Communication and Information Aggregation in Committees

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Abstract

We explore the work of committees that aggregate private information of their heterogeneous members under: (a) standard monotone preference; (b) non-monotones preferences reflecting fit between The equilibria in case (a) have a partition structure but may be non-monotone, while in case (b) they can be either "exclusive" or "overlapping." We provide a sufficient condition for uniqueness of equilibria with a fixed number of categories bunching the members provate information. More equilibrium categories imply ex-ante better decisions. Collective mistakes are possible even under the most informative equilibrium with the highest number of categories: The committee may make decisions to the contrary of the preferences of its members. The worst mistakes are made when the committee members observe opposite signals, confirming their inherent biases.

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1 Introduction

The paper studies decision-making procedures for committees consisting of members who have similar preferences, but differ in their assessment of the costs os making mistakes and taking wrong decisions. In a related paper, Li, Rosen, and Suen (2001, hereafter LRS) have shown that even a small difference between the individuals' preferences limits how a committee can aggregate their private information. Although each member observes a realization of a continuous random variable correlated with the state of nature, the committee's decision procedure bunches different realizations into coarse *categories*.

LRS identify the equilibrium conditions for deterministic and monotone decision mechanisms where each member's signal space is partitioned into a number of categories. The difference in the members' assessment of the costs of type-1 and type-2 errors provides a bound on how many categories can be supported in an equilibrium.

In this paper, we provide a sufficient condition for the uniqueness of equilibrium with a given number of categories. The significance of establishing uniqueness goes beyond a theoretical point. LRS argue that committees make better decisions by using mechanisms allowing for more equilibrium categories. Their argument is based on the demonstration that, assuming that such equilibria exist, each N-category equilibrium is dominated by *one* N+1-category equilibrium in terms of the committee members' ex-ante expected costs for wrong decisions. Hence, our uniqueness result strengthens the case for the efficiency of more equilibrium categories without the need for a selection argument.

Under this sufficient condition, we provide a procedure to identify the maximum number of categories that can be supported in an equilibrium of a decision mechanism. Even in this equilibrium, the committee sometimes makes Pareto inefficient decisions. By processing only the signal categories, the committee may end up choosing one decision whereas its members would prefer the alternative decision in light of the raw signals. We can quantify the inefficiency of a committee decision by calculating the likelihood that the committee's decision is wrong as a function of the raw signals of the committee members. One question of interest here is what signal combinations will lead to the most inefficient decisions by the committee.

We show that, in the equilibrium with the maximum number of categories, the committee makes its most inefficient decisions when the *progressive* element within the committee (whose cost of maintaining the status-quo incorrectly is relatively high) receives a signal within the weakest category for the status-quo, whereas the *conservative* element (with a low relative cost of maintaining the status quo) receives a signal strongly supporting the status-quo. This observation applies to both type-1 and type-2 errors. Mistakes would still happen as the progressive element's signal lends more support to the status-quo and simultaneously the conservative element's signal becomes less supportive. Yet, the magnitudes of the worst mistakes decline monotonically as the signals change in these directions.

2 Preliminaries

Following LRS, we consider a committee of two members A and B. There are two states of nature and two decisions which we, for brevity, denote in the same way: the null state and the corresponding null decision, and the alternative state and corresponding alternative decision. The committee is tasked with accepting the null decision or rejecting it and accepting the alternative (i.e., maintaining or abandoning the status-quo). The prior of member i that the null is true is γ^i, her personal cost of type-1 error (accepting the null when it is false) is λ_1^i, and her personal cost of type-2 error (rejecting the null when it is correct) is λ_2^i. Let $k_1^i = \lambda_1^i (1 - \gamma^i)$ and $k_2^i = \lambda_2^i \gamma^i$. As LRS argued, what is relevant for the decisions is the cost of false acceptance relative to the cost of false rejection $k^i = k_1^i / k_2^i$. In order to study the conflict of interest between the committee members, we assume that $k^a < k^b$: member A is more prone to accepting the null (e.g., more conservative).

