Market Supply with Variable Participants: An Analysis of the U.S. Cement Industry from 1993-1998*

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Abstract

We characterize the necessary and sufficient conditions of profit maximization for aggregate market behavior when participants on the supply side vary and individual firm supply is unobservable. Our approach is non-parametric. We use the conditions on market behavior to examine whether the United States cement industry could have been profit maximizing between 1993 and 1998. We find that the U.S. cement industry always rejects profit maximization when firms are required to make weakly positive profits. We also provide a measure of necessary profit loss to measure how far the industry is from profit maximization. We find errors from profit maximization that are as large as $755.1 million.

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1 Introduction

The theory of competitive markets requires individual firms to maximize profits. However, due to data limitations, it is often difficult or impossible to evaluate whether individual firms actually *are* profit maximizing. For example, data on individual firm behavior are often proprietary. On the other hand, market level data on the participants in an industry and the aggregate net outputs are often obtainable. Responding to data limitations, we study necessary and sufficient conditions for market supply to be generated by profit maximizing individual firms. We assume that each observation consists of:

1. The observed prices of production net outputs,

2. The observed market supply (aggregate production),

3. The firms participating in the market.

It is well known that supply at the economy level exhibits the same behavior as individual supply, due to the fact that profit maximization aggregates.[1] This would seem to imply that testing profit maximization at the economy level is a trivial exercise, since there are well-known tests of profit maximization for individual firms (see Afriat (1972); Hanoch and Rothschild (1972); Diewert and Parkan (1983); Varian (1984)); at least, if we had no data on market participants. However, when the set of participants changes, the aggregate technology also necessarily changes. When a firm goes bankrupt, or exits the market, their production technology "disappears" from the marketplace. So, in this context, we would not expect economy-level production to mimic the behavior of an individual producer.

Our innovation is to take seriously the fact that individual firms participating in the market may vary from period to period. Were individual production observable, the well-known weak axiom of profit maximization (WAPM) tests

[1] This is in stark contrast to individual demand, whereby aggregate demand is known to carry very few of the properties that individual demand enjoys. See Sonnenschein (1972); Mantel (1974); Debreu et al. (1974). Brown and Matzkin (1996) use "rich" data and show how to set up some empirical implications of market demand.

whether individual firms profit maximize. It is an easy test, only asking that no observed production bundle generate higher profits at given prices than the observed production at those prices. A difficulty in our case is obviously that individual firm production is unobservable, and must be somehow *inferred*. In this paper, we provide necessary and sufficient conditions for when it is possible that *there exist* hypothetical firm production levels which are consistent with profit maximization and aggregate to the observed market supply.

We introduce a condition which is closely related to the Weak Axiom of Profit Maximization (WAPM), and indeed; if the set of market participants is identical in every period, it coincides with WAPM. The conditions for market supply differ from WAPM because we have no method of directly observing individual production. Instead, we only know which firms were operating in a given period. Roughly, the test asserts that if market supply is profit maximizing, then there could not exist an arbitrager who could profit from buying and selling the market supply as prices vary in the dataset. The difference from a test of WAPM is that the arbitrager must have been able to make these trades with *any* firm that is present in the market.

We also provide additional tests for alternative hypotheses. For example, we examine the additional restrictions imposed when there are known constraints on the production technology (such as those requiring outputs be nonnegative) and profit is required to be non-negative (which is the same as requiring that firms can do nothing). Further, with a dynamic structure, we propose a test which allows for the possibility that technology of market participants increases throughout time. In the case of increasing technology, the condition that characterizes market profit maximization rules out the existence of an arbitrager who can only sell at prices at some later period. This differs from the case of a static technology which rules out the existence of an arbitrager who makes profits selling at any set of the observed prices.

This paper also develops a notion of *necessary profit loss* to quantify the size of violations of profit maximization within the market. We consider two measures: One for the market as a whole and one for firms. The first measure computes the minimum profit an arbitrager would make by trading a unit of the market supply. The measure for firms gives the smallest weighted profit an arbitrager could make by selling the market supply to a single firm. Both measures of necessary profit loss essentially quantify how large the deviation of profits are expected to be for *any* set of production technologies used to estimate firm supply in the market.

Lastly, we show that these conditions *can* and *do* refute profit maximization in practice. In the empirical application, we examine whether the United States cement industry could have been profit maximizing between 1993 and 1998 using yearly data on net-outputs, average yearly prices, and firm participants. The United States cement industry is a concentrated industry and there is a large amount of regional heterogeneity in prices and demand. Thus, *a priori* we hypothesize that the cement market in the United States is a likely candidate to refute profit maximization using market supply data. We confirm this hypothesis in the empirical section. We find from 1993-1998 that the market necessary profit loss is $755.1 million for conditions that most closely resemble the cement market.[2] In particular, this occurs for a test where technology is static, firms make non-negative profits, and there are restrictions on what is an input/output.[3]

It is interesting to compare the magnitude of these errors with those found in a related meta-analysis. In particular, Ryan (2012) examines how welfare estimates change by estimating a structural model with and without dynamic components. Ryan (2012) finds that a static model of the cement industry would underestimate welfare costs by at least $300 million. Although we do not know the sign of the errors in aggregate profit maximization, we show that the errors from using aggregate data are at least $755.1 million. This suggests that it is important to use information on regional competition and heterogeneity when it exists. Moreover, one should be cautious when interpreting estimates assuming profit maximization behavior for aggregate data when there is regional competition or pricing heterogeneity. In particular, the errors

[2]Within the paper, we use 1996 dollars for comparison to the work of Ryan (2012).

[3]Surprisingly, we find that if we allow firm technology to weakly increase over time, then the data can be rationalized by profit maximization. However, for the time period we observe there is very little technological change as we discuss in Section 5.2.

that result by failing to incorporate regional competition and heterogeneity for the U.S. cement industry are more than twice as large as the discrepeancies between static and dynamic models.

While we are the first to consider the revealed preference conditions for market supply datasets, similar conditions have been studied in models of household behavior. For example the work of Cherchye, De Rock, and Vermeulen (2007), Cherchye, De Rock, and Vermeulen (2009), and Cherchye, De Rock, and Vermeulen (2011) study models when household consumption is observed, but the individual consumption within the household is unobserved. This is similar to the case of market supply, except that one must now account for how households aggregate their preferences. This paper is also related to work examining the testable restrictions on market behavior. For example, Carvajal, Deb, Fenske, and Quah (2013) and Carvajal, Deb, Fenske, and Quah (2014) study testable restrictions for models of oligopolistic behavior. To the best of our knowledge, this paper is one of the first to examine conditions where the set of participants change in revealed preference tests. In contrast, there are statistical models that study how market participants change allowing entry and exit following the work of Tamer (2003).

The remainder of the paper proceeds as follows. Section 2 defines the model market profit maximization, characterizes the model, and provides intuition and examples for the result. Section 3 provides characterizations when there is *a priori* knowledge on the technology of firms, firms are required to make non-negative profits, and when the technology is increasing. Section 4 defines a measure of necessary profit loss that measures in dollars how large errors of profit maximization would be for *any* technology. Section 5 outlines the empirical analysis on the cement industry and provides the results. Section 6 contains our final remarks.

2 Description of Model and Main Result

We consider a model of profit maximization with market level data. Let there be a finite set of commodities given by $k \in \{1, \ldots, K\}$. There is also a finite set of *potential firms*, indexed by F. A *market supply observation* is a triple consisting of $\langle P, \pi, y \rangle$, where $P \subseteq F$ and $P \neq \varnothing$ comprises the participants in the market, $\pi \in \Re^K_+$ lists the *market prices*, and $y \in \Re^K$ gives the *net outputs* in the market. We will sometimes refer to y as the *market supply.* A *market supply dataset* is a finite collection of market supply observations, $\{\langle P^j, \pi^j, y^j \rangle\}_{j=1}^J$.

In this paper, we examine whether a market supply dataset could have been generated by profit maximizing firms. For a market supply dataset and a firm $f \in F$, we define the set of observations in which firm f is a participant as $\mathcal{O}^f = \{j : f \in P^j\}$. This object is important since a firm will only have a chance to violate profit maximization when it participates in the market.

Definition 1. *We say a market supply data set is* profit rationalizable *if for every $f \in F$, there is a production possibility set $Y_f \subseteq \Re^k$ such that firm f is profit maximizing for each $j \in \mathcal{O}^f$, so there is $y^j_f \in Y_f$ where*

$$\pi^j \cdot y^j_f \in \arg\max_{y \in Y_f} \pi^j \cdot y$$

and the sum of net outputs accross all firms equals the market supply so for all $j \in \{1, \ldots, J\}$

$$\sum_{f \in P^j} y^j_f = y^j.$$

Below, we provide a characterization of the benchmark model of profit rationalizability since it is easiest to understand. Later in the paper, we provide characterizations when there are constraints on the technology, profits are non-negative, and technology can be weakly increasing. The condition for the benchmark model of profit rationalizability requires that there is now way to shift industry production across firms in a way that violates a “no arbitrage” condition. In particular, the statement of the result gives a condition on transition matrices over the periods each firm participates. A *transition matrix* is a nonnegative matrix whose rows sum to 1; for example, $\Lambda \in \Re^n \times \Re^n$ is a transition matrix if for all i, $\sum_l \Lambda_{i,l} = 1$. Below is the formal statement of the main result.

Theorem 1. *A market supply dataset is profit rationalizable if and only if for every $(f, j) \in F \times J$, every $\mu^j \in \Re^K$, and every transition matrix $\Lambda(f) \in \Re^{\mathcal{O}^f \times \mathcal{O}^f}$, if for every $j \in J$ and every $f \in P^j$,*

$$\sum_{\ell \in \mathcal{O}^f} \Lambda(f)_{l,j} \pi^l = \pi^j + \mu^j, \tag{1}$$

then

$$\sum_{j=1}^{J} \mu^j \cdot y^j \leq 0.$$

The result in Theorem 1 is essentially a "no arbitrage" condition. To see this intuitively, suppose there is some firm, $\tilde{f}$, that is present in every period, so that $\mathcal{O}^{\tilde{f}} = \{1, \ldots, J\}$. Moreover, consider the existence of an arbitrager in the market who is trying to make a profit by buying y^j for prices in the j-th period and selling $\Lambda(f)_{\ell,j}$ percent of y^j in the ℓ-th period. The arbitrager can attempt to do this in every period, but will only make a profit when

$$\sum_{j=1}^{J} \sum_{\ell=1}^{J} \Lambda(f)_{\ell,j} \left(\pi^\ell - \pi^j\right) \cdot y^j > 0.$$

The ability of an arbitrager to make a profit is prevented by Theorem 1 by plugging in the expression of μ^j from Equation 1.

