Optimal Limited Authority for Principal*

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Abstract

This paper studies a principal-agent problem where the only commitment for the uninformed principal is to restrict the set of decisions she makes following a report by the informed agent. Compared to no commitment, the principal improves the quality of communication from the agent. An ex ante optimal equilibrium for the principal corresponds to a finite partition of the state space, and each retained decision is sub-optimal for the principal, biased toward the agent's preference. Generally an optimal equilibrium does not maximize the number of decisions the principal can credibly retain.

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A principal needs to elicit information from an agent in order to make decisions, but their inherent conflict of interest makes truthful communication difficult. When the principal cannot credibly give up her authority to make the final decision, the seminal paper by **?** (hereafter CS) shows that the principal's decisions suffer from the agent's incentive to distort his information in favor of his bias. When the principal can credibly delegate her decision-making authority, the agent uses his information efficiently but his decision is biased. In reality, however, the principal may be able to give up certain aspects of her decision-making authority, but not all, due to institutional or technological reasons.

This paper presents a model of *limited authority*: ex ante, the principal can credibly rule out certain decisions as infeasible; but for the remaining decisions, she cannot commit to any particular decision rule such as adopting the agent's recommendation without change ex post. Real life examples of this type of limited authority abound. For instance, in a typical university tenure system, the university has only two decisions given a department's recommendation on a tenure case: promote the assistant professor or fire him. In the US House of Representatives, the Rules Committee can establish a set of special rules to limit the amendment process when a bill is introduced. In particular, the committee can adopt a structured rule that specifies the amendments to be considered and the time for debate. Finally, in a factory setting, the owner's choice of one type of an assembly line may make the production of certain products impossible, but she can still choose the final product within the capacity of the assembly line after hearing the manager's recommendation.

Our model of limited authority presents an ex ante tradeoff for the principal in deciding how much ex post authority to retain. On one hand, by retaining more decisions the principal can make better use of the agent's reported information. On the other hand, more retained decisions creates a bigger credibility problem: the information content of the report is lower because the agent anticipates the principal's incentive to exploit it. Using the same general framework as CS, we show that under the optimal limited authority, finitely many decisions are retained. The agent partitions the state space and makes a recommendation from the set of retained decisions for each partition element, and the principal always follows the recommendation and never randomizes. Moreover, the principal is strictly better off under the optimal limited authority than in any CS equilibrium. Intuitively, by ruling out some decisions, the principal reduces her incentives to distort decisions recommended by the agent, which allows the biased agent to make more precise recommendations than in the cheap talk game.

To better understand the tradeoff for the principal under limited authority, in particular the properties of the retained decisions, we turn to the example with uniformly distributed state and convex loss function, which is a slight generalization of the uniform-quadratic example commonly used in the communication literature. We fully characterize the principal's optimal limited authority in this case. To begin with, in the optimal limited authority, all the retained decisions are above the principal's ex post optimal decisions—in the direction of the agent's bias—given that she learns the partitional elements. Second, retained decisions are more evenly distributed under the optimal limited authority than the induced decisions in a CS equilibrium. Intuitively, in a CS equilibrium each induced decision is ex post optimal because the principal has no commitment power, and thus the agent induces decisions that grow in distance between each other in the direction of his bias. In contrast, under the optimal limited authority the principal restricts the set of decisions she can choose from, which reduces the agent's incentive to distort his recommendations due to their conflict of interest. This increases the possible number of decisions that can be credibly retained, and decreases the distance between them. Third, we show that, contrary to the predictions of both the cheap talk and delegation models, the principal does not necessarily maximize the number of decisions that can be credibly retained. Intuitively, allowing more decisions ex ante can make the principal worse off by reducing the communication quality because the credibility problem distorts the choices of the decisions. In particular, some decisions may be used with almost zero probability, but their presence still forces the principal to move other retained decisions away from the ex ante optimal ones.

This paper is directly related to the literature on delegation initiated by Holmstrom (1984). He shows that the optimal outcome under full commitment of the principal is achieved by restricting the set of decisions and delegating decision-making authority to the agent. Our paper analyzes the environment in which the principal cannot delegate authority to the agent, but can restrict the set of decisions. Closely related are Dessein (2002) and Marino (2007), who study the optimal delegation problem where the principal can veto the agent's decision and replace it with some default decision, and Mylovanov (2008), who instead assumes that the principal can choose the default decision ex ante. Less related to our work, Milgrom and Roberts (1988) and Szalay (2005) analyze how restricting the set of decisions affects influence activities and information acquisition respectively. Sections 1.3 and 4.1 discuss the related literature in greater detail.

The rest of this paper is organized as follows. Section 1 sets up the limited authority model by adding to the cheap talk game a first move by the principal choosing the set of retained decisions. Section 1.3 provides detailed motivations for our limited authority assumption. Section 2 derives general properties about the optimal limited authority by first characterizing it as a solution to a constrained maximization problem. This characterization provides an equivalent interpretation of our limited authority model as a delegation game in which the principal chooses the delegation set but cannot commit to not changing the agent's decision within the set. Section 3 provides a full characterization of the example with uniformly distributed state, convex loss functions for both the principal and the agent, and a state-independent bias for the agent. Section 4 compares the principal's welfare under optimal limited authority to various organizational forms studied in the existing literature. We find that the principal's ex ante expected payoffs are similar under the optimal limited authority and optimal delegation, and both are significantly higher than that under the most informative cheap talk equilibrium. Section 5 briefly discusses extensions of the model. All proofs can be found in the appendix.

1 The model

1.1 Setup

This paper analyzes the CS model with one modification. In CS the set of decisions is a real line; we instead assume that, ex ante, the principal can credibly restrict the set of decisions from which she makes decisions ex post. The model specified in this section is called a model of *limited authority* throughout the paper. It has two natural interpretations suitable for different environments, namely, the cheap talk game and the delegation game. We first present the model using the cheap talk game, and then comment briefly on the delegation game.

Formally, there is an informed agent A (he) and an uninformed principal P (she). Payoffs of A and P, denoted by $u^A(y,\theta)$ and $u^P(y,\theta)$, are both functions of the decision y and the state of the world θ. The timing of the cheap talk game is as follows:

1. P chooses a decision set Y, a compact subset of the real line.

2. A observes Y and privately learns θ, drawn from the interval $(0,1]$ according to a positive probability density function $f(\theta)$.

3. A sends a cheap talk message m from the interval $[0,1]$.

4. P receives m and makes a decision $y \in Y$.

All aspects of the game are common knowledge. We make the CS assumptions on functions $u^A(y,\theta)$ and $u^P(y,\theta)$, which are maintained throughout the paper:

Assumption 1 *There exists a function u and a scalar $b > 0$ such that $u^A(y,\theta) = u(y,\theta,b)$ and $u^P(y,\theta) = u(y,\theta,0)$. Moreover,*

1. *u is twice continuously differentiable in all variables.*

2. *$u_{yy}(y,\theta,\beta) < 0$ for all $y \in \mathbb{R}$, $\theta \in [0,1]$, and $\beta \in [0,b]$.*

3. $u_y(y^*(\theta), \theta, \beta) = 0$ *for some function* $y^*(\theta)$*, and for all* $\theta \in [0,1]$ *and* $\beta \in [0,b]$.

4. $u_{y\theta}(y, \theta, \beta) > 0$ *for all* $y \in \mathbb{R}$*,* $\theta \in [0,1]$*, and* $\beta \in [0,b]$.

5. $u_{y\beta}(y, \theta, \beta) > 0$ *for all* $y \in \mathbb{R}$*,* $\theta \in [0,1]$*, and* $\beta \in [0,b]$.

Parts 2 and 3 imply that both A and P's preferences are single-peaked. Parts 1-3 together imply that $y^i(\theta) \equiv \arg\max_{y \in \mathbb{R}} u^i(y, \theta)$ is well defined and continuous in θ for all $\theta \in [0,1]$ and $i = A, P$. Part 4 is a sorting condition, which ensures that both $y^A(\theta)$ and $y^P(\theta)$ are increasing in θ for all $\theta \in [0,1]$. Finally, part 5 guarantees that $y^P(\theta) < y^A(\theta)$ for all $\theta \in [0,1]$.

In the delegation game interpretation of the model, first P chooses a delegation set Y, and then A chooses some y from Y, which P can approve or change to some other $\tilde{y}$ in Y. The only formal difference between this interpretation and the above cheap talk interpretation is that in the delegation game A makes a choice y from Y, instead of sending a cheap talk message. The reduction in A's strategy space, from the set of messages $[0,1]$ in the cheap talk game to the set Y in the delegation game, turns out to be immaterial to our characterization of the optimal limited authority. This claim will be formally established as part of the proof of Proposition 1 in the next section.[1] As a result, the delegation game and the cheap talk game are two interpretations of the same limited authority model.

1.2 Solution concept and definitions

The solution concept we use is Perfect Bayesian Equilibria (hereafter PBE). A PBE is P's choice of Y, A's report strategy $\sigma : 2^{\mathbb{R}} \times (0,1] \to \Delta[0,1]$, P's decision strategy $\rho : 2^{\mathbb{R}} \times [0,1] \to \Delta\widetilde{Y}$, and P's belief $p : 2^{\mathbb{R}} \times [0,1] \to \Delta(0,1]$, such that strategies are optimal given players' beliefs, and beliefs are derived from Bayes' rule whenever possible.[2] Formally, the equilibrium

[1] The claim is obviously true if we restrict the attention to equilibria where P uses a pure strategy on the equilibrium path. The proof of Proposition 1 establishes the claim allowing for the possibility of random decisions by P.

[2] A technical issue arises with the existence of the conditional distribution function, $p(\theta|Y, m)$, which can be bypassed using the notion of distributional strategies (see Milgrom and Weber (1985)) and Theorem 33.3 of Billingsley (1995).

conditions are, for all $\widetilde{Y} \subset \mathbb{R}$, $m \in [0,1]$, for m^* in the support of $\sigma(\cdot|\widetilde{Y},\theta)$, and for y^* in the support of $\rho\left(\cdot|\widetilde{Y},m\right)$:

$$Y \in \arg\max_{\widetilde{Y}\subset\mathbb{R}} \int_{\widetilde{Y}\times[0,1]\times[0,1]} u^P(y,\theta)\,\rho\left(y|\widetilde{Y},\tilde{m}\right)\sigma(\tilde{m}|\widetilde{Y},\theta)f(\theta)\,dy\,d\theta\,d\tilde{m},$$

$$m^* \in \arg\max_{\tilde{m}\in[0,1]} \int_{\widetilde{Y}} u^A(y,\theta)\,\rho\left(y|\widetilde{Y},\tilde{m}\right)dy,$$

$$y^* \in \arg\max_{y\in\widetilde{Y}} \int_0^1 u^P(y,\theta)\,p\left(\theta|\widetilde{Y},m\right)d\theta,$$

$$p\left(\theta|\widetilde{Y},m\right) = \frac{\sigma(m|\widetilde{Y},\theta)f(\theta)}{\int_0^1 \sigma\left(m|\widetilde{Y},\theta\right)f(\theta)d\theta}.$$

A PBE of the delegation game is defined analogously, with the only difference being that A's mixed strategy is a mapping from the set of states $[0,1]$ to the set of probability distributions over the set Y chosen by P, instead of to the set of distributions over the message space $[0,1]$.[3]

Regardless of the interpretation of the limited authority model, we adopt the following definitions. The decision y is *induced by* θ (or equivalently θ *induces* y) in a PBE if y is chosen by P with positive probability when the state is θ in this PBE, or

$$\int_{\{m:\rho(y|Y,m)>0\}} \sigma(m|Y,\theta)dm > 0.$$

The decision y is *induced in a PBE* if y is induced in at least one state. A PBE is *informative* if there are at least two induced decisions, and *uninformative* otherwise. The *uninformative decision* y^P is defined as $y^P \equiv \arg\max_{y\in\mathbb{R}} \int_0^1 u^P(y,\theta)f(\theta)d\theta$. Finally, a PBE is a *partition equilibrium* $(\{\theta_i\}_{i=0}^n, \{y_i\}_{i=1}^n)$ if $\{\theta_i\}_{i=0}^n$ is a partition of $(0,1]$, and $\{y_i\}_{i=1}^n \subset Y$ is a set of induced decisions where

$$\begin{aligned} &0 = \theta_0 < \theta_1 < \ldots < \theta_n = 1, \\ &y_1 < \ldots < y_n, \end{aligned} \tag{1}$$

such that any $\theta \in (\theta_{i-1},\theta_i]$ induces decision y_i for all $i = 1,\ldots,n$. Condition (1) is called the *partition* condition. Clearly, a partition equilibrium can be supported as a PBE of the

[3]In the cheap talk game, it is without loss of generality to restrict the set of messages to $[0,1]$, as in CS. In the delegation game, we have implicitly assumed that A cannot choose lotteries. This assumption is non-consequential to our analysis, as established later in Proposition 1.

delegation game where the delegation set Y chosen by P on the equilibrium path has the following properties. First, it is *minimal*, in that each decision $y \in Y$ is induced; and second, it is *veto-free*, in that P chooses the same y chosen by A.

Two remarks are in order. First, all CS equilibria can be supported as a PBE in this framework. Indeed, consider any CS equilibrium. The following strategies and beliefs constitute a PBE. If P chooses $\widetilde{Y} = \mathbb{R}$, then both A and P's strategies and beliefs are given by the CS equilibrium. If P chooses $\widetilde{Y} \neq \mathbb{R}$, then A sends uninformative messages; P believes so and makes the best decision out of $\widetilde{Y}$ based on her prior belief. This observation implies that a PBE always exists.

Second, similar to the CS model, for each PBE there exists an outcome equivalent PBE in which all messages in $[0, 1]$ are sent on the equilibrium path. Therefore, we cannot refine the set of PBE using standard equilibrium refinements such as those of Cho and Kreps (1987) which restrict out-of-equilibrium beliefs.[4] This paper mostly focuses on PBE that maximizes P's expected payoff, which we refer to as the *optimal* PBE. Such a refinement is natural if P does not only choose Y at the first stage, but also announces the outcome she plans to implement with the chosen decision set Y.

1.3 Discussion of the model

The imperfect commitment assumption that ex ante the principal can credibly restrict the set of decisions available to her ex post, but she cannot commit to any specific decision rule deserves further discussion. Below we provide three motivations. Our first motivation is formal, and is based on the incomplete contracting approach initiated by Grossman and Hart (1986) and Hart and Moore (1988). Our limited authority model describes a contracting environment in which the authority to make final (irreversible) decisions resides with the

[4] Some refinements for cheap talk games have been proposed in the literature but they do not generally select a unique equilibrium. A notable exception is due to Chen et al. (2008), which selects the most informative equilibrium under some regularity conditions.

principal, but *only* these final decisions are verifiable.[5] In this environment, all the principal can do is to ex ante exclude some decisions from what she may choose ex post. In the same spirit of the observable-but-not-verifiable assumption in the incomplete contracting literature, our restrictions on contractibility are certainly severe. In particular, in our limited authority model neither communication by the agent to the principal, such as reports on his information or recommendations to the principal, nor decision rights is verifiable. Allowing reports or recommendations by the agent to be verifiable would of course turn our model into an exercise in mechanism design without transfers; likewise, allowing the decision rights to be contractible would change our model into an optimal delegation problem. Both these problems have been extensively studied in the literature; see for example the more recent works by Kovac and Mylovanov (2009) and Alonso and Matouschek (2008). The innovation in our paper is instead to study a more primitive contracting environment than the full-commitment framework initiated by Holmstrom (1984), while at the same time demonstrate what "simple" contracts can achieve relative to the no-contracting, cheap talk framework of CS. Furthermore, from an applied point of view, there are contracting situations for which our limited authority model is appropriate. For example, it may be prohibitively costly for the agent to present physical evidence of his communication with the principal in the court. Similarly, to delegate formal authority to the agent, the principal may need to sell relevant productive assets to the agent, which may be impractical because the same assets are used by the principal for other purposes.