We propose a transformation of the private observations of the committee members, which would make our points easier to demonstrate. In the work of LRS, each committee member i receives a private observation y^i which is a realization of a random variable distributed on the subset $\left[\underline{y}^i, \overline{y}^i\right]$ of the real line. The cumulative distribution function for this random variable is F_q^i if the null is true, and F_u^i if the null is false. Both these functions are continuously differentiable. We transform this random variable to the domain $[0,1]$ and ensure that its distribution is uniform when the null is false. That is, for each $y^i \in \left[\underline{y}^i, \overline{y}^i\right]$, we define variable $x^i = F_u^i\left(y^i\right)$. Accordingly, when the null is true, x^i is the realization of a random variable with the cumulative distribution function

$$G^i\left(x^i\right) = F_q^i\left[\left(F_u^i\right)^{-1}\left(x^i\right)\right].$$

It will be easier to concentrate on the density of this distribution which gives us the *likelihood* ratio for the null after observing x^i:

$$g^i\left(x^i\right) = \frac{f_q^i\left[\left(F_u^i\right)^{-1}\left(x^i\right)\right]}{f_u^i\left[\left(F_u^i\right)^{-1}\left(x^i\right)\right]},$$

where f_q^i and f_u^i are the density functions for the original observation y^i, when the null is true and when it is false respectively. The monotone likelihood ratio property that LRS impose is identical to the assumption that function g^i is strictly increasing on $[0,1]$. This is an assumption that we will maintain for our analysis.

As LRS indicate, in light of the collective evidence x^a, x^b, whether accepting the null is a good decision for member i depends on comparing the likelihood of the null $g^a\left(x^a\right) \times g^b\left(x^b\right)$ to the relative cost of rejecting the null for this member $\left(k^i\right)$. The fact that the two members do not have the same relative cost for false decisions hinders effective communication.

To simplify our analysis by avoiding some corner conditions, we assume that either committee member is willing to reject the null if her own signal is the lowest possible one. Moreover, she is willing to accept the null upon receiving the highest possible signal, regardless of the signal of the other member. That is, we assume that $g^a(0) < k^a < g^a(1)\, g^b(0)$ and $g^b(0) < k^b < g^a(0)\, g^b(1)$.

3 Equilibrium Conditions

LRS show that, any deterministic and monotone equilibrium outcome of a decision mechanism, which gives each individual member of the committee the power to *unilaterally accept* the null,[1] is a *partition outcome*: For each committee member, there are N thresholds $t_1^i, t_2^i, ..., t_N^i$ partitioning the observation space $[0,1]$ into $N+1$ categories. We can imagine such a mechanism as a voting game with a scoring rule, where each member can show the intensity of her support by choosing an integer between 0 and N (including 0 and N), and the null is accepted if the sum of the integers is at least N. In equilibrium, each partition cell for each member corresponds to a different integer. Committee member A who has observed a signal exactly equal to threshold t_n^a must be indifferent between choosing integer $n-1$ and integer n. The decision between these two options would matter only if member B chooses integer $N-n$, which would happen when she has observed a signal x^b between t_{N-n}^b and t_{N-n+1}^b. The probability of such an observation equals $t_{N-n+1}^b - t_{N-n}^b$ if the null is incorrect (recall that our transformation implies a uniform distribution of the signal in this case), or $\int_{t_{N-n}^b}^{t_{N-n+1}^b} g^b\left(x^b\right) dx^b$ if the null is correct. Accordingly, the increase in member A's expected cost of false acceptance after voting n is

$$\eta \int_{t_{N-n}^b}^{t_{N-n+1}^b} k_1^a dx^b,$$

and the increase in her expected cost of a false rejection after voting $n-1$ is

$$\eta \int_{t_{N-n}^b}^{t_{N-n+1}^b} k_2^a g^a\left(t_n^a\right) g^b\left(x^b\right) dx^b,$$

where $\eta = \left(\gamma^a g\left(t_n^a\right) + (1-\gamma^a)\right)^{-1}$ is the normalization factor under Bayesian updating. Comparing type-1 and type-2 errors reveals the following equilibrium no-arbitrage con-

[1]The *unilateral rejection* setting would be similar to the unilateral acceptance setting that is discussed here, after the relabeling of the null and alternative states.

dition:

$$(A_n): \int_{t^b_{N-n}}^{t^b_{N-n+1}} \left[g^a\left(t^a_n\right) g^b\left(x^b\right) - k^a\right] dx^b = 0,$$

with the convention that if $t^b_{N-n} = t^b_{N-n+1}$, then t^a_n is solution to $g^a\left(t^a_n\right) g^b\left(t^b_{N-n}\right) = k^a$.

An identical condition arises for the thresholds of member B:

$$(B_n): \int_{t^a_{N-n}}^{t^a_{N-n+1}} \left[g^a\left(x^a\right) g^b\left(t^b_n\right) - k^b\right] dx^a = 0.$$

The following conditions ensure that the lower bounds of the lowest categories match with the lower bounds of the supports of the signal distributions:

$$\begin{aligned}(A_0) &: \ t^a_0 = 0, \\ (B_0) &: \ t^b_0 = 0.\end{aligned}$$

Thresholds $t^a_0 \leq t^a_1 < ... < t^a_N \leq 1$ and $t^b_0 \leq t^b_1 < ... < t^b_N \leq 1$ identify an *equilibrium solution* with length N if they satisfy conditions (A_n) and (B_n) for $n = 0, 1, ..., N$. These conditions together correspond to equilibrium conditions in (13) in LRS. A partition equilibrium divides the space of signal pairs $[0,1] \times [0,1]$ into two areas according to whether the null is accepted or rejected in equilibrium. These areas are separated by the graph of a step function, whose kink points are given by pairs $\left(t^a_n, t^b_m\right)$ such that $n + m$ equals either N or $N + 1$. See Figure 1 for an equilibrium solution with length 2, partitioning the signals of each committee member into three categories.