While the intuition above is for a single firm, a similar intuition follows for the general problem. The difference is it must be possible to carry out the arbitrage for *any* firm in the dataset. To see this, note that Equation 1 says that the difference in trades depends only on the date, j, the trade takes place, but it does not depend on the firm. Thus, the arbitrager must have been able to make the same series of trades with any firm and still be able to make a profit.

To better understand the result of Theorem 1 in practice, we also consider some structured examples.

Example 1. If there is only one firm that participates in every period, then Theorem 1 implies the weak axiom of profit maximization. Suppose there is exactly one firm, f_1, that participates in every period. For observations $\ell, j \in \{1, \dots, J\}$ with $\ell \neq j$, let $\Lambda(f_1)_{j,\ell} = 1$ and for all $m \neq j$ let $\Lambda(f_1)_{m,m} = 1$. This choice of transition matrix implies that

$$\pi^j \cdot y^\ell \leq \pi^j \cdot y^j.$$

Any other statement of the weak axiom of profit maximization follows through choosing an appropriate transition matrix. Therefore, this condition is equivalent to the standard profit maximization condition.

Example 2. Suppose there are two firms f_1 and f_2 where $P^1 = \{f_1\}$ and $P^2 = \{f_2\}$. Note here both conditions are trivially satisfied since Λ matrix is a one for each individual. This is essentially the case of one observation per firm. Even if $P^2 = \{f_1, f_2\}$, there are no restrictions on supply data since market supply can act as though f_1 does not participate in P^2.

Example 3. We now consider a novel application of Theorem 1. Suppose we see two firms produce separately and a researcher wants to examine if they are profit maximizing when they both participate in the market. Let $F = \{f_1, f_2\}$ and suppose there are three observations, with $P^1 = \{f_1\}$, $P^2 = \{f_2\}$, and $P^3 = \{f_1, f_2\}$. In the following, cone$Y$ indicates the convex cone spanned by a set Y.

Corollary 1. *Suppose that π^1, π^2, π^3 are distinct vectors with the property that cone$\{\pi^1, \pi^3\} \cap$ cone$\{\pi^2, \pi^3\}$ = cone$\{\pi^3\}$. Then the market supply dataset is profit rationalizable if and only if $\pi^3 \cdot (y^1 + y^2) \leq \pi^3 \cdot y^3$.*

Proof. Because of the cone condition, the only transition matrices for which for each $i = 1, 2$, $\sum_{l \in \mathcal{O}^{f_i}} \Lambda(f_i)_{l,3} \pi^l = \pi^3 + \mu^3$ are those for which $\Lambda(f_i)_{l,3} = 0$ when $l \neq 3$. To see this, suppose false, so that $\Lambda(f_1)_{1,3}\pi^1 + \Lambda(f_1)_{3,3}\pi^3 = \pi^3 + \mu^3 = \Lambda(f_2)_{2,3}\pi^2 + \Lambda(f_2)_{3,3}\pi^3$. Suppose without loss that $\Lambda(f_2)_{3,3} \geq \Lambda(f_1)_{3,3}$ and observe that then $\Lambda(f_1)_{1,3}\pi^1 = \Lambda(f_2)_{2,3}\pi^2 + (\Lambda(f_2)_{3,3} - \Lambda(f_1)_{3,3})\pi^3$, demonstrating that $\Lambda(f_1)_{1,3}\pi^1$ is in the convex cone spanned by π^2 and π^3, from which we conclude by hypothesis that $\Lambda(f_1)_{1,3} = 0$. Similarly then, as π^2 and π^3 are distinct, it follows that $\Lambda(f_2)_{2,3} = 0$ and that $\Lambda(f_2)_{3,3} = \Lambda(f_1)_{3,3}$.

Let us refer to this number, $\Lambda(f_1)_{3,3}$ as α, where obviously $\alpha \in [0,1]$.

Hence, for any such matrix, the constraint reads: $(1-\alpha)\pi^3 \cdot y^1 + (1-\alpha)\pi^3 \cdot y^2 - (1-\alpha)\pi^3 \cdot y^3 \leq 0$. □

This simple result can be generalized.[4] Suppose that there are n firms, and $n+1$ observations. Suppose that for $i = 1, \ldots, n$, $P^i = \{f_i\}$ and that $P^n = \{f_1, \ldots, f_n\}$. If each of $\pi^1, \ldots, \pi^{n+1}$ are distinct, and there is a pair of firms, i, j, for which $\text{cone}\{\pi^i, \pi^{n+1}\} \cap \text{cone}\{\pi^j, \pi^{n+1}\} = \text{cone}\{\pi^{n+1}\}$, then the only implication of the model is that $\pi^{n+1} \cdot (\sum_{i=1}^n y^i) \leq \pi^{n+1} \cdot y^{n+1}$.

Of course, Corollary 1 fails absent its hypotheses. Here, we present some simple examples. As a first trivial example, suppose that $\pi^1 = \pi^2$. Suppose data are profit rationalizable. Then for any $y^3_{f_1}, y^3_{f_2}$ for which $y^3_{f_1} + y^3_{f_2} = y^3$ and which rationalize the observations, we must have $\pi^1 \cdot y^1 \geq \pi^1 \cdot y^3_{f_1}$ and $\pi^2 \cdot y^2 \geq \pi^2 \cdot y^3_{f_2}$; but since $\pi^1 = \pi^2$, we obtain the requirement that $\pi^1 \cdot (y^1 + y^2) \geq \pi^1 \cdot y^3$.

This example is generated by the transition matrices:

$$\Lambda(f_1) = \begin{array}{c c} & \begin{array}{cc} 1 & 3 \end{array} \\ \begin{array}{c} 1 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \end{array}$$

and

$$\Lambda(f_2) = \begin{array}{c c} & \begin{array}{cc} 2 & 3 \end{array} \\ \begin{array}{c} 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \end{array}.$$

A more complicated example is as follows. Suppose, for example, that $\pi^2 + \pi^3 = \pi^1$. Up to normalization, this means that π^1 lies in the relative interior of the convex cone spanned by π^2 and π^3. This is the basic case where the hypotheses of Corollary 1 fail.

Consider the following transition matrices:

[4] The conditions for this result are similar to the conditions in Cherchye, Demuynck, and De Rock (2018) that characterize what datasets allow GARP to differ from WARP.

$$\Lambda(f_1) = \begin{array}{c c} & \begin{array}{cc} 1 & 3 \end{array} \\ \begin{array}{c} 1 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \end{array}$$

and

$$\Lambda(f_2) = \begin{array}{c c} & \begin{array}{cc} 2 & 3 \end{array} \\ \begin{array}{c} 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \end{array}.$$

In this case, the antecedent condition of Theorem 1 is satisfied with $\mu^1 = -\pi^2$, $\mu^2 = -\pi^2$, and $\mu^3 = \pi^2$. Therefore, Theorem 1 implies that

$$\pi^2 \cdot y^3 \leq \pi^2 \cdot (y^1 + y^2).$$

We can work out why this inequality is a necessary implication of profit rationalizability. If $y^3_{f_1} + y^3_{f_2} = y^3$ and is chosen to rationalize the data, then by profit maximization, the following three inequalities must hold:

$$\begin{aligned} \pi^1 \cdot (y^3_{f_1} - y^1) &\leq 0 \\ \pi^3 \cdot (y^1 - y^3_{f_1}) &\leq 0 \\ \pi^2 \cdot (y^3_{f_2} - y^2) &\leq 0. \end{aligned}$$

Summing these inequalities, and substituting $\pi^1 = \pi^2 + \pi^3$ and $y^3 = y^3_{f_1} + y^3_{f_2}$ yields the desired result.

Example 4. Finally, let us construct one more example, where there are four observations: $F = \{f_1, f_2\}$ and $P^1 = \{f_1\}$, $P^2 = \{f_1\}$, $P^3 = \{f_2\}$, $P^4 = \{f_1, f_2\}$.

Suppose now that $\pi^1 + \pi^2 = \pi^3$. Consider the transition matrices:

$$\Lambda(f_1) = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \end{array} & \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

and

$$\Lambda(f_2) = \begin{array}{c c} & \begin{array}{cc} 3 & 4 \end{array} \\ \begin{array}{c} 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \end{array}.$$

Observe that these transition matrices imply, by virtue of Theorem 1, that

$$(\pi^1 + \pi^2) \cdot y^4 \leq \pi^1 \cdot y^1 + \pi^2 \cdot y^2 + \pi^3 \cdot y^3.$$

Similarly, for the transition matrices:

$$\Lambda(f_1) = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \end{array} & \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

and

$$\Lambda(f_2) = \begin{array}{c c} & \begin{array}{cc} 3 & 4 \end{array} \\ \begin{array}{c} 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \end{array}$$

we obtain the necessary inequality

$$\pi^4 \cdot y^1 + \pi^1 \cdot y^4 + \pi^2 \cdot y^4 + \pi^4 \cdot y^3 \leq \pi^1 \cdot y^1 + \pi^2 \cdot y^2 + \pi^3 \cdot y^3 + \pi^4 \cdot y^4.$$

Thus, the transition matrix places restrictions on the data that differ from the traditional weak axiom of profit maximization.

3 Additional Structure on Technology

We are often interested in imposing additional structure on the notion of profit rationalizability. Three further conditions follow. Here, $\overline{\Re}$ denotes the extended real numbers (that is, the real numbers together with $-\infty$ and ∞).

- *Firm constraints for $a, b \in \overline{\Re}^K$, $a \leq b$:*We ask that for all $f \in F$ and all

$j \in \mathcal{O}^f$, $y_f^j \in [a,b]$.[5]

- *Nonnegative profits:* We ask that for all $f \in F$, all $j \in \mathcal{O}^f$, $\pi^j \cdot y_f^j \geq 0$.
- *Technological Change:* We as that for all f and $\ell < j$ that technologies satisfy $Y_f^\ell \subseteq Y_f^j$.

The firm constraints condition allows us to impose exogenous technological constraints on inputs and outputs. For example, the definition of profit rationalizability allows for the possibility that an output could be produced in negative amounts. To eliminate this possibility, we simply ask that outputs are nonnegative. Similarly, we would ask that inputs be nonpositive.

The nonnegative profits constraint is equivalent to allowing each firm the possibility of doing nothing. In other words, it requires $0 \in Y^f$ or that the firm can choose to costlessly do nothing. See Corollary 6.5 of Chambers and Echenique (2016). Lastly, the technological change condition treats the market supply dataset similar to a time series and imposes that there are technological innovations.