The second motivation for the imperfect commitment assumption is technological, and presumes a contracting environment where verifiability of any reports or decisions is com-

[5] Hart and Moore (2004) impose a similar contractibility assumption. They assume that ex ante the parties can restrict the set of outcomes over which they bargain ex post. However, the parties cannot commit to any specific mechanism according to which the outcome from this restricted set is chosen ex post. Also, Hermalin et al. (2007) propose a similar approach to model situations in which a contract has ambiguous provisions. That is, each contingency in a contract is associated with a set of outcomes from which the final outcome is chosen. In this context, the imperfect commitment assumption requires that the same set of possible outcomes should be associated with each contingency.

pletely absent. For example, making certain decisions may require a specialized equipment which is prohibitively expensive to procure ex post. In this case, not procuring the equipment ex ante commits the principal to not making the decisions that use the equipment. Similarly, in many organizations, managers make decisions using software packages, such as SAP ERP. This software is typically adjusted to the specific needs of each organization so that certain decisions are made unavailable, such as trading of some products at certain prices in a financial company. In addition, it may be impractical to give control over this software to those who have relevant information for decision making in an organization. This technology thus allows the principal to restrict her decision set without making it possible for her to commit to decisions based on the agent's reports. It is worth noting that between the two aspects in the standard full-commitment model, the principal's ability to make certain decisions infeasible and her ability to commit to not changing the decision made by the agent, the first one may be accomplished through some technology while the second one is often technologically harder or even impossible.

The third and final motivation for the imperfect commitment assumption is institutional, and is based on realistic assumptions about how decision rights are allocated in organizations. In many organizations, managers typically make critical decisions based on information supplied by their subordinates, but are held accountable for the final decisions. Organization rules often prohibit managers from delegating their decisions to their subordinates, but allow managers to credibly commit to not taking certain decisions. This is the kind of organizational setting that makes our limited authority model applicable. The same situation may also arise in a multi-level hierarchy in which contracts can be written only among certain parties. For instance, our limited authority model applies in a multi-divisional organization with the headquarters, a division manager, and the manager's subordinate, where enforceable contracts can be written only between the headquarters and the manager.

2 General analysis

In this section we provide a general analysis of the optimal PBE in our limited authority model. We start by characterizing the optimal PBE as a solution to a constrained maximization problem in Proposition 1. This is a useful result that we exploit further in the uniform-convex loss setup in Section 3 to completely characterize the optimal PBE. Here we use it to establish the main result of the section, Proposition 2, that the optimal PBE strictly improves the principal's welfare relative to the most informative equilibrium of CS. Under further assumptions on the preference functions u^A and u^P, Proposition 3 provides a tight upper bound on the agent's bias parameter b for the principal to benefit from limited authority relative to the CS model.

Our first result establishes the existence of optimal PBE under limited authority and characterizes its basic properties. In particular, it shows that the optimal PBE is a partition equilibrium with a finite number of induced decisions.

Proposition 1 *An optimal PBE exists and is a partition equilibrium with a finite number of elements. Moreover, among all partition equilibria* $(\{\theta_i\}_{i=0}^n, \{y_i\}_{i=1}^n)$ *with a finite* n, *it maximizes* $\sum_{i=1}^n \int_{\theta_{i-1}}^{\theta_i} u^P(y_i,\theta)f(\theta)d\theta$ *subject to, for each* $i = 2, \ldots, n$,

$$u^A(y_i, \theta_{i-1}) = u^A(y_{i-1}, \theta_{i-1}), \tag{2}$$

$$\int_{\theta_{i-1}}^{\theta_i} u^P(y_i, \theta) f(\theta) d\theta \geq \int_{\theta_{i-1}}^{\theta_i} u^P(y_{i-1}, \theta) f(\theta) d\theta. \tag{3}$$

We start the proof of Proposition 1 by characterizing all PBE. First, we exploit the assumptions on the payoff functions u^A and u^P to show that any PBE in the cheap talk game can be supported as one in the delegation game, meaning that we can restrict attention to PBE's in which P's equilibrium decision set Y is minimal and veto-free. Second, we prove that any PBE is a partition equilibrium with a finite number of elements. The proof of the finiteness is quite standard, except that the distance between three rather than two adjacent induced lotteries is bounded away from zero. Third, in any PBE, the adjacent incentive conditions are sufficient for all incentive conditions of P because P's payoff function is strictly concave. Further, in any PBE in which P does not randomize over the set of decisions upon receiving a message, P never has incentives to deviate to higher decisions, because A has an upward bias and even he weakly prefers not to deviate to higher decisions.

To complete the proof of Proposition 1, we characterize optimal PBE. First, we show that in any optimal PBE, P never randomizes. The proof is involved because of the finiteness of decision set Y.[6] We show that for any PBE with non-degenerate lotteries, P can increase her payoff by replacing each non-degenerate lottery with the higher decision in the lottery. The observation above that in any non-degenerate lottery $y < y'$, A strictly prefers the higher decision y' implies that P can improve the quality of communication by A by increasing the probability weight on y'. By putting all the weight on y' instead, she makes the (now degenerate) lottery more attractive so that A recommends y' for a set of higher states than in the old PBE. Since P prefers a lower decision for the same set of states, she is strictly better off. Finally, the problem of finding the optimal PBE reduces to a constrained maximization problem where the set of feasible choices is all partition equilibria with a finite number of elements that satisfy A's indifference conditions (2) and P's adjacent downward incentive conditions (3). The solution to this problem exists by the maximum theorem.

Clearly, P cannot do worse than in any CS equilibrium, as she can replicate any CS equilibrium outcome by restricting the set of decisions to those induced in the CS equilibrium. Our second result shows that P can in fact do strictly better.

Proposition 2 *P's expected payoff is strictly higher in the optimal PBE than in any informative CS equilibrium.*

In a CS equilibrium, each induced decision is ex post optimal for P in that it maximizes P's expected payoff over all possible decisions $y \in \mathbb{R}$ given P's belief about the state after receiving a message from A. Therefore, P's incentive conditions (3) are not binding in an informative CS equilibrium, and she can marginally increase any induced decision y_i without

[6] In CS P's payoff function is strictly concave in a decision and the set of decisions is convex. Thus, upon receiving a message, P has a unique optimal decision.

violating (3). As P increases y_i, by the envelope theorem, her expected payoff is unaffected by the introduction of ex post inefficiencies, but it increases marginally due to an increase in the partition thresholds θ_{i-1} and θ_i. For example, as θ_{i-1} increases to θ'_{i-1}, an upwardly biased A induces y_{i-1} instead of a higher decision y_i for states $\theta \in (\theta_{i-1}, \theta'_{i-1}]$, which increases P's expected payoff.

We can strengthen Proposition 2 by showing that P can still strictly improve her expected payoff by restricting the set of decisions even when there does not exist an informative CS equilibrium. More formally, suppose that an informative CS equilibrium exists whenever b is less than b^*, with two decisions y_1 and y_2. Then there exists ε such that for all b less than $b^* + \varepsilon$, ε sufficiently small, P's expected payoff is strictly higher in the optimal PBE than in any CS equilibrium. By Proposition 2, for b less than b^*, P can increase either y_1 or y_2 to achieve the desired PBE. By continuity of u, these new induced decisions still constitute a PBE in which P's expected payoff is strictly higher than that in the uninformative CS equilibrium at $b = b^* + \varepsilon$.

Under additional assumptions on the function u, we can further strengthen Proposition 2. We show that P's expected payoff is strictly higher in the optimal PBE than a babbling equilibrium if and only if delegation is valuable under full commitment. Adopting a definition from Alonso and Matouschek (2008), we say that *delegation is valuable* if P can improve on the uninformed decision y^P by committing to letting A choose between at least two decisions.

Proposition 3 *Let $u^P(y, \theta) = -(y - y^P(\theta))^2$ and $u^A(., \theta)$ be symmetric around $y^A(\theta)$. P's expected payoff is strictly higher in the optimal PBE than in any CS equilibrium if and only if delegation is valuable.*

The "only if" part is immediate, because by Proposition 1 the optimal PBE is a partition equilibrium and any partition equilibrium can be implemented through delegation as the incentive conditions (3) for P are absent in delegation under full commitment. The proof of the "if" part is based on a result due to Alonso and Matouschek (2008). They show that if delegation is valuable, then P can improve on implementing the uninformed decision y^P by letting A choose between exactly two decisions. We show that these two decisions satisfy P's incentive condition (3), and thus can be induced in a PBE.

3 The uniform-convex loss example

This section focuses on a slight generalization of the leading example of CS. In particular, we assume:

Assumption 2 *$f(\theta) = 1$ for $\theta \in (0, 1]$, $u(y, \theta, \beta) = -l(|y - (\theta + \beta)|)$, where l is increasing and convex with $l(0) = l'(0) = 0$.*

Assumption 2 includes the leading uniform-quadratic example as a special case with $l(z) = z^2$. Clearly, Assumption 2 satisfies Assumption 1, so Propositions 1 and 2 hold. We focus on this example because it is widely used in applications as a building block.[7]

This example is particularly well-behaved to apply the constrained maximization program given in Proposition 1. In leading to the main result of this section, a complete characterization of the optimal PBE in Proposition 5, we provide a few results that have independent interests and a solution approach that yields further insights about the optimal PBE. We first establish that in the optimal PBE each induced decision is higher than what is ex post optimal conditional on P learning the corresponding partition element. As a result, P's incentive conditions (3) take a particularly simple form. In Proposition 4, we show that binding these conditions yields both an upper bound on the number of induced decision in the optimal PBE, and a PBE that achieves this upper bound. We then solve the full-commitment problem of maximizing P's expected payoff with a fixed number of induced decisions, subject

[7] There is another uniform-quadratic example that has been analyzed in recent papers including Gordon (2007) and Alonso et al. (2008). In this example, A has an outward rather than upward bias such that his payoff is given by $u^A(y, \theta) = -(y - b - c\theta)^2$ where $b < 0$ and $b + c > 1$. Intuitively, an outwardly biased A prefers extreme decisions when the state of the world is extreme. In the example with outwardly biased A, there exists an equilibrium with a countable number of induced decisions which eliminates an integer problem peculiar to the leading example of CS. Therefore, in some applications, an example with outwardly biased A is simpler to analyze.

only to the partition conditions (1) and the agent's indifference conditions (2). The solution provides a lower bound on the number of induced decisions in the optimal PBE when it satisfies P's incentive conditions (3). The optimal PBE can be then characterized by considering all partition equilibria with a number of induced decisions between the lower bound from the full-commitment problem and the upper bound from Proposition 4.

3.1 Maximal limited authority

From Proposition 1, an optimal PBE exists and it is a partition equilibrium that satisfies A's indifference conditions (2) and P's adjacent downward incentive conditions (3). In the present uniform-convex loss model, these conditions can be rewritten as: for all $i = 2, \ldots, n$,

$$\theta_{i-1} + b - y_{i-1} = y_i - \theta_{i-1} - b; \tag{4}$$

$$|y_i - y_i^*| \leq |y_i^* - y_{i-1}|, \tag{5}$$

where $y_i^* = \frac{1}{2}(\theta_{i-1} + \theta_i)$ is P's *ex post optimal* decision conditional on the interval $(\theta_{i-1}, \theta_i]$. We now show that induced decisions y_i are higher than ex post optimal y_i^* for all $i = 1, \ldots, n$.

Lemma 1 *In any optimal PBE* $(\{\theta_i\}_{i=0}^n, \{y_i\}_{i=1}^n)$*,* $y_i > y_i^*$ *for each* $i = 1, \ldots, n$*.*

The first part of the proof of the above result establishes that if $y_i \leq y_i^*$ for some $i = 1, \ldots, n$ in the optimal PBE, then P's $(i+1)$-th incentive condition binds. This holds for the general model set up in Section 2, not just the uniform-convex loss model here. The intuition is that if $y_i \leq y_i^*$ and the $(i+1)$-th condition is slack, P can obtain a greater expected payoff by marginally increasing y_i without affecting any incentive condition. Her payoff gain is clear: a higher y_i moves her closer to her ex post optimal decision given the same belief of the states; and the resulting increases in thresholds θ_{i-1} and θ_i mean that A now recommends y_i for a set of higher states, making P better off by the same argument as in Proposition 2.[8] The

[8] In the general model, this result implies that in an optimal PBE the highest decisions y_n and y_{n-1} satisfy $y_n > y_n^*$ and $y_{n-1} > y_{n-1}^*$, and that no two adjacent decisions y_i and y_{i+1} are below y_i^* and y_{i+1}^* respectively. Further, in the hypothetical problem of full commitment with a fixed number of decisions introduced in Section 3.2, every decision y_i is strictly higher than y_i^*.

second part of the proof of Lemma 1 uses the special structure of the uniform-convex loss model to show that $y_i \leq y_i^*$ is incompatible with binding P's $(i+1)$-th incentive condition.

We now restate the constrained maximization problem of Proposition 1 for the uniform-convex loss setup by substituting out A's indifference conditions (4). The choice variables are $\{y_i\}_{i=1}^n$. By partition condition (1), $\theta_0 = 0$ and $\theta_n = 1$, so the objective function becomes

$$U_n^P = -\sum_{i=1}^{n} \int_{\theta_{i-1}}^{\theta_i} l\left(|\theta - y_i|\right) d\theta \tag{6}$$

The constraints are

$$y_1 < ... < y_n, \tag{7}$$

$$y_1 + y_2 > 2b, \tag{8}$$

$$y_{i+1} - y_{i-1} \geq 4b, \text{ for each } i = 2, \ldots, n-1, \tag{9}$$

$$y_n + y_{n-1} \leq 2\left(1 - b\right). \tag{10}$$

Conditions (7) are part of the partition condition (1). Condition (8) ensures that $\theta_1 > 0$; $\theta_{n-1} < 1$ is implied by condition (10); and $\theta_1 < \theta_2 < ...\theta_{n-2} < \theta_{n-1}$ follow from (7). Conditions (9) and (10) are equivalent to (5) for $i = 2, \ldots, n$ by Lemma 1. Condition (10) takes a different form because $\theta_n = 1$ by partition condition (1), instead of being determined by A's indifference condition in (4).

Observe that conditions (9) and (10) place constraints on the distance between decisions, and thus the possible number of decisions, in the optimal PBE. We now proceed to characterize the maximum number of decisions that P can possibly use in an optimal PBE for a given bias b. More importantly, we show that there always exists a PBE in which the maximum number of decisions is induced.

