Strategic Information Acquisition and Transmission*

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Abstract

This paper explores the implications of costly information acquisition in the strategic communication models that follow Crawford and Sobel (1982). We show that equilibrium decisions based on a biased expert's advice may be more precise than in the case when information is directly acquired by the decision maker, even if the expert is not more efficient than the decision maker at acquiring information. Consequently, we find that communication by an expert to the decision maker may often outperform delegation of the decision making authority to an expert, as well as centralization by the decision maker of both information acquisition and decision making authority. This result bears important implications for organization design, and especially for the study of the optimal authority allocation in the presence of incomplete information. In particular, it provides an endogenous justification for the emergence of signalling as a preferred organizational form (paradigm).

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1 Introduction

Strategic information transmission, also known as cheap talk, has been extensively analyzed in the economics literature under various assumptions on the underlying primitives, information and payoff structure.[1] Starting from the seminal contribution of Crawford and Sobel (1982), this literature has pointed out that the possibility of credible information transmission by an informed expert (sender) to an uninformed decision-maker (receiver), when the sender and the receiver disagree about the optimal decision, is limited: Some information will necessarily be lost in transmission. Put otherwise, the sender will transmit a noisy signal of her true information. Further, because information is lost in communication, the decisions based on a biased expert's advice need to be imprecise.

In turn, equilibrium loss of information in transmission implies that decision making based on the advice of a biased expert may not be an optimal organizational form. When an uninformed decision-marker can choose how to structure her interaction with an informed expert when their preferences are misaligned, she will often find that the cheap talk outcome and the associated sender-receiver structure are suboptimal for her. Ideally, the decision-maker would prefer to have direct access to the information, rather than relying on an expert. If that is not possible, the decision maker would be better off if she delegated the decision-making authority to the informed expert, (Dessein (2002), Ottaviani (2000)), rather than retaining the decision-making authority while relying on the imperfect information communicated by the expert.

The above results about the possibility of credible information transmission in a cheap talk game are obtained for an exogenously given informational structure. The existing literature typically assumes that the decision-maker is completely uninformed, and the sender is perfectly informed.[2] The latter had, presumably, acquired his information at

[1]This literature includes Austen-Smith (1993), Gilligan and Krehbiel (1987, 1989), Krishna and Morgan (2001a, 2001b), Wolinsky (2002), Battaglini (2002, 2004), Ambrus and Takahashi (2008), and other works.

[2]Some exceptions are Austen-Smith (1994), Ottaviani (2000) and Ivanov (2010). In all these papers, the sender may or may not be informed. In Austen-Smith (1994), the sender may either acquire full information or remain completely ignorant. In the model by Ottaviani (2000), the amount of information available to the expert is exogenously given. In the model by Ivanov (2010), it can be selected costlessly by the decision-maker.

no cost. This informational structure is clearly an extreme point in the set of feasible possibilities. For one thing, information is typically costly. Expertise and knowledge are obtained as a result of often time-consuming work and research.

Departing from the previous literature, we consider a framework where information is costly and endogenously acquired and investigate how this affects strategic communication. Our analysis uncovers several novel effects. Importantly, we demonstrate the possibility of overinvestment in information acquisition by the sender. As a result, the decisions based on a biased expert's advice may be more precise than optimal choices based on direct information acquisition, even if the expert does not possess a better information acquisition technology than the decision maker.[3]

The insight that the sender may overinvest in information acquisition allows us to revisit the analysis of the relative payoff ranking, for a decision-maker, of different organizational forms such as the sender-receiver structure, delegation, and centralization. Contrary to the previous literature, we find that communication by an expert to the decision maker may often outperform delegation of the decision making authority to the expert.

Let us now briefly describe our set-up, the results and the intuitions behind them. We consider an environment in which both parties - the decision-maker and her agent - are initially uninformed about the state of the world and share a common prior. Information can be acquired by performing costly "experiments," or trials. The more trials are performed, the higher the precision of the information about the state of the world. In the limit, with infinitely many experiments, the state of the world is learned precisely and perfectly.[4]

We first analyze the sender-receiver structure: The agent acquires the information and then makes an announcement to the principal, who makes a decision. We consider both overt

[3]A recent paper by Che and Kartik (2009) studies a similar problem of information acquisition and communication. But while we consider cheap talk, their analysis is set in the opposite benchmark case of disclosure of verifiable information. Further, also the focus of the analysis is different. We compare the precision of decisions under communication and when the receiver acquires information directly. Che and Kartik (2009) do not consider this possibility. Instead, they show that divergence in prior beliefs between sender and receiver, while stifling communication, delivers better incentives for information acquisition.

[4]Out set-up is related to the Bernoulli-Uniform model of cheap talk analyzed by Morgan and Stocken (2008).

information acquisition (henceforth, the *overt game)*, where the number of trials performed by the expert is observed by the decision-maker, and the covert information acquisition case in which this number is only known to the expert (henceforth, the *covert game*). Our focus is on the amount of information *acquired and credibly transmitted* by the expert in equilibrium, and hence on the quality of the final action made by the decision maker.

Next, we compare the outcome of both the overt and covert sender-receiver games to alternative organizational forms, i.e. delegation of both the information acquisition and the decision-making tasks to an expert, or centralization of both tasks.

Our central results are the following. First, we provide sufficient conditions such that both the overt and the covert sender-receiver games have at least an equilibrium (the one preferred by the decision maker) in which the precision of the final decision is strictly larger than in the case of centralization. The expert acquires more information than what the decision-maker would if she had direct access to information. This more than offsets the tendency to lose some of it in transmission. These conditions are satisfied for moderate values of the bias and of the information acquisition cost. Second, we find tighter conditions guaranteeing that in all the Pareto-undominated equilibria of both the overt and the covert game, the precision of the final decision is at least as large as in centralization.

These results have immediate implications for the comparison of organizational structures. Suppose that the decision maker can obtain for free (as the cost is borne by the expert) an amount of information at least as large as what she would optimally choose to acquire if she acquired it personally and bore the cost. Then, she will clearly prefer the sender-receiver structure to centralization. Similarly, she will prefer the sender-receiver structure to full delegation: The precision of the final decision is weakly higher in the former case, and the decision is biased by the expert's preferences in the latter case. These results stand in contrast with the findings of Ottaviani (2000) and Dessein (2002) that delegation usually outperforms communication, when the amount of information held by the expert sender is exogenously given. Instead, in our model, communication often outperfoms delegation, and this result is independent of the information acquisition cost of the decision maker.[5]

Let us now illustrate the two key intuitions that drive our results. The first strategic effect that our setup allows us to highlight is the possibility of overinvestment in information by the sender. This is based on the fact that the presence of strategic considerations makes the value of information for the expert in a communication game different from the value of information for a decision maker who directly acquires information. The mechanism through which this leads to overinvestment is different in the overt and the covert game.

Consider the overt game first. The cheap talk stage that follows information acquisition is characterized by a multiplicity of equilibria endemic to most cheap talk games. In particular, there is always a "babbling equilibrium" in which the principal completely ignores the signal sent by the agent. This multiplicity of equilibria provides the principal with an instrument allowing her to induce the agent to acquire more information than the latter would like. Precisely, the principal could and would credibly threaten the agent to play a "babbling" equilibrium unless the agent performs a certain number of trials. An expert's advice would only be taken into account if the expert had invested a large amount in information acquisition. Since the principal does not bear the cost of information acquisition, she would be disposed in favor of a larger number of experiments. Hence, this factor works in favor of overinvestment.

We find that the latter factor (overinvestment) dominates the loss of information in transmission for moderate values of the bias and of the investment cost. As a result, the decision-maker's information and, hence her action, would be more precise than under centralization (or delegation). As a consequence, in those cases the decision-maker would prefer the sender-receiver structure to the alternative organizational forms.

When information acquisition is covert, deviations at the information acquisition stage are not observable. Hence, reversion to the babbling equilibrium in case of underinvestment

[5] In a recent paper, Thordal-Le Quement (2010) analyzes a model in which the decision maker can choose between retaining and delegating the decision power *after* observing the amount of information acquired by the expert. He shows that this provides the expert with an incentive to acquire a large amount of information in order to guarantee that the decision power will be delegated to him.

is no longer possible. However, overinvestment in information may still arise due to a different and novel factor appearing in our model. This factor stems from the inflexibility of the equilibrium language when information acquisition is covert. Note that the equilibrium language i.e., the mapping between the set of feasible messages and the choices of the receiver, is determined by the receiver's equilibrium expectations about the number of experiments to be performed by the sender. Therefore, it is not sensitive to a sender's unobservable deviation to a different number of experiments at the information acquisition stage. This property limits the attractiveness of a deviation to a non-equilibrium number of signals for the sender: Regardless of the number of trials that she conducts, any message she sends will be interpreted under the belief that she acquired the equilibrium (rather than the actual) number of signals.[6,7]

The second strategic effect that we uncover concerns the incentives of the sender. When information is costly, it does not make sense for the sender to pay for pieces of information that would be lost in transmission with a high probability. This consideration makes the sender reluctant to acquire more information than she transmits in the equilibrium and creates a strong incentive against the loss of information in transmission. As a result, under certain parameter values very little, or even no information is lost in transmission in an equilibrium of the communication game.

In combination, the incentive for overinvestment in information acquisition and the effect of cost considerations limiting the loss of information in transmission stand behind the central results of our paper. In particular, we establish the existence of equilibria of the overt and covert games that are characterized by a higher precision of the final decision than under centralization.

Moreover, we also establish a stronger result. We show that under more restrictive conditions on the parameters of the model, the precision of the final decision in all Pareto

[6]The latter intuition is related to the analysis in Eso and Szalay (2011) of the incentives of an uninformed sender to acquire full information about the state of the world in a cheap talk game. In their paper, information acquisition is covert, and it is a binary choice: The sender can either stay uninformed, or learn the state of the world at a cost.

[7]In Section 4.4 we discuss the robustness of our results in a setting where the expert can also send a message to the decision maker about the number of trials performed.

undominated equilibria of both the overt and covert game is at least as large as under centralization. In particular, this result holds in the equilibrium of the overt game which selects the most informative equilibrium in every communication subgame, on and off the equilibrium path. More restrictive conditions on the parameters are required because in establishing this result we can no longer rely on the effect of the receiver off equilibrium behavior inducing the sender to overinvest in information acquisition.

For both results, we provide sufficient conditions that involve full revelation in equilibrium. This simplifies the analysis, but full revelation is by no means a necessary condition, as we show numerically.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 discusses the characterization of the equilibria of the covert and overt sender-receiver games. Section 4 focuses on the precision of the decision in the equilibrium outcomes of both games, and its comparison with the precision of the decision in the centralized outcome. Section 5 derives the implications for organization design. Section 6 concludes.

