Multi Agent Information Acquisition and Sharing.

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Abstract

How should a manager optimally choose transfers to incentivize multiple agents both to collect and to share costly information? To answer this question we study a simple model with a principal and two agents. The agents can obtain costly signals and communicate with each another via non-verifiable messages (cheap talk). A principal offers a contract which is separable in the performances of the agents. We characterize the optimal transfers and show a surprising result that for sufficiently correlated information and not too high costs of information acquisition an agent's optimal transfer should depend mainly on the performance of the other agent.

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1 Introduction

How should incentives be optimally designed for multiple agents both to collect and to share costly information that is useful to optimize each agent's performance? If agents are paid for their own individual performances, this might lead to inefficiently high information rents and insufficient information sharing. Remunerating agents on the basis of their relative performance would facilitate information acquisition but harm incentives to share information. Instead, remunerating the agents as a "team" facilitates communication but harms individual incentives to collect information in the first place.

The object of our study is an explicit manifestation of the general question of whether rewarding individual performance (piece-meal rates), team performance (joint bonuses and penalties) or relative performance (tournaments) is the most effective way to incentivize teams of agents.[1] Earlier theoretical papers have studied this question, which is considered fundamental in personnel economics, within models that did not precisely identify the form of individual efforts to be remunerated, nor the form of the uncooperative behavior that makes wage spread undesirable. We explicitly posit that individual effort is placed in costly information acquisition, and that the uncooperative behavior consists of not sharing the information collected. Remarkably, this is the real world example provided by Lazear (1989) to motivate his study of cooperation in teams.[2]

The model we formulate comprises a principal (she) and two agents (he). The former can be thought as the headquarter manager and the latter as local division managers. There are two unobserved local states, drawn from two continuous and correlated distributions. Each agent takes a decision in his division. The principal's profit is separable across the agents' choices, and increasing in how closely each agent's decision matches his division's local state.[3]

[1]It is an age-old insight that piece-meal payments may be more a potent incentive to foster individual effort than fixed wages. Seminal work by Lazear and Rosen (1981) identified a role for tournaments to provide high-powered incentives to workers. But Lazear (1989) later remarked that increasing the wage spread fosters uncooperative behavior among agents, providing an efficiency argument for the desirability of equitable wage structures.

[2]"The term 'sabotage' [i.e., uncooperative behavior] is used as shorthand for any (costly) decisions that one worker takes that adversely affect output of another. For example, erecting barriers so that co-workers cannot obtain useful information falls under this definition." Lazear (1989), page 563.

[3]For tractability each agent's performance net loss due to the mismatch of his action with his local state

Prior to making his decision, each agent can obtain a costly private signal about his local state. The agents can inform each other about their signals using (simultaneous) cheap talk messages. Then the agents simultaneously choose their decisions. The profit determined by each agent's choice is verifiable in a court of law, but the agents' decisions and the local states are not.

The principal offers and commits to a linear transfer scheme which is separable in the performances of both agents: it may remunerate each agent for his own performance and/or for the other agent's performance. The contracts can feature a competitive, tournament element if better performance by the co-worker leads to a worse payoff for the agent, or a cooperative, team element if better performance by the other agent increases the agent's own payoff. The agents are protected by limited liability and negative transfers are ruled out. We ask, what is the optimal pattern of communication and signal acquisition from the principal's perspective and what are the cheapest incentives to achieve it?

It is intuitive that if the signal acquisition cost is sufficiently low, the principal finds it optimal to incentivize both agents to collect and share information. The corresponding optimal linear transfers depend on the correlation between the local states: if it is below a threshold, each agent's remuneration is only responsive to the agent's own performance. Surprisingly, we find that if the state correlation is above the threshold, the optimal linear transfers remunerate each agent mainly for the performance of the other agent.

To understand this result, suppose momentarily that the local states are perfectly correlated, so that each agent's researched information is equally valuable for both agents' decisions. Suppose that agent 1 shirks and does not exert effort in collecting information. In the optimal equilibrium, agent 2 exerts effort researching information, reports this information to 1, and expects 1 to do the same. To fulfill this expectation, agent 1 has to put together something to report to 2. Because it is not based on research, what 1 reports is just noise. Agent 2 takes 1's report seriously and uses it to make his decisions, together with his own costly and valuable research. Agent 2 would make a better decision if he simply ignored agent 1's report, and based his decision only on his own research. This is exactly what agent

is measured as a quadratic loss function.

1 does: He bases his decision on 2's report only. The shirking agent 1 does more damage to his peer 2's performance than to his own: on top of not providing useful information, 1 also biases 2's decision. As a result, the most potent incentive to prevent an agent i from shirking information acquisition is making i's payment sensitive not to his own performance, but to the performance of his peer j.

This finding holds as long as the states are sufficiently correlated. When they are not, the value of the non-shirking agent j's information for the shirking agent i's decision is too low, and the bias induced on j's decision by the noise in agent i's report is too low, so that if an agent i shirks information acquisition, he damages his own performance more than the other agent j's performance. The most potent incentive to prevent shirking is then making each agent i's payment depend on i's own performance.

The logic behind this characterization is not limited to the case of low information acquisition costs, and it is valid as long as the research cost is not so high that profit maximization requires the agents to not collect information. The only difference is that, for such intermediate research costs, the profit maximizing contract is such that only one agent collects and shares information. But again, if the states are sufficiently correlated, this is optimally achieved by making that agent's payment depend mainly on the other agent's performance.

Interestingly, for intermediate research costs and low correlation among states, the optimal linear contract is such that both agents collect information without sharing it. The reason for this counterintuitive result is that, in this parameter range, it is too costly (relative to the expected profit increase) to motivate an agent i to collect information to improve his own performance if i already expects valuable information to be reported from the other agent j. Profits turn out to be maximized by asking agents to research information without sharing it, in exchange for lower expected payments.

We complete the paper by generalizing our results to consider non-linear contracts. We show that allowing the transfers to depend on a direct interaction between the losses of both divisions does not change the optimal contract. Further, we allow for the possibility that the principal can be biased towards one of the divisions. In this case, if the correlation between the local states is not too hight, the principal can prefer an allocation with two acquired signal but only one sided information where an agent of her least preferred division communicates his signal to the other agent.

In terms of the empirical/testable implications of our analysis, it should not be surprising that the incentivation of information acquisition and sharing by multiple agents displays a feature of joint performance remuneration. As pointed out earlier, information sharing is possibly the most obvious form of cooperation in teams of agents. And there is empirical evidence that incentive schemes based in part on joint performance evaluation improve worker's productivity and profit relative to fixed wage structures and individual performance evaluations. As we detail in the literature review, Kruse (1993) document a productivity increase of about 4.5%-5.5% in companies that adopt profit sharing plans in the forms of cash transfers. Further, Kandel and Lazear (1992), Che and Yoo (1999) and Alonso, Dessein and Matouscheck (2008) discuss case studies of major corporations that experienced productivity increments following the restructuring of managerial incentives to include elements of joint performance evaluation.

Pushing these lines of reasoning further, this paper provocatively suggests that, in some cases, optimal incentives may be provided by making an agent's remuneration depend mainly on the performance of other agents. We demonstrate this suggestion in a model of information acquisition and sharing. When each agent's information is equally useful for each agent's decisions, making an agent responsible for the other agent's performance is a more potent incentive to prevent shirking than remunerating the agent for his own performance.

2 Related Literature

Our paper fits into the literature on contract design with multiple agents. A contract can foster either a competitive or cooperative behavior (or both). The competition element is usually associated with a tournament-based contract structure. In a seminal contribution, Lazear and Rosen (1981) show that with risk-neutral agents the tournaments are optimal and result in the same outcomes as piece rates. Green and Stokey (1983) study risk-averse agents and show that tournaments can outperforms piece rates when agents performances are influenced by a common shock unobservable to the principal. There is no common shock in the agents' performances in our model, and there is no role for tournaments in the optimal contracts.

The cooperation element in contracts is studied in Holmström and Milgrom (1990), Lazear (1989), Kandel and Lazear (1992), and Itoh (1991), among others. These papers show in general that with production externalities the principal finds it optimal to reward agents according to the team output. In those cases, rewards which are only contingent on individual performances or on relative performances can harm cooperation and so the principalâĂŹs objectives. Itoh (1993) shows that optimal contracts include team bonuses, when the agents can monitor each other and stipulate self-enforceable side-contracts, and Che and Yoo (2001) study the merits of relative versus joint performance evaluation in a repeated game in which such self-enforcing contracts arise as an equilibrium phenomenon. By considering a model of information acquisition and sharing in teams of agents, we identify a precise case in which joint performance evaluations improve workers' productivity. Our analysis pushes the argument one step further suggesting that, in some cases, agents' remunerations should be based mainly on their co-workers' performances.

Empirical studies find a strong positive relationship between the adoption of profit sharing schemes and a productivity increase, there is no negative post-adoption trend.[4]. For instance, Kruse (1993) presents evidence based on a survey of 500 U.S. companies with publicly traded stock. He documents a productivity increase of about 4.5%-5.5% in companies that adopt profit sharing plans in the forms of cash transfers. This effect is more pronounced in smaller firms, and for larger shares of profit. (The mechanism is not determined: it may be because of increased monitoring in small teams, but also because of workers' cooperation.) There are several case studies that compare the effectiveness of different incentive schemes and show the value of joint bonuses. For example, Alonso, Dessein and Matouscheck (2008) discuss the case of the management restructuring of BPX, the oil and gas exploration division of British Petroleum, in the early '90s by the then head of BPX (and future CEO of BP) John

[4]The empirical importance of joint performance evaluation is, e.g., in Ichniowski and Shaw (2003)

Browne.[5]

Costly acquisition of endogenous information in organizations in studied in Argenziano, Severinov and Squintani (2016) and Pei (2015). These papers study a setting with a single agent who can collect information at a cost and communicate it to the decision maker using a cheap talk message. The setting with multiple agents and cheap talk communication is studied in Alonso, Dessein and Matouscheck (2008), Alonso Dessein and Matouscheck (2010) who assume exogenous information. Angelucci (2016) studies a model with two agents and endogenous costly information. Different to our setting, in their papers the transfers are absent and the principal has to rely on different instruments than transfers to achieve second-best.