Proposition 4 *The number of decisions induced in an optimal PBE is strictly less than* $1/(2b) + 1$*. Conversely, there exists a PBE with* n *induced decisions for any positive integer* $n < 1/(2b) + 1$.

Note that conditions (8) and (10) require that $b < \frac{1}{2}$. This is consistent with the above proposition, as the maximum number of decisions that P can possibly use in any PBE is

1 if $b \geq \frac{1}{2}$. The second part of Proposition 4 is established by construction. A PBE that achieves the upper bound on the number of decisions in an optimal PBE in the present uniform-convex loss model is called *maximal limited authority.* The construction binds all P's incentive conditions (9), with symmetric and equidistant decisions.[9] It is instructive to compare the maximal limited authority with the most informative CS equilibrium. In a CS equilibrium, the distance between subsequent induced decisions grows at the rate $4b$, that is, $y_{i+1} - y_i = y_i - y_{i-1} + 4b$ for $i = 2, ..., n-1$. Therefore, the number of induced decisions n in any CS equilibrium has to satisfy $2n(n-1)b < 1$. In contrast, under limited authority, the distance between two subsequent induced decisions does not grow. In fact, under the maximal limited authority with N induced decisions, binding P's i-th incentive condition we have $y_{i+1} - y_{i-1} = 4b$ for all $i = 2, \ldots, N-1$. As a result, N is the largest integer n satisfying $2(n-1)b < 1$, which is greater than the number of induced decisions in the most informative CS equilibrium.

Although both the upper bound on the number of decisions in an optimal PBE and the construction of a PBE that achieves the upper bound are specific to the uniform-convex loss setup, there is a more general logic behind the result that the number of decisions under maximal limited authority is larger than that in the most informative CS equilibrium. In a CS equilibrium, each induced decision y_i is ex post optimal conditional on the corresponding interval $(\theta_{i-1}, \theta_i]$ because P has no commitment power. In contrast, because our limited authority model gives P some commitment power, in a partition equilibrium, P's incentive conditions are given by (3), requiring only that P prefer y_i to the adjacent lower decision y_{i-1} conditional on $(\theta_{i-1}, \theta_i]$. Therefore the partitioning of the state space $(0, 1]$ under limited authority need not be as rightward skewed as in a CS equilibrium, and more decisions can be induced as a result.

[9]When $n = 2$, the maximum limited authority coincides with the full commitment solution introduced in Lemma 2, so P's incentive condition does not bind. See the proof of the proposition in the appendix.

3.2 Optimal limited authority

Before characterizing the optimal PBE, it is useful to consider the hypothetical problem of maximizing (6) subject to constraints (7), (8), and

$$y_{n-1} + y_n < 2(1 + b), \tag{11}$$

where we have dropped P's incentive conditions (9) and (10), but added (11) to ensure that $\theta_{n-1} < 1$. This hypothetical problem has the interpretation of the case of full commitment, with the restriction to a finite number n of decisions, and is informative about the limited authority model.[10] The solution also provides an upper bound on what P can achieve in an optimal PBE if it has n decisions. Denote the solution as $Y^{FC}(n) = \{y_i^{FC}\}_{i=1}^n$. (We write y_i^{FC} instead of $y_i^{FC}(n; b)$ whenever it can be done without loss of clarity.) The following lemma provides a characterization of $Y^{FC}(n)$.

Lemma 2 *Suppose that $b < \frac{1}{2}$. For any natural n, $Y^{FC}(n)$ is given by $y_i^{FC} = \frac{1}{2} + \Delta\left(i - \frac{n+1}{2}\right)$, $i = 1, \ldots, n$, where $\Delta > 0$ is uniquely determined by*

$$2l(y_1^{FC}) = l(b + \Delta/2) + l(|b - \Delta/2|).$$

For $b \geq \frac{1}{2}$, the optimal full commitment solution has the single decision of $\frac{1}{2}$. In contrast, for $b < \frac{1}{2}$, under full commitment for any n optimal decisions y_i^{FC} are equidistant, that is, all decisions $i = 1, \ldots, n$ are Δ apart and symmetric around $\frac{1}{2}$. In order to minimize P's expected payoff given by (6), the decisions need to be equidistant to make the partition of the state space uniform in that $\theta_i - \theta_{i-1}$ is the same for all $i = 2, \ldots, n-1$. The uniform partition in turn minimizes the loss of information, which can be loosely understood as the average residual uncertainty of the state of the world provided that P learns the partition elements (see Section 4 for more details when the loss function is quadratic). That decisions

[10] Melumad and Shibano (1991) show that the optimal decision set is equal to $[b, 1-b]$ in the full commitment model in which P can commit to not to change A's recommendation without the restriction to a finite number of decisions. The solution given in Lemma 2 becomes arbitrarily close to $[b, 1-b]$ as $n \to \infty$. To see this, note that as $n \to \infty$, $\Delta \to 0$ and $(n-1)\Delta \to 1 - 2b$.

y_i^{FC} are symmetric around $\frac{1}{2}$ is also intuitive, because P is unbiased, that is, $y^P(\theta) = \theta$.[11] Finally, Lemma 2 implies that as n increases, the maximized value of U_n^P in the hypothetical full-commitment problem strictly increases. In fact, a robust feature of models with full commitment is that more decisions can only improve A's communication quality because P commits to not using the information A revealed strategically.

Solution $Y^{FC}(n)$, however, may violate P's incentive conditions (9) and thus become infeasible under limited authority. We now turn to the problem of maximizing P's expected payoff (6) by choosing a set of a fixed finite number n of decisions, subject to all constraints (7)-(10). Denote the solution to this problem as $Y^{LC}(n) = \left\{y_i^{LC}(n)\right\}_{i=1}^n$, and we refer to it as the *n-optimal limited authority* as it takes n as given. A few observations are immediate. First, obviously, $Y^{LC}(1) = Y^{FC}(1)$ for any b, with $y_1^{LC}(1) = \frac{1}{2}$. Second, for each $n \geq 2$, by Proposition 4, $Y^{LC}(n)$ exists only if and only if

$$b < b^{LC}(n) \equiv \frac{1}{2(n-1)}. \tag{12}$$

Third, for $n \geq 3$, the solution $Y^{FC}(n)$ to the full commitment problem satisfies P's incentive condition (9), and hence $Y^{LC}(n) = Y^{FC}(n)$, if and only if $b \leq b^{FC}(n)$, where $b^{FC}(n)$ is uniquely determined by

$$2l(1/2 - (n-1)b^{FC}(n)) = l(2b^{FC}(n)). \tag{13}$$

This follows because the above definition of b^{FC} is simply the evaluation of the condition in Lemma 2 at $\Delta = 2b$, so the distance Δ between two adjacent decisions y_{i+1}^{FC} and y_i^{FC} is greater than or equal to $2b$ if and only if $b \leq b^{FC}(n)$.[12] Intuitively, when b or n is small, the decisions under $Y^{FC}(n)$ are sufficiently far apart from each other so that P would not want to deviate to the lower decision y_{i-1} when A recommends y_i. For $n = 2$, it turns out that

[11] If we focused on PBE that maximized A's expected payoff instead of P's, then the optimal decisions would tend to be symmetric around $\frac{1}{2} + b$ because A has the upward bias $b > 0$.

[12] Since l is convex, the right-hand side of the condition in Lemma 2 is increasing in Δ regardless of whether $\Delta \geq 2b$ or $\Delta < 2b$. The remaining incentive condition (10) of P can be shown to be equivalent to $\theta_1 \geq 0$, which is always satisfied under $Y^{FC}(n)$. For details see the proof of Lemma 2 in the appendix.

$Y^{FC}(2)$ given in Lemma 2 is always incentive compatible for P, and thus $Y^{LC}(2) = Y^{FC}(2)$.[13] Since $Y^{FC}(2)$ exists if and only if $b \leq \frac{1}{2}$, we write $b^{FC}(2) = \frac{1}{2}$.

The following lemma characterizes the n-optimal limited authority $Y^{LC}(n)$ for $n \geq 3$ and $b \in (b^{FC}(n), b^{LC}(n))$. The crucial feature is that P's incentive conditions (9) all bind under $Y^{LC}(n)$. That is, under any $Y^{LC}(n)$, whenever some of P's incentive conditions must bind because $b > b^{FC}(n)$, P is indifferent between implementing each recommended decision y_i^{LC} and replacing it with the adjacent lower decision y_{i-1}^{LC} for each $i = 2, \ldots, n-1$. Otherwise, if some, but not all, incentive conditions (9) bind, then it would be possible to modify the decisions to make them more equidistant and P better off. For example, if $y_{i+1} - y_{i-1} > 4b$ for some i, then we could increase y_{i-1} or decrease y_{i+1} without violating any incentive condition of P. That all incentive conditions of P must bind if any of them binds is an intuitive result due to the assumption of uniform distribution of the state, and this result is what makes the characterization of $Y^{LC}(n)$ relatively straightforward. It turns out that the characterization of $Y^{LC}(n)$ depends on whether n is odd or even. In both cases, the decisions $\{y_i^{LC}\}_{i=1}^n$ are symmetric around $\frac{1}{2}$, which is intuitive because the state is uniformly distributed and the loss function l is convex. When n is odd, the decisions are all equidistant at $2b$. When n is even, the decisions are equidistant in an alternating manner, with $y_{i+1}^{LC} - y_i^{LC}$ equal for all odd i and for all even i respectively but strictly smaller for odd i.[14]

Lemma 3 *Fix any $n \geq 3$ and $b \in (b^{FC}(n), b^{LC}(n))$. The n-optimal limited authority $Y^{LC}(n)$ is given by $y_i^{LC} = \frac{1}{2} + 2b\left(i - \frac{n+1}{2}\right) + \left(b - \frac{1}{2}\Delta_1\right)$ for odd i, and $y_i^{LC} = \frac{1}{2} + 2b\left(i - \frac{n+1}{2}\right) - \left(b - \frac{1}{2}\Delta_1\right)$ for even i, where $\Delta_1 = 2b$ if n is odd and $\Delta_1 < 2b$ determined by*

$$2l(y_1^{LC}) = l(b + \Delta_1/2) + l(b - \Delta_1/2) - \frac{n-2}{2}\left[l(3b - \Delta_1/2) - l(b + \Delta_1/2)\right] \tag{14}$$

if n is even.

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[13]The only incentive condition of P is (10). This is equivalent to $\theta_1 \geq 0$, which is satisfied because in this case $\theta_1 = \frac{1}{2} - b$ and $b \leq \frac{1}{2}$. See the proof of Proposition 4 in the appendix for details.

[14]Having all $y_{i+1}^{LC} - y_i^{LC}$ equal to $2b$ is not optimal when n is even, because the number of such differences is not a multiple of the number of incentive conditions in (9). Starting from a set of decisions $\{y_i\}_{i=1}^n$ that are equidistant at $2b$ and symmetric around $\frac{1}{2}$, we can increase P's expected payoff by increasing y_i for all odd i and decreasing it for even i by the same amount.

Given the above characterization of the n-optimal limited authority decision set $Y^{LC}(n)$ for each fixed n and for all $b \in \left(b^{FC}(n), b^{LC}(n)\right)$, we can now present the main result of this section. This is a characterization of the optimal PBE, achieved by comparing P's expected payoffs under all feasible decision sets. By (12) and (13),[15]

$$b^{FC}(n) < b^{LC}(n+1) < b^{FC}(n-1).$$

Since P prefers $Y^{FC}(n-1)$ to $Y^{FC}(i)$ and $Y^{LC}(i)$ for all $i < n-1$, we can restrict the search for the optimal PBE in the interval $[b^{FC}(n), b^{FC}(n-1))$ to $Y^{FC}(n-1)$, $Y^{LC}(n)$ and $Y^{LC}(n+1)$.

Proposition 5 *Suppose that $l(z) = z^2$. For each $n \geq 3$, there exists $b(n, n-1) \in (b^{LC}(n+1), b^{FC}(n-1))$ such that the induced decisions in the optimal PBE are given by $Y^{LC}(n)$ for all $b \in \left[b^{FC}(n), b(n, n-1)\right)$, and by $Y^{FC}(n-1)$ for all $b \in \left[b(n, n-1), b^{FC}(n-1)\right)$.*

The logic behind the comparison among $Y^{FC}(n-1)$, $Y^{LC}(n)$ and $Y^{LC}(n+1)$, which holds for all loss function l that satisfies Assumption 2, can be seen as follows. First, observe that at $b = b^{FC}(n)$, the optimal decision sets under full commitment and limited authority are identical: $Y^{FC}(n) = Y^{LC}(n)$. P's expected payoff jumps down discontinuously at $b^{FC}(n)$ if decisions change from $Y^{FC}(n)$ to $Y^{FC}(n-1)$. In contrast, $Y^{LC}(n)$ changes continuously with b at $b^{FC}(n)$, consequently P is strictly better off with $Y^{LC}(n)$ than with $Y^{FC}(n-1)$ if b is sufficiently close to and greater than $b^{FC}(n)$. Second, at b just below the cutoff value $b^{LC}(n+1)$, P strictly prefers $Y^{LC}(n)$ to $Y^{LC}(n+1)$. This follows because under $Y^{LC}(n+1)$, the lowest partition threshold $\theta_1^{LC}(n+1)$ equals 0 at $b^{LC}(n+1)$, so effectively only n decisions are recommended by A and approved by P. Since $Y^{LC}(n)$ is available at $b^{LC}(n+1)$, $Y^{LC}(n+1)$ is dominated for P: the additional decision in $Y^{LC}(n+1)$ does nothing to improve her expected payoff, but distorts the quality of her decisions, making her worse off. Therefore,

[15] Note that the function $g(k, b) \equiv 2l(1/2 - (k-1)b) - l(2b)$ is decreasing in b for $b \in \left(0, b^{LC}(k)\right)$ and is equal to 0 at $b^{FC}(k)$ for all k. The first inequality holds because $g\left(n, b^{LC}(n+1)\right) = 2l\left(\frac{1}{2n}\right) - l\left(\frac{1}{n}\right) < 0$ for any convex loss function l. The second inequality holds because $g\left(n-1, b^{LC}(n+1)\right) = l\left(\frac{1}{n}\right) > 0$.

the cutoff value $b^{LC}(n+1)$ is not relevant for P's search for optimal PBE in the interval $\left[b^{FC}(n), b^{FC}(n-1)\right)$.

As b increases in the interval $(b^{FC}(n), b^{FC}(n-1))$, P's expected payoff decreases under each of $Y^{FC}(n-1)$, $Y^{LC}(n)$ and $Y^{LC}(n+1)$. Under the assumption of $l(z) = z^2$, the proof of Proposition 5 in the appendix ranks the rate of decrease for the three sets of decisions. In particular, we show that P's expected payoff decreases slower under $Y^{FC}(n-1)$ than under $Y^{LC}(n)$ for any $b \in (b^{FC}(n), b^{FC}(n-1))$, and in turn slower under $Y^{LC}(n)$ than under $Y^{LC}(n+1)$ for any $b \in (b^{FC}(n), b^{LC}(n+1))$. We then show that at $b^{FC}(n-1)$, P strictly prefers $Y^{FC}(n-1)$ to $Y^{LC}(n)$. Shifting the indices forward by 1 and noting that $Y^{LC}(n) = Y^{FC}(n)$ at $b^{FC}(n)$, we have that P strictly prefers $Y^{LC}(n)$ to $Y^{LC}(n+1)$. This evaluation then allows us to establish the proposition.