2 The Model and Preliminaries

We start by introducing our model of cheap talk with endogenous acquisition of costly information by the sender. It is a natural extension of the classic Crawford and Sobel (1982) uniform-quadratic model. There are two players, the expert and the decision maker. At the end of the game, the decision maker chooses an action $y \in [0, 1]$ to maximize her payoff

$$U^R(y, \theta) = -(y - \theta)^2, \tag{1}$$

where θ is an unknown state of the world, with uniform common prior distribution on $[0, 1]$. At the beginning of the game, the expert can purchase some information on θ. Specifically, he acquires $n \in \mathbb{N} \cup \{0\}$ i.i.d. binary trials, with probability of success equal to θ, at a linear cost $c(n) = cn$. The players' preferences are misaligned. The expert's payoff is:

$$U^S(y, \theta, b) - c(n) = -(y - \theta - b)^2 - cn, \tag{2}$$

where the bias $b > 0$ measures the preference discrepancy among players. After simultaneously acquiring n trials, the expert observes the number k of success and communicates a message $m \in [0, 1]$ to the decision maker. If information acquisition is overt, n is observed by the decision maker and we refer to this case as the *overt game*. If information acquisition is covert, n is private, unverifiable information of the expert and we refer to this case as the *covert game*.

The number of trials n measures the precision of the expert's information about θ. Note that k, the number of successes, is distributed according to the binomial distribution:

$$f(k; n, \theta) = \frac{n!}{k!(n-k)!}\theta^k (1-\theta)^{n-k}, \text{ for } 0 \leq k \leq n.$$

For future reference, we also note that the unconditional distribution of k is uniform:

$$\Pr(k; n) = \int_0^1 \frac{n!}{k!(n-k)!}\theta^k (1-\theta)^{n-k} d\theta = \frac{1}{n+1}.$$

We note that the posterior distribution of θ after observing k successes in n trials is a Beta distribution with parameters $k+1$ and $n-k+1$:

$$f(\theta; k, n) = \frac{(n+1)!}{k!(n-k)!}\theta^k (1-\theta)^{n-k}, \text{ if } 0 \leq \theta \leq 1.$$

The corresponding posterior expectation is $E[\theta|k, n] = \dfrac{k+1}{n+2}$.

2.1 The Overt Game

A pure strategy Perfect Bayesian equilibrium of the overt game is described by $\left(\hat{n}, \{P_{n'}\}_{n'=0,1,...,\infty}, \{\mathbf{y}(P_{n'})\}_{n'=0,1,...,\infty}\right)$, where $\hat{n}$ is the equilibrium number of trials conducted by the expert; $P_{n'}$ is the partition of the set of the expert's types $\{0, 1, ..., n'\}$

describing the information communicated by the expert if the expert has chosen to conduct $n' \in \{0, 1, ..., \infty\}$ trials; and $\{\mathbf{y}(P_{n'})\} \equiv \left(y_{p_1}^{n'}, ..., y_{p_{\#P_{n'}}}^{n'}\right)$ is the decision maker's action profile under type space partition $P_{n'}$ where $y_{p_i}^{n'} \in [0,1]$ denotes the decision-maker's action after the expert signals the element p_i of the partition (i.e. after the expert announces that the number of successes k in n' trials is s.t. $k \in p_i$). Note that $P_{n'}$ describes both the expert's equilibrium strategy profile following the acquisition of n' trials and the set of feasible equilibrium posterior beliefs of the decision maker (implying the obvious Bayesian updating of the decision-maker's prior conditional on the expert's announcement $p_i \in \{P_{n'}\}$ given observed n').

Equilibrium requires the following conditions to hold:

(i) Action profile $\mathbf{y}(P_{n'})$ has to be sequentially rational for all n' i.e., $y_{p_i}^{n'}$ has to maximize the decision-maker's expected payoff given her beliefs that the sender's type belongs to p_i, for every $p_i \in P_{n'}$; [8]

(ii) For every, $n' \in \{0, 1, \infty\}$, the partition $P_{n'}$ must be incentive compatible i.e., the following condition must hold for any $k \in \{0, 1, ..., n'\}$ and $p_i \in P_{n'}$ s.t. $k \in p_i$:

$$\int_0^1 U^S\left(y_p^{n'}, \theta, b\right) f\left(\theta; k, n'\right) d\theta \geq \int_0^1 U^S(y_q^{n'}, \theta, b) f\left(\theta; k, n'\right) d\theta, \text{ for all } q \in P_{n'}. \quad (3)$$

(iii) $\hat{n}$ maximizes the decision-maker's expected payoff given the equilibrium profiles $\{P_{n'}\}$ and $\{\mathbf{y}(P_{n'})\}$, $n' = 0, 1, ..., \infty$.

To characterize the equilibria, we start from the decision-maker's optimal action. Given the quadratic payoff function, the optimal, sequentially rational, action $y_p^{n'}$ is equal to the receiver's posterior expectation of θ given p_i and n' (see Appendix for more details) i.e.,

$$y_p^{n'} = E\left[\theta | p, n'\right] = \frac{1}{|p|} \sum_{k \in p} \frac{k+1}{n'+2}, \quad (4)$$

where $|p|$ denotes the cardinality of p.

Let us now describe incentive-compatible partitions $P_{n'}$ of the type space. First, we

[8] Note that sequential rationality implies that the action $y_{p_i}^{n'}$ depends only on p_i and n', and not on the whole partition $P_{n'}$. The latter can therefore be omitted from the argument of $y_{p_i}^{n'}$.

show that each element p -which we henceforth refer to as "pool"- of an incentive compatible partition $P_{n'}$ consists of consecutive types. That is, $p = \{k_1, k_1+1, ..., k_1+r\}$ for some $k_1 \in \{0, 1, ..., n'\}$, $r \in \{0, n'-k_1\}$. Then, exploiting the fact that the lowest and highest types in a pool must have no incentives to deviate to adjacent pools we obtain the following Lemma:

Proposition 1 *For any n', any incentive compatible communication partition $P_{n'} \equiv \{p_1, ..., p_I\}$ is such that each pool $p_i \in P_{n'}$ consists of consecutive types.*

For each $i \in \{1, ..., I-1\}$, the number of types in the pool p_i -denoted by $|p_i|$- and and the number of types in the pool p_{i+1} -denoted by $|p_i|$- are such that the the following condition holds:

$$4b\left(n'+2\right) - 2 \leq |p_{i+1}| - |p_i| \leq 4b\left(n'+2\right) + 2. \tag{5}$$

Proposition 1 has th e following immediate Corollary:

Corollary 1 *A fully separating partition is incentive compatible if and only if $b\,(n'+2) \leq 1/2$.*

If $b \geq 0.25$ the only incentive compatible communication partition is a trivial one consisting of a single element.

To compare and contrast our model with the original model of Crawford and Sobel (1982), note that incentive-compatible communication strategies in their model are characterized by a partition of the type space such that any element i of this partition is an interval (a_i, a_{i+1}), and each marginal type a_i is exactly indifferent between the two sequentially rational actions y_i and y_{i+1} associated with the intervals (a_{i-1}, a_i) and (a_i, a_{i+1}). This implies the so-called "arbitrage condition", $a_{i+1} - a_i = a_i - a_{i-1} + 4b$, which pins down all equilibria.

In our model, the expert is not perfectly informed and the type space is finite. For this reason, there are typically no exactly indifferent marginal types in our incentive compatible partition. However, as $n' \to \infty$, any incentive compatible partition $P_{n'}$ of our model converges to an equilibrium partition of the Crawford and Sobel (1982) in which the expert is perfectly informed. Precisely, $|p_i|/(n'+1) \to a_i - a_{i-1}$ for any i.

Since the decision-maker's optimal action, $y_p^{n'}$, is equal to the conditional expectation of θ given the pool p and the number of trials n', we have for any incentive compatible partition $P_{n'}$:

$$E\left[-\left(y_p^{n'} - \theta - b\right)^2 |P_{n'}\right] = E\left[-\left(y_p - \theta\right)^2 |P_{n'}\right] - b^2. \tag{6}$$

Thus, the expert and the decision-maker share the same payoff function at the ex-ante stage, after the cost of trials $c(n')$ has been incurred but the number of successes had not been realized yet. Hence, for any number of trials n' the equilibria in the ensuing communication game can be Pareto-ranked based on the associated communication partitions $P_{n'}$. Pareto-efficient equilibrium. minimizes the expected residual variance of θ, $E\left[\left(y_p^{n'} - \theta\right)^2 |P_{n'}\right]$.

Note that a fully separating equilibrium of a communication subgame, if such exists, is Pareto efficient. One way to see this is to note that full separation corresponds to the finest partition $P_n = \{\{0\}, \{1\}, ..., \{n\}\}$ each pool of which is a singleton. If a partition $P_{n'}$ contains a non-singleton pool p, then for this pool we have $E[\theta|p] = \sum_{k\in p} E[\theta|k]/|p|$. Since quadratic function is convex, by Jensen's inequality P_n is associated with a lower variance than $P_{n'}$.

More generally, the following Proposition characterizes Pareto efficient incentive compatible partitions. According to this Proposition, such partition is the one with the highest possible number of elements and the smallest heterogeneity in the cardinalities of the elements.

Proposition 2 *For any n and b, the Pareto-efficient incentive compatible partition is $P^* = \{p_1^*, ..., p_K^*\}$ such that $K = \max\{k \in \mathbb{N} | k + \lceil 4b(n+2) - 2\rceil \times \frac{k(k-1)}{2}) \leq n+1\}$. For all $i = 1, ..., K$, the element p_i^* of the equilibrium partition consists of consecutive types and has cardinality $|p_i^*| = 1 + \lceil 4b(n+2) - 2\rceil \times (i-1) + \left\lfloor \frac{r}{K} \right\rfloor + \mathbb{I}\left\{r - \left(\left\lfloor \frac{r}{K} \right\rfloor + 1\right) K + i > 0\right\}$, where $r \equiv n + 1 - \left[K + \lceil 4b(n+2) - 2\rceil \times \frac{K(K-1)}{2}\right]$, and $\mathbb{I}$ denotes the indicator function.*

Finally, let us consider the agent's choice of the number of trials. The key to understanding this decision lies in the fact that there are typically several incentive compatible partitions associated with the same number of trials. So, if the agent had performed such number of trials, then a particular incentive compatible partition characterizing subsequent communication is determined by equilibrium considerations.

This feature of the information acquisition game leads to multiple equilibria. Importantly, as we will show below, the expert's choice of a certain number of trials can be supported in equilibrium only by associating the crudest partition with any other choice of the number of trials i.e., a "babbling" continuation must be played off the equilibrium path.

