Free-riding and Unequal Pay in Symmetric Teams*

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Abstract

This paper studies optimal incentive pay in teams where agents exert substitute efforts. It shows that the principal may unequally reward identical agents for collective success to control the free-rider problem. The unequal pay effectively creates a "team leader" who overworks and symmetric "followers" who underwork. Such leader-follower teams are more likely to emerge if: (1) agents are more patient, fewer in number, or have larger productivity spillovers; (2) the principal cares more about or is less patient for project success. These findings contrast with the recent literature pinpointing complementary efforts as the source of unequal pay in symmetric teams.

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1 Introduction

The potential trade-off between incentive provision and equal pay in organizations has received much scholarly attention. In an influential paper that builds on Segal (2003), Winter (2004) shows the following in a static setting: an organization aiming to induce everyone to work may unequally reward identical individuals for their team success if and only if their efforts are strategic complements – the marginal return to one's effort increases with the others'.[1] Intuitively, the optimal rewards are set so generously for some team members that they are guaranteed to work hard. And motivated by their enthusiasm, others are paid progressively less for their effort. In other words, the organization takes advantage of the positive feedback in the team and saves on its incentive cost. This intuition is reinforced in similar contracting environments by Bose, Pal, and Sappington (2010), Winter (2010), Halac, Kremer, and Winter (2020), and Camboni and Porcellacchia (2022), among others.[2]

Besides explaining the distribution of various organizational ranks and job titles to comparable individuals, the above-mentioned literature carries another important message: unequal treatment of identical team members is not optimal if their efforts are strategic substitutes, which will be the case if free-riding as opposed to effort coordination is the main obstacle to team performance. The reason is that with strategic substitution, eliciting high effort from a team member via a high reward would only de-motivate others, requiring even higher rewards for them.[3] This observation is intriguing as it contrasts with that of the vast literature on the voluntary provision of a pure public good whose level is simply the total contribution. Most notably, in their seminal paper, Bergstrom, Blume, and Varian (1986) find that the free-rider problem is often the worst among homogenous contributors in that the total equilibrium contribution is the lowest.[4] After all, whatever one can do, so can identical others. The equal-treatment implication of the aforementioned literature is also important because, beginning with Holmström (1982), a large and growing literature (reviewed below) has mainly studied

[1] In Winter's setting, the organization first announces the reward or bonus profile for a successful project. Then, team members simultaneously choose whether to work or shirk. As Winter notes, asymmetric rewards for identical individuals would be less surprising if they were asymmetrically informed of each other's actions or if only a subset of them sufficed for the project's success.

[2] In particular, Bose et al. (2010) reach the same "if and only if" conclusion as Winter (2004) despite considering continuous effort choices. However, the nature of unequal pay in Bose et al., as well as Winter (2010), is slightly different from Winter (2004) in that, working sequentially on the project, the first-mover is paid less than the second-mover even though the former contributes more to the success.

[3] Goerg, Kube, and Zultan (2010) experimentally support this reasoning.

[4] Specifically, their Theorem 5(iii) states that "an equalizing wealth redistribution among donors of identical tastes will never increase the voluntary equilibrium supply of the public good."

the free-rider problem in teamwork.

This paper aims to reconcile the role of free-riding in team incentive pay. Specifically, I will present a parsimonious dynamic model of teamwork in which the participants' efforts are strategic substitutes due solely to the free-rider problem. Yet, the organization will optimally introduce a pay differential by essentially picking a "team leader" who overworks and symmetric "followers" who underwork, albeit not shirk, from the social standpoint. The dynamics in my model help generate sufficient free-riding incentives in the form of procrastination. However, as I will discuss later, they are not necessary for my main result: less intuitive static settings could yield the same conclusion.

My model builds on the familiar continuous-time R&D race framework, e.g., Lee and Wilde (1980), except that the identical agents are asked to work together, not compete, to achieve a single breakthrough. Consider, for instance, a pharmaceutical company hiring a group of scientists to develop a new drug or a firm forming a problem-solving team to deal with its supply chain issue. As in the previous studies, unable to distinguish agents' contributions to it, the principal can contract only on the team outcome: success or failure. Given her reward profile for success,[5] each agent privately chooses his (stationary) flow effort, which is his exponential rate of discovery. Therefore, the team's rate of breakthrough is the sum of the agents' efforts, making them perfect productive substitutes and in turn, strategic substitutes.

I first show that under the standard, convex cost of effort, the team has a unique (Nash) equilibrium in effort choices for any reward profile. *Conversely*, any effort profile can be uniquely engendered as equilibrium by a unique reward profile. The profit-maximizing principal's initial task is then to minimize the reward sum or her wage bill by choosing agents' efforts that aggregate to her desired success rate. As expected, the necessary condition for this minimization problem equates the *marginal pay* across agents. With identical agents, an equal-effort allocation would always satisfy this necessary condition but not the sufficiency. Indeed, the equal-effort allocation might even (locally) *maximize* the reward sum, indicating Bergstrom et al.'s insight at work. This is because in general, an agent's reward function is

[5]The reward structure describes a success bonus for each agent. I assume these bonuses are time-invariant. As Mason and Välimäki (2015) demonstrated for a single agent, this assumption is not without loss, but it makes our model easily comparable to static settings referenced above. Moreover, success bonuses that depend on the completion date appear less common in practice for innovative projects with a flexible timeline, which our model features. For instance, technology companies often pay predetermined bonuses for patent filings by their employees (https://www.wired.com/story/big-tech-patent-intellectual-property). Companies in other industries, such as manufacturing and banking, similarly reward their frontline workers for generating and implementing new ideas. (https://hbr.org/2022/09/how-your-company-can-encourage-innovation-from-all-employees).

S-shaped in his effort (convex turning concave) if the cost of effort is not too convex, the agent is relatively patient, and the desired aggregate effort is sufficiently large. To understand, notice that an agent's reward compensates him for his marginal cost *and* for not delaying his effort, i.e., not procrastinating. As such, the marginal pay increases at low levels of individual effort since much is left to free-ride on. In contrast, the marginal pay decreases at high individual effort levels since little is left for others to rely on.

An immediate implication of the S-shaped reward function is that the principal will have at most one team member, whom I call the "leader," with a high effort level, i.e., on the concave part of the reward function. If there were two, they would optimally be induced the same effort (to equate their marginal pay). But this would be suboptimal because, given concavity, the principal could strictly lower the incentive cost by slightly re-allocating the total effort between them, i.e., creating a slight pay heterogeneity. This intuition parallels Bergstrom et al.'s: the free-rider problem is the worst among homogenous agents, though only if they are significant contributors to the team project in our setting. Otherwise, the S-shaped reward function also implies that being on the convex part of the reward function, the rest of the agents, whom I call the "followers," will be induced the same low effort via the same low reward for the joint success.[6]

Next, I show that a leader-follower team is more likely to emerge if: (**a**) agents are more patient and thus more prone to procrastinating, (**b**) the principal cares more about or is less patient to achieve the breakthrough; hence, she targets a larger team effort, or (**c**) the team is smaller so that equal effort would mean significant effort for each member, intensifying the procrastination concerns for the principal. In a simple extension, I identify another factor in favor of the leader-follower team: positive effort spillovers à la Kamien, Muller, and Zang (1992), whereby agents may get inspired by each other's attempts at the breakthrough. Although agents' efforts remain perfect productive substitutes in this extension, their increased motivation to inspire each other worsens the free-rider problem, making unequal pay more likely.

Related Literature. Aside from those cited above, this paper relates to the strand of the literature that views teamwork as a dynamic public good provision with substitute efforts.

[6]Since agents take simultaneous actions, there is no leader or follower in the usual (Stackelberg) sense in my model. I use the terminology for convenience and similarly to Winter (2004), given that the organization may designate the highest-paid agent as the "project head" or "team captain." The difference is that the leader in my model also works disproportionately harder than the rest, though his pay need not increase at the same rate. It is also distinct from Hermalin's (1998) theory of leadership since no agent holds private information about the project's prospects.

See, for instance, Admati and Perry (1991), Fershtman and Nitzan (1991), Bonatti and Hörner (2011), Georgiadis (2015), Bonatti and Rantakari (2016), Cetemen, Hwang, and Kaya (2020), Ozerturk and Yildirim (2021), and Yildirim (2023).