An important feature of a solution to the system of equations above is that knowing the value of t^a_1 is sufficient for calculating the rest of the equilibrium thresholds iteratively: Equation (B_N) yields t^b_N given the values of t^a_1 and $t^a_0 = 0$. Then, for all $n \geq 1$, equation (A_n) yields t^b_{N-n} given t^b_{N-n+1} and t^a_n. And equation (B_{N-n}) yields t^a_{n+1} given t^a_n and t^b_{N-n}. If t^a_1 is indeed the equilibrium threshold for the lowest category, then the other thresholds identified by this iterative procedure would be within set $[0,1]$ and the final threshold t_0^b equals zero to satisfy (B_0). We refer to thresholds $t_0^a \leq t_1^a < ... < t_N^a \leq 1$ and $t_0^b \leq t_1^b < ... < t_N^b \leq 1$ as a *partial solution* with length N if they satisfy (A_n) and (B_n) for $n = 1, ..., N$ as well as $(A_0 : t_0^a = 0)$, but not necessarily $\left(B_0 : t_0^b = 0\right)$.

4 Uniqueness

LRS discuss conditions for existence and uniqueness of an equilibrium outcome with two categories where each of the two members vote either yes or no and the null is accepted if there is at least one yes vote. But there is no known condition that guarantees existence and/or uniqueness for arbitrary number of categories. LRS also point to the similarities between the construction of the equilibrium partition in this setting and the construction of equilibria in a cheap talk game played by an informed sender and an uninformed receiver à la Crawford and Sobel (1982). The differences between the two approaches are that asymmetric information is double sided within a committee (as opposed to the single-sided asymmetry between the sender and the receiver) and the final decision to be made is binary (whereas there is a continuum of available alternatives for the receiver in Crawford and Sobel's work). However, each committee member's *Bayesian* strategy can be thought as setting an acceptance cutoff on the $[0, 1]$ continuum for each category reported by the other committee member, leading to the resemblance of the equilibrium conditions in the two settings. Therefore, as discussed by Crawford and Sobel, the following monotonicity condition on the partial solutions (satisfying equations (A_n) and (B_n) for $n \geq 1$ and (A_0)) would ensure uniqueness of a partition equilibrium with N thresholds.

(M) : Suppose $t = \left(t_0^a, t_1^a, ..., t_N^a; t_0^b, t_1^b, ..., t_N^b\right)$ and $\tilde{t} = \left(\tilde{t}_0^a, \tilde{t}_1^a, ..., \tilde{t}_N^a; \tilde{t}_0^b, \tilde{t}_1^b, ..., \tilde{t}_N^b\right)$ are two partial solutions as defined above. If $t_1^a > \tilde{t}_1^a$, then $t_n^a > \tilde{t}_n^a$ for $n > 1$ and $t_n^b < \tilde{t}_n^b$ for $n \geq 0$.

Crawford and Sobel provide a sufficient condition on the primitives of their model for (M) in their Theorem 2. They show this by proving that, once their sufficient condition is satisfied, an increase in the initial condition t_1^a of the iterative process defining the other thresholds implies an even higher increase for t_2^a, and even higher one on t_3^a, etc. This leads to the equilibrium structure that each category is larger than the preceding one, which is a familiar feature from their uniform-quadratic example. In the context of the current two-sided asymmetric information / binary decision setting however, their sufficient condition corresponds to a likelihood (of the null) which is a function of the *sum* of the observations of the two members. This is not satisfied by the likelihood $g^a(x^a)\, g^b\left(x^b\right)$ under the conditional independence of the observations in the LRS model.

Below, we will demonstrate that the concavity of $g^i(\cdot)/g^{i\prime}$ for $i = a, b$ is an alternative sufficient condition for (M) and therefore for the uniqueness within the class of N-threshold equilibria.[2] Under this concavity condition, we can establish not only the monotonicity of the partial solution's thresholds in the initial threshold t_1^a, but also the monotonicity of the evolution of the likelihood of the null state corresponding to the threshold pairs of the two members. This concavity condition is satisfied by a broad range of distributions, e.g. *power functions* where the random variable x is governed by the cumulative distribution function x^m for $m \geq 2$. The corresponding density function $g^i(x) = mx^{m-1}$ yields a linear (and therefore concave) $g^i(x)/g^{i\prime}(x)$.