Proposition 5 makes it clear that the optimal limited authority does not generally coincide with the maximal limited authority. This is reflected in two ways. First, when b falls in $\left[b^{FC}(n), b^{LC}(n+1)\right)$, $Y^{LC}(n+1)$ is available but is never optimal. Indeed, as observed above, for any loss function l that satisfies Assumption 2, P strictly prefers $Y^{LC}(n)$ to $Y^{LC}(n+1)$ for b sufficiently close to and lower than $b^{LC}(n+1)$. Second, when b falls in $\left(b(n, n-1), b^{FC}(n-1)\right)$, $Y^{LC}(n)$ is available but P strictly prefers $Y^{FC}(n-1)$. Thus, unlike in the solution to the hypothetical full-commitment problem, P's payoff does not necessarily increase with the number of decisions under limited authority. Intuitively, in an optimal PBE, P retains fewer decisions in order to relax the incentive conditions due to limited authority.[16]

Our result that the optimal PBE under limited authority does not always maximize the number of induced decisions contrasts strongly with CS and models with full commitment. This may be counterintuitive, but recall that we have restricted the search for the optimal PBE under limited authority to decision sets that are minimal and veto-free. Thus, in characterizing the decision set $Y^{LC}(n)$ for each fixed n, we have imposed the condition that all n decisions are induced in some states, and precluded the standard reasoning that adding a

[16] By Proposition 5, in the familiar uniform-quadratic case, the number of induced decisions under the maximal and optimal limited authority is the same only for $b \in \left[b^{LC}(n+1), b(n-1, n)\right)$.

decision cannot make P worse off. Instead, each additional decision in our model presents P with a credibility problem. In a PBE with a larger number of induced decisions there is better information transmission, which would benefit P, everything else being equal. This better information transmission, however, is achieved due to P's commitment to ex-post suboptimal decisions. As a result, P may prefer a PBE with worse information transmission, but better decision-making.

4 Welfare comparison across organizational forms

Many existing papers have analyzed extensions of the CS model that improve communication quality and thus P's welfare. We categorize these papers into six organizational forms: cheap talk, delegation, persuasion, informational control, noisy talk, and limited authority. Then we compare P's ex ante expected payoffs under these organizational forms. All the payoff comparisons are based on specializing the uniform-convex loss example of Section 3.2 to the quadratic loss function.

4.1 Organizational forms

Our first organizational form is CS's *cheap talk* model in which neither P nor A has any commitment power. In the CS model, there are essentially three players: nature, A, and P. Nature draws the state of the world $\theta \in \Theta$. A privately observes the state θ and sends a message $m \in M$ to P, who then makes a decision $y \in Y$.

Let us introduce a fourth non-strategic player to the CS model who takes an input $i \in I$ and returns a possibly stochastic output $o \in O$ according to some pre-specified mapping $\phi : I \to \Delta O$, where ΔO denotes the set of lotteries over outcomes O. The fourth player can either replace one of the players or be a mediator. Each possible way that a fourth player could be introduced into the game corresponds to a different *organizational form.* Note that a strategic fourth player is just a particular non-strategic player who uses a certain (equilibrium) mapping. Thus we can analyze either a fourth player designed by P, whose mapping maximizes P's expected payoff; or a fourth player with an equilibrium mapping, which corresponds to a strategic player or to a modification of the CS environment.

We first discuss organizational forms where the fourth player replaces another player (Figure 1, left panel). There are two such forms: *delegation* and *persuasion*, which correspond to the replacement of P and A respectively by a fourth player. Delegation refers to P's commitment power: under delegation, A sends a message m to the fourth player instead of to P, and the fourth player then makes a decision y according to $\phi_D : M \to \Delta Y$. Such delegation encompasses both optimal delegation and full delegation: the former corresponds to a fourth player designed by P and the latter corresponds to A being the fourth player respectively.[17] Persuasion refers to A's commitment power: under persuasion, the fourth player observes the state θ and sends a message m to P according to $\phi_{PM} : \Theta \to \Delta M$.[18]

We turn next to organizational forms where the fourth player is a mediator (Figure 1, right panel). There are three such forms: *informational control*, *noisy talk*, and *limited authority*, which correspond to information, message, and decision mediation, respectively. Informational control refers to A's information structure: under informational control, the fourth player privately observes the state θ and returns a signal $\widehat{\theta} \in \Theta$ according to $\phi_{IC} : \Theta \to \Delta\Theta$; A privately observes the signal $\widehat{\theta}$ and sends a message to P, who makes a decision.[19] Noisy talk refers to the quality of the communication channel between A and P: under noisy talk, the fourth player receives a message m from A and sends a perturbed message $\widehat{m} \in M$

[17]Holmstrom (1984), Melumad and Shibano (1991), Martimort and Semenov (2006), and Alonso and Matouschek (2008) study optimal delegation in which a third player is restricted to deterministic decision rules. Goltsman et al. (2009), and Kovac and Mylovanov (2009) study optimal stochastic delegation. Dessein (2002) studies full delegation and analyzes how it compares to cheap talk. Gilligan and Krehbiel (1987), Krishna and Morgan (2001), and Mylovanov (2008) study veto-delegation.

[18]Kamenica and Gentzkow (2011) analyze optimal persuasion that maximizes A's expected payoff. Clearly, full information transmission maximizes P's expected payoff.

[19]Ivanov (2010b) studies optimal informational control, while Austen-Smith (1994), and Fischer and Stocken (2001) study a non-optimal informational control. The extensive literature on information acquisition and reputational cheap talk is also related to informational control, but there are additional elements added to the CS model.

to P according to $\phi_{NT} : M \to \Delta M$.[20]

Finally, limited authority refers to the set of decisions available to P: under limited authority, P receives a message from A and then recommends a decision y to the fourth player, who implements a decision $\widehat{y} \in Y$ according to $\phi_{RA} : Y \to \Delta Y$. This paper focuses on P's optimal deterministic limited authority (deterministic in that a decision mediator is bound to use a deterministic mapping $\phi_{RA} : Y \to Y$). Note that limited authority is equivalent to letting P make any decision from the restricted set $\phi_{RA}(Y)$, which justifies the setup of our model.

4.2 Payoff comparisons

In this section, we compare P's highest ex ante expected payoffs within each organizational form. To begin with, Proposition 6 calculates P and A's expected payoffs for a small bias b under optimal limited authority.

Proposition 6 *The first term in Taylor expansion around $b = 0$ of P and A's expected payoffs under optimal limited authority is equal to $-\frac{4}{3}b^2$ and $-\frac{1}{3}b^2$ respectively.*

To understand the intuition behind Proposition 6, it is useful to decompose P and A's expected payoffs as the sum of the loss of information and the loss of control:

$$U^P = -\mathbb{E}\left[(y-\theta)^2\right] = -\underbrace{\mathbb{E}\left[(y_m - \mathbb{E}[\theta|m])^2\right]}_{P\text{'s Loss of Control}} - \underbrace{\mathbb{E}\left[Var(\theta|m)\right]}_{\text{Loss of Information}},$$

$$U^A = -\mathbb{E}\left[(y-(\theta+b))^2\right] = -\underbrace{\mathbb{E}\left[(y_m - \mathbb{E}[\theta|m] - b)^2\right]}_{A\text{'s Loss of Control}} - \underbrace{\mathbb{E}\left[Var(\theta|m)\right]}_{\text{Loss of Information}},$$

where y_m, $\mathbb{E}[\theta|m]$, and $Var(\theta|m)$ are the decision taken, the expectation, and the variance of the state θ given a message m (under P's beliefs), respectively. The loss of information

[20] Goltsman et al. (2009) characterize optimal noisy talk. Furthermore, there are a number of papers that can be represented as a particular type of a message mediator: Krishna and Morgan (2004) analyze back and forth communication between the agent and the principal; Blume et al. (2007) study communication with perturbed messages; Ivanov (2010a), Li (2010), and Ambrus et al. (2010) study communication with a strategic message mediator and a sequence of strategic message mediators, respectively.

is defined as the expected conditional variance of the state given P's belief at the time of decision making. Therefore, the loss of information captures the residual uncertainty that P has after communication took place. P and A' loss of control is defined as the expected loss from making decision y_m instead of the ex post optimal decision given the message m.

Now we calculate the loss of information and the loss of control under optimal limited authority. By Proposition 5, adjacent induced decisions are symmetric around $\frac{1}{2}$ and approximately $2b$ apart from each other ($y_i \approx \frac{1}{2} + \left(i - \frac{n+1}{2}\right) 2b$). Therefore, the loss of information can be approximated as

$$\sum_{i=1}^{n} \Pr\left(\left(\theta_{i-1}, \theta_i\right]\right) Var\left(\theta \middle| \left(\theta_{i-1}, \theta_i\right]\right) \approx Var\left(\theta \middle| \left(\theta_{i-1}, \theta_i\right]\right) \approx \frac{(2b)^2}{12} = \frac{1}{3} b^2,$$

and P's loss of control can be approximated as

$$\sum_{i=1}^{n} \Pr\left(\left(\theta_{i-1}, \theta_i\right]\right) \left(y_i - \mathbb{E}\left[\theta \middle| \left(\theta_{i-1}, \theta_i\right]\right]\right)^2 \approx \left(y_i - \frac{\theta_{i-1} + \theta_i}{2}\right)^2 \approx b^2$$

where the last equality follows from A's indifference conditions (4). Analogous calculations show that A has essentially no loss of control. Summing up the loss of information and the loss of control yields Proposition 6.

Next we compare P and A's expected payoffs for a small bias b under all organizational forms (see Table 1).[21]

	Persuasion	Informational Control	Optimal Delegation	Full Delegation	Limited Authority	Noisy Talk	Cheap Talk
U^P	0	$-\frac{1}{3}b^2$	$-b^2$	$-b^2$	$-\frac{4}{3}b^2$	$-\frac{1}{3}b$	$-\frac{1}{3}b$
U^A	$-b^2$	$-\frac{4}{3}b^2$	$-\frac{8}{3}b^3$	0	$-\frac{1}{3}b^2$	$-\frac{1}{3}b$	$-\frac{1}{3}b$

[21] We believe that these results hold more generally with a caveat that each row of Table 1 is multiplied by some constant. In particular, we expect them to hold if P's and A's payoffs are given by arbitrary smooth loss functions $u^P(y, \theta) = -l^P(|y - \theta|)$, $u^A(y, \theta) = -l^A(|y - (\theta + b)|)$. Intuitively, as the bias goes to zero, the distance between any two subsequent decisions also goes to zero. Therefore, the loss functions can be approximated by quadratic functions, and the distribution function can be approximated by a piecewise uniform distribution.

Table 1. P and A's expected payoffs under all organizational forms.

To understand these payoff comparisons, we decompose P's expected payoff into the loss of information and the loss of control. In terms of loss of control, in all organizational forms, either P or A has essentially no loss of control, and thus the other party has a loss of control equal to b^2. Under delegation and limited authority, P has commitment power and effectively delegates authority to A to improve information transmission, and her loss of control b^2 is simply due to A's bias. Next, we turn to the loss of information. There is essentially no loss of information under delegation and persuasion because the state is almost fully revealed. The loss of information is approximately $\frac{1}{3}b^2$ under informational control and limited authority because induced decisions are approximately $2b$ apart from each other.[22] Under cheap talk and noisy talk, however, the partition is coarse such that the distance between adjacent decisions grows at the approximate rate $4b$, leading to a much larger loss of information of approximately $\frac{1}{3}b$. Combining these two parts lead to the payoff comparisons in Table 1.

The above intuition suggests that P and A's expected payoffs are still given by Table 1 under A's optimal organizational forms, except for A's optimal delegation, which coincides with full delegation. It is also straightforward to characterize P and A's expected payoffs under combinations of different organizational forms. However, the first term in Taylor expansion of P and A's expected payoffs will depend on whether we are looking at P or A's optimal combination of organizational forms. For example, if both P and A have full-commitment power (a combination of persuasion and delegation), then they can eliminate the loss of information and split the loss of control arbitrarily such that P and A's expected payoffs on the Pareto frontier are given by $U^A = -(\alpha b)^2$ and $U^P = -((1-\alpha)b)^2$, where $\alpha \in [0,1]$.

[22]This connection between limited authority and informational control is due to limited commitment power of both A and P. Under limited authority, P makes the decision space discrete to relax her incentive conditions, whereas under informational control, P makes the state space discrete to relax A's incentive conditions.

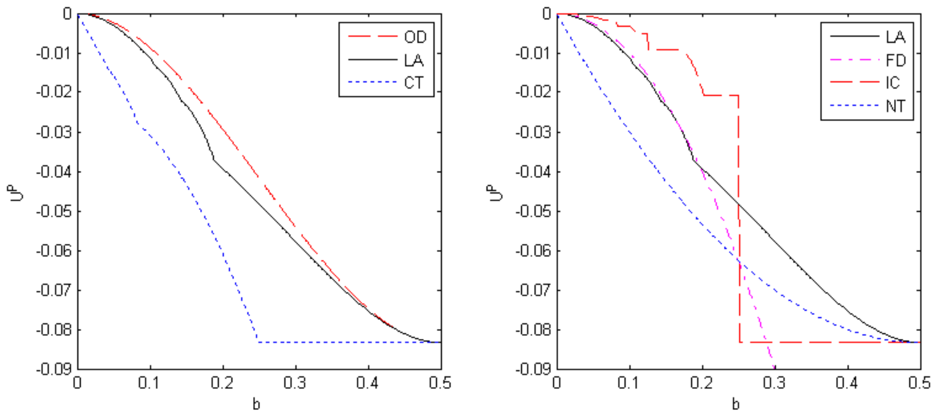

Figure 2 illustrates P's expected payoff under all organizational forms for all possible values of the bias $b \in \left(0, \frac{1}{2}\right)$. As we can see from the figure, for the whole range of A's bias, P's expected payoffs are considerably lower under cheap talk and noisy talk than under all other organizational forms. P's payoff is of the same order of the bias under informational control, delegation, and limited authority. In particular, P's payoff under limited authority is almost as high as P's payoff under optimal delegation despite the fact that only finite decisions can be induced under limited authority. Moreover, P receives a higher payoff under informational control than under limited authority when A's bias is small whereas the opposite is true when A's bias is large. Although these payoff comparisons alone do not indicate which organizational form should be chosen due to differences in environment, they do suggest that organizations can potentially benefit from credibly ruling out some decisions ex ante even if it cannot fully delegate the decision-making authority.

5 Concluding remarks

Our model of limited authority aims to explore and understand the environment in which the principal has some degree of commitment power, but not all. The remainder of this section contains a discussion of how the optimal equilibrium may be affected by different assumptions about the communication process and contracting environment (see Kolotilin (2011) for more detailed expositions) as well as some thoughts for further research.