To describe the expert's equilibrium choice of the number of trials formally, for any $n \in \{0, 1, 2, ..., \infty\}$ fix P_n, an incentive compatible partition satisfying condition (5) and associated with n' trials. Also, let $\mathbf{y}(P_{n'}) \equiv (y_n^0, y_n^1, ..., y_n^n)$ be a sequentially rational action profile induced by the partition $P_{n'}$.

Then the equilibrium number of trials $n*$ chosen by the agent has to maximize her ex-ante expert's utility i.e.,

$$n^* \in \arg \max_{n \in \{0,1,\ldots,\infty\}} \sum_{k=0}^{n} \left(\int_0^1 U^S \left(y_n^k, \theta, b \right) f\left(\theta; k, n\right) d\theta \times \Pr\left(k; n\right) \right) - c(n). \tag{7}$$

Given the multiplicity of equilibria in our game, it is reasonable to focus on a subset of equilibria with some attractive properties. One implication of this requirement is that on equilibrium path, that is in the subgame that follows the acquisition of the equilibrium number of trials n, players play the communication equilibrium which minimizes the expected residual variance. But it is possible that players coordinate on sub-optimal equilibria off the equilibrium path, i.e. for communication subgames that follow the acquisition of n' trials, for any $n' \neq n$.

2.2 Covert Game

Let us now consider the covert game. A pure-strategy Perfect Bayesian Equilibrium of the covert game is described by a triple $(n, P_n, \mathbf{y}(P_n))$, where n is the equilibrium number of trials which maximizes the expert's ex-ante expected utility, P_n is communication partition of the set of expert's types $\{0, 1, ..., n\}$ and $\mathbf{y}(P_n)$ is the decision-maker's equilibrium action profile.

As in the overt game. P_n must be incentive compatible given $\mathbf{y}(P_n)$, and the decision maker's action profile $\mathbf{y}(P_n)$ must be sequentially rational.

The difference between covert and overt game lies in the fact that in the covert game the decision maker does not observe the number of trials performed by the expert. An expert's deviation from the equilibrium number of trials at the information acquisition stage remains unobservable. Therefore, in a putative equilibrium with n trials performed on the equilibrium path, the decision-maker's belief about the expert's information acquisition decision is not sensitive to the actual number of trials performed by the latter. Therefore, even if the expert performs n' trials such that $n' \neq n$, the communication game has to proceed on the basis of the equilibrium partition P_n and any message by the expert in the communication stage can only induce one of the actions in the equilibrium list $\mathbf{y}(P_n)$. After a deviation to n' trials, a type $k \in \{0, 1, ..., n'\}$ of the expert will choose optimally which action from this list to induce. So, a triple $(n, P_n, \mathbf{y}(P_n))$, constitutes an equilibrium only if the number of trials n maximizes the following expression:

$$\sum_{k=0}^{n'} \left[\max_{y_p \in \mathbf{y}(P_n)} \int_0^1 U^S(y_p, \theta, b) f(\theta; k, n') \, d\theta \right] \Pr(k; n') - c(n'). \tag{8}$$

As with overt case, in the analysis of the covert case we will focus on Pareto undominated equilibria. However, we cannot rely on the result that the equilibrium partition P_n of any Pareto undominated equilibrium with n trials is the most informative, that is the one that minimizes $E\left[(y_p - \theta)^2 | P_n\right]$.

It is possible that in the candidate equilibrium with n trials and the most informative communication partition there is a profitable deviation for the expert at the information acquisition stage, while in a candidate equilibrium with n trials and a less informative communication partition there are no profitable deviations.

2.3 Benchmark

As a benchmark for the over and covert games, let us now consider the optimization problem of a decision-maker who chooses how many trials n to perform at cost $c(n) = cn$, and then decides the action y_k^* on the basis of the signal realization $k = 0, ..., n$. In the Appendix we show that that

$$E\left[U^R(\mathbf{y}^*, \theta)\,|n\right] - c(n) = E\left[-(y_k^* - \theta)^2\,|n\right] - cn = -\frac{1}{6(n+2)} - cn. \tag{9}$$

Hence, the optimal number of trials $n^*(c)$ acquired by the decision maker for a given unitary cost c is such that, for generic values of c,

$$\begin{aligned} n^*(c) &= \max\left\{n : -\frac{1}{6(n+2)} - cn - \left(-\frac{1}{6(n-1+2)} - c(n-1)\right) > 0\right\} \\ &= \left\lfloor \sqrt{\frac{2+3c}{12c}} - \frac{3}{2} \right\rfloor. \end{aligned} \tag{10}$$

Combining (9) and (10), we obtain:

$$E\left[(y_{n^*} - \theta)^2\,|n^*\right] = \frac{1}{6\left(\left\lfloor \sqrt{\frac{2+3c}{12c}} - 1.5\right\rfloor + 2\right)}. \tag{11}$$

3 Oversinvestment and Decision Precision

The fundamental insight of standard communication models á la Crawford and Sobel (1982) is that decisions based on information communicated by a biased expert are less efficient than those that would be made if the decision maker had direct access to the information. We revisit this issue in the context of our model, in which information is costly and is endogenously acquired. The information associated to an equilibrium as minus the residual *variance of the equilibrium action* $E\left[-\left(y_p-\theta\right)^2|P_n\right]$.

In this section, we establish an overinvestment result: plausible equilibria of the information acquisition/communication game are characterized by a higher level of investment in information acquisition than in the benchmark direct acquisition case. Hence, the decisions based on the advice of a biased expert can be more precise than the decisions based on information directly acquired by the decision-maker.

This property of the communication equilibria emerges as a result of a tradeoff between two factors. On the one hand, the fact that the cost of information acquisition is borne by the expert and is not internalized by the decision-maker implies that the latter has an incentive to induce the agent to acquire as much information as possible. The decision-maker acts on this incentives through her off-equilibrium beliefs and actions.

On the other hand, the expert prefers to acquire less information than in the benchmark direct-acquisition case. This happens for the following reasons. First, some information is lost in transmission. So, if the outcome of the n-th trial is lost in transmission with a high probability, then the expert does not want to incur the cost of this trial. Second, since the preferences of the expert and the decision-maker are misaligned, the decision-maker "misuses" the expert's information by taking a "biased" decision, from the expert's point of view. This further reduces the incentives of the expert to acquire information.

At first glance, it would appear that the expert's incentives to acquire less information should dominate and lead to underinvestment. Nevertheless, we show that this is not the case. Both in the overt and covert game the exists a class of equilibria in which the aforementioned tradeoff is resolved in favor of the decision-maker and overinvestment does occur, although the exact way in which this happens is different in the overt and covert game.

3.1 Overinvestment in the Overt Game

An important aspect of the overt game is the observability of the number of trials. Hence, any deviation from the equilibrium number of trials by the expert may cause the principal to change her behavior and, in particular, punish the expert for the deviation. The question, however, is which decision-maker's off equilibrium beliefs and reaction strategies are credible i.e., constitute a Bayesian equilibrium in the continuation game.

Below we explore two kinds of credible off equilibrium behavior by the decision-maker both of which lead to overinvestment in equilibrium. The first type of behavior involves the decision-maker stopping to trust the expert entirely. The second type of behavior involves a more measured response and requires behavior to be Pareto-undominated.

Let us consider equilibria of the first type. It is intuitive and natural that an observable deviation by the expert from the equilibrium number of trials would cause the decision-maker to lose any trust in the expert. In this case, the decision-maker will consider any message send by the deviating expert to be non-credible and uninformative, and so a babbling equilibrium will be played off the equilibrium path. This behavior is credible, as babbling is an equilibrium in any communication game. It also constitutes the worst possible punishment for the expert for a deviation at the information acquisition stage since such deviation is followed by a complete loss of information in transmission.

In the sequel we focus on equilibria that are best from the decision-maker's point of view (we refer to them as best-receiver equilibria). To characterize such equilibria we find the number of trials n and incentive compatible communication partition P_n that maximize the decision maker's ex-ante utility $E\left[-\left(y_p-\theta\right)^2|P_n\right]$ subject to the constraint that the expert does not wish to deviate to $n' \neq n$ trials followed by a "babbling" equilibrium in the communication game.

The threat of babbling turns out to be sufficiently strong to induce overinvestment by the expert. In the following Proposition we demonstrate the existence of equilibria with babbling after a deviation in the information acquisition stage, and with overinvestment and full revelation of the information in the communication game on the equilibrium path, so that no information is lost in transmission. Such equilibria exist when the cost of information acquisition and the expert's bias are not too large. The overinvestment in information acquisition compared to a single-person decision problem implies that the decision taken in equilibrium is strictly more precise than when the decision maker acquires information directly. It is important to note that this equilibrium is not the first-best, as the amount of information acquisition is not socially optimal. This results is stated in the following Proposition.

Proposition 3 *Suppose that the information the expert and the decision maker have the same information acquisition cost.*

If $b \leq \left(\sqrt{1+\frac{2}{3c}}+3\right)^{-1}$ *and* $c \leq \frac{5-\sqrt{17}}{48}$*, then there exists an equilibrium of the overt game in which the final decision is more precise than the decision based on direct information acquisition by the decision maker.*

A couple of remarks are in order.

First, the result of Proposition 3 holds a fortiori if the expert is less efficient than the decision maker at acquiring information.

Second, while in the equilibrium that we construct no information is lost in transmission, this feature is not necessary to induce a more informed decision in the overt game than with direct information acquisition by the decision maker. What is necessary is that the information loss in the communication stage following (over)investment is not too severe.

To illustrate this, we provide a numerical characterization of the region of the parameter space for which the receiver-best equilibrium has a higher precision of action than under optimal direct information acquisition.

We performed the analysis for $b \in [0, 0.25]$, $c \in [0, 0.027]$ and $n \leq 100$. This is the relevant parameter range, since for $b \geq 0.25$ the unique equilibrium in the communication game is uninformative, while for $c > 0.02\overline{7}$ the unique solution of the decision maker's optimization problem involves $n = 0$. The exercise involved computing equilibrium residual variance $E\left[(y_n - \theta)^2 |n\right]$ in the best-receiver equilibrium of the communication game with overt information acquisition and comparing to expression (11).

The results presented in Figure 1 confirm the insights of Proposition 3 that overinvestment more than compensates the information loss due to communication and hence results in more precise decisions when the cost on information acquisition and the bias are not too large. Figure (1a) depicts the region where the sufficient conditions in Proposition 3 are satisfied. Figure (1b) reports the results of the numerical analysis.

Figure 1: (1a) In the white region, the sufficient conditions in Proposition 3 are satisfied. (1b) In the white region the decision in the most informative equilibrium of the overt game is strictly more precise than with direct information acquisition. In the grey region it is as precise. In the black region it is strictly less precise.