3 Model

An organization consists of a headquarter manager and two division managers. For simplicity we refer to the division managers as agents 1 and 2 (he), and to the headquarter manager simply as a principal (she). The principal wants each agent $i = 1, 2$ to take an decision $y_i \in [0, 1]$ which matches an unobserved local state $\theta_i \in [0, 1]$. We assume that the profit function π is separable in the agent's performances and takes the form:[6] $\pi = \pi_1(y_1, \theta_1) + \pi_2(y_2, \theta_2)$, where $\pi_i(y_i, \theta_i) = \overline{\pi} - \ell_i(y_i, \theta_i)$ with $\ell_i(y_i, \theta_i) = -(y_i - \theta_i)^2$ for either agent i. Each agent i's decision y_i gives a higher profit $\pi_i(y_i, \theta_i)$ the more precisely it matches the local state θ_i. The profit π_i consists of $\overline{\pi} \geq 1$ minus the loss $\ell_i(y_i, \theta_i)$ expressed in a simple quadratic form that is standard in communication games since Crawford and Sobel (1982).

The states θ_1, θ_2 are correlated: with probability q both states are perfectly correlated

[5]"Browne decentralized BPX in the early 1990s, creating almost 50 semi-autonomous business units. Initially, since "business unit leaders were personally accountable for their units' performance, they focused primarily on the success of their own businesses rather than on the success of BPX as a whole." (Hansen and von Oetinger, 2001) To encourage coordination between the business units, BPX established changed in the implicit and explicit incentives of business unit leaders to reward and promote them, not just based on the success of their own division, but also for contributing to the successes of other business units. As a result, "Lone stars' those who deliver outstanding business unit performance but engage in little cross-unit collaboration can survive within BP, but their careers typically plateau." (Hansen and von Oetinger, 2001)." Alonso, Dessein and Matouscheck (2008), page 164-165.

[6]The profit formula is generalized in section 6.

and randomly drawn from the uniform distribution $U[0,1]$. With probability $1-q$ each state is drawn independently from $U[0,1]$. Prior to choosing y_i, each agent i can exert a costly effort $c>0$ which enables him to observe a private binary signal about the local state, $s_i \in \{0,1\}$ such that $\Pr(s_i = 1) = \theta_i$. If an agent exerts no effort he cannot obtain an informative signal. After the signals are received and before the decisions (y_1, y_2) are taken the agents can communicate with each other using cheap talk messages. We assume that each agent i has an arbitrary large set of messages M_i with a typical element $m_i \in M_i$.

The principal is not able to observe either agent's messages or the signal acquisition decisions, nor to verify in court the agents' final decisions and the two states of the world. Contracting is thus based only on the agents' performances, determined by the losses ℓ_1 and ℓ_2, that can be verified in court. At the beginning of the game the principal offers (and commits to) a contract t_i for each agent $i=1,2$. For ease of exposition, we focus on linear contracts:[7]

$$t_i(\ell_i, \ell_j) := z_i - a_i\ell_i - b_i\ell_j = z_i - a_i(y_i-\theta_i)^2 - b_i(y_j-\theta_j)^2,$$

where $z_i \in \mathbb{R}$, $a_i \in \mathbb{R}$, $b_i \in \mathbb{R}$, and $j=1,2$, $j \neq i$ denotes the other agent.

As standard in the literature we assume that both agents are protected by limited liability, and cannot be paid negative transfers: it must be the case that $t_i(\ell_i, \ell_j) \geq 0$ for all possible loss realizations $\ell_i \in [0,1]$ and $\ell_j \in [0,1]$. Normalizing the value of their outside option to zero, it must also be the case that each agent i is willing to accept the contract t_i ex-ante, i.e., before deciding whether to collect and share signal s_i and before choosing y_i.

Because $\ell_i = -(y_i-\theta_i)^2$ is the loss determined by agent i's imprecise matching of y_i with θ_i, the contracts t_i are piece-wise linear contracts, composed of a fixed wage $w_i = z_i - a_i - b_i$, a bonus payment $a_i(1-\ell_i)$ that depends on agent i's performance, and a payment $b_i(1-\ell_j)$ based on the other agent j's performance.[8] The contracts t_i can also be interpreted as a mixture of relative performance evaluation and joint performance evaluation based transfers, by letting $\overline{a}_i = (a_i - b_i)/2$ and $\overline{b}_i = (a_i+b_i)/2$ and obtaining: $t_i(\ell_i, \ell_j) = z_i - \overline{a}_i(\ell_i - \ell_j) -$

[7] Optimal contracts are considered in section 6

[8] We will later show that in the optimal linear contracts, it is the case that $a_i < 1$, $b_i < 1$, and that the limited liability constraint binds, so that $t_i(\ell_i, \ell_j) = 0$ for $\ell_i = 1$ and $\ell_j = 1$, and hence $z_i - a_i - b_i = 0$. Then, the contracts $t_i(\ell_i, \ell_j)$ can be decomposed as $t_i(\ell_i, \ell_j) = w_i + a_i(1-\ell_i) + b_i(1-\ell_j)$.

$\bar{b}_i(\ell_i + \ell_j)$. The parameter $\bar{a}_i$ can be understood as a weighting factor for agent i's relative performance and $\bar{b}_i$ as a weighting factor for the team performance: agent i's payment $t_i(\ell_i, \ell_j)$ is more sensitive to the relative loss $(\ell_i - \ell_j)$ the higher is $\bar{a}_i$ and to the aggregate loss $(\ell_i + \ell_j)$ the higher is $\bar{b}_i$.

The game proceeds as follows: First, nature (privately) chooses θ_1, θ_2 and the principal offers (and commits to) contracts (t_1, t_2). Second, the agents $i = 1, 2$ decide whether to collect signals s_i. Then they send simultaneous cheap talk messages m_i to each other. Finally, each agent i chooses y_i, losses ℓ_1 and ℓ_2 are publicly observed and the transfers are paid as specified in the contracts. All elements of the model are common knowledge apart from the private signals. Given the contracts (t_1, t_2), multiple equilibria may exist in the agents' game, for example there is always an equilibrium in which agents do not communicate with each other. As customary, we consider the equilibria that are most informative, and it turns out that these equilibria are also optimal for the principal.

4 Conditional optimal linear transfers

In the first step we fix the profile of decisions over the signal acquisition and communication which the principal wants to implement and show the corresponding optimal payment schemes. We begin by considering the case in which the principal wants both agents to collect information and to communicate it to the other agent. The principal wants to minimize the expected sum of transfers $E[t_1(\ell_1, \ell_2) + t_2(\ell_2, \ell_1)]$ subject to the limited liability constraint and to the constraints that each agent $i = 1, 2$ chooses decision y_i so as to minimize the loss $\ell_i = (y_i - \theta_i)^2$ given her equilibrium information (s_i, m_j), that i collects the costly signal s_i and that i truthfully communicates $m_i = s_i$ to the other agent j.

We begin noting that each agent $i = 1, 2$ is motivated to choose decision y_i so as to minimize the loss $\ell_i = (y_i - \theta_i)^2$ with any $a_i \geq 0$. Because the agent's utility is independent of y_i and θ_i, an arbitrary small but positive a_i ensures that i chooses y_i to minimize ℓ_i.[9] At

[9]Instead, the coefficient a_i needs to be strictly bounded above zero if agent i's effort has a direct effect on the profit π_i, instead of just an indirect effect through information acquisition. In order to motivate agent i to exert such productive effort, the transfer $t_i(\ell_i, \ell_j)$ needs to be significantly sensitive to the loss ℓ_i. In a

the decision stage, he matches y_i to $E_i(\theta_i|s_i, m_j)$, the posterior expectation of θ given his signal s_i and the presumed truthful message m_j by agent j.

Proceeding backwards, we consider how to motivate agents to share their information. The following Lemma formalizes the result that, given that $a_i \geq 0$ for both $i = 1, 2$, each agent i is motivated to truthfully report $m_i = s_i$ by setting $b_i \geq 0$.

Lemma 1. *In the equilibrium with two signals and two-sided truthful communication, agent* $i \in \{1, 2\}$ *does not deviate from truthtelling if* $b_i \geq 0$.

The result is intuitive: to incentivize truthful communication by, say, agent 1, the principal has to make agent 1's payoff dependent on the performance of agent 2. Because $a_2 \geq 0$, agent 2 chooses y_2 to match his expectation of θ_2 given her signal s_2 and the equilibrium belief that $m_1 = s_1$. Since communication is costless and agent 1's utility is independent of y_2 and θ_2, an arbitrary small b_1 ensures that an informed agent 1 sends a truthful message to agent 2.

We consider, next, a deviation of agent 1 at the effort stage. If agent 1 decides to collect a signal which he, then, truthfully communicates to agent 2, and there is a common belief that both signals are collected and truthfully communicated, the expected payoff of agent 1 is calculated as follows. We proceed backwards, and determine agent 1's equilibrium payoff conditional on his signal $s_1 = 0, 1$:

$$\begin{aligned} u_1(s_1) &= z_1 - a_1 \sum_{s_2=0,1} \Pr(s_2|s_1) E[(y_1(s_1, s_2) - \theta_1)^2 | s_1, s_2] \\ &\quad - b_1 \sum_{s_2=0,1} \Pr(s_2|s_1) E[(y_2(s_1, s_2) - \theta_2)^2 | s_2, s_1] - c, \end{aligned} \tag{1}$$

using the fact that agents are truthful in equilibrium, so that $m_2 = s_2$ and $m_1 = s_1$.

supplementary appendix available upon request, we consider the case in which each agent i needs to pay an implementation cost $c_0 > 0$ to precisely choose his implemented action $y_i \in [0, 1]$, and else y_i is random draw from a uniform distribution on $[0, 1]$. We show that our results generalize in a qualitative sense, as long as this 'precise implementation cost' c_0 is not larger than the research cost c.