Because only finitely many decisions can be induced in any optimal equilibrium under limited authority, one may wonder whether the principal benefits from a finite decision set per se, that is, when the principal's decision space is discrete. It can be shown that in the uniform-quadratic setup in Section 3, the equilibria in the CS model with a discrete set of decisions are similar to equilibria in the CS model with a continuous set of decisions, but with a modified smaller bias. Therefore, by discretizing the set of available decisions, the principal can effectively decrease the agent's bias and thereby improve communication.[23] In fact, under the optimal limited authority given in Proposition 5, the agent's effective bias disappears such that a uniform partition becomes feasible.

One possible critique of the limited authority model is that parties can renegotiate, after the agent's recommendation, to a decision not in the pre-specified set if both prefer to do so. To address this issue, we can strengthen our solution concept and look for the optimal renegotiation-proof equilibrium. In our model the scope for renegotiation is limited for two reasons. First, because transfers between the parties are not allowed, both the principal and the agent must prefer the renegotiated decision to the optimal equilibrium decision. Second, in any PBE there is unresolved uncertainty about the state of the world after the agent's recommendation, and thus the principal does not know to which decision, if any, she should renegotiate. In the uniform-quadratic setup, we can show that a renegotiation-proof equilibrium exists, and has a maximal possible number of induced decisions that can be supported as an equilibrium.

Another critique is that if the principal can credibly restrict the set of decisions ex ante, then she may also be able to commit to transfers contingent on decisions. Following the literature on communication and delegation, we rule out such transfers in the limited authority model.[24] Note that with transfers, the principal can do at least as well as in the limited

[23] This result resembles that of Alonso and Matouschek (2007) who show that the principal's commitment power reduces the agent's "effective" bias.

[24] Monetary transfers may be explicitly ruled out by law, or implicitly ruled out if the parties involved are very risk averse with respect to money. However, the assumption that there are no transfers is strong. Even though explicit transfers between parties may be ruled out, the parties can effectively "burn" money, which generally improves their welfare (see, for example, Austen-Smith and Banks, 2000).

authority model by committing to no transfers for decisions in the pre specified set and very large transfers for decisions outside the set. In fact, with transfers, the principal can achieve the first best outcome, which maximizes the sum of the principal and agent's expected payoffs, if there are no monetary frictions. However, not all decision rules can be supported with transfers. For example, the principal's and the agent's optimal decision rules are not achievable. Further, if there are frictions, such as when the agent is protected by limited liability or if the principal can "burn" money, then there is a tradeoff between making efficient decisions and leaving a quasi rent to the agent. Due to this tradeoff, there is incomplete information revelation for high states of the world, even though full information revelation is feasible.[25]

In the current model, the principal never vetoes the agent's recommendation in equilibrium. A further topic of research is to extend our model to allow veto to happen in equilibrium. One way to model this is to imagine that there is some small, exogenous probability that the principal can observe the true state after hearing the agent's recommendation, and may consequently desire to change her decision (still within the pre-specified decision set) given this information. In this case, it is without loss of generality to restrict to equilibria in which the principal follows the agent's recommendation when she does not learn the state and otherwise makes a choice independent of the recommendation. Thus, any equilibrium characterized in Proposition 1 remains feasible, and further, the principal can do better by adding any decision that is chosen with zero probability in equilibrium so long as her incentive conditions (when she does not learn the state) are unaffected. The interesting question is how the principal optimally modifies the decisions that are used with positive probabilities when she does not learn the state, in order to retain more decisions that she will use when she does. Answering this question can further our understanding of the principal's tradeoff between maintaining the flexibility of responding to new information and establishing the credibility of letting the

[25]This result is analogous to that of Krishna and Morgan (2008) who consider a communication game in which the principal can commit to transfers contingent on messages and the agent is protected by limited liability. Kartik (2009) obtains a somewhat similar result. He shows that in a model of communication with lying costs, there is pooling for high states of the world.

agent best use his private information.

6 Appendix: Proofs

Proof of Proposition 1: We prove the proposition through a series of lemmas.

First, we establish that it is without loss of generality to restrict attention to PBE's in which all decisions in P's equilibrium choice Y are induced, each message from A's is a *recommendation* of some probability distribution over Y, and no recommendation is vetoed by P. This is a version of the revelation principle adapted to our setting. Fix any PBE and the subgame after P has made the equilibrium choice Y. We refer to any response by P to a message m from A as a *lottery*, and a particular choice from Y as a *degenerate lottery*. We say that two PBE's are *outcome equivalent* if they both result in the same (random) mapping from states to decisions on the equilibrium path.

Lemma 4 *Consider a PBE with P's equilibrium choice Y. There exists an outcome-equivalent PBE with P's equilibrium choice $\tilde{Y} \subseteq Y$, where $\tilde{Y}$ is the union of the supports of all induced lotteries and for any induced lottery there is a unique y in its support chosen by A as a message.*

Proof: Fix any PBE and the subgame after P has chosen the equilibrium Y. Since $u^P(\cdot,\theta)$ is strictly concave, there are at most two decisions y and y' in Y that are optimal given the equilibrium belief of P conditional on any m. Thus, a non-degenerate lottery has exactly two decisions. Moreover, if y and y' in Y satisfying $y < y'$ are in the support of some lottery, then $(y, y') \cap Y = \emptyset$; otherwise, strict concavity of $u^P(\cdot,\theta)$ implies that the lottery would not be optimal for P. Finally, no two induced lotteries have the same support. Otherwise, if $y, y' \in Y$ with $y < y'$ are in the common support of two distinct lotteries induced after A chooses m and m' respectively, then one of them, say the lottery following m', first order stochastically dominates the other. Since $u_{y\beta} > 0$, P being indifferent between y and y' given the belief conditional on m implies that A strictly prefers y' to y given the same belief. Thus, there are states in which A is supposed to choose m but would find it profitable to deviate to m' to induce the lottery following m', a contradiction. By the same argument, if $y, y' \in Y$ with $y < y'$ are the support of some induced lottery, y' is not induced as a degenerate lottery.

Let $\tilde{Y}$ be the union of the supports of all induced lotteries following Y. We construct an outcome-equivalent PBE where P chooses $\tilde{Y}$ instead of Y on the equilibrium path and A's message space is restricted to P's choice of set of decisions on and off the equilibrium path. For any choice of P that is not $\tilde{Y}$, including Y, let the continuation in the new PBE be such that A chooses the lowest decision in the set chosen by P regardless of realized θ and P chooses a decision that is optimal in the set given her prior belief. It remains to specify the continuation equilibrium in the new PBE following $\tilde{Y}$ that is outcome-equivalent to the continuation equilibrium in the original PBE following Y. For each degenerate lottery $y \in Y$ induced in the continuation equilibrium following Y after A chooses some message m, let A choose y in the subgame following $\tilde{Y}$ and let P's belief be the same as in the original PBE conditional on m; and for each non-degenerate lottery where P randomizes between y and y' with $y < y'$ following Y after A chooses some message m', let A choose y' in the subgame following $\tilde{Y}$ and let P's belief be the same as in the original PBE conditional on m'. All equilibrium conditions are satisfied in the new PBE following $\tilde{Y}$ as they are a subset of the equilibrium conditions in the original PBE following Y. Further, by construction $\tilde{Y}$ is part of the new PBE, because Y is part of the original PBE, and the equilibrium payoff for P is greater than or equal to the payoff from choosing y^P. **QED**

Second, we show that in any PBE the number of induced lotteries is finite. Denote $\{y, y'; w\}$ as a lottery induced in some continuation game after P has chosen Y, with P choosing y with probability $w \in (0, 1)$ and $y' \geq y$ with probability $1 - w$. We adopt the convention that a degenerate lottery is represented by $y' = y$. The proof of Lemma 4 implies that any two distinct lotteries $\{y_1, y_1'; w_1\}$ and $\{y_2, y_2'; w_2\}$ can be ordered, with the first *lower* than the latter, such that $y_1 \leq y_1' \leq y_2 \leq y_2'$, with at least one strict inequality and $y_1' = y_2$ implying that $y_2' > y_2$.

Lemma 5 *The number of decisions induced in any PBE is finite.*

Proof: Fix some PBE and the subgame after P has chosen the equilibrium Y. Let $\{y_i, y_i'; w_i\}$, $i = 1, 2, 3$, be three distinct induced lotteries, in increasing order. Since both $\{y_2, y_2'; w_2\}$ and $\{y_3, y_3'; w_3\}$ are induced, there is a state $\hat{\theta}$ such that

$$w_2 u^A(y_2, \hat{\theta}) + (1 - w_2) u^A(y_2', \hat{\theta}) = w_3 u^A(y_3, \hat{\theta}) + (1 - w_3) u^A(y_3', \hat{\theta}).$$

Since $u^A(\cdot, \hat{\theta})$ is strictly concave, $y^A(\hat{\theta}) \in (y_2, y_3')$. Further, since $u^A_{y\theta} > 0$, the lottery $\{y_2, y_2'; w_2\}$ is not induced for any $\theta > \hat{\theta}$, as

$$\begin{aligned}
& w_3(u^A(y_3, \theta) - u^A(y_3, \hat{\theta})) + (1 - w_3)(u^A(y_3', \theta) - u^A(y_3', \hat{\theta})) \\
& \geq u^A(y_3, \theta) - u^A(y_3, \hat{\theta}) \\
& \geq u^A(y_2', \theta) - u^A(y_2', \hat{\theta}) \\
& \geq w_2(u^A(y_2, \theta) - u^A(y_2, \hat{\theta})) + (1 - w_2)(u^A(y_2', \theta) - u^A(y_2', \hat{\theta})),
\end{aligned}$$

with at least one inequality being strict. This implies that $\{y_2, y_2'; w_2\}$ can only be induced if the state θ is smaller than $\hat{\theta}$. As a result, we have $y^P(\hat{\theta}) > y_1$; otherwise, since $u^P_{y\theta} > 0$, a similar argument as above would imply that P prefers the lottery $\{y_1, y_1'; w_1\}$ to $\{y_2, y_2'; w_2\}$ for all $\theta < \hat{\theta}$ but then $\{y_2, y_2'; w_2\}$ would never be induced. It then follows that $y_1 < y^P(\hat{\theta}) < y^A(\hat{\theta}) < y_3'$. Since $y^P(\theta) < y^A(\theta)$ for all $\theta \in [0, 1]$ and are both continuous, there exists $\varepsilon > 0$ such that $y^A(\theta) - y^P(\theta) \geq \varepsilon$ for all $\theta \in [0, 1]$. There can be at most one induced decision greater than $y^P(1)$ and one lower than $y^P(0)$. The lemma then follows immediately. **QED**

By the first two lemmas, for any PBE, it is without loss of generality to assume that the equilibrium Y has a finite number of decisions, and each decision $y \in Y$ is induced either in a degenerate lottery or in a lottery with another decision $y' \in Y$. Denote the induced lotteries as $\{y_i, y_i'; w_i\}$, $i = 1, \ldots, n$, in increasing order. Since $u^A_{y\theta} > 0$, there is a partition $\{\theta_i\}_{i=0}^n$ of the state space $[0, 1]$, with $\theta_0 = 0$ and $\theta_n = 1$, such that each $\{y_i, y_i'; w_i\}$, $i = 1, \ldots, n$, is induced in state $\theta \in (\theta_{i-1}, \theta_i]$. The necessary equilibrium conditions are A's indifference conditions: for each partition threshold θ_i, $i = 1, \ldots, n-1$,

$$w_i u^A(y_i, \theta_i) + (1 - w_i) u^A(y_i', \theta_i) = w_{i+1} u^A(y_{i+1}, \theta_i) + (1 - w_{i+1}) u^A(y_{i+1}', \theta_i); \qquad (15)$$

and P's incentive condition for each lottery $\{y_i, y_i'; w_i\}$, $i = 1, \ldots, n$,

$$\int_{\theta_{i-1}}^{\theta_i} (w_i u^P(y_i, \theta) + (1 - w_i) u^P(y_i', \theta)) f(\theta) d\theta \geq \int_{\theta_{i-1}}^{\theta_i} u^P(\widetilde{y}, \theta) f(\theta) d\theta \tag{16}$$

for $\widetilde{y} = y_j, y_j'$ and all $j = 1, \ldots, n$. If in addition,

$$\sum_{i=1}^{n} \int_{\theta_{i-1}}^{\theta_i} (w_i u^P(y_i, \theta) + (1 - w_i) u^P(y_i', \theta)) f(\theta) d\theta \geq \int_0^1 u^P(y^P, \theta) f(\theta) d\theta$$

so that P's expected payoff is greater than that from making an uninformed decision, then the above necessary conditions are also sufficient for PBE.

Third, we simplify the incentive conditions for P.

Lemma 6 *In any PBE, P's incentive conditions (16) must all be slack except for $\widetilde{y} = y_{i-1}', y_i, y_i', y_{i+1}$. Further, if $y_i = y_i'$ for all i, then P's incentive conditions except for (3) are all slack.*

Proof: We first argue that adjacent incentive conditions are sufficient for all incentive conditions. Consider all P's incentive conditions for $\{y_i, y_i'; w_i\}$. Since P prefers $\{y_i, y_i'; w_i\}$ to y_{i-1}' conditional on $(\theta_{i-1}, \theta_i]$, the most preferred decision conditional on the interval is higher than y_{i-1}'. By the strict concavity of u^P, P strictly prefers y_{i-1}', and hence $\{y_i, y_i'; w_i\}$, to all decisions lower than y_{i-1}' conditional on $(\theta_{i-1}, \theta_i]$. By the same argument, P strictly prefers $\{y_i, y_i'; w_i\}$ to all decisions higher than y_{i+1} conditional on $(\theta_{i-1}, \theta_i]$.

Next, we argue that the adjacent upward incentive condition is satisfied if all induced lotteries are degenerate. To see this, note that in any partition equilibrium A's indifference conditions (2) hold. Since $u_{y\beta} > 0$, A being indifferent between y_i and y_{i+1} in state θ_i implies that P strictly prefers y_i to y_{i+1} in the same state. By $u^P_{y\theta} > 0$, P then prefers y_i to y_{i+1} for all $\theta < \theta_i$, and in particular, for any $\theta \in (\theta_{i-1}, \theta_i]$. **QED**

Fourth, we show that if an optimal PBE exists, then on the equilibrium path, P never randomizes over the set of decisions.

