 0.116 |
| Employment | 0.208 | 0.124 | 0.097 | 0.111 | 0.126 |
| Unemployment | 0.226 * | 0.171 | 0.175 | 0.192 | 0.158 |
| GDP inflation | 0.047 *** | 0.029 *** | 0.010 *** | 0.010 *** | 0.000 |
| Bond yield | 0.038 *** | 0.057 *** | 0.019 *** | 0.081 | 0.063 ** |
| Bank rate | 0.047 *** | 0.076 *** | 0.097 | 0.182 | 0.126 |
| BVAR | | | | | |
| GDP growth | 0.066 *** | 0.057 *** | 0.058 *** | 0.061 ** | 0.063 ** |
| Consumption | 0.085 * | 0.057 *** | 0.097 | 0.091 | 0.095 |
| Investment | 0.179 | 0.152 | 0.117 | 0.101 | 0.105 |
| Employment | 0.198 | 0.114 | 0.117 | 0.131 | 0.137 |
| Unemployment | 0.179 | 0.171 | 0.165 | 0.182 | 0.189 |
| GDP inflation | 0.047 *** | 0.010 *** | 0.010 *** | 0.000 | 0.000 |
| Bond yield | 0.038 *** | 0.057 *** | 0.097 | 0.071 * | 0.084 * |
| Bank rate | 0.047 *** | 0.067 *** | 0.107 | 0.121 | 0.137 |
| BVAR-SV | | | | | |
| GDP growth | 0.094 * | 0.114 | 0.097 | 0.111 | 0.105 |
| Consumption | 0.170 | 0.143 | 0.155 | 0.152 | 0.200 |
| Investment | 0.123 | 0.114 | 0.097 | 0.101 | 0.105 |
| Employment | 0.179 | 0.095 | 0.107 | 0.131 | 0.126 |
| Unemployment | 0.236 * | 0.229 | 0.243 | 0.253 | 0.326 |
| GDP inflation | 0.085 ** | 0.086 *** | 0.029 *** | 0.020 *** | -0.000 |
| Bond yield | 0.123 | 0.124 | 0.117 | 0.121 | 0.147 |
| Bank rate | 0.075 *** | 0.086 * | 0.117 | 0.152 | 0.158 |
| BVAR-CSV | | | | | |
| GDP growth | 0.085 * | 0.095 | 0.078 * | 0.091 | 0.084 |
| Consumption | 0.123 | 0.114 | 0.146 | 0.141 | 0.168 |
| Investment | 0.189 | 0.152 | 0.155 | 0.141 | 0.147 |
| Employment | 0.226 * | 0.171 | 0.117 | 0.152 | 0.147 |
| Unemployment | 0.236 ** | 0.238 * | 0.243 * | 0.232 | 0.305 |
| GDP inflation | 0.094 ** | 0.086 ** | 0.019 *** | 0.010 *** | 0.000 |
| Bond yield | 0.066 *** | 0.105 | 0.107 | 0.111 | 0.116 |
| Bank rate | 0.075 *** | 0.076 *** | 0.117 | 0.121 | 0.137 |
| BVAR-stationary CSV | | | | | |
| GDP growth | 0.085 * | 0.086 | 0.087 | 0.101 | 0.095 |
| Consumption | 0.132 | 0.114 | 0.155 | 0.152 | 0.179 |
| Investment | 0.170 | 0.143 | 0.146 | 0.111 | 0.116 |
| Employment | 0.236 ** | 0.143 | 0.126 | 0.141 | 0.137 |
| Unemployment | 0.189 | 0.238 * | 0.233 | 0.222 | 0.274 |
| GDP inflation | 0.104 * | 0.076 *** | 0.019 *** | 0.000 | 0.000 |
| Bond yield | 0.057 *** | 0.067 *** | 0.107 | 0.081 * | 0.095 |
| Bank rate | 0.066 *** | 0.067 *** | 0.097 | 0.111 | 0.158 |