To provide the uniqueness argument, we start with defining the likelihood levels at the kink points of the step function separating the acceptance and rejection regions: $\underline{g}_n = g^a(t_n^a)\, g^b\left(t_{N-n}^b\right)$ and $\overline{g}_n = g^a(t_n^a)\, g^b\left(t_{N-n+1}^b\right)$. Now, let us use a change of variable to rewrite the conditions (A_n) and (B_{N-n}) as follows:

$$(A_n) : \frac{1}{g^a(t_n^a)} \int_{\underline{g}_n}^{\overline{g}_n} \frac{z - k^a}{g^{b\prime}(x^b)} dz = 0, \tag{1}$$

where x^b is defined implicitly as the unique solution to equation $g^a(t_n^a)\, g^b\left(x^b\right) = z$ which yields the following relation between x^b and t_n^a for fixed z:

$$\frac{dx^b}{dt_n^a} = -\frac{g^{a\prime}(t_n^a)\, g^b\left(x^b\right)}{g^a(t_n^a)\, g^{b\prime}(x^b)}.$$

[2] Where $g^{i\prime}(\cdot)$ refers to the first derivative of the density function $g^i(\cdot)$.

Similarly,

$$(B_{N-n}) : \frac{1}{g^b\left(t^b_{N-n}\right)} \int\limits_{\underline{g}_n}^{\overline{g}_{n+1}} \frac{z-k^b}{g^{a\prime}\left(x^a\right)} dz = 0, \tag{2}$$

where x^a is the unique solution to equation $g^a\left(x^a\right) g^b\left(t^b_{N-n}\right) = z$, and so for fixed z we have $\frac{dx^a}{dt^b_{N-n}} = -\frac{g^a(x^a)g^{b\prime}\left(t^b_{N-n}\right)}{g^{a\prime}(x^a)g^b\left(t^b_{N-n}\right)}$.

The following Lemma is based on the analysis of conditions (1) and (2):

Lemma 1 *1. Suppose $g^b\left(\cdot\right)/g^{b\prime}\left(\cdot\right)$ is concave. If both $\overline{g}_n$ and t^a_n increase, and at least one of them increases strictly, then $\underline{g}_n$ and t^b_{N-n} decrease strictly for condition (A_n) to continue to hold.*

2. Suppose $g^a\left(\cdot\right)/g^{a\prime}\left(\cdot\right)$ is concave. If both $\underline{g}_n$ and t^b_{N-n} decrease, and at least one of them decreases strictly, then $\overline{g}_{n+1}$ and t^a_{n+1} increase strictly for condition (B_{N-n}) to continue to hold.

Using Lemma 1 we establish the following uniqueness result.

Proposition 1 *If $g^a\left(\cdot\right)/g^{a\prime}\left(\cdot\right)$ and $g^b\left(\cdot\right)/g^{b\prime}\left(\cdot\right)$ are both concave, then condition (M) holds and there is at most one equilibrium partitioning the signal space of each member with N thresholds, for any given N.*

In the rest of our analysis we assume that the concavity conditions of Proposition 1 are satisfied by the distributions of the members' signals.

As highlighted in the Introduction, this uniqueness result together with the analysis of LRS (2001) allow to rank different equilibria in terms of efficiency. As the number of equilibrium categories increases, ex-ante expected cost of type-1 and type-2 errors decrease for both committee members. This implies that the most efficient equilibrium has the largest possible number of categories. In the next section, we introduce a procedure to identify the highest number of categories.

5 The Partition Equilibrium with Most Categories

We start this section with an important property of the step function separating the acceptance and rejection areas in the data space. Each (horizontal and vertical) step of this function passes through the *disagreement zone* where the two members of the committee cannot agree on accepting or rejecting the null. In Figure 1, the disagreement zone corresponds to the area between the two downward-sloping curves. For an environment with bounded support and bounded likelihood functions for the distributions, this observation implies an upper bound on the number of categories that can be supported in a partition equilibrium.