Lemma 7 *For each PBE in which lotteries are induced, there exists another PBE in which only degenerate lotteries are induced and P obtains a higher expected payoff.*

Proof: Fix some PBE with induced lotteries $\{y_i, y'_i; w_i\}$, $i = 1, \ldots, n$, in increasing order. We prove this lemma in two steps. First, we show that there is another PBE in which P's equilibrium choice of Y contains only y'_i, $i = 1, ..., n$, and each decision y'_i is induced in a degenerate lottery. Each new threshold $\widehat{\theta}_i$ is given by (15) where w_i and w_{i+1} are set to 0. Since $y_i \le y'_i \le y_{i+1} \le y'_{i+1}$, the concavity of $u^A(\cdot, \widehat{\theta}_i)$ and A's indifference condition at $\widehat{\theta}_i$ between y'_i and y'_{i+1} imply that $u^A(y_i, \widehat{\theta}_i) \le u^A(y'_i, \widehat{\theta}_i)$ and $u^A(y_{i+1}, \widehat{\theta}_i) \ge u^A(y'_{i+1}, \widehat{\theta}_i)$. Then, since $u^A_{y\theta} > 0$, using the implicit function theorem applied to (15) gives that the solution to (15) decreases in w_i and w_{i+1}, which implies that each new threshold $\widehat{\theta}_i$ is higher than the original threshold θ_i. The distribution function of the state θ conditional on $[\widehat{\theta}_{i-1}, \widehat{\theta}_i]$, given by $(F(\theta) - F(\widehat{\theta}_{i-1}))/(F(\widehat{\theta}_i) - F(\widehat{\theta}_{i-1}))$, first-order stochastically dominates the distribution function of the state θ conditional on $[\theta_{i-1}, \theta_i]$, because it is decreasing in $\widehat{\theta}_{i-1}$ and $\widehat{\theta}_i$. Since the difference $u^P(y'_i, \theta) - u^P(y'_{i-1}, \theta)$ is increasing in θ by the assumption of $u^P_{y\theta} > 0$, P prefers y'_i to y'_{i-1} conditional on $[\widehat{\theta}_{i-1}, \widehat{\theta}_i]$, because in the original PBE, P prefers y'_i to y'_{i-1} conditional on $[\theta_{i-1}, \theta_i]$. Since the downward incentive conditions are satisfied, Lemma 6 implies that we indeed constructed a new PBE.

Second, we show that P obtains a higher expected payoff in the new PBE than in the original PBE by transforming the original PBE into the new PBE in such a way that P's expected payoff continuously increases. We continuously decrease each lottery weight $\tilde{w}_i$ from w_i to 0, one lottery at a time starting at $i = 1$ and ending at $i = n$, while increasing thresholds $\tilde{\theta}_i$ and $\tilde{\theta}_{i-1}$ to always satisfy A's indifference conditions (15). Note that all other partition thresholds are unchanged when we continuously decrease $\tilde{w}_i$ alone. The partial derivative of P's expected payoff with respect to $\tilde{w}_i$ is given by

$$\int_{\tilde{\theta}_{i-1}}^{\tilde{\theta}_i} (u^P(y_i, \theta) - u^P(y'_i, \theta)) f(\theta) d\theta,$$

which is negative because P prefers y'_i to y_i conditional on $\left(\tilde{\theta}_{i-1}, \tilde{\theta}_i\right]$ (recall that P is indifferent between y'_i and y_i conditional on $(\theta_{i-1}, \theta_i]$, so the argument from the previous paragraph applies). Thus, as we decrease $\tilde{w}_i$ continuously, the direct effect on P's expected payoff is positive. The partial derivative of P's expected payoff with respect to $\tilde{\theta}_i$ is equal to $f\left(\tilde{\theta}_i\right)$

multiplied by

$$
\begin{aligned}
&\tilde{w}_i u^P(y_i, \tilde{\theta}_i) + (1-\tilde{w}_i) u^P(y_i', \tilde{\theta}_i) - (w_{i+1} u^P(y_{i+1}, \tilde{\theta}_i) + (1-w_{i+1}) u^P(y_{i+1}', \tilde{\theta}_i)) \\
&= \tilde{w}_i (w_{i+1}(u^P(y_i, \tilde{\theta}_i) - u^P(y_{i+1}, \tilde{\theta}_i)) + (1-w_{i+1})(u^P(y_i, \tilde{\theta}_i) - u^P(y_{i+1}', \tilde{\theta}_i))) \\
&\quad + (1-\tilde{w}_i)(w_{i+1}(u^P(y_i', \tilde{\theta}_i) - u^P(y_{i+1}, \tilde{\theta}_i)) + (1-w_{i+1})(u^P(y_i', \tilde{\theta}_i) - u^P(y_{i+1}', \tilde{\theta}_i))) \\
&> \tilde{w}_i (w_{i+1}(u^A(y_i, \tilde{\theta}_i) - u^A(y_{i+1}, \tilde{\theta}_i)) + (1-w_{i+1})(u^A(y_i, \tilde{\theta}_i) - u^A(y_{i+1}', \tilde{\theta}_i))) \\
&\quad + (1-\tilde{w}_i)(w_{i+1}(u^A(y_i', \tilde{\theta}_i) - u^A(y_{i+1}, \tilde{\theta}_i)) + (1-w_{i+1})(u^A(y_i', \tilde{\theta}_i) - u^A(y_{i+1}', \tilde{\theta}_i))) \\
&= \tilde{w}_i u^A(y_i, \tilde{\theta}_i) + (1-\tilde{w}_i) u^A(y_i', \tilde{\theta}_i) - (w_{i+1} u^A(y_{i+1}, \tilde{\theta}_i) + (1-w_{i+1}) u^A(y_{i+1}', \tilde{\theta}_i)) \\
&= 0,
\end{aligned}
$$

where the inequality follows from $u_{y\beta} > 0$, and the last equality follows from A's indifference condition between $\{y_i, y_i'; \tilde{w}_i\}$ and $\{y_{i+1}, y_{i+1}'; w_{i+1}\}$ in state $\tilde{\theta}_i$. Because we replace one lottery at a time starting at $i = 1$, the lottery $\{y_{i-1}, y_{i-1}'; w_{i-1}\}$ must be degenerate. By construction $\widetilde{w}_{i-1} = 0$ when we decrease $\widetilde{w}_i$, so analogously the partial derivative of P's expected payoff with respect to $\tilde{\theta}_{i-1}$ is equal to $f\left(\tilde{\theta}_{i-1}\right)$ multiplied by

$$
\begin{aligned}
&u^P(y_{i-1}', \tilde{\theta}_{i-1}) - (\tilde{w}_i u^P(y_i, \tilde{\theta}_{i-1}) + (1-\tilde{w}_i) u^P(y_i', \tilde{\theta}_{i-1})) \\
&= \tilde{w}_i (u^P(y_{i-1}', \tilde{\theta}_{i-1}) - u^P(y_i, \tilde{\theta}_{i-1})) + (1-\tilde{w}_i)(u^P(y_{i-1}', \tilde{\theta}_{i-1}) - u^P(y_i', \tilde{\theta}_{i-1})) \\
&> \tilde{w}_i (u^A(y_{i-1}', \tilde{\theta}_{i-1}) - u^A(y_i, \tilde{\theta}_{i-1})) + (1-\tilde{w}_i)(u^A(y_{i-1}', \tilde{\theta}_{i-1}) - u^A(y_i', \tilde{\theta}_{i-1})) \\
&= u^A(y_{i-1}', \tilde{\theta}_{i-1}) - (\tilde{w}_i u^A(y_i, \tilde{\theta}_{i-1}) + (1-\tilde{w}_i) u^A(y_i', \tilde{\theta}_{i-1})) \\
&= 0.
\end{aligned}
$$

Thus, as we decrease $\tilde{w}_i$ continuously, the indirect effects of increased $\tilde{\theta}_{i-1}$ and $\tilde{\theta}_i$ on P's expected payoff are also positive. Finally, if we suppose that at least one induced lottery in the original PBE is non-degenerate, then the direct effect will be strictly positive, which implies that P's expected payoff is strictly higher in the new PBE. **QED**

Fifth and last, we show that an optimal PBE exists. Combining the above lemmas, we have already established that an optimal PBE, if one exists, is a solution to the constrained maximization problem where the objective is P's expected payoff and the feasible choices are all partition equilibria with a finite number of elements.

Lemma 8 *An optimal PBE exists.*

Proof: Let us consider a relaxed problem in which strict inequalities of the partition condition (1) are replaced with weak inequalities. By Lemma 5, the number of induced decisions n is uniformly bounded. Thus, the relaxed problem is a constrained maximization problem with finitely many variables. There exists $\overline{y}$ such that we can impose $|y_i| \leq \overline{y}$ for all $i = 1, \ldots, n$ without affecting the maximization problem.[26] These constraints, $|y_i| \leq \overline{y}$, together with a finite number of constraints (2) and (3) determine the compact set for variables $\{\theta_i\}_{i=0}^{n}$, $\{y_i\}_{i=1}^{n}$ over which the continuous function $\sum_{i=1}^{n} \int_{\theta_{i-1}}^{\theta_i} u^P(y_i, \theta) f(\theta) d\theta$ is maximized. Clearly, there exists a solution to this relaxed problem. Finally, we need to show that the value of the relaxed problem is achievable with strict inequalities (1), which will prove the existence of an optimal PBE. If some of θ_i or y_i coincide, we can take the maximal subset $\{\theta_i'\}_{i=0}^{n'} \subset \{\theta_i\}_{i=0}^{n}$ and a corresponding subset of induced decisions $\{y_i'\}_{i=1}^{n'} \subset \{y_i\}_{i=1}^{n}$ such that all θ_i' and y_i' are distinct. These $\{\theta_i'\}_{i=0}^{n'}$ and $\{y_i'\}_{i=1}^{n'}$ will satisfy (2)-(3) and strict inequalities of the partition condition (1). Moreover, this modification does not change P's expected payoff. **QED**

This concludes the proof of Proposition 1. **QED**

Proof of Proposition 2: Consider a CS equilibrium $(\{\theta_i\}_{i=0}^{n}, \{y_i\}_{i=1}^{n})$ with $n \geq 2$. We prove that for any sufficiently small δ, there exists a PBE with P's equilibrium choice $\left\{y_i^{\delta,j}\right\}_{i=1}^{n} \equiv \{y_1, ..., y_j + \delta, ..., y_n\}$, and the corresponding partition $\left\{\theta_i^{\delta,j}\right\}_{i=0}^{n} \equiv \{\theta_0, ..., \theta_{j-1}(\delta), \theta_j(\delta), ..., \theta_n\}$. Moreover, we prove that P's expected payoff in this PBE is strictly higher than in the CS equilibrium. By the implicit function theorem applied to A's indifference condition (2), $\theta_{j-1}(\delta)$

[26] There can be at most one induced decision above $y(1)$ and one induced decision below $y(0)$. Moreover, there is at least one induced decision in $[y^P(0), y^P(1)]$. Let us define $g_1(y_2, \theta_1)$ as y_1 that solves $u^A(y_1, \theta_1) = u^A(y_2, \theta_1)$ and $g_n(y_{n-1}, \theta_{n-1})$ as y_n that solves $u^A(y_{n-1}, \theta_{n-1}) = u^A(y_n, \theta_{n-1})$. The functions g_1 and g_n are decreasing in the first argument and increasing in the second argument which implies that $y_1 \geq g_1(y^P(1), 0)$ and $y_n \leq g_n(y^P(0), 1)$. Therefore, $|y_i| \leq \max\{|g_1(y^P(1), 0)|, |g_n(y^P(0), 1)|\} \equiv \overline{y}$ for all i.

and $\theta_j(\delta)$ are continuous functions in a neighborhood of $\delta = 0$ with

$$\left.\frac{d\theta_{j-1}(\delta)}{d\delta}\right|_{\delta=0} = \frac{-u_y^A(y_j,\theta_{j-1})}{u_\theta^A(y_j,\theta_{j-1}) - u_\theta^A(y_{j-1},\theta_{j-1})} \text{ for } j \neq 1,$$

$$\left.\frac{d\theta_j(\delta)}{d\delta}\right|_{\delta=0} = \frac{u_y^A(y_j,\theta_j)}{u_\theta^A(y_{j+1},\theta_j) - u_\theta^A(y_j,\theta_j)} \text{ for } j \neq n.$$

For $j = 1$ and $j = n$ we have $\left.\frac{d\theta_0(\delta)}{d\delta}\right|_{\delta=0} = \left.\frac{d\theta_n(\delta)}{d\delta}\right|_{\delta=0} = 0$ because $\theta_0(\delta) = 1 - \theta_n(\delta) = 0$.

In the CS equilibrium, $\int_{\theta_{i-1}}^{\theta_i} u^P(y_i,\theta)f(\theta)d\theta > \int_{\theta_{i-1}}^{\theta_i} u^P(y_{i-1},\theta)f(\theta)d\theta$ for all i because $y_i = \arg\max_{y\in\mathbb{R}} \int_{\theta_{i-1}}^{\theta_i} u^P(y,\theta)f(\theta)d\theta$. Therefore, incentive conditions (3), $\int_{\theta_{i-1}^{\delta,j}}^{\theta_i^{\delta,j}} u^P(y_i^{\delta,j},\theta)f(\theta)d\theta > \int_{\theta_{i-1}^{\delta,j}}^{\theta_i^{\delta,j}} u^P(y_{i-1}^{\delta,j},\theta)f(\theta)d\theta$, hold for all i because functions $u^P(y,\theta)$, $\theta_{j-1}(\delta)$, $\theta_j(\delta)$ are continuous, and δ is sufficiently small.

The derivative of P's expected payoff with respect to δ at $\delta = 0$ is given by

$$\left(u^P(y_{j-1},\theta_{j-1}) - u^P(y_j,\theta_{j-1})\right) f(\theta_{j-1}) \left.\frac{d\theta_{j-1}(\delta)}{d\delta}\right|_{\delta=0} +$$
$$\left(u^P(y_j,\theta_j) - u^P(y_{j+1},\theta_j)\right) f(\theta_j) \left.\frac{d\theta_j(\delta)}{d\delta}\right|_{\delta=0} + \int_{\theta_{j-1}}^{\theta_j} u_y^P(y_j,\theta)f(\theta)d\theta$$

The last term in the above expression is 0 because $y_j = \arg\max_{y\in\mathbb{R}} \int_{\theta_{j-1}}^{\theta_j} u^P(y,\theta)f(\theta)d\theta$. The second term is positive for $j \neq n$ because $u^P(y_j,\theta_j) - u^P(y_{j+1},\theta_j) > 0$ holds by (2) and $u_{y\beta} > 0$; and $\left.\frac{d\theta_j(\delta)}{d\delta}\right|_{\delta=0} > 0$ holds by $u_y^A(y_j,\theta_j) > 0$ and $u_\theta^A(y_{j+1},\theta_j) - u_\theta^A(y_j,\theta_j) > 0$. More specifically, $u_y^A(y_j,\theta_j) > 0$ holds by (2) and $u_{yy}^A > 0$, whereas $u_\theta^A(y_{j+1},\theta_j) - u_\theta^A(y_j,\theta_j) > 0$ holds by $u_{y\theta}^A > 0$. Analogously, the first term is positive for $j \neq 1$. To sum up, the above expression is positive for all j. **QED**

Proof of Proposition 3: For the "only if" part, recall from Proposition 1 that any optimal PBE is a partition equilibrium. The claim then follows immediately, because any partition equilibrium can be implemented by delegation under full commitment.

