# A Rational Inattention Theory of Echo Chamber

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#### Abstract

Finite players gather information about an uncertain state before making decisions. Each player allocates his limited attention capacity between biased sources and the other players, and the resulting stochastic attention network facilitates the transmission of information from primary sources to him either directly or indirectly through the other players. The scarcity of attention leads the player to focus on his own-biased source, resulting in occasional crosscutting exposures but most of the time a reinforcement of his predisposition. It also limits his attention to like-minded friends who, by attending to the same primary source as his, serve as secondary sources in case the information transmission from the primary source to him is disrupted. A mandate on impartial exposures to all biased sources disrupts echo chambers but entails ambiguous welfare consequences. Inside an echo chamber, even a small amount of heterogeneity between players can generate fat-tailed distributions of public opinion, and factors affecting the visibility of sources and players could have unintended consequences for public opinion and consumer welfare.

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# 1 Introduction

The Cambridge English Dictionary defines echo chamber as "an environment in which a person encounters only beliefs or opinions that coincide with their own, so that their existing views are reinforced and alternative ideas are not considered." Examples that fit this definition have recently flourished on the Internet and social media (Adamic and Glance (2005); Bakshy, Messing, and Lada (2015); Mosleh et al. (2021)), and they could prove dire for political polarization, public health, and efforts to contain misinformation and fake news (Del Vicario et al. (2016); Barbera (2020); Cossard et al. (2020)). Existing theories of echo chamber focus on its behavioral roots (Levy and Razin (2019)). This paper develops a rational theory of echo chamber and examines its normative implications.

Our premise is Rational Inattention (RI), i.e., the rational and flexible allocation of one's limited attention span across the various news sources. Such a premise has become increasingly relevant in today's digital age, as more people get news from the Internet and social media where the amount of available information (2.5 quintillion bytes) is vastly greater than what any individual can process in a lifetime (Matsa and Lu (2016)). Constrained by attentional bottlenecks, news consumers must be selective about which websites to read and which people to follow on Facebook and Twitter. Enabled by technology advancements, they can now personalize how much time to spend on the various news sources based on their preferences, needs, demographic and psychographic attributes, digital footprints, etc (Pariser (2011)).

In this paper, we take a simplistic view on news sources, modeling them as potentially biased technologies rather than self-interest entities with persuasion or profitmaximizing motives (see, however, an earlier draft of this paper for an extension). Instead, we focus on how RI gives rise to echo chambers and affects the opinion distribution inside an echo chamber. We also examine regulations of information platforms in terms of their efficacy in disrupting echo chambers and in shaping public opinion and consumer welfare.

Our analysis is conducted in a simple environment of decision-making under uncertainties. To fix ideas, suppose there are two possible states of the world L and R that occur with equal probability, depending on whether the Democratic or the Republican candidate has a better quality. There are finitely many players, each of whom can express support to one candidate by taking action L or R, and his utility is the highest when his action matches the true state of the world. If the two objects mismatch, then the player's disutility depends on whether he is taking his *default action*, i.e., supporting the candidate he horizontally prefers. Taking the default action is optimal given the prior belief about the state distribution, and two players are *like-minded friends* if they share the same default action.

There are two primary sources, e.g., newspapers, that specialize in the reporting of different state realizations. In each state L or R, the source that specializes in its reporting generates news about it, whereas the other source is deactivated. To make informed decisions, players pay attention to primary sources and to other players by spending valuable time with them. Attention is limited, and yet its allocation is fully flexible: a *feasible attention strategy* for a player specifies the amount of attention he pays to each source and to any other player, subject to the constraint that the total amount of attention he pays mustn't exceed his *bandwidth*.

After players decide on their attention strategies, the state is realized, and news is circulated in the society for two rounds according to independent Poisson processes. In the first round, the activated primary source disseminates news about the realized state to players, and the probability that such news reaches a player increases with the amount of attention the latter pays to the primary source. In the second round, each player who learned about the state previously passes along the news as a *secondary source*, and the probability that such news reaches another player increases with the sender's *visibility parameter* (e.g. domain-specific knowledge, socioeconomic status, personality strength, desire to influence others, tech-savviness (Winter and Neubaum (2016)) and the amount of attention the receiver pays to the sender. After that, players update beliefs and take actions. We analyze pure strategy perfect Bayesian equilibria of this game.

We interpret specialized reporting as a kind of source bias. Specifically, we define player's *own-biased source* as the source that generates disapproving news of his default action, so that hearing no news from it reinforces the player's belief that the state favors his default action.<sup>1</sup> We note that all like-minded friends share the same own-biased source, and say that *echo chambers arise in equilibrium* if all players limit attention to their own-biased sources and like-minded friends. In an echochamber equilibrium, each player remains uninformed of the state with more than fifty percent probability and reinforces his belief that the state favors his default action in that event. That is, he is occasionally exposed to cross-cutting content and updates his belief accordingly, but most of the time he gets his predisposition reinforced. The coexistence of belief polarization and occasional belief reversal is a hallmark of Bayesian rationality, and its presence after (social) media consumption has recently been documented by the empirical literature.<sup>2</sup>

<sup>&</sup>lt;sup>1</sup>Our notion of source bias isn't new to the literature (see Section 1.1 for further discussions). According to it, a left-wing voter's own-biased source is NY Times, because hearing no endorsement by NY Times for the Republican candidate reinforces the belief that the state favors the Democratic candidate. According to Chiang and Knight (2011), newspaper endorsements of presidential candidates are most effective in shaping voters' beliefs and decisions if they go against the bias of the newspaper.

<sup>&</sup>lt;sup>2</sup>Flaxman, Goel, and Rao (2016) compare subjects' ideological positions before and after Internet and social media consumption in a large data set. They find that social media consumption increases subjects' propensities to support their own-party candidates, as well as their opinion intensities when supporting opposite-party candidates. Allcott et al. (2020) and Balsamo et al. (2019) provide separate accounts for belief polarization and belief reversal after social media consumption, respectively.

We examine when and how echo chambers could arise in equilibrium. Since a player can always take his default action without paying attention, paying attention is only useful if it sometimes disapproves of him from taking that action. When attention is scarce, a player would focus on the disapproving news generated by his own-biased source. He would also limit attention to his like-minded friends who, by attending to the same primary source as his, could serve as secondary sources in case the transmission of news from the primary source to him is disrupted. Such a configuration arises as the unique equilibrium outcome when players have sufficiently strong preferences for taking default actions or when the number of players is large. It is in general inefficient, as mandating players to attend to the primary sources that are biased against their predispositions quality more of them as secondary sources and could generate significant efficiency gains.

Inside an echo chamber, we define the amount of attention a player pays to his ownbiased source as his *resourcefulness* as a secondary source. In order for a player to be attended by his like-minded friends, his resourcefulness must cross a threshold, after which he receives a constant amount of attention from each of his like-minded friends (hereafter, his *influence* on public opinion). We name those players who manage to cross their visibility thresholds as *core* players and those who fail to do so as *peripheral* players. When both types of players exist, a *core-periphery architecture* emerges, whereby core players acquire information from the primary source and share results among each other, whereas peripheral players tap into the core for second-hand news but are themselves ignored by any other player. Such a configuration is most likely to arise when players are heterogeneous, so that those players with large bandwidths and high visibility parameters form the core, whereas the remaining players form the

Evidence for Bayesian rationality in traditional media consumption is surveyed by DellaVigna and Gentzkow (2010).

periphery.

Our comparative statics exercises exploit the fact that players' resourcefulness as secondary sources are *strategic substitutes*. As a player becomes more resourceful, his like-minded friends pay more attention to him and less attention to the primary source. By tracing out how the resulting effects reverberate in the echo chamber using a new toolkit, we demonstrate that increasing a player's bandwidth improves his resourcefulness and influence while diminishing that of *any* other player. As a result, even a small amount of initial heterogeneity among players can be magnified into a very uneven distribution of public opinion, with some players acting as opinion leaders and others as opinion followers. An important source of heterogeneity stems from today's high-choice media environment, which enables most people to abandon hard news consumption for entertainment and causes their separations from a small number of news junkies (Prior (2007)). According to our theory, this widening gap between people's availability to consume hard news may give rise to fat-tailed distributions of public opinion, whose presences on the social media sphere have been detected by Farrell and Drezner (2008), Lu et al. (2014), and Néda, Varga, and Biró (2017).

Our comparative statics exercises also shed light on the efficacy of regulating information platforms. For one thing, we find the comparative statics regarding players' visibility parameters ambiguous, which suggests that recent calls for Internet platforms to tighten content controls in response to the storming of the U.S. Capitol (see, e.g., Romm (2021)) could have unintended consequences for public opinion and consumer welfare. For another, we find that as an echo chamber grows in size, each of its members has access to more secondary sources and so acquires less information from the primary source. Depending on whether the efficiency loss from free-riding dominates the efficiency gain from having a larger population, the net effect on players' equilibrium utilities can go either way. Recent attempts by tech companies such as Allsides.com to battle against the rising polarization mandate that platform users must be impartially exposed to both liberal and conservative viewpoints. We find that while mandatory exposure does disrupt echo chambers by forcing people on different sides of the political spectrum to attend to each other, it makes more secondary sources available and so entails an ambiguous welfare consequence in general.

### 1.1 Related literature

**Rational inattention** The literature on rational inattention (RI) pioneered by Sims (1998) and Sims (2003) has traditionally refrained players from acquiring information about other players' signals. Denti (2015) demonstrates the limitation of this assumption when players are coordination motives. The current paper abstracts away from coordination motives. Instead, a player is attended by his like-minded friends if he is more capable of spreading information to the other players than the primary source is.

The idea that even a rational decision maker can exhibit a preference for biased news when constrained by limited information processing capacities dates back to Calvert (1985) and is later expanded on by Suen (2004), Zhong (2017), and Che and Mierendorff (2019). While the last three papers study the dynamic information acquisition problems faced by a single decision maker, the current paper examines the trade-off between attending to primary sources and to other players in a static game. Our model becomes equivalent to the stage decision problem studied by Che and Mierendorff (2019) if players are forbidden from attending to each other. The assumption of Poisson attention technology is also made by Dessein, Galeotti, and Santos (2016) to analyze the efficient attention network among the nonstrategic members of an organization. Zhong (2017) provides a foundation for Poisson attention technologies by studying a dynamic RI problem with two terminal actions.

**Rational information segregation** A small yet growing literature examines the rational origins of information segregation. The closest work to ours is done by Baccara and Yariv (2013) (BY), though their theory has three major differences from ours.<sup>3</sup> First, BY model information as a local public good that is automatically shared among group members, and players' preferences for public goods are encoded in their utility functions. Here, acquiring second-hand news from other players is a costly private decision, and the preference for one kind of news over another arises endogenously from RI. Second, BY generate conglomerations via sorting, i.e., by limiting group sizes and letting players sort into the groups that produce the closest information goods to their tastes. In contrast, we impose no restriction on the sizes of echo chambers and instead determine them endogenously through RI. Finally, BY's assumption about players' preferences makes their analysis of group composition interesting. Such an analysis is uninteresting here, as players' preference parameters do not affect what happen inside echo chambers.

**Network theory** Inside an echo chamber, our players play a network game where their investments in resourcefulness as secondary sources constitute strategic substitutes. Meanwhile, the attention network between them is formed endogenously based on their resourcefulness. Bala and Goyal (2000) pioneer the study of non-cooperative network formation without endogenous investments. There is also a large literature on

<sup>&</sup>lt;sup>3</sup>Other rational theories of information segregation are proposed by: Galeotti, Ghigilino, and Squintani (2013), who examine factors that facilitate strategic communication among multiple players; Sethi and Yildiz (2016), who demonstrate how past interactions beget future ones by allowing players to infer each others' biases; Meng (2021), who studies a model of matching followed by Bayesian persuasion and demonstrates how the desire to persuade dissolves partnerships between people holding significantly different priors. In contrast, Levy and Razin (2019) survey the behavioral roots of echo chambers.

games with negative local externalities played on fixed networks (see, e.g., Bramoullé, Kranton, and D'Amours (2014)). To trace out how changing in one player's characteristics affects the opinion distribution inside an echo chamber, we develop a new toolkit that is to our best knowledge new to this literature.

Galeotti and Goyal (2010) (GG) analyze a game in which players can acquire information from a primary source and share information through endogenous network formation. By modeling information as a scalar and working with homogeneous players, GG abstract away from issues of our interest such as news bias and echo chamber formation. And by working with different information acquisition and network formation technologies from ours, GG manage to predict the *law of the few* among homogeneous players,<sup>45</sup> whereas we can do so most easily among heterogeneous players. Kinateder and Merlino (2017) investigate how making GG's players heterogeneous affects their predictions.

The remainder of the paper proceeds as follows: Section 2 introduces the baseline model; Section 3 gives equilibrium characterizations; Section 4 conducts comparative statics analysis; Section 5 investigates extensions of the baseline model; Section 6 concludes. Proofs and additional materials can be found in Appendices A-C.

<sup>&</sup>lt;sup>4</sup>The law of the few refers to the phenomenon that information is disseminated by a few key players to the rest of the society. It was originally discovered by Lazarsfeld, Berelson, and Gaudet (1948) in their classical study of how personal contacts facilitate the dissemination of political news in the age of mass media, and it has since then been rediscovered in numerous areas such as marketing and the organization of online communities.

 $<sup>{}^{5}</sup>$ GG's reasoning hinges on the tension between two properties of their information technology: (1) the total amount of information acquisition is independent of players' population size; and (2) links for information sharing are discrete. Together, they limit the number of core players acquiring information from the primary source and spreading information to the rest of the society.

# 2 Baseline model

### 2.1 Setup

A finite set  $\mathcal{I}$  of players faces an uncertain state  $\omega$  of the world that is evenly distributed on  $\{L, R\}$ . Each player  $i \in \mathcal{I}$  can take an action  $a_i \in \{L, R\}$ , and his utility in state  $\omega$  is the highest and is normalized to zero if  $a_i = \omega$ . If  $a_i \neq \omega$ , then the player experiences a disutility whose magnitude depends on whether he is taking his *default action*  $d_i \in \{L, R\}$  or not. This disutility equals  $-\beta_i$  if  $a_i = d_i$  and -1 if  $a_i \neq d_i$ , where the assumption  $\beta_i \in (0, 1)$  implies that taking the default action is optimal given the prior belief about the state distribution. For convenience, we refer to a player's default action as his *type* and say that two players of the same type are *like-minded friends*. Denote the sets of type-L and type-R players by  $\mathcal{L}$  and  $\mathcal{R}$ , respectively, and assume that  $|\mathcal{L}|, |\mathcal{R}| \in \mathbb{N} - \{1\}$ .

There are two primary sources: L-revealing and R-revealing. In state  $\omega \in \{L, R\}$ , the  $\omega$ -revealing source is activated and generates news that fully reveals  $\omega$ , whereas the other source is deactivated. To gather information about the state, a player must pay attention to sources or to the other players by spending valuable time with them. The total amount of attention he pays mustn't exceed his bandwidth, and yet the allocation of attention across the various sources and players is fully flexible. For each player  $i \in \mathcal{I}$ , let  $C_i = \{\text{L-rev}, \text{R-rev}\} \cup \mathcal{I} - \{i\}$  denote the set of the parties he can attend to,  $x_i^c \geq 0$  denote the amount of attention he pays to party  $c \in C_i$ , and  $\tau_i > 0$  denote his bandwidth. An attention strategy  $x_i = (x_i^c)_{c \in C_i}$  is feasible for player i if it satisfies his bandwidth constraint  $\sum_{c \in C_i} x_i^c \leq \tau_i$ , and the set of feasible attention strategies for him is  $\mathcal{X}_i = \{x_i \in \mathbb{R}_+^{|C_i|} : \sum_{c \in C_i} x_i^c \leq \tau_i\}$ .

After players choose their attention strategies, the state  $\omega$  is realized, and news about  $\omega$  is circulated in the society for two rounds. In the first round, news is disseminated by the  $\omega$ -revealing source to players according to independent Poisson processes with rate one, and the probability  $1 - \exp(-x_i^{\omega\text{-rev}})$  that such news reaches each player  $i \in \mathcal{I}$  increases with the amount of attention the latter pays to the source. In the second round, each player i who learned about  $\omega$  in the first round passes along the news to the other players as a *secondary source*. The dissemination of news follows independent Poisson processes with rate  $\lambda_i > 0$  (hereafter, player i's *visibility parameter*), and the probability  $1 - \exp(-\lambda_i x_j^i)$  that each player  $j \in \mathcal{I} - \{i\}$  hears from player i increases the sender's visibility parameter and the amount of attention the recipient pays to the sender. After that, players update their beliefs about the state and take actions. The game sequence is summarized as follows.

- 1. Players choose attention strategies.
- 2. The state  $\omega$  is realized.
- 3. (a) The  $\omega$ -revealing source disseminates news to players.
  - (b) Players who learned about  $\omega$  in Stage 3(a) pass along the news to other players.
- 4. Players take actions.

Our solution concept is *pure strategy perfect Bayesian equilibrium* (PSPBE).