**Appendix Table A24. Coverage Rates, right tail, 8-variable BVARs,
UK data, 1985:Q1-2011:Q2**

(probability of outcome in 15 percent right tail)

| | $h = 1Q$ | $h = 2Q$ | $h = 4Q$ | $h = 8Q$ | $h = 12Q$ |
|----------------------------|-----------|-----------|-----------|-----------|-----------|
| AR | | | | | |
| GDP growth | 0.009 *** | 0.010 *** | 0.010 *** | 0.010 *** | 0.000 |
| Consumption | 0.057 *** | 0.057 *** | 0.068 *** | 0.051 *** | 0.042 *** |
| Investment | 0.132 | 0.114 | 0.117 | 0.121 | 0.095 ** |
| Employment | 0.274 *** | 0.190 | 0.146 | 0.152 | 0.116 |
| Unemployment | 0.170 | 0.143 | 0.078 * | 0.091 | 0.074 * |
| GDP inflation | 0.009 *** | 0.010 *** | 0.000 | 0.000 | 0.000 |
| Bond yield | 0.019 *** | 0.038 *** | 0.019 *** | 0.000 | 0.000 |
| Bank rate | 0.019 *** | 0.019 *** | 0.019 *** | 0.030 *** | 0.000 |
| BVAR | | | | | |
| GDP growth | 0.019 *** | 0.000 | 0.000 | 0.000 | 0.000 |
| Consumption | 0.047 *** | 0.038 *** | 0.039 *** | 0.030 *** | 0.021 *** |
| Investment | 0.170 | 0.143 | 0.146 | 0.111 | 0.105 |
| Employment | 0.264 *** | 0.200 | 0.165 | 0.242 | 0.211 |
| Unemployment | 0.104 | 0.076 ** | 0.078 * | 0.111 | 0.105 |
| GDP inflation | 0.028 *** | 0.010 *** | 0.010 *** | 0.000 | 0.000 |
| Bond yield | 0.047 *** | 0.038 *** | 0.029 *** | 0.010 *** | 0.011 *** |
| Bank rate | 0.019 *** | 0.010 *** | 0.019 *** | 0.020 *** | 0.000 |
| BVAR-SV | | | | | |
| GDP growth | 0.179 | 0.171 | 0.184 | 0.141 | 0.105 |
| Consumption | 0.104 | 0.105 | 0.126 | 0.111 | 0.105 |
| Investment | 0.160 | 0.133 | 0.146 | 0.152 | 0.137 |
| Employment | 0.245 ** | 0.181 | 0.175 | 0.222 | 0.232 |
| Unemployment | 0.142 | 0.124 | 0.097 | 0.121 | 0.158 |
| GDP inflation | 0.104 * | 0.038 *** | 0.019 *** | 0.010 *** | -0.000 |
| Bond yield | 0.123 | 0.086 ** | 0.039 *** | 0.051 *** | 0.021 *** |
| Bank rate | 0.038 *** | 0.029 *** | 0.058 *** | 0.051 *** | -0.000 |
| BVAR-CSV | | | | | |
| GDP growth | 0.066 *** | 0.057 *** | 0.058 *** | 0.051 *** | 0.063 ** |
| Consumption | 0.066 *** | 0.067 *** | 0.068 *** | 0.091 | 0.074 |
| Investment | 0.189 | 0.171 | 0.194 | 0.152 | 0.147 |
| Employment | 0.274 *** | 0.219 | 0.214 | 0.303 ** | 0.347 ** |
| Unemployment | 0.142 | 0.124 | 0.087 | 0.111 | 0.168 |
| GDP inflation | 0.094 ** | 0.048 *** | 0.029 *** | 0.010 *** | 0.000 |
| Bond yield | 0.057 *** | 0.057 *** | 0.029 *** | 0.030 *** | 0.000 |
| Bank rate | 0.019 *** | 0.019 *** | 0.039 *** | 0.040 *** | 0.011 *** |
| BVAR-stationary CSV | | | | | |
| GDP growth | 0.075 ** | 0.076 ** | 0.078 ** | 0.101 | 0.084 |
| Consumption | 0.075 *** | 0.076 *** | 0.087 * | 0.081 | 0.074 |
| Investment | 0.179 | 0.171 | 0.204 | 0.172 | 0.126 |
| Employment | 0.274 *** | 0.229 * | 0.214 | 0.283 * | 0.305 ** |
| Unemployment | 0.160 | 0.124 | 0.087 | 0.131 | 0.168 |
| GDP inflation | 0.085 ** | 0.048 *** | 0.019 *** | 0.010 *** | 0.000 |
| Bond yield | 0.066 *** | 0.057 *** | 0.029 *** | 0.010 *** | 0.011 *** |
| Bank rate | 0.019 *** | 0.010 *** | 0.010 *** | 0.030 *** | 0.000 |