Bilateral Communication and Joint Decision-Making

Gorkem Celik[1] and Sergei Severinov[2]

[1]ESSEC Business School and Thema Research Center

[1]Vancouver School of Economics, University of British Columbia

February 21, 2018

Outline

1. Introduction

2. The Model

3. Analysis and Main Results

Introduction: Description of the Problem

- Two parties need to take a joint decision: "Yes" or "No"
  - matching: marriage, merger, partnership formation.
  - public good provision;
  - joint project.
- Before the decision is made, the parties engage in communication exchanging information about the value of the project.
- The paper focuses on:
  1. *the role and scope of pre-match communication.*
  2. communication protocols
  3. the set of outcomes implementable without a mediator or a mechanism designer.

The Environment

- No Transfers
- The value of the match/project is uncertain.
- The parties possess private information about it.
- Non-alignment of interests creates a friction: ex-post one party may want to implement the project ("match") when the other does not.
- But there is a joint interest to implement very profitable matches and avoid very unprofitable one. Hence, there is scope for "negotiations."

Main Goals

- Characterize the set of implementable decision rules.
- Understand how communication should be organized and structured to implement the set of desirable decision rules.

Communication Structure

- Focus on cheap-talk unmediated communication and consider various communication protocols.
- No fixed "sender-receiver" relationship. Generally, the parties may exchange messages in various sequences and order.
- First, study "short and "simple" communication protocols which turn out to be sufficient for monotone decision rules;
- Longer communication protocols - necessary for implementing non-monotone decision rules.

Related Literature

- Crawford & Sobel (1982)
- Melumad & Shibano (1991)
- Aumann & Hart (2003)
- Gerardi (2004)
- Krishna & Morgan (2004)
- Gerardi& Yariv (2008)
- Li & Madarász (2008)
- Chen (2009)
- Moreno de Barreda (2010)
- Forges (2010)
- Li, Rosen, Suen (2001)

Model

- Two players, A and B, have to decide whether to execute a joint project/match or not match.
- Common payoff from matching $v(\alpha, \beta)$ - continuous, increasing in both arguments, with bounded first-order derivatives.
- α and β are privately known types of players A and B, respectively, uniformly and independently distributed on $[0, 1]$. Uniform distribution is without loss of generality. Can relabel the distribution and the value function.
- Voluntary participation. Outside options of A and B are common knowledge and equal to R_a and R_b, respectively.
- $R_a > R_b$: Asymmetry between players comes from their outside options.

Matching/Decision-Making Process

- A match ("Yes" Decision) is formed only if both agree. So the decision has to be interim individually rational for each player.
- The parties engage in negotiations by exchanging cheap-talk messages.
- No transfers

Technical Assumptions

- Lowest types do not want to match at all:

$$v(1,0) < R_b, \quad v(0,1) < R_a.$$

- Highest types would like to match at least with some types:

$$v(1,1) > R_a.$$

Stylized example.

- Two parties contemplating a “marriage.”
- Their outside options can be deduced from the market or by doing “due diligence.”
- Each party knows what they can contribute to the marriage, but the other party cannot easily ascertain this.
- The parties negotiate by exchanging messages.
- The decision to “get married” has to be unanimous.

Decision Rules and Outcome Functions

- The decisions taken by the parties can be formally described by an outcome function.
  Outcome function $f(\alpha, \beta) \in [0, 1]$ is a probability that the project is undertaken when the parties have types α and β, respectively.
- A's ex-post payoff from $f(.)$ is
  $v(\alpha, \beta)f(\alpha, \beta) + (1 - f(\alpha, \beta))R_a$,
  B's ex-post payoff from $f(.)$ is
  $v(\alpha, \beta)f(\alpha, \beta) + (1 - f(\alpha, \beta))R_b$.
- The expected payoff of type α of A is:
  $\int_{\beta \in [0,1]} v(\alpha, \beta)f(\alpha, \beta) + (1 - f(\alpha, \beta))R_a d\beta$.
  The expected payoff of type β of B is
  $\int_{\alpha \in [0,1]} v(\alpha, \beta)f(\alpha, \beta) + (1 - f(\alpha, \beta))R_b d\alpha$
- Parties take a decision by engaging in communication.
  Formally, we model communication and the decision making