In particular, while mainly focusing on the optimal team size, Georgiadis (2015) also discovers in his Proposition 7 the value of asymmetric contracts for two identical agents and quadratic cost of effort when the project is sufficiently close to completion; hence, the free-rider problem is severe enough. Besides the differences in our formal setups, I allow for more general cost functions and arbitrary team size, which are crucial for understanding the optimal team structure. Employing the same model as in this paper, Ozerturk and Yildirim (2021) examine in a second-best benchmark how a utilitarian planner would allocate the total "credit" of one for the breakthrough among agents of heterogeneous abilities. With convex *marginal* cost assumed in this paper, however, the planner would treat identical agents equally in their analysis. This difference is unsurprising because the planner is more biased toward minimizing the team's total cost and, in turn, equalizing marginal costs across the agents than a profit-maximizing principal. Using the same setup, Yildirim (2023) assumes identical exogenous rewards for team success and demonstrates, among other results, how the planner can efficiently allocate effort, i.e., equalize marginal costs in equilibrium by carefully teaming up agents of heterogeneous abilities. Although the ability-grouping is also done to alleviate the free-rider problem, it is not isomorphic to heterogenous rewards considered here. For instance, no heterogeneous-ability team can be efficient for log-concave effort cost, such as the iso-elastic specification, in Yildirim (2023). In contrast, the principal can create an asymmetric team via unequal rewards in the present paper.[7] In this sense, my paper also complements Franco, Mitchell, and Vereshchagina (2011), Kaya and Vereshchagina (2014), and Glover and Kim (2021), among others, who highlight the role of team composition in mitigating the free-rider problem.[8]

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 establishes the unique equilibrium in effort choices and characterizes the principal's cost-minimizing reward structure, followed by her profit-maximization problem in Section 4. Section 5 extends the analysis to effort spillovers. Finally, Section 6 concludes. The proofs of all formal results are contained in the Appendix.

[7] Of course, a large body of work following Holmström (1982) deals with the optimal pay in teams, but it is also primarily concerned with inducing efficiency.

[8] Prat (2002) also argues in favor of workforce heterogeneity but in a model with no effort choice.

2 Model

A risk-neutral principal hires $n \in \{2, 3, ...\}$ identical and risk-neutral agents to work on a joint project toward a single breakthrough. Agents exert unobservable effort over a continuous and infinite time horizon, $t \in \mathbb{R}_+$. Let $x_i(t) \in \mathbb{R}_+$ denote agent i's flow effort and discovery rate at time t. Hence, project completion follows a Poisson process with the aggregate rate $X(t) = \sum_i x_i(t)$. The cost of effort, $c(x_i(t))$, satisfies some standard convexity assumptions:

$$c',\, c'',\, c''' > 0, \text{ with } c(0) = c'(0) = c''(0) = 0 \text{ and } c'(\infty) = \infty. \tag{C1}$$

(C1) will guarantee a unique and interior equilibrium in the team for any positive reward schedule described below. As such, the analysis will require no equilibrium selection, and with every agent exerting effort, team size will be meaningful. Notice also that the convexity of the marginal cost favors equal pay, which will help identify the strategic reason for any unequal pay in our setup. Among others, (C1) holds for the iso-elastic cost specification: $c(x) = x^k/k$, $k > 2$.[9]

As is common in agency settings, the principal can contract only on the project outcome. In particular, she cannot observe the team's effort profile or distinguish who has made the breakthrough. To motivate the agents, the principal publicly announces a nonnegative and time-invariant reward schedule $\mathbf{v} = (v_1, ..., v_n) \in \mathbb{R}^n_+$ where v_i represents the bonus payment promised to agent i upon project's success.[10] There is no penalty for an unsuccessful project, as agents are assumed to be protected by limited liability. The principal values the breakthrough at $\pi > 0$. Furthermore, the discount rates for the agents and the principal are $r > 0$ and $r_p > 0$, respectively.

To keep the model close to its static counterparts, I restrict attention to stationary equilibrium strategies and drop the time index in the sequel.[11] Under stationary strategies, agent

[9] It is perhaps worth mentioning that the linearity of the individual discovery rate in the effort level is without loss because the aggregate rate is their sum. In general, agent i's rate could be some strictly increasing function $\psi(x_i)$ where $\psi(0) = 0$. However, any nonlinearity in i's rate could be subsumed in the cost of effort, c, by a change of variables: $x_i := \psi^{-1}(x_i)$.

[10] As mentioned in Footnote 5, the assumption of time-invariant rewards is not without a loss, but it suits the purpose of this paper, and such stationary rewards are used in practice.

[11] In Appendix B, I elaborate on this restriction. I establish that given any time-invariant reward profile, (1) if others follow stationary strategies, it is the best response for an agent to do the same, and (2) stationary strategies are without loss among continuously differentiable strategies in 2- and 3-agent teams. Although I conjecture the latter holds for any team size, I was unable to prove it mainly because the Jacobian matrix for laws of motion for the agents' equilibrium utilities is not symmetric due to heterogeneous rewards. Nonetheless, the point of this paper is made even with teams of 2 and 3 members.

i's program at any instant without the breakthrough can be recursively written as

$$ru_i = \max_{x_i} \left[X(v_i - u_i) - c(x_i) \right], \tag{1}$$

where u_i denotes agent i's expected discounted payoff. The RHS of (1) says that with no success at time t, agent i receives the reward v_i but forgoes his continuation payoff if the team succeeds with probability $X dt$ in the next instant. Regardless, he bears the flow cost $c(x_i)dt$. The LHS of (1) reflects the opportunity cost of staying in the team. From (1), agent i's program reduces to maximizing his expected discounted payoff expressed in terms of the team's effort profile:

$$u_i = \frac{X}{r+X} v_i - \frac{c(x_i)}{r+X}. \tag{2}$$

Remark 1 *(2) can be interpreted as follows if the (exponential) discount rate, r, is viewed as nature's fixed flow effort to "grab" the breakthrough before the team. Then, the term $p(X) \equiv X/(r+X)$ becomes the team's probability of winning against nature, in which case agent i receives the promised reward v_i. Furthermore, $d(X) \equiv 1/(r+X)$ corresponds to the expected duration the agent bears his effort cost until the team or nature makes the discovery. With this interpretation, (2) can be written: $u_i = p(X)v_i - d(X)c(x_i)$.*

The principal's expected discounted payoff conditional on the team's effort profile at $t = 0$ is verified to be

$$\begin{aligned} u_P &= \int_0^\infty e^{-r_P t} e^{-Xt} \left(\pi - \sum_i v_i \right) dt \\ &= \frac{X}{r_P + X} \left(\pi - \sum_i v_i \right), \end{aligned} \tag{3}$$

where the term $\pi - \sum_i v_i$ is her net return from the completed project.

It is evident from (3) that the principal cares only about the team's aggregate effort, X, and that her optimal reward schedule can be characterized in two intuitive steps in the spirit of Grossman and Hart (1983). First, we determine the minimum reward sum, $\sum_i v_i$, to implement a fixed aggregate effort X. Then, we find the optimal X that maximizes (3). As we will see below, the first step carries most of the insight in this investigation.

3 Cost-minimizing rewards

To fix ideas, I begin the analysis by establishing the first-best benchmark. This benchmark will help isolate the role of free-riding in the optimal rewards.

3.1 First-best: observable effort and equal pay

Suppose that the principal could contract on agents' effort levels. To elicit the total effort X at the minimum cost, she would then solve the following program:

$$\min_{\mathbf{v},\mathbf{x}} \sum v_i \tag{FB}$$

$$\text{s.t. } u_i \geq 0 \text{ for all } i \tag{IR}$$

$$\sum_i x_i = X \text{ (given } X) \tag{BC}$$

$$v_i \geq 0 \text{ for all } i. \tag{LL}$$

where the (IR) constraint ensures each agent's participation in the project, and the (BC) and (LL) refer to the "budget" and limited liability constraints. The unique solution to this program is equal pay.

Lemma 1 *Fix X. With contractible efforts, the optimal pay is equal across agents:*

$$x_i^{FB} = \frac{X}{n} \text{ and } v_i^{FB} = \frac{c(\frac{X}{n})}{X}.$$

If the principal could dictate an agent's effort level, she would only pay for its cost so that the agent breaks even. Formally, setting $u_i = 0$ in (2) would imply the reward function:

$$\widehat{v}(x_i) = \frac{c(x_i)}{X}. \tag{4}$$

Since the cost of effort is strictly convex, so is each agent's reward *given* the aggregate effort, X. Therefore, the total reward that elicits X is minimized when agents' marginal pays and, in turn, their marginal costs are equalized, which implies equal effort and equal pay for all. Note that the first-best reward schedule is socially efficient as it minimizes the total cost, $\sum c(x_i)$, of eliciting X.