### 2.2 Model discussion

This section walks through the main modeling assumptions.

**Decision environment** We consider a simple decision environment in which a player's utility depends the state and his action, but not on other players' actions. In this way, we are able to single out the role of RI (rather than the direct payoff

externalities between players' actions) in generating echo chambers. The assumption of binary states and actions will be relaxed in Appendix B.6.

**Sources** We will interpret specialized reporting as a kind of primary source bias in Section 3.2. The assumption that there is a single primary source of each kind has no consequence for our analysis and will be dispensed with in Section 5. For each player, we assume that his visibility as a secondary source is constant across potential recipients of his information. This assumption is crucial for technical reasons (see Section 4.1), and it will be relaxed in Appendix B.5. Finally, neither Poisson attention technologies nor a single round of information transmission among players matters much for the rise of echo chambers, yet they are crucial for the opinion distribution inside an echo chamber (see Section 3.3).

## 3 Analysis

### 3.1 Players' problems

This section formalizes the problem faced by any player  $i \in \mathcal{I}$ , taking the attention strategies  $x_{-i} \in \mathcal{X}_{-i} := \times_{j \in \mathcal{I} - \{i\}} \mathcal{X}_i$  of the other players as given. In case player *i* uses attention strategy  $x_i \in \mathcal{X}_i$ , the attention channel between him and the  $\omega$ -revealing source is disrupted with probability

$$\delta_i^{\omega\text{-rev}} = \exp\left(-x_i^{\omega\text{-rev}}\right),$$

and the attention channel between him and player  $j \in \mathcal{I} - \{i\}$  is disrupted with probability

$$\delta_i^j = \exp\left(-\lambda_j x_i^j\right).$$

Define  $\mathcal{U}_i$  as the event in which player *i* remains uninformed of the state at Stage 4 of the game (hereafter, the *decision-making stage*). In state  $\omega$ ,  $\mathcal{U}_i$  happens if all attention channels that connect player *i* to the  $\omega$ -revealing source either directly or indirectly through the other players are disrupted. Its probability equals

$$\mathbb{P}_{x}\left(\mathcal{U}_{i} \mid \omega\right) = \delta_{i}^{\omega \operatorname{-rev}} \prod_{j \in \mathcal{I} - \{i\}} \left(\delta_{j}^{\omega \operatorname{-rev}} + \left(1 - \delta_{j}^{\omega \operatorname{-rev}}\right) \delta_{i}^{j}\right),$$

where  $x = (x_i, x_{-i})$  denotes the joint attention strategy across all players.

At the decision-making stage, player *i* matches his action with the state realization and earns zero payoff in event  $\mathcal{U}_i^c$ . In event  $\mathcal{U}_i$ , he prefers to take his default action  $d_i$  if the likelihood ratio that the state favors his default action rather than the other action exceeds his preference parameter:

$$\frac{\mathbb{P}_x\left(\mathcal{U}_i \mid \omega = d_i\right)}{\mathbb{P}_x\left(\mathcal{U}_i \mid \omega \neq d_i\right)} \ge \beta_i.$$

At Stage 1 of the game (hereafter, the *attention-paying stage*), player i's expected utility equals

$$\max\left\{-\frac{\beta_{i}}{2}\mathbb{P}_{x}\left(\mathcal{U}_{i}\mid\omega\neq d_{i}\right),-\frac{1}{2}\mathbb{P}_{x}\left(\mathcal{U}_{i}\mid\omega=d_{i}\right)\right\},\$$

where the first and second terms in the above expression represent the expected disutilities generated by the state-contingent plans of taking  $a_i = d_i$  and  $a_i \neq d_i$  in event  $\mathcal{U}_i$ , respectively. The player chooses a feasible attention strategy to maximize his expected utility, taking the other players' strategies  $x_{-i}$  as given.

### 3.2 Key concepts

This section defines key concepts to the upcoming analysis. We first interpret specialized reporting as a kind of *source bias*.

**Definition 1.** A primary source is a-biased for  $a \in \{L, R\}$  if hearing no news from it increases the belief that the state favors action a compared to the prior belief.

By definition, the *R*-revealing source is *L*-biased (hereafter denoted by l), whereas the *L*-revealing source is *R*-biased (hereafter denoted by r). To better understand these concepts, imagine, in the leading example detailed in the introduction, that l is NY Times and r is Fox News. According to Chiang and Knight (2011), an endorsement of the Republican candidate by NY Times, which goes against its bias, is most effective in shaping voters' beliefs and voting decisions. Absent such an endorsement, voters update their beliefs in favor of the Democratic candidate.

We next define players' own-biased sources.

**Definition 2.** A player's own-biased source is the primary source that is biased towards his default action.

By definition, a type-L player's own-biased source is l (NY Times), and a type-R player's own-biased source is r (Fox News). Moreover, hearing no news from one's own-biased source reinforces his belief that the state favors his default action.

We next define what it means for *echo chambers* to arise in equilibrium.

**Definition 3.** An equilibrium is an echo-chamber equilibrium if all players attend only to their own-biased sources and like-minded friends at the attention-paying stage and take their default actions in case they remain uninformed of the state at the decision-making stage. An echo-chamber equilibrium has two defining features. The first feature is the "chamber," i.e., the confinement of one's attention to his own-biased source and likeminded friends. The second feature follows from the first one: as a result of playing confined attention allocation strategies, each player remains uninformed of the state with more than fifty percent probability and reinforces his belief that the state favors his default action in that event. Thus in an echo-chamber equilibrium, players are occasionally exposed to cross-cutting content and update their beliefs accordingly, but most of the time they get their predispositions reinforced, just like what Flaxman, Goel, and Rao (2016) find among social media consumers. Combining these features gives rise to the name of echo-chamber equilibrium.

We finally define two useful functions.

**Definition 4.** For each  $\lambda \geq 0$ , define

$$\phi(\lambda) = \begin{cases} \log\left(\frac{\lambda}{\lambda-1}\right) & \text{if } \lambda > 1, \\ +\infty & \text{if } 0 \le \lambda \le 1 \end{cases}$$

For each  $\lambda > 1$  and  $x \in [\phi(\lambda), +\infty)$ , define

$$h(x; \lambda) = \frac{1}{\lambda} \log \left[ (\lambda - 1) \left( \exp \left( x \right) - 1 \right) \right].$$

**Lemma 1.**  $\phi' < 0$  on  $(1, +\infty)$ . For each  $\lambda > 1$ ,  $h(\cdot; \lambda)$  satisfies (i)  $h(\phi(\lambda); \lambda) = 0$ and  $h_x(\phi(\lambda); \lambda) = 1$ ; (ii)  $h_x(x; \lambda) \in (0, 1)$  and  $h_{xx}(x, \lambda) < 0$  on  $(\phi(\lambda), +\infty)$ .

### 3.3 Equilibrium analysis

Consider first a benchmark case in which players can attend only to primary sources but not to each other. The next lemma solves for players' decision problems in this case.

**Lemma 2.** Let everything be as in Section 2 except that Stage 3(b) is removed from the game. Then each player attends only to his own-biased source at the attentionpaying stage and takes his default action in case he remains uninformed of the state at the decision-making stage.

The idea behind Lemma 2 is straightforward and standard. Since a player can always take his default action without paying attention, paying attention is useful only if it sometimes disapproves of him from taking that action. When attention is scarce, it is optimal to focus on the disapproving news generated by one's own-biased source and then increase the belief that the state favors his default action in case he doesn't hear such news. The last event happens with more than fifty percent probability, implying that rational and flexible attention allocation reinforces one's predisposition most of the time.

We next allow players to attend to each other. The next theorem shows that if we keep increasing players' preferences for taking default actions while holding everything else constant, then echo chambers will eventually emerge as the unique equilibrium outcome.

**Theorem 1.** Fix the population sizes  $|\mathcal{L}|, |\mathcal{R}| \in \mathbb{N} - \{1\}$  and characteristic profiles  $(\lambda_i, \tau_i)_{i \in \mathcal{L}} \in \mathbb{R}^{2|\mathcal{L}|}_{++}, (\lambda_i, \tau_i)_{i \in \mathcal{R}} \in \mathbb{R}^{2|\mathcal{R}|}_{++}$  of type-L and type-R players, respectively. Then there exists  $\underline{\beta} \in (0, 1)$  such that if  $\beta_i \in (0, \underline{\beta}) \ \forall i \in \mathcal{I}$ , then any PSPBE of our game must be an echo-chamber equilibrium, and such an equilibrium exists.

When a player has a sufficiently strong preference for taking his default action, doing so becomes a dominant strategy in case he remains uninformed of the state at the decision-making stage. This implies that at the attention-paying stage, the player would focus on his own-biased source and ignore the other source for the reason articulated above. He would also limit attention to his like-minded friends who, by attending to the same primary source as his, could serve as secondary sources in case the attention channel between him and the primary source is disrupted. This completes the proof that any equilibrium must be an echo-chamber equilibrium, which doesn't exploit the exact functional form of Poisson attention technologies or the assumption that news is transmitted among players for a single round. The existence of an equilibrium will soon become clear. In Appendix B.1, we show that echo chambers would also arise in equilibrium if the numbers of type-L and type-R players are sufficiently large.

We next take a closer look at what happen inside an echo chamber. Without loss of generality (w.l.o.g.), consider the echo chamber among type-L players.

**Theorem 2.** Any echo-chamber equilibrium must satisfy the following properties.

(i) For any  $i \in \mathcal{L}$ ,

$$x_{i}^{l} = \max\left\{\tau_{i} - \sum_{j \in \mathcal{L} - \{i\}} \frac{1}{\lambda_{j}} \log \max\left\{(\lambda_{j} - 1)(\exp(x_{j}^{l}) - 1), 1\right\}, 0\right\}.$$

(ii) In case all type-L players attend to source l, i.e., x<sup>l</sup><sub>i</sub> > 0 ∀i ∈ L, the following are equivalent for any i ∈ L: (a) x<sup>i</sup><sub>j</sub> > 0 for some j ∈ L − {i}; (b) x<sup>l</sup><sub>i</sub> > φ(λ<sub>i</sub>);
(c)

$$x_j^i = h(x_i^l; \lambda_i) \ \forall j \in \mathcal{L} - \{i\}.$$

(iii) In case all type-L players attend to each other, i.e.,  $x_j^i > 0 \ \forall i \in \mathcal{L}$  and  $j \in$ 

 $\mathcal{L} - \{i\}$ , the Stage 1-expected utility of any  $i \in \mathcal{L}$  equals

$$-\frac{\beta_i}{2} \exp\left(-\sum_{j\in\mathcal{L}} x_j^l + \sum_{j\in\mathcal{L}-\{i\}} \phi\left(\lambda_j\right)\right).$$

Part (ii) of Theorem 2 shows that in order for player *i* to be attended by his like-minded friends, he must first cross his threshold of being visible, i.e., pay at least  $\phi(\lambda_i)$  units of attention to the primary source that is decreasing in his visibility parameter  $\lambda_i$ . After that, he receives a constant amount of attention  $h(x_i^l; \lambda_i)$  from each of his like-minded friends that is increasing in the amount of attention  $x_i^l$  he pays to the primary source. For this reason, we shall hereafter refer to  $x_i^l$  as player *i*'s resourcefulness as a secondary source. To pin down the strategy profile(s) that can arise in equilibrium, we can first solve the system of equations that governs equilibrium resourcefulness as in Theorem 2(i), and then substitute the solution(s) into Theorem 2(ii-c) to back out the attention network between like-minded friends. The existence of a solution to the system of equations and, hence, that of an equilibrium, follows from Brouwer's fixed point theorem.

The following properties of equilibrium attention network(s) are noteworthy.

Core-periphery architecture For any equilibrium, let  $COR = \{i \in \mathcal{L} : x_i^l > \phi(\lambda_i)\}$  to denote the set of the players who are attended by their like-minded friends and  $\mathcal{PER} = \{i \in \mathcal{L} : x_i^l \leq \phi(\lambda_i)\}$  denote the set of the players who are ignored by their like-minded friends. When both sets are nonempty, a *cor-periphery architecture* emerges, whereby COR players acquire information from the primary source and share results among each other, whereas  $\mathcal{PER}$  players tap into COR for second-hand news but are themselves ignored by any player. A necessary condition for player *i* to belong to COR is  $\tau_i > \phi(\lambda_i)$ , which holds if the player has a large bandwidth and a high visibility parameter, and only if he is more visible than the primary source, i.e.,  $\lambda_i > 1.^6$  If, instead,  $\tau_i \leq \phi(\lambda_i)$ , then the player belongs to  $\mathcal{PER}$  in any equilibrium. Note that a core-periphery architecture is most likely to arise when players are heterogeneous, so that those players with large bandwidths and high visibility parameters form  $\mathcal{COR}$  and the remaining players  $\mathcal{PER}.^7$  Also note that players' preference parameter  $\beta_i$ 's do not affect the division between  $\mathcal{COR}$  and  $\mathcal{PER}$ and, indeed, what happen inside an echo chamber.

Strategic substitutability between resourcefulness For any COR player, we define his *influence* on public opinion as the amount  $h(x_i^l; \lambda_i)$  of attention he receives from any other player. From  $h_x > 0$ , it follows that players' resourcefulness as secondary sources are *strategic substitutes*: as a player becomes more resourceful, his like-minded friends pay more attention to him and less attention to the primary source. In the next section, we examine the consequences of this observation for public opinion distribution in great detail.

Equilibrium utility When all players belong to COR, the equilibrium expected utility of any player depends positively on the total amount of attention the echo chamber pays to the primary source, and it depends negatively on the visibility thresholds of his like-minded friends. Intuitively, members of an echo chamber become better off as they together acquire more information from the primary source and as they become more capable of disseminating news to each other.

<sup>&</sup>lt;sup>6</sup>The phenomenon that a person like Oprah Winfrey is more capable of disseminating (political) information to the audience than primary news sources are is called the "Oprah effect" by political scientists (Baum and Jamison (2006)).

<sup>&</sup>lt;sup>7</sup>When players are homogeneous, we may not be able to sustain a core-periphery architecture in any equilibrium (as suggested by the numerical analysis that has so far been conducted). In Appendix B.3, we provide sufficient conditions for the game among homogeneous players to exhibit a unique (and, hence, symmetric) equilibrium.

We close this section by pointing out that echo chambers are in general inefficient. Intuitively, mandating that players attend to the sources that are biased against their predispositions qualify more of them as secondary sources. The resulting efficiency gain can be significant, but it cannot be realized in any equilibrium when the preference for taking default action is sufficiently strong (see Appendix B.2 for further details).

### 4 Comparative statics

This section examines the comparative statics of echo-chamber equilibrium. The analysis is conducted among type-L players under the following assumption.

**Assumption 1.** The game among type-L players has a unique equilibrium, and all type-L players attend to each other in that equilibrium.

Assumption 1 has two parts. The first part on the uniqueness of equilibrium is substantial and will be expanded on in Appendix B.3. At this stage, it suffices to notice that since the attention network between like-minded friends is endogenous, we cannot directly impose restrictions on it when proving the uniqueness of equilibrium (as is done in the existing studies of network games with negative externalities such as Bramoullé, Kranton, and D'Amours (2014)). The second part of Assumption 1, which stipulates that all players belong to COR, is meant to ease the exposition: as shown in Appendix B.4, introducing PER players to the analysis wouldn't affect any of our qualitative predictions.

In the remainder of this section, we write  $\mathcal{L} = \{1, \dots, N\}, \theta_i = (\lambda_i, \tau_i) \ \forall i \in \mathcal{L}$ , and  $\boldsymbol{\theta} = [\theta_1, \dots, \theta_N]^{\top}$ . We conduct two comparative statics exercises: the first exercise varies players' characteristic  $\theta_i$ 's while holding their population size fixed, whereas

the second exercise does the opposite. We exclude players' preference parameter  $\beta_i$ 's from our analysis because they do not affect what happen inside the echo chamber.

### 4.1 Individual characteristics

The next theorem examines how varying a single player's characteristics affects equilibrium outcomes.<sup>8</sup>

**Theorem 3.** Fix any  $N \in \mathbb{N} - \{1\}$ , and let  $\Theta$  be any neighborhood in  $\mathbb{R}^{2N}_{++}$  such that for any  $\theta \in \Theta$ , the game among a set  $\mathcal{L}$  of type-L players with population size N and characteristic profile  $\theta$  satisfies Assumption 1. Then the following must hold for any  $i \in \mathcal{L}, j \in \mathcal{L} - \{i\}$ , and  $k \in \mathcal{L} - \{j\}$  at any  $\theta^{\circ} \in int(\Theta)$ .