Our next result shows that overinvestment is sufficiently robust and, in particular, "babbling" off the equilibrium is not necessary to support it. Indeed, it holds weakly in all Pareto undominated equilibria[9]

Proposition 4 *If* $b \leq \left(\sqrt{1 + \frac{2}{3c}} + 1\right)^{-1}$ *then the level of information precision under direct information acquisition regime does not exceed the information precision in any Pareto undominated equilibrium of the overt game.*

[9] An equilibrium is Pareto undominated if there is no other equilibrium in which both players attain greater ex ante expected utilities, with at least one of the players attaining a strictly greater expected utility.

Proposition 4 holds when the bias is not too large relative to the information acquisition cost c. The proof consists of two steps. The first step shows that, under the conditions of the Theorem, the expert's preferred equilibrium involves the same number of trials that is optimal in the direct acquisition regime and which is given by (10), and then fully reveals the outcome in the communication game. Hence, this equilibrium of the overt game has the same precision as under direct information acquisition.

The second step of the proof shows that the precision of the decision in any other Pareto-undominated equilibrium must be greater or equal than under direct acquisition. To see why this is so, note that the expert's preferred equilibrium is by construction Pareto-undominated. In any other Pareto-undominated equilibrium the ex-ante utility of the decision maker must be weakly larger than in this equilibrium, while the utility of the expert must be weakly smaller.

The analysis of the expert's preferred equilibrium of the game is particularly illuminating. The derivation of this equilibrium can be viewed as a two-stage maximization process. First, for an arbitrary fixed number n' of experiments, we select an incentive compatible partition and a profile of equilibrium actions $(y_{n'})_{n' \geq 0}$ that maximize the expert's expected payoff $E\left[-(y_{n'} - \theta)^2 \,|n'\right]$. Second, choose the number of experiments n that maximizes the expert's expected payoff as derived in the first step.

If the condition in Proposition 3 holds, then in the expert's preferred equilibrium the expert runs $n^*(c)$ trials and fully reveals their realization. His expected equilibrium utility, $E\left[U^S(\mathbf{y}^*, \theta, b)\,|n\right] - c(n)$ is equal to $E\left[U^R(\mathbf{y}^*, \theta)\,|n\right] - c(n) - b^2$, the expected payoff of a decision maker who directly conducts $n^*(c)$ less b^2.

If the expert deviates from this equilibrium at the information acquisition stage by running n' trials, then her expected payoff in the most informative continuation equilibrium of the communication game is less than the expected payoff of the decision maker who directly conducts n' trials by an amount greater than b^2. Because $n^*(c)$ solves the decision maker's optimization problem, such deviation to n' is not profitable for the expert.

3.2 Covert Game Results

Let us now turn to the covert information acquisition. This case is substantially different from the overt information because the principal can no longer respond to the expert's deviation in the information acquisition. Still, the next Proposition shows that the overinvestment result extends to the covert game when the bias is not too large relative to the trial cost c.

Proposition 5 *If* $b \leq \left(2\sqrt{1+\frac{2}{3c}}+2\right)^{-1}$*, then the decision precision in any Pareto undominated equilibrium of the covert game is at least as large as under optimal direct information acquisition.*

Notice that it is sufficient to prove that the result holds in the expert's preferred equilibrium, since then it would also hold true in any Pareto-undominated equilibrium.

Under the condition of Proposition 5, in her preferred equilibrium the expert performs $n^*(c)$ trials, where $n^*(c)$ is given by (10) and fully reveals their outcome. To establish that this is, indeed, an equilibrium we need to show two things. First, we show that full revelation of the information after $n^*(c)$ trials is incentive compatible. This follows from the conditions on the parameters.

Second, we need to show that the expert will not benefit by deviating to a different number of experiments n', $n' \neq n^*(c)$. We establish this by comparing the change in expected profits, caused by performing n' trials rather than $n^*(c)$, or the expert in the covert game and for the decision-maker who directly performs information acquisition. We show that the former experiences a larger loss in expected payoff from such a deviation than the latter. But since $n^*(c)$ is optimal under direct information acquisition and since full revelation is optimal for an expert who had acquired $n^*(c)$ trials in the covert game, it follows that the expert would not want to deviate from $n^*(c)$ trials.

Our next result shows that the covert game possesses Pareto undominated equilibria with strictly higher precision of decision is more precise than under optimal direct information acquisition. information. Suppose, in fact, that the decision maker directly acquiring information chooses to run n^* trials. Then $[6(n^*+2)(n^*+3)]^{-1} = \underline{c}(n^*) < c < \bar{c}(n^*) = [6(n^*+1)(n^*+2)]^{-1}$. Then, there exists an equilibrium where the expert at least runs n^*+1 trials and fully reveals the trial realizations to the decision maker, as long as it is the case that $\bar{n}(b) \geq n^*+1$ and that the cost c is smaller than the following threshold:

$$\hat{c}(n^*, b) = \min\left\{[6(n^*+1)(n^*+3)]^{-1}, \frac{2n^*+5}{6(n^*+3)} - b^2 + 2\sum_{j=0}^{n^*} \frac{y_j(n^*, b) - b}{n^*+1}\left(\frac{y_j(n^*, b) - b}{2} - \frac{j+1}{n^*+2}\right)\right\},$$

where

$$y_j(n^*, b) = \frac{\left\lfloor (n^*+3)\left(b + \frac{j+1}{n^*+2}\right) - \frac{1}{2}\right\rfloor + 1}{n^*+3}.$$

We present our findings as follows.

Proposition 6 *If $b \leq \left(2\sqrt{1+\frac{2}{3c}}+6\right)^{-1}$ and $\underline{c}(n^*) < c < \hat{c}(n^*, b)$, for any arbitrary n^*, then there is at least one equilibrium of the covert game in which the decision is more precise than the decision based on direct information acquisition by the decision maker.*

The intuition for the result is as follows. Fix an arbitrary number of trials n^*, and suppose that if the decision maker directly acquires information, she optimally runs exactly n^* trials, so that $\underline{c}(n^*) < c < \bar{c}(n^*)$. The first condition in the Proposition implies that $\bar{n}(b) \geq n^*+1$. Because $\widehat{n}(b) \geq \bar{n}(b)$, this guarantees that full revelation of the realization of n^*+1 trials is incentive compatible, as long as the choice of n^*+1 trials is is part of the equilibrium in the first place. The second condition guarantees that this is indeed the case. In particular, $c > \underline{c}(n^*)$ guarantees that the expert does not deviate from n^*+1 trials to any larger trial number, with the same logic used in the Proof of Proposition 5. The condition that c is smaller than $[6(n^*+1)(n^*+3)]^{-1}$ guarantees that even a decision maker directly acquiring information would not deviate from running n^*+1 trials to run n^*-1 trials. This is true also for any number of trials smaller than n^*-1, due to the concavity of the function $E\left[-(y_k^*-\theta)^2 \,|n\right] = -[6(n+2)]^{-1}$. Finally, the condition that c is smaller than the second argument in the minimum defining $\hat{c}(n^*, b)$ implies that the expert does not deviate from running n^*+1 trials to run exactly n^* trials.

Similarly to Proposition 3, the conditions of Proposition 6 are sufficient, but not necessary. To show this, we have numerically identified the region of the parameter space where the precision of the decision maker's action in her preferred equilibrium of the covert game is strictly higher than under optimal direct information acquisition. The results are reported in Figure 2. Figure (2a) represents the region where the sufficient conditions of Proposition 6 hold. Figure (2b) reports the results of the numerical analysis.

Figure 2: (2a) In the white region, the sufficient conditions in Proposition 6 hold. (2b) In the white region the decision in the most informative equilibrium of the covert game is strictly more precise than under optimal direct information acquisition. In the grey region the precision is the same in the most informative equilibrium of the overt game and under optimal direct information acquisition. In the black region, the decision under optimal direct information is more precise than in the most informative equilibrium of the overt game.

4 Organization Design

Our results have strong implications for the growing literature started by Dessein (2002), that uses communication games to explore issues of information aggregation and authority allocation in organizations. We introduce endogenous information acquisition in this literature. Specifically, we consider an organization where a principal would like to make an informed decision, and information acquisition is costly. The principal (decision-maker) may operate with an agent (expert), who has biased preferences. The principal may allocate the tasks of information acquisition and decision (authority) to the agent, or retain them for herself. Hence, there are three task allocations available to the principal:

- Centralization: The principal acquires information, paying the cost of information acquisition, and makes an informed decision
- Communication: The principal delegates the costly information acquisition task to the agent, but retains authority on the decision task
- Delegation: The principal delegates both the information acquisition and the decision task to the agent.

The trade-off between communication and delegation has been previously explored in the literature, but without information acquisition. The key result identified by Dessein (2002) and Ottaviani (2000) is that delegation outperforms communication: the principal is better off delegating the decision-making authority to the agent, than requesting his advice while retaining the decision-making authority. We now show that this result may be reversed when accounting for the fact that the level of expertise of the agent is often endogenous. Specifically, Corollary 2 below shows that communication outperforms delegation under the conditions of Propositions 4 and 5. Further, Corollary 2 shows that communication outperforms also centralization; and this is despite the assumption that the expert's cost of information acquisition is not larger than the principal's direct information acquisition cost.[10]

In order to introduce our results, we first report the principal's expected utility under centralization:

$$E\left[-\left(y_k^*-\theta\right)^2 |n^*(c)\right]-cn^*(c). \tag{12}$$

and under delegation:

$$E\left[-\left(y_k^*-\theta\right)^2 |n^*(c)\right]-b^2 \tag{13}$$

[10] To our knowledge, communication and centralization have not been previously explicitly compared. In fact, the previous literature typically assumed the agent is perfectly informed, whereas the principal's cost of information acquisition is infinite.

In both cases, the party acquiring information will run $n^*(c)$ trials as calculated in equation (10).

Turning to communication, in a given equilibrium of the overt or covert game with n trials and communication partition P_n, the principal's expected utility is:

$$E\left[-\left(y_p-\theta\right)^2|P_n\right]. \tag{14}$$

Comparing expressions (12), (13) and (14) it is immediate to see that delegation and centralization are dominated by any equilibrium of the overt or covert game for which $E\left[-\left(y_p-\theta\right)^2|P_n\right]$ is at least as large as $E\left[-\left(y_k^*-\theta\right)^2|n^*\left(c\right)\right]$. Equivalently, delegation and centralization are dominated by any equilibrium of the overt or covert game such that the final decision is at least as precise as with delegation or centralization. Holding precision constant, in fact, communication is cheaper than centralization as the cost of information acquisition is off-loaded to the agent; and communication dominates delegation as it avoids making biased decisions.