3.2 Second-best: free-riding and unequal pay

In a second-best world, each agent privately chooses his effort: x_i is unobservable to his teammates and the principal at any time. Therefore, the latter's program is also subject to a team incentive compatibility constraint. Formally, the principal now solves

$$\min_{\mathbf{v},\mathbf{x}} \sum v_i \tag{SB1}$$
$$\text{s.t. (IR), (BC), (LL), and}$$
$$x_i = \arg\max_{\widehat{x}_i} u_i \text{ given } X_{-i} \text{ for all } i. \tag{IC}$$

The unobservability of effort engenders a simultaneous-move game within the team. Hence, the (IC) describes a "team equilibrium:" given the reward schedule $\mathbf{v} \in \mathbb{R}_+^n$, agent i best responds to his teammates' effort, X_{-i}. To understand i's equilibrium incentive and simplify (SB1), we differentiate (2) with respect to x_i and obtain the following first-order condition:[12]

$$c'(x_i)\,(r+X) - c(x_i) = rv_i. \tag{5}$$

The left-hand side of (5) is a *dynamic* marginal cost. Re-arranging its terms as

$$c'(x_i)r + [c'(x_i)x_i - c(x_i)] + c'(x_i)X_{-i},$$

notice that the first term, $c'(x_i)r$, refers to the marginal cost of increasing effort now rather than in the next instant. The second term, $[c_i'(x_i)x_i - c_i(x_i)]$, is the net increase in the flow cost of exerting effort x_i, and the last term, $c'(x_i)X_{-i}$, is the marginal opportunity cost of effort in case teammates make the breakthrough. The agent trades off the dynamic marginal cost against the dynamic marginal benefit rv_i on the right-hand side of (5), which is the time value of receiving the reward, v_i.

Remark 2 *Building on Remark 1, (5) can also be written as:* $c'(x_i) + d'(X)c(x_i) = p'(X)v_i$*. Since* $d'(X) < 0$*, the LHS represents agent i's net marginal cost per unit time after taking into account the expected reduction in his total cost due to a marginally faster arrival of breakthrough. The RHS is the expected marginal benefit.*

Inspecting (5) reveals that agent i's dynamic marginal cost is increasing in his effort, x_i, (since $c'' > 0$) and in the others', X_{-i}, which implies $\partial x_i/\partial X_{-i} < 0$. That is, teamwork is prone to free-riding: the breakthrough being a public good, each agent would slack if his teammates worked harder. The nature of the free-riding is, however, only intertemporal in our model, i.e.,

[12] The second-order condition is easily verified: $\frac{\partial^2 u_i}{\partial x_i^2} = -c''(x_i)(r+X) < 0$ whenever $\frac{\partial u_i}{\partial x_i} = 0$. The first-order condition in (5) could also be obtained from the recursive program (1) by noting $c'(x_i) = v_i - u_i$ and substituting for u_i from (2).

the agent's ability to delay the discovery by relying on the team's future effort. To see this, we insert (5) into (2) and find i's equilibrium payoff to be:

$$u_i = v_i - c'(x_i) = \frac{c'(x_i)X - c(x_i)}{r}.$$

Clearly, $u_i \to 0$ as $r \to \infty$. That is, unable to delay their actions, infinitely impatient agents would choose $x_i = c'^{-1}(v_i)$ regardless of others and command no positive expected payoffs or "rents." Building on this insight, the following result characterizes the (IC) and helps streamline (SB1).

Lemma 2 ***(team equilibrium)*** *For any reward structure* $\mathbf{v} \in \mathbb{R}_+^n$*, there is a unique team equilibrium. Conversely, for any effort vector* $\mathbf{x} \in \mathbb{R}_+^n$*, there exists a unique reward structure that implements* $\mathbf{x}$ *as team equilibrium. This reward structure is as follows:*

$$\overline{v}(x_i) = c'(x_i) + \underbrace{\frac{c'(x_i)X - c(x_i)}{r}}_{\overline{u}(x_i)}. \tag{6}$$

Moreover, given the total effort X*,*

(a) $\overline{v}'(x_i) > 0$*, with* $\overline{v}(0) = \overline{v}'(0) = 0$*.*

(b) $\overline{u}'(x_i) > 0$*, with* $\overline{u}(0) = 0$*.*

(c) $\overline{v}(x_i) > \overline{v}(x_j)$ *implies* $c'(x_i) > c'(x_j)$*.*

For a given reward schedule, the uniqueness of team equilibrium obtains because, as argued above, agents view their efforts as strategic substitutes, i.e., $\partial x_i / \partial X_{-i} < 0$. Conversely, solving for the reward v_i in (5), the reward schedule $\overline{v}(x_i)$ uniquely elicits a fixed effort profile, $\mathbf{x}$, as equilibrium. (6) indicates that the principal pays agent i for his marginal cost of effort, $c'(x_i)$, as she would in a static, single-agent moral hazard problem, *and* for not postponing this effort, which, as mentioned above, is also the source of the agent's positive rent. Parts (a) and (b) of Lemma 2 simply show that the principal needs to pay a more diligent agent more generously and leave a greater rent. Hence, equal work deserves equal pay in our framework. Part (a) further shows that it is virtually costless for the principal to elicit a small effort from each agent, which will later ensure an all-active team.[13] Part (c) adds to part (a) that anytime an agent is paid more than another, he is also induced to work inefficiently harder.

[13] Here, $\overline{v}'(0) = 0$ obtains since $c'(0) = c''(0) = 0$ by (C1).

Comparing (6) with (4), it is worth observing that unlike in the second-best, the first-best reward is independent of agents' discount rate, which makes sense in the absence of a free-rider problem. Consequently, an agent's first-best reward decreases with the others' effort, X_{-i}, whereas his second-best reward increases to prevent his procrastination.

Armed with Lemma 2 and the fact that (IC) implies (IR) due to limited liability,[14] (SB1) reduces to:

$$\min_{\mathbf{x}} \sum_i \overline{v}(x_i) \text{ s.t. } \sum_i x_i = X \text{ (given } X\text{).} \tag{SB}$$

A solution $\mathbf{x}^*$ for (SB) exists because $\overline{v}(x_i)$ is continuous over a compact set, $x_i \in [0, X]$ for all i. Since $\overline{v}(0) = \overline{v}'(0) = 0$ by Lemma 2(a), *every* solution must be interior: $x_i^* > 0$ for all i, ensuring an all-active team. Therefore, as with the first-best, the first-order condition of (SB) requires that the principal equate agents' marginal pays:

$$\overline{v}'(x_i^*) = \overline{v}'(x_j^*). \tag{7}$$

Clearly, the symmetric effort profile $\left(\frac{X}{n}, ..., \frac{X}{n}\right)$ always satisfies (7). It will be the solution to (SB) if, as in the first-best, the reward function in (6) is strictly convex given X. And this is the case for the agents who are sufficiently impatient or have sufficiently convex cost of effort, as confirmed in the next result.

Lemma 3 ***(equal pay)*** *Fix X. Then, $\overline{v}''(x) > 0$, and thus, equal pay is the unique solution to (SB) if r is sufficiently large or $c^{(4)}(x) \geq 0$ in $[0, X]$.*

Intuitively, very impatient agents hardly procrastinate. Similarly, when the cost of effort is sufficiently convex in the sense of a nonnegative fourth derivative (e.g., cubic power or higher for the iso-elastic specification), agents know they cannot rely much on their teammates. In each case, worried little about free-riding in the team, the principal pays agents primarily for their marginal costs of effort, $c'(x_i)$, which is strictly convex by our assumption (C1). Thus, the principal optimally induces equal effort via equal pay.

Lemma 3 implies that unequal pay can be optimal only when agents are relatively patient *and* their costs are not too convex, so free-riding is significant in the team. Formally, a necessary condition for unequal pay is that the reward function $\overline{v}$ not be convex everywhere in $[0, X]$. Indeed, if $\overline{v}(x_i^*) \neq \overline{v}(x_j^*)$, i.e., the optimal pay differs for some agents i and j, then $x_i^* \neq x_j^*$ in

[14] This follows because, with a nonnegative payment, an agent can always guarantee himself a nonnegative utility by exerting zero effort.