For the "if" part, we use a result obtained by Alonso and Matouschek (2008). They show that if delegation is valuable then there exists $\theta^* \in (0,1)$ such that $S(\theta^*) < 0 < T(\theta^*)$, where $T(\theta^*) \equiv \int_0^{\theta^*} \left(y^A(\theta^*) - y^P(\theta)\right) f(\theta)\,d\theta$ and $S(\theta^*) \equiv \int_{\theta^*}^1 \left(y^A(\theta^*) - y^P(\theta)\right) f(\theta)\,d\theta$. The decision rule $\widetilde{y}(\theta)$ given by

$$\widetilde{y}(\theta) = \begin{cases} y^A(\theta^*) + T(\theta^*) - S(\theta^*) & \text{if } \theta > \theta^*, \\ y^A(\theta^*) + S(\theta^*) - T(\theta^*) & \text{if } \theta \leq \theta^*, \end{cases}$$

satisfies A's indifference condition (2) at θ^* because A's payoff function is symmetric around $y^A(\theta)$. Moreover, the difference in P's expected payoff under $\widetilde{y}(\theta)$ and the uninformative decision y^P is equal to $-4T(\theta^*)S(\theta^*)$, and is positive. The decision rule $\widetilde{y}(\theta)$ satisfies P's incentive condition (3) that P prefers decision $y^A(\theta^*)+T(\theta^*)-S(\theta^*)$ to $y^A(\theta^*)+S(\theta^*)-T(\theta^*)$, as otherwise she would prefer decision $y^A(\theta^*)+S(\theta^*)-T(\theta^*)$ to $\widetilde{y}(\theta)$. Thus, $\widetilde{y}(\theta)$ can be supported as a PBE. **QED**

Proof of Lemma 1: Suppose that in the optimal PBE, $y_i \leq y_i^*$ for some $i = 2, \ldots, n$. We first show by contradiction that P's $(i+1)$-th incentive condition binds. Suppose not. Consider marginally increasing y_i, keeping all other decisions unchanged. We know from the proof of Proposition 2 that θ_{i-1} and θ_i both increase, with all other thresholds unaffected. By the concavity of $u^P(\cdot, \theta)$, P's i-th incentive condition is slack and hence unaffected because $y_{i-1} < y_i \leq y_i^*$. Similarly, her $(i-1)$-th incentive condition remains satisfied because θ_{i-1} increases as y_i increases. The proof of Proposition 2 has already established that P's expected payoff is increased when either θ_i or θ_{i-1} increases. Her expected payoff is further increased because y_i moves closer to her ex post optimal decision y_i^* on $(\theta_i, \theta_{i+1}]$. A contradiction.

The above result immediately implies that $y_n > y_n^*$. For each $i = 2, ..., n-2$, note that P's $(i+1)$-th incentive condition binding implies that $y_i < y_{i+1}^* < y_{i+1}$, so we can rewrite it as $y_{i+1} + y_i = \theta_{i+1} + \theta_i$. Using (4) for θ_i and θ_{i+1}, we then have

$$y_{i+2} - y_i = 4b.$$

However, if $y_i \leq y_i^*$, from A's indifference conditions (4) we would have

$$y_{i+1} - y_i \geq 4b + (y_i - y_{i-1}) > 4b,$$

which contradicts P's binding $(i+1)$-th incentive condition. This establishes the lemma for $i = 2, \ldots, n-2$.

Next, we show that $y_{n-1} > y_{n-1}^*$. Suppose not. Consider marginally increasing y_{n-1} and decreasing y_n in such a way that θ_{n-1} remains unchanged. Then P's n-th incentive condition is unaffected. However, this increases P's expected payoff because by assumption $y_{n-1} \leq y_{n-1}^*$, $y_n > y_n^*$, and because θ_{n-2} increases as a result of increasing y_{n-1}. A contradiction.

Finally, it can be verified using the proof of Proposition 5 that $y_1 > y_1^*$ at the optimal PBE. Note that this result is not needed for rewriting P's incentive conditions (5). **QED**

Proof of Proposition 4: Adding up condition (9) for $i = 2, \ldots, n-1$, we have

$$y_n + y_{n-1} - (y_1 + y_2) \geq 4b(n-2).$$

Also, using conditions (8) and (10), we have $2b(n-1) < 1$, or $n < 1/(2b) + 1$.

For the converse, let n be a positive integer strictly less than $1/(2b) + 1$. By definition of N, $1/(2N) \leq b < 1/(2(N-1))$. Note that $N \geq 2$ if and only $b < \frac{1}{2}$.

If $n = 1$, then there exists a babbling equilibrium with the induced decision $\overline{y}_1(1) = \frac{1}{2}$.

If $n = 2$, suppose that $b < \frac{1}{2}$ and consider the "full commitment" problem of choosing two decisions y_1 and y_2 with $0 \leq y_1 \leq y_2 \leq 1$ that maximizes P's expected payoff

$$U(2) = -\int_0^{\theta_1} l(|y_1 - \theta|)\, d\theta - \int_{\theta_1}^1 l(|y_2 - \theta|)\, d\theta,$$

subject only to A's indifference condition $\theta_1 = \frac{1}{2}(y_1 + y_2) - b$. The first order conditions with respect to y_1 and y_2 are

$$\frac{\partial U(n)}{\partial y_1} = \frac{1}{2}(l(y_2 - \theta_1) + l(|y_1 - \theta_1|)) - l(y_1) = 0;$$
$$\frac{\partial U(n)}{\partial y_2} = -\frac{1}{2}(l(y_2 - \theta_1) + l(|y_1 - \theta_1|)) + l(1 - y_2) = 0.$$

The above conditions imply that $y_1 = 1 - y_2$. It is straightforward to verify that the second order condition is satisfied. The above first order conditions become identical, and we can rewrite it as

$$2l((1 - \Delta)/2) = l(b + \Delta/2) + l(|b - \Delta/2|),$$

where $\Delta = y_2 - y_1$. By the convexity of l, the right hand side is increasing in Δ, so there is a unique $\Delta \in (0, 1)$ satisfying the above condition. The solution to the full commitment problem is then given by $\overline{y}_1(2) = \frac{1}{2}(1 - \Delta)$ and $\overline{y}_2(2) = \frac{1}{2}(1 + \Delta)$, with $\theta_1 = \frac{1}{2} - b$. Note that $b < \frac{1}{2}$ implies that $\theta_1 > 0$. The incentive condition of P is satisfied at this solution, because $\overline{y}_1(2) + \overline{y}_2(2) = 1 < 1 + \theta_1$. We can thus take the solution to the full commitment problem to be a limited authority PBE. Since the solution has $\overline{y}_2(2) - \overline{y}_1(2) = \Delta > 0$, it gives P a strictly higher payoff than making the uninformed decision of $\frac{1}{2}$.

Finally, for any $n \geq 3$, consider the set of n decisions $\overline{Y}(n)$ given by $\overline{y}_i(n) = \frac{1}{2}+2b\left(i - \frac{n+1}{2}\right)$ for each $i = 1, \ldots, n$. Then, by A's indifference condition, $\theta_i = \overline{y}_i(n)$ for each $i = 1, \ldots, n-1$. It is straightforward to verify that conditions (8), (9) and (10) are all satisfied. The expected payoff for P under this construction is given by

$$\overline{U}(n) = -2\int_0^{\overline{y}_1(n)} l(\overline{y}_1(n) - \theta)\ d\theta - (n-1)\int_{\overline{y}_1(n)}^{\overline{y}_2(n)} l(\overline{y}_2(n) - \theta)\ d\theta. \tag{17}$$

It is straightforward to show that $U(n) > U(n-2)$: the difference is given by

$$\overline{U}(n) - \overline{U}(n-2) = 2\int_0^{\overline{y}_1(n)} \left(l(\overline{y}_2(n) - \theta) - l(\overline{y}_1(n) - \theta)\right)\ d\theta > 0,$$

where $\overline{y}_1(n)$ is the smallest decision under the construction for n, and $\overline{y}_2(n)$ the second smallest decision for n and the smallest for $n-2$ (that is, $\overline{y}_2(n) = \overline{y}_1(n-2)$).

The above argument immediately implies that the expected payoff to P under the above construction with n decisions is greater than making the uninformed decision of $\frac{1}{2}$ for all $n \geq 3$ and odd. To complete the proof of the proposition, we only need to show that the above construction $\overline{Y}(4)$ for $n = 4$ dominates making the uninformed decision of $\frac{1}{2}$ for P. (This step is necessary because the payoff formula (17) does not apply to the case of $n = 2$.) It is straightforward to show that

$$\begin{aligned}
&\int_0^1 l(|1/2 - \theta|)\ d\theta - \sum_{i=1}^{4}\int_{\theta_{i-1}}^{\theta_i} l(\overline{y}_i - \theta)\ d\theta \\
> \ & \left[\int_{\overline{y}_3}^{\overline{y}_3+b} l(\theta - 1/2)\ d\theta - \int_{\overline{y}_2}^{1/2} l(\overline{y}_3 - \theta)\ d\theta\right] + \left[\int_{1/2}^{\overline{y}_3} (l(\theta - 1/2) - l(\overline{y}_3 - \theta))\ d\theta\right] \\
&\quad + \left[\int_{\overline{y}_2}^{1/2} l(1/2 - \theta)\ d\theta - \int_{\overline{y}_3}^{\overline{y}_3+b} l(\theta - \overline{y}_3)\ d\theta\right] \\
= \ & 0,
\end{aligned}$$

where the first line follows because $\frac{1}{2}$ is a more extreme decision than the corresponding decisions $\overline{y}_1$, $\overline{y}_2$ and $\overline{y}_4$ outside the interval $[\overline{y}_2, \overline{y}_3 + b]$, and the second line follows because each term in the bracket is zero.[27] **QED**

[27]One integral that appears in $\overline{U}(4)$ is $\int_{\overline{y}_3}^{\overline{y}_4} l(\overline{y}_4 - \theta)\, d\theta$. It is equal to $\int_{\overline{y}_3}^{\overline{y}_4} l(\theta - \overline{y}_3)\, d\theta$ by a change of

Proof of Lemma 2: For $n = 1$, it is trivially true that $y_1^{FC}(1) = \frac{1}{2}$. For $n = 2$, $Y^{FC}(2)$ is derived in the proof of Proposition 4. Fix any $n \geq 3$. Arguments similar to the proof of Lemma 8 in Proposition 1 can show that $Y^{FC}(n)$ exists. We will guess and verify later that conditions (7), (8), and (11) are not binding at $Y^{FC}(n)$. Denote $\Delta_i = y_{i+1}^{FC} - y_i^{FC}$.

To get a contradiction, suppose that there is i such that $\Delta_i \neq \Delta_{i-1}$. The derivative of P's expected payoff with respect to y_i is

$$\begin{aligned} \frac{\partial U(n)}{\partial y_i} &= \frac{1}{2}\left[l(y_{i+1} - \theta_i) - l(y_i - \theta_{i-1})\right] - \frac{1}{2}\left[l(|y_{i-1} - \theta_{i-1}|) - l(|y_i - \theta_i|)\right] \\ &= \frac{1}{2} l\left(\frac{\Delta_i}{2} + b\right) + \frac{1}{2} l\left(\left|\frac{\Delta_i}{2} - b\right|\right) - \frac{1}{2} l\left(\frac{\Delta_{i-1}}{2} + b\right) - \frac{1}{2} l\left(\left|\frac{\Delta_{i-1}}{2} - b\right|\right) \quad (18) \end{aligned}$$

where we used A's indifference conditions. Since l is convex, $\frac{\partial U(n)}{\partial y_i}$ has the same sign as $\Delta_i - \Delta_{i-1}$ regardless of whether $\Delta \geq 2b$ or $\Delta < 2b$. Thus, P's expected payoff can be increased by changing y_i to decrease $|\Delta_i - \Delta_{i-1}|$. A contradiction.

Thus, the optimal decisions satisfy $y_i^{FC} - y_{i-1}^{FC} = \Delta > 0$ for all $i = 2, \ldots, n$, so the optimum is interior. From A's indifference conditions, we have $\theta_i - \theta_{i-1} = \Delta$ for all $i = 2, \ldots, n-1$. Since the state is uniformly distributed, we can rewrite P's expected payoff as

$$U(n) = -\int_0^{\theta_1} l(|y_1 - \theta|)d\theta - (n-2)\int_{\theta_1}^{\theta_2} l(|y_2 - \theta|)d\theta - \int_{\theta_{n-1}}^{1} l(|y_n - \theta|)d\theta. \quad (19)$$

To find Δ, we differentiate (19) with respect to y_1 and y_n. From the two first order conditions we immediately have $l(y_1) = l(1 - y_n)$, and thus $y_1 = 1 - y_n = (1 - (n-1)\Delta)/2$. The two conditions then become identical, and are given by

$$\frac{\partial U(n)}{\partial y_1} = \frac{1}{2}\left(l(b + \Delta/2) + l(|b - \Delta/2|)\right) - l((1 - (n-1)\Delta)/2) = 0. \quad (20)$$

We claim that there exists a unique $\Delta \in (0, 1/(n-1))$ that solves (20). Since l is convex, $\partial U(n)/\partial y_1$ is strictly increasing in Δ regardless of whether $\Delta \geq 2b$ or $\Delta < 2b$, so there can be at most one value of Δ that solves (20). At $\Delta = 0$, we have $\partial U(n)/\partial y_1 < 0$ because by assumption $b < \frac{1}{2}$; and at $\Delta = 1/(n-1)$, we have $\partial U(n)/\partial y_1 > 0$. Thus, a unique

variables. The first part of the latter integral, from $\overline{y}_3$ to $\overline{y}_3 + b$, is the integral that appears in the last bracket.

$\Delta \in (0, 1/(n-1))$ exists that solves (20). Condition (20) is a necessary condition for Δ to be optimal. Since there exists a unique solution Δ, (20) is also sufficient.

To complete the derivation of $Y^{FC}(n)$, we verify that the dropped constraints are satisfied. Condition (7) is satisfied because $\Delta > 0$. Condition (8) is equivalent to $y_1 > b - \frac{1}{2}\Delta$. This is satisfied if $\Delta \geq 2b$ since $\Delta < 1/(n-1)$ implies that $y_1 > 0$; it also holds if $\Delta < 2b$, because in that case it is implied by (20). Finally, condition (11) is satisfied because given $y_n = 1 - y_1$ it is implied by (7). **QED**

Proof of Lemma 3: The lemma follows immediately from the three claims below.

Claim 1 *P's incentive conditions (9) bind at* $Y^{LC}(n)$ *for* $b \in \left[b^{FC}(n), b^{LC}(n)\right)$.

Proof: To get a contradiction, without loss of generality suppose that there exists i, $i = 2, ..., n-2$, such that $y_{i+2} - y_i = 4b$ and $y_{i+1} - y_{i-1} > 4b$ at $Y^{LC}(n)$. Denote $\Delta_i = y_{i+1} - y_i$. Below we will change one decision y_k in such a way that all conditions (7)-(10) are still satisfied and $|\Delta_k - \Delta_{k-1}|$ is decreased. Condition (18) then implies that P's expected payoff increases with this change, leading to a contradiction. If $\Delta_i \geq \Delta_{i-1}$ (and thus $\Delta_i > 2b > \Delta_{i+1} = 4b - \Delta_i$), then decrease y_{i+1} slightly. If $\Delta_i > \Delta_{i-1}$, then there are two cases. If $i - 1 = 1$ or $y_i - y_{i-2} > 4b$, then decrease y_i slightly, otherwise ($y_i - y_{i-2} = 4b$), increase y_{i-1} slightly. **QED**

Claim 2 *For any* $n \geq 3$ *and odd, and* $b \in \left(b^{FC}(n), b^{LC}(n)\right)$, $Y^{LC}(n)$ *is given by* $y_i^{LC} = \frac{1}{2} + 2b\left(i - \frac{n+1}{2}\right)$ *for all* $i = 1, ..., n$.