3.1 Firm Constrained Production

Within a given industry, one may want to put constraints on which goods are produced and used as inputs. In our application, we consider profit maximization of the United States cement industry. Within the cement industry most firms produce cement, and use raw materials, energy, and labor to create cement. Therefore, we need conditions that hold when the inputs and outputs are known *a priori* for the firms.

The next result characterizes profit maximization with constraints on net outputs. These conditions strengthen those of Theorem 1. These new terms effectively increase the set of possible transition matrices that need to be checked for the inequality that generalizes the weak axiom of profit maximization. Therefore, this places additional requirements on profit maximization. The formal statement of the result is below. For a vector $x \in \overline{R}^K$, we denote by

[5]Here, $[a,b] \equiv \{x \in \Re^K : a_k \leq x_k \leq b_k\}$

$S_x \equiv \{k \in K : x_k \in \Re\}$; that is, S_x denotes the coordinates for which x is real-valued.

Theorem 2. *A market supply dataset is profit rationalizable and satisfies firm constraints for $a \leq b$ if and only if for every $(f, j) \in F \times J$, for every $\mu^j \in \Re^K$, for every $\alpha_f^j \in \mathbb{R}_+^{S_a}$ and $\beta_f^j \in \mathbb{R}_+^{S_b}$, and every transition matrix $\Lambda(f) \in \Re^{\mathcal{O}^f \times \mathcal{O}^f}$, if for every $j \in J$ and every $f \in P^j$,*

$$\sum_{\ell \in \mathcal{O}^f} \Lambda(f)_{l,j} \pi^l = \pi^j + \mu^j + (\alpha_f^j, 0_{-S_a}) - (\beta_f^j, 0_{-S_b}),$$

then

$$\sum_{j \in J} \mu^j \cdot y^j + \sum_{j \in J} \sum_{f \in P^j} \alpha_f^j \cdot a_{S_a} - \sum_{j \in J} \sum_{f \in P^j} \beta_f^j \cdot b_{S_b} \leq 0.$$

Of course, in Theorem 2, we could impose firm-specific constraints. A careful reader may go through the proof and observe exactly how the algebraic manipulations would change. For our purposes, it would only serve to complicate the equations. Theorem 2 differs from Theorem 1 by the presence of α_f^j and β_f^j. The addition of the α and β terms to the equation in Theorem 2 increases the set of transition matrices that need to be verified over in the inequality.

An important application of Theorem 2 is the case in which we have knowledge that some commodities operate as outputs, and some as inputs. In this case, an input would naturally be restricted to have $a_k = -\infty$ and $b_k = 0$, and an output would naturally be restricted to have $a_k = 0$ and $b_k = +\infty$. For example in our main application, it is reasonable to assume that cement is an output, rather than an input. On the other hand, it is reasonable to assume labor is an input, rather than an output.

Let us label the inputs as $\mathrm{IN} \subseteq \{1, \ldots, K\}$ and the outputs as $\mathrm{OUT} \subseteq \{1, \ldots, K\}$; for convenience suppose that $\mathrm{IN} \cap \mathrm{OUT} = \varnothing$. We say that the data set is rationalizable with constraints induced by $(\mathrm{IN}, \mathrm{OUT})$ if it is rationalizable with constraints such that for every $k \in \mathrm{IN}$, $a_k = -\infty$ and $b_k = 0$, for every $k \in \mathrm{OUT}$, $a_k = 0$ and $b_k = +\infty$, and for every $k \notin \mathrm{IN} \cup \mathrm{OUT}$, $a_k = -\infty$ and $b_k = +\infty$. For the vector $x \in \Re^K$ and a set $S \subseteq \{1, \ldots, K\}$, we use the notation $x|_S$ to be all entries of x for dimensions in the set S. We have the following immediate corollary of Theorem 2.

Corollary 2. *A market supply dataset is profit rationalizable and satisfies firm constraints induced by (IN, OUT) if and only if for every $(f, j) \in F \times J$, for every $\mu^j \in \Re^K$ and every transition matrix $\Lambda(f) \in \Re^{\mathcal{O}^f \times \mathcal{O}^f}$, if for every $j \in J$ and every $f \in P^j$,*

$$(\sum_{\ell \in \mathcal{O}^f} \Lambda(f)_{l,j} \pi^l)|_{IN} \geq (\pi^j + \mu^j)|_{IN}$$

$$(\sum_{\ell \in \mathcal{O}^f} \Lambda(f)_{l,j} \pi^l)|_{OUT} \leq (\pi^j + \mu^j)|_{OUT}$$

$$(\sum_{\ell \in \mathcal{O}^f} \Lambda(f)_{l,j} \pi^l)|_{\{1,\ldots,K\} \setminus IN \cup OUT} = (\pi^j + \mu^j)|_{\{1,\ldots,K\} \setminus IN \cup OUT}$$

then

$$\sum_{j \in J} \mu^j \cdot y^j \leq 0.$$

3.2 Non-negative profits

Another sensible restriction to place on firms is that they have non-negative profit. This condition is identical to allowing a firm to produce nothing in any given observation. In practice, this may also help place enough structure on the technology to align with intuition. In the case of examining the U.S. cement industry, non-negative profits will help insure that the technology of each firm present in the market produces some cement.

Definition 2. *We say a market supply data set is* non-negative profit rationalizable *when there is a production possibility set $Y_f \subseteq \Re^K$ such that firm f is profit maximizing for each $j \in \mathcal{O}^f$ with non-negative profits, so there is $y^j_f \in Y_f$ where*

$$\pi^j \cdot y^j_f \in \arg\max_{y \in Y_f} \pi^j \cdot y \quad \textit{and} \quad \pi^j \cdot y^j_f \geq 0$$

and the sum of net outputs accross all firms equals the market supply so for *all* $j \in \{1, \ldots, J\}$

$$\sum_{f \in P^j} y_f^j = y^j.$$

As in the other cases, there is a simple dual characterization for non-negative profit rationalization. The result bellow differs from Theorem 1 and Theorem 2. The main difference is that each the constraint on transition matrices can now be any scale of π^j where the scale can vary for each firm. Intuitively, this means that an arbitrager could now purchase more than one unit of the market net output in an attempt to make a profit from some series of trades.

Theorem 3. *A market supply dataset* $\{\langle P^j, \pi^j, y^j \rangle\}_{j=1}^J$ *is non-negative profit rationalizable if and only if for every* $(f, j) \in F \times J$*, for every* $\mu^j \in \Re^K$*, for every* $\gamma_f^j \geq 1$*, and every transition matrix* $\Lambda(f) \in \Re^{\mathcal{O}^f \times \mathcal{O}^f}$*, if for every* $j \in J$ *and every* $f \in P^j$*,*

$$\sum_{\ell \in \mathcal{O}^f} \Lambda(f)_{l,j} \pi^l = \gamma_f^j \pi^j + \mu^j,$$

then

$$\sum_{j \in J} \mu^j \cdot y^j \leq 0.$$

3.3 Technological Change

Lastly, we allow for unobserved technological change at each firm. As for the case of non-negative profit maximization, there is no change in the dataset required for testing. However, this will restrict what entries of the transition matrix $\Lambda(f) \in \Re^{\mathcal{O}^f \times \mathcal{O}^f}$ can be non-zero. For the following definition, we allow each firm to have some unobserved technological shift at each time period. Therefore, let $\mathcal{O}^f$ be considered as an ordered object so $\ell, j \in \mathcal{O}^f$ with $\ell < j$ can be read as observation ℓ occurs before observation j.

Definition 3. *We say a market supply data set is* profit rationalizable with technological schocks *when there exist production possibility sets* $Y_f^j \subseteq \Re^K$ *such that firm* f *is profit maximizing for each* $j \in \mathcal{O}^f$*, so there is* $y_f^j \in Y_f^j$

where

$$\pi^j \cdot y_f^j \in \arg\max_{y \in Y_f^j} \pi^j \cdot y,$$

the sum of net outputs accross all firms equals the market supply so for all $j \in \{1, \ldots, J\}$

$$\sum_{f \in P^j} y_f^j = y^j,$$

and technology is increasing so that for all $\ell, j \in \mathcal{O}^f$ *with* $\ell < j$ *then* $Y_f^\ell \subseteq Y_f^j$.

This requires that the firm technology is improving with each time period. The result below of Theorem 4 now requires that the transition matrix is lower triangular. This means that for every firm $f \in F$ that if $j > \ell$ that $\Lambda_{\ell,j} = 0$. In terms of the story of an arbitrager, this condition requires that the arbitrager can only sell the market net output at periods in the future. Thus, this condition prevents the arbitrager from making sales for prices that occured in the past and is a weaker condition than Theorem 1.

Theorem 4. *A market supply dataset* $\{\langle P^j, \pi^j, y^j \rangle\}_{j=1}^J$ *is profit rationalizable with technolofical shocks if and only if for every* $(f, j) \in F \times J$, *for every* $\mu^j \in \Re^K$ *and every lower triangular transition matrix* $\Lambda(f) \in \Re^{\mathcal{O}^f \times \mathcal{O}^f}$, *if for every* $j \in J$ *and every* $f \in P^j$,

$$\sum_{\ell \in \mathcal{O}^f} \Lambda(f)_{l,j} \pi^l = \pi^j + \mu^j,$$

then

$$\sum_{j \in J} \mu^j \cdot y^j \leq 0.$$

4 Necessary Profit Loss

Before performing the empirical analysis, we develop the notion of an approximate profit maximizing firm and show how to use this to find measures of *necessary profit loss* for the market and firms. We define *necessary profit loss* to be the smallest amount of profit lost from imperfect optimization. This gives a measure of how far the market is from profit maximization in a similar spirit to efficiency indexes developed by Afriat (1967), Varian (1990), and Färe and Grosskopf (1995). The main difference of necessary profit loss from other efficiency measures is that necessary profit loss is an additive measure of imperfect optimization. Our focus on the minimal amount of necessary profit loss is primarily for convenience since it can be checked by linear programming and is easy to interpret. For example, any welfare measure that incorporates firm profits and assumes profit maximization will necessarily have errors that are at least as large as the necessary profit loss in the market.

Before defining an *approximate profit maximizer*, recall that a profit maximizing firm $f \in F$ with technology Y_f is profit maximizing at observation $j \in \mathcal{O}^f$ when there is a $y_f^j \in Y_f$ such that

$$\pi^j \cdot y_f \leq \pi^j \cdot y_f^j$$

for all $y \in Y_f$. An *approximate profit maximizer* is similar to the a standard profit maximizer, except we allow the firm to make profit maximization errors. Formally, the firm $f \in F$ with technology Y_f at observation $j \in \mathcal{O}^f$ is approximately profit maximizing at level $\varepsilon_f^j \in \Re_+$ when

$$\pi^j \cdot y \leq \pi^j \cdot y_f^j + \varepsilon_f^j$$

for all $y \in Y^f$. In other words, the firm may only approximately profit maximize with errors up to ε_f^j dollars. This is related to other work studying approximate maximizers in revealed preference theory (Dziewulski, 2018; Allen and Rehbeck, 2018), but we consider these ideas in the case of profit maximization. Note that the approximation error is both observation and firm specific.

We now use the approximation errors when firms profit maximize to definition a notion of *neccessary profit loss* (NPL). We consider NPL that depend on the whole market and those the depend on individual firms. The *necessary profit loss in the market* (m-NPL) will be the total amount of profit that is necessarily lost by all firms. The *firm level necessary profit loss* (f-NPL) is defined to be the smallest worst case profit loss of a firm in the market. These measures address how far the market and firms are respectively from the conditions of profit maximizers. We now give formal definitions of the m-NPL and f-NPL.

Definition 4. *The market necessary profit loss (m-NPL) of a market supply dataset* $\{\langle P^j, \pi^j, y^j \rangle\}_{j=1}^J$ *is defined as*

$$\begin{aligned} \min_{y_f^j \in \Re^K, \varepsilon_f^j \in \Re_+} & \sum_{f \in F} \sum_{j \in \mathcal{O}^f} \varepsilon_f^j \\ s.t.\ & \pi^j \cdot y_f^\ell \leq \pi^j \cdot y_f^j + \varepsilon_f^j \quad \forall f \in F \ and\, \forall j, \ell \in \mathcal{O}^f \\ & \sum_{f \in P^j} y_f^j = y^j \quad \forall j \in J \end{aligned}$$

Definition 5. *The firm necessary profit loss (f-NPL) of a market supply dataset* $\{\langle P^j, \pi^j, y^j \rangle\}_{j=1}^J$ *is defined as*

$$\begin{aligned} \min_{y_f^j \in \Re^K, \varepsilon_f^j \in \Re_+} & \max_{f \in F} \left\{ \sum_{j \in \mathcal{O}^f} \varepsilon_f^j \right\} \\ s.t.\ & \pi^j \cdot y_f^\ell \leq \pi^j \cdot y_f^j + \varepsilon_f^j \quad \forall f \in F \ and\, \forall j, \ell \in \mathcal{O}^f \\ & \sum_{f \in P^j} y_f^j = y^j \quad \forall j \in J \end{aligned}$$

The m-NPL and f-NPL are both zero when firms are profit maximizing and non-zero otherwise. One interpretation of these numbers is that the m-NPL gives a lower bound on errors for industry level welfare comparisons which include profit and use market data. Similarly the f-NPL provides a bound on errors for any welfare comparisons which include profits made by an individual firm. One can impose nonnegative profit maximization, increasing technology, and firm constraints when checking for either NPL by varying the constraints.[6]

Before proceeding, we give dual formulations of both the m-NPL and the

[6]This is easily incorporated since non-negative profits require that $\pi^j \cdot y_f^j \geq 0$. Similarly the constraints on inputs require $y_{f,k} \leq 0$ and outputs are constrained so $y_{f,k} \geq 0$.

f-NPL. In particular, the dual formulation of the m-NPL affords a meaningful use of the variables in Theorem 1.

Theorem 5. *The m-NPL is given by:*

$$\max_{\mu^j \in \Re^K} \sum_{j=1}^{J} \mu^j \cdot y^j$$

subject to

$$\sum_{\ell \in \mathcal{O}^f} \Lambda(f)_{l,j} \pi^l = \pi^j + \mu^j, \quad \forall j \in J \text{ and } \forall f \in P^j$$

where each $\Lambda(f) \in \Re^{\mathcal{O}^f \times \mathcal{O}^f}$ *is a transition matrix.*

In terms of interpretation, the m-NPL is the most profit an arbitrager can make from a series of trades as described after Theorem 1. Mathematically, the variables $\Lambda(f)$ and μ^j are dual variables (Lagrange multipliers). At optimal solutions to the original mNPL problem (say y^*, ε^*), or to the dual problem (say $\Lambda^*(f)$, μ^*), the standard complementary slackness conditions hold, so that for $f \in F$ and $j, l \in \mathcal{O}^f$ with $j \neq l$, $\Lambda(f)_{j,l} > 0$ only in the case the constraint binds; that is when the optimal y^* and ε^* has

$$\pi^j \cdot y_f^{j*} + \varepsilon_f^{j*} = \pi^j \cdot y_f^{l*}.$$

Further, $\Lambda(f)_{j,l}$ specifies the rate at which the m-NPL would decrease were we to allow a small violation of this particular profit maximization constraint. Thus, if $\Lambda(f)_{j,\ell} > 0$ and it were possible to increase production of outputs holding other inputs fixed, then the m-NPL would decrease. Likewise, the variable μ_k^j specifies the rate at which the m-NPL would decrease were we to decrease y_k^j. In particular, if $\mu_k^j < 0$, this means that decreasing y_k^j would actually *increase* the m-NPL, so that increasing y_k^j would decrease the m-NPL.

The next theorem gives the dual characterization of the f-NPL. Here, the notation $\beta \in \Delta(F)$ is a member of $\Re_+^F$ whose coordinates sum to one (so, a probability on F). Relative to the arbitrager interpretation after Theorem 1, this gives a particular weighted average of a cycle of trades across firms. In particular, this gives the lowest weighted average of profit that an arbitrager could earn.

Theorem 6. *The f-NPL is given by*

$$\max_{\mu^j \in \Re^K} \sum_{j=1}^{J} \mu^j \cdot y^j$$

subject to

$$\beta_f \sum_{\ell \in \mathcal{O}^f} \Lambda(f)_{l,j} \pi^l = \beta_f \pi^j + \mu^j, \quad \forall j \in J \text{ and } \forall f \in P^j$$

where each $\Lambda(f) \in \Re^{\mathcal{O}^f \times \mathcal{O}^f}$ *is a transition matrix and* $\beta \in \Delta(F)$.

5 Empirical Analysis

We use the tests developed in Section 2 to examine whether the cement industry in the United States between 1993 to 1998 is profit maximizing.[7] We chose to examine the cement industry because *a priori* we believe it is a good candidate to refute profit maximization. The cement industry is well known to be a concentrated industry and prices can vary depending on the region, thus one would not expect market level data to satisfy profit maximization.[8]

However, since the conditions for a market supply profit maximization place no restrictions on technology, it is possible that the U.S. cement industry is profit rationalized by some set of technologies. If we were able to find such a rationalization, then it would say that the conditions for profit maximization are weak and may not be informative in practice. Empirically, we find that the US cement industry from 1993-1998 is not profit maximizing with any technology when assuming firms make non-negative profits. This shows the

[7] Additionally we have data from 1980-1998. For data quality reasons, we focus on the time period from 1993-1998. Results for the full set of data are in Appendix C.

[8] For example, the largest four firms accounted for 32.5% of production in 1997 (Ryan, 2012). That prices vary by state can be seen looking at the cement entry of the United States Geological Survey Minerals Yearbook.

conditions are strong enough to refute profit maximization within an industry where violations are thought to occur. The rest of this section details these results and provides details on the size of violations from profit maximization using the necessary profit loss measure introduced in Section 4.

5.1 U.S. Cement Industry Overview

As mentioned before, the United States cement industry is a good candidate to refute profit maximization since it is a concentrated industry and the price can vary with the region. At a high level, the cement manufacturing process uses inputs of raw materials, energy, and labor to product cement as an output. We include information from all of these inputs in the main analysis. We treat cement as a homogeneous good as it has strict standards of production.[9] As a percentage of total mass, the main raw material is limestone (approximately 84%) while other materials make up the rest of the physical inputs.[10]

In addition to the cement industry being a good candidate to refute profit maximization, it is also of economic importance. The cement industry accounted for 1.3% of all U.S. anthropogenic carbon dioxide emissions in 2000 (Van Oss and Padovani, 2003). This fact has lead to the cement industry receiving wide attention when studying environmental policy (See Ryan (2012) and Fowlie, Reguant, and Ryan (2016)). This literature studies responses of cement production to changes in environmental policy using regional data since the market is concentrated and there is variation in prices across regions. To gain traction on these problems, the economic models often impose functional form restrictions on the production technology for each firm.[11] This paper complements the existing literature by showing that even without specifying

[9] For the study, we examine sales of all cement which includes both Portland and masonry cement. Both Portland and masonry cement have strict standards of production by ASTM International (International, 2018a,b).

[10] The percentage of total mass of limestone in the production of cement is derived from Table 3 in Van Oss and Padovani (2002). The interested reader can find additional details on the cement industry in Van Oss and Padovani (2002).

[11] The restrictions of Ryan (2012) and Fowlie, Reguant, and Ryan (2016) are on the cost function which effectively limits the technology of each firm.

structure on the technology at each firm, industry wide cement production is not profit maximizing.

We now discuss the data used to conduct the empirical analysis. We include information on output, raw material inputs, energy inputs, and labor inputs. The complete list of goods we include in the analysis is summarized in Table 1. We examine the cement industry using yearly aggregate data for the cement industry. The data on the amounts of inputs and outputs are readily available from the U.S. Mines Geological Yearbook and the Portland Cement Association. The U.S. Mines Geological Yearbook also contains information on the prices of cement and raw materials. The average yearly price of energy inputs was collected from the American Energy Review. We use average yearly manufacturing wages from the St. Louis Federal Reserve as the price of labor inputs. Lastly, we gathered information on the firms that participate in the cement industry from the Portland Cement Association. Additional details on data collection are collected in Appendix B.

Final Product	Raw Materials	Energy	Labor
Cement	Limestone	Coal	Hours Worked
	Marl	Oil	
	Clay/Shale	Natural Gas	
	Sand	Electricity	
	Iron Ore		
	Gypsum		

Table 1: Goods Included in Model

From Theorem 1, the main difference of a test of market level profit maximization is that firms may leave/enter the industry and their production is unobserved. We provide some descriptive details on firm entry between 1993-1998. For this time period, there are 118 different firms that participated in the cement industry. We display in Table 2 the number of firms that participate in the cement industry each year and how many entered/left the industry relative to the previous year. There is some entry/exit in the industry during this time period, but not much. Also, there are a large number of firms (118) relative to the number of time periods (6) so that one might expect the test to be weak.

Year	1993	1994	1995	1996	1997	1998
Number of Firms	115	115	117	115	115	115
Entering Firms	-	1	2	0	0	0
Exiting Firms	-	1	0	2	0	0

Table 2: Firm Participation from 1993-1998

5.2 Results

We examine a variety of different structural conditions when examining profit maximization. We examine both when there is a static technology and when technology improves. We examine each of these conditions with the restrictions of non-negative profits and restricting goods to be inputs/outputs. In particular cement is restricted to be an output while the other goods are inputs. The results on the m-NPL are presented in Table 3. We note that the weakest test of this model with improving technology is able to profit rationalize the model without restricting profits to be non-negative. However, the restriction of allowing *all* firms to have weakly improving technology every period is likely too weak. The reason this test is likely too weak is that the main technology used in the production of cement are large kilns to produce heat that facilitates the chemical reactions. During the time period from 1993-1998, only seven kilns were updated and 1/7 is present for the entire period from 1993-1998. Thus, we believe the static technology better represents the data.

	Unrestricted	Input/Output	Input/Output and Non-negative Profits
Static Technology	348.0	348.0	755.1
Improving Technology	0	0	109.0

Table 3: m-NPL in millions of 1996 dollars

For models that assume a static technology, the m-NPL is $755.1 million when one has restrictions on inputs/outputs and non-negative profit maximization. One interesting feature of the test is that the constraints on which goods are inputs and outputs do not affect the analysis. For some comparison on the magnitude of the m-NPL, the dynamic structural work of Ryan (2012) finds welfare errors of $300 million when comparing the results to a static structural model that incorporates regional pricing and competition. These values are not directly comparable since Ryan (2012) uses a structural model while the analysis here is non-parametric. However, the magnitude of error from assuming profit maximization of the industry is more than twice the size of the errors from dyanamic vs. static considerations. Since most welfare calculations include industry profit, this could have large effects on welfare comparisons if one assumes profit maximization at the aggregate when there is regional price variation and competition.

Next, we examine the f-NPL in Table 4. The f-NPL is substantially smaller than the m-NPL, which is expected as it is a measure for a single firm. Also, we note that the m-NPL is not too far from the number of firms times the f-NPL. This suggests that the best way to distribute profit maximizing errors is to give about the same amount of error to each firm. The error in profit maximization to a firm is $6.566 million.

	Unrestricted	Restricted	Restricted and Non-negative Profits
Static Technology	3.026	3.026	6.566
Improving Technology	0	0	0.948

Table 4: f-NPL in millions of 1996 dollars

6 Conclusion

In this paper, we show how to conduct a test of market profit maximization when a researcher has knowledge of market supply, market prices, and firm participation. Roughly the test examines whether there could be an arbitrager who could make a profit by selling a unit of the market supply to any firm in the industry at prices in the other periods. We extend the test to examine restrictions on technologies, non-negative profit maximization, and weakly increasing technologies. We then develop a notion of an approximate profit maximizer and necessary profit loss that measure how far the market and firms are from profit maximization in terms of optimization error. We use this test and measure to show that the U.S. Cement industry is not profit maximizing when assuming non-negative profits and that the necessary profit loss for the market is $755.1 million dollars for the conditions that most closely match the market (static technology, input/output, and non-negative profits).

Finally, a word on our notion of data. It is possible that, over time, firms merge or change names, or otherwise change structure. If we believe two firms consolidate to form a larger one, we can provide a test of the merger hypothesis (though we have not done so here) by assuming their two technologies are merged into the (setwise) sum of the individual technologies. But our test is specifically for given data, in terms of given firms. Our point is that one can investigate more general models (where firms change names or merge), but one has to specify these relationships in the data from the outset.

Appendix A Proofs

We first prove the result of Theorem 2. We note that Theorem 1 is a special case of Theorem 2 as the constraints could all be $a = -\infty$ and $b = \infty$.

Proof of Theorem 2. First, let us establish that a profit rationalizable market supply dataset with firm constraints a, b satisfies the condition. Let the profit rationalizable market supply dataset be given, and suppose that for each $j \in J$ and $f \in P^j$, $y_f^j \in \Re^K$ profit rationalizes the dataset and satisfies the constraints $a \leq y_f^j \leq b$.

So, for each $j \in J$, let $\mu^j \in \Re^k$ and for each $f \in F$, let $\Lambda(f) \in \Re^{\mathcal{O}^f \times \mathcal{O}^f}$

such that

$$\sum_{l \in P^f} \Lambda(f)_{l,j} \pi^l = \pi^j + \mu^j + ((\alpha^j_f)_{S_a}, 0_{-S_a}) - ((\beta^j_f)_{S_b}, 0_{-S_b}).$$

Observe that

$$\begin{aligned}
&\sum_{j=1}^{J} \mu^j \cdot y^j \\
=&\sum_{j=1}^{J} \mu^j \cdot (\sum_{f \in P^j} y^j_f) \\
=&\sum_{j=1}^{J} \sum_{f \in P^j} \mu^j \cdot y^j_f \\
=&\sum_{f=1}^{F} \sum_{j \in \mathcal{O}^f} \mu^j \cdot y^j_f \\
=&\sum_{f=1}^{F} \sum_{j \in \mathcal{O}^f} \left(\left(\sum_{l \in \mathcal{O}^f} \Lambda(f)_{l,j} \pi^l \right) - \pi^j - ((\alpha^j_f)_{S_a}, 0_{-S_a}) + ((\beta^j_f)_{S_b}, 0_{-S_b}) \right) \cdot y^j_f.
\end{aligned}$$

Observe then that this expression is the same as:

$$\begin{aligned}
&\sum_{f=1}^{F} \sum_{j \in \mathcal{O}^f} \sum_{l \in \mathcal{O}^f} \Lambda(f)_{l,j} \pi^l \cdot y^j_f - \sum_{f=1}^{F} \sum_{j \in \mathcal{O}^f} \left(\pi^j + ((\alpha^j_f)_{S_a}, 0_{-S_a}) - ((\beta^j_f)_{S_b}, 0_{-S_b}))\right) \cdot y^j_f \\
=&\sum_{f=1}^{F} \sum_{l \in \mathcal{O}^f} \sum_{j \in \mathcal{O}^f} \Lambda(f)_{l,j} \pi^l \cdot y^j_f - \sum_{f=1}^{F} \sum_{j \in \mathcal{O}^f} \left(\pi^j + ((\alpha^j_f)_{S_a}, 0_{-S_a}) - ((\beta^j_f)_{S_b}, 0_{-S_b}))\right) \cdot y^j_f \\
=&\sum_{f=1}^{F} \sum_{j \in \mathcal{O}^f} \sum_{l \in \mathcal{O}^f} \Lambda(f)_{j,l} \pi^j \cdot y^l_f - \sum_{f=1}^{F} \sum_{j \in \mathcal{O}^f} \left(\pi^j + ((\alpha^j_f)_{S_a}, 0_{-S_a}) - ((\beta^j_f)_{S_b}, 0_{-S_b}))\right) \cdot y^j_f.
\end{aligned}$$

The first equality here results from interchanging the summation of l and j, and the second by relabelling the summation via l and j as j and l (using the fact that they are dummy variables).

Finally, observe that for each $f \in F$ and $j \in \mathcal{O}^f$, we have $\sum_{l \in \mathcal{O}^f} \Lambda(f)_{j,l} \pi^j \cdot$

$y^l_f \leq \pi^j \cdot y^j_f$, since $\Lambda(f)$ is a transition matrix, and by profit maximization, for any $l \in \mathcal{O}^f$, $\pi^j \cdot y^l_f \leq \pi^j \cdot y^j_f$. Consequently, the entire expression is less than:

$$-\sum_{f=1}^{F} \sum_{j \in \mathcal{O}^f} (((\alpha^j_f)_{S_a}, 0_{-S_a}) - ((\beta^j_f)_{S_b}, 0_{-S_b})) \cdot y^j_f$$

Using the fact that each of $(\alpha^j_f)_{S_a} \geq 0$ and $(\beta^j_f)_{S_b} \geq 0$, we then obtain that this expression is less than or equal to:

$$-\sum_{f=1}^{F} \sum_{j \in \mathcal{O}^f} (\alpha^j_f)_{S_a} \cdot a_{S_a} + \sum_{f=1}^{F} \sum_{j \in \mathcal{O}^f} (\beta^j_f)_{S_b} \cdot b_{S_b}$$

establishing the result.

This concludes one direction.

Conversely, let $(\langle P^j, \pi^j, y^j \rangle)_{j=1}^{J}$ be a market supply data set. We will show that if it satisfies the condition of the theorem, it is profit rationalizable with constraints.

To this end, observe that by Varian (1984), $(\langle P^j, \pi^j, y^j \rangle)_{j=1}^{J}$ is profit rationalizable with constraints if and only if for each $j \in J$ and each $f \in P^j$, there is $y(f)^j \in \Re^K$ such that the following inequalities are satisfied:

1. For all $j \in J$, $\sum_{f \in P^j} y^j_f = y^j$
2. For all $f \in F$, and all $j, l \in \mathcal{O}^f$, $\pi^j \cdot y^l_f \leq \pi^j \cdot y^j_f$
3. For all $f \in F$ and all $j \in \mathcal{O}^f$, $a \leq y^j_f \leq b$

The first equation, for each j, can be rewritten thusly: For all $j \in J$ and all $k \in \{1, \ldots, K\}$, $\sum_{f \in P^j} y^j_{f,k} = y^j_k$. We therefore have a list of real-valued linear inequalities whose compatibility is necessary and sufficient for profit rationalizability.

Therefore, suppose by means of contradiction that the data are not profit rationalizable with constraints. In particular, then, there is no solution to the list of linear inequalities; and by a Theorem of the Alternative (*e.g.*

Lemma 1.14 of Chambers and Echenique (2016)), for each $j \in J$ and each $k \in \{1, \dots, K\}$, there is $\nu(j,k) \in \Re$ for each $f \in F$ and $j, l \in \mathcal{O}^f$, there is $\theta(f,j,l) \in \Re_+$, for each $j \in J$, each $k \in S_a$, and each $f \in P^j$, there is $\eta(j,k,f) \in \Re_+$, and for each $j \in J$, each $k \in S_b$, and each $f \in P^j$, there is $\delta(j,k,f) \in \Re_+$ such that:

1. For each $j \in J$, $k \in S_a \cap S_b$, and $f \in P^j$: $\nu(j,k) + \sum_{l \in \mathcal{O}^f, l \neq j} [\theta(f,j,l)\pi^j_k - \theta(f,l,j)\pi^l_k] + \eta(j,k,f) - \delta(j,k,f) = 0$
2. For each $j \in J$, $k \in S_a \setminus S_b$, and $f \in P^j$: $\nu(j,k) + \sum_{l \in \mathcal{O}^f, l \neq j} [\theta(f,j,l)\pi^j_k - \theta(f,l,j)\pi^l_k] + \eta(j,k,f) = 0$
3. For each $j \in J$, $k \in S_B \setminus S_A$, and $f \in P^j$: $\nu(j,k) + \sum_{l \in \mathcal{O}^f, l \neq j} [\theta(f,j,l)\pi^j_k - \theta(f,l,j)\pi^l_k] - \delta(j,k,f) = 0$
4. For each $j \in J$, $k \in \{1, \dots, K\} \setminus (S_a \cup S_b)$, and $f \in P^j$: $\nu(j,k) + \sum_{l \in \mathcal{O}^f, l \neq j} [\theta(f,j,l)\pi^j_k - \theta(f,l,j)\pi^l_k] = 0$
5. $\sum_{j \in J} \sum_{k=1}^K \nu(j,k) y^j_k + \sum_{j \in J} \sum_{k \in S_a} \sum_{f \in P^j} \eta(j,k,f) a_k - \sum_{j \in J} \sum_{k \in S_a} \sum_{f \in P^j} \delta(j,k,f) b_k > 0$.

Observe first that we may write $\nu^j \equiv \nu(j, \cdot) \in \Re^k$, $\eta^j_f \in \Re^{S_a}_+$, $\delta^j_f \in \Re^{S_b}_+$. The equations then read:

1. For each $j \in J$ and $f \in P^j$, $\sum_{l \in \mathcal{O}^f, l \neq j} [\theta(f,l,j)\pi^l - \theta(f,j,l)\pi^j] = \nu^j + (\eta^j_f, 0_{S-a}) - (\delta^j_f, 0_{S-b})$
2. $\sum_{j \in J} \nu^j \cdot y^j + \sum_{j \in J} \sum_{f \in P^j} \eta^j_f \cdot (a|_{S_a}) - \sum_{j \in J} \sum_{f \in P^j} \delta^j_f \cdot (b|_{S_b}) > 0$.

Now, the existence of ν, θ, η, δ terms satisfying these inequalities implies (by rescaling) the existence of μ and λ terms such that:

1. For each $j \in J$ and $f \in P^j$, $\sum_l \lambda(f,j,l) < 1$
2. For each $j \in J$, $\sum_{l \in \mathcal{O}^f, l \neq j} [\lambda(f,l,j)\pi^l - \lambda(f,j,l)\pi^j] = \mu^j + (\alpha^j_f, 0_{S-a}) - (\beta^j_f, 0_{S-b})$

3. $\sum_{j\in J} \mu^j \cdot y^j + \sum_{j\in J} \sum_{f\in P^J} \alpha^j_f \cdot a|_{S_a} - \sum_{j\in J} \sum_{f\in P^j} \beta^j_f \cdot b|_{S_b} > 0.$

So, clearly we define $\Lambda(f)_{j,l} = \lambda(f, j, l)$ for $\ell \neq j$, and for any $j \in \mathcal{O}^f$, $\Lambda(f)_{j,j} = 1 - \sum_{l\in\mathcal{O}^f, l\neq j} \lambda(f, j, l)$. Observe that each $\Lambda(f) \in \Re^{\mathcal{O}^f\times\mathcal{O}^f}$ is a transition matrix; yet violates the inequalities listed in the theorem for μ^j terms.

□

Proof of Theorem 3. First, let us establish that a non-negative profit rationalizable market supply dataset satisfies the condition. Let the market supply dataset be given, and suppose that for each $j \in J$ and $f \in P^j$, $y^j_f \in \Re^K$ profit rationalizes the dataset and y^j_f satisfies non-negative profit conditions.

For all $j \in J$ and for all $f \in F$, let $\mu^j \in \Re^k$ and $\gamma^j_f \geq 1$, and $\Lambda(f) \in \Re^{\mathcal{O}^f\times\mathcal{O}^f}$ such that

$$\sum_{l\in\mathcal{O}^f} \Lambda(f)_{l,j}\pi^l = \gamma^j_f \pi^j + \mu^j.$$

Observe that

$$\begin{aligned}
&\sum_{j=1}^{J} \mu^j \cdot y^j \\
=&\sum_{j=1}^{J} \mu^j \cdot \left(\sum_{f\in P^j} y^j_f\right) \\
=&\sum_{j=1}^{J} \sum_{f\in P^j} \mu^j \cdot y^j_f \\
=&\sum_{j=1}^{J} \sum_{f\in P^j} \left(\sum_{l\in\mathcal{O}^f} \Lambda(f)_{l,j}\pi^l - \pi^j + (1-\gamma^j_f)\pi^j\right) \cdot y^j_f.
\end{aligned}$$

by adding and subtracting $\pi^j \cdot y^j_f$ for each firm.

Performing the inner product of the γ terms with y, we obtain

$$\sum_{j=1}^{J} \sum_{f\in P^j} (1-\gamma^j_f)\pi^j \cdot y^j_f \leq 0$$

since $\gamma_f^j \geq 1$ for all $f \in F$ and $j \in \{1, \ldots, J\}$ and profit is non-negative for each firm. It follows that

$$\sum_{j=1}^{J} \sum_{f \in P^j} \left(\sum_{l \in \mathcal{O}^f} \Lambda(f)_{l,j} \pi^l - \pi^j + (1 - \gamma_f^j) \pi^j \right) \cdot y_f^j$$
$$\leq \sum_{j=1}^{J} \sum_{f \in P^j} \left(\left(\sum_{l \in \mathcal{O}^f} \Lambda(f)_{l,j} \pi^l \right) - \pi^j \right) \cdot y_f^j$$

by using the above inequality and re-arrangement. The remainder of the proof follows from re-arranging the summations as in Theorem 2.

Conversely, let $\{\langle P, \pi, y \rangle\}_{j=1}^J$ be a market supply dataset. We will show that if it satisfies the condition of Theorem 3, it is non-negative profit rationalizable. To this end, observe that by Varian (1984), $(\langle P^j, \pi^j, y^j \rangle)_{j=1}^J$ is non-negative profit rationalizable if and only if for each $j \in J$ and each $f \in P^j$, there is $y_f^j \in \Re^K$ such that the following inequalities are satisfied:

1. For all $j \in J$, $\sum_{f \in P^j} y_f^j = y^j$
2. For all $f \in F$, and all $j, l \in \mathcal{O}^f$, $\pi^j \cdot y_f^l \leq \pi^j \cdot y_f^j$.
3. For all $f \in F$, all $j \in \mathcal{O}^f$ that $0 \leq \pi^j \cdot y_f^j$.

The first equation, for each j, can be rewritten thusly: For all $j \in J$ and all $k \in \{1, \ldots, K\}$, $\sum_{f \in P^j} y_{f,k}^j = y_k^j$. Similarly, the inequalities from three can be rewritten for all $j \in J$ as $0 \leq \sum_{k=1}^K \pi_k^j y_{f,k}^j$. Therefore, we have a list of real-valued linear inequalities whose compatibility is necessary and sufficient for non-negative profit rationalizability. Therefore, suppose by means of contradiction that the data are not profit rationalizable. In particular, then, there is no solution to the list of linear inequalities; and by a Theorem of the Alternative, for each $j \in J$ and each $k \in \{1, \ldots, K\}$, there is $\nu(j,k) \in \Re$, for each $f \in F$ and $j, l \in \mathcal{O}^f$, there is $\theta(f, j, l) \in \Re_+$, and for each $f \in F$ and $j \in \mathcal{O}^f$ there exists $\gamma(f, j)$ such that

1. For each $j \in J$, $k \in \{1, \ldots, K\}$, and $f \in P^j$: $\nu(j,k) + \sum_{l \in \mathcal{O}^f, l \neq j} [\theta(f, j, l) \pi_k^j - \theta(f, l, j) \pi_k^l] + \gamma(f, j) \pi_k^j = 0$

2. $\sum_{j\in J}\sum_{k=1}^{K}\nu(j,k)y_k^j > 0$.

Relative to Theorem 2, this adds the restriction of profit maximization. Similar to the case of known net inputs and net outputs, it only changes the restrictions on the Lagrange multipliers. We may write $\nu^j \equiv \nu(j,\cdot) \in \Re^k$. Thus, the equations read:

1. For each $j \in J$, $\sum_{l\in\mathcal{O}^f, l\neq j}[\theta(f,l,j)\pi^l - \theta(f,j,l)\pi^j] = \nu^j + \gamma(f,j)\pi^j$
2. $\sum_{j\in J}\nu^j \cdot y^j > 0$.

Now, the existence of ν, θ, and γ terms satisfying these inequalities implies (by rescaling) the existence of μ, λ, and $\tilde{\gamma}$ terms such that:

1. For each $j \in J$ and $f \in P^j$, $\sum_l \lambda(f,j,l) < 1$
2. For each $j \in J$, $\sum_{l\in\mathcal{O}^f, l\neq j}[\lambda(f,l,j)\pi^l - \lambda(f,j,l)\pi^j] = \mu^j + \tilde{\gamma}(f,j)\pi^j$
3. $\sum_{j\in J}\mu^j \cdot y^j > 0$.

So, clearly we define $\Lambda(f)_{j,l} = \lambda(f,j,l)$ for $\ell \neq j$, and for any $j \in \mathcal{O}^f$, $\Lambda(f)_{j,j} = 1 - \sum_{l\in\mathcal{O}^f, l\neq j}\lambda(f,j,l)$. Observe that each $\Lambda(f) \in \Re^{\mathcal{O}^f\times\mathcal{O}^f}$ is a transition matrix; yet violates the inequalities listed in the theorem for μ^j terms.

$\square$

Proof of Theorem 4. First, let us establish that a non-negative profit rationalizable market supply dataset satisfies the condition. Let the market supply dataset be given, and suppose that for each $j \in J$ and $f \in P^j$, $y_f^j \in \Re^K$ profit rationalizes with technology shocks the dataset.