(i) 
$$\partial x_i^l / \partial \tau_i \big|_{\boldsymbol{\theta} = \boldsymbol{\theta}^\circ} > 0, \ \partial x_j^i / \partial \tau_i \big|_{\boldsymbol{\theta} = \boldsymbol{\theta}^\circ} > 0, \ \partial x_j^l / \partial \tau_i \big|_{\boldsymbol{\theta} = \boldsymbol{\theta}^\circ} < 0, \ and \ \partial x_k^j / \partial \tau_i \big|_{\boldsymbol{\theta} = \boldsymbol{\theta}^\circ} < 0.$$

(ii) One of the following situations happens:

- (a)  $\partial x_i^l / \partial \lambda_i |_{\boldsymbol{\theta} = \boldsymbol{\theta}^\circ} > 0, \ \partial x_j^i / \partial \lambda_i |_{\boldsymbol{\theta} = \boldsymbol{\theta}^\circ} > 0, \ \partial x_j^l / \partial \lambda_i |_{\boldsymbol{\theta} = \boldsymbol{\theta}^\circ} < 0, \ and \ \partial x_k^j / \partial \lambda_i |_{\boldsymbol{\theta} = \boldsymbol{\theta}^\circ} < 0;$
- (b) all inequalities in Part (a) are reversed;
- (c) all inequalities in Part (a) are replaced with equalities.
- (iii) If N = 2 or if players are homogeneous, then  $\partial \sum_{n \in \mathcal{L}} x_n^l / \partial \tau_i \Big|_{\theta = \theta^\circ} > 0$  and  $\operatorname{sgn}\left(\partial \sum_{n \in \mathcal{L}} x_n^l / \partial \lambda_i \Big|_{\theta = \theta^\circ}\right) = \operatorname{sgn}\left(-\partial x_i^l / \partial \lambda_i \Big|_{\theta = \theta^\circ}\right).$

Part (i) of Theorem 3 shows that increasing a player's bandwidth raises his resourcefulness and influence while diminishing that of any other player, suggesting that even a small amount of initial heterogeneity among players can be magnified into a

<sup>&</sup>lt;sup>8</sup>It is easy to extend our analysis to common shocks to players' characteristics. In Section 5, we demonstrate that increasing the visibility of sources effectively scales down all players' visibility parameters.

very uneven distribution of public opinion. An important source of heterogeneity stems from today's high-choice media environment, which enables most people to abandon hard news consumption for entertainment and causes their separation from news junkies who constitute only a small fraction of the population (Prior (2007)). According to Theorem 3(i), this widening gap between people's availability to consume news may give rise to the law of the few whereby news junkies consume most first-hand news, whereas the majority of us relies on the second-hand news passed along by them. Recently, patterns consistent with the law of the few, such as fat-tailed distributions of public opinion, have been detected among political blogs, retweets, and Facebook shares (Farrell and Drezner (2008); Lu et al. (2014); Néda, Varga, and Biró (2017)). In the future, it will be interesting to quantify the role of our amplifying mechanism in generating the above patterns.

Part (ii) of Theorem 3 shows that increasing a player's visibility parameter by a small amount either raises his resourcefulness and influence while diminishing that of any other player, or vice versa. On the one hand, such a perturbation reduces the player's visibility threshold, which must be crossed before he can start attracting attention from other players. On the other hand, there is a countervailing effect stemming from the player's enhanced efficacy in spreading information, which makes other players less responsive to changes in his resourcefulness. In general, either effect can dominate the other (as depicted in Figures 1 and 2 in Appendix C), which renders the comparative statics ambiguous. In the aftermath of the 2021 storming of the U.S. Capitol, there have been calls to modify Section 230 of the Communications Decency Act of 1996 so as to enable Internet platforms to exercise more content controls (see, e.g., Romm (2021)). According to Theorem 3(ii), augmenting the visibility of Internet or social media accounts could have unintended consequences for public opinion and so must be strictly scrutinized before they are put into practice.

Part (iii) of Theorem 3 concerns the total amount of attention the echo chamber as a whole pays to the primary source, which is a crucial determinant of players' equilibrium utilities. Unfortunately but unsurprisingly, nothing clear-cut can be said except in two special cases: when the echo chamber is small or when its members are homogeneous. In both cases, increasing a player's bandwidth makes everyone in the echo chamber better off. As for the consequences of increasing a player's visibility parameter, our result depends on whether that player ends up being an opinion leader or an opinion follower. In the first (resp. second) situation, the echo chamber as a whole pays less (resp. more) attention to the primary source.

**Proof sketch** We only sketch the proof for  $\partial x_1^l / \partial \tau_1 |_{\theta=\theta^\circ} > 0$  and  $\partial x_j^l / \partial \tau_1 |_{\theta=\theta^\circ} < 0$  $\forall j \neq 1$ , starting off from the case of two players. In that case, differentiating players' bandwidth constraints (which must be binding in equilibrium) against  $\tau_1$  yields

$$\begin{bmatrix} 1 & \frac{\partial x_1^2}{\partial x_2^l} \\ \frac{\partial x_2^1}{\partial x_1^l} & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial x_1^l}{\partial \tau_1} \\ \frac{\partial x_2^l}{\partial \tau_1} \end{bmatrix} \bigg|_{\boldsymbol{\theta} = \boldsymbol{\theta}^{\circ}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

where the term  $\partial x_i^j / \partial x_j^l |_{\theta = \theta^\circ}$  in the above equation captures how perturbing player *j*'s resourcefulness as a secondary source affects his influence on player *i*. Write  $g_j$  for  $h_x(x_j^l;\lambda_j)|_{\theta=\theta^\circ}$ , and recall that

$$\frac{\partial x_i^j}{\partial x_j^l} \bigg|_{\boldsymbol{\theta} = \boldsymbol{\theta}^{\circ} \text{ Theorem } 2} \underbrace{g_j}_{\text{Lemma } 1} \underbrace{\in}_{\text{Lemma } 1} (0, 1),$$

i.e., increasing player j's resourcefulness by one unit raises his influence on player i by less than one unit. From  $g_j > 0$ , it follows that one and only one player ends up paying more attention to the primary source as we increase  $\tau_1$  by a small amount, so that the net effect on player 2's bandwidth constraint equals zero. Then from  $g_j < 1$ , it follows that that player must be player 1, as the direct effect stemming from increasing his bandwidth dominates the indirect effects that he and player 2 can exert on each other.

Extending the above argument to more than two players require that we trace out the effects of our perturbation across a large attention network. Mathematically, we must solve

$$\left[\mathbf{I}_N + \mathbf{G}_N\right] \nabla_{\tau_1} \left[ x_1^l \cdots x_N^l \right]^\top \Big|_{\boldsymbol{\theta} = \boldsymbol{\theta}^\circ} = \left[1, 0, \cdots, 0\right]^\top$$

where  $\mathbf{I}_N$  is the  $N \times N$  diagonal matrix, and  $\mathbf{G}_N$  is the marginal influence matrix defined as

$$\left[\mathbf{G}_{N}\right]_{i,j} = \begin{cases} 0 & \text{if } i = j, \\ \frac{\partial x_{i}^{j}}{\partial x_{j}^{j}} \Big|_{\boldsymbol{\theta} = \boldsymbol{\theta}^{\circ}} & \text{else }. \end{cases}$$

From  $x_i^j = h(x_j^l; \lambda_j) \ \forall i \neq j$ , i.e., the influence exerted by player j is constant across his like-minded friends, it follows that  $\partial x_i^j / \partial x_j^l |_{\theta=\theta^\circ} = g_j \ \forall i \neq j$ , i.e., the off-diagonal entries of  $\mathbf{G}_N$  are constant column by column:

$$\mathbf{G}_{N} = \begin{bmatrix} 0 & g_{2} & \cdots & g_{N} \\ g_{1} & 0 & \cdots & g_{N} \\ \vdots & \vdots & \ddots & \vdots \\ g_{1} & g_{2} & \cdots & 0 \end{bmatrix}$$

Based on this property, as well as  $g_j \in (0, 1) \ \forall j \in \{1, \dots, N\}$ , we develop a methodology for solving  $[\mathbf{I}_N + \mathbf{G}_N]^{-1}$  and determining the signs of its entries. Our finding are reported in the next lemma, from which Theorem 3 follows.

**Lemma 3.** Fix any  $N \in \mathbb{N} - \{1\}$  and any  $g_1, \dots, g_N \in (0, 1)$ , and let  $[\mathbf{G}_N]_{i,j} = g_j$  $\forall i \neq j$  in the marginal influence matrix. Then  $\mathbf{A}_N \coloneqq \mathbf{I}_N + \mathbf{G}_N$  is invertible, and the following must hold  $\forall i \in \{1, \dots, N\}$ : (i)  $[\mathbf{A}_N^{-1}]_{i,i} > 0$ ; (ii)  $[\mathbf{A}_N^{-1}]_{i,j} < 0 \ \forall j \neq i$ ; (iii)  $\sum_{j=1}^N [\mathbf{A}_N^{-1}]_{i,j} > 0$ .

Lemma 3 is to our best knowledge new to the literature on network games with negative externalities, as it allows us to trace out the effects of changing a player's characteristics across the equilibrium attention network without assuming linear bestresponses or a symmetric influence matrix. While the assumption a player's visibility parameter is constant across his like-minded friends is certainly important, we can relax it without affecting our qualitative predictions, provided that the influence matrix satisfies the properties stated in Lemma 3 (see Appendix B.5 for further details).

### 4.2 Population size

This section examines the comparative statics regarding players' population size N. To best illustrate the main idea, we abstract away from individual-level heterogeneity and instead assume that all players have the same visibility parameter and bandwidth. Under this assumption, it is easy to see that if our game has a unique equilibrium (as required by Assumption 1), then it must be symmetric across players. Let x(N)denote the amount of attention each player pays to the primary source in equilibrium. Its comparative statics regarding N are as follows.

**Proposition 1.** Take any  $\lambda, \tau > 0$  and  $N'' > N' \ge 2$  such that the game among  $N \in \{N', N''\}$  type-L players with visibility parameter  $\lambda$  and bandwidth  $\tau$  satisfies Assumption 1. Then as N increases from N' to N'', x(N) decreases, whereas Nx(N) may either increase or decrease.

As an echo chamber grows in size, each member of it has access to more secondary sources and so pays less attention to the primary source. Depending on which effect dominates the other, the echo chamber as a whole may pay more or less attention to the primary source, and its members' utilities may either increase or decrease. Proposition 1 sheds light on the recent attempts by tech companies such as Allsides.com to disrupt echo chambers, which mandate that readers on their platforms must be impartially exposed to both left-leaning and right-leaning articles. The welfare consequence of this social experiment is the topic we next turn to.

### 5 Extensions

This section reports main extensions of the baseline model. See Appendix B for further extensions.

**Merging sources** So far we have allowed players to selectively expose themselves to biased primary sources and, hence, to like-minded friends. Now consider a content regulation akin to that advocated by Allsides.com, which merges sources l and r into a mega source m. The next proposition establishes the isomorphism between the resulting game and a game we are more familiar with.

**Proposition 2.** Let everything be as in Section 2 except that a single source m disseminates both the L-revealing news and the R-revealing news to players. Compare the resulting game (a) to a game (b) among a set  $\mathcal{I}$  of type-L players with characteristic profile  $(\beta_i, \lambda_i, \tau_i)$ 's and access to source l.

(i) If a joint attention strategy (x<sup>m</sup><sub>i</sub>, (x<sup>j</sup><sub>i</sub>)<sub>j∈I-{i}</sub>)<sub>i∈I</sub> can arise in an equilibrium of game (a), then the joint attention strategy (y<sup>l</sup><sub>i</sub>, (y<sup>j</sup><sub>i</sub>)<sub>j∈I-{i}</sub>)<sub>i∈I</sub> where y<sup>l</sup><sub>i</sub> = x<sup>m</sup><sub>i</sub> and y<sup>j</sup><sub>i</sub> = x<sup>j</sup><sub>i</sub> ∀i ∈ I and j ∈ I - {i} can arise in an equilibrium of game (b). Moreover, the expected utility of any player i ∈ I in the first equilibrium equals its counterpart in the second equilibrium.

### (ii) The converse of Part (i) is also true.

Two takeaways are immediate. First, the content regulation advocated by Allsides.com does disrupt echo chambers as it claims, because it forces people on different sides of the political spectrum to attend to each other as secondary sources. And yet its welfare consequence is in general ambiguous because making more secondary sources available discourages information acquisition from the primary source. In the case where type-*L* and type-*R* players are symmetric, i.e.,  $|\mathcal{L}| = |\mathcal{R}|$  and  $(\beta_i, \lambda_i, \tau_i)$ is constant across  $i \in \mathcal{I}$ , merging *l* and *r* into *m* is mathematically equivalent to doubling the population size in game (b), which according to Proposition 1 has an ambiguous effect on players' equilibrium utilities.

Multiple independent sources So far we have restricted players to facing a single source of each kind. Now suppose there are  $K_1 \in \mathbb{N}$  independent *L*-biased sources  $l_1, \dots, l_{K_1}$  and  $K_2 \in \mathbb{N}$  independent *R*-biased sources  $r_1, \dots, r_{K_2}$ . For each player  $i \in \mathcal{I}$ , redefine  $\mathcal{D}_i = \{l_1, \dots, l_{K_1}, r_1, \dots, r_{K_2}\} \cup \mathcal{I} - \{i\}$  as the set of the parties he can attend to, and modify the set of feasible attention strategies for him accordingly. The next proposition shows that introducing multiple independent sources of the same kind to the analysis affects neither the total amount of attention any player pays to each kind of sources nor the attention network between players.

**Proposition 3.** Let everything be as in Section 2 except that there are  $K_1 \in \mathbb{N}$ independent L-biased sources  $l_1, \dots, l_{K_1}$  and  $K_2 \in \mathbb{N}$  independent R-biased sources  $r_1, \dots, r_{K_2}$ . Compare the resulting game (a) and the game (b) in Section 2.

(i) If a joint attention strategy  $((y_i^d)_{d \in \mathcal{D}_i})_{i \in \mathcal{I}}$  can arise in an equilibrium of game (a), then the joint attention strategy  $((x_i^c)_{c \in \mathcal{C}_i})_{i \in \mathcal{I}}$  where  $x_i^l = \sum_{k=1}^{K_1} y_i^{l_k}$ ,  $x_i^r =$   $\sum_{k=1}^{K_2} y_i^{r_k}$ , and  $x_i^j = y_i^j \quad \forall i \in \mathcal{I}$  and  $j \in \mathcal{I} - \{i\}$  can arise in an equilibrium of game (b).

 (ii) If a joint attention strategy ((x<sup>c</sup><sub>i</sub>)<sub>c∈C<sub>i</sub></sub>)<sub>i∈I</sub> can arise in an equilibrium of game (b), then there exists a joint attention strategy ((y<sup>d</sup><sub>i</sub>)<sub>d∈D<sub>i</sub></sub>)<sub>i∈I</sub> as in Part (i) that can arise in an equilibrium of game (a).

Since its establishment, the Federal Communications Comission (FCC) has long promoted viewpoint diversity through its diversity objectives, e.g., limiting the crossownership between media outlets, mandating that at least eight independent media outlets be must operating in a digital media area (i.e., the eight voices test), etc. (Ho and Quinn (2009)). According to Proposition 3, these rulings may neither foster nor disrupt echo chambers and could have limited impacts on consumer welfare. The same can be said about recent technology advances that have flooded the Internet with user-generated content while causing massive layoffs in the traditional media sector (Newman (2009)). The former trend tends to increase the number of independent viewpoints while the latter trend tends to reduce it.

**Source visibility** So far we have normalized the visibility parameter of primary sources to one. The next proposition extends Theorems 1 and 2 to an arbitrary source visibility parameter  $\nu > 0$ .

**Proposition 4.** Let everything be as in Section 2 except that the visibility parameter of sources l and r is  $\nu > 0$ . Then Theorems 1 and 2 remain valid after we replace  $x_i^c$ ,  $\lambda_i$ , and  $\tau_i$  with  $\tilde{x}_i^c = \nu x_i^c$ ,  $\tilde{\lambda}_i = \lambda_i / \nu$ , and  $\tilde{\tau}_i = \nu \tau_i \ \forall i \in \mathcal{I}$  and  $c \in C_i$ .

Among other things, increasing the visibility of sources effectively scales down all players' visibility parameters by the same magnitude. This observation, together with Theorem 2(ii), implies that the comparative statics regarding source visibility are in general ambiguous. In practice, factors affecting source visibility/quality include, but are not limited to: media companies' investments in digital distributions and their increasing reliance on AI to boost traffic and reach (Smith (2016)); the rising time and financial pressures faced by journalists that result in a deterioration in their reporting qualities (Fürst (2020)).

# 6 Conclusion

We conclude by posing open questions. While the current work takes players' utilities from taking the various actions as given, in reality these utilities could depend on factors such as candidates' policymaking. In a recent paper, Hu, Li, and Segal (2019) develop a method of computing the outcome of electoral competition between officemotivated candidates when voters' signals constitute voting recommendations they strictly prefer to obey. The last property is satisfied by the current model, which, if embedded into Hu, Li, and Segal (2019), might yield insights into how echo chambers affect policy polarization, and vice versa.