Because the optimal decisions are $y_i(s_i, s_j) = E(\theta_i|s_i, s_j)$, we can simplify (1) as

$$\begin{aligned} u_1(s_1) &= z_1 - a_1 \sum_{s_2=0,1} \Pr(s_2|s_1) E[(E(\theta_1|s_1, s_2) - \theta_1)^2|s_1, s_2] \\ &\quad - b_1 \sum_{s_2=0,1} \Pr(s_2|s_1) E[(E(\theta_2|s_2, s_1) - \theta_2)^2|s_2, s_1] - c. \end{aligned}$$

Because of symmetry across agents, $E[(E(\theta_2|s_1, s_2)-\theta_2)^2|s_1, s_2] = E[(E(\theta_1|s_1, s_2)-\theta_1)^2|s_1, s_2]$ when $s_1 = s_2$. Adding also symmetry across signal realizations, it is also the case that $E[(E(\theta_2|s_2, s_1) - \theta_2)^2|s_2, s_1] = E[(E(\theta_1|s_1, s_2) - \theta_1)^2|s_1, s_2]$ when $s_1 \neq s_2$. We then rewrite (1) as

$$u_1(s_1) = z_1 - (a_1 + b_1) \sum_{s_2=0,1} \Pr(s_2|s_1) E[(E(\theta_1|s_1, s_2) - \theta_1)^2|s_1, s_2].$$

Suppose that $s_1 = 1$ (the case $s_1 = 0$ is symmetric). As shown in Appendix, the conditional values and densities are

$$\begin{aligned} E(\theta_1|s_1 = 0, s_2 = 0) &= \frac{1}{3+q},\ f(\theta_1|s_1 = 0, s_2 = 0) = \frac{6(1-\theta_1)(1+q-2q\theta_1)}{3+q}, \\ E(\theta_1|s_1 = 0, s_2 = 1) &= \frac{1}{3-q},\ f(\theta_1|s_1 = 0, s_2 = 1) = \frac{6(1-\theta_1)(1+q-2q\theta_1)}{3-q}. \end{aligned}$$

Substituting in the expected losses definitions and simplifying, we obtain, for $s_1 = s_2 = 0$,

$$\begin{aligned} E[(E(\theta_1|s_1, s_2) - \theta_1)^2|s_1, s_2] &= \int_0^1 (E(\theta_1|s_1, s_2) - \theta_1)^2 f(\theta_1|s_1, s_2) d\theta_1 \\ &= \frac{5+2q-q^2}{10(3+q)^2}. \end{aligned}$$

and for $s_1 = 0$, $s_2 = 1$,

$$E[(E(\theta_1|s_1, s_2) - \theta_1)^2|s_1, s_2] = \frac{5-2q-q^2}{10(3-q)^2}.$$

Substituting the expected losses in (1), together with the conditional posteriors which agent 1 assigns to the signal realizations of agent 2 (shown in the appendix)

$$\Pr(s_2 = 1|s_1 = 1) = \frac{3+q}{6},\ \Pr(s_2 = 0|s_1 = 1) = \frac{3-q}{6},$$

and simplifying, we obtain:

$$u_1(s_i) = z_1 - (a_1 + b_1) \frac{3-q^2}{6(9-q^2)} - c. \qquad (2)$$

Because the signals $s_1 = 0$ and $s_1 = 1$ are equally likely ex-ante, $u_1(s_1 = 0) = u_1(s_1 = 1)$ and this is also the expected equilibrium payoff u_i of agent 1 before observing s_1 if collecting his signal.

Now, suppose that agent 1 deviates at the signal acquisition stage and exerts no effort. He, thus, does not obtain a signal about θ_1. In the most informative equilibrium, agent 1 cannot communicate to agent 2 that she did not exert effort.[10] Agent 2 believes that 1 collected s_1 and hence interprets any possible message realization $m_1 \in M_1$ as either meaning that $s_1 = 0$ or that $s_1 = 1$. Like in Argenziano, Severinov and Squintani (2016), the equilibrium language is fixed by the on-path communication strategy. Assume that after the deviation at the effort stage agent 1 sends the message $m_1 = 0$ (sending $m_1 = 1$ results in the same constraint). The corresponding off-path payoff of agent 1 is

$$u_i^o = z_1 - a_1 \sum_{s_2=0,1} \frac{1}{2} E[(y_1(s_2) - \theta_1)^2 | s_2] d\theta_1 - b_1 \sum_{s_2=0,1} \frac{1}{2} \int_0^1 E[y_2(s_2, m_1) - \theta_2)^2 | s_2] d\theta_2, \quad (3)$$

using $m_2 = s_2$. Agent 1's optimal decisions is $y_1(s_2) = E(\theta_1 | s_2)$. To calculate the expected loss

$$E[(y_1(s_2) - \theta_1)^2 | s_2] = E[(E(\theta_1 | s_2) - \theta_1)^2 | s_2],$$

suppose that $s_2 = 0$: by symmetry across signal realizations, $E[(y_1(s_2) - \theta_1)^2 | s_2]$ is the same for $s_2 = 0$ and $s_2 = 1$. As shown in the appendix, the densities and the expected values of θ_1 depending on the realization of $s_2 = 0$ are

$$f(\theta_1 | s_2 = 0) = 1 + q(1 - 2\theta_1), \; E(\theta_1 | s_2 = 0) = \frac{3 - q}{6}.$$

Substituting in the expected loss definition and simplifying, we obtain

$$E[(y_1(s_2) - \theta_1)^2 | s_2] = \int_0^1 (E(\theta_1 | s_2) - \theta_1)^2 f(\theta_1 | s_2) d\theta_1 = \frac{3 - q^2}{36}.$$

Agent 2's decision $y_2(s_2, m_1)$ equal $E(\theta_2 | s_2, s_1 = m_1)$ under the mistaken belief that 1 collected signal s_1 and truthfully reported $m_1 = s_1$. Using again symmetry across signal realizations, we assume that $s_1 = 0$ and calculate the expected loss $E[(E(\theta_2 | s_2, m_1) - \theta_2)^2 | s_2]$ associated with agent 2's decision.

[10] Later, we discuss other equilibria and their plausibility.

When $m_1 = 0$ and $s_2 = 0$, because the conditional densities and expected values (shown in the appendix) are

$$f(\theta_2|s_2) = 2(1-\theta_2) \text{ and } E(\theta_2|s_2, m_1) = \frac{1}{3+q},$$

the expected loss is:

$$\begin{aligned} E[(E(\theta_2|s_2, m_1) - \theta_2)^2|s_2] &= \int_0^1 (E(\theta_2|s_2, m_1) - \theta_2)^2 f(\theta_2|s_2) d\theta_2 \\ &= \frac{3+2q+q^2}{6(3+q)^2}. \end{aligned}$$

When $m_1 = 0$ and $s_2 = 1$, the expected loss is:

$$\begin{aligned} E[(E(\theta_2|s_2, m_1) - \theta_2)^2|s_2] &= \int_0^1 (\frac{2-q}{3-q} - \theta_2)^2 2\theta_2 d\theta_2 \\ &= \frac{3-2q+q^2}{6(3-q)^2}. \end{aligned}$$

Plugging the obtained formulas into (3) and simplifying, we obtain

$$u_i^o = z_1 - a_1 \frac{3-q^2}{36} - b_1 \frac{27+q^4}{6(9-q^2)^2}. \tag{4}$$

To understand the expected losses notice, first, that agent 1 chooses y_1 only based on the signal of agent 2. Moreover, his conditional posterior distribution of θ_1 is only based on the truthful message $m_2 = s_2$. However, as agent 2 mistakenly believes that agent 1 collected s_1 and sent a truthful message m_1, the former chooses y_2 based on both his own signal s_2 and the message m_1 of agent 1. The misled decision y_2 determines a larger loss that if agent 2 knew that agent 1 did not collect s_1. Whether agent 1's shirking at the information acquisition stage induces a larger expected loss $E\ell_1 = \frac{3-q^2}{36}$ or $E\ell_2 = \frac{27+q^4}{6(9-q^2)^2}$ depends on the correlation q across the states θ_1 and θ_2, i.e. on how informative the truthful message $m_2 = s_2$ is for the optimal choice of y_1, and on how misleading is the mistaken belief that 1 collected s_1 and sent a truthful message $m_1 = s_1$ for the choice of y_2. There exists a threshold q' (later determined precisely) such that $\frac{3-q^2}{36} > \frac{27+q^4}{6(9-q^2)^2}$ if and only if $q < q'$. That is, for highly correlated states θ_1 and θ_2, the largest expected loss determined by agent 1 not collecting his signal s_1 consists in misleading the choice y_2 of agent 2, and not in agent 1 choosing y_1 with less information.