(7) and, in turn, $\overline{v}''(x^I) = 0$ for some x^I between x_i^* and x_j^* by the mean-value theorem, i.e., $\overline{v}$ has at least one inflection point. In fact, by the same argument, if the optimal pay schedule has m different rewards in the team, then the reward function must have at least $m-1$ inflection points. This suggests an intimate relationship between the number of inflection points in the reward function and the pay heterogeneity in the team. Lemma 4 indicates that $\overline{v}$ has at most one inflection if the effort cost is not too convex.

Lemma 4 *Fix X, and suppose $c^{(4)}(x) < 0$ in $[0, X]$. Then, the reward function $\overline{v}$ has at most one inflection point (convex turning concave) in $[0, X]$.*

According to Lemma 4, if the cost of effort is not too convex in the sense of $c^{(4)}(x) < 0$ in the relevant region, then, unlike in the first-best, the reward function $\overline{v}(x)$ can have a concave part but at most be S-shaped (convex turning concave), as depicted in Figure 1. For instance, under the iso-elastic cost, $c(x) = x^k/k$, it is readily verified from (6) that the unique inflection point is: $x^I = (k-2)(r+X)$.[15]

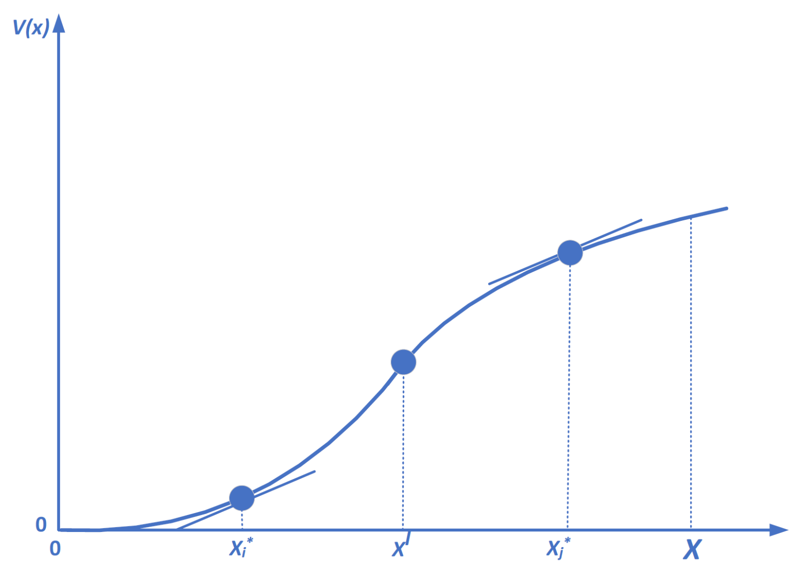

Figure 1. S-shaped reward function, $\overline{v}(x)$

The intuition behind an S-shaped reward function is that at low effort levels, the marginal pay accounts for an agent's increasing marginal cost and his strong incentive to free ride on

[15] The following is another cost family that satisfies $c^{(4)}(x) < 0$ and (C1): $c''(x) = \Pi_{j=0}^{J} \frac{x+2j}{x+2j+1}$ for any $J \in \{0, 1, ...\}$. For example, $c(x) = \frac{x(x+2)}{2} - (x+1)\ln(x+1)$ for $J = 0$. Using *Wolfram Mathematica*, others can be easily generated in closed form.

others' high residual effort. Although the marginal cost is increasing at any effort level ($c'' > 0$), the marginal pay must be decreasing at sufficiently high effort since the work left for others is too low to free ride on.[16,17]

The importance of an S-shaped reward function and how the principal manages free-riding in the team become evident in the next finding.

Proposition 1 *Fix X, and suppose $\overline{v}(x)$ is S-shaped in $[0, X]$. Then, the optimal reward schedule either pays $(n-1)$ agents equally and one higher, or it pays all n agents equally.*

To understand Proposition 1, note from Figure 1 that the optimal pay schedule cannot have multiple agents on the concave or high-effort part of the reward function. If it did, then the equal marginal pay condition in (7) would require equal effort, say x^*, for such agents. But, by the concavity of $\overline{v}$ in this region,

$$\overline{v}(x^* - \varepsilon) + \overline{v}(x^* + \varepsilon) < 2\overline{v}\,(x^*)\,.$$

This would imply that the principal could implement the total effort of $2x^*$ at a strictly lower cost between two agents on the concave part by slightly reallocating the effort between them. In other words, the principal optimally manages the severe free-riding problem among high-effort agents by having only one. A similar argument reveals that the optimal reward schedule must treat all agents equally in the convex region of $\overline{v}$.

Although agents move simultaneously in our model, the unequal pay schedule seems to create a "leader-follower team" with one agent being paid so generously that in the unique equilibrium, everyone expects him to work significantly harder – in fact, inefficiently harder per Lemma 2(c). As in Winter (2004), the "leader" could be the "project head" or "team captain." In contrast, the equal pay schedule creates a "horizontal team" with equal stakes participants. Proposition 2 identifies the conditions under which each team structure emerges. Corollary 1 parametrizes these conditions for the iso-elastic cost.

[16] Put differently, when the principal considers eliciting marginally more effort from an agent, she must simultaneously consider eliciting marginally less from some others to keep the total effort X fixed in her reward-minimization problem (SB). Therefore, what matters for an agent's marginal pay is the difference between the two considerations. Note that X being fixed plays a role in this intuition, but it is without loss per the two-step approach to the principal's problem. Otherwise, the reader will observe that v in (6) is always convex given X_{-i}: $v''(x_i) = \frac{1}{r}c''(x_i)\,(r + X) > 0$. However, it would be erroneous to conclude from this convexity that the optimal reward must be equal for all since $X_{-i} + x_i$ would no longer be fixed under such partial differentiation, invalidating the two-step approach and calling for another, which is bound to yield the same result.

[17] In light of Footnote 9, it is perhaps worth remarking that the free-riding incentive being significant only when $c^{(4)}(x) < 0$ is an artifact of our assumption that the discovery rate is linear in one's effort. To see this, suppose agent i's rate were $\sqrt{x_i}$ (instead of x_i), and his actual cost of effort were $x_i^{1.1}$. By a change of variables, this setting would be strategically equivalent to ours with the cost function $c(x_i) = x_i^{2.2}$.

Proposition 2 ***(unequal pay)*** *Fix X, and suppose $\overline{v}(x)$ is S-shaped with an inflection point x^I in $[0, X]$. Then, the optimal pay creates*

$$\begin{cases} \textit{a leader-follower team} & \textit{if} \quad X > nx^I \\ \textit{a horizontal team} & \textit{if} \quad X \le x^I \\ \textit{either team} & \textit{if} \quad x^I < X \le nx^I. \end{cases}$$

Corollary 1 ***(iso-elastic cost)*** *Fix X, and let $c(x) = x^k/k$, with $k \in (2, 2 + \frac{1}{n})$. Then, the optimal pay creates*

$$\begin{cases} \textit{a leader-follower team} & \textit{if} \quad X > r\frac{k-2}{2+\frac{1}{n}-k} \\ \textit{a horizontal team} & \textit{if} \quad X \le r\frac{k-2}{3-k} \\ \textit{either team} & \textit{if} \quad r\frac{k-2}{3-k} < X \le r\frac{k-2}{2+\frac{1}{n}-k}. \end{cases}$$

Proposition 2 says that if the principal seeks sufficiently high team effort in that the average per agent lies on the concave part of the reward function, it is optimal to have a leader-follower team. In fact, since $\overline{v}''(X/n) < 0$ in this case, equal pay would locally *maximize* the principal's wage bill in (SB). If, on the other hand, the principal seeks moderate team effort, $X \in (x^I, nx^I)$, so that $\overline{v}''(X/n) > 0$, a horizontal team is locally optimal. However, the principal can consider a non-local deviation, which ramps up the effort for one agent, pushing him to the concave part of the reward function, while scaling everyone else's effort down in the convex region. Such a non-local deviation, resulting in a leader-follower team, may or may not dominate the horizontal team for the principal, depending on the targeted level of team effort. Nevertheless, it would be inferior to a horizontal team if the principal targeted sufficiently low team effort, $X \le x^I$, in which case even the leader's effort would lie in the convex region. Using the iso-elastic cost, Corollary 1 reveals that a leader-follower team is more likely when agents have lower discount rate and less elastic cost. Example 1 illustrates Corollary 1.[18]

Example 1 *Let $n = 3$, $r = 1.85$, $k = 2.23$, and $X = 2.99$ Then, $x^I \approx 1.11$, and the leader and each follower's rewards are approximately 5.29 and $.90$, inducing efforts 2.11 and $.44$, respectively. Here, $\overline{v}''(X/3) \approx .08 > 0$, so the equal effort profile is only a local minimizer.*

Our last result in this section performs comparative statics with respect to team size.