In order to identify the highest number of equilibrium categories, we introduce the following partial solution. Partial solution $\hat{t}(N)$ with arbitrary length N is constructed with the initial condition $\hat{t}_1^a(N) = \hat{t}_0^a(N) = 0$.[3] Thanks to condition (M) such a partial solution would be well-defined as long as its final element is in the signal space: $\hat{t}_0^b(N) \geq 0$. (It follows from $g^a(1)\, g^b(0) > k^a$ that threshold $\hat{t}_N^a(N)$ is bounded away from 1 as long as $\hat{t}_0^b(N)$ satisfying (A_N) exists.) Because the first category of voting for member A has zero measure, $\hat{t}(N)$ is essentially identical to a partial solution $\tilde{t}(N-1)$ with length $N-1$, where $\tilde{t}_n^a(N-1) = \hat{t}_{n+1}^a(N)$ and $\tilde{t}_n^b(N-1) = \hat{t}_n^b(N)$ for $1 \leq n \leq N-1$.

The maximum number of thresholds that can be sustained in a partition equilibrium is determined by the maximum length $\bar{N}$ that partial solution $\hat{t}(N)$ can have: Existence of a partition equilibrium that has more thresholds than $\bar{N}$ would violate condition (M), and we can construct an equilibrium with length $\bar{N}$ by using a *continuous deformation* argument as in Crawford and Sobel (1982). To see this construction, consider partial solution $t^x = \left(t_0^{xa}, t_1^{xa}, ..., t_{\bar{N}}^{xa}; t_0^{xb}, t_1^{xb}, ..., t_{\bar{N}}^{xb}\right)$ with length $\bar{N}$ satisfying (A_0), (A_n), and (B_n) with the initial condition $t_1^{xa} = x$. If $x = 0$ then $t^x = \hat{t}\left(\bar{N}\right)$. As x increases, condition (M) implies that thresholds t_n^{xa} increase for $n > 0$ and thresholds t_n^{xb} decrease for $n \geq 0$ continuously. Consider the most extreme threshold t_0^{xb}. For a large enough

[3] Recall that, when $t_0^a = t_1^a$, we define t_N^b as the solution to the equation $g^a(t_0^a)\, g^b\left(t_N^b\right) = k^b$. This maintains continuity of t_N^b in t_0^a and t_1^a.

value of x, threshold t_0^{xb} assumes value 0 and the corresponding t^x gives us $\bar{N}$ pairs of equilibrium thresholds.

We can also see that the value of x that ensures $t_0^{xb} = 0$ is smaller than $\hat{t}_2^a\left(\bar{N}\right)$. Otherwise, $t_0^{xb} > 0$ for $x = \hat{t}_2^a\left(\bar{N}\right) = \tilde{t}_1^a\left(\bar{N}-1\right)$ and partial solution t^x with length $\bar{N}$ would correspond to $\tilde{t}\left(\bar{N}\right)$, indicating existence of $\hat{t}\left(\bar{N}+1\right)$, in contradiction to $\bar{N}$ being the maximum length for such a partial solution. This observation will be crucial in the following section, when deriving the result on the magnitudes of the committee's mistakes.

Once the maximum number of thresholds $\bar{N}$ that can be supported in an equilibrium is identified, the same continuous deformation procedure can be repeated to establish that there exists an equilibrium with N thresholds as long as $1 < N \leq \bar{N}$.

6 Worst Mistakes

Because each step of the step function separating the equilibrium acceptance and rejection regions is crossing the disagreement zone between the two members, the equilibrium decision of the committee is ex-post inefficient. In other words, the committee will make decisions to the contrary of the preferences of its members k^a, k^b and the aggregation of their observations x^a, x^b. These mistakes will be made at signal pairs in the neighborhood of the kink points of the step function separating the acceptance and rejection regions, indicated by the shaded areas in Figure 1.[4]

The magnitudes of the committee's mistakes can be identified by the likelihood of the null state on the graph of this step function. Under the concavity of $\frac{g^a(\cdot)}{g^{a\prime}(\cdot)}$ and $\frac{g^b(\cdot)}{g^{b\prime}(\cdot)}$, concentrating on the most informative (and therefore most efficient) partition equilibrium with $\bar{N}$ thresholds, we show that the worst mistakes are made when member A indicates a high preference for acceptance after observing her individual signal and member B

[4]At the kink points of the graph of the step function bordering the equilibrium acceptance and rejection regions, the likelihood levels are either lower than both members' relative false-acceptance cost ($g^a\left(t_n^a\right) g^b\left(t_m^b\right) < k^a < k^b$ for $n+m=N$) or higher than both members' relative false-acceptance cost ($g^a\left(t_n^a\right) g^b\left(t_m^b\right) > k^b > k^a$ for $n+m=N+1$).

indicates a high preference for rejection: If the outcome is implemented with a scoring mechanism, the worst errors are committed when B votes 0 and A votes either $\bar{N}$ (for a false acceptance with the lowest likelihood) or $\bar{N}-1$ (for a false rejection with the highest likelihood). Suppose that partition in Figure 1 is the partition of the equilibrium with the highest number of categories. According to our next result, the false acceptance of the null with the lowest likelihood happens around signal pair $(t_2^a, 0)$ and the false rejection of the null with the highest likelihood is around $\left(t_2^a, t_1^b\right)$. The magnitudes of both types of errors decline as we move both upwards and leftward on the graph to the other kink points of the step function.