The information acquisition incentive constraints are derived by subtracting (3) from (2), and given the payoff symmetry for both agents, we obtain

Lemma 2. *In the equilibrium with two signals and two-sided truthful communication, agent* $i \in \{1, 2\}$ *does not deviate from collecting the signal* s_i *if*

$$\frac{1}{36}\frac{(3-q^2)^2}{9-q^2}a_1 + 2\frac{q^2}{(9-q^2)^2}b_1 \geq c. \tag{5}$$

The final constraint to consider is limited liability.[11] Because the maximal possible loss is $\ell_i = 1$, which occurs when $y_i = 0$ and $\theta_i = 1$ or vice-versa, limited liability requires only that $z_i - a_i - b_i \geq 0$ for $i = 1, 2$. Because the earlier considered equilibrium constraints do not affect z_i, it is obvious that $z_i = a_i + b_i$ in the optimal linear contracts t_i.

Continuing, we note that restricting attention to symmetric pair of linear contracts $t_1 = t_2$ is without loss of generality, because the principal's cost minimization program is linear.[12] Imposing symmetry across agents, we have reduced the principal's cost-minimization program to the following program:

$$\begin{aligned} & \min_{a_1 \geq 0,\, b_1 \geq 0} E[a_1(1 - E[(y_1 - \theta_1)^2 | s_1, s_2]) + b_1(1 - E[(y_2 - \theta_2)^2 | s_2, s_1])] \\ = & \min_{a_1 \geq 0,\, b_1 \geq 0} (a_1 + b_1)[1 - \frac{3 - q^2}{6(9 - q^2)}] \end{aligned} \tag{6}$$

subject to the incentive compatibility constraint (5).

Because the objective function is linear in a_1 and b_1 (with "marginal rate of substitution" equal to one), and the constraint (5) is also linear in a_1 and b_1, the solution of program (6) is generically "bang-bang": either $a_1 > 0$ and $b_1 = 0$, or $a_1 = 0$ and $b_1 > 0$. Also, this is determined solely by whether $\frac{1}{36}\frac{(3-q^2)^2}{9-q^2}$, the coefficient of a_1 in the constraint (5), is larger or smaller than $2\frac{q^2}{(9-q^2)^2}$, the coefficient of b_1.

[11] We show in the proof of Proposition 1 in appendix that the optimal linear contract characterized by the equilibrium constraints considered here also satisfy the ex-ante participation constraint.

[12] Each agent i's constraints are linear in the maximization arguments a_i and b_i. Thus, the constraint set is convex. Hence, suppose that an asymmetric pair of linear contracts $t_1 \neq t_2$ minimized the sum of expected transfers to the agents. Because the model is symmetric, the antisymmetric pair of contracts $t'_1 = t_2$, $t'_2 = t_1$ is also optimal. But then, the constraint set being convex, it contains also the symmetric pair of contracts obtained by averaging these two pairs. As the objective is linear, this symmetric pair contracts is also optimal.

We obtain the following

Proposition 1. *The optimal linear contracts t_1, t_2 to make each agent $i \in \{1,2\}$ collect information and transmit it to the other agent j in the most informative equilibrium depend on q and c as follows:*[13]

1. For $q < q' \approx 0.803$ the optimal contract t_i features $z_i = a_i = 36\frac{9-q^2}{(3-q^2)^2}c$ and $b_i = 0$, which leads to the expected principal's profit

$$E\pi = 2\overline{\pi} - \frac{3-q^2}{3(9-q^2)} - \frac{12}{(3-q^2)^2}(51-5q^2)c.$$

2. For $q > q'$ the optimal contract t_i features $a_i = 0$ and $z_i = b_i = \frac{c(9-q^2)^2}{2q^2}$, which leads to the expected principal's profit

$$E\pi = 2\overline{\pi} - \frac{3-q^2}{3(9-q^2)} - (51-5q^2)\frac{9-q^2}{6q^2}c.$$

For future reference, we note that the expected profit can be written as:

$$E\pi_{22} = 2\overline{\pi} - \frac{3-q^2}{3(9-q^2)} - (51-5q^2)\min\left\{\frac{12}{(3-q^2)^2}, \frac{9-q^2}{6q^2}\right\}c.$$

As earlier anticipated, we revisit the matter of equilibrium selection in the agents' game. For every pair of contracts t_1, t_2 we have assumed that agents coordinate on the most informative equilibrium, which is also the one that maximizes the principal's profit. In the equilibrium we considered, after an agent i deviated by not collecting his signal s_i, he cannot communicate to agent j that he shirked. For the optimal linear contracts t_1, t_2 with $z_i = a_i > 0$ and $b_i = 0$ for $i = 1, 2$, there also exists an equilibrium in which agent i, as well as revealing agent j the realization of s_i, would also be able to communicate to j that he did not collect signal s_i off path. But this equilibrium fails to exist for the optimal contracts t_1, t_2 of Proposition 1 with $a_i = 0$ and $z_i = b_i > 0$ for $i = 1, 2$. Selecting this equilibrium would imply that $z_i = a_i > 0$ and $b_i = 0$ for all q in the optimal contract, hence lowering the principal's profit for $q > q'$.

[13] The precise value of the threshold q' is:

$$q' \equiv \sqrt{5 - 10(\frac{2}{9\sqrt{29}})^{1/3} + 2^{2/3}(9\sqrt{29}-43)^{1/3}}.$$

The plausibility of the equilibrium selected in Proposition 1 relative to the more demanding equilibrium we just described ultimately depends on the intended application. It may sound appealing that agent i be capable to "collude" by alerting j that he does not have information useful for j's decision y_j. However, it is not plausible that i would reveal his co-worker j that he shirked and did not collect the information he was supposed to. Agent j does not shirk in equilibrium, and would feel "cheated" by i and retaliate, possibly reporting to the principal that i shirked. Further, the equilibrium selected in Proposition 1 is collusive in the sense that it maximizes the agents' payoffs when the optimal linear contracts t_i are stipulated.

Having determined the optimal linear contracts to make both agents i collect signal s_i and share it with the other agent j, we calculate the optimal linear contracts for the other two cases in which both agents collect information. In the first one, both agents collect information and only one of them shares with the other agent. In the second case, both agents collect information and neither of them shares it with the other agent.

The optimal linear contract t_i for the agent(s) i who are not intended to share signal s_i with j is such that $a_i > 0$ and $b_i = 0$ for all values of c and q. The optimal contract is piece rate as the principal does not want to make i share his information with j. Again, the information acquisition constraints leads to a trade-off within the optimal linear contract for an agent i incentivized to share signal s_i. For low values q of correlation among the states θ_1 and θ_2, optimality requires that $a_i > 0$ and $b_i = 0$. When θ_1 and θ_2 are highly correlated, the optimal linear contract is such that $a_i = 0$ and $b_i > 0$.

Proposition 2. *The optimal linear contracts t_1, t_2 to induce both agents $i = 1, 2$ to collect information and only one of them, say agent 1, to transmit it to the other agent j is such that $z_2 = a_2 = 36\frac{9-q^2}{(q^2-3)^2}c$, $b_2 = 0$, and that $z_1 = a_1 = 36c$, $b_1 = 0$ for $q < \sqrt{\frac{(28\sqrt{3}-\sqrt{2267})3\sqrt{3}}{5}} \approx 0.958$, and $a_1 = 0$, $z_1 = b_1 = \frac{(9-q^2)^2}{2q^2}c$ for $q > \sqrt{\frac{(28\sqrt{3}-\sqrt{2267})3\sqrt{3}}{5}}$. This yields the principal's expected profit:*

$$E\pi_{21} = 2\overline{\pi} - \frac{9-2q^2}{9(9-q^2)} - \min\left\{34, (51-5q^2)\frac{(9-q^2)}{12q^2}\right\} c - 6\frac{51-5q^2}{(3-q^2)^2}c.$$

Optimal linear contracts t_1, t_2 to induce both agents $i = 1, 2$ to collect information and not *to transmit it to the other agent j are such that $z_i = a_i = 36c$, $b_i = 0$,*[14] *and yield expected profit:*

$$E\pi_{20} = 2\overline{\pi} - \frac{1}{9} - 68c.$$

The next result shows the optimal linear contracts to induce a single agent i to collect information, and either then share it with agent j or not. Agent j is not asked to collect information. The characterization in Proposition 3 mirrors the case for two agents (Propositions 1 and 2), but the optimal linear for agent j is irresponsive of the performance losses ℓ_i and ℓ_j, i.e., with $a_i = 0$ and $b_j = 0$.

Proposition 3. *Optimal linear contracts t_1, t_2 to make one agent, say agent 1, collect information and transmit it to the other agent 2, and agent 2 to not collect information, are such that $z_2 = a_2 = b_2 = 0$, and that $z_1 = a_1 = 36c$, $b_1 = 0$ for $q < \sqrt{\frac{33}{67}} \approx 0.701$ and $a_1 = 0$, $z_1 = b_1 = \frac{18}{q^2}c$ for $q > \sqrt{\frac{33}{67}}$. They yield the principal's expected profit:*

$$E\pi_{11} = 2\overline{\pi} - \frac{1}{36}(5 - q^2) - \min\left\{34, \frac{1}{2}\frac{q^2 + 33}{q^2}\right\} c.$$

Optimal linear contracts t_1, t_2 to induce only one agents, say agent 1 to collect signal s_1, but not to transmit to agent 2 are such that $z_1 = a_1 = 36c$, $b_1 = 0$ and $z_2 = a_2 = b_2 = 0$. They yield expected payoff:

$$E\pi_{10} = 2\overline{\pi} - \frac{5}{36} - 34c.$$

To conclude, it is obvious that the optimal linear contracts t_1, t_2 in the case that both agents $i = 1, 2$ are not supposed to collect information are such that $z_i = a_i = b_i = 0$. The only role played by contracts is to ensure that each agent i matches y_i with the state θ_i to the best of the shared knowledge that θ_i is uniformly distributed on $[0, 1]$. Because neither agent i derives any (dis-)utility from the decision y_i and state θ_i, this objective can be achieved with any $a_i \geq 0$ as earlier pointed out. It is easily shown in the appendix that such optimal

[14] The optimal linear contracts are such that $a_i = 36c$ and the parameters z_i and b_i are undetermined, under the constraint that $a_i + b_i = z_i$. In the interest of simplicity, we henceforth only report linear contracts with $a_i = 0$ and/or $b_i = 0$, when such contracts are optimal. The idea is that the principal stipulates a simple contract unless bonuses are useful to provide incentives to the agents.

linear contracts yield to the principal profit:

$$E\pi_{00} = 2\overline{\pi} - \frac{1}{6}.$$

5 Optimal linear contracts

For different values of the information acquisition cost c and state correlation q parameters, the principal's expected profit may be maximized by different information acquisition and communication agents' choices, together with the associated optimal linear transfers. We begin the analysis by showing that some of the agents' choices are dominated for all parameter c and q values. We consider the optimal linear contracts to make only one agent i collect and share his signal s_i with the other agent j, and j not collect his signal s_j. We show that these contracts yield a higher profit than the optimal linear contracts that make i collect his signal s_i without sharing it with j, and j not collect s_j.