Proof: By Claim 1, $y_{i+2} - y_i = 4b$ for all $i = 1, \ldots, n-2$. Then, $y_i = y_1 + 2b(i-1)$ for i odd, and $y_i = y_2 + 2b(i-2)$ for i even. Further, $\theta_i - \theta_{i-1} = 2b$ for all $i = 2, \ldots, n-1$. Using the assumption that the state is uniformly distributed, we can rewrite P's expected payoff (6) as

$$\begin{aligned} U(n) &= -\int_0^{\theta_1} l(|y_1 - \theta|)d\theta - \frac{n-1}{2}\int_{\theta_1}^{\theta_2} l(|y_2 - \theta|)d\theta \\ &\quad -\frac{n-3}{2}\int_{\theta_1}^{\theta_2} l(|y_1 + 2b - \theta|)d\theta - \int_{\theta_{n-1}}^{1} l(|y_n - \theta|)d\theta. \qquad (21) \end{aligned}$$

The first order conditions with respect to y_1 and y_2 are

$$\frac{\partial U(n)}{\partial y_1} = -l(y_1) + l(1-y_n) - \frac{n-1}{4}[l(3b-\Delta_1/2) - l(b+\Delta_1/2)] = 0;$$
$$\frac{\partial U(n)}{\partial y_2} = \frac{n-1}{4}[l(3b-\Delta_1/2) - l(b+\Delta_1/2)] = 0$$

where $\Delta_1 = y_2 - y_1$. It follows immediately that $y_1 = 1 - y_n$ and $\Delta_1 = 2b$. Furthermore, it is straightforward to verify that the second order condition with respect to y_1 and y_2 are satisfied at $y_1 = 1 - y_n$ and $\Delta_1 = 2b$. Finally, (7) is satisfied because $\Delta_1 \in (0, 4b)$, and (8) and (10) are equivalent to $2b(n-1) < 1$, and thus are satisfied because $b < b^{LC}(n)$. **QED**

Claim 3 *For any $n \geq 2$ and even, and $b \in \left(b^{FC}(n), b^{LC}(n)\right)$, $Y^{LC}(n)$ is given by $y_i^{LC} = \frac{1-\Delta_1}{2} + 2b\left(i - \frac{n}{2}\right)$ for odd i, and $y_i = \frac{1+\Delta_1}{2} + 2b\left(i - \frac{n+2}{2}\right)$ for even i, where $\Delta_1 < 2b$ is uniquely determined by (14).*

Proof: Similar to Claim 2, we can rewrite P's expected payoff (6) as:

$$U(n) = -\int_0^{\theta_1} l(|y_1-\theta|)d\theta - \frac{n-2}{2}\int_{\theta_1}^{\theta_2} l(|y_2-\theta|)d\theta - \frac{n-2}{2}\int_{\theta_1}^{\theta_2} l(|y_1+2b-\theta|)d\theta - \int_{\theta_{n-1}}^{1} l(|y_n-\theta|)d\theta. \quad (22)$$

The first order conditions with respect to y_1 and y_2 are

$$-l(y_1) + \frac{1}{2}[l(b+\Delta_1/2) + l(|b-\Delta_1/2)] - \frac{n-2}{4}[l(3b-\Delta_1/2) - l(b+\Delta_1/2)] = 0;$$
$$l(1-y_n) - \frac{1}{2}[l(b+\Delta_1/2) + l(|b-\Delta_1/2)] + \frac{n-2}{4}[l(3b-\Delta_1/2) - l(b+\Delta_1/2)] = 0$$

where $\Delta_1 = y_2 - y_1$. It follows immediately that $y_1 = 1 - y_n$ and Δ_1 satisfies (14). Furthermore, we can easily verify that the second order condition with respect to y_1 and y_2 are satisfied. Finally, (7) is satisfied because $\Delta_1 \in (0, 4b)$, and (8) and (10) are equivalent to $2b(n-1) < 1$, and thus are satisfied because $b < b^{LC}(n)$.

To see that there is a unique $\Delta_1 \in (0, 2b)$ that satisfies (14), note that since $b > b^{FC}(n)$, the left-hand side of (14) is strictly smaller than the right-hand side at $\Delta_1 = 2b$. As Δ decreases, the left-hand side of (14) increases while the right-hand side decreases because l

is convex. At $\Delta = 0$, the left-hand side of (14) is strictly greater than the right-hand side because $b < b^{LC}(n)$. It follows that there exists a unique $\Delta_1 \in (0, 2b)$ that satisfies condition (14). **QED**

Proof of Proposition 5: First we establish a series of claims.

Claim 4 *Suppose that* $l(z) = z^2$*. For each* $n \geq 3$*,* $dU^{FC}(n-1)/db > dU^{LC}(n)/db$ *for all* $b \in (b^{FC}(n), b^{FC}(n-1))$*, where* $U^{FC}(n-1)$ *and* $U^{LC}(n)$ *are* P*'s expected payoff under* $Y^{FC}(n-1)$ *and under* $Y^{LC}(n)$ *respectively.*

Proof: Consider $Y^{FC}(n-1)$. For $b < b^{FC}(n-1)$, from condition (13) for $n-1$ we have Δ given in Lemma 2 is strictly greater than $2b$. From (19) for $n-1$, using the Envelope Theorem we have

$$\frac{dU^{FC}(n-1)}{db} = -(n-2)[l(\Delta/2 + b) - l(\Delta/2 - b)].$$

It is straightforward to see from the first order condition (20) that $y_1^{FC}(n)$ is decreasing in n for fixed b and increasing in b for fixed n. Since $y_1^{FC}(n) = \frac{1}{2}(1 - 2b(n-1))$ at $b = b^{FC}(n)$, we have $y_1^{FC}(n-1) > \frac{1}{2}(1 - 2b(n-1))$ for all $b > b^{FC}(n)$. It then follows from the convexity of l that

$$\frac{dU^{FC}(n-1)}{db} > -(n-2)[l(2b + b/(n-2)) - l(b/(n-2))].$$

Using the assumption of $l(z) = z^2$, we immediately have

$$\frac{dU^{FC}(n-1)}{db} > -(n-1)l(2b).$$

Next, suppose that n is odd and consider $Y^{LC}(n)$. From (21), using the Envelope Theorem we have

$$\frac{dU^{LC}(n)}{db} = -2(n-1)[l(2b) - l(1/2 - (n-1)b)]. \tag{23}$$

Since $b > b^{FC}(n)$, from condition (13) we have $2l(1/2 - (n-1)b) < l(2b)$, and thus

$$\frac{dU^{LC}(n)}{db} < -(n-1)l(2b),$$

establishing the claim for the case of n odd.

Lastly, suppose that n is even and consider $Y^{LC}(n)$. From (22), using the Envelope Theorem we have

$$\frac{dU^{LC}(n)}{db} = -\frac{1}{2}(n-2)(n+1)l(3b - \Delta_1/2) + \frac{1}{2}n(n-3)l(b+\Delta_1/2) + (n-1)l(b-\Delta_1/2). \quad (24)$$

For fixed b, the above is clearly increasing in Δ_1. Evaluating the above at $\Delta_1 = 2b$, we then have

$$\frac{dU^{LC}(n)}{db} < -(n-1)l(2b),$$

establishing the claim for the case of n even. **QED**

Claim 5 *For any l that satisfies Assumption 2, and for each $n \geq 3$, $dU^{LC}(n)/db > dU^{LC}(n+1)/db$ for all $b \in (b^{FC}(n), b^{LC}(n+1))$.*

Proof: First, suppose that n is odd. Using the first order condition (14) for $n+1$ we can rewrite (24) for $n+1$ as

$$\frac{dU^{LC}(n+1)}{db} = -(n-1)[l(3b-\Delta_1/2) + l(b+\Delta_1/2)] - 2l(b+\Delta_1/2) + 2nl(y_1^{LC}(n+1)).$$

Since l is convex, we have

$$l(3b - \Delta_1/2) + l(b + \Delta_1/2) > 2l(2b).$$

Further, (14) implies that $l(b + \Delta_1/2) > l(y_1^{LC}(n+1))$. Thus,

$$\frac{dU^{LC}(n+1)}{db} < -2(n-1)[l(2b) - l(y_1^{LC}(n+1))].$$

The lemma then follows from $y_1^{LC}(n+1) < \frac{1}{2} - (n-1)b$ and (23).

Second, suppose that n is even. Using the first order condition (14) we can rewrite (24) as

$$\frac{dU^{LC}(n)}{db} = -(n+1)[l(b+\Delta_1/2)+l(b-\Delta_1/2)-2l(y_1^{LC}(n))]-(n-1)[l(b+\Delta_1/2)-l(b-\Delta_1/2)].$$

Since $\Delta_1 < 2b$, from (14) we have

$$l(b+\Delta_1/2) + l(b-\Delta_1/2) - 2l(y_1^{LC}(n)) < l(2b) - 2l(1/2 - (n-1)b).$$

Thus

$$\frac{dU^{LC}(n)}{db} > -(n+1)[l(2b) - 2l(1/2 - (n-1)b)] - (n-1)[l(b + \Delta_1/2) - l(b - \Delta_1/2)].$$

The lemma then follows from $\Delta_1 < 2b$ and (23) for $n+1$. **QED**

Claim 6 *Suppose that $l(z) = z^2$. Then, for any $n \geq 3$, $U^{FC}(n-1) > U^{LC}(n)$ at $b = b^{FC}(n-1)$.*

Proof: From Lemma 2 and condition (13), at $b^{FC}(n-1)$ all $n-1$ decisions in $Y^{FC}(n-1)$ are $2b^{FC}(n-1)$ apart, that is, Δ given in Lemma 2 is equal to $2b^{FC}(n-1)$. Further, from A's indifference conditions we have $\theta_i = y_i^{FC}(n-1)$ for all $i = 1, \ldots, n-2$. We distinguish two cases.

First, suppose that n is odd. By Lemma 3, all n decisions in $Y^{LC}(n)$ are also $2b^{FC}(n-1)$ apart, with $\theta_i = y_i^{LC}(n)$ for all $i = 1, \ldots, n-1$. Note that $y_1^{FC}(n-1) - y_1^{LC}(n) = b^{FC}(n-1)$. Using (19) and (21), we can show that the difference between P's expected payoff $U^{FC}(n-1)$ under $Y^{FC}(n-1)$ and $U^{LC}(n)$ under $Y^{LC}(n)$ is given by

$$U^{FC}(n-1) - U^{LC}(n) = \int_0^{2b^{FC}(n-1)} l(\theta)d\theta - 2\int_{y_1^{FC}(n-1)-b^{FC}(n-1)}^{y_1^{FC}(n-1)} l(\theta)d\theta.$$

Using the assumption of $l(z) = z^2$, we can explicitly compute $y_1^{FC}(n-1)$ in terms of $b^{FC}(n-1)$ and use it to show that the above is strictly positive.

Second, suppose that n is even. In this case, under $Y^{LC}(n)$ the thresholds θ_i remain evenly spaced, with $\theta_{i+1} - \theta_i = 2b^{FC}(n-1)$. Note that $y_1^{FC}(n-1) - y^{LC}(n) = \frac{1}{2}\Delta_1$ where Δ_1 as defined by condition (14). As in the case of odd n, using (19) and (22) we can show that the difference between P's expected payoff $U^{FC}(n-1)$ under $Y^{FC}(n-1)$ and $U^{LC}(n)$ under $Y^{LC}(n)$ is given by

$$\begin{aligned} U^{FC}(n-1) - U^{LC}(n) &= \int_{b^{FC}(n-1)-\Delta_1/2}^{b^{FC}(n-1)+\Delta_1/2} l(\theta)d\theta - 2\int_{y_1^{LC}(n)}^{y_1^{LC}(n)+\Delta_1/2} l(\theta)d\theta \\ &\quad +(n/2-1)\left[\int_{2b^{FC}(n-1)}^{3b^{FC}(n-1)-\Delta_1/2} l(\theta)d\theta - \int_{b^{FC}(n-1)+\Delta_1/2}^{2b^{FC}(n-1)} l(\theta)d\theta\right]. \end{aligned}$$

Using the assumption of $l(z) = z^2$, we can explicitly compute Δ_1 from equation (14), and use it to show that the above is strictly positive. **QED**

Now, observe that $Y^{LC}(n) = Y^{FC}(n)$ at $b = b^{FC}(n)$, and recall from Lemma 2 that $Y^{FC}(n) > Y^{FC}(n-1)$. Then, from Claim 4 and Claim 6, we have that for any $n \geq 3$, there exists $b(n, n-1)$ such that $U^{FC}(n-1) < U^{LC}(n)$ for $b \in (b^{FC}(n), b(n, n-1))$ and $U^{FC}(n-1) > U^{LC}(n)$ for $b \in (b(n, n-1), b^{FC}(n-1)]$. Next, since Claim 6 implies that $U^{LC}(n) = U^{FC}(n) > U^{LC}(n+1)$ at $b = b^{FC}(n)$, it follows from Claim 5 that $U^{LC}(n) > U^{LC}(n+1)$ for all $b \in [b^{FC}(n), b^{LC}(n+1))$. Finally, using the assumption of $l(z) = z^2$, we can easily show that for any $n \geq 3$, $\overline{U}(n) > U^{FC}(n-1)$ at $b = b^{LC}(n+1)$ where $\overline{U}(n)$ is P's expected payoff under $\overline{Y}(n) = \left\{\frac{1}{2} + 2b\left(i - \frac{n+1}{2}\right)\right\}_{i=1}^{n}$. Since $U^{LC}(n) \geq \overline{U}(n)$ by the definition of $Y^{LC}(n)$, from Claim 4 we have $b(n, n-1) > b^{LC}(n+1)$. This concludes the proof of Proposition 5. **QED**

Proof of Proposition 6: As it is clear from the proof of Proposition 5, P's expected payoff (6) can be bounded below and above by substituting $Y^{FC}(n)$ and $Y^{FC}(n+1)$, respectively, where n is the largest integer smaller than $\frac{1}{2b} - \sqrt{2} + 1$. Since $\Delta(n; b)$ and $\Delta(n+1; b)$ given by Lemma 2 are equal to $2b + O(b^2)$, P's expected payoff for $k = n, n+1$ is

$$\begin{aligned} U_n^P &= -\frac{1}{12}(k-1)\Delta^3 - (k-1)b^2\Delta - \frac{1}{12}(1-(k-1)\Delta)^3 \\ &= -\frac{(2b)^2}{12} - b^2 - 0 + o\left(b^2\right) = -\frac{4}{3}b^2 + o\left(b^2\right). \end{aligned}$$

Similarly, A's expected payoff is

$$U_n^A = -\frac{1}{12}(k-1)\Delta^3 - b^2(1-(k-1)\Delta) - \frac{1}{12}(1-(k-1)\Delta)^3 = -\frac{1}{3}b^2 + o\left(b^2\right).$$

QED

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