Propositions 4 and 5 identify sufficient conditions under which the final decision precision $E\left[-\left(y_p-\theta\right)^2|P_n\right]$ in all Pareto undominated equilibria of the overt and covert game, respectively, is at least as large as $E\left[-\left(y_k^*-\theta\right)^2|n^*\left(c\right)\right]$. It is an immediate corollary that, under the same conditions, all Pareto undominated equilibria of the overt and covert game outperform delegation and centralization.

Corollary 2 ***(a)*** *If* $b\leq\left(\sqrt{1+\frac{2}{3c}}+1\right)^{-1}$, *then the principal strictly prefers any Pareto undominated equilibrium of the overt game to centralization and delegation.*
(b) *If* $b\leq\left(2\sqrt{1+\frac{2}{3c}}+2\right)^{-1}$, *then the principal strictly prefers any Pareto undominated equilibrium of the covert game to centralization and delegation.*

While for expositional simplicity, we have supposed that the information acquisition cost is the same for the expert and the decision maker, the comparison between delegation and the Pareto-undominated equilibria of the overt and covert game holds regardless of any information acquisition cost difference across these two subjects. The reason is that, both when communicating information to the decision maker and when delegated the final decision, it is the expert who bears the cost of information acquisition, and hence the decision maker's information cost is irrelevant in the comparison. Relaxing the assumption that the information acquisition cost is the same for the expert and for the decision maker, suppose that the cost for the expert is c^E. In the case of delegation, the expert will optimally perform $n^*\left(c^E\right)$ trials. The conditions in corollary 2, when expressed with respect to c^E rather than c, guarantee that in any Pareto-undominated equilibrium of the overt and covert game, respectively, the information transmitted is at least equal to full revelation of the outcome of $n^*\left(c^E\right)$ trials. Hence, these equilibria are preferred to the delegation outcome.

Observe that as c converges to zero, the maximum bias for which the conditions in Corollary 2 is satisfied converges to zero. Intuitively, the amount of trials performed in both delegation and centralization, $n^*(c)$, converges to infinity, therefore the highest bias for which the outcome of $n^*(c)$ trials can be credibly revealed in equilibrium converges to zero.

Corollary 2 provides sufficient conditions for communication to outperform delegation and centralization, vindicating the frequent use of this task allocation in organizations. We complete our comparison of centralization, communication and delegation by means of numerical analysis. Before presenting our results, we briefly discuss the pros and cons of the three possible task allocations.

First, we note that the principal faces a clear trade-off when choosing between delegation and centralization. By delegating, the principal off-loads the information acquisition cost to the agent, but simultaneously loses authority over the final decision. As the principal's loss from allowing the agent to take a decision increases in the agent's bias b, the principal will prefer delegation over centralization only when the bias b is small relative to the cost of information acquisition c.

Second, also the trade-off between centralization and communication is simple. Again, when the principal chooses communication over centralization, she off-loads the information acquisition cost to the agent. But, due to imperfect transmission of information, the principal bears the cost of making a less informed decision. Because the informational loss increases with the bias b, the principal prefers communication over centralization when the bias is small relative to the cost of information acquisition c.

Third, we note that the comparison between delegation and communication is more complex. In both cases, the principal's payoff decreases in the bias b, either because of the agent's biased action (under delegation), or because more information is lost in transmission with higher values of the bias (under communication). Also, in both cases the principal does not pay the cost of the information acquired.

We now present our numerical results comparing communication with delegation and centralization. Figures (3a) and (3b) respectively consider the decision maker's and the expert's preferred equilibrium of the overt game. It is remarkable that communication is the best task allocation for a large set of the cost-bias parameters. In particular, communication is optimal unless the bias is significantly large. These findings are markedly different from those in the existing literature which does not consider information acquisition and typically finds that delegation is the best task allocation.

Figure 3: In the white region, the best task allocation is Communication. In the grey region, it is Delegation. In the black region, it is Centralization. Panel (3a) considers the equilibrium of the overt game preferred by the decision maker. Panel (3b) considers the one preferred by the expert.

Despite the prevalence of communication, we also observe that, for small costs of information acquisition, any of the three task allocations can be optimal, depending on the bias. Specifically, the principal prefers communication if the bias is small, delegation if the bias is intermediate, and centralization if the bias is large. But the region where delegation is optimal disappears as the information acquisition cost increases. Perhaps not surprisingly, centralization dominates when the bias is large, regardless of the cost of information acquisition. Not unexpectedly, communication is more likely to be preferred by the principal under her preferred equilibrium than under the expert's preferred equilibrium. But interestingly, this gain is mostly at the expenses of delegation, rather than centralization. Figure 4 illustrates the region of the parameter space where the sufficient condition in Corollary 2(a) is satisfied. The comparison with figure 3 shows that clearly it is only sufficient but not necessary.

Figure 4: In the white region, the sufficient condition in Corollary 2a is satisfied.

The case of covert information acquisition is considered in Figure 5b, which compares the decision maker's preferred equilibrium to delegation and centralization. (The figure for the expert's preferred equilibrium is very similar, and hence omitted.) The qualitative features of the comparison are similar to the case of overt information acquisition. There are two main differences. First, the area where delegation is the best task allocation significantly expands; and, second, it includes also the case of large information acquisition costs. Figure 5a illustrates the region of the parameter space where the sufficient condition in Corollary 2(b) is satisfied. The comparison with figure 5b highlights that it is only sufficient but not necessary.

Figure 5: (5a) In the white region, the sufficient condition in Corollary 2b is satisfied. (5b) In the white region, the best task allocation is Communication. In the grey region, it is Delegation. In the black region, it is Centralization.

5 Conclusion

This paper has constructed a simple, yet rich statistical model to explore the implications of costly endogenous information acquisition in the strategic communication paradigm that was earlier developed by Crawford and Sobel (1982). One fundamental insight, if not the most important one, of previous work on strategic communication is that because information is lost in communication, the decisions based on a biased expert's advice need to be imprecise. By including costly endogenous information acquisition in the modellization, we have revisited this fundamental insight and surprisingly established that decisions based on a biased expert's advice may be more precise than optimal choices based on direct information acquisition. Strikingly, this is the case, we have shown, even if the expert is not more efficient than the decision maker at acquiring information.

As argued in the paper, this result bears important implications for organization design. A growing literature started by Ottaviani (2000) and Dessein (2002) has in fact used the strategic communication framework to give an account of authority and of the optimal authority allocation in the presence of incomplete information. Contrary to earlier results, we have found that communication by an expert to the decision maker may often outperform delegation of the decision making authority to the expert. Crucially, this result holds regardless of whether the expert is more or less efficient than the decision maker at acquiring information.

As our paper is exploratory in nature, the construction admits several directions that could lead to fruitful extension.

One natural question is for example how do overt and covert information acquisition compare from the receiver's stand point. Would a decision maker prefer knowing the amount of information acquired by an expert, although of course she cannot inspect its content? As proved by Austen-Smith (1994), indeed, the question is not transparent. His set-up, in fact, yields the result that, when the receiver is uncertain of whether the sender is informed or not, informative communication is possible for a wider range of parameter values than is possible when the receiver is sure that the sender is informed.

A second possible complement of the current analysis consists in studying the value of information in our version of the canonical example by Crawford and Sobel (1982). In fact, the work by Ivanov (2010) has in fact proved the value of information may be negative, in a statistical model where the sender may observe to which element of a partition the state of the world belongs. One natural question is whether this result would also extend to our canonical Beta-Binomial statistical model.

Appendix

Calculations leading to Expression (4). The decision maker chooses y_P so as to maximize

$$-\int_0^1 (y_p - \theta)^2 f(\theta | k \in p)\, d\theta.$$

Taking the first-order condition, we obtain $y_p = \int_0^1 \theta f(\theta | k \in p)\, d\theta = E[\theta | p]$. Simplifying:

$$E[\theta|p] = E[E[\theta|k]\,|k \in p] = \sum_{k \in p} E[\theta|k] \frac{f(k)}{\sum_{k \in p} f(k)} = \frac{1}{|p|} \sum_{k \in p} \frac{k+1}{n+2} = \frac{k+1}{n+2}$$

because $E\left[\theta|k\right] = \frac{k+1}{n+2}$, and

$$\begin{aligned} f(k) &= \int_0^1 f(k;n,\theta)\,d\theta = \frac{n!}{k!\,(n-k)!}\int_0^1 \theta^k (1-\theta)^{n-k}\,d\theta \\ &= \frac{n!}{k!\,(n-k)!}\frac{k!\,(n-k)!}{(n+1)!} = \frac{1}{n+1}. \end{aligned}$$

Proof of Proposition 1 First, we show that the incentive compatibility constraint (3) can be rewritten as

$$(y_q - y_p)\left[(y_p + y_q) - 2E\left[\theta/k, n\right] + 2b\right] \geq 0 \textit{ for all } q \in P_n.$$

The calculations are as follows:

$$\int_0^1 U^S\left(y_p\left(P_{n'}\right), \theta, b\right) f\left(\theta; k, n'\right) d\theta \geq \int_0^1 U^S(y_q\left(P_{n'}\right), \theta, b) f\left(\theta; k, n'\right) d\theta$$

$$\begin{aligned} -\int_0^1 \left[(y_p - \theta - b)^2 - (y_q - \theta - b)^2\right] f(\theta;k,n)\,d\theta &\geq 0 \\ -\int_0^1 \left[y_p^2 + (\theta+b)^2 - 2y_p(\theta+b) - y_q^2 - (\theta+b)^2 + 2y_q(\theta+b)\right] f(\theta;k,n)\,d\theta &\geq 0 \\ -\int_0^1 \left[y_p^2 - y_q^2 - 2(y_p - y_q)(\theta+b)\right] f(\theta;k,n)\,d\theta &\geq 0 \\ -(y_p - y_q)\left[(y_p + y_q) - 2E\left[\theta/k, n\right] - 2b\right] &\geq 0 \end{aligned}$$

Next, we prove that in any pure-strategy equilibrium of the communication subgame, each element of the equilibrium partition is connected. Suppose by contradiction that there exists an equilibrium where at least one element of the partition is not connected. Then, there exists at least a triple of types (k, k', k'') such that $k < k'' < k'$, k and k' belong to the same element of the partition, which we denote by p_a, and k'' belongs to a different element, which we denote by p_b. Let y_a and y_b be the equilibrium actions associated to p_a

and p_b respectively. By incentive compatibility, the following inequalities must hold:

$$\begin{aligned}
(y_b - y_a)\left(y_a + y_b - \frac{2(k+1)}{n+2} - 2b\right) &\geq 0 \\
(y_b - y_a)\left(y_a + y_b - \frac{2(k'+1)}{n+2} - 2b\right) &\geq 0 \\
(y_a - y_b)\left(y_a + y_b - \frac{2(k''+1)}{n+2} - 2b\right) &\geq 0
\end{aligned}$$

Because the first two expressions are positive, then $y_a + y_b - \dfrac{2(k+1)}{n+2} - 2b$ and $y_a + y_b - \dfrac{2(k'+1)}{n+2} - 2b$ have the same sign. But then, also $y_a + y_b - \dfrac{2(k''+1)}{n+2} - 2b$ has the same sign, because $k < k'' < k$. And hence, the last expression is negative: A contradiction.