Communication Protocol

- Parties take a decision by engaging in communication. Formally, we model communication and the decision making process via a "communication protocol:" a game form $(S_a, S_b, g(.))$ where S_a and S_b are strategy spaces of players A and B, respectively, with the outcome function $g : S_a \times S_b \mapsto [0, 1]$ mapping the strategy profiles into the probabilities of matching.
- An outcome function $f(.)$ is *implementable* if there exists a communication protocol $(S_a, S_b, g(.))$ and a Perfect Bayesian equilibrium of this protocol $(s_a(\alpha), s_b(\beta)), \mu_a, \mu_b))$ such that $g(s_a(\alpha), s_b(\beta))) = f(\alpha, \beta)$;
- A feasible communication protocol must preserve a veto power (ability to reject the project) for each player. This is built into the outcome function $g(.)$.

Implementable Outcome Functions.

An implementable outcome function $f(\alpha, \beta)$ must be incentive compatible and individually rational for both players in the following sense:

Definition

(i) Outcome function $f(\cdot, \cdot)$ is incentive compatible for player A if, for all $(\alpha, \hat{\alpha}) \in [0,1]^2$, we have:

$$\int_0^1 [v(\alpha, \beta) - R_\alpha] f(\alpha, \beta) \, d\beta \geq \int_0^1 [v(\alpha, \beta) - R_\alpha] f(\hat{\alpha}, \beta) \, d\beta \quad (1)$$

(ii) Outcome function $f(\cdot, \cdot)$ is incentive compatible for player B if, for all $\left(\beta, \hat{\beta}\right) \in [0,1]^2$, we have:

$$\int_0^1 [v(\alpha, \beta) - R_\beta] f(\alpha, \beta) \, d\alpha \geq \int_0^1 [v(\alpha, \beta) - R_\alpha] f\left(\alpha, \hat{\beta}\right) d\alpha \quad (2)$$

Individual Rationality

Definition

Outcome function $f(\cdot,\cdot)$ is individually rational for player A if, for all $\alpha \in [0,1]$, we have:

$$V_a(f,\alpha) \equiv \int_0^1 \left[v\left(\alpha,\beta\right) - R_\alpha\right] f(\alpha,\beta) d\beta \geq 0 \tag{3}$$

Outcome function $f(\cdot,\cdot)$ is individually rational for player B if, for all $\beta \in [0,1]$, we have:

$$V_b(f,\beta) \equiv= \int_0^1 \left[v\left(\alpha,\beta\right) - R_\beta\right] f(\alpha,\beta) d\alpha \geq 0 \tag{4}$$

Pareto Efficient and Ex-post Efficient outcome functions

Lemma

If an outcome function $f^p(\cdot,\cdot)$ is Pareto efficient, then:

1. $f^p(\alpha, \beta) = 1$ *(i.e. a match is formed with probability 1) if* $v(\alpha, \beta) > R_a$*;*
2. $f^p(\alpha, \beta) = 0$ *(i.e. a match is formed with probability 0) if* $v(\alpha, \beta) < R_b$*.*

Definition

An ex-post efficient outcome function $f^*(.,.)$ is such that $f^*(\alpha, \beta) = 1$ (i.e. probability of a match is 1), when $2v(\alpha, \beta) \geq R_a + R_b$ and $f^*(\alpha, \beta) = 0$ (i.e. probability of a match is 0) if $2v(\alpha, \beta) < R_a + R_b$.

Impossibility of Pareto Efficient and Ex-Post Efficient Implementation

Theorem

A Pareto efficient outcome function is not incentive compatible.

Theorem

An ex-post efficient outcome is Pareto efficient. So it is not incentive compatible either.

Focus on Monotone Deterministic Outcome Functions

Definition

A matching outcome function $f(\cdot, \cdot)$ is deterministic if it takes values 0 and 1 only.

So a deterministic outcome function specifies a non-random outcome - any pair of types (α, β) is either ‘matched for sure or with zero probability
Consider also the following definition:

Definition

An outcome function $f(\cdot, \cdot)$ is monotonic if it is nondecreasing on $[0, 1]^2$.

Main Results

- Impossibility of Pareto effcient and ex-post efficient outcomes.
- Focus on IC and IR monotone deterministic decision rules.
- Partitional structure of feasible outcomes.
- "Short" communication protocols are sufficient.
- Invariance of the outcome to the assignment of the sender-receiver roles.
- Non-monotone decision-rules can be implemented via "longer" communication protocols.

Deterministic Monotone outcome function $f(.)$.