The role of echo chamber in spreading misinformation and fake news has recently received attention from the scientific community and public health officials (Del Vicario et al. (2016)), though existing theories on this subject matter take a homophilic social network structure as given (see, e.g., Bloch, Demange, and Kranton (2018)). By developing an RI theory of echo chamber, the current work uncovers plausible channels through which people's preferences and attention capacities could facilitate or impede the spread of misinformation and fake news. Formalizing these channels is a step we plan to take next.<sup>9</sup>

We view our theory as complementary to non-Bayesian theories of belief polar-

<sup>&</sup>lt;sup>9</sup>See also Jackson (2014)'s survey about how homophily affects diffusion and contagion in general, and why micro-founding homophily (as we do) is important.

ization and information segregation. The best way to test these theories apart is to look for evidence for or against Bayes' rule on the social media sphere. Recently, a few scholars including Allcott et al. (2020) and Mosleh et al. (2021) have started to conduct controlled experiments on social media. We hope someone, maybe us, will study the RI origin of echo chambers using this new method. An interesting pattern to look for is the coexistence of belief polarization and occasional belief reversal, which Flaxman, Goel, and Rao (2016) have already discovered using large data sets.

# A Proofs

### A.1 Preliminaries

This appendix prepares the reader for replicating the proofs omitted from the main text, starting off by reformulating the problem faced by any type-L player  $i \in \mathcal{L}$  using logarithms. Recall that for any joint attention strategy  $x \in \times_{j \in \mathcal{I}} \mathcal{X}_j$ , we have

$$\mathbb{P}_{x}(\mathcal{U}_{i} \mid \omega = L) = \delta_{i}^{r} \prod_{j \in \mathcal{I} - \{i\}} (\delta_{j}^{r} + (1 - \delta_{j}^{r})\delta_{i}^{j})$$
  
and  $\mathbb{P}_{x}(\mathcal{U}_{i} \mid \omega = R) = \delta_{i}^{l} \prod_{j \in \mathcal{I} - \{i\}} (\delta_{j}^{l} + (1 - \delta_{j}^{l})\delta_{i}^{j}),$ 

where

$$\delta_i^c = \exp(-x_i^c) \ \forall c \in \{l, r\} \text{ and } \delta_i^j = \exp(-\lambda_j x_i^j) \ \forall j \in \mathcal{I} - \{i\}.$$

If player *i* plans to take action *L* in event  $\mathcal{U}_i$ , then his Stage-1 expected utility equals  $-\beta_i \mathbb{P}_x(\mathcal{U}_i \mid \omega = R)/2$ , and maximizing this expected utility is equivalent to solving

$$\max_{x_i \in \mathcal{X}_i} -\log \delta_i^l - \sum_{j \in \mathcal{I} - \{i\}} \log(\delta_j^l + (1 - \delta_j^l) \delta_i^j).$$
(1)

Meanwhile, if player *i* plans to take action *R* in event  $\mathcal{U}_i$ , then his Stage-1 expected utility equals  $-\mathbb{P}_x(\mathcal{U}_i \mid \omega = L)$ , and maximizing this expected utility is equivalent to solving

$$\max_{x_i \in \mathcal{X}_i} -\log \delta_i^r - \sum_{j \in \mathcal{I} - \{i\}} \log(\delta_j^r + (1 - \delta_j^r) \delta_i^j).$$
(2)

At Stage 1 of the game, player i solves Problems (1) and (2) separately and then chooses a solution that generates the highest expected utility.

We next state a useful lemma.

**Lemma 4.** Fix any  $\lambda > 1$  and  $\tau > \phi(\lambda)$ . For each  $N \in \mathbb{N} - \{1\}$  and  $x \in [\phi(\lambda), +\infty)$ , define

$$\varphi^{N}(x) = \tau - (N-1)h(x;\lambda).$$

Then  $\varphi^N$  has a unique fixed point x(N) that belongs to  $(\phi(\lambda), \tau)$  and satisfies  $\lim_{N \to +\infty} x(N) = \phi(\lambda)$ .

### A.2 Proofs of lemmas

**Proof of Lemma 1** Differentiating  $\phi$  w.r.t.  $\lambda$  and h w.r.t. x gives desired results.

**Proof of Lemma 2** We only prove the result for an arbitrary type-L player  $i \in \mathcal{L}$ . The proofs for type-R players are analogous and hence are omitted. Under the assumption that player i can attend only to sources l and r but not to the other players, simplifying Problems (1) and (2) yields

$$\max_{x_i^l, x_i^r} x_i^l \text{ s.t. } x_i^l, x_i^r \ge 0 \text{ and } \tau_i \ge x_i^l + x_i^r$$
  
and 
$$\max_{x_i^l, x_i^r} x_i^r \text{ s.t. } x_i^l, x_i^r \ge 0 \text{ and } \tau_i \ge x_i^l + x_i^r,$$

respectively, and solving these problems yields  $(x_i^l, x_i^r) = (\tau_i, 0)$  and  $(x_i^l, x_i^r) = (0, \tau_i)$ , respectively. Since the Stage 1-expected utility  $-\beta_i \exp(-\tau_i)/2$  generated by the first solution exceeds that  $-\exp(-\tau_i)/2$  generated by the second solution, the first solution solves player *i*'s Stage-1 problem.

**Proof of Lemma 3** We proceed in three steps.

**Step 1.** Solve for  $\mathbf{A}_N^{-1}$ . We conjecture that

$$\det\left(\mathbf{A}_{N}\right) = 1 + \sum_{s=1}^{N} \left(-1\right)^{s-1} \left(s-1\right) \sum_{\left(k_{l}\right)_{l=1}^{s} \in \{1, \cdots, N\}} \prod_{l=1}^{s} g_{k_{l}}$$
(3)

and that  $\forall i \in \{1, \cdots, N\}$  and  $j \in \{1, \cdots, N\} - \{i\}$ :

$$\left[\mathbf{A}_{N}^{-1}\right]_{i,i} = \frac{1}{\det\left(\mathbf{A}_{N}\right)} \left[1 + \sum_{s=1}^{N-1} \left(-1\right)^{s-1} \left(s-1\right) \sum_{\left(k_{l}\right)_{l=1}^{s} \in \{1, \cdots, N\} - \{i\}} \prod_{l=1}^{s} g_{k_{l}}\right]$$
(4)

and

$$\left[\mathbf{A}_{N}^{-1}\right]_{i,j} = \frac{1}{\det\left(\mathbf{A}_{N}\right)} \left(-1\right)^{N-1} g_{j} \prod_{k \in \{1, \cdots, N\} - \{i, j\}} \left(g_{k} - 1\right).$$
(5)

Our conjecture is clearly true when N = 2, because det  $(\mathbf{A}_2) = 1 - g_1 g_2$  and

$$\mathbf{A}_2^{-1} = \frac{1}{1 - g_1 g_2} \begin{bmatrix} 1 & -g_2 \\ -g_1 & 1 \end{bmatrix}.$$

For each  $N \ge 2$ , define

$$\mathbf{B}_N = [g_{N+1} \ g_{N+1} \ \cdots \ g_{N+1}]^\top$$

and

$$\mathbf{C}_N = [g_1 \ g_2 \ \cdots \ g_N].$$

Then

$$\mathbf{A}_{N+1} = egin{bmatrix} \mathbf{A}_N & \mathbf{B}_N \ \mathbf{C}_N & 1 \end{bmatrix},$$

and it can be inverted blockwise as follows:

$$\mathbf{A}_{N+1}^{-1} = \begin{bmatrix} \mathbf{A}_{N}^{-1} + \mathbf{A}_{N}^{-1}\mathbf{B}_{N} \left(1 - \mathbf{C}_{N}\mathbf{A}_{N}^{-1}\mathbf{B}_{N}\right)^{-1}\mathbf{C}_{N}\mathbf{A}_{N}^{-1} & -\mathbf{A}_{N}^{-1}\mathbf{B}_{N} \left(1 - \mathbf{C}_{N}\mathbf{A}_{N}^{-1}\mathbf{B}_{N}\right)^{-1} \\ - \left(1 - \mathbf{C}_{N}\mathbf{A}_{N}^{-1}\mathbf{B}_{N}\right)^{-1}\mathbf{C}_{N}\mathbf{A}_{N}^{-1} & \left(1 - \mathbf{C}_{N}\mathbf{A}_{N}^{-1}\mathbf{B}_{N}\right)^{-1} \end{bmatrix}.$$

Tedious but straightforward algebra shows that  $\mathbf{A}_N^{-1}\mathbf{B}_N$  is a column vector of size N whose  $i^{th}$  entry equals

$$\frac{g_{N+1}}{\det\left(\mathbf{A}_{N}\right)}\prod_{k\in\{1,\cdots,N\}-\{i\}}\left(1-g_{k}\right),$$

and that  $\mathbf{C}_N \mathbf{A}_N^{-1}$  is a row vector of size N whose  $i^{th}$  entry equals

$$\frac{g_i}{\det\left(\mathbf{A}_N\right)}\prod_{k\in\{1,\cdots,N\}-\{i\}}\left(1-g_k\right).$$

Moreover,

$$\mathbf{C}_{N}\mathbf{A}_{N}^{-1}\mathbf{B}_{N} = \frac{g_{N+1}}{\det(\mathbf{A}_{N})} \sum_{s=1}^{N} (-1)^{s-1} s \sum_{(k_{l})_{l=1}^{s} \in \{1, \cdots, N\}} \prod_{l=1}^{s} g_{k_{l}},$$

which, after simplifying, becomes

$$1 - \mathbf{C}_N \mathbf{A}_N^{-1} \mathbf{B}_N = \frac{\det\left(\mathbf{A}_{N+1}\right)}{\det\left(\mathbf{A}_N\right)}.$$

Substituting these results into the expression for  $\mathbf{A}_{N+1}^{-1}$  and doing a lot of algebra verify our conjecture for N + 1.

**Step 2.** Show that  $\det(\mathbf{A}_N) > 0$ , i.e,

$$\sum_{s=1}^{N} (-1)^{s-1} (s-1) \sum_{(k_l)_{l=1}^s \in \{1, \cdots, N\}} \prod_{l=1}^s g_{k_l} > -1.$$

Denote the left-hand side of the above inequality by LHS  $(g_1, \dots, g_N)$ . Since the function LHS :  $[0, 1]^N \to \mathbb{R}$  is linear in each  $g_i$ , holding  $(g_j)_{j \neq i}$  constant, its minimum is attained at an extremal point of  $[0, 1]^N$ . Moreover, since the function is symmetric across  $g_i$ s, the following must hold for any  $(g_1, \dots, g_N) \in \{0, 1\}^{N-1}$  such that  $\sum_{i=1}^N g_i = n$ :

LHS 
$$(g_1, \cdots, g_N) = f(n) \coloneqq \sum_{k=1}^n (-1)^{k-1} (k-1) \binom{n}{k}.$$

It remains to show that  $f(n) \ge -1$   $\forall n = 0, 1, \dots, N$ , which is clearly true when n = 0 and 1 (in both cases f(n) = 0). For each  $n \ge 2$ , define

$$p(n) = \sum_{k=1}^{n} (-1)^{k} k \binom{n}{k}.$$

Below we prove by induction that f(n) = -1 and  $p(n) = 0 \forall n = 2, \dots, N$ .

Our conjecture is clearly true for n = 2:

$$f(2) = -\binom{2}{2} = -1$$
 and  $p(2) = -\binom{2}{1} + 2\binom{2}{2} = 0.$ 

Now suppose it is true for some  $n \ge 2$ . Then

$$\begin{split} f(n+1) \\ &= \sum_{k=1}^{n+1} (-1)^{k-1} (k-1) \binom{n+1}{k} \\ &= \sum_{k=1}^{n} (-1)^{k-1} (k-1) \binom{n+1}{k} + (-1)^{n} n \binom{n+1}{n+1} \\ &= \sum_{k=1}^{n} (-1)^{k-1} (k-1) \binom{n}{k} + \binom{n}{k-1} + (-1)^{n} n \quad (\because \binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}) \\ &= f(n) + \sum_{k=1}^{n-1} (-1)^{k} k\binom{n}{k} + (-1)^{n} n \\ &= f(n) + \sum_{k=1}^{n} (-1)^{k} k\binom{n}{k} \\ &= f(n) + p(n) \\ &= -1, \end{split}$$

and

$$p(n+1) = \sum_{k=1}^{n+1} (-1)^k k \binom{n+1}{k}$$
  
=  $\sum_{k=1}^n (-1)^k k \left(\binom{n}{k} + \binom{n}{k-1}\right) + (-1)^{n+1} (n+1)$  (::  $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$ )  
=  $p(n) + \sum_{k=0}^{n-1} (-1)^{k+1} (k+1) \binom{n}{k} + (-1)^{n+1} (n+1)$   
=  $0 + \sum_{k=1}^{n-1} (-1)^{k+1} k \binom{n}{k} + \sum_{k=0}^{n-1} (-1)^{k+1} \binom{n}{k} + (-1)^{n+1} (n+1)$ .

Then from

$$\sum_{k=1}^{n-1} (-1)^{k+1} k \binom{n}{k} = \sum_{k=1}^{n} (-1)^{k+1} k \binom{n}{k} - (-1)^{n+1} n$$
$$= -p (n) - (-1)^{n+1} n$$
$$= -(-1)^{n+1} n,$$

it follows that

$$p(n+1) = \sum_{k=0}^{n-1} (-1)^{k+1} \binom{n}{k} + (-1)^{n+1},$$

hence p(n+1) = 0 if and only if

$$q(n) \coloneqq \sum_{k=0}^{n-1} (-1)^{k+1} \binom{n}{k} = (-1)^n.$$

The last conjecture is clearly true when n is odd (simplifying q(n) using  $\binom{n}{k} = \binom{n}{n-k}$  gives the desired result). When n is even, expanding q(n) yields

$$q(n) = \sum_{k=1}^{n-1} (-1)^{k-1} \left( \binom{n-1}{k-1} + \binom{n-1}{k} \right) - \binom{n}{0}$$
$$= \sum_{k=1}^{n-1} (-1)^{k-1} \binom{n-1}{k-1} + \sum_{k=0}^{n-1} (-1)^{k-1} \binom{n-1}{k}.$$

Then from

$$\sum_{k=1}^{n-1} (-1)^{k-1} \binom{n-1}{k-1} = \sum_{k=0}^{n-2} (-1)^k \binom{n-1}{k} = -q (n-1) = 1 \quad (\because q(n-1) = -1)$$

$$\sum_{k=0}^{n-1} (-1)^{k-1} \binom{n-1}{k} = \sum_{k=0}^{n-2} (-1)^{k-1} \binom{n-1}{k} + (-1)^{n-2} \binom{n-1}{n-1}$$
$$= q (n-1) + 1$$
$$= 0,$$

it follows that  $q(n) = 1 = (-1)^n$ , which completes the proof.

**Step 3.** Verify that  $[\mathbf{A}_N^{-1}]_{i,i} > 0$ ,  $[\mathbf{A}_N^{-1}]_{i,j} < 0$ , and  $\sum_{j=1}^N [\mathbf{A}_N^{-1}]_{i,j} > 0 \quad \forall i \in \{1, \dots, N\}$  and  $j \in \{1, \dots, N\} - \{i\}$ . A careful inspection of Equations (3) and (4) reveals that

$$\left[\mathbf{A}_{N}^{-1}\right]_{i,i} = \frac{\det\left(\mathbf{D}_{N,i}\right)}{\det\left(\mathbf{A}_{N}\right)},$$

where  $\mathbf{D}_{N,i}$  is the principal minor of  $\mathbf{A}_N$  that results from deleting its  $i^{th}$  row and  $i^{th}$  column and so satisfies all properties stated in Lemma 3. Then from det  $(\mathbf{A}_N) > 0$ , it follows that det  $(\mathbf{D}_{N,i}) > 0$  and, hence,  $[\mathbf{A}_N^{-1}]_{i,i} > 0$ . Meanwhile, a close inspection of Equation (5) reveals that  $[\mathbf{A}_N^{-1}]_{i,j} < 0 \ \forall j \in \{1, \dots, N\} - \{i\}$ . Finally, summing  $[\mathbf{A}_N^{-1}]_{i,j}$  over  $j \in \{1, \dots, N\}$  and doing a lot of algebra yields

$$\sum_{j=1}^{N} \left[ \mathbf{A}_{N}^{-1} \right]_{i,j} = \frac{1}{\det \left( \mathbf{A}_{N} \right)} \prod_{k \in \{1, \cdots, N\} - \{i\}} \left( 1 - g_{k} \right) > 0.$$

**Proof of Lemma 4** From Lemma 1, i.e.,  $h(\phi(\lambda); \lambda) = 0$  and  $h_x(x; \lambda) > 0$  on  $(\phi(\lambda), +\infty)$ , it follows that  $\varphi^N(x) = \tau - (N-1)h(x; \lambda)$  has a unique fixed point  $x(N) \in (\phi(\lambda); \tau)$ . For  $x(N) = \varphi^N(x(N)) > \phi(\lambda)$  to hold,  $h(x(N); \lambda)$  must converge to zero as N grows to infinity, hence  $\lim_{N \to +\infty} x(N) = \phi(\lambda)$ .

and

### A.3 Proofs of theorems and propositions

**Proof of Theorems 1 and 2** Our proof fixes an arbitrary type-*L* player  $i \in \mathcal{L}$ . We first show that taking action *L* in event  $\mathcal{U}_i$  is a dominant strategy for player *i* when  $\beta_i$  is sufficiently small. By attending only to source *l* and taking action *L* in event  $\mathcal{U}_i$ , player *i* can guarantee himself an expected utility  $-\beta_i \exp(-\tau_i)/2$ . If, instead, he takes action *R* in event  $\mathcal{U}_i$ , then his Stage-1 problem is Problem (2). Since that problem maximizes a strictly concave function over a compact convex set, its solution is unique for any given  $x_{-i} \in \mathcal{X}_{-i}$  (hereafter denoted by  $BR(x_{-i})$ ). Also note that for any given  $x_i \in \mathcal{X}_i$ , the maximand of Problem (2) is strictly increasing in each  $x_j^r$ ,  $j \in \mathcal{I} - \{i\}$ , so it is maximized when  $x_j^r = \tau_j \,\forall j \in \mathcal{I} - \{i\}$  (hereafter denote this strategy profile among players -i by  $\overline{x}_{-i}$ ). Taken together, we obtain an upper bound for player *i*'s Stage-1 expected utility if he plans to take action *R* in event  $\mathcal{U}_i$ :

$$-\frac{1}{2}\mathbb{P}_{(\overline{x}_{-i},BR(\overline{x}_{-i}))}(\mathcal{U}_i \mid \omega = L),$$

which is a strictly negative number. Thus when

$$\beta_i < \underline{\beta_i} \coloneqq \exp(\tau_i) \mathbb{P}_{(\overline{x}_{-i}, BR(\overline{x}_{-i}))}(\mathcal{U}_i \mid \omega = L),$$

taking action L in event  $\mathcal{U}_i$  is a dominant strategy.