Lemma 3. *For all cost c and correlation values q, it is the case that $E\pi_{11} \geq E\pi_{10}$, and the inequality is strict for almost all c and q. The expected profit $E\pi_{11}$ of the optimal linear contracts t_1, t_2 inducing only one agent i to collect s_i and share it with the other agent j is larger than $E\pi_{10}$, the optimal profit obtained when i collects s_i and does not share it with j.*

This result is intuitive. Given that the optimal linear contracts t_1, t_2 induce only one agent, say agent 1 to collect his signal s_1, there is no reason not to make him also share s_1 with the other agent 2. The only consequence of transmitting s_1 to agent 2 is that the precision of agent 2's decision y_2 improves, and this reduces the loss ℓ_2 borne by the principal. Further, it is very cheap to make agent 1 share s_1, as this can be achieved with any $b_1 \geq 0$.

However, this simple logic does not extend to the optimal contracts that make both agents collect information. For some c and q, it is not true that the expected profit $E\pi_{22}$ of the optimal linear contracts that makes both agents i collect s_i and share it with the other agent j is larger than $E\pi_{21}$, the expected profit of optimally inducing both agents i to collect s_i but only one of them to share it, nor that $E\pi_{22}$ is larger than $E\pi_{20}$, the expected profit of optimally making both agents i collect s_i without sharing it.

The reason for this result is subtle. Suppose that both agents $i = 1, 2$ are asked to collect their signals s_i by the principal. Consider an agent, say agent 2, and suppose that he expects to receive signal s_1 from agent 1 in equilibrium. Then, the informational benefit of collecting signal s_2 is smaller than when he does not expect to receive s_1. As a result, the contractual transfer needed to make agent 2 collect s_2 needs to reward agent 2's decision precision more than when 2 does not receive s_1 in equilibrium. When the cost of information c is sufficiently high, it becomes so expensive to simultaneously make agent 1 send s_1 to agent 2 and agent 2 collect s_2, that the principal is better off not asking agent 1 to share s_1 with agent 2.

Of course, this intuition does not apply to the comparison between $E\pi_{11}$ and $E\pi_{10}$, because in this case agent 2 is not asked to collect s_2 by the principal. Further, this intuition does not entirely invalidates the possibility of comparing expected profit in the three cases in which both agents are asked to collect their signals by the principal. It turns out that for every information cost value c and every correlation value q, the choice of asking both agents i to collect s_i and only one of them to share s_i with the other agent j is either dominated by asking both i to collect and also share s_i, or by asking both i to collect s_i without sharing it.

Lemma 4. *For all cost c and correlation values q, it is either the case that $E\pi_{21} \leq E\pi_{22}$ or that $E\pi_{21} \leq E\pi_{20}$ or both, and the inequalities are strict for almost all c and q. The expected profit $E\pi_{21}$ of the optimal linear contracts t_1, t_2 inducing both agents i to collect s_i and only of them to share it with the other agent j is either smaller than $E\pi_{22}$, the optimal profit obtained when both agents $i = 1, 2$ collect and share s_i, or smaller than $E\pi_{20}$, the optimal profit obtained when both agents $i = 1, 2$ collect s_i without sharing it with the other agent j.*

The intuition for this result lies in the separability of the principal's expected profit and symmetry across players. Given that both agents i are asked to collect their signal s_i, it is either the case that it is more advantageous to also ask both to share s_i so as to improve j's decision precision, or that this would increase the reward needed to make j collect s_j so much that it is better not to ask either agent i to share s_i. But it cannot be the case that it is optimal to ask one agent to share his signal and the other not to.

The optimal linear contracts inducing the remaining four possible course of actions (both

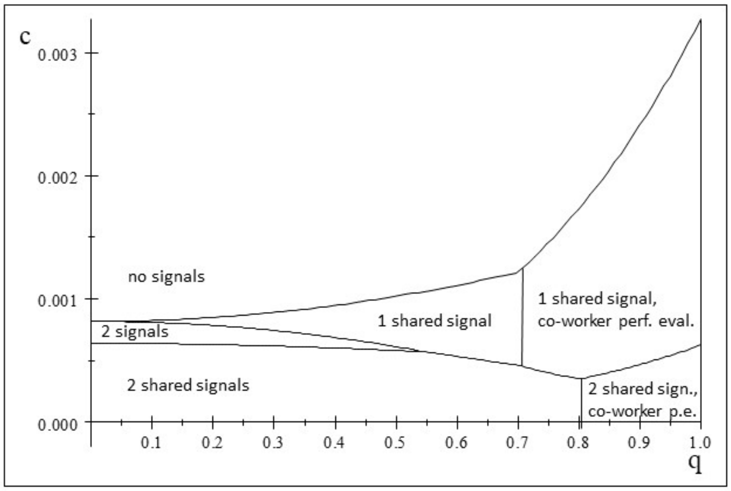

Figure 1: Optimal Linear Contracts

agents $i = 1, 2$ collecting and sharing signals s_i, both agents $i = 1, 2$ collecting s_i without sharing it, only one agent i collecting and sharing s_i, and neither agent $i = 1, 2$ collecting s_i) turn out to maximize the principal's profit in different areas of the parameter space defined by the information acquisition cost c and state correlation q. The complete characterization is summarized in the proposition that follows, and depicted in Figure 1, which also identifies the areas in which the optimal linear contracts t_1, t_2 reward information acquisition and transmission by an agent i by making i's payment depend on the performance of the other agent j.

Proposition 4. *The profit maximizing agents' actions achieved through the optimal linear* *contracts are as follows.*[15]

1. *For research cost* $c < c_{22-20}(q)$ *and correlation* $q < \tilde{q} \approx 0.553$, *and for* $q > \tilde{q}$ *and* $c < c_{22-11}(q)$, *both agents* $i = 1, 2$ *collect signal* s_i *and share it with the other agent* j.[16]

2. *When* $q < \tilde{q}$ *and* $c_{22-20}(q) < q < c_{20-11}(q)$, *both agents* $i = 1, 2$ *collect signal* s_i *but do not share it with* j.

3. *When* $c_{22-11}(q) < c < c_{11-00}(q)$, *for all* q, *only one agent* i *collects signal* s_i *and shares it with* j.

4. *When* $c > c_{11-00}(q)$, *for all* q, *neither agent* i *collects signal* s_i.

We conclude this section by combining Propositions 1, 3 and 4 to present the main result of our analysis. We describe the optimal linear contracts t_1, t_2 that induce agent(s) i to collect and share information by making i's payment depend on the other agent j's performance.

Corollary 1. *If the states are sufficiently correlated* ($q > q' \approx 0.803$) *and signal acquisition cheap enough* ($c < c(q)_{22-11}$), *then* $a_i = 0$ *and* $z_i = b_i > 0$ *for both* $i = 1, 2$. *Each agent* $i = 1, 2$ *is induced to collect signal* s_i *and share it with the other agent* j *with a reward based on the other agent* j*'s performance.*

For sufficient state correlation ($q > \sqrt{\frac{33}{67}} \approx 0.701$) *and intermediate signal costs* ($c(q)_{22-11} < c < c(q)_{11-00}$), *only one agent* i *is induced to collect* s_i *and share with* j *with a reward based* j*'s performance. (The other agent receives a flat payment.)*

For all other values of q *and* c, *each agent* i *is induced to collect* s_i *(and possibly share* s_i *with* j*) only with rewards based on agent* i*'s own performance.*

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[15] The threshold functions used in the statement are:

$$c_{22-20}(q) = \frac{\frac{1}{9} - \frac{3-q^2}{3(9-q^2)}}{\min\left\{\frac{12}{(3-q^2)^2}, \frac{9-q^2}{6q^2}\right\}(51 - 5q^2) - 68}, \quad c_{20-11}(q) = \frac{1 - q^2}{2448 - 36\min\left\{34, \frac{q^2+33}{2q^2}\right\}}$$

$$c_{22-11}(q) = \frac{\frac{5-q^2}{36} - \frac{3-q^2}{3(9-q^2)}}{\min\left\{\frac{12}{(3-q^2)^2}, \frac{9-q^2}{6q^2}\right\}(51 - 5q^2) - \min\left\{34, \frac{1}{2}\frac{q^2+33}{q^2}\right\}}, \quad c_{11-00}(q) = \frac{1 + q^2}{36\min\left\{34, \frac{1}{2}\frac{q^2+33}{q^2}\right\}}$$

[16] The precise value of the threshold is: $\tilde{q} \equiv \sqrt{\frac{63}{17} - \sqrt[3]{\frac{54\,576}{4913} - \frac{1}{289}\sqrt{9914\,048}} - \frac{484}{289\sqrt[3]{\frac{54\,576}{4913} - \frac{1}{289}\sqrt{9914\,048}}}}$

This section has determined the profit maximizing linear contracts for our baseline model. We have uncovered an important role for joint performance evaluations. Making one agent's remuneration depend on his co-worker's performance may be a very potent incentive for information acquisition and sharing in teams of agents. In the extreme case in which each agent's information is equally useful for both agents tasks, we have shown that such an incentive is even more potent than remunerating the agent for his own performance. The model solved in this section is kept simple with the purpose of presenting our results in the cleanest manner. The next section explores how our results would generalize when considering robustness exercises.