- A monotone deterministic outcome function $f(.,.)$ induces a partition of the type space $[0,1]^2$ into two regions: match region $M(f) \equiv \{(\alpha,\beta) \in [0,1]^2 | f(\alpha,\beta) = 1\}$ and non-match region $N(f) \equiv \{(\alpha,\beta) \in [0,1]^2 | f(\alpha,\beta) = 0\}$.
- Let $c_f(.) : [0,1] \mapsto [0,1]$ denote the boundary between these regions:

$$c_f(\alpha) = \begin{cases} 1 & \text{if } f(\alpha,\beta) = 0 \text{ for all } \beta \in [0,1], \\ \sup\{\beta \in [0,1] | f(\alpha,\beta) = 0\} & \text{otherwise.} \end{cases}$$

Deterministic Monotone outcome function $f(.)$.

Lemma

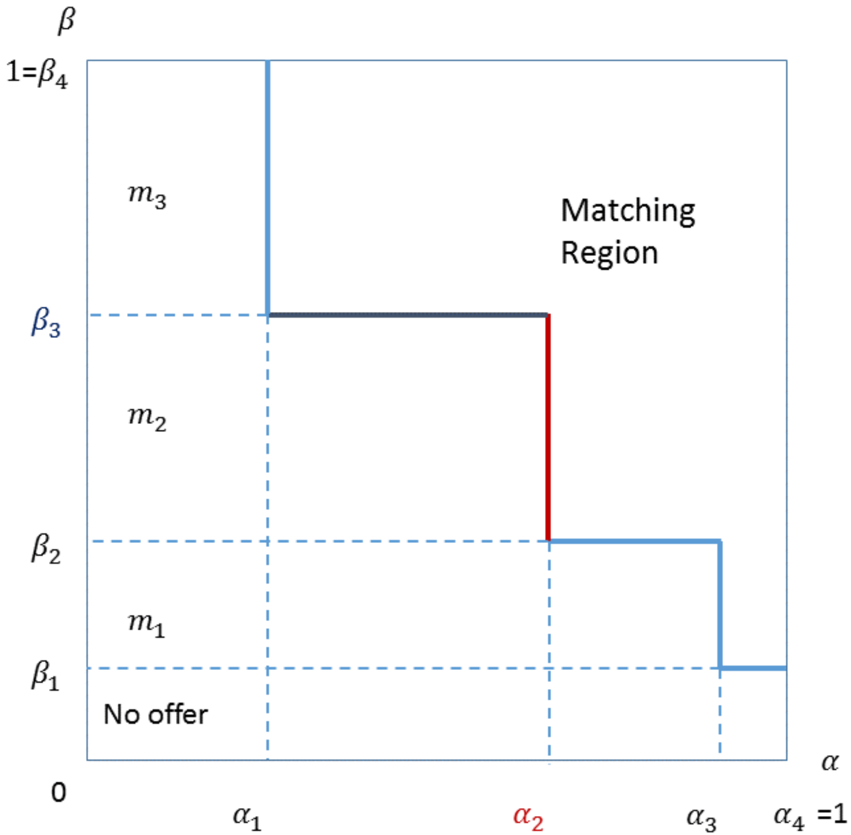
Consider any implementable (i.e. incentive compatible and individually rational), monotone and deterministic outcome function $f(.,.)$. Then there is no $\alpha \in (0,1)$ such that $c_f(.)$ is strictly decreasing at α.

Lemma

If $f(.)$ is implementable (i.e. incentive compatible and individually rational), then $c_f(.)$ can have at most finitely many points of discontinuity.

Conclusion: Any implementable monotone and determinstic outcome function has a partitional structure.

Implementable (IC and IR) monotone deterministic outcome function.



Implementation of Monotone Deterministic Outcomes: Short One-Sided Communication Protocol (SC)

- A single message is sent by one of the players.
- The other player receives the message and decides whether to match or not.
  If the latter agrees, a match is concluded.
  If the latter rejects, the parties get their outside options.

1. Protocol $SC(A)$: A sends a message, B takes matching decision.
2. Protocol $SC(B)$: B sends a message, A takes matching decision.

Short one-sided communication protocol SC.

- The set of messages under SC protocol is sufficiently large and includes a "null" message which allows the sender to refuse matching.
- Any non-null message under SC protocol contains: (a) agreement to match; (b) information about the sender's type.
- After a non-null message, the receiver makes a matching decision based on: (i) her type; (ii) information inferred from the sender's message.