[18] *Wolfram Mathematica* was used for simulations.

Proposition 3 ***(team size)*** *Fix X. If the optimal pay is unequal in a team, it is also unequal in a smaller team. Moreover, among the leader-follower teams,*

(a) *the pay range is larger in a smaller team.*

(b) *each follower's effort is increasing while the leader's effort is decreasing in team size.*

Perhaps contrary to conventional wisdom, Proposition 3 indicates that a leader-follower structure is more likely to be optimal in smaller teams. The reason is that given the fixed aggregate effort, equal pay would require each agent to do substantial work in a small team, which would then exacerbate the free-riding incentive and call for a leader-follower structure, as explained in Proposition 1. Part (a) indicates that the pay difference in leader-follower teams decreases with size. As part (b) highlights, such pay compression in larger teams occurs not only because the leader needs to exert lower effort when there is one more participant but also because each follower is paid higher to *increase* his effort with size. To understand the latter, refer to Figure 1 and consider adding one more agent i. Clearly, the principal would not keep x_i^* as it is; otherwise, given the fixed aggregate effort, this would mean a lower effort for j (the leader) and unequal marginal pays within the team, violating cost-minimization. If, on the other hand, the principal were to induce a lower effort than x_i^*, i.e., reduce the followers' effort and, in turn, reduce their marginal pay in a larger team, she would have to do so substantially to match the leader's marginal pay. But this would mean too much work for the leader. Thus, the principal increases each follower's effort in a larger team.

4 Profit-maximizing rewards

Although the principal's cost minimization problem (SB) contains most of the insight into her reward structure, I also briefly examine her profit-maximizing choice of aggregate effort in this section. Notice that in light of Proposition 1, (SB) reduces to:

$$w(X) = \min_{x \in [0, \frac{X}{n}]} (n-1)\overline{v}(x) + \overline{v}(X - (n-1)x), \tag{SB2}$$

where x denotes the effort level for the $(n-1)$ followers, and $w(X)$ denotes the principal's minimum wage bill to elicit X.

We know from the previous section that (SB2) has a unique and interior solution for any X. Moreover, it is verified that $w(X)$ is strictly increasing in X, with $w(0) = w'(0) = 0$.[19]

[19] These claims are proved within Proposition 4.

Using (SB2) and recalling (3), the principal finds her optimal X by solving:

$$\max_{X \geq 0} \frac{X}{r_P + X} \left(\pi - w(X)\right). \tag{PM}$$

Proposition 4 *Let X^* denote a solution to (PM). Then, (1) X^* exists, and it is positive, and (2) X^* is strictly increasing in π and r_P. Thus, under an iso-elastic cost of effort, if the principal chooses a leader-follower team for some $k \in (2, 2 + \frac{1}{n})$, r, r_P, and π, she also does so for all $k' \in (2, k]$, $r' \leq r$, $r'_P \geq r_P$, and $\pi' \geq \pi$.*

As expected, the principal will elicit greater team effort if, all else equal, she cares more about the breakthrough or has less patience to get it. Generally, the principal's desire for greater team effort has two opposing effects on her optimal pay structure. While cost efficiency favors equal pay, the agents' increased free-riding incentives call for unequal pay. Proposition 4 reveals that when the cost of effort is iso-elastic, the latter effect dominates, as evident from Corollary 1.[20] To illustrate, consider Example 1 with $r_P = 1.85$ and $\pi = 40$. Then, $X^* \approx 2.99$, implying the leader-follower structure in Example 1.

In the next section, I show that the principal will also lean toward a leader-follower structure when there are positive effort spillovers among agents.

5 An extension: effort spillover and unequal pay

In the base model, an agent's rate of discovery depends only on his effort. What makes agents a team is the fact that regardless of who achieves the breakthrough, all benefit from it. In practice, by working closely, team members may also benefit from each other's effort toward the breakthrough. Following Kamien, Muller, and Zang (1992) on research joint ventures, I formalize such effort spillovers by augmenting agent i's rate of discovery as:

$$y_i = x_i + \beta \sum_{j \neq i} x_j, \tag{8}$$

where $\beta \in [0, 1]$ is the commonly known probability that agent i gets inspired by his teammates' attempts at the breakthrough. In general, β may depend on how closely the agents work and communicate.

[20] The opposing effects can also be seen in Figure 1. First, the inflection point, x^I, grows with X, widening the reward function's convex region. This signifies the cost-efficiency effect. Second, given x^I, a higher X makes X/n more likely to lie in its concave part, highlighting the increased free-riding incentive.

Given (8), the team's breakthrough rate becomes

$$\sum_i y_i = \alpha X,$$

where $\alpha = 1 + (n-1)\beta$ represents each agent's marginal contribution to the team's rate of discovery.

The baseline analysis seamlessly extends to this case by substituting αX for X in (2). In particular, agent i's first-order condition becomes

$$c'(x_i)\left(\frac{r}{\alpha} + X\right) - c(x_i) = rv_i. \tag{9}$$

It is evident from (9) that agent i's dynamic marginal cost on the left-hand side of (9) is decreasing in the spillover rate, which enables the principal to decrease his reward, v_i. Intuitively, the amplified impact of his action on team's success motivates the agent, requiring less inducement. However, such increased motivation also exacerbates the free-rider problem in the team, leading the principal to favor a leader-follower (unequal pay) structure. Graphically, with the spillover, the reward function and its inflection point would both lie below those in Figure 1. For instance, under the iso-elastic cost, $x^I = (k-2)(\frac{r}{\alpha}+X)$, which is decreasing in α. Thus, applying Proposition 2 (or substituting αX for X in Corollary 1), it is immediate that the $X-$ regions for a leader-follower team expand with α, which is reported in Proposition 5.

Proposition 5 *Fix X. If , all else equal, the principal chooses a leader-follower team for the spillover rate β, she also does so for $\beta' \geq \beta$.*

6 Concluding remarks

The objective of this paper was to make a simple point: contrary to what is implied by the recent literature, e.g., Winter (2004) on teamwork with complementary efforts, an organization may also introduce asymmetric pay among symmetric team members solely to mitigate the free-rider problem when their efforts are (perfect) substitutes. Although it plays an important role, the difference in prediction is more than our modeling of effort as a continuous choice variable since Bose et al. (2010) reached the same conclusion as Winter (2004) with this feature. The contrast also stems from our model's dynamics that can create sufficient intertemporal free-riding incentives without inducing some team members to remain idle, which would obscure the meaning of team size.

I show that when sufficiently concerned about the free-riding incentive, the organization may alleviate it by promising one agent a substantially higher reward for the team's success than the rest of his teammates, leaving the latter much less work and, in turn, room to delay their actions. Thus, our model predicts a team with one "leader" who overworks and equally rewarded "followers" who underwork from the social viewpoint. Accordingly, the leader "deserves" his higher pay by working harder overall and at the margin than his teammates in our model. This observation is consistent with the general understanding that project leaders assume greater responsibility for the project's success.

Our investigation has also produced some testable implications. For instance, it predicts that a pharmaceutical company will more likely employ a leader-follower structure for its scientific team when it urgently needs a drug discovery, perhaps because of the competitive pressure. The same is true for manufacturers facing critical supply chain issues and forming problem-solving teams. Our investigation further implies that leader-follower teams are more likely in organizations whose employees work more closely, inspiring each other for success.

As previously stated, the model presented in this paper should be viewed as a reasonable set of sufficient conditions to convey its main point. In particular, unequal pay in symmetric teams may also emerge in static settings, to the extent that they are easy to interpret. For instance, consider the static payoff function: $u_i = p(X)v_i - c(x_i)$ for agent i, where $p(X) \in [0,1]$ is the team's probability of success, and $p'(X) > 0$ and $p''(X) \leq 0$. The first-order condition for agent i yields the reward function: $v_i = c'(x_i)/p'(X) \equiv \overline{v}(x_i)$. Thus, if $c''' > 0$, as assumed in (C1), equal pay would minimize the total reward to induce a target X. If, on the other hand, $c''' < 0$, then the principal would optimally employ a single agent, rendering the team size irrelevant. This means that unequal pay that induces every agent to work can be optimal only if c''' changes sign on $[0, X]$, e.g., when c' is S-shaped in $[0, X]$.[21] Even so, free-riding would play no role in this conclusion.

In closing, I want to point out that while employees appear to receive fixed success bonuses for their creative ideas in innovation-driven industries (see Footnote 5), it may be worth exploring the optimal time-dependent bonuses, as suggested by Mason and Välimäki (2015), and examining how such bonuses treat identical team members across time.

[21] To illustrate, consider the marginal cost function $c'(x) = \frac{x^3}{10x^3 + (1-x)^3}$, which is increasing and S-shaped in $[0, 1]$, with the inflection point $x^I \approx .3$. It is numerically verified that in a 2-agent team, $x_1^* \approx .52$ and $x_2^* \approx .13$ for $X = .65$.

Appendix A: Proofs

Proof of Lemma 1. When efforts are contractible, the principal optimally sets each reward so that (IR) binds. This reveals

$$v_i = \frac{c(x_i)}{X} \equiv \widehat{v}(x_i),$$

where $v', v'' > 0$ given X and $c', c'' > 0$. Then, (FB) reduces to:

$$\min_{\mathbf{x}} \sum_i \widehat{v}(x_i) \text{ s.t. } \sum_i x_i = X.$$

The solution equates the marginal pays: $\widehat{v}'(x_i^{FB}) = \widehat{v}'(x_j^{FB})$, implying $x_i^{FB} = x_j^{FB} = X/n$ for all i and j. ■

Proof of Lemma 2. Fix a reward schedule $\mathbf{v} \in \mathbb{R}^n_+$. Since $v_i = 0$ implies $x_i = 0$ by (5), it suffices to consider $\mathbf{v} \in \mathbb{R}^n_{++}$ in this part. Define

$$\Omega(x, X) = c'(x)\,(r + X) - c(x). \tag{A-1}$$

Then, the first-order condition (5) reads

$$\Omega(x_i, X) = rv_i, \tag{A-2}$$

where Ω has the following properties:

$$\Omega(0, X) = 0$$

since $c'(0) = c(0) = 0$;

$$\Omega(\infty, X) = \infty$$

since $c'' > 0$, $c'(\infty) = \infty$, and $\Omega(x, X) \geq c'(x)r$; and

$$\Omega_X = c'(x) > 0 \text{ and } \Omega_x = c''(x)\,(r + X) - c'(x) > 0 \text{ for } x > 0$$

since $c''(x) > 0$, and $c''(x)x - c'(x) \geq 0$ by $c'''(x) \geq 0$.

Then, the properties of Ω imply a unique solution for x_i to (A-2), which we denote by

$$x_i = f_i(X). \tag{A-4}$$

Summing (A-4) over all i, the equilibrium team effort X^e must solve the following fixed-point equation:

$$g(X) \equiv \sum_i f_i(X) - X = 0. \tag{A-5}$$

It is verified that

$$f_i'(X) = -\frac{\Omega_X}{\Omega_x} < 0 \text{ and } \lim_{X\to\infty} f_i(X) = 0, \tag{A-6}$$

where the limit obtains because the left-hand side of (A-2) would diverge if $\lim_{X\to\infty} f_i(X) > 0$.

Hence,

$$g'(X) < 0 \text{ and } \lim_{X\to\infty} g(X) = -\infty. \tag{A-7}$$

The equilibrium is established if $g(X) > 0$ for some $X \geq 0$. To this end, I consider two cases:

Case 1. $c'(r)r - c(r) > r \max_i v_i$.

Then, $f_i(0) > 0$ for all i by (A-2), and, in turn, $g(0) = \sum_i f_i(0) > 0$.

Case 2. $c'(r)r - c(r) \leq r \max_i v_i$.

Let $i_{\max} = \arg\max_i v_i$. Note that $(c'(x)x - c(x))' = c''(x)x > 0$ for $x > 0$. Thus, there is some $\widehat{X} \geq 0$ such that

$$c'(r+\widehat{X})\left(r+\widehat{X}\right) - c(r+\widehat{X}) = rv_{i_{\max}},$$

which implies $f_{i_{\max}}(\widehat{X}) = r + \widehat{X} > 0$ since $r > 0$. Then,

$$\begin{aligned} g(\widehat{X}) &= \sum_i f_i(\widehat{X}) - \widehat{X} \\ &= \left(r+\widehat{X}\right) + \sum_{i\neq i_{\max}} f_i(\widehat{X}) - \widehat{X} \\ &> 0. \end{aligned}$$

Given (A-7) and the two cases, there is a unique $X^e > 0$ that solves (A-5). Therefore, from (A-4) and the fact that $f_i' < 0$, there is a unique team equilibrium: $x_i^e = f_i(X^e)$ for all i.

Conversely, take an arbitrary effort profile $\mathbf{x} \in \mathbb{R}^n_+$, and using (A-2), define the reward $\overline{v}(x_i) = \frac{\Omega(x_i,X)}{r} \geq 0$, as in (6). Clearly, $\overline{v}'(x_i) > 0$ for $x_i > 0$ because $\Omega_x > 0$, and $\overline{v}(0) = 0$ because $\Omega(0,X) = 0$. Hence, by the previous existence result, the reward profile $(\overline{v}(x_1), ..., \overline{v}(x_n))$ uniquely engenders $\mathbf{x}$ as team equilibrium.

To prove the rest, fix X. Part (a) is completed by observing $\overline{v}'(0) = \frac{\Omega_x(0,X)}{r} = 0$ since $c'(0) = c''(0) = 0$ by (C1). The proof of part (b) mimics part (a). Finally, part (c) follows because $\overline{v}(x_i) > \overline{v}(x_j)$ implies $x_i > x_j$, which, in turn, implies $c'(x_i) > c'(x_j)$, given $c'' > 0$. ■

Proof of Lemma 3. Fix X. From (6),

$$\overline{v}''(x) = \frac{1}{r}\left[c'''(x)\left(r+X\right) - c''(x)\right]. \tag{A-8}$$

Clearly, $\overline{v}''(0) \geq 0$ since $c''(0) = 0$ and $c'''(x) > 0$ for $x > 0$. And, (C1) implies $\lim_{r\to\infty} \overline{v}''(x) = c'''(x) > 0$ for $x > 0$.

Next, suppose $c^{(4)}(x) \geq 0$ in $[0, X]$. Then,

$$\begin{aligned}
\overline{v}''(x) &> \frac{1}{r}\left[c'''(x)x - c''(x)\right] \text{ (since } x \leq X \text{ and } r > 0\text{)} \\
&= \frac{x}{r}\left[c'''(x) - \frac{c''(x)}{x}\right] \\
&\geq 0 \text{ (since } c''(0) = 0 \text{ and } c^{(4)}(x) \geq 0\text{)}.
\end{aligned}$$

Hence, $\overline{v}''(0) \geq 0$ and $\overline{v}''(x) > 0$ for $x \in (0, X]$.

Given the strict convexity of $\overline{v}(x)$ in each case, the unique solution to (SB) is the equal effort profile and thus, equal pay. ■

Proof of Lemma 4. Fix X. Suppose $\overline{v}''(x) = 0$ for some $x^I \in (0, X)$ where $sgn\left[\overline{v}''(x)\right] = sgn\left[\frac{c'''(x)}{c''(x)} - \frac{1}{r+X}\right]$ by (A-8). Clearly, if $c^{(4)}(x) < 0$ in $[0, X]$, then $\left(\frac{c'''(x)}{c''(x)}\right)' < 0$. Hence, if $x^I \in (0, X)$ exists, it must be unique and imply that $\overline{v}''(x) > 0$ for $x < x^I$ and $\overline{v}''(x) < 0$ for $x > x^I$. ■

Proof of Proposition 1. Suppose $\overline{v}(x)$ is S-shaped in $[0, X]$, i.e., it has an (interior) inflection point x^I (convex turning concave). As argued in the text, under the optimal reward schedule, there can be at most one agent such that $x_i^* > x^I$, i.e., $\overline{v}''(x_i^*) < 0$. By the same argument, if x_i^*, $x_j^* \leq x^I$ for some i and j, then $x_i^* = x_j^*$. Otherwise, $x_i^* < x_j^*$ would imply $\overline{v}''(x) > 0$ for $x \in [x_i^*, x_j^*]$. But, by Jensen's inequality, we would have

$$2\overline{v}\left(\frac{x_i^* + x_j^*}{2}\right) < \overline{v}(x_i^*) + \overline{v}(x_j^*),$$

revealing that the principal could implement $x_i^* + x_j^*$ at a strictly lower reward sum for i and j, contradicting the optimality of the initial pay. From here, the statement of the proposition is confirmed. ■