In the remainder of the proof, suppose  $\beta_j < \underline{\beta} := \min_{j \in \mathcal{I}} \underline{\beta_j} \quad \forall j \in \mathcal{I}$ , so that all players take their default actions in case they remain uninformed at Stage 4 of the game. Then at Stage 1 of the game, player *i* faces Problem (1). Let  $\eta_{x_i^c} \ge 0$ and  $\gamma_i \ge 0$  denote the Lagrange multipliers associated with the constraints  $x_i^c \ge 0$ and  $\tau_i \ge \sum_{c \in \mathcal{C}_i} x_i^c$ , respectively. The first-order conditions regarding  $x_i^l$ ,  $x_i^r$ , and  $x_i^j$ ,  $j \in \mathcal{I} - \{i\}$  are then

$$1 - \gamma_i + \eta_{x_i^l} = 0, \qquad (FOC_{x_i^l})$$

$$-\gamma_i + \eta_{x_i^r} = 0, \qquad (\text{FOC}_{x_i^r})$$

and 
$$\frac{\lambda_j (1 - \delta_j^l) \delta_i^j}{\delta_j^l + (1 - \delta_j^l) \delta_i^j} - \gamma_i + \eta_{x_i^j} = 0.$$
 (FOC<sub>x\_i^j</sub>)

A careful inspection of  $\text{FOC}_{x_i^l}$  and  $\text{FOC}_{x_i^r}$  reveals that  $\gamma_i = \eta_{x_i^r} \ge 1$  and, hence,  $\sum_{c \in C_i} x_i^c = \tau_i$  and  $x_i^r = 0$ . That is, player *i* must exhaust his bandwidth but ignore source *r*. For the same reason, all type-*R* players must ignore source *l*, i.e.,  $x_j^l = 0$ and, hence,  $\delta_j^l = \exp(-x_j^l) = 1 \ \forall j \in \mathcal{R}$ . Substituting the last observation into  $\text{FOC}_{x_i^j}$ and simplifying yield  $\eta_{x_i^j} = \gamma_i > 0$  and, hence,  $x_i^j = 0 \ \forall j \in \mathcal{R}$ . Taken together, we obtain that player *i* must ignore source *r* and all type-*R* players, thus proving that any PSPBE must be an echo-chamber equilibrium.

Consider next what happens inside the echo chamber  $\mathcal{L}$ . For any  $j \in \mathcal{L} - \{i\}$ , simplifying  $\text{FOC}_{x_i^j}$  shows that  $x_i^j > 0$  if and only if  $x_i^j = \frac{1}{\lambda_j} \log \left\{ \left( \frac{\lambda_j}{\gamma_i} - 1 \right) (\exp(x_j^l) - 1) \right\}$ , or equivalently

$$x_i^j = \frac{1}{\lambda_j} \log \max\left\{ \left(\frac{\lambda_j}{\gamma_i} - 1\right) (\exp(x_j^l) - 1), 1 \right\}.$$
 (6)

Then from the fact that  $\gamma_i \ge 1$  and the inequality is strict if and only if  $x_i^l = 0$ , it follows that

$$x_{i}^{l} = \max\left\{\tau_{i} - \sum_{j \in \mathcal{L} - \{i\}} \frac{1}{\lambda_{j}} \log \max\{(\lambda_{j} - 1)(\exp(x_{j}^{l}) - 1), 1\}, 0\right\},$$
(7)

which completes the proof of Theorem 2(i).

Solving the system of Equations (6) and (7) among type-L players yields the

attention strategies that can arise among them in all echo-chamber equilibria. Below we simplify these equations in two special cases.

**Case 1.** All type-*L* players attend to source *l* as in Theorem 2(ii). In that case, we have  $\gamma_j = 1 \ \forall j \in \mathcal{L}$ , so Equation (6) becomes

$$x_i^j = \frac{1}{\lambda_j} \log \max\{(\lambda_j - 1)(\exp(x_j^l) - 1), 1\}.$$
(8)

A close inspection of Equation (8) reveals the equivalence between (a)  $x_j^l > \phi(\lambda_j)$ , (b)  $x_i^j > 0$ , and (c)  $x_k^j = h(x_j^l; \lambda_j) \ \forall k \in \mathcal{L} - \{j\}$ , thus proving Theorem 2(ii).

**Case 2.** All like-minded friends attend to each other as in Theorem 2(iii). In that case, we must have  $x_j^l > \phi(\lambda_j) \ \forall j \in \mathcal{L}$ , so Equations (6) and (7) become

$$x_i^j = h(x_j^l; \lambda_j) \tag{9}$$

and 
$$x_i^l = \tau_i - \sum_{j \in \mathcal{L} - \{i\}} h(x_j^l; \lambda_j),$$
 (10)

respectively. Simplifying  $\mathbb{P}_x (\mathcal{U}_i \mid \omega = R)$  accordingly yields

$$\begin{aligned} &\mathbb{P}_{x}\left(\mathcal{U}_{i}\mid\omega=R\right)\\ &=\delta_{i}^{l}\prod_{j\in\mathcal{L}-\{i\}}\left(\delta_{j}^{l}+(1-\delta_{j}^{l})\delta_{i}^{j}\right)\\ &=\exp(-x_{i}^{l})\prod_{j\in\mathcal{L}-\{i\}}\left\{\exp(-x_{j}^{l})+(1-\exp(-x_{j}^{l}))\exp\left(-\lambda_{j}\cdot\frac{1}{\lambda_{j}}\log(\lambda_{j}-1)(\exp(x_{j}^{l})-1)\right)\right\}\\ &=\exp\left(-\sum_{j\in\mathcal{L}}x_{j}^{l}+\sum_{j\in\mathcal{L}-\{i\}}\phi\left(\lambda_{j}\right)\right),\end{aligned}$$

thus proving Theorem 2(iii).

We finally demonstrate that the game among type-L players has an equilibrium. From the above argument, we know that all equilibria can be obtained from (i) solving the system of Equation (7) among type-L players and (ii) substituting the solution(s) into Equation (6) to back out the attention network between them. To show that the first system of equations has a solution, we write  $\{1, \dots, N\}$  for  $\mathcal{L}$  and define, for each  $\mathbf{x}^l \coloneqq [x_1^l \cdots x_N^l]^\top \in \times_{i=1}^N [0, \tau_i], T(\mathbf{x}^l)$  as the N-vector whose  $i^{th}$  entry is given by the right-hand side of Equation (7). Rewrite the first system of equations as  $T(\mathbf{x}^l) = \mathbf{x}^l$ . Since  $T : \times_{i=1}^N [0, \tau_i] \to \times_{i=1}^N [0, \tau_i]$  is a continuous mapping from a compact convex set to itself, it has a fixed point according to Brouwer's fixed-point theorem.

**Proof of Theorem 3** Write  $\{1, \dots, N\}$  for  $\mathcal{L}$ . In order to satisfy Assumption 1, the equilibrium profile  $(x_i^l)_{i=1}^N$  must solve the system of Equations (9) and (10) among type-L players, and it must satisfy  $x_i^l > \phi(\lambda_i)$  and, hence,  $g_i \coloneqq h_x(x_i^l;\lambda_i) \in (0,1)$  $\forall i \in \{1, \dots, N\}$  by Lemma 1. Then from  $[\mathbf{G}_N]_{i,j} = g_j \ \forall j \in \{1, \dots, N\}$  and  $i \in \{1, \dots, N\} - \{j\}$ , it follows that  $\mathbf{A}_N \coloneqq \mathbf{I}_N + \mathbf{G}_N$  is invertible, and the signs of  $\mathbf{A}_N^{-1}$ are as in Lemma 3.

Part (i): We only prove the result for  $\tau_1$ . Differentiating the system of Equation (10) among  $i \in \{1, \dots, N\}$  with respect to  $\tau_1$  yields

$$\nabla_{\tau_1} \mathbf{x}^l = \mathbf{A}_N^{-1} \begin{bmatrix} 1 \ 0 \ \cdots \ 0 \end{bmatrix}^\top$$

where  $\mathbf{x}^l \coloneqq [x_1^l \cdots x_N^l]^\top$ . Then from Lemma 3, it follows that

$$\frac{\partial x_1^l}{\partial \tau_1} = \left[\mathbf{A}_N^{-1}\right]_{1,1} > 0 \text{ and } \frac{\partial x_j^l}{\partial \tau_1} = \left[\mathbf{A}_N^{-1}\right]_{j,1} < 0 \ \forall j \neq 1,$$

which together with Equation (9) implies that

$$\frac{\partial x_j^1}{\partial \tau_1} = g_1 \frac{\partial x_1^l}{\partial \tau_1} > 0 \text{ and } \frac{\partial x_k^j}{\partial \tau_1} = g_j \frac{\partial x_j^l}{\partial \tau_1} < 0 \ \forall j \neq 1 \text{ and } k \neq j.$$

Part (ii): We only prove the result for  $\lambda_1$ . Differentiating the system of Equation (10) among  $i \in \{1, \dots, N\}$  with respect to  $\lambda_1$  yields

$$\nabla_{\lambda_1} \mathbf{x}^l = \kappa \mathbf{A}_N^{-1} \begin{bmatrix} 0 \ 1 \ \cdots \ 1 \end{bmatrix}^\top,$$

where  $\kappa \coloneqq -h_{\lambda}(x_1^l; \lambda_1)$  has an ambiguous sign in general. Then from Lemma 3, it follows that

$$\operatorname{sgn}\left(\frac{\partial x_1^l}{\partial \lambda_1}\right) = \operatorname{sgn}\left(\kappa \underbrace{\sum_{i \neq 1} \left[\mathbf{A}_N^{-1}\right]_{1,i}}_{<0}\right) = \operatorname{sgn}\left(-\kappa\right)$$

and

$$\operatorname{sgn}\left(\frac{\partial x_{j}^{l}}{\partial \lambda_{1}}\right) = \operatorname{sgn}\left(\kappa\left(\underbrace{\sum_{i=1}^{N}\left[\mathbf{A}_{N}^{-1}\right]_{j,i}}_{>0} - \underbrace{\left[\mathbf{A}_{N}^{-1}\right]_{j,1}}_{<0}\right)\right) = \operatorname{sgn}\left(\kappa\right) \ \forall j \neq 1,$$

where the second result and Equation (9) together imply that

$$\operatorname{sgn}\left(\frac{\partial x_k^j}{\partial \lambda_1}\right) = \operatorname{sgn}\left(g_j \frac{\partial x_j^l}{\partial \lambda_1}\right) = \operatorname{sgn}\left(\kappa\right) \ \forall j \neq 1 \text{ and } k \neq j.$$

Finally, differentiating  $x_j^1$  with respect to  $\lambda_1$  and simplifying yield

$$\operatorname{sgn}\left(\frac{\partial x_{j}^{1}}{\partial \lambda_{1}}\right) = \operatorname{sgn}\left(\kappa \left[g_{1} \underbrace{\sum_{i \neq 1} \left[\mathbf{A}_{N}^{-1}\right]_{i,1}}_{<0} - 1\right]\right) = \operatorname{sgn}\left(-\kappa\right).$$

Thus in total, only three situations can happen:

- (a)  $\kappa < 0$  and, hence,  $\partial x_1^l / \partial \lambda_1 > 0$ ,  $\partial x_j^1 / \partial \lambda_1 > 0$ ,  $\partial x_j^l / \partial \lambda_1 < 0$ , and  $\partial x_k^j / \partial \lambda_1 < 0$  $\forall j \neq 1$  and  $k \neq j$ ;
- (b)  $\kappa > 0$  and, hence, all inequalities in case (a) are reversed;
- (c)  $\kappa = 0$  and, hence, all inequalities in case (a) are replaced with equalities.

Part (iii): Write  $\overline{x}$  for  $\sum_{i=1}^{N} x_i^l$ , and note that

$$\frac{\partial \overline{x}}{\partial \tau_1} = \begin{bmatrix} 1 \ 1 \ \cdots \ 1 \end{bmatrix} \nabla_{\tau_1} \mathbf{x}^l = \underbrace{\sum_{i=1}^N \begin{bmatrix} \mathbf{A}_N^{-1} \end{bmatrix}_{i,1}}_{\mathbf{A}}$$
  
and  $\frac{\partial \overline{x}}{\partial \lambda_1} = \begin{bmatrix} 1 \ 1 \ \cdots \ 1 \end{bmatrix} \nabla_{\lambda_1} \mathbf{x}^l = \kappa \left( \underbrace{\sum_{i,j=1}^N \begin{bmatrix} \mathbf{A}_N^{-1} \end{bmatrix}_{i,j} - \sum_{i=1}^N \begin{bmatrix} \mathbf{A}_N^{-1} \end{bmatrix}_{i,1}}_{\mathbf{B}} \right).$ 

From sgn  $(\partial x_1^l / \partial \lambda_1) = \text{sgn}(-\kappa)$  (as shown in Part (ii)), it follows that sgn  $(\partial \overline{x} / \partial \lambda_1) = \text{sgn}(-\partial x_1^l / \partial \lambda_1)$  if and only if B > 0. The remainder of the proof shows that A, B > 0 in two special cases.

**Case 1.** N = 2 In this case, solving  $\mathbf{A}_2^{-1}$  explicitly yields

$$\mathbf{A}_2^{-1} = \frac{1}{1 - g_1 g_2} \begin{bmatrix} 1 & -g_2 \\ -g_1 & 1 \end{bmatrix},$$

so  $A = (1 - g_1)/(1 - g_1g_2) > 0$  and  $B = (1 - g_2)/(1 - g_1g_2) > 0$ .

**Case 2.**  $(\lambda_i, \tau_i) \equiv (\lambda, \tau)$  As we will demonstrate in the proof of Proposition 1, in this case  $\mathbf{A}_N$  is a symmetric matrix, so  $A = \sum_{i=1}^N \left[\mathbf{A}_N^{-1}\right]_{1,i}$  and  $B = (N-1) \sum_{i=1}^N \left[\mathbf{A}_N^{-1}\right]_{1,i}$ , which are both positive by Lemma 3.

**Proof of Proposition 1** Fix any  $N \in \mathbb{N} - \{1\}$ , and consider the game among N type-L players with visibility parameter  $\lambda$  and bandwidth  $\tau$ . In order to satisfy Assumption 1, the equilibrium profile  $(x_i^l)_{i=1}^N$  must solve the system of Equations (9) and (10) among type-L players, and it must satisfy  $x_i^l > \phi(\lambda) \quad \forall i \in \{1, \dots, N\}$ . The last condition holds only if  $\lambda > 1$  and  $\tau > \phi(\lambda)$ .

The remainder of the proof proceeds in two steps. We first show that all players pay x(N) units of attention to source l and, hence,  $h(x(N); \lambda)$  units of attention to any other player in equilibrium, where x(N) is the unique fixed point of  $\varphi^N(x) =$  $\tau - (N-1)h(x; \lambda)$ . Consider first the case N = 2. In that case, (10) is simply

$$x_1^l = \varphi^2(x_2^l) \text{ and } x_2^l = \varphi^2(x_1^l),$$
 (11)

which has a unique solution  $(x_1^l, x_2^l) = (x(2), x(2)).$ 

For each  $N \geq 3$ , we fix any pair  $i \neq j$  and define  $\hat{\tau} = \tau - \sum_{k \neq i,j} h(x_k^l; \lambda)$ . Rewrite the system of Equation (10) among players i and j as  $x_i^l = \hat{\tau} - h(x_j^l; \lambda)$  and  $x_j^l = \hat{\tau} - h(x_i^l; \lambda)$ , where  $\hat{\tau} < \tau$  by assumption, and  $\hat{\tau}$  must exceed  $\phi(\lambda)$  in order for players *i* and *j* to attend to each other (as required by the proposition). Thus  $(x_i^l, x_j^l)$  is the solution to Equation (11) with  $\tau$  replaced with  $\hat{\tau}$ , so it must satisfy  $x_i^l = x_j^l \in (\phi(\lambda), \hat{\tau})$ . Repeating the above argument for all pairs of (i, j) shows that  $x_i^l = x_j^l \in (\phi(\lambda), \tau) \, \forall i, j \in \{1, \dots, N\}$ . Simplifying (10) accordingly yields  $x_i^l = \varphi^N(x_i^l)$  and, hence,  $x_i^l = x(N) \, \forall i \in \{1, \dots, N\}$ .

We next examine the comparative statics of x(N). Differentiating both sides of  $x(N) = \varphi^N(x(N))$  with respect to N yields

$$\frac{dx(N)}{dN} = -\frac{1}{\lambda} \left[ 1 + \frac{N-1}{\lambda} \frac{\exp(x(N))}{\exp(x(N)) - 1} \right]^{-1} \log\left[ (\lambda - 1) \left( \exp(x(N)) - 1 \right) \right] < 0,$$

where the last inequality exploits the fact that  $x(N) > \phi(\lambda)$ . Meanwhile, solving Nx(N) numerically yields non-monotonic solutions in N. For example, when  $\tau = 2$  and  $\lambda = 5$ , Nx(N) equals 4.25 when N = 4, 3.85 when N = 5, and 4.00 when N = 6.

**Proof of Proposition 2** With a single mega source m, the set of the parties that any player  $i \in \mathcal{I}$  can attend to becomes  $C_i = \{m\} \cup \mathcal{I} - \{i\}$ , and the set  $\mathcal{X}_i$  of feasible attention strategies for him must be modified accordingly. For any feasible joint attention strategy  $x \in \times_{j \in \mathcal{I}} \mathcal{X}_j$ , we have

$$\mathbb{P}_x(\mathcal{U}_i \mid \omega) = \delta_i^m \prod_{j \in \mathcal{I} - \{i\}} (\delta_j^m + (1 - \delta_j^m)) \ \forall \omega \in \{L, R\}$$

and, hence,

$$\frac{\mathbb{P}_x(\mathcal{U}_i \mid \omega = d_i)}{\mathbb{P}_x(\mathcal{U}_i \mid \omega \neq d_i)} = 1 > \beta_i,$$

where the last inequality implies that player i strictly prefers to take his default action in event  $\mathcal{U}_i$ . Given this, player i's Stage-1 problem becomes

$$\max_{x_i \in \mathcal{X}_i} -\log \delta_i^m - \sum_{j \in \mathcal{I} - \{i\}} \log(\delta_j^m + (1 - \delta_j^m) \delta_i^j).$$

Solving this problem simultaneously for all  $i \in \mathcal{I}$  is the same as solving a game among a set  $\mathcal{I}$  of type-L players with visibility parameter  $\lambda_i$ 's, bandwidth  $\tau_i$ 's, and access to source l.

**Proof of Proposition 3** Part (ii) is obvious. To show Part (i), recall that with  $K_1$  independent *L*-biased sources  $l_1, \dots, l_{K_1}$  and  $K_2$  independent *R*-biased sources  $r_1, \dots, l_{K_2}$ , the set of the parties that any player  $i \in \mathcal{I}$  can attend to becomes  $\mathcal{D}_i = \{l_1, \dots, l_{K_1}, r_1, \dots, r_{K_2}\} \cup \mathcal{I} - \{i\}$ , and the set of feasible attention strategies for him becomes  $\{y_i \in \mathbb{R}^{|\mathcal{D}_i|}_+ : \sum_{d \in \mathcal{D}_i} y_i^d \leq \tau_i\}$ . Write  $y_i^l$  for  $\sum_{k=1}^{K_1} y_i^{l_k}$  and  $y_i^r$  for  $\sum_{k=1}^{K_2} y_i^{r_k}$ . Replacing  $x_i^l$  with  $y_i^l$  in (1) yields the Stage-1 problem faced by player i if he plans to take action L in event  $\mathcal{U}_i$ , and replacing  $x_i^r$  with  $y_i^r$  in (2) yields the Stage-1 problem he faces if he plans to take action R in event  $\mathcal{U}_i$ . Thus for any profile  $((y_i^d)_{d \in \mathcal{D}_i})_{i \in \mathcal{I}}$  that can arise in an equilibrium with multiple independent sources, the corresponding profile  $((x_i^c)_{c \in \mathcal{C}_i})_{i \in \mathcal{I}}$  where  $x_i^l = y_i^l$ ,  $x_i^r = y_i^r$ , and  $x_i^j = y_i^j \ \forall j \in \mathcal{I} - \{i\}$  can arise in an equilibrium of the baseline model.

**Proof of Proposition 4** In case the visibility parameter of sources l and r equals  $\nu > 0$ , replacing  $x_i^c$ ,  $\lambda_i$  and  $\tau_i$  with  $\tilde{x}_i^c \coloneqq \nu x_i^c$ ,  $\tilde{\lambda}_i \coloneqq \lambda_i / \nu$  and  $\tilde{\tau}_i \coloneqq \nu \tau_i \quad \forall i \in \mathcal{I}$  and  $c \in \mathcal{C}_i$  in the proofs of Theorems 1 and 2 gives the desired result. Uninteresting algebra are available upon request.

# **B** Additional materials

This appendix presents omitted materials from the main text. All mathematical proofs are gathered towards the end.

### B.1 Echo chamber in a large society

Suppose, in the baseline model, that type-L and type-R players have the same population size  $N \in \mathbb{N} - \{1\}$ , preference parameter  $\beta \in (0, 1)$ , visibility parameter  $\lambda > 1$ , and bandwidth  $\tau > \phi(\lambda)$ . A PSPBE is *symmetric* if players of the same type adopt the same strategy. The next theorem shows that if we let the population size N grow while keeping everything else constant, then echo chambers will eventually emerge as the unique symmetric equilibrium outcome.

**Theorem 4.** For any  $\beta \in (0,1)$ ,  $\lambda > 1$ , and  $\tau > \phi(\lambda)$ , there exists  $\underline{N} \in \mathbb{N}$  such that for any  $N > \underline{N}$ , our game has a unique symmetric PSPBE that is also an echo-chamber equilibrium.

To develop intuition, note that if everyone except a type-L player i acts as above, then player i must choose between acting like a type-L player and mimicking a type-Rplayer. In the first case, he attends to his own-biased source and N - 1 like-minded friends and takes his default action in event  $\mathcal{U}_i$ . In the second case, he attends to source r and N type-R players and takes action R in event  $\mathcal{U}_i$ . When N is small, the gain from mimicking a type-R player could be significant. Yet such a gain vanishes and eventually becomes negative as N grows.

### **B.2** Efficient attention network

In this appendix, we solve for the attention profile that maximizes the utilitarian welfare among players and demonstrate that echo chambers are in general inefficient. To best illustrate the main idea, suppose type-L and type-R players are symmetric, i.e., they have the same population size N and characteristics  $\beta$ ,  $\lambda$ , and  $\tau$ . Consider the case where  $\beta$  is small, so that taking one's default action in event  $\mathcal{U}_i$  is not only a dominant strategy but is also efficient. The social planner's problem is then

$$\max_{x \in \times_{i \in \mathcal{I}} \mathcal{X}_i} -\frac{\beta}{2} \sum_{i \in \mathcal{I}} \mathbb{P}_x(\mathcal{U}_i \mid \omega \neq d_i),$$

whose solution doesn't exhibit echo chambers when  $\lambda$  and  $\tau$  are sufficiently large.

**Theorem 5.** Suppose  $|\mathcal{L}| = |\mathcal{R}| = N$  and  $(\beta_i, \lambda_i, \tau_i) = (\beta, \lambda, \tau) \quad \forall i \in \mathcal{I}$ . Then for any  $N \in \mathbb{N} - \{1\}$  and  $\lambda > N/(N-1)$ , there exists  $\tau(\lambda, N)$  such that for any  $\tau > \tau(\lambda, N)$ , there exists  $\beta(\lambda, \tau, N) > 0$  such that echo chambers are inefficient for any  $\beta \in (0, \beta(\lambda, \tau, N))$ .

Theorem 5 stands in sharp contrast to Theorem 1, which shows that any equilibrium must be an echo-chamber equilibrium when  $\beta$  is sufficiently small. Intuitively, when players are capable of disseminating news to each other, maximizing social welfare requires that they attend to the primary sources that are biased against their predispositions. Doing so qualifies more players as secondary sources for opposite-type players, yet the resulting efficiency gain cannot be sustained in any equilibrium.

### **B.3** Uniqueness of equilibrium

This appendix provides sufficient conditions for the game among type-L players to admit a unique equilibrium (as required by Theorem 3). Our starting observation is that only members of  $\mathcal{PV} \coloneqq \{i \in \mathcal{L} : \tau_i > \phi(\lambda_i)\}$  are potentially visible to their like-minded friends in equilibrium. Simplifying Theorem 2 accordingly yields the following observation.

**Observation 1.** The game among type-L players has a unique equilibrium if and only the system of Equation (7) among members of  $\mathcal{PV}$  has a unique solution. If  $|\mathcal{PV}| \leq 1$ , then the game among type-L players has a unique equilibrium.

*Proof.* Equilibria of the game among type-L players can be computed as follows.

**Step 1.** Solve the system of Equation (7) among members of  $\mathcal{PV}$ . For each solution  $(x_i^l)_{i\in\mathcal{PV}}$ , define  $\mathcal{COR} = \{i\in\mathcal{PV}: x_i^l > \phi(\lambda_i)\}$  and  $\mathcal{PER} = \mathcal{L} - \mathcal{COR}$ .

**Step 2.** For each pair  $i, j \in COR$ , let  $x_i^j = h(x_j^l; \lambda_j)$ . For each pair  $i \in \mathcal{L}$ and  $j \in \mathcal{PER}$ , let  $x_i^j = 0$ . For each pair  $i \in \mathcal{PER}$  and  $j \in COR$ , let  $x_i^j = \frac{1}{\lambda_j} \log \max \left\{ \left( \frac{\lambda_j}{\gamma_i} - 1 \right) \left( \exp(x_j^l) - 1 \right), 1 \right\}$  and  $x_i^l = \tau_i - \sum_{j \in COR} x_i^j$ , where  $\gamma_i \ge 1$  is the Lagrange multiplier associated with the constraint  $x_i^l \ge 0$ , and  $\gamma_i > 1$  if and only if  $x_i^l = 0$ .

The remainder of this appendix assumes that  $|\mathcal{PV}| \geq 2$ . The analysis differs depending on whether members of  $\mathcal{PV}$  are homogeneous or not. The next theorem concerns the case of homogeneous players.

**Theorem 6.** In the case where  $(\lambda_i, \tau_i) = (\lambda, \tau) \quad \forall i \in \mathcal{PV}$ , the game among type-L players has a unique equilibrium if  $\tau - (|\mathcal{PV}| - 2)h(\tau; \lambda) > \phi(\lambda)$ .

With heterogeneous players, we cannot guarantee that the game in the baseline model has a unique equilibrium. The reason is that when  $x_i^l \approx \phi(\lambda_i)$ , the marginal influence  $h_x(x_i^l; \lambda_i)$  that player *i* exerts on the other players is close to 1, which is too big for the contraction mapping theorem to work. To bound players' marginal influence, we enrich the baseline model by assuming that each player i has  $\overline{\tau}_i > 0$ units of attention to spare and yet must pay at least  $\underline{\tau}_i \in [0, \overline{\tau}_i)$  units of attention to his own-biased source. The next proposition establishes the counterparts of Theorems 1-3 in this new setting.

**Proposition 5.** Fix the population sizes  $|\mathcal{L}|, |\mathcal{R}| \in \mathbb{N} - \{1\}$  and characteristic profiles  $(\lambda_i, \underline{\tau}_i, \overline{\tau}_i)_{i \in \mathcal{L}}, (\lambda_i, \underline{\tau}_i, \overline{\tau}_i)_{i \in \mathcal{R}}$  of type-L and type-R players, respectively. Then there exists  $\underline{\beta} > 0$  such that in the case where  $\beta_i \in (0, \underline{\beta}) \quad \forall i \in \mathcal{I}$ , any PSPBE of our game must be an echo-chamber equilibrium and must satisfy the following properties.

(i) For any  $i \in \mathcal{L}$ ,

$$x_i^l = \max\left\{\overline{\tau}_i - \sum_{j \in \mathcal{L} - \{i\}} \frac{1}{\lambda_j} \log \max\left\{(\lambda_j - 1)(\exp(x_j^l) - 1), 1\right\}, \underline{\tau}_i\right\}.$$

- (ii) If  $x_i^l > \underline{\tau}_i \ \forall i \in \mathcal{L}$ , then Theorem 2(ii) remains valid.
- (iii) If  $x_i^l > \underline{\tau}_i$  and  $x_i^j > 0 \ \forall i \in \mathcal{L}$  and  $j \in \mathcal{L} \{i\}$ , then Theorem 2(iii) and Theorem 3 remain valid.

Redefine  $\mathcal{PV} = \{i \in \mathcal{L} : \overline{\tau}_i > \phi(\lambda_i)\}$ . Suppose  $\underline{\tau}_i > \phi(\lambda_i) \ \forall i \in \mathcal{PV}$ , and define  $\overline{g} = \max_{i \in \mathcal{PV}} h_x(\underline{\tau}_i; \lambda_i)$ . From Lemma 1, it follows that  $\overline{g} < 1$  is a uniform upper bound for the marginal influence that members of  $\mathcal{PV}$  exert on the other players, and that  $\overline{g}$  is non-increasing in each  $\underline{\tau}_i$ ,  $i \in \mathcal{PV}$ . The next theorem shows that the game among type-L players has a unique equilibrium when  $\overline{g}$  is sufficiently small.

**Theorem 7.** Let everything be as above. Then the game among type-L players has a unique equilibrium if  $\overline{g} < 1/(|\mathcal{PV}| - 1)$ .

# B.4 Adding peripheral players to comparative statics exercises

When proving our main comparative statics result Theorem 3, we focused on the situation where all type-L players attend to each other, i.e.,  $\mathcal{L} = COR$ . In this appendix, we replace this assumption with a weaker one.

Assumption 2. The game among type-L players has a unique equilibrium whereby all players attend to source l,  $|COR| \ge 2$  (to make the analysis interesting), and no PER player is a borderline player, i.e.,  $x_i^l < \phi(\lambda_i) \ \forall i \in PER$ .

Under the above assumption, perturbing the characteristics of a  $\mathcal{PER}$  player has no impact on any other player. The next proposition examines what happens to a  $\mathcal{PER}$  player as we perturb the characteristics of a  $\mathcal{COR}$  player.

**Proposition 6.** Fix any  $N \in \mathbb{N} - \{1\}$ , and let  $\Theta$  be any neighborhood in  $\mathbb{R}^{2N}_{++}$  such that for any  $\theta \in \Theta$ , the game among a set  $\mathcal{L}$  of type-L players with population size N and characteristic profile  $\theta$  satisfies Assumption 2. Then at any  $\theta^{\circ} \in int(\Theta)$ . the following must hold for any  $i, j \in COR$  and any  $k \in PER$ .

$$\begin{array}{l} (i) \ \operatorname{sgn}\left(\left.\frac{\partial x_k^l}{\partial \tau_i}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}^\circ}\right) = \operatorname{sgn}\left(-\left.\frac{\partial x_i^l}{\partial \tau_i}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}^\circ}\right) \ and \ \operatorname{sgn}\left(\left.\frac{\partial x_k^j}{\partial \tau_i}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}^\circ}\right) = \operatorname{sgn}\left(\left.\frac{\partial x_j^l}{\partial \tau_i}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}^\circ}\right). \\ (ii) \ \operatorname{sgn}\left(\left.\frac{\partial x_k^l}{\partial \lambda_i}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}^\circ}\right) = \operatorname{sgn}\left(-\left.\frac{\partial x_i^l}{\partial \lambda_i}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}^\circ}\right) \ and \ \operatorname{sgn}\left(\left.\frac{\partial x_k^j}{\partial \lambda_i}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}^\circ}\right) = \operatorname{sgn}\left(\left.\frac{\partial x_j^l}{\partial \lambda_i}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}^\circ}\right). \end{aligned}$$

### B.5 Pairwise visibility parameter

In this appendix, we parameterize the visibility of player j to player  $i \in \mathcal{I} - \{j\}$  by  $\lambda_i^j > 0$  and write  $\lambda_i$  for  $(\lambda_i^j)_{j \in \mathcal{I} - \{i\}}$ . The next proposition extends Theorem 1 and 2 to encompass pairwise visibility parameters.

**Proposition 7.** Fix the population sizes  $|\mathcal{L}|, |\mathcal{R}| \in \mathbb{N} - \{1\}$  and characteristic profiles  $(\lambda_i, \tau_i)_{i \in \mathcal{L}}, (\lambda_i, \tau_i)_{i \in \mathcal{R}}$  of type-L and type-R players, respectively. Then there exists  $\underline{\beta} > 0$  such that in the case where  $\beta_i \in (0, \underline{\beta}) \forall i \in \mathcal{I}$ , any PSPBE of our game must be an echo-chamber equilibrium and must satisfy the following properties.

(i) For any  $i \in \mathcal{L}$ ,

$$x_{i}^{l} = \max\left\{\tau_{i} - \sum_{j \in \mathcal{L} - \{i\}} \frac{1}{\lambda_{i}^{j}} \log \max\left\{(\lambda_{i}^{j} - 1)(\exp(x_{j}^{l}) - 1), 1\right\}, 0\right\}$$

- (ii) If all type-L players attend to source l, then the following are equivalent for any  $i \in \mathcal{L}$  and  $j \in \mathcal{L} - \{i\}$ : (a)  $x_j^l > \phi(\lambda_i^j)$ ; (b)  $x_i^j > 0$ ; (c)  $x_i^j = h(x_j^l; \lambda_i^j)$ .
- (iii) If all type-L players attend to each other, then the Stage-1 expected utility of any player  $i \in \mathcal{L}$  equals

$$-\frac{\beta_i}{2} \exp\left(-\sum_{j \in \mathcal{L}} x_j^l + \sum_{j \in \mathcal{L} - \{i\}} \phi(\lambda_i^j)\right)$$

With pairwise visibility parameters, player j must cross a personalized visibility threshold  $\phi(\lambda_i^j)$  in order to be attended by player i. After that, the amount of attention  $h(x_j^l; \lambda_i^j)$  he receives from player i depends on his resourcefulness  $x_j^l$  as a secondary source and his visibility  $\lambda_i^j$  to player i. The equilibrium expected utility of player i depends positively on the total amount of attention that the entire echo chamber pays to the primary source, and it depends negatively on the visibility threshold  $\phi(\lambda_i^j)$ 's that prevent his like-minded friends from spreading information to him.

We next examine the comparative statics regarding individual players' characteristics, writing  $\mathcal{L} = \{1, \dots, N\}, \ \theta_i = (\lambda_i, \tau_i) \ \forall i \in \mathcal{L}, \ \text{and} \ \boldsymbol{\theta} = [\theta_1 \ \cdots \ \theta_N]^\top$  as before. Note that with pairwise visibility parameters, the influence exerted by a player is no longer constant across his like-minded friends. Mathematically, this means that the off-diagonal entries of the marginal influence matrix may not be constant column by column. Nevertheless, if that matrix still satisfies the properties stated in Lemma 3, then all results we've so far obtained would carry over qualitatively.

**Proposition 8.** Fix any  $N \in \mathbb{N} - \{1\}$ . Let  $\Theta$  be any neighborhood in  $\mathbb{R}_{++}^{N^2}$  such that for any  $\boldsymbol{\theta} \in \Theta$ , the game among a set  $\mathcal{L}$  of type- $\mathcal{L}$  players with population size Nand characteristic profile  $\boldsymbol{\theta}$  satisfies Assumption 1, and the matrix  $\mathbf{A}_N := \mathbf{I}_N + \mathbf{G}_N$ satisfies the properties stated in Lemma 3. Then at any  $\boldsymbol{\theta}^\circ \in int(\Theta)$ , the following must hold for any  $i \in \{1, \dots, N\}, j, k \in \{1, \dots, N\} - \{i\}$  and  $m \in \{1, \dots, N\} - \{k\}$ .

(i) 
$$\partial x_i^l / \partial \tau_i \Big|_{\boldsymbol{\theta} = \boldsymbol{\theta}^\circ} > 0, \ \partial x_k^i / \partial \tau_i \Big|_{\boldsymbol{\theta} = \boldsymbol{\theta}^\circ} > 0, \ \partial x_k^l / \partial \tau_i \Big|_{\boldsymbol{\theta} = \boldsymbol{\theta}^\circ} < 0, \ and \ \partial x_m^k / \partial \tau_i \Big|_{\boldsymbol{\theta} = \boldsymbol{\theta}^\circ} < 0.$$

- (ii) One of the following situations happen:
  - (a)  $\partial x_i^l / \partial \lambda_i^j |_{\boldsymbol{\theta} = \boldsymbol{\theta}^\circ} > 0, \ \partial x_k^i / \partial \lambda_i^j |_{\boldsymbol{\theta} = \boldsymbol{\theta}^\circ} > 0, \ \partial x_k^l / \partial \lambda_i^j |_{\boldsymbol{\theta} = \boldsymbol{\theta}^\circ} < 0, \ and \ \partial x_m^k / \partial \lambda_i^j |_{\boldsymbol{\theta} = \boldsymbol{\theta}^\circ} < 0;$
  - (b) all inequalities in Part (a) are reversed;
  - (c) all inequalities in Part (a) are replaced with equalities.

### **B.6** More than two states and actions

In this appendix, suppose the state  $\omega$  is uniformly distributed on a finite set  $\{1, \dots, M\}$ with  $M \in \mathbb{N} - \{1\}$ . There are M types of players, each of whom has a population size  $N \in \mathbb{N} - \{1\}$  and can take one of the actions in  $\{1, \dots, M\}$ . In case a type-mplayer takes action a, his utility in state  $\omega$  equals zero if  $a = \omega, -1$  if  $\omega = m$  and  $a \neq m$ , and  $-\beta$  if  $\omega \neq m$  and a = m. Assume  $\beta \in (0, 1)$ , so that action m is type-m players' default action. Also assume that all players, regardless of their types, have the same visibility parameter  $\lambda > 0$  and bandwidth  $\tau > 0$ .

There are M sources: 1-revealing,  $\cdots$ , M-revealing. In state  $\omega \in \{1, \cdots, M\}$ , the  $\omega$ -revealing source is activated and generates news that fully reveals  $\omega$ . To make informed decisions, players attend to sources and to other players as in the baseline model. Our solution concept is symmetric PSPBE, whereby players of the same type adopt the same strategy. Moreover, we parameterize the attention strategy of any type-m player by four quantities:

- (i)  $\Delta^*$ : the amount of attention he pays to the *m*-revealing source.
- (ii)  $x^*$ : the amount of attention he pays to any other source.
- (iii)  $y^*$ : the amount of attention he pays to each of his like-minded friends.
- (iv)  $z^*$ : the amount of attention he pays to any other player.

A symmetric PSPBE of the above kind constitutes a generalized echo-chamber equilibrium if  $\Delta^* = 0$  and  $y^* > z^*$ . That is, no player wastes time on learning that the state favors his default action, and all players prioritize like-minded friends over other players when deciding whom to pay attention to.

The next theorem proves an analog of Theorem 1: when players have sufficiently strong preferences for taking their default actions, any symmetric PSPBE of the above kind must be a generalized echo-chamber equilibrium, and such an equilibrium exists.

**Theorem 8.** For any  $M, N \in \mathbb{N} - \{1\}, \lambda > 1/(M-1)$  and  $\tau > (M-1)\phi(\lambda(M-1))$ , there exist  $\underline{\beta} \in (0,1)$  such that for any  $\beta < \underline{\beta}$ , our game has a unique symmetric PSPBE of the above kind that is also a generalized echo-chamber equilibrium.

### B.7 Proofs

**Proof of Theorem 4** In any symmetric PSPBE, either all players take their default actions in event  $\mathcal{U}_i$ 's, or type-L players take action R and type-R players take action L in event  $\mathcal{U}_i$ 's. Below we solve for the unique equilibrium of the first kind and demonstrate that no equilibrium of the second kind exists when N is large.

For any equilibrium of the first kind, we can show, based on the arguments made in the proof of Theorem 1, that all players must attend only to their own-biased sources and like-minded friends at Stage 1 of the game. Then based on the arguments made in the proof of Proposition 1, we can show that each player pays x(N) units of attention to his own-biased source and  $h(x(N); \lambda)$  units of attention to each of his N - 1 likeminded friends, where x(N) is the unique fixed point of  $\varphi^N(x) = \tau - (N - 1)h(x; \lambda)$ . His Stage 1-expected utility equals  $-\beta \exp(-Nx(N) + (N - 1)\phi(\lambda))/2$ .

To sustain the above outcomes on the equilibrium path, we must show that no type-*L* player benefits from attending to source *r* and type-*R* players and then taking action *R* in event  $\mathcal{U}_i$ . In case a type-*L* player  $i \in \mathcal{L}$  commits such a deviation, his best response is to pay  $y_i^j = h(x(N); \lambda)$  units of attention to each type-*R* player  $j \in \mathcal{R}$ and  $y_i^r = \tau - Nh(x(N); \lambda)$  units of attention to source *r*, where  $y_i^r = \varphi^N(x(N)) - h(x(N); \lambda) = x(N) - h(x(N); \lambda) \to \phi(\lambda) > 0$  as  $N \to \infty$  by Lemma 4. His Stage 1expected utility equals  $-\exp(-y_i^r - Nx(N) + N\phi(\lambda))/2$ , which, after simplifying, becomes

$$-\frac{1}{2}\exp\left(-\tau + \frac{N\left(\tau - x\left(N\right)\right)}{N-1} - Nx\left(N\right) + N\phi\left(\lambda\right)\right)$$
(12)

Comparing (12) and the on-path expected utility, we find that the above deviation is unprofitable if and only if

$$\frac{\tau}{N-1} + \phi(\lambda) - \frac{N}{N-1}x(N) > \log\beta.$$

Since the left-hand side of the above inequality converges to zero as N grows to infinity by Lemma 4, it must exceed the right-hand side when N is sufficiently large.

We next argue that no equilibrium of the second kind exists when N is large. If the contrary is true, then we can show, based on the arguments made in the proof of Proposition 1, that on the equilibrium path, each type-L player (resp. type-Rplayer) pays x(N) units of attention to source r (resp. source l) and  $h(x(N); \lambda)$  units of attention to each of his N - 1 like-minded friends. The Stage-1 expected utility of any type-L player equals  $-\exp(-Nx(N) + N\phi(\lambda))/2$ , which falls short of what he could earn from attending to source r and type-R players when N is large: by committing this deviation, the player could increase his Stage 1-expected utility to  $\beta \cdot (12)$  for the reason given in the preceding paragraph.

**Proof of Theorem 5** Let  $|\mathcal{L}| = |\mathcal{R}| = N \ge 2$  and  $(\beta_i, \lambda_i, \tau_i) = (\beta, \lambda, \tau) \quad \forall i \in \mathcal{I}$ . When  $\beta$  is sufficiently small, it's efficient for players to take their default actions in event  $\mathcal{U}_i$ 's. Given this, the social planner's problem becomes

$$\max_{x \in \times_{i \in \mathcal{I}} \mathcal{X}_i} - \sum_{i \in \mathcal{L}} \delta_i^l \prod_{j \in \mathcal{I} - \{i\}} \left( \delta_i^l + \left( 1 - \delta_j^l \right) \delta_i^j \right) - \sum_{i \in \mathcal{R}} \delta_i^r \prod_{j \in \mathcal{I} - \{i\}} \left( \delta_i^r + \left( 1 - \delta_j^r \right) \delta_i^j \right).$$

Since the above problem has a strictly concave maximand and a compact convex choice set, its solution is unique, and any interior solution to it must be characterized by first-order conditions. Consider a particular kind of interior solution that parameterizes the attention profile of any player by four quantities:

- (i)  $x^* > 0$ : the amount of attention he pays to his own-biased source;
- (ii)  $y^* > 0$ : the amount of attention he pays to the other source;
- (iii)  $z^* > 0$ : the amount of attention he pays to each of his like-minded friends;

(iv)  $\Delta^* > 0$ : the amount of attention he pays to any other player.

Such a solution, if it exists, must solve the planner's original problem, and it doesn't exhibit echo chambers.

The remainder of the proof provides sufficient conditions for a solution of the above kind to exist. For ease of notation, write A for  $\exp(-x^*) + (1 - \exp(-x^*)) \exp(-\lambda z^*)$ , B for  $\exp(-y^*) + (1 - \exp(-y^*)) \exp(-\lambda \Delta^*)$ , a for  $\exp(x^*) - 1$ , b for  $\exp(y^*) - 1$ , cfor  $\exp(\lambda z^*) - 1$ , and d for  $\exp(\lambda \Delta^*) - 1$ . Fix any type-L player  $i \in \mathcal{L}$ , and let  $\gamma > 0$ denote the Lagrange multiplier associated with his bandwidth constraint, which must be binding. The first-order conditions regarding  $x_i^l$ ,  $x_i^r$ ,  $x_i^j$ ,  $j \in \mathcal{L}$ , and  $x_i^k$ ,  $i \in \mathcal{R}$  are

$$\delta_i^l A^{N-1} B^N + \sum_{j \in \mathcal{L} - \{i\}} \delta_i^l \delta_j^l (1 - \delta_j^i) A^{N-2} B^N = \gamma$$
 (FOC<sub>xi</sub>)

$$\sum_{j \in \mathcal{R}} \delta_i^r \delta_j^r (1 - \delta_j^i) A^{N-1} B^{N-1} = \gamma \qquad (\text{FOC}_{x_i^r})$$

$$\lambda \delta_i^l (1 - \delta_j^l) \delta_i^j A^{N-2} B^N = \gamma \qquad (\text{FOC}_{x_i^j})$$

and 
$$\lambda \delta_i^l (1 - \delta_j^l) \delta_i^j A^{N-1} B^{N-1} = \gamma,$$
 (FOC<sub>x<sub>i</sub><sup>k</sup>)</sub>

and setting  $x_i^l = x^*, \, x_i^r = y^*, \, x_i^j = z^*$ , and  $x_i^k = \Delta^*$  in these FOCs yields

$$\begin{cases} (\lambda - 1)a = Nc + 1\\ Nd = \lambda b\\ \lambda a(b + d + 1) = Nd(a + c + 1)\\ \log(a + 1) + \log(b + 1) + \frac{N-1}{\lambda}\log(c + 1) + \frac{N}{\lambda}\log(d + 1) = \tau. \end{cases}$$

Solving the first three linear equations yields

$$b = \frac{Na}{N-1-a}, \ c = \frac{(\lambda-1)a-1}{N}, \ \text{and} \ d = \frac{\lambda a}{N-1-a}.$$

Substituting these solutions into the last equation yields

$$\log(a+1) + \log\left(\frac{Na}{N-1-a} + 1\right) + \frac{N-1}{\lambda}\log\left(\frac{(\lambda-1)a-1}{N} + 1\right) + \frac{N}{\lambda}\log\left(\frac{\lambda a}{N-1-a} + 1\right) = \tau.$$
(13)

It remains to find conditions on  $(\lambda, \tau, N)$  such that (13) admits a solution  $a(\lambda, \tau, N)$ satisfying

$$a(\cdot) > 0, \ \frac{Na(\cdot)}{N-1-a(\cdot)} > 0, \ \frac{(\lambda-1)a(\cdot)-1}{N} > 0, \ \text{and} \ \frac{\lambda a(\cdot)}{N-1-a(\cdot)} > 0,$$

or equivalently

$$\lambda > \frac{N}{N-1}$$
 and  $a(\cdot) \in \left(\frac{1}{\lambda-1}, N-1\right)$ 

To see when  $a(\cdot) \in (1/(\lambda - 1), N - 1)$  is satisfied, note that the left-hand side of (13) as a function of a (i) is well-defined on (0, N - 1), (ii) is negative when  $a \approx 0$ , (iii)  $\rightarrow +\infty$  as  $a \rightarrow N - 1$ , (iv) is strictly increasing in a, and (v) is independent of  $\tau$ . Thus for any  $N \geq 2$  and  $\lambda > N/(N - 1)$ , there exists a threshold  $\tau(\lambda, N)$  such that the solution to (13) belongs to  $(1/(\lambda - 1), N - 1)$  for any  $\tau > \tau(\lambda, N)$ , which completes the proof.

**Proof of Theorem 6** Write  $\{1, \dots, N\}$  for  $\mathcal{PV}$ , and recall that  $\tau_i > \phi(\lambda_i)$  (which only holds if  $\lambda_i > 1$ )  $\forall i \in \mathcal{PV}$ . Given this, we can simplify the system of Equation

(7) among members of  $\mathcal{PV}$  to

$$x_i^l = \max\left\{\tau_i - \sum_{j \in \mathcal{PV}-\{i\}} \max\left\{h(x_j^l; \lambda_j), 0\right\}, 0\right\} \quad \forall i \in \mathcal{PV}.$$
 (14)

Below we demonstrate that if  $(\lambda_i, \tau_i) = (\lambda, \tau) \ \forall i \in \mathcal{PV}$  and if  $\tau - (|\mathcal{PV}| - 2)h(\tau; \lambda) > \phi(\lambda)$ , then (14) has a unique solution  $x_i^l = x(N) \ \forall i \in \mathcal{PV}$ , where  $x(N) \in (\phi(\lambda), \tau)$  is the unique fixed point of  $\varphi^N(x) = \tau - (N-1)h(x; \lambda)$ .

Consider first the case N = 2. In that case, (14) is simply

$$x_{1}^{l} = \max \left\{ \tau - \max \left\{ h \left( x_{2}^{l}; \lambda \right), 0 \right\}, 0 \right\}$$
  
and  $x_{2}^{l} = \max \left\{ \tau - \max \left\{ h \left( x_{1}^{l}; \lambda \right), 0 \right\}, 0 \right\},$ 

and it has a unique solution  $(x_1^l, x_2^l) = (x(2), x(2))$ . For each  $N \ge 3$ , we fix any pair  $i \ne j$ . Define  $\hat{\tau} = \tau - \sum_{k \ne i,j} \max \{h(x_k^l; \lambda), 0\}$ , and note that  $\hat{\tau} \in (\phi(\lambda), \tau)$  by assumption. Then from

$$x_{i}^{l} = \max\left\{\hat{\tau} - \max\left\{h\left(x_{j}^{l};\lambda\right),0\right\},0\right\}$$
  
and  $x_{j}^{l} = \max\left\{\hat{\tau} - \max\left\{h\left(x_{i}^{l};\lambda\right),0\right\},0\right\},$ 

it follows that  $(x_i^l, x_j^l)$  is the unique solution to the system of equations in the case of N = 2 with  $\tau$  replaced with  $\hat{\tau}$ , so it must satisfy  $x_i^l = x_j^l \in (\phi(\lambda), \hat{\tau})$ . Repeating the above argument for all pairs of i, j shows that  $x_i^l = x_j^l \in (\phi(\lambda), \tau) \ \forall i, j \in \mathcal{PV}$ . Simplifying (14) accordingly yields  $x_i^l = \varphi^N(x_i^l)$  and, hence,  $x_i^l = x(N) \ \forall i \in \mathcal{PV}$ .  $\Box$ 

**Proof of Proposition 5** The proof closely parallels that of Theorems 1-3 and hence is omitted.  $\hfill \Box$ 

**Proof of Theorem 7** Write  $\{1, \dots, N\}$  for  $\mathcal{PV}$  and define  $y_i = x_i^l - \underline{\tau}_i$  and  $\Delta \tau_i = \overline{\tau}_i - \underline{\tau}_i$  for each  $i \in \mathcal{PV}$ . By definition, any  $i \in \mathcal{PV}$  must satisfy  $\overline{\tau}_i > \underline{\tau}_i > \phi(\lambda_i)$  and, hence,  $\lambda_i > 1$ . Given this, we can simplify the best response function of any  $i \in \mathcal{PV}$ :

$$x_i^l = \max\left\{\overline{\tau}_i - \sum_{j \in \mathcal{PV}-\{i\}} \frac{1}{\lambda_i} \log \max\{(\lambda_j - 1)(\exp(x_j^l) - 1), 1\}, \underline{\tau}_i\right\}$$

to:

$$y_i = \max\left\{ \Delta \tau_i - \sum_{j \in \mathcal{PV} - \{i\}} h(y_j^l + \underline{\tau}_j; \lambda_j), 0 \right\}.$$

For each  $\mathbf{y} = [y_1 \cdots y_N]^\top \in Y \coloneqq \times_{i=1}^N [0, \Delta \tau_i]$ , define  $F(\mathbf{y})$  as the *N*-vector whose  $i^{th}$  entry is given by  $y_i + \sum_{j \in \mathcal{PV} - \{i\}} h(y_j^l + \underline{\tau}_j; \lambda_j) - \Delta \tau_i$ . Note that the function  $F: Y \to \mathbb{R}^N$  is strongly monotone,<sup>10</sup> because for any  $\mathbf{y}, \mathbf{y}' \in Y$ :

$$\begin{aligned} (\mathbf{y} - \mathbf{y}')^{\top} (F(\mathbf{y}) - F(\mathbf{y}')) \\ &= \sum_{i=1}^{N} (y_i - y_i')^2 - \sum_{i=1}^{N} \sum_{j \neq i} (y_i - y_i') [h_x(y_j; \lambda_j) - h_x(y_j'; \lambda_j)] \\ &\geq \|\mathbf{y} - \mathbf{y}'\|^2 - \overline{g} \sum_{i=1}^{N} \sum_{j \neq i} |y_i - y_i'| |y_j - y_j'| \qquad (\because h_x(\cdot; \cdot) \in (0, \overline{g})) \\ &\geq \underbrace{[1 - (N - 1)\overline{g}]}_{>0 \text{ by assumption}} \|\mathbf{y} - \mathbf{y}'\|^2. \end{aligned}$$

Then from Proposition 1 of Naghizadeh and Liu (2017), it follows that the game among members of  $\mathcal{PV}$  has a unique equilibrium.

**Proof of Proposition 7** The proof closely parallels that of Theorems 1 and 2 and hence is omitted for brevity.  $\Box$ 

<sup>&</sup>lt;sup>10</sup>A function  $f: K \to \mathbb{R}^n$  defined on a closed convex set  $K \subset \mathbb{R}^n$  is strongly monotone if there exists c > 0 such that  $(\mathbf{x} - \mathbf{y})^\top (F(\mathbf{x}) - F(\mathbf{y})) > c ||\mathbf{x} - \mathbf{y}||^2 \quad \forall \mathbf{x}, \mathbf{y} \in K.$ 

**Proof of Proposition 8** Write  $\{1, \dots, N\}$  for  $\mathcal{L}$ . Under the assumption stated in Proposition 8, the following must hold  $\forall i \in \{1, \dots, N\}$  and  $j \in \{1, \dots, N\} - \{i\}$ : (i)

$$x_i^l = \tau_i - \sum_{j \in \mathcal{L} - \{i\}} h(x_j^l; \lambda_i^j)$$
(15)

and 
$$x_i^j = h(x_j^l; \lambda_i^j),$$
 (16)

(ii)  $x_j^l > \phi(\lambda_i^j)$  and, hence,  $h_x(x_j^l; \lambda_i^j) \coloneqq [\mathbf{G}_N]_{i,j} \in (0, 1)$ ; (iii)  $\mathbf{A}_N \coloneqq \mathbf{I}_N + \mathbf{G}_N$  satisfies the properties stated in Lemma 3.

Part (i): The proof is the exact same as that of Theorem 3(i) and hence is omitted.

Part (ii): We only prove the result for i = 1 and j = 2. Differentiating the system of Equation (15) among  $i \in \{1, \dots, N\}$  with respect to  $\lambda_1^2$  yields

$$\nabla_{\lambda_1^2} \mathbf{x}^l = \kappa_1^2 \mathbf{A}_N^{-1} \begin{bmatrix} 1 \ 0 \ \cdots \ 0 \end{bmatrix}^\top,$$

where  $\mathbf{x}^{l} \coloneqq [x_{1}^{l} \cdots x_{N}^{l}]^{\top}$ , and  $\kappa_{1}^{2} \coloneqq -h_{\lambda}(x_{2}^{l}; \lambda_{1}^{2})$  has an ambiguous sign in general. Then from the assumption that  $[\mathbf{A}_{N}^{-1}]_{1,1} > 0$  and  $[\mathbf{A}_{N}^{-1}]_{k,1} < 0 \ \forall k \neq 1$ , it follows that

$$\operatorname{sgn}\left(\frac{\partial x_{1}^{l}}{\partial \lambda_{1}^{2}}\right) = \operatorname{sgn}\left(\kappa_{1}^{2}\left[\mathbf{A}_{N}^{-1}\right]_{1,1}\right) = \operatorname{sgn}\left(\kappa_{1}^{2}\right)$$

and

$$\operatorname{sgn}\left(\frac{\partial x_k^l}{\partial \lambda_1^2}\right) = \operatorname{sgn}\left(\kappa_1^2 \left[\mathbf{A}_N^{-1}\right]_{k,1}\right) = \operatorname{sgn}\left(-\kappa_1^2\right) \ \forall k \neq 1.$$

The second result and Equation (16) together imply that

$$\operatorname{sgn}\left(\frac{\partial x_k^1}{\partial \lambda_1^2}\right) = \operatorname{sgn}\left(h_x(x_1^l;\lambda_k^1)\frac{\partial x_1^l}{\partial \lambda_1^2}\right) = \operatorname{sgn}\left(\kappa_1^2\right) \ \forall k \neq 1$$

and

$$\operatorname{sgn}\left(\frac{\partial x_m^k}{\partial \lambda_1^2}\right) = \operatorname{sgn}\left(h_x(x_k^l;\lambda_m^k)\frac{\partial x_k^l}{\partial \lambda_1^2}\right) = \operatorname{sgn}\left(-\kappa_1^2\right) \ \forall k \neq 1 \text{ and } (m,k) \neq (1,2).$$

Finally, differentiating  $x_1^2 = h(x_2^l; \lambda_1^2)$  with respect to  $\lambda_1^2$  yields

$$\operatorname{sgn}\left(\frac{\partial x_1^2}{\partial \lambda_1^2}\right) = \operatorname{sgn}\left(\kappa_1^2 \left[h_x(x_2^l;\lambda_1^2)\left[\mathbf{A}_N^{-1}\right]_{2,1} - 1\right]\right) = \operatorname{sgn}\left(-\kappa_1^2\right),$$

where the second equality follows from the assumption that  $[\mathbf{A}_N^{-1}]_{2,1} < 0$ . Thus in total, only three situations can happen:

- (a)  $\kappa_1^2 > 0$  and, hence,  $\partial x_1^l / \partial \lambda_1^2 > 0$ ,  $\partial x_k^l / \partial \lambda_1^2 > 0$ ,  $\partial x_k^l / \partial \lambda_1^2 < 0$  and  $\partial x_m^k / \partial \lambda_1^2 < 0$  $\forall k \neq 1$  and  $m \neq k$ ;
- (b)  $\kappa_1^2 < 0$  and, hence, all inequalities in case (a) are reversed;
- (c)  $\kappa_1^2 = 0$  and, hence, all inequalities in case (a) are replaced with equalities.

**Proof of Proposition 6** We only prove that  $\operatorname{sgn} \left( \partial x_k^l / \partial \tau_i \right) = \operatorname{sgn} \left( -\partial x_i^l / \partial \tau_i \right)$  and that  $\operatorname{sgn} \left( \partial x_k^l / \partial \lambda_i \right) = \operatorname{sgn} \left( -\partial x_1^l / \partial \lambda_i \right)$  for arbitrary  $k \in \mathcal{PER}$  and  $i \in \mathcal{COR}$ . The remaining results follow immediately from what we already know and so won't be proven again. Write  $\{1, \dots, N\}$  for  $\mathcal{COR}$ , and let  $\mathbf{G}_N$  be the marginal influence matrix among  $\mathcal{COR}$  players. Note that  $[\mathbf{G}_N]_{i,j} = h(x_j^l; \lambda_j)$  (hereafter written as  $g_j$ )  $\forall i \neq j$ , and that any  $j \in \mathcal{COR}$  must satisfy  $x_j^l > \phi(\lambda_j)$  and, hence,  $g_j \in (0, 1)$ . Thus  $\mathbf{A}_N \coloneqq \mathbf{I}_N + \mathbf{G}_N$  is invertible, and the signs of its entries are as in Lemma 3.

W.l.o.g. let i = 1. Under the assumption that player k attends to source l, we

must have

$$x_{k}^{l} = \tau_{k} - \sum_{j=1}^{N} \frac{1}{\lambda_{j}} \log \left[ (\lambda_{j} - 1)(\exp(x_{j}^{l}) - 1) \right].$$
(17)

Differentiating both sides of Equation (17) with respect to  $\tau_1$  yields

$$\frac{\partial x_k^l}{\partial \tau_1} = \sum_{j=1}^N -g_j \frac{\partial x_j^l}{\partial \tau_1},$$

and simplifying this result using  $\nabla_{\tau_1} [x_1^l \cdots x_N^l]^\top = \mathbf{A}_N^{-1} [1 \ 0 \cdots 0]^\top$  (as shown in the proof of Theorem 3) and doing a lot of algebra yield

$$\frac{\partial x_k^l}{\partial \tau_1} = (1 - g_1) \frac{\partial x_1^l}{\partial \tau_1} - 1.$$

Then from  $\partial x_1^l / \partial \tau_1 > 0$  (Theorem 3), it follows that  $\operatorname{sgn}(\partial x_k^l / \partial \tau_1) = \operatorname{sgn}(-\partial x_1^l / \partial \tau_1)$ if and only if  $\partial x_1^l / \partial \tau_1 < 1/(1-g_1)$ . To establish the last inequality, recall that  $\partial x_1^l / \partial \tau_1 = [\mathbf{A}_N^{-1}]_{1,1}$ , whose expression is given by Equation (4). Tedious but straightforward algebra shows that

$$\left[\mathbf{A}_{N}^{-1}\right]_{1,1} - \frac{1}{1-g_{1}} = \frac{-g_{1}}{\det\left(\mathbf{A}_{N}\right)\left(1-g_{1}\right)} \prod_{j=2}^{N} \left(1-g_{j}\right) < 0,$$

which completes the proof.

Meanwhile, differentiating both sides of Equation (17) with respect to  $\lambda_1$  yields

$$\frac{\partial x_k^l}{\partial \lambda_1} = -\sum_{j=1}^N g_j \frac{\partial x_j^l}{\partial \lambda_1} + \kappa$$

where  $\kappa = -h_{\lambda}(x_1^l; \lambda_1)$ . Simplifying this result using  $\nabla_{\lambda_1}[x_1^l \cdots x_N^l]^{\top} = \kappa \mathbf{A}_N^{-1}[0 \ 1 \ \cdots \ 1]^{\top}$ 

(as shown in the proof of Theorem 3) and doing a lot of algebra yield

$$\begin{aligned} \frac{\partial x_k^l}{\partial \lambda_1} &= (1 - g_1) \frac{\partial x_1^l}{\partial \lambda_1} + \kappa \\ &= \kappa \left( (1 - g_1) \sum_{j=2}^N \left[ \mathbf{A}_N^{-1} \right]_{1,j} + 1 \right) \\ &= \frac{\kappa}{\det \left( \mathbf{A}_N \right)} \prod_{j=2}^N \left( 1 - g_j \right). \end{aligned}$$

Then from sgn  $(\partial x_1^l / \partial \lambda_1) = \text{sgn}(-\kappa)$  (as shown in the proof of Theorem 3), it follows that sgn  $(\partial x_k^l / \partial \lambda_1) = \text{sgn}(-\partial x_1^l / \partial \lambda_1)$ , which completes the proof.

**Proof of Theorem 8** In the setting laid out in Appendix B.6, the set of the parties that any player *i* can attend to is  $C_i = \{1\text{-rev}, \dots, m\text{-rev}\} \cup \mathcal{I} - \{i\}$ , and the set  $\mathcal{X}_i$ of feasible attention strategies for him must be modified accordingly. When  $\beta$  is sufficiently small, we note, for the reason given in the proof of Theorem 1, that all players must take their default actions in case they remain uninformed of the state at Stage 4 of the game. Given this, we can formulate the Stage 1-problem faced by any type-*m* player as follows:

$$\max_{x_i \in \mathcal{X}_i} -\frac{\beta}{M} \sum_{\omega \neq m} \delta_i^{\omega - \text{revealing}} \prod_{j \in \mathcal{I} - \{i\}} \left( \delta_j^{\omega - \text{revealing}} + \left( 1 - \delta_j^{\omega - \text{revealing}} \right) \delta_i^j \right).$$

Using the parameterization specified in Appendix B.6 and solving, we obtain  $\Delta^* = 0$ ,  $(M-1)x^* + (N-1)y^* + (M-1)Nz^* = \tau$ ,  $y^* = g_1(x^*)$  and  $z^* = g_2(x^*)$ , where

$$g_1(x) \coloneqq \frac{1}{\lambda} \log \max \left\{ (\lambda (M-1) - 1) (\exp (x) - 1), 1 \right\}$$
  
and 
$$g_2(x) \coloneqq \frac{1}{\lambda} \log \max \left\{ (\lambda (M-2) - 1) (\exp (x) - 1), 1 \right\}.$$

Thus  $x^*$  is the unique fixed point of  $\frac{1}{M-1}[\tau - (N-1)g_1(x) - (M-1)Ng_2(x)]$ , and the following are equivalent: (i)  $y^* > 0$ ; (ii)  $y^* > z^*$ ; (iii)  $\lambda > 1/(1-M)$  and  $\tau > (M-1)\phi(\lambda(M-1))$ .

# C Figures



Figure 1: A and B represent equilibrium  $(x_1^l, x_2^l)$ 's when  $\lambda_1 = 1.5$  and 2, respectively:  $\lambda_2 = 1.5, \tau_1 = \tau_2 = 3.$ 



Figure 2: A and B represent equilibrium  $(x_1^l, x_2^l)$ 's when  $\lambda_1 = 1.5$  and 4, respectively:  $\lambda_2 = 1.5, \tau_1 = \tau_2 = 3.$ 

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