Implementation and Equilibria under SC protocol.

Theorem

Any deterministic, monotone, incentive compatible and individually rational outcome function can be supported as an equilibrium of a short one-sided communication protocol SC, either $SC(A)$, or $SC(B)$.

Implications:

- If we focus on monotone and deterministic decision rules, we do not need a matchmaker (mechanism designer), nor extended negotiations and communication.
- Communication protocols other than SC are of interest only if we want to implement non-monotone or non-deterministic outcome functions.

Sketch of the proof:

- We showed that the boundary between the match and non-match regions *under any IC monotone deterministic decision rule* must be a step function.
- For the types at the end-points of the steps indifference conditions hold both in the outcome function and *SC* protocol. These indifference conditions imply incentive compatibility.

Equivalence of $SC(A)$ and $SC(B)$.

Theorem

The sets of equilibrium outcomes under protocols $SC(A)$ and $SC(B)$ are identical.
Namely, if $\{\alpha_0, \alpha_1, ..., \alpha_n\}$ and $\{\beta_0, \beta_1, ..., \beta_n\}$ are equilibrium partitions under communication protocol $SC(B)$, then these partitions also constitute an equilibrium under protocol $SC(A)$ and vice versa.

Implications: under a short one-sided communication protocol, the set of equilibrium outcomes is invariant to the assignment of the roles of sender and receiver.
We will extend this invariance further in what follows.

Equilibria under SC protocol.

All equilibria under SC protocol have a partitional structure.
This follows from the following results:

Theorem

(No separation) There is no equilibrium under the communication protocol $SC(B)$ in which every type of B in some interval of types $[\beta_1, \beta_2]$ such that $v(1, \beta_1) - R_a > 0$ sends a separating message not sent by any other type of B.
A similar statement holds for communication protocol $SC(A)$.

Lemma

(Convexity) In any equilibrium of $SC(B)$, if types β_1 and β_2 send the same message m, then any type $\beta \in (\beta_1, \beta_2)$ also sends this message.
Similarly, for $SC(A)$.

Structure of Equilibria under SC protocol.

- Any equilibrium under communication protocols $SC(B)$ can be represented by two partitions of equal size, $\{\alpha_0, \alpha_1, ..., \alpha_n\}$ and $\{\beta_0, \beta_1, ..., \beta_n\}$ such that $\alpha_0 = \beta_0 = 0$, $\alpha_n = \beta_n = 1$, $\alpha_i < \alpha_{i+1}$, $\beta_i < \beta_{i+1}$ for all $i \in \{0, ..., n-1\}$.
- Every type of player B in (β_k, β_{k+1}), $k \in \{1, ..., n-1\}$, sends message m_k^b. In response, the match is accepted by any type of A in $[\alpha_{n-k}, 1]$. Any type of B in $[\beta_0, \beta_1)$ does not send any message and withdraws from matching.
- A symmetrical statement holds for communication protocol $SC(A)$.

Equilibrium Conditions under SC protocol.

(i) Player A of type α_{n-k} must be indifferent between matching types (β_k, β_{k+1}) and her outside option:

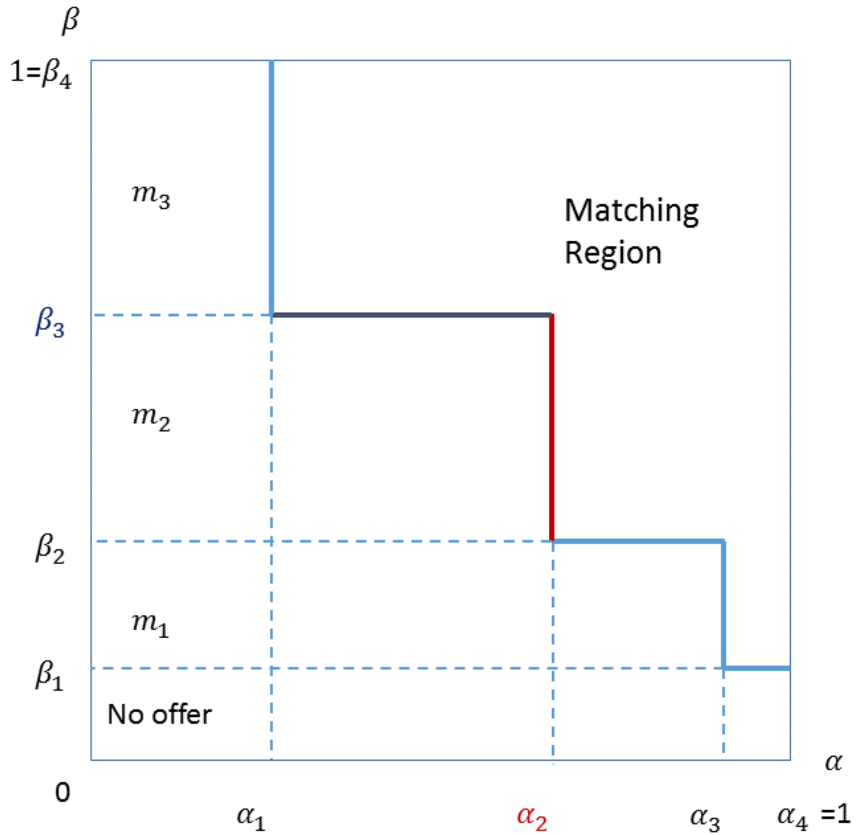
$$\int_{\beta_k}^{\beta_{k+1}} \left[V\left(\alpha_{n-k}, \beta\right) - R_\alpha\right] d\beta = 0 \text{ for } k = 1, ..., n-1. \tag{5}$$