Proof of Proposition 2. Suppose $\overline{v}(x)$ is S-shaped in $[0, X]$, with an inflection point x^I. Also suppose $x^I < \frac{X}{n}$. If all agents were paid equally, then each would exert effort $\frac{X}{n}$ and be on the concave part of $\overline{v}$. But, by Proposition 1, there cannot be multiple agents in this part, implying the optimality of a "leader-follower" team. Next suppose $X \leq x^I$. Then, $x_i^* \leq x^I$ for all i, i.e., all agents operate on the convex part of $\overline{v}$, implying an optimal equal pay or a "horizontal" team. ■

Proof of Corollary 1. For $c(x) = x^k/k$, with $k \in (2, 2 + \frac{1}{n})$, (A-8) implies $x^I = (k-2)(r + X)$ (which solves $\overline{v}''(x) = 0$). From Proposition 2, we then observe that $\frac{X}{n} >$

$x^I \Longleftrightarrow X > r\frac{k-2}{2+\frac{1}{n}-k}$, and $X \leq x^I \Longleftrightarrow X \leq r\frac{k-2}{3-k}$. ■

Proof of Proposition 3. In light of Proposition 1, the principal's reward-minimization program (SB) reduces to:

$$\min_{x\in[0,\frac{X}{n}]} (n-1)\overline{v}(x) + \overline{v}(X-(n-1)x) \equiv \phi(x;X,n),$$

where $(n-1)$ agents are induced to exert the (low) effort $x \in [0, \frac{X}{n}]$.

The first- and second-order conditions are given by:

$$\text{FOC: } \phi_x(x^*;.) = 0 \Longleftrightarrow \overline{v}'(x^*) - \overline{v}'(X-(n-1)x^*) = 0$$

and

$$\text{SOC: } \phi_{xx}(x^*;.) = \overline{v}''(x^*) + (n-1)\overline{v}''(X-(n-1)x^*) \geq 0.$$

Treating n as a continuous parameter here, the implicit differentiation of the FOC implies

$$\phi_{xx}(x^*;.)\frac{\partial x^*}{\partial n} + \phi_{xn}(x^*;.) = 0$$

$\Longleftrightarrow$

$$\phi_{xx}(x^*;.)\frac{\partial x^*}{\partial n} + x^*\overline{v}''(X-(n-1)x^*) = 0. \tag{A-9}$$

Suppose $x^* < \frac{X}{n}$, i.e., a leader-follower team is optimal. Then, $\phi_{xx}(x^*;.) > 0$. Otherwise, by (A-9), $\phi_{xx}(x^*;.) = 0$ would imply

$$\overline{v}''(X-(n-1)x^*) = 0.$$

That is, the leader's effort, $X-(n-1)x^*$, would be at the inflection of the reward function, $\overline{v}$. But then, all agents' efforts would be on the convex part of $\overline{v}$, implying the optimality of equal effort, contradicting the hypothesis $x^* < \frac{X}{n}$.

Given $\phi_{xx}(x^*;.) > 0$ and the fact that $\overline{v}''(X-(n-1)x^*) < 0$ when $x^* < \frac{X}{n}$, (A-9) reveals

$$\frac{\partial x^*}{\partial n} > 0. \tag{A-10}$$

Suppose $x^*(n+1,.) < \frac{X}{n+1}$, i.e, unequal pay in a team of size $n+1$. Then, by (A-10), $x^*(n,.) < x^*(n+1,.)$, implying that $x^*(n,.) < \frac{X}{n}$. Hence, unequal pay in a team of size $n+1$ means unequal pay in a team of size n, as claimed.

To prove part (a), suppose an n-agent team has unequal pay, i.e., $x^*(n,.) < \frac{X}{n}$. Then, it trivially has a wider pay range than a larger team of size $m > n$ if the latter has equal pay, i.e., $x^*(m,.) = \frac{X}{m}$. If, instead, $x^*(m,.) < \frac{X}{m}$, then $x^*(n,.) < x^*(m,.)$ by (A-10), and, in turn, $X - (n-1)x^*(n,.) > X - (m-1)x^*(m,.)$. Together with the fact that $\overline{v}'(x_i) > 0$ by Lemma 2(a), these reveal that the smaller team has a wider pay range.

Finally, part (b) follows from (A-10) since each follower exerts effort x^*, leaving $X-(n-1)x^*$ to the leader. ■

Proof of Proposition 4. We first prove the properties of $w(X)$ as claimed in the text. Notice that using (6), (SB2) can be re-written as:

$$w(X) = \min_{x\in[0,\frac{X}{n}]} (n-1)[c'(x) + \overline{u}(x)] + \left[c'(X-(n-1)x) + \overline{u}(X-(n-1)x)\right],$$

where $\overline{u}(x) = \frac{c'(x)X - c(x)}{r}$.

Clearly, $w(0) = 0$ since $c(0) = c'(0) = 0$ by (C1). Moreover, by applying the Envelope Theorem, we find

$$w'(X) = \left[(n-1)c'(x^*) + c''(X-(n-1)x^*)(r+X)\right]/r.$$

Thus, $w'(X) > 0$ for $X > 0$, and $w'(0) = 0$ since $c'' > 0$, with $c''(0) = 0$. The Envelope Theorem further reveals that $w(X)$ is strictly decreasing in r for $X > 0$ since $\overline{u}(x^*) > 0$ by Lemma 2. Finally, $w(X)$ must be strictly decreasing in n for $X > 0$ because the solution to (SB2) is interior for any n by Lemma 2, i.e., each additional team member is utilized by the principal.

To prove the proposition, define $\overline{u}_P(X) = \frac{X}{r_P + X}(\pi - w(X))$. A solution to (PM), denoted by X^*, exists because $\overline{u}_P(X)$ is continuous in X, and $X \in [0, w^{-1}(\pi)]$. Next, note that

$$\overline{u}'_P(X) = \frac{r_p[\pi - w(X) - Xw'(X)] - X^2 w'(X)}{(r_P + X)^2},$$

implying that $\overline{u}'_P(0) = \frac{\pi}{r_P} > 0$. Hence, $X^* > 0$ and, in turn, $\overline{u}'_P(X^*) = 0$ in the first-order condition.

Now take $\pi < \pi'$, and let X^* and $X^{*\prime}$ be the corresponding solutions to (PM). Then, by revealed preference arguments,[22] we have

$$\frac{X^{*\prime}}{r_P + X^{*\prime}}\left(\pi - w(X^{*\prime})\right) \le \frac{X^*}{r_P + X^*}\left(\pi - w(X^*)\right)$$

and

$$\frac{X^*}{r_P + X^*}\left(\pi' - w(X^*)\right) \le \frac{X^{*\prime}}{r_P + X^{*\prime}}\left(\pi' - w(X^{*\prime})\right).$$

[22] I use revealed preference arguments instead of Calculus for comparative statics because the second-order properties of $w(X)$ are not immediate or needed here.

Adding the terms side by side and arranging them, we find

$$\frac{X^*}{r_P + X^*}\left(\pi' - \pi\right) \leq \frac{X^{*\prime}}{r_P + X^{*\prime}}\left(\pi' - \pi\right),$$

which implies $X^* \leq X^{*\prime}$. Notice that $X^* \neq X^{*\prime}$ since $\pi < \pi'$ and $\overline{u}'_P(X^*) = 0 = \overline{u}'_P(X^{*\prime})$.

Similar revealed preference arguments also prove that X^* is strictly increasing in r_P. The rest of the proposition obtains from Corollary 1: under the iso-elastic cost, the principal optimally chooses a leader-follower if $X^* > r\frac{k-2}{2+\frac{1}{n}-k}$. ■

Proof of Proposition 5. Fix X. Solving for v_i in (9), we amend (6) as:

$$\overline{v}(x_i) = \frac{c'(x_i)\left(\frac{r}{\alpha} + X\right) - c(x_i)}{r}. \tag{A-11}$$

Hence, as in the proof of Lemma 4, $sgn\left[\overline{v}''(x)\right] = sgn\left[\frac{c'''(x)}{c''(x)} - \frac{1}{\frac{r}{\alpha}+X}\right]$.