For all $j \in J$ and for all $f \in F$, let $\mu^j \in \Re^k$ and $\Lambda(f) \in \Re^{\mathcal{O}^f\times\mathcal{O}^f}$ be a lower triangular transition matrix such that

$$\sum_{l\in\mathcal{O}^f}\Lambda(f)_{l,j}\pi^l = \pi^j + \mu^j.$$

Observe that

$$\begin{aligned}
&\sum_{j=1}^{J} \mu^j \cdot y^j \\
=&\sum_{j=1}^{J} \mu^j \cdot \left(\sum_{f \in P^j} y^j_f \right) \\
=&\sum_{j=1}^{J} \sum_{f \in P^j} \mu^j \cdot y^j_f \\
=&\sum_{j=1}^{J} \sum_{f \in P^j} \left(\sum_{l \in \mathcal{O}^f} \Lambda(f)_{l,j} \pi^l - \pi^j \right) \cdot y^j_f .
\end{aligned}$$

The remainder of the proof follows from re-arranging the summations as in Theorem 2.

Conversely, let $\{\langle P, \pi, y \rangle\}_{j=1}^J$ be a market supply dataset. We will show that if it satisfies the condition of Theorem 4, it is profit rationalizable with technological shocks. To this end, observe that by Varian (1984), $(\langle P^j, \pi^j, y^j \rangle)_{j=1}^J$ is non-negative profit rationalizable if and only if for each $j \in J$ and each $f \in P^j$, there is $y^j_f \in \Re^K$ such that the following inequalities are satisfied:

1. For all $j \in J$, $\sum_{f \in P^j} y^j_f = y^j$
2. For all $f \in F$, and all $j, \ell \in \mathcal{O}^f$ with $\ell < j$, $\pi^j \cdot y^l_f \leq \pi^j \cdot y^j_f$

where the main difference if y^j_f must create more profit than earlier observed relations since $Y^\ell_f \subseteq Y^j_f$

The first equation, for each j, can be rewritten thusly: For all $j \in J$ and all $k \in \{1, \ldots, K\}$, $\sum_{f \in P^j} y^j_{f,k} = y^j_k$. Therefore, we have a list of real-valued linear inequalities whose compatibility is necessary and sufficient for non-negative profit rationalizability. Therefore, suppose by means of contradiction that the data are not profit rationalizable. In particular, then, there is no solution to the list of linear inequalities; and by a Theorem of the Alternative, for each $j \in J$ and each $k \in \{1, \ldots, K\}$, there is $\nu(j,k) \in \Re$ and for each $f \in F$ and $j, l \in \mathcal{O}^f$ with $\ell < j$, there is $\theta(f, j, l) \in \Re_+$ such that

1. For each $j \in J$, $k \in \{1, \dots, K\}$, and $f \in P^j$: $\nu(j,k) + \sum_{\ell \in \mathcal{O}^f, \ell \leq j} \theta(f,j,l)\pi_k^j - \sum_{\ell \in \mathcal{O}^f, j \leq \ell} \theta(f,l,j)\pi_k^l = 0$
2. $\sum_{j \in J} \sum_{k=1}^{K} \nu(j,k) y_k^j > 0$.

Relative to Theorem 2, this relaxes the number of comparisons made across time periods by the θ multipliers.

1. For each $j \in J$, $\sum_{\ell \in \mathcal{O}^f, j \leq \ell} \theta(f,l,j)\pi_k^l - \sum_{\ell \in \mathcal{O}^f, \ell \leq j} \theta(f,j,l)\pi_k^j = \nu^j$
2. $\sum_{j \in J} \nu^j \cdot y^j > 0$.

Now, the existence of ν, θ terms satisfying these inequalities implies (by rescaling) the existence of μ and λ terms such that:

1. For each $j \in J$ and $f \in P^j$, $\sum_{l \neq j} \lambda(f,j,l) < 1$
2. For each $j \in J$, $\sum_{l \in \mathcal{O}^f, l \neq j} [\lambda(f,l,j)\pi^l - \lambda(f,j,l)\pi^j] = \mu^j$
3. $\sum_{j \in J} \mu^j \cdot y^j > 0$.

So, clearly we define $\Lambda(f)_{j,l} = \lambda(f,j,l)$ for $\ell < j$, $\Lambda(f)_{j,\ell} = 0$ if $j > \ell$, and for any $j \in \mathcal{O}^f$, $\Lambda(f)_{j,j} = 1 - \sum_{l \in \mathcal{O}^f, l \neq j} \lambda(f,j,l)$. Observe that each $\Lambda(f) \in \Re^{\mathcal{O}^f \times \mathcal{O}^f}$ is a lower triangular transition matrix; yet violates the inequalities listed in the theorem for μ^j terms.

$\square$

Proof of Theorem 5. We apply Theorem 3.1 of Gale (1989). We seek, for all $j \in \{1, \dots, J\}$, all $f \in P^j$, and all $k \in \{1, \dots, K\}$, ε_f^j and $y_{f,k}^j$ such that

1. For each $j \in J$ and $f \in P^j$, $\varepsilon_f^j \geq 0$
2. For each $f \in F$ and binary $\{j, l\} \subseteq \mathcal{O}^f$, $\pi^j \cdot y_f^j + \varepsilon_f^j - \pi^j \cdot y_l^f \geq 0$
3. For each $j \in J$, $f \in P^j$, and $k \in \{1, \dots, K\}$, $\sum_{f \in P^j} y_{f,k}^j = y_k^j$

to maximize $\sum_{f \in F} \sum_{j \in \mathcal{O}^f} \varepsilon_f^j$.

Applying Theorem 3.1 of Gale (1989), we obtain, for each $\varepsilon_f^j \geq 0$ constraint, a multiplier $\alpha_f^j \geq 0$, for each constraint of type 2 (an ordered pair j, l with $j \neq l$), a multiplier $\Lambda(f)_{j,l} \geq 0$, and for constraints of type 3, a multiplier $\mu_k^j \in \Re$.

Our goal is then to maximize $\sum_{j \in J} \mu^j \cdot y^j$ subject to for all $j \in J$ and $f \in P^j$, $\sum_l \Lambda(f)_{j,l} \pi^j + \mu^j = \sum_{l \in \mathcal{O}^f, l \neq j} \Lambda(f)_{l,j} \pi^l$ and $\alpha_f^j + \sum_{l \in \mathcal{O}^f, l \neq j} \Lambda(f)_{j,l} \leq 1$, or removing the α_f^j constraint, $\sum_{l \in \mathcal{O}^f, l \neq j} \Lambda(f)_{j,l} \leq 1$. By creating a term $\Lambda(f)_{j,j}$ for each $j \in J$ and $f \in P^j$, we see we obtain exactly the optimization problem in Theorem 5.

□

Proof of Theorem 6. We remark that the proof relies on the same technique as in the proof of Theorem 5. First, the original problem can be reformulated as the linear program

$$\min \quad \varepsilon$$

subject to:

1. For all $f \in F$ and $j \in \mathcal{O}^f$, $\varepsilon_f^j \geq 0$.
2. For all $f \in F$ and $j, l \in \mathcal{O}^f$, $\pi^j \cdot y_f^j + \varepsilon_f^j \geq \pi^j \cdot y_f^l$.
3. For all $j \in J$, $\sum_{f \in P^j} y_f^j = y^j$.
4. For all $f \in F$, $\varepsilon - \sum_{j \in \mathcal{O}^f} \varepsilon_f^j \geq 0$.

Applying Theorem 3.1 of Gale (1989), we obtain, for each $\varepsilon_f^j \geq 0$ constraint, a multiplier $\alpha_f^j \geq 0$, for each constraint of type 2 (an ordered pair j, l with $j \neq l$), a multiplier $\Lambda(f)_{j,l} \geq 0$, for constraints of type 3, a multiplier $\mu_k^j \in \Re$, and for constraints of type 4 we obtain a multiplier of $\beta_f \geq 0$.

Our goal is to maximize $\sum_{j \in J} \mu^j \cdot y^j$ subject to for all $j \in J$ and $f \in P^j$, $\sum_l \Lambda(f)_{j,l} \pi^j + \mu^j = \sum_{l \in \mathcal{O}^f, l \neq j} \Lambda(f)_{l,j} \pi^l$ and $\alpha_f^j - \beta_f + \sum_{l \in \mathcal{O}^f, l \neq j} \Lambda(f)_{j,l} \leq 0$, and $\sum_{f=1}^F \beta_f \leq 1$. Note that at least one $\beta_f > 0$. To see this, suppose all β_f are zero so by complementary slackness, it would follow for each $f \in F$ that

$\varepsilon - \sum_{j \in \mathcal{O}^f} \varepsilon_f^j > 0$. In this case though, ε would not be at a minimum. Since at least one $\beta_f > 0$ we can assume the inequality is equal since we can divide by $\sum_{f=1}^{F} \beta_f$ without changing the solution so that $\beta \in \Delta(F)$. Similarly, we can remove the α_f^j term to get that $\sum_{l \in \mathcal{O}^f, l \neq j} \Lambda(f)_{j,l} \leq \beta_f$. We can now create a term $\Lambda(f)_{j,j}$ for each $j \in J$ and $f \in P^j$. We now see that the conditions from Theorem 6 match those for the dual problem.

□

Appendix B Data Collection Methods

To perform the analysis in the main text, we use data from a variety of sources. In particular, data from 1980-1998 was collected from: the United States Geological Survey (USGS) Minerals Yearbook, the American Energy Review, the Portland Cement Association (PCA), the United States Bureau of Labor Statistics (US BLS), and the St. Louis Federal Reserve. In the following paragraphs, we describe what data was used from each source and how the data was processed for use in the test of profit maximization of aggregate cement production.

Much of the data was collected from the United States Geological Survey (USGS) Minerals Yearbooks published between 1980-1998. Most entries for a given material (e.g. cement) in the USGS Mineral Yearbook have information on various industry and regional level statistics for the current year and several previous years. There are often inconsistencies with the data if one looks across different years since the Mineral Yearbook is often published before firms in the industry have responded to the surveys issued by the USGS.[12] For this reason, when we use information from the USGS Minerals Yearbook we take the information from the latest year it appears in a yearbook entry.

The data on industry wide production of cement was gathered from the USGS Minerals Yearbook entry on cement. The entry on cement contains data on output, price, raw materials inputs, and energy inputs. For example, the

[12]We are grateful for Henrick vanOss for pointing out this detail in a personal correspondence.

information on production and unit value of cement for 1995 were collected from Table 1 in the USGS cement entry from 1999. We treat the unit value as the average price of cement in the United States for a given year in the main analysis.

The data on raw material input quantities for cement was also collected from the USGS Mineral Yearbook entry on cement. For example, the information on input quantity in 1998 was collected from Table 6 in the 1999 yearbook entry on cement. The data categories are not always consistent across years so we discuss how inputs are grouped into those mentioned in Table 1 in the main text. First, when inputs are split between clinker and cement production, we add these entries to produce the total quantity inputs for a given year. The inputs of sand and gypsum match the associated labels in the yearbook entry on cement. We differ from the yearbook entry since we treat all cement rock as marl and all ferrous material as iron ore. We make this distinction to match the data from the cement yearbook entry to the associated prices we use for the different materials. We also include coral into the category "limestone" for analysis since later entries in the yearbook do not make a distinction between coral and limestone. Similarly, we treat clay and shale as the same good since the price information is on the price of common clay and shale.

The cement entry in the USGS mineral yearbook also contains the information on the quantity of different energy inputs. For example, we collect data on fuel usage for 1998 from Table 7 in the cement yearbook entry from 1999. Quantities for oil and natural gas are recorded directly from the table. The quantity of "coal" used in the analysis is the sum of coal, coke, and petroleum coke. We aggregate these quantities together since earlier data does not always make these distinctions. Therefore, we have comparable amounts of "coal" across different time periods. Also, we make the assumption that all coal is bituminous to match the amount of coal to a single price from the American Energy Review. This seems a reasonable first approximation since looking at previous yearbook entries, we see that virtually all coal used is bituminous (e.g. over 94% in 1995). We note that in 1991 there is no record of energy usage in the cement industry. This is the only missing data of all materials from 1980-1998. Therefore, the the largest dataset we use to examine profit maximization includes data from 1980-1990 and 1992-1998.

The final piece of information we gather from the cement entry of the USGS yearbook is electricity usage. For example, electricity usage for 1998 was collected from Table 8 of the cement yearbook entry in 1999. We treat the electricity category as the sum of all purchased energy by cement plants in a given year. For example, the electricity usage is the sum of purchased energy from all plants plus the energy purchased for plants that grind materials.

So far, we have accounted for all quantity amounts with the exception of labor. We measure labor as total hours worked which is derived using the information on the amount of cement produced with information from the Portland Cement Association (PCA). In particular, PCA records the average number of labor hours needed to produce a thousand metric tons of cement. Total labor hours for a given year is generated by multiplying the quantity of cement produced by the labor hours per metric ton from PCA.

The above paragraphs documents how we obtained data on the quantities produced from various data sources. However, we have not mentioned the unit of measurement for each input/output. From 1980-1999, the USGS changed the units that inputs/outputs were measured in from English/U.S. engineering units to metric units. For the analysis, we measure all inputs/outputs in metric units when appropriate. Therefore, we often had to apply a conversion factor to these measurements for different years. All the information on the change of units and measurement units used in different time periods are recorded in Table 5. There are four main types of unit changes those for mass (cement, limestone, marl, clay/shale, sand, iron ore, gypsum, and coal), liquid volume (oil), and gaseous volume (natural gas). We also report the degree of precision to which the units are rounded. The measurement of electricity is constant in million-kilowatt hours. Finally, we report labor to the nearest hour after transforming the amount of cement from short tons to metric tons.

We now discuss how we collect data on raw material input prices. Raw material input prices are treated as the unit value or freight on board price of the different goods. These values are collected from the USGS Minerals

Type	Units	Reporting Years	Metric Conversion	Rounding
Mass	Thousand Short Tons	1980-1992	0.907185 $\frac{\text{metric ton}}{\text{short ton}}$	Thousand Metric Ton
	Thousand Metric Tons	1993-1998		
Liquid	Thousand 42-gallon Barrels	1980-1992	3.78541 $\frac{\text{liter}}{\text{gallon}}$	Thousand Liter
Volume	Thousand Liters	1993-1998		
Gaseous	Thousand Cubic Feet	1980-1992	0.0283168 $\frac{\text{cubic meter}}{\text{cubic foot}}$	Thousand Cubic Meter
Volume	Thousand Cubic Meters	1993-1998		

Table 5: Units of Measurement in Cement Yearbook

Yearbook. Limestone and calcareous marl unit values were collected from the crushed stone/stone yearbook entries. For example, the unit values for 1998 were collected from Table 2 in the 1999 crushed stone yearbook entry. The information on prices of both limestone and calcareous marl are not always available before 1993 which is why we restrict to analysis to the years 1993-1998 in the main text. From 1980-1992, the unit values of limestone and calcareous marl are only available on odd years. However, even years from 1980-1992 still contain information on the unit value of all crushed stone. Thus, when we analyze profit maximization for the larger period from 1980-1998, we treat the price of limestone and calcareous marl as the average unit value for all crushed stone in the even years between 1980-1992.

The price of common clay/shale is obtained from the Clay and Shale/Clay yearbook entry from the USGS Minerals yearbook. The yearbook makes no distinction between the prices of these goods, so they are aggregated into a single commodity in the analysis. This information is not recorded in a table and the yearbook entry only contains one year of data. For an example, the unit value of clay/shale from 1998 is collected from the section on Prices under the heading "Common Clay and Shale" in the 1998 yearbook entry.

The price of ferrous material is treated as the average freight-on-board mine value of usable iron ore. This number is obtained from the Iron Ore entry of the USGS Minerals yearbook. For example, the unit value for 1998 is gathered from the first paragraph under the heading "Prices" in the 1998 yearbook entry. The price of sand is treated as freight on board (f.o.b.) price of sand. We obtain the average f.o.b. price of sand from the Construction Sand and Gravel entry of the USGS Minerals Yearbook. This information is generally in a separate section on prices. For example, the 1996 price of sand is the f.o.b. price of sand collected from the subsection on prices in the 1996 yearbook entry. The prices of sand were not reported in even years up to 1992 but were instead estimated. This is another data limitation that motivates looking at the restricted sample from 1993-1998 in the main text.

Lastly, the price of gypsum was taken as the per unit value of uncalcined gypsum used in the portland cement industry. The yearbook entry on gypsum has additional information that allows us to look more closely at the unit value of gypsum for the cement industry. For example, the price of gypsum in 1993 was taken calculating the unit value of uncalcined gypsum used in for portland cement derived from Table 4 in 1994 yearbook entry. As in the case of the cement yearbook entry, many entries switched measurement units from the English/U.S. engineering units to metric units. Most of the conversions are from dollars per short ton to dollars per metric ton, where we use the conversion from Table 5. We also make use of the conversion from long tons to metric tons for iron ore from 1980-1983 where the conversion is $1.01605 \frac{\text{metric ton}}{\text{long ton}}$.

We obtain energy prices from the American Energy Reviews Published in 2011, 1998, and 1982. This includes prices for coal, oil, natural gas, and electricity purchase. As with the other data sources, there are discrepancies in prices when one looks across the different publications. We take the entry from the most recent publication in keeping with the previous analysis. The prices of oil for 1995-1998 are recovered from the No. 4 Residual Fuel Oil prices to end users in Table 5.22 of the 2011 American Energy Review. Other prices are recovered from the analogous tables in different American Energy Reviews. There is an exception where prices of oil from 1980-1981 are wholesale prices of No. 4 residual fuel oil as the price to end user was not recorded. These prices are recorded in dollars per gallon which we convert to dollars per liter using the conversion factor in Table 5.

We take the price of coal to be the price of bituminous coal. The prices of coal from 1980-1998 are all recovered from the nominal prices of bituminous coal from Table 7.9 in the 2011 American Energy Review. The prices of coal are recorded in dollars per short ton which we convert to dollars per metric ton using the conversion from Table 5. For electricity consumption, we treat price as the average retail prices of electricity in the industrial sector. The prices from 1980-1998 are all collected from Table 8.10 of the American Energy Review of 2011. These prices are recorded in terms of dollars per kilowatthour which agree with the units from the cement yearbook entry.

The final set of prices needed is the price of labor. We treat the price of labor as the yearly average hourly wage for manufacturing employees from the St. Louis Federal Reserve. In particular, we use monthly level data to generate an average yearly hourly wage using data that is not seasonally adjusted. We choose to use this data rather than the seasonally adjusted data so that all units begin in nominal unscaled dollars and then are converted to real prices by a common conversion factor. This data can be obtained from of Labor Statistics (2018a).

Thus far, all of the entries for prices have been nominal prices. To convert prices into real purchasing power, we use the information of the Consumer Price Index (CPI) acquired from the U.S. Beaurau of Labor and Statistics (of Labor Statistics, 2018b). We normalize the value to 1996 dollars.

Appendix C Extended Analysis 1980-1990 & 1992-1998

We repeat the analysis from Section 5.2, but for data from 1980-1990 & 1992-1998. We leave out 1991 since we do not have energy inputs for this year. We note that over this period there is more entry/exit than in the years from just 1993-1998. In particular, there are 166 distinct firms in the market during this extended time period. All of the information on entry and exit from 1980-1998 is present in Table 6. We include 1991 even though we do not use it in the analysis for completeness. For the year 1980-1990, the entry and exit is much more common than 1993-1998. Since there is more variation from firms, we hypothesize adding these years may have little effect to the analysis.

We show the results replicating the analysis from Section 5.2 below in Table 7 and Table 8. We see that the additional data only changes the measures of NPL for the static model of profit maximization with input/output constraints. However, the errors more than double in magnitude when we increase the range on the data. For this expanded dataset, we find a m-NPL of \$1.7 billion and a f-NPL of \$12.8 million. This shows that as one expands the time period over which the analysis is performed, then the errors will increase

Year	1980	1981	1982	1983	1984	1985
Number of Firms	146	144	140	140	138	133
Entering Firms	-	7	2	3	1	0
Exiting Firms	-	9	6	3	3	5
Year	1986	1987	1988	1989	1990	1991
Number of Firms	130	129	124	120	116	116
Entering Firms	0	4	1	0	0	0
Exiting Firms	3	5	6	4	4	0
Year	1992	1993	1994	1995	1996	1997
Number of Firms	116	115	115	117	115	115
Entering Firms	1	0	1	2	0	0
Exiting Firms	1	1	1	0	2	0
Year	1998					
Number of Firms	115					
Entering Firms	0					
Exiting Firms	0					

Table 6: Firm Participation from 1980-1998

in non-trivial magnitudes.

	Unrestricted	Input/Output	Input/Output and Non-negative Profits
Static Technology	348.0	348.0	1,746
Improving Technology	0	0	109.0

Table 7: m-NPL for full dataset in millions of 1996 dollars

	Unrestricted	Restricted	Restricted and Non-negative Profits
Static Technology	3.026	3.026	12.83
Improving Technology	0	0	0.948

Table 8: f-NPL for full dataset in millions of 1996 dollars

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