(ii) Player B of type β_k must be indifferent between matching types $(\alpha_{n-k}, \alpha_{n-k+1})$ and her outside option:

$$\int_{\alpha_{n-k}}^{\alpha_{n-k+1}} \left[V\left(\alpha, \beta_k\right) - R_\beta\right] d\alpha = 0 \text{ for } k = 1, ..., n-1. \tag{6}$$

Note that equilibrium conditions under $SC(A)$ are exactly the same. These are the same as incentive conditions for an outcome function $f(.)$.

Feasible (IC and IR) monotone deterministic outcome function.



Existence of a non-trivial equilibrium

Let $\underline{\beta}$ be the type of B who is indifferent to matching with the highest type of A i.e.

$$v(1, \underline{\beta}) = R_b$$

Theorem

There exists a non-trivial matching equilibrium under communication protocols $SC(A)$ and $SC(B)$, in which there are non-zero sets of types of A and B who match, if the following condition holds:

$$\int_{\underline{\beta}}^{1} v(1, \beta) - R_a d\beta > 0,$$

i.e., if type 1 of player A is happy to match with the set of types of B who are happy to match with her.

Size of an equilibrium partition, SC protocol.

Lemma

The size of a "pool" of types of A and B is bounded from below, and depends on R_a, R_b and $v(.)$.
The number of elements of an equilibrium partition is bounded from above.

Theorem

Suppose that there exists an equilibrium with a partition consisting of n elements. Then there is an equilibrium partition with any n' elements, $1 \leq n' < n$.

Comparative Statics

The equilibria under SC protocol are sensitive to the outside options pair (R_a, R_b).

Theorem

As $R_a - R_b$ becomes small, the maximal number of elements n^m in an equilibrium partition increases, and the efficiency of this maximal equilibrium increases.
As $R_a - R_b$ converges to zero, n^m grows to infinity.
When $R_a = R_b$, there exists a fully separating equilibrium