Next take $\beta < \beta'$, or equivalently, $\alpha < \alpha'$. Clearly,

$$sgn\left[\overline{v}''(x)|\alpha'\right] < sgn\left[\overline{v}''(x)|\alpha\right]$$

for all $x > 0$. In particular, if an inflection point in $[0, X]$ exist for α', then $x^{I\prime} < x^I$. The conclusion follows from Proposition 2. ■

Appendix B: On stationary strategies

In this appendix, I prove the claim made in Footnote 11 that the restriction to stationary strategies is without loss for the 2- and 3-agent teams. To this end, fix a positive reward profile, i.e., $v_i > 0$ for all i. Recall that $x_i(t)$ and $u_i(t)$ denote agent i's continuously differentiable effort level and expected discounted payoff at time t, respectively. Then, as is standard by now, agent i's dynamic program can be written as an Hamilton-Jacobi-Bellman equation:

$$ru_i(t) = \max_{x_i(t)}\left[(x_i(t) + \sum_{j\neq i} x_j(t))(v_i - u_i(t)) - c(x_i(t)) + u'_i(t)\right]. \tag{B-1}$$

Clearly, in equilibrium, $u_i(t) \in (0, v_i)$ due to costly effort and the cost assumption (C1).[23] The first-order condition requires that

$$x_i(t) = h(v_i - u_i(t)), \tag{B-2}$$

[23] (B-1) reduces to (1) under stationary strategies.

where $h = c'^{-1}$, and $h' > 0$ and $h'' < 0$, with $h(0) = 0$ and $h'(\infty) = \infty$ by (C1). Plugging (B-2) into (B-1) and arranging terms, we obtain the law of motion for i's equilibrium utility:

$$u_i'(t) = ru_i(t) + c(h(v_i - u_i(t))) - (v_i - u_i(t)) \sum_j h(v_j - u_j(t)). \tag{B-3}$$

Finding an equilibrium amounts to solving this system of n ODEs. Evidently, the steady state of this system, i.e., its solution when $u_i'(t) = 0$ for all i, corresponds to the stationary equilibrium analyzed in the main text. By Lemma 2, the steady state uniquely exists, which we denote by $u^* = (u_1^*, u_2^*, ..., u_n^*)$.

To rule out any other equilibrium, I will argue that if $u_i(\tau) \neq u_i^*$ for some agent i at time τ, $u_j(t)$ would diverge for some agent j as $t \to \infty$, violating its feasible values $(0, v_j)$.[24] To do so, I write the Jacobian matrix for the system in (B-3) and show that its eigenvalues are all real and positive.

Consider first a 2-agent team. Omitting the time index for conciseness below, notice from (B-3) that

$$\begin{aligned} \frac{\partial u_i'}{\partial u_i} &= r - c'(h(v_i - u_i))h'(v_i - u_i) + [h(v_i - u_i) + h(v_j - u_j)] + (v_i - u_i)h'(v_i - u_i) \\ &= r + h(v_i - u_i) + h(v_j - u_j), \end{aligned}$$

since $c'(h(.)) = c'(c'^{-1}(.)) = v_i - u_i$. Similarly,

$$\frac{\partial u_i'}{\partial u_j} = (v_i - u_i)h'(v_j - u_j).$$

To further simplify the notation, let

$$z_i = v_i - u_i.$$

Then, the Jacobian matrix for 2 agents evaluated at utility values at time τ becomes

$$J_2 = \begin{bmatrix} \frac{\partial u_1'}{\partial u_1} & \frac{\partial u_1'}{\partial u_2} \\ \frac{\partial u_2'}{\partial u_1} & \frac{\partial u_2'}{\partial u_2} \end{bmatrix} = \begin{bmatrix} r + h(z_1) + h(z_2) & z_1 h'(z_2) \\ z_2 h'(z_1) & r + h(z_1) + h(z_2) \end{bmatrix}.$$

The eigenvalues of this matrix admit closed forms:

$$\lambda_{1,2} = r + h(z_1) + h(z_2) \pm \sqrt{z_1 z_2 h'(z_1) h'(z_2)}.$$

[24] In their working paper version, Bonatti and Rantakari (2016) argue similarly for eliminating nonstationary equilibrium in their 2-agent benchmark. However, their "phase-diagram" approach is difficult to extend to more than 2 agents.

Since $h'' < 0$ and $h(0) = 0$, we have $h'(z) < \frac{h(z)}{z}$ for $z > 0$. Hence,

$$r + h(z_1) + h(z_2) - \sqrt{z_1 z_2 h'(z_1) h'(z_2)} \geq r + h(z_1) + h(z_2) - \sqrt{h(z_1) h(z_2)} > 0,$$

implying that *both eigenvalues are real and positive* for the 2-agent team.

Next consider a 3-agent team. In this case, the Jacobian matrix is given by

$$J_3 = \begin{bmatrix} r + \sum_i h(z_i) & z_1 h'(z_2) & z_1 h'(z_3) \\ z_2 h'(z_1) & r + \sum_i h(z_i) & z_2 h'(z_3) \\ z_3 h'(z_1) & z_3 h'(z_2) & r + \sum_i h(z_i) \end{bmatrix}.$$

Unfortunately, the eigenvalues of J_3 do not admit tractable closed forms. Moreover, J_3 is not symmetric for arbitrary $z-$values, nor is it diagonally dominant or totally positive, each of which would help argue all real and positive eigenvalues. However, J_3 is a positive matrix for which the Perron-Frobenius Theorem would imply that the largest eigenvalue in magnitude is real and positive. Still, that theorem is silent about the real parts of the other eigenvalues. To determine them, I work directly with the characteristic equation of J_3, which, in general, is of the form

$$\lambda^3 - tr(J_3)\lambda^2 + p\lambda - \det(J_3) = 0. \qquad \text{(B-4)}$$

Clearly, $tr(J_3) = 3\left(r + \sum_i h(z_i)\right) > 0$. It can also be verified that

$$\begin{aligned} \det(J_3) &= \left(r + \sum_i h(z_i)\right)\left[\left(r + \sum_i h(z_i)\right)^2 - \left(z_1 z_2 h'(z_1)h'(z_2) + z_1 z_3 h'(z_1)h'(z_3) + z_2 z_3 h'(z_2)h'(z_3)\right)\right] \\ &\quad + 2 z_1 z_2 z_3 h'(z_1)h'(z_2)h'(z_3) \\ &> 0, \end{aligned}$$

because $h'(z) < \frac{h(z)}{z}$ by $h''(z) < 0$. Lastly, by the same argument,

$$\begin{aligned} p &= 3\left(r + \sum_i h(z_i)\right)^2 - \left(z_1 z_2 h'(z_1)h'(z_2) + z_1 z_3 h'(z_1)h'(z_3) + z_2 z_3 h'(z_2)h'(z_3)\right) \\ &> 0. \end{aligned}$$

Then, Descartes' rule of signs reveals that (B-4) has either 1 or 3 real positive roots, and no negative real root. To determine the number of real roots, we find, after some algebra, the discriminant of the cubic polynomial in (B-4):

$$108\left[\left(\frac{\left(z_1 z_2 h'(z_1)h'(z_2) + z_1 z_3 h'(z_1)h'(z_3) + z_2 z_3 h'(z_2)h'(z_3)\right)}{3}\right)^3 - \left(z_1 z_2 z_3 h'(z_1)h'(z_2)h'(z_3)\right)^2\right]$$

which is positive by the arithmetic-geometric mean inequality. Hence, all three roots of (B-4) and thus, all eigenvalues of J_3, are real, which, by Descartes' rule of signs, are also all positive.

Conclusion 1 *For 2- and 3-agent teams, if the initial point is not exactly its steady state, the solutions to the system of ODEs described by (B-3) are divergent, i.e.,* $u_i(t) \notin (0, v_i)$ *as* $t \to \infty$ *for some* i*. Therefore, there is no equilibrium with continuously differentiable strategies other than the stationary one considered in the main text.*

Although I strongly conjecture that Conclusion 1 extends to any team size, I have been unable to prove it. Nevertheless, it is evident that for an arbitrary team size, if all but agent i followed stationary strategies, so would agent i. Otherwise, since $\frac{\partial u_i'}{\partial u_i} > 0$, agent i's utility would diverge. Hence, we reach

Conclusion 2 *For an arbitrary team size, if agent* i *expects all others to adopt stationary strategies, he does the same.*

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