6 Robustness exercises

Non-linear contracts. This section shows the robustness of the optimal linear contract for sufficiently low costs and a simple extension to non-linear contracts. Consider a contract with an additional interaction term $d_i l_i l_j$ to linear contracts such that a non-linear contract can be written as $t_i(\ell_i, \ell_j) = z_i - a_i\ell_i - b_i\ell_j - d_i\ell_i\ell_j$. We focus on sufficiently low costs such that the principal wants to implement an allocation with complete information acquisition and sharing. We ask, which profile (a_i, b_i, d_i) for $i = 1, 2$ minimizes principal's costs to implement the most informative equilibrium? The next proposition shows that the non-linear contract specified above does not improve upon the optimal linear contract.

Proposition 5. *Conditional on sufficiently low costs, the optimal non-linear contract t_1, t_2 to make each agent $i \in \{1, 2\}$ to collect information and transmit it to the other agent j in the most informative equilibrium is the same as the optimal linear contract.*

To understand this result, we start with the incentives in the non-linear case. The functional form of the transfers implies immediately that each agent $i = 1, 2$ optimally chooses $y_i = E_i(\theta_i | s_i, m_j)$ upon receiving a message m_j from the other agent. Given the previously obtained expected payoffs, the following incentive constraint ensures that agent i

truthfully reveals his signal to agent j:

$$z_i - (a_i + b_i)\frac{3-q^2}{6(9-q^2)} - d_i\Big(\frac{3-q^2}{6(9-q^2)}\Big)^2 \geq$$
$$z_i - a_i\frac{3-q^2}{6(9-q^2)} - b_i\frac{(3+q^2)(9+q^2)}{6(9-q^2)^2} - d_i\Big(\frac{3-q^2}{6(9-q^2)}\Big)\Big(\frac{(3+q^2)(9+q^2)}{6(9-q^2)^2}\Big)$$

which results in $b_i \geq -\frac{d(3-q^2)}{6(9-q^2)}$. Notice how the additional interaction between the losses of both agents allows either for $(b_i > 0, d_i < 0)$ or $(b_i < 0, d_i > 0)$. This finding is intuitive as truthful communication of a signal to another agent only requires a decreasing pay in the losses of the other division. Since those losses appear twice, the constraint allows for one of the parameters (b_i, d_i) to be negative as long as it is "compensated" by a sufficiently positive another parameter.

Next, we study the incentives which ensure costly signal acquisition. Since the equilibrium language is fixed, the deviating agent not only conceals the absence of the signal, but additionally misleads the other agent by communicating either $m_i = 0$ or $m_i = 1$. Since the problem is symmetric for each message, we focus on $m_i = 0$. Signal acquisition requires

$$z_i - (a_i + b_i)\frac{3-q^2}{6(9-q^2)} - d_i\Big(\frac{3-q^2}{6(9-q^2)}\Big)^2 - c \geq$$
$$z_i - a_i\frac{3-q^2}{36} - b_i\frac{27+q^4}{6(9-q^2)^2} - d_i\Big(\frac{3-q^2}{36}\Big)\Big(\frac{27+q^4}{6(9-q^2)^2}\Big).$$

which holds for

$$a_i \geq \frac{-432b_iq^2 + 216c\left(9-q^2\right)^2 - d_i\left(3-q^2\right)\left(q^2+3\right)^2}{6\left(9-q^2\right)\left(3-q^2\right)^2}.$$

This result coincides with the linear case for $d_i = 0$. Choosing $d_i > 0$ ($d_i < 0$) allows to decrease (increase) either a_i or b_i (or both) such that the above constraint still holds. Finally, notice that the we can express limited liability assuming the following form for the transfers

$$t_i(\ell_i, \ell_j) = a_i(1-\ell_i) + b_i(1-\ell_j) + d_i(1-\ell_i\ell_j) + z_i'$$

where

$$z_i' = \begin{cases} 0 & (a_i, b_i, d_i) \geq 0 \\ a_i \mathbb{1}_{a_i<0} + b_i \mathbb{1}_{b_i<0} + d_i \mathbb{1}_{d_i<0} & \text{otherwise.} \end{cases}$$

It is easy to check that the above functional form guarantees non-negative transfers for every combination of $(y_1, y_2, \theta_1, \theta_2)$ where the parameters a_i, b_i, d_i can take negative values.

To understand the result in Proposition 5 think, first, of a contract which only depends on d_i. Given the constraint at the effort stage, it requires $d_i > 0$. Given the limited liability, the transfer is $t_i = d_i(1 - \ell_i \ell_j)$. Since $E(1 - \ell_i) < E(1 - \ell_i \ell_j)$, the principal pays higher rents compared to an optimal linear contract. In particular, for $q < q'$ the linear case conditions the transfers on the full losses of the agent's own division whereas a contract which only depends on d_i conditions the agent's pay on the interaction of both losses which is a fraction of the agent's own losses. As a result, the combination of limited liability and the costly incentives to acquire information results in higher losses for the principal. The argument is similar for $q > q'$.

The same logic applies for $d_i \geq 0$ and either $(a_i > 0, b_i = 0)$ or $(a_i = 0, b_i > 0)$. The principal does not benefit allowing for $d_i > 0$ as this results in higher rents which cannot be compensated by a corresponding decrease in either a_i or b_i to satisfy the constraint at the effort acquisition stage.

Notice that the communication constraint allows for either $(b_i < 0, d_i > 0)$ or $(b_i > 0, d_i < 0)$. However, as shown in the proof, the net change in transfers to make all constraints hold is strictly positive compared to the optimal linear transfer. Thus, when allowing for the above non-linear specification, the principal optimally chooses $d_i = 0$.

Asymmetric weights on agents. Suppose that the principal's objective is $\pi = \lambda_1(\pi_1 - \ell_1) + \lambda_2(\pi_2 - \ell_2)$ with $\ell_i = -(y_i - \theta_i)^2$ for each agent i. The transfer for each agent is the same as before:

$$t_i(l_i, l_{-i}) = z_i - a_i(y_i - \theta_i)^2 - b_i(y_i - \theta_i)^2, \; i = 1, 2,$$

which means that the conditional optimal transfers remain the same. However, as we show below, for $\lambda_1 \neq \lambda_2$ an asymmetric contract with two signals and one-sided communication is optimal even if the costs of information acquisition are very low.

Proposition 6: *An allocation in which both agents acquire information and only agent j informs agent i dominates all other allocations if for $\lambda_j > \lambda_i$ the following conditions on (q, c) are satisfied:*

1. *For $q < q' \approx 0.803$:*

$$c \in \left(\lambda_i \frac{(3-q^2)^2}{18(9-q^2)(87-17q^2)}, \lambda_j \frac{(3-q^2)^2}{18(9-q^2)(87-17q^2)}\right].$$

2. *For $q' < q < q_1 \approx 0.855$:*

$$c \in \left(\lambda_i \frac{2q^4\left(q^2-3\right)^2}{3\left(9-q^2\right)\left(5q^8-330q^6+2484q^4-7290q^2+4131\right)}, \lambda_j \frac{(3-q^2)^2}{18(9-q^2)(87-17q^2)}\right].$$

First, notice that, different to the case of an unbiased principal ($\lambda_1 = \lambda_2$), whenever the correlation coefficient q is sufficiently low and $\lambda_j > \lambda_i$, there is a range of costs where the principal wants to implement an asymmetric allocation with two signals and one-sided information. The principal wants both agents to obtain a signal and agent i to communicate his signal to the principal's most prefered division j. The larger the weight which the principal assigns to his favorite division, relative to the other division, the larger the range of cost which rationalizes the above asymmetric allocation as the most preferred for the principal. For the intermediate values of correlation, the value of communicating a signal increases. Thus, the principal only prefers an asymmetric allocation with two signals and i communicating to j if the gap $\lambda_j - \lambda_i$ is sufficiently large.

7 Conclusion

This paper studies a model of an organization where the principal designs transfers to incentivize both acquisition and communication of costly information between two agents. We show that whenever the agent's local information is sufficiently correlated with the local information of the other agent and the costs of information acquisition are not too high, the principal chooses a transfer scheme using a threshold strategy. If the correlation between the local states is below a threshold, the principal links the agents' transfers only to the agent's own performances. Otherwise she links the agent's transfer only to the performance of the other agent. For a low probability of correlation and not too high costs of information acquisition the principal wants to implement an allocation where both agents collect information but not communicate with each other. For sufficiently high costs of signal acquisition the principal prefers a single agent to collect and to communicate his signal if the correlation probability between the states is sufficiently high. Otherwise a message is not informative enough to justify incentivizing communication and so the principal prefers a no-signal allocation. For sufficiently low costs of information acquisition we showed the robustness of the results to a non-linear contract.

We studied a tractable beta-binomial model. It would be interesting to look at a more general information environment and see how it affects the optimal mixture between the team and the tournament element. Another interesting question which can be addressed in current framework is, how should a principal design an organization if she is not able to commit to contracts. In this case she has to rely on further instruments such as delegation of decision rights or structuring the sequence of communication.