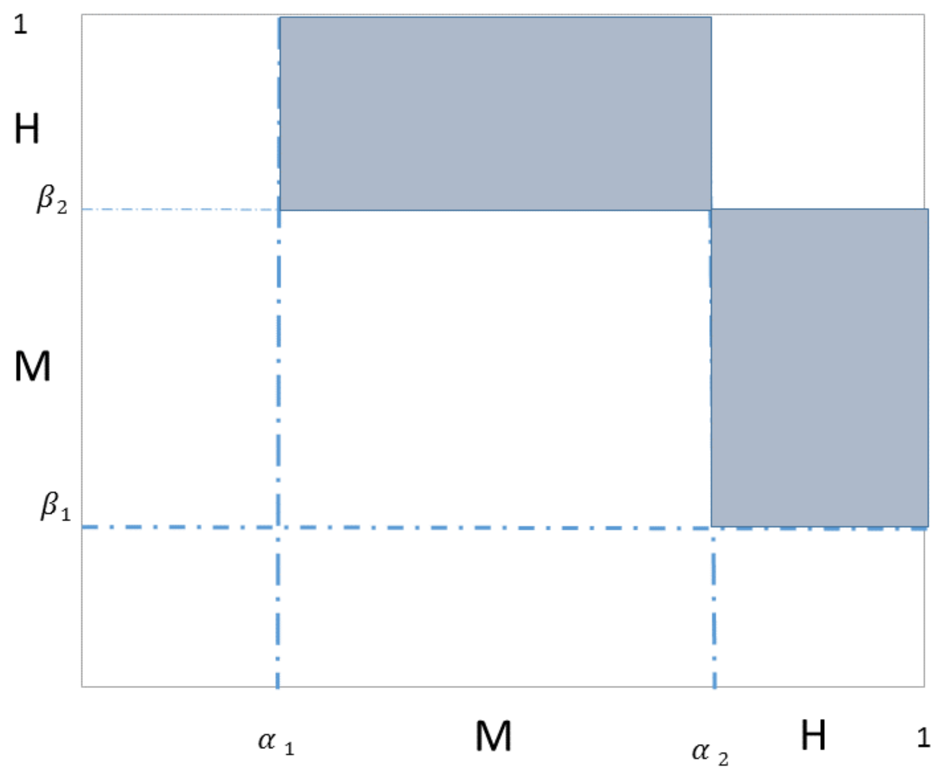
Other Communication Protocols

- Many alternative communication protocols: multiple rounds of communication, intermediate commitments, etc.
- Question: Can the players use other communication protocols to implement matching outcomes not implementable via short protocol SC?
  - The answer is negative, if we restrict attention to monotone deterministic matching rules. In this sense, SC protocol is a compelling implementation instrument.
  - But alternative communication protocols allow to implement non-monotone matching rules, which cannot be implemented via SC.

Non-monotone Outcome functions

- There exist incentive compatible and individually rational *non-monotone* match functions.
- Under these match functions, some types face a tradeoff: either match with a higher probability with lower types or match with a lower probability with higher types.
- Higher types prefer the first alternative, lower types prefer the second alternative.

Non-monotone Outcome Function. Example 1.



IC and IR Non-Monotone Outcome Function. Example.

Linear value function: $v(\alpha, \beta) = \alpha + \beta$. Outside options: $R_a = 1.55$, $R_b = 1.45$.

- "Low" types of A in $[0, 0.58)$ and "low" types of B in $[0, 0.52)$ do not match.
- "Middle" types of A in $[0.58, 0.86)$ match with "high" types of B in $[0.94, 1]$.
- "Middle" types of B in $[0.52, 0.94)$ match with "high" types of A in $[0.86, 1]$.

Non-Monotone Outcome Function

- An IR and IC outcome function can be implemented with a mediator recommending an action or a mechanism designer.
- Can it be implemented via a communication protocol WITHOUT a mediator?
  - Not with an SC protocol which can implement ONLY monotone outcome functions.
  - But longer bilateral communication protocols work.

BCD Protocol

BCD (bilateral communication and decision-making):

- In stage 1, the parties simultaneously send messages to each other.
- In stage 2, the parties simultaneously take decisions to match or not.

Outcome functions implementable via BCD Protocol.

We say that outcome function $f(.)$ induces a 1-to-1 partition structure if the following is true:

- Consider any set Z of types of player A. Then there exists a set I of player B such that: $f(\alpha, \beta) = 1$ for all $\beta \in I$ and $f(\alpha, \beta) = 0$ for all $\beta \notin I$.
  If $\alpha' \notin Z$ then $f(\alpha', \beta) = 0$ for all $\beta \in I$.
- A similar condition holds for any set W of types of player B.
- Informally, if the types of A in some set Z match with the types of B in the set I only, then types in I match only with types in Z on;y.