Next, we prove that incentive compatibility implies expression (5). Let k be the expert's type. Denote by y the equilibrium action associated to k, and by $\widetilde{y}$ any other equilibrium action. The incentive compatibility constraint is:

$$(\widetilde{y} - y)\left(\widetilde{y} + y - \frac{2(k+1)}{n+2} - 2b\right) \geq 0. \tag{15}$$

First, we consider the possibility that a type k deviates by inducing an equilibrium action $\widetilde{y}$ larger than y. Hence, incentive compatibility is satisfied if and only if

$$\widetilde{y} + y - \frac{2(k+1)}{n+2} - 2b \geq 0. \tag{16}$$

Because the expression is increasing in $\widetilde{y}$ and decreasing in k, it immediately follows that the tightest incentive compatibility constraints concern the highest type k in any element p_i of the equilibrium partition, entertaining the possibility of deviating and inducing the equilibrium action $\widetilde{y}$ associated to p_{i+1}, the element of the partition immediately to the right of p.

Hence, we now consider such constraints. Letting z be the cardinality of p_i and j be the cardinality of p_{i+1}, the explicit expression for y and $\tilde{y}$ are:

$$y = \frac{1}{z}\left[\frac{k+1}{n+2} + \frac{k-1+1}{n+2} + ... + \frac{k-(z-1)+1}{n+2}\right] = \frac{2k-z+3}{2(n+2)}$$

$$\widetilde{y} = \frac{1}{j}\left[\frac{k+1+1}{n+2} + \frac{k+2+1}{n+2} + ... + \frac{k+j+1}{n+2}\right] = \frac{2k+j+3}{2(n+2)}$$

Hence, condition (16) simplifies as:

$$\frac{2k+j+3}{2(n+2)} + \frac{2k-z+3}{2(n+2)} - \frac{2(k+1)}{n+2} - 2b \geq 0,$$

or,

$$j \geq z + 4b(n+2) - 2. \tag{17}$$

Proceeding in the same fashion, we prove that when $\widetilde{y} < y$, the tightest incentive compatibility constraints concern the lowest type k in any element p_i of the equilibrium partition, entertaining the possibility of deviating and inducing the equilibrium action $\widetilde{y}$ associated to p_{i-1}, the element of the partition immediately to the left of p_i. Again, letting j be the cardinality of p_i, and z be the cardinality of p_{i-1}, we obtain

$$y = \frac{2k+j+1}{2(n+2)} = \frac{1}{j}\left[\frac{k+1}{n+2} + \frac{k+1+1}{n+2} + ... + \frac{k+j-1+1}{n+2}\right] = \frac{2k+j+1}{2(n+2)}$$

$$\widetilde{y} = \frac{1}{z}\left[\frac{k-1+1}{n+2} + \frac{k-2+1}{n+2} + ... + \frac{k-z+1}{n+2}\right] = \frac{2k-z+1}{2(n+2)}$$

Hence, condition (16) simplifies as:

$$\frac{2k-z+1}{2(n+2)} + \frac{2k+j+1}{2(n+2)} - \frac{2(k+1)}{n+2} - 2b \leq 0$$

which implies

$$j \leq z + 4b(n+2) + 2. \tag{18}$$

Putting together the inequalities (17) and (18), we obtain condition (5). This characterization implies that a fully separating equilibrium exists if and only if $4b(n+2) - 2 \leq 0$, i.e. $b \leq \frac{1}{2(n+2)}$. Further, for $b \geq 1/4$, it follows that $4b(n+2) \geq n+2$, and hence condition (17) cannot be satisfied by any partition, other than the trivial partition $P = \{\{0, 1, ..., n+1\}\}$: The unique equilibrium is the babbling equilibrium. Finally, as $n \to \infty$, any equilibrium partition P converges to an equilibrium partition of the model by Crawford and Sobel (1982) where the expert is perfectly informed, in the sense that, for any i, $|p_i|/(n+1) \to a_i - a_{i-1}$. In fact, condition (5) implies that

$$\frac{4b(n+2)-2}{n+1} \leq \frac{|p_{i+1}| - |p_i|}{n+1} \leq \frac{4b(n+2)+2}{n+1},$$

and, taking limits for $n \to \infty$,

$$4b \leq a_i - a_{i-1} + a_{i+1} - a_i \leq 4b,$$

which is exactly the incentive compatibility condition of Crawford and Sobel (1982).

Calculations leading to Expression (6). A mean-variance decomposition yields:

$$\begin{aligned}
E\left[-(y_p - \theta - b)^2\right] &= -\int_0^1 (y_p - \theta - b)^2 \, d\theta \\
&= -\int_0^1 \left[(y_p - \theta)^2 + b^2 - 2b(y_p - \theta)\right] d\theta \\
&= E\left[-(y_p - \theta)^2\right] - b^2 + 2bE[y_p - \theta] \\
&= E\left[-(y_p - \theta)^2\right] - b^2,
\end{aligned}$$

because $E_p[y_p] = E_p[E_\theta[\theta|p]] = E_\theta[\theta]$, by the law of iterated expectations.

Proposition 7 *For any n and b, the Pareto-efficient incentive compatible partition is $P^* = \{p_1^*, ..., p_K^*\}$ such that $K = \max\{k \in \mathbb{N} | k + \lceil 4b(n+2) - 2 \rceil \times \frac{k(k-1)}{2}) \leq n+1\}$. For all $i = 1, ..., K$, the element p_i^* of the equilibrium partition consists of consecutive types and has cardinality $|p_i^*| = 1 + \lceil 4b(n+2) - 2 \rceil \times (i-1) + \lfloor \frac{r}{K} \rfloor + \mathbb{I}\left\{r - \left(\lfloor \frac{r}{K} \rfloor + 1\right) K + i > 0\right\}$, where $r \equiv n + 1 - \left[K + \lceil 4b(n+2) - 2 \rceil \times \frac{K(K-1)}{2}\right]$, and $\mathbb{I}$ denotes the indicator function.*

Proof. The equilibrium partition P identified in the Proposition is the one with the largest cardinality K and with the smallest difference in the cardinality of subsequent elements, subject to the incentive compatibility condition (5).

First, we show that minus the expected residual variance $E_\theta\left[-(y_p-\theta)^2\right]$ can be rewritten as $-\frac{1}{3}+E\left[E(\theta|p)^2\right]$.

By the law of iterated expectations,

$$\begin{aligned}
E_\theta\left[-(y_p-\theta)^2\right] &= -E_\theta\left[(E[\theta|p]-\theta)^2\right] \\
&= -E_p\left[E_\theta\left[(E[\theta|p]-\theta)^2|p\right]\right] \\
&= -E_p\left[Var[\theta|p]\right].
\end{aligned}$$

Because $Var[\theta]=E_p[Var[\theta|p]]+Var_p[E(\theta|p)]$, we thus obtain:

$$\begin{aligned}
E_\theta\left[-(y_p-\theta)^2\right] &= -Var[\theta]+Var_p[E(\theta|p)] \\
&= -Var[\theta]+E\left[E(\theta|p)^2\right]-E[E(\theta|p)]^2 \\
&= -Var[\theta]+E\left[E(\theta|p)^2\right]-E[\theta]^2 \\
&= -\frac{1}{12}+E\left[E(\theta|p)^2\right]-\left(\frac{1}{2}\right)^2 \\
&= -\frac{1}{3}+E\left[E(\theta|p)^2\right].
\end{aligned}$$

Next, we show that among the equilibrium partitions with the largest number of elements, the equilibrium with the smallest difference between the cardinalities of any two subsequent elements is the one that maximizes $E\left[-(y_p-\theta)^2\right]=-\frac{1}{3}+E\left[E[\theta|p]^2\right]$.

Consider an equilibrium partition P with I elements $\{k_i,...,k_{i+1}-1\}_{i=1}^{I}$. Denoting the associated expected residual variance by $E\left[-(y_p-\theta)^2;P\right]$ we obtain:

$$E\left[-(y_p-\theta)^2;P\right]=-\frac{1}{3}+E\left[E(\theta|p)^2\right]=-\frac{1}{3}+\sum_{i=1}^{I}\frac{k_{i+1}-k_i}{n+1}\left(\frac{k_{i+1}+k_i+1}{2(n+2)}\right)^2$$

where $k_{i+1}\equiv n+1$. Next, consider a different equilibrium partition $P'=\left\{k_i',...,k_{i+1}'-1\right\}_{i=1}^{I}$, such that there is a unique $i\in I$ with $k_i'=k_i+1$, and $k_j'=k_j$ for all $j\neq i$. Denoting the associated expected residual variance by $E\left[-\left(y_p-\theta\right)^2;P\right]$ we obtain:

$$\begin{aligned}
& E\left[-\left(y_p-\theta\right)^2;P'\right]-E\left[-\left(y_p-\theta\right)^2;P\right] \\
= & \frac{k_{i+1}-(k_i+1)}{n+1}\left(\frac{k_{i+1}+(k_i+1)+1}{2(n+2)}\right)^2+\frac{k_i+1-k_{i-1}}{n+1}\left(\frac{k_i+1+k_{i-1}+1}{2(n+2)}\right)^2 \\
& -\frac{k_{i+1}-k_i}{n+1}\left(\frac{k_{i+1}+k_i+1}{2(n+2)}\right)^2-\frac{k_i-k_{i-1}}{n+1}\left(\frac{k_i+k_{i-1}+1}{2(n+2)}\right)^2 \\
= & \frac{(k_{i+1}-k_{i-1})\left[(k_{i+1}-k_i)-(k_i+1-k_{i-1})\right]}{4(n+2)^2(n+1)}>0.
\end{aligned}$$

where the last inequality holds because P' is an equilibrium partition, hence it must be that $k'_{i+1}-k'_i=k_{i+1}-k_i-1>k_i+1-k_{i-1}=k'_i-k'_{i-1}$.

To conclude the proof, we need to show that, among the equilibrium partitions with the smallest difference in the cardinality of subsequent elements, the one which maximizes welfare is the equilibrium partition with the largest number of elements. Specifically, denoting the best equilibrium partition among those with m elements by $P(m)$, we prove that $P(j)$ dominates $P(j-1)$. Repeating the argument proves the statement.

To prove that $P(j)$ dominates $P(j-1)$ we describe an algorithm to construct a sequence of partitions with the following features:

(a) the first term of the sequence is $P(j)$

(b) the last term of the sequence is $P(j-1)$

(c) each term of the sequence, except for the last one, is a partition with j elements

(d) each term of the sequence is preferred by both players to the next one

The algorithm is the following. Given the n-th term of the sequence (the n-th partition), the $(n+1)$-th is constructed as follows:

(i) If the sub-partition that includes the largest $(j-2)$ elements of n-th partition is identical to the sub-partition that includes the largest $(j-2)$ elements of $P(j-1)$, then let the $n+1$-th partition be $P(j-1)$; i.e., let the first element of the $n+1$-th partition be equal to the union of the first *two* elements of the n-th partition. This step concludes the algorithm, and satisfies condition (d), because, for any k_1, k_2 with $k_1 > 1$, and $k_2 > k_1 + 1$,

$$\begin{aligned} & \frac{k_2 - k_1}{n+1}\left(\frac{k_2 + k_1 + 1}{2(n+2)}\right)^2 + \frac{k_1 - 1}{n+1}\left(\frac{k_1 + 1 + 1}{2(n+2)}\right)^2 - \frac{k_2 - 1}{n+1}\left(\frac{k_2 + 1 + 1}{2(n+2)}\right)^2 \\ = \; & \frac{1}{4}\frac{(k_2 - k_1)(k_2 - 1)(k_1 - 1)}{(n+2)(n+1)} > 0. \end{aligned}$$

(ii) If the sub-partition that includes the last $(j-2)$ elements of n-th partition is *not* identical to the sub-partition that includes the largest $(j-2)$ elements of $P(j-1)$, then the $(n+1)$-th partition is obtained from the n-th by moving the highest type included in the k-th element p_k^n into the $(k+1)$-th element p_{k+1}^n, where $k < j$ is the highest index that satisfies the following conditions:

(iia) for $l < j-2$, if the sub-partition that includes the last l elements of n-th partition is identical to the sub-partition that includes the last l elements of $P(j-1)$, then $k < j - l$.[11]

(iib) the cardinality of p_{k+1}^n is strictly smaller than the cardinality of the k-th element of $P(j-1)$.

(iic) if the union of p_1^n and p_2^n is equal to the first element of $P(j-1)$, then $k > 2$.

Because the number of types is finite, the algorithm has an end.

The type-(ii) step can be repeated exactly until the condition for the type-(i) step is satisfied because, by construction, the cardinality of the l-th element of $P(j-1)$ is weakly larger than the cardinality of the $(l+1)$-th element of $P(j)$, hence the union of the first two elements of $P(j)$ has cardinality weakly larger than the cardinality of the first element of $P(j-1)$.

Calculations leading to Expression (9). First we note that for any number of trials

[11] For example, if $j = 10$, if the last three elements of the $n-th$ partition in the sequence are identical to the last three elements of the target partition, then they shouldn't be changed anymore, hence $k < 7$, so that "at most" a type is taken from the 6-th element and moved into the 7-th.

n and realization k, the optimal action y_k equals $E[\theta|k] = (k+1)/(n+2)$. Hence,

$$
\begin{aligned}
E\left[-\left(y_k^* - \theta\right)^2 |n\right] - cn &= -\sum_{k=0}^{n} \Pr(k;n) \int_0^1 \left(E[\theta|k] - \theta\right)^2 f(\theta;k,n) - cn \\
&= -\sum_{k=0}^{n} \frac{1}{n+1} \int_0^1 \left(\frac{k+1}{n+2} - \theta\right)^2 \frac{(n+1)!}{k!\,(n-k)!} \theta^k (1-\theta)^{n-k} d\theta - cn \\
&= -\sum_{k=0}^{n} \frac{1}{n+1} \int_0^1 \left[\left(\frac{k+1}{n+2}\right)^2 + \theta^2 - 2\theta\left(\frac{k+1}{n+2}\right)\right] \frac{(n+1)!}{k!\,(n-k)!} \theta^k (1-\theta)^{n-k} d\theta - cn \\
&= -\sum_{k=0}^{n} \frac{1}{n+1} \left[\int_0^1 \theta^2 \frac{(n+1)!}{k!\,(n-k)!} \theta^k (1-\theta)^{n-k} d\theta - \left(\frac{k+1}{n+2}\right)^2\right] - cn \\
&= -\sum_{k=0}^{n} \frac{1}{n+1} \left[\frac{(k+2)(k+1)}{(n+3)(n+2)} - \left(\frac{k+1}{n+2}\right)^2\right] - cn \\
&= -\frac{1}{6(n+2)} - cn.
\end{aligned}
$$

Proof of Proposition 3. The proof proceeds as follows. First, we find the maximal number of trials $\widetilde{n}(c)$ such that, under a given cost of trial c, the utility that the expert obtains by conducting $\widetilde{n}(c)$ trials and fully revealing their realization to the decision maker is higher than the utility from running any other number of trials and playing the babbling equilibrium. Formally, $\widetilde{n}(c)$ is the highest integer that satisfies

$$
-\frac{1}{6(n+2)} - b^2 - cn \geq -\frac{1}{12} - b^2.
$$

Further, as shown before, $\widehat{n}(b) = \left\lfloor \frac{1}{2b} - 2 \right\rfloor$ is the maximal number of trials for which full revelation in the communication game is incentive compatible. Hence, is it an equilibrium for the expert to run $n^*(c)$ trial to fully reveal the information to the decision maker whenever the following condition holds:

$$
n^*(c) \leq \max\{\widehat{n}(b), \widetilde{n}(c)\}. \tag{19}
$$

The condition $n^*(c) + 1 \leq \widetilde{n}(c)$ is satisfied if $\sqrt{\frac{2+3c}{12c}} - \frac{3}{2} + 1 \leq \frac{1}{12c} - 2$, i.e., $c \leq \frac{5-\sqrt{17}}{48}$, whereas the condition $n^*(c) + 1 \leq \widehat{n}(b)$ is satisfied if $\sqrt{\frac{2+3c}{12c}} - \frac{3}{2} + 1 \leq \frac{1}{2b} - 2$, or $b \leq \left(\sqrt{1 + \frac{2}{3c}} + 3\right)^{-1}$.

If $\widehat{n}(b) \geq n^*(c)+1$ and $\widetilde{n}(c) \geq n^*(c)+1$, then there exists an equilibrium of the overt information acquisition game in which the expert runs $n^*(c)+1$ trials and fully reveals their realizations, while the babbling equilibrium is played in any subgame in which $n' \neq n$ trials are run. Hence, the decision maker's utility $E\left[-(y_p-\theta-b)^2 |P_n\right]$ in the decision maker's preferred equilibrium must be at least $-1/[6(n^*+1+2)]$ which is strictly strictly bigger than the decision maker's utility $-1/[6(n^*+2)]$ if she had direct access to information. The proof is then concluded by noting that the final decision precision coincides with the decision maker's utility.

Proof of Proposition 4. The condition $b \leq \left(\sqrt{1+\frac{2}{3c}}+1\right)^{-1}$ implies that $\left\lfloor \frac{1}{2b}-2 \right\rfloor \geq \left\lfloor \sqrt{\frac{2+3c}{12c}}-1.5 \right\rfloor$. Consider the equilibrium in which the Pareto-efficient incentive compatible partition is played in the communication stage on and off the equilibrium path and the expert, correctly anticipating this, selects the number of trials that maximizes his expected payoff. We prove that if $\left\lfloor \frac{1}{2b}-2 \right\rfloor \geq \left\lfloor \sqrt{\frac{2+3c}{12c}}-1.5 \right\rfloor$, then in equilibrium the number of trials is exactly $n^*(c)$ and full revelation occurs. First, notice that $\left\lfloor \frac{1}{2b}-2 \right\rfloor \geq \left\lfloor \sqrt{\frac{2+3c}{12c}}-1.5 \right\rfloor$ implies that $\widehat{n}(b) \geq n^*(c)$, which in turn implies that fully revealing the outcome of $n^*(c)$ trials is incentive compatible. Next, consider deviations at the information acquisition stage. If the expert purchases less than $n^*(c)$ trials, full revelation occurs. Since $E\left[-(y_p-\theta-b)^2\right]-cn = E\left[-(y_p-\theta)^2\right]-b^2-cn$, the difference between equilibrium payoff and deviation payoff is then equal to the payoff difference that the decision maker would receive in the single agent decision problem if he purchased less than $n^*(c)$ trials rather than $n^*(c)$. This payoff difference is negative, by definition of $n^*(c)$. If instead the expert deviates to purchasing more than $n^*(c)$ trials, full separation might or might not be incentive compatible in the subsequent communication stage. Hence, the deviation gain is weakly smaller than the payoff difference that the decision maker would receive in the single agent decision problem and again the result is implied by the definition of $n^*(c)$.

Proof of Proposition 5

The condition implies that $\bar{n}(b) \geq n^*(c)$. Because $\widehat{n}(b) \geq \bar{n}(b)$, it follows that

$\widehat{n}(b) \geq n^*(c)$. Hence, full revelation is incentive compatible at the communication stage if the expert runs $n^*(c)$ trials. Further, because $n^*(c)$ is the optimal centralized number of trials, it follows that, if there is an equilibrium where $n^*(c)$ trials are run, and the realizations are fully revealed, then this equilibrium is the expert preferred equilibrium. Such equilibrium yields a higher ex-ante decision maker utility than both centralization and delegation. Because this is the expert preferred equilibrium, a fortiori, all Pareto-undominated equilibria yield a higher ex-ante decision maker utility than both centralization and delegation.

Hence, we only need to show that running $n^*(c)$ and fully revealing the realizations is an equilibrium. In order to do so, we show that for any deviation to $n \neq n^*(c)$, the expert collects a payoff $\hat{W}(n)$ —net of information acquisition costs, which is smaller than the full revelation payoff $W(n) = 1/[6(n+2)]$. Then, the definition of n^* immediately implies that $W(n^*) \geq W(n) > \hat{W}(n)$.

First, note that, for any deviation n, the expected utility of a type $j = 0, ..., n$ for inducing the outcome y_j is:

$$\begin{aligned} W(i, n; y) &= -\int_0^1 (y_j - \theta - b)^2 f(\theta|j, n) \, d\theta \\ &= -\int_0^1 \left(y_j^2 + (\theta + b)^2 - 2(\theta + b) y_j\right) f(\theta|j, n) \, d\theta \\ &= -\left[y_j^2 - 2y_j (E[\theta|j, n] + b)\right] - \int_0^1 (\theta + b)^2 f(\theta|j, n) \, d\theta, \end{aligned}$$

which is quadratic in y_j, with bliss point $E[\theta|j, n] = \frac{j+1}{n+2} + b$. The outcomes that type j may induce after the deviation are the outcomes $y_k = \frac{k+1}{n^*+2}$, for $k = 0, ..., n^* + 1$ which are compatible with the belief that n^* trials were run. Hence type j will choose the outcome $y_k(j) = \frac{k+1}{n^*+2}$ which minimizes

$$\left| \frac{k+1}{n^*+2} - \left(\frac{j+1}{n+2} + b\right) \right|.$$

In the full revelation outcome instead, he would induce action $\frac{j+1}{n+2}$. Because the expected utility is quadratic, and hence symmetric around the bliss point, the type j is better off by inducing action $y_k(j)$ instead of action $\frac{j+1}{n+2}$ if and only if there exists $k = 0, ..., n^*$ such that:

$$\left| \frac{k+1}{n^*+2} - \left(\frac{j+1}{n+2} + b \right) \right| < b.$$

So, in the case that $b \leq 1/\left[4(n^*(c)+2)\right]$, type j is better off with $y_k(j)$ relative to $\frac{j+1}{n+2}$ if and only if there is $k = 0, ..., n^*$ such that:

$$\frac{k+1}{n^*+2} - 2b \leq \frac{j+1}{n+2} \leq \frac{k+1}{n^*+2}.$$

Taking the worse-case scenario, let's set henceforth $b = 1/\left[4(n^*(c)+2)\right]$, so that type j is better off with $y_k(j)$ relative to $\frac{j+1}{n+2}$ if and only if there is $k = 0, ..., n^*$ such that:

$$\frac{k+1}{n^*+2} - \frac{1}{2(n^*+2)} \leq \frac{j+1}{n+2} \leq \frac{k+1}{n^*+2}.$$

But note that these inequalities implies that type $n-j$ is worse off by inducing $y_k = \frac{k+1}{n^*+2}$, for any $k = 0, ..., n^*$, relative to $\frac{n-j+1}{n+2}$, because they imply

$$1 - \left(\frac{k+1}{n^*+2} - \frac{1}{2(n^*+2)} \right) \geq 1 - \frac{j+1}{n+2} \geq 1 - \frac{k+1}{n^*+2},$$

and hence

$$\frac{n^*-k+1}{n^*+2} \leq \frac{n-j+1}{n+2} \leq \frac{n^*-k+1}{n^*+2} + \frac{1}{2(n^*+2)}.$$

Under full revelation, type j's utility is:

$$\begin{aligned}
W(j,n) &= -\int_0^1 \left(E\left[\theta|j,n\right] - \theta - b\right)^2 f\left(\theta|j,n\right) \\
&= -\int_0^1 \left[\left(E\left[\theta|j,n\right] - \theta\right)^2 - b\left(E\left[\theta|j,n\right] - \theta\right) + b^2\right] f\left(\theta|j,n\right) \\
&= -\int_0^1 \left(E\left[\theta|j,n\right] - \theta\right)^2 f\left(\theta|j,n\right) - b^2 \\
&= -\left[\left(E\left[\theta|j,n\right]\right)^2 - 2(E\left[\theta|j,n\right])\frac{j+1}{n+2} + \frac{(j+2)(j+1)}{(n+3)(n+2)}\right] - b^2 \\
&= -\left[\left(\frac{j+1}{n+2}\right)^2 - 2\left(\frac{j+1}{n+2}\right)\frac{j+1}{n+2} + \frac{(j+2)(j+1)}{(n+3)(n+2)}\right] - b^2 \\
&= -\left[\frac{(j+2)(j+1)}{(n+3)(n+2)} - \left(\frac{j+1}{n+2}\right)^2\right] - b^2
\end{aligned}$$

If he induces action y_j, his welfare is:

$$
\begin{aligned}
\hat{W}(j,n;y_j) &= -\int_0^1 (y_j-\theta-b)^2 f(\theta|j,n)\,d\theta \\
&= -\int_0^1 \left[(y_j-b)^2+\theta^2-2\theta(y_j-b)\right]\frac{(n+1)!}{j!\,(n-j)!}\theta^j(1-\theta)^{n-j}\,d\theta \\
&= -\left[(y_j-b)^2+\int_0^1 \frac{(n+1)!}{j!\,(n-j)!}\theta^{j+2}(1-\theta)^{n-j}\,d\theta-2(y_j-b)\int_0^1 \frac{(n+1)!}{j!\,(n-j)!}\theta^{j+1}(1-\theta)^{n-j}\,d\theta\right] \\
&= -\left[(y_j-b)^2+\frac{(n+1)!}{j!\,(n-j)!}\frac{(2+j)!\,(n-j)!}{(n+3)!}-2(y_j-b)\frac{(n+1)!}{j!\,(n-j)!}\frac{(1+j)!\,(n-j)!}{(n+2)!}\right] \\
&= -\left[(y_j-b)^2-2(y_j-b)\frac{j+1}{n+2}+\frac{(j+2)(j+1)}{(n+3)(n+2)}\right].
\end{aligned}
$$

Because the types j and $n-j$ are equally likely, we only need to show that, for arbitrary j,

$$
D(j,n)\equiv W(j,n)+W(n-j,n)-\hat{W}\left(j,n;\frac{k+1}{n^*+2}\right)-\hat{W}\left(j,n;\frac{n^*-k+1}{n^*+2}\right)>0.
$$

We distinguish two cases, depending on whether $\frac{n-j+1}{n+2}+b$ is closer to $\frac{n^*-k+1}{n^*(c)+2}$ or to $\frac{n^*-k+2}{n^*(c)+2}$. In the first case, type $n-j$ chooses $y_{n^*-k}=\frac{n^*-k+1}{n^*(c)+2}$, and we calculate:

$$
D(j,n)=\frac{2\left(2k-2j+n+kn-n^*-jn^*\right)^2}{(n^*+2)^2(n+2)^2}.
$$

In the second case, type $n-j$ chooses $y_{n^*-k+1}=\frac{(n^*-k+1)+1}{n^*(c)+2}$. In this case, the expression of $D(j,n)$ is very cumbersome. But it turns out that:

$$
\frac{\partial}{\partial b}D(j,n)=-2<0,
$$

hence we can bound $D(j,n)$ by setting b equal to its upper bound $1/\left[4(n^*(c)+2)\right]$. We obtain:

$$
D(j,n)=\frac{\left(4k-4j+n+2kn-2n^*-2jn^*-2\right)^2}{2(n+2)^2(n^*+2)^2}>0.
$$

This concludes the proof.

Proof of Proposition 6. Fix any number of trials n^* run in centralization. The first condition implies that $b\le 1/\left[4(n^*+3)\right]$. Hence, it implies that $b\le 1/\left[2\left((n^*+1)+2\right)\right]$ and full revelation is incentive compatible when running n^*+1 trials.

Because $\frac{1}{6(n^*+2)(n^*+3)} = \underline{c}(n^*) < c < \overline{c}(n^*) = \frac{1}{6(n^*+1)(n^*+2)}$, and $b \leq 1/[4(n^*+3)]$ the proof of Proposition 5 —interchanging n^* with n^*+1 —- implies that deviating from n^*+1 trials to run $n > n^*+1$ trials is not profitable.

Further, $\frac{W(n^*+1)-W(n^*-1)}{2} = \frac{1}{6(n^*+1)(n^*+3)}$, and by concavity of W, $\frac{W(n^*+1)-W(n^*-k)}{k+1} > \frac{W(n^*+1)-W(n^*-1)}{2}$. Hence, requiring that $c < \frac{W(n^*+1)-W(n^*-1)}{2}$ deters all deviations from n^*+1 to n^*-k, $k = 1, ..., n^*$.

To rule out the possibility to deviate to n^* trials from n^*+1 trials, we want that $c < W(n^*+1) - \hat{W}(n^*)$. Of course, $W(n^*+1) = -\frac{1}{6((n^*+1)+2)} - b^2$. Turning to calculate $\hat{W}(n^*)$, we proceed as follows. For any $j = 0, ..., n^*$, it is easy to see that type j will send the message $\frac{k+1}{n+3}$ associated with the largest integer k such that

$$\frac{k}{n+3} + \frac{1}{2(n+3)} \leq \frac{j+1}{n+2} + b,$$

i.e.

$$k = \left\lfloor (n+3)\left(b + \frac{j+1}{n+2}\right) - \frac{1}{2} \right\rfloor .$$

This fact is immediate for $j > 1$, and for $j = 1$ note that

$$\begin{aligned} \left\lfloor (n+3)\left(b + \frac{1+1}{n+2}\right) - \frac{1}{2} \right\rfloor &\geq (n+3)\left(b - \frac{1}{2(n+3)} + \frac{1+1}{n+2}\right) - 1 \\ &= \frac{1}{2}\frac{\left(12b + n + 10bn + 2bn^2 + 6\right)}{n+2} > 0. \end{aligned}$$

Hence,

$$y_j = \frac{k+1}{n+3} = \frac{\left\lfloor (n+3)\left(b + \frac{1+1}{n+2}\right) - \frac{1}{2} \right\rfloor + 1}{n+3},$$

as requested by the sufficient condition.

Further, averaging across $j = 0, ..., n^*$, we obtain:

$$\hat{W}(n^*) = \frac{1}{n^*+1}\sum_{j=0}^{n^*} \hat{W}(j, n^*; y_j) = -\sum_{j=0}^{n^*} \frac{1}{n^*+1}(y_j - b)^2 + 2\sum_{j=0}^{n^*} \frac{j+1}{(n^*+1)(n^*+2)}(y_j - b) - \frac{1}{3}$$

$$= -\frac{1}{3} - \sum_{j=0}^{n^*} \frac{y_j - b}{n^* + 1} \left[y_j - b - 2\frac{j+1}{n^*+2} \right].$$

Hence,

$$\begin{aligned} W(n^*+1) - \hat{W}(n^*) &= -\frac{1}{6(n^*+3)} - b^2 + \frac{1}{3} + \sum_{j=0}^{n^*} \frac{y_j - b}{n^*+1} \left[y_j - b - 2\frac{j+1}{n^*+2} \right] \\ &= \frac{2n^*+5}{6(n^*+3)} - b^2 + \sum_{j=0}^{n^*} \frac{y_j - b}{n^*+1} \left[y_j - b - 2\frac{j+1}{n^*+2} \right]. \end{aligned}$$

This concludes the proof of Part (b).

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