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Appendix

Update about the signals. It is useful to see how the players update their beliefs based on obtained signals and received messages. Suppose, first, that only agent 1 obtains a signal. The posterior density of θ_1 given s_1 is obtained via Bayes rule:

$$f(\theta_1|s_1) = \frac{f(\theta_1)f(s_1|\theta_1)}{\int_0^1 f(s_1|\theta_1)d\theta_1}$$

where

$$f(s_1|\theta_1) = \theta^{s_1}(1-\theta_1)^{1-s_1}.$$

Thus, for $s_1 = 0$ the density is $f(\theta_1|s_1) = 2(1-\theta_1)$ with the expected value $E[\theta_1|s_1] = \frac{1}{3}$ and for $s_1 = 1$ the density is $f(\theta_1|s_1) = 2\theta_1$ with the expected value $E[\theta_1|s_1] = \frac{2}{3}$.

Next, suppose that only agent 2 obtains a signal and truthfully communicates it to agent 1. The posterior density of agent 1 is

$$f(\theta_1|s_2) = \frac{f(\theta_1)f(s_2|\theta_1)}{\int_0^1 f(s_2|\theta_1)d\theta_1}$$

with

$$f(s_2|\theta_1) = q\underbrace{\theta_1^{s_2}(1-\theta_1)^{1-s_2}}_{Pr(s_2|\theta_1)|\theta_1=\theta_2} + (1-q)\underbrace{(1/2)}_{Pr(s_2|\theta_1)|\theta_1\neq\theta_2}.$$

The densities and the expected values of θ_1 depending on the realization of $s_2 \in \{0,1\}$ are

$$f(\theta_1|s_2=0) = 1 + q(1-2\theta_1) \text{ with } E(\theta_1|s_2=0) = \frac{3-q}{6}, \text{ and}$$

$$f(\theta_1|s_2=1) = 1 - q(1-2\theta_1) \text{ with } E(\theta_1|s_2=1) = \frac{3+q}{6}.$$

Naturally, if $q = 0$ then the posterior $f(\theta_1|s_2)$ is equal to the prior. For $q > 0$ and $s_2 = 0$ ($s_2 = 1$) the posterior puts a larger mass to the left (right) of $\frac{1}{2}$. As q increases, the expected value converges to $\frac{1}{3}$ ($\frac{2}{3}$).

The conditional distributions that agent 1 assigns to the signal realization of agent 2 are

$$\begin{aligned}
\Pr(s_2=1|s_1=1) &= q\Pr(s_2=1|s_1=1,\theta_1=\theta_2) + (1-q)\Pr(s_2=1|s_1=1,\theta_1\neq\theta_2) \\
&= q\frac{2}{3} + (1-q)\frac{1}{2} = \frac{3+q}{6}, \\
\Pr(s_2=0|s_1=1) &= q\frac{1}{3} + (1-q)\frac{1}{2} = \frac{3-q}{6}.
\end{aligned}$$

Suppose that both agents collect and truthfully communicate their signals. We consider θ_1, the case for θ_2 is symmetric. The density of θ_1 after obtaining s_1 and receiving $m_2 = s_2$ is

$$f(\theta_1|s_1,s_2) = \frac{f(\theta_1,s_1,s_2)}{f(s_1,s_2)} = \frac{f(s_1,s_2|\theta_1)f(\theta_1)}{\int_0^1 f(s_1,s_2|\theta_1)f(\theta_1)d\theta_1}.$$

To derive $f(s_1, s_2|\theta_1)$ notice that the following. First, the ex ante probability of $s_1+s_2 = 0$ and $s_1+s_2 = 1$ (it means when $s_1 = s_2$) is $\frac{1}{3}$, whereas the ex ante probability of both signals being different is $\frac{1}{6}$. To see this notice that $Pr(l|n = 2) = \int_0^1 Pr(l|\theta_1, n = 2)d\theta_1 = \frac{1}{n+1}$ and that *conditional* on a particular l all sequences of signals which result in the same sum of signals l are equiprobable. Second, if both states are correlated which happens with probability q, the probability of $l = s_1 + s_2$ is $\frac{n!}{l!(n-l)!}\theta_1^l(1-\theta_1)^{n-l_1}$. With the converse probability $1-q$ the probability of s_1 is $\theta_1^{s_1}(1-\theta_1)^{1-s_1}$ and the realization of s_2 is independent of θ_1 (and so of s_1) and is equal to $\frac{1}{2}$.

Therefore, for $s_1 + s_2 = l \in \{0, 2\}$ we have

$$f(s_1, s_2|\theta_1) = q \underbrace{[\theta_1^l(1-\theta_1)^{2-l_1}]}_{Pr(s_1,s_2|(\theta_1,\theta_1=\theta_2))} + (1-q) \underbrace{[\theta_1^{s_1}(1-\theta_1)^{1-s_1}]\frac{1}{2}}_{Pr(s_1,s_2|(\theta_1,\theta_1\neq\theta_2))}$$

and for $s_1 + s_2 = 1$ we have

$$f(s_1, s_2|\theta_1) = q \underbrace{\frac{1}{2}[2\theta_1(1-\theta_1)]}_{Pr(s_1,s_2|(\theta_1,\theta_1=\theta_2))} + (1-q) \underbrace{[\theta_1^{s_1}(1-\theta_1)^{1-s_1}]\frac{1}{2}}_{Pr(s_1,s_2|(\theta_1,\theta_1\neq\theta_2))}$$

The corresponding densities of the posterior are, for $s_1 + s_2 = l \in \{0, 2\}$

$$f(\theta_1|s_1, s_2) = \frac{q\theta_1^l(1-\theta_1)^{2-l_1} + (1-q)\theta_1^{s_1}(1-\theta_1)^{1-s_1}\frac{1}{2}}{\int_0^1 q\theta_1^l(1-\theta_1)^{2-l_1} + (1-q)\theta_1^{s_1}(1-\theta_1)^{1-s_1}\frac{1}{2}d\theta_1}$$

and

$$f(\theta_1|s_1, s_2) = \frac{q\frac{1}{2}2\theta_1(1-\theta_1) + (1-q)\theta_1^{s_1}(1-\theta_1)^{1-s_1}\frac{1}{2}}{\int_0^1[q\frac{1}{2}2\theta_1(1-\theta_1) + (1-q)\theta_1^{s_1}(1-\theta_1)^{1-s_1}\frac{1}{2}]d\theta_1}.$$

The calculations for θ_2 are symmetric.

Assume two efforts and truthful communication. The corresponding posteriors, and the expected values for agent 1 (the analysis for agent 2 is analogous) are:

$$\begin{aligned} f(\theta_1|s_1 = s_2 = 0) &= \frac{q[\frac{2!}{0!(2-0)!}\theta_1^0(1-\theta_1)^{2-0}] + (1-q)[\frac{1!}{0!(1-0)!}\theta_1^0(1-\theta_1)^{1-0}]\frac{1}{2}}{\frac{3+q}{12}} \\ &= \frac{6(1-\theta_1)(1+q-2q\theta_1)}{3+q}, \end{aligned}$$

with

$$E(\theta_1|s_1 = s_2 = 0) = \int_0^1 \theta_1 \frac{6(1-\theta_1)(1+q-q2\theta_1)}{3+q}d\theta_1 = \frac{1}{3+q}.$$

Further,

$$f(\theta_1|s_1 = 0, s_2 = 1) = \frac{6(1-\theta_1)(1-q+q2\theta_1)}{3-q}, \text{ with } E(\theta|s_1 = 0, s_2 = 1) = \frac{1}{3-q}.$$

Further,

$$f(\theta_1|s_1=1,s_2=0)=\frac{6\theta_1(1+q-2q\theta_1)}{3-q},\text{ with } E(\theta_1|s_1=1,s_2=0)=\frac{2-q}{3-q}.$$

Then,

$$f(\theta_1|s_1=s_2=1)=\frac{6\theta_1(1-q+2q\theta_1)}{3+q},\text{ with } E(\theta_1|s_1=s_2=1)=\frac{2+q}{3+q}.$$

Proof of Lemma 1: Given the optimal choices (y_1, y_2) consider the incentives of agent 1 at the communication stage if he holds a signal $s_1 = 0$. For a common belief that both agents are truthful (which is correct in equilibrium) agent 2 chooses an decision which matches his posterior of θ_2 given his own private signal s_2 and the message from agent 1, m_1. The expected payoff of agent 1 if he reveals his signal truthfully to agent 2 is given by (2), calculated in section 4 in the analysis that leads to Lemma 2:

$$u_1(s_1)=z_1-(a_1+b_1)\frac{3-q^2}{6(9-q^2)}.$$

If agent 1 decides to deviate at the communication stage and to inform agent 2 that his signal is 1 instead of the true signal $s_1 = 0$ he expects the payoff

$$\begin{aligned} u_1^D(s_1) &= z_1-a_1\sum_{s_2=0,1}\Pr(s_2|s_1)E[(y_1(s_1,s_2)-\theta_1)^2|s_1,s_2] \\ &\quad -b_1\sum_{s_2=0,1}\Pr(s_2|s_1)E[(y_2(s_2,1-s_1)-\theta_2)^2|s_2,s_1]. \end{aligned}$$

We calculate the expected losses associated with agent 1 deviating from equilibrium and sending a message $m_1 = 1 - s_1$ which is mistakenly believed to be truthful by agent 2. When $s_2 = 1$, using

$$y_2(s_2=1,m_1=1)=E(\theta_2|s_2=1,s_1=1)=\frac{2+q}{3+q}$$

$$f(\theta_2|s_2=1,s_2=0)=\frac{6\theta_2(1+q-2q\theta_2)}{3-q},$$

we obtain:

$$\begin{aligned} E[(y_2(s_2,1-s_1)-\theta_2)^2|s_2,s_1] &= \int_0^1 E(\theta_2|s_2=1,s_1=1)f(\theta_2|s_2=1,s_2=0)d\theta_2 \\ &= \int_0^1 (\frac{2+q}{3+q}-\theta_2)^2\frac{6\theta_2(1+q-2q\theta_2)}{3-q}d\theta_2 \\ &= \frac{15+9q+11q^2+q^3}{10(3-q)(3+q)^2}. \end{aligned}$$