Proposition 2 *Suppose* $\frac{g^a(\cdot)}{g^{a\prime}(\cdot)}$ *and* $\frac{g^b(\cdot)}{g^{b\prime}(\cdot)}$ *are both concave. If* $t^* = \left(t_0^{*a}, t_1^{*a}, ..., t_{\bar{N}}^{*a}; t_0^{*b}, t_1^{*b}, ..., t_{\bar{N}}^{*b}\right)$ *is the most informative equilibrium solution with the highest possible number of thresholds* $\bar{N}$*, then* $\underline{g}_n(t^*)$ *is decreasing and* $\overline{g}_n(t^*)$ *is increasing in* n*.*

7 Appendix

Proof of Lemma 1

Let us begin by proving the statement of part (1). Totally differentiating the left-hand-side of (A_n) yields:

$$\frac{\overline{g}_n - k^a}{g^{b\prime}\left(t_{N-n+1}^b\right)} d\overline{g}_n + \frac{k^a - \underline{g}_n}{g^{b\prime}\left(t_{N-n}^b\right)} d\underline{g}_n + \left[\int_{\underline{g}_n}^{\overline{g}_n} \left[z - k^a\right] \frac{d\left(g^{b\prime}\left(x^b\right)\right)^{-1}}{dt_n^a} dz\right] dt_n^a = 0. \tag{3}$$

The coefficients of $d\overline{g}_n$ and $d\underline{g}_n$ are positive. To see that coefficient of dt_n^a is positive, notice that

$$d\left(g^{b\prime}\left(x^b\right)\right)^{-1} = -\frac{g^{b\prime\prime}\left(x^b\right)}{\left(g^{b\prime}\left(x^b\right)\right)^2} dx^b = \frac{g^{a\prime}\left(t_n^a\right) g^b\left(x^b\right) g^{b\prime}\left(x^b\right)}{g^a\left(t_n^a\right)\left(g^{b\prime}\left(x^b\right)\right)^3} dt_n^a.$$

Therefore the integral in (3) can be rewritten as follows:

$$\int_{\underline{g}_n}^{\overline{g}_n} [z - k^a] \frac{g^{a\prime}(t_n^a) g^b(x^b) g^{b\prime\prime}(x^b)}{g^a(t_n^a)(g^{b\prime}(x^b))^3} dz = \frac{g^{a\prime}(t_n^a)}{g^a(t_n^a)} \int_{t_{N-n}^b}^{t_{N-n+1}^b} \left[g^a(t_n^a) g^b(x^b) - k^a\right] \frac{g^b(x^b) g^{b\prime\prime}(x^b)}{(g^{b\prime}(x^b))^2} dx^b.$$

The ratio $\frac{g^{a\prime}(t_n^a)}{g^a(t_n^a)}$ is positive. Concavity of $\frac{g^b(\cdot)}{g^{b\prime}(\cdot)}$ implies $\frac{g^b(x^b)g^{b\prime\prime}(x^b)}{(g^{b\prime}(x^b))^2}$ is non-decreasing in x^b. Moreover, we know that $\int_{t_{N-n}^b}^{t_{N-n+1}^b} \left[g^a(t_n^a) g^b(x^b) - k^a\right] dx^b = 0$ and the integrand $g^a(t_n^a) g^b(x^b) - k^a$ is increasing in x^b. This establishes that the coefficient of dt_n^a is positive. Hence, if both t_n^a and $\overline{g}_n$ are increasing (at least one of the strictly), it must be that $\underline{g}_n$ is strictly decreasing for equation (A_n) to continue to hold. Because $\underline{g}_n = g^a(t_n^a) g^b(t_{N-n}^b)$, this also implies that t_{N-n}^b is strictly decreasing. This completes the proof of the statement in part (1) of the Lemma.

The proof of the statement in part (2) is analogous and is therefore omitted. *Q.E.D.*

Proof of Proposition 1

The proof follows from Lemma 1. Start with an arbitrary partial solution with length N. (If there is no such partial solution, the proposition is trivially correct for N.) An increase in t_1^a implies a decrease in t_N^b and $\underline{g}_0$ and an increase in $\overline{g}_1$ (from (B_N)). Then part (1) of the lemma implies a decrease in $\underline{g}_1$ and t_{N-1}^b, and part (2) in turn implies an increase in $\overline{g}_2$ and t_2^a. Continuing in this fashion up to t_N^a and t_0^b establishes (M). *Q.E.D.*

Proof of Proposition 2

We prove this result in two steps, each of which relies on an induction argument. The first step establishes the monotonicity of the likelihood levels calculated at the threshold pairs of the partial solution $\hat{t}(\bar{N})$. The second step shows that this monotonicity is maintained by the continuous deformation of $\hat{t}(\bar{N})$ that yields the equilibrium solution t^*. For brevity, we drop the argument $(\bar{N})$ of $\hat{t}$.

Step 1. $\underline{g}_n(\hat{t})$ is decreasing and $\overline{g}_n(\hat{t})$ is increasing in n.

Given that $\hat{t}_0^a = \hat{t}_1^a = 0$ and $\hat{t}_N^b$ is the solution to $g^a(0) g^b(\hat{t}_N^b) = k^b$, we know that

$\underline{g}_0\left(\hat{t}\right) = \overline{g}_1\left(\hat{t}\right) = k^b$. It follows from (A_1) that $\underline{g}_1\left(\hat{t}\right) < k^a < k^b = \underline{g}_0\left(\hat{t}\right)$, and it follows from (B_{N-1}) that $\underline{g}_2\left(\hat{t}\right) < k^b$. This constitutes the first step of the induction argument. Comparing (B_{N-n-1}) with (B_{N-n}) for arbitrary $n \geq 0$, part (2) of Lemma 1 implies that if $\underline{g}_{n+1}\left(\hat{t}\right) < \underline{g}_n\left(\hat{t}\right)$, then $\overline{g}_{n+2}\left(\hat{t}\right) > \overline{g}_{n+1}\left(\hat{t}\right)$. Similarly, comparing (A_{n+2}) to (A_{n+1}), part (1) of Lemma 1 implies that if $\overline{g}_{n+2}\left(\hat{t}\right) > \overline{g}_{n+1}\left(\hat{t}\right)$, then $\underline{g}_{n+2}\left(\hat{t}\right) < \underline{g}_{n+1}\left(\hat{t}\right)$.

Step 2. $\underline{g}_n\left(t^x\right)$ is decreasing and $\overline{g}_n\left(t^x\right)$ is increasing in n, where t^x is the continuous deformation of $\hat{t}$ defined by the initial condition $t_1^{xa} = x$ for $0 \leq x \leq \hat{t}_2^a$ as above.

If $x = 0$ then $t^x = \hat{t}$. As x increases in the interval $\left[0, \hat{t}_2^a\right]$, monotonicity condition implies that thresholds t_n^{xa} increase for $n > 0$ and thresholds t_n^{xb} decrease for $n \geq 0$ in x. A decreasing t_N^{xb} implies that $\underline{g}_0\left(t^x\right)$ is also continuously decreasing in x. By Lemma 1, $\overline{g}_n\left(t^x\right)$ is increasing and $\underline{g}_n\left(t^x\right)$ is decreasing in x for $n \leq \bar{N}$. At the upper bound of this continuous deformation where $x = \hat{t}_2^a$, the first $\bar{N}-1$ pairs of t^x are identical to the last $\bar{N}-1$ pairs of $\hat{t}$, i.e. $t_n^{xa} = \hat{t}_{n+1}^a$ and $t_n^{xb} = \hat{t}_n^b$ for $1 \leq n \leq \bar{N}-1$ if $x = \hat{t}_2^a$. For $x < \hat{t}_2^a$, this upper bound implies that the value of $\overline{g}_n\left(t^x\right)$ remains smaller than $\overline{g}_{n+1}\left(\hat{t}\right)$ and the value of $\underline{g}_n\left(t^x\right)$ remains larger than $\underline{g}_{n+1}\left(\hat{t}\right)$ for $n < N$. That is, for any $x \in \left[0, \hat{t}_2^a\right)$, and in particular for the value of x yielding the $\bar{N}$ pairs of equilibrium thresholds, we have:

$$
\begin{aligned}
\overline{g}_n\left(\hat{t}\right) &\leq \overline{g}_n\left(t^x\right) < \overline{g}_{n+1}\left(\hat{t}\right) \text{ for } n < N \text{ and } \overline{g}_N\left(\hat{t}\right) \leq \overline{g}_N\left(t^x\right), \\
\underline{g}_n\left(\hat{t}\right) &\geq \underline{g}_n\left(t^x\right) > \underline{g}_{n+1}\left(\hat{t}\right) \text{ for } n < N \text{ and } \underline{g}_N\left(\hat{t}\right) \leq \underline{g}_N\left(t^x\right).
\end{aligned}
$$

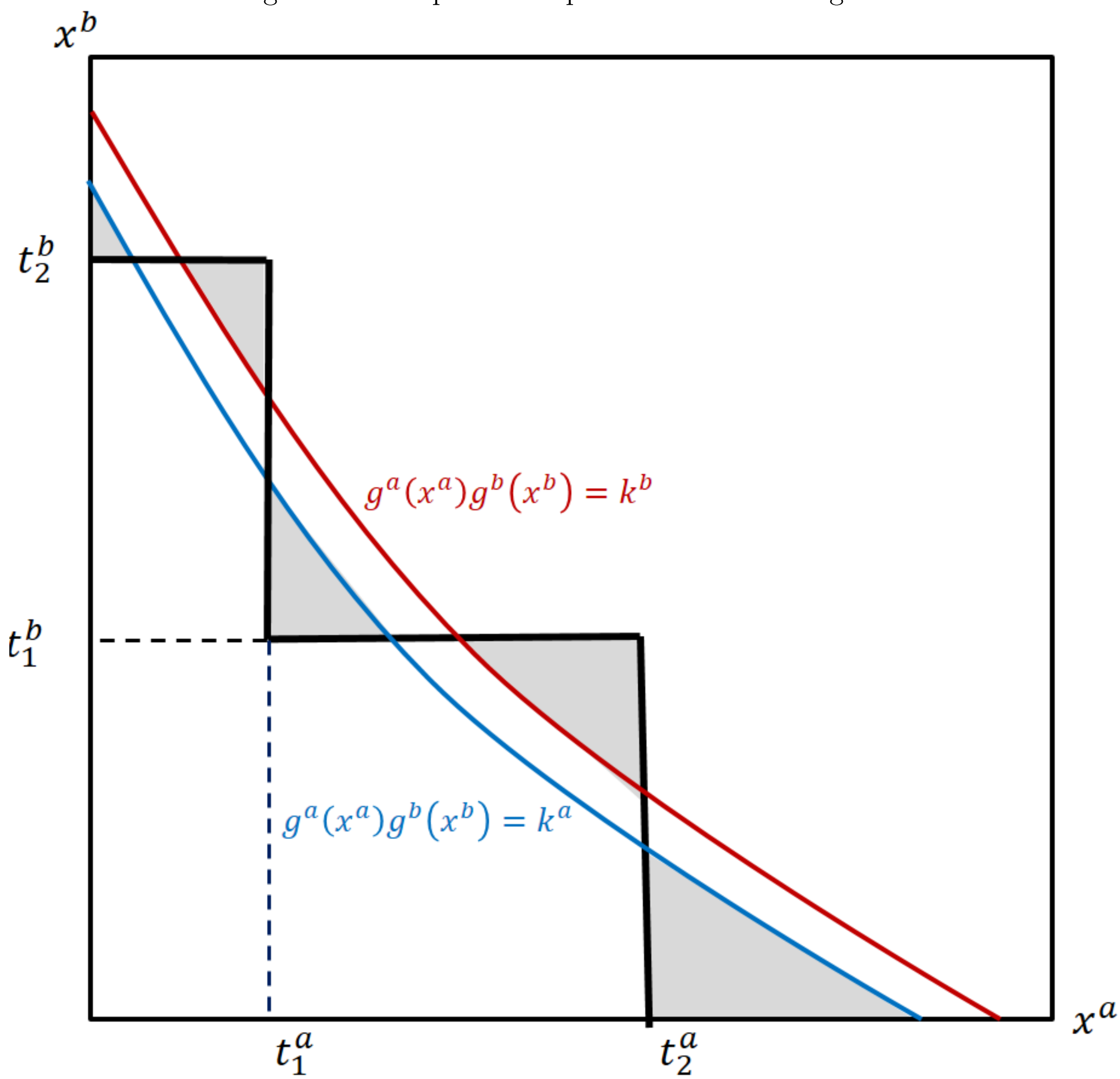
Q.E.D.

References

[1] Crawford, Vincent P., and Joel Sobel. 1982. "Strategic Information Transmission" *Econometrica* 50(6): 1431-1451.

[2] Li, Hao, Sherwin Rosen, and Wing Suen. 2001. "Conflicts and Common Interests in Committees" *American Economic Review* 91(5): 1478-1497.

Figure 1: An equilibrium partition with 3 categories



The step function separates the acceptance (of the null) and rejection regions in equilibrium. The two downward-sloping curves ($g^a(x^a)\, g^b\left(x^b\right) = k^a$ and $g^a(x^a)\, g^b\left(x^b\right) = k^b$) indicate the preferences of the two committee members. Each step of the step function crosses the disagreement zone between these two curves, implying that the committee can make decisions to the contrary of the preferences of both members. The shaded areas give the signal pairs for which the committee makes such wrong decisions.