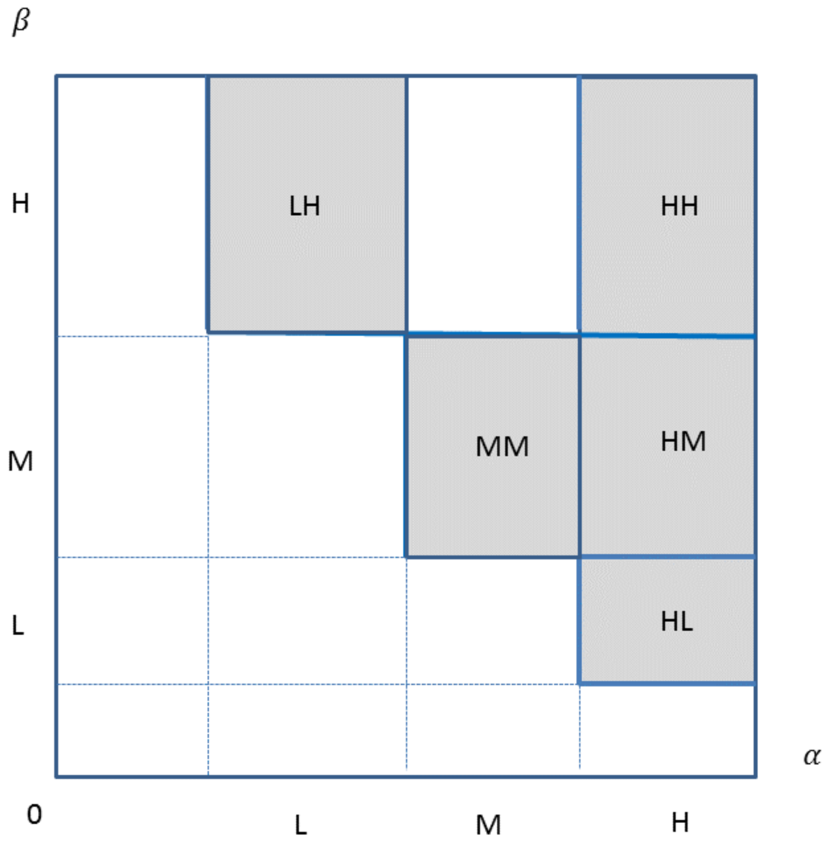
Implementation via BCD protocol.

Theorem

Consider an IC and IR non-monotone deterministic outcome function $f(.)$ that induces a 1-to-1 partition structure. Such outcome function can be implemented via a BCD protocol.

Equilibrium Strategies: In stage 1, each type sends a message revealing truthfully the element of the equilibrium partition to which it belongs. In stage 2, each type votes to match if only if the types are compatible i.e., she receives a message signalling that the other player's type belongs to the set with whom she is supposed to match in equilibrium.
Coordination equilibrium is played in the acceptance stage 2.
This is essential.

Non-Monotone Outcome Function. Example 2.



“Long” Communication Protocol ($LBCD$).

- The outcome function in Example 2 is not implementable via BCD protocol, or with SC protocol because it does not induce a 1-to-1 partition structure.
  Failure to match in region HL: If a type of player A in H learns that the type of player B is in L, then player A will refuse to match.
- In fact, this outcome function is not implementable via any protocol with one round of communication.
- To implement this outcome function need to use a longer communication protocol, with two rounds of communication.

Long communication protocol $LBCD$.

- In Stage 1, player A sends a message to player B.
- In Stage 2, Player B can accept or reject.
- If player B accepts, then the match if formed. If player B rejects, then proceed to stages 3 and 4.
- In stages 3 and 4, BCD protocol is played.
  Stage 3: players simultaneously send messages.
  Stage 4: players simultaneously take acceptance decisions.
  A match is formed after stage 4 if and only if both players accept in this stage.

Implementation via $LBCD$ protocol.

Theorem

There exists an IC and IR non-monotone deterministic outcome function $f(.)$ that induces the partition in Example 2. Such outcome function is implementable via protocol $LBCD$.

Equilibrium Strategies.

- In stage 1, player A sends a message revealing whether her type is in the set H or not.
- In stage 2, player B accepts if and only if A's message is H and B's type is in L, M, or H.
- In stage 3, player A sends a message revealing whether her type is in M, L, or below. Player B sends a message revealing whether her type is in H, M or below.
- In stage 4, both accept iff the combination of messages sent in stage 3 is "LH" or "MM." Otherwise, both reject.

Implications.

- Sequential multistage communication is necessary to implement certain outcome function.
- More complex outcome functions than in Example 2 require additional rounds of communications.

Conclusions

- We analyze cheap talk communication between privately informed parties who take a joint decision.
- Any monotone deterministic outcome function can be implemented via a short communication protocol in which only one party sends a message.
  Invariance: The set of implementable outcomes is invariant to the assignments of the roles of the sender and receiver.
- Bilateral communication is necessary for implementing non-monotone decision rules.
- Non-monotone outcome functions which induce 1-to-1 partition structure are implementable with one round communication via a BCD protocol.
- Non-monotone outcome functions that do not satisfy 1-to-1 partition structure property require longer protocols, with multiple stages of communication and decision nodes.