When $s_2 = 0$, using,

$$E(\theta|s_2 = 0, s_1 = 1) = \frac{1}{3-q}, \; f(\theta_2|s_2 = s_1 = 0) = \frac{6(1-\theta_2)(1+q-2q\theta_2)}{3+q},$$

we obtain:

$$\begin{aligned} E[(E(\theta_2|s_2, 1-s_1) - \theta_2)^2|s_1, s_2] &= \int_0^1 E(\theta_2|s_2 = 0, s_1 = 1) f(\theta_2|s_2 = s_1 = 0) d\theta_2 \\ &= \int_0^1 (\frac{1}{3-q} - \theta_2)^2 \frac{6(1-\theta_2)(1+q-2q\theta_2)}{3+q} d\theta_2 \\ &= \frac{15 - 9q + 11q^2 - q^3}{10(3-q)^2(3+q)} \end{aligned}$$

The expected deviation payoff can be written as

$$\begin{aligned} u_1^D(s_1) &= z_1 - a_1 \frac{3-q^2}{6(9-q^2)} - b_1 [\frac{3-q}{6} \cdot \frac{15+9q+11q^2+q^3}{10(3-q)(3+q)^2} + \frac{3+q}{6} \cdot \frac{15-9q+11q^2-q^3}{10(3-q)^2(3+q)}] \\ &= z_1 - a_1 \frac{3-q^2}{6(9-q^2)} - b_1 \frac{(9+q^2)(3+q^2)}{6(9-q^2)^2} \end{aligned}$$

so that agent 1 does not deviate at the communication stage if

$$z_1 - (a_1 + b_1) \frac{3-q^2}{6(9-q^2)} \geq z_1 - a_1 \frac{3-q^2}{6(9-q^2)} - b_1 \frac{(9+q^2)(3+q^2)}{6(9-q^2)^2}$$

which implies

$$b_1 \frac{4q^2}{(9-q^2)^2} \geq 0$$

or $b_1 \geq 0$.

The constraint for agent 1 given $s_1 = 1$, and for agent 2 given either $s_2 = 0$ or $s_2 = 1$ results in the same constraint, by symmetry across signal realizations and across players. Q.E.D.

Proof of Lemma 2: The information acquisition incentive constraint is satisfied if

$$z_1 - (a_1 + b_1) \frac{3-q^2}{6(9-q^2)} - c \geq z_1 - a_1 \frac{(3-q^2)}{36} - b_1 \frac{27+q^4}{6(9-q^2)^2}.$$

Appropriate rearranging yields the constraint (5).
Q.E.D.

Proof of Proposition 1: When $\frac{1}{36} \frac{(3-q^2)^2}{(9-q^2)}$, the coefficient of a_1 in the constraint (5), is strictly larger than $2 \frac{q^2}{(9-q^2)^2}$, the optimal linear contract t_1 is such that $b_1 = 0$ and, using the binding constraint (5), $a_1 = 36\frac{9-p^2}{(3-p^2)^2}c$. The constraint $z_1 = (a_1 + b_1)$ yields $z_1 = a_1$. The expected payoff for the principal is:

$$\begin{aligned}\pi &= 2\overline{\pi} - 2E[(E(\theta|s_1,s_2)-\theta)^2] - 2E[t_1(\ell_1,\ell_2)] = 2\overline{\pi} - \frac{3-p^2}{3(9-p^2)} - \frac{72(9-p^2)}{(3-p^2)^2}[1-\frac{3-p^2}{6(9-p^2)}]c \\ &= 2\overline{\pi} - \frac{3-p^2}{3(9-p^2)} - (51-5p^2)\frac{12}{(3-p^2)^2}c.\end{aligned}$$

Conversely, when $\frac{1}{36}\frac{(3-q^2)^2}{9-q^2} < 2\frac{q^2}{(9-q^2)^2}$, the optimal linear contract t_1 is such that $a_1 = 0$, $b_1 = \frac{(9-q^2)^2}{2q^2}c$ and $z_1 = b_1$. The expected payoff for the principal is:

$$\begin{aligned}\pi &= 2\overline{\pi} - 2E[(E(\theta_1|s_1,s_2)-\theta_1)^2] - 2E[t_1(\ell_1,\ell_2)] = 2\overline{\pi} - \frac{3-p^2}{3(9-p^2)} - \frac{(9-p^2)^2}{p^2}[1-\frac{3-p^2}{6(9-p^2)}]c \\ &= 2\overline{\pi} - \frac{3-p^2}{3(9-p^2)} - (51-5p^2)\frac{9-p^2}{6p^2}c.\end{aligned}$$

The condition $\frac{1}{36}\frac{(3-q^2)^2}{(9-q^2)} > 2\frac{q^2}{(9-q^2)^2}$ simplies to $q^2 < (q'^2 \equiv \sqrt[3]{\sqrt{37\,584}-172} - \frac{20}{\sqrt[3]{\sqrt{37\,584}-172}} + 5$ or

$$q < q' \equiv \sqrt{5 - 10(\frac{2}{9\sqrt{29}})^{1/3} + 2^{2/3}(9\sqrt{29}-43)^{1/3}}.$$

We conclude by showing that the ex-ante participation constraint

$$z_1 - a_1(1 - E[(E(\theta_1|s_1,s_2)-\theta_1)^2]) + b_1(1 - E[(E(\theta_2|s_2,s_1)-\theta_2)^2])] - c \geq 0$$

is satisfied in the optimal contract t_1. For $b_1 = 0$, and $a_1 = 36\frac{9-q^2}{(3-q^2)^2}c$, the ex-ante participation constraint is

$$\frac{36(9-q^2)}{(3-q^2)^2}c(1-\frac{3-q^2}{6(9-q^2)}) - c = (q^2+33)\frac{9-q^2}{(q^2-3)^2}c > 0$$

For $a_1 = 0$ and $b_1 = \frac{(9-q^2)^2}{2q^2}c$, this constraint is

$$\frac{(9-q^2)^2}{2q^2}c(1-\frac{3-q^2}{6(9-q^2)}) - c = \frac{459-108q^2+5q^4}{12q^2}c > 0.$$

Q.E.D.

Proof of Proposition 2: To determine the optimal linear contracts t_1, t_2 to induce both agents $i = 1, 2$ to collect information and only agent, say 1, to transmit it to the other agent, first note that, again, each agent $i = 1, 2$ is motivated to choose decision y_i so as to minimize the loss $\ell_i = (y_i - \theta_i)^2$ by setting $a_i \geq 0$.

The equilibrium payoff of agent 2 is, using $m_1 = s_1$,

$$\begin{aligned} & z_2 - a_2 E[(y_2(s_2, s_1) - \theta_2)^2] - b_2 E[(y_1(s_1) - \theta_1)^2] - c \\ = \quad & z_1 - a_2 \frac{3 - q^2}{6(9 - q^2)} - b_2 \frac{1}{18} - c. \end{aligned}$$

If agent 2 deviates at the information acquisition stage, her payoff is

$$\begin{aligned} & z_2 - a_2 E[(y_2(s_1) - \theta_2)^2] - b_2 E[(y_1(s_1) - \theta_1)^2] \\ = \quad & z_2 - a_2 \frac{3 - q^2}{36} - b_2 \frac{1}{18}. \end{aligned}$$

using

$$\begin{aligned} E[(y_2(s_1) - \theta_2)^2] &= \sum_{s_1=0,1} \frac{1}{2} \int_0^1 (E[\theta_2|s_1] - \theta_2)^2 f(\theta_2|s_1) d\theta_2 \\ &= \int_0^1 (E[\theta_2|s_1 = 0] - \theta_2)^2 f(\theta_2|s_1 = 0) d\theta_2 = \frac{3 - q^2}{36}, \end{aligned}$$

because $f(\theta_2|s_1 = 0) = 1 + q(1 - 2\theta_2)$ with $E(\theta_2|s_1 = 0) = \frac{3-q}{6}$.

So, the constraint at the information acquisition stage is:

$$a_2 \frac{1}{36} \frac{(3 - q^2)^2}{9 - q^2} \geq c.$$

This yields the optimal linear contract $z_2 = a_2 = 36 \frac{9-q^2}{(3-q^2)^2} c$ and $b_2 = 0$.

Then, we consider agent 1 to note that she does not deviate from truthtelling if and only if $b_1 \geq 0$. Turning to the information acquisition constraint, we note that the equilibrium payoff of agent 1 is:

$$\begin{aligned} & z_1 - a_1 E[(y_1(s_1) - \theta_1)^2] - b_1 E[(y_2(s_2, m_1) - \theta_2)^2] - c \\ = \quad & z_1 - a_1 \frac{1}{18} - b_1 \frac{3 - q^2}{6(9 - q^2)} - c, \end{aligned}$$

using the fact that

$$\begin{aligned} E[(y_1(s_1) - \theta_1)^2] &= \sum_{s_1=0,1} \frac{1}{2} \int_0^1 (E(\theta_1|s_1) - \theta_1)^2 f(\theta_1|s_1) d\theta_1 \\ &= \int_0^1 (E(\theta_1|s_1 = 0) - \theta_1)^2 f(\theta_1|s_1 = 0) d\theta_1 \\ &= \int_0^1 (\frac{1}{3} - \theta_1)^2 2(1 - \theta_1) d\theta_1 = \frac{1}{18}. \end{aligned}$$

If agent 1 deviates and does not collect information, her payoff is:

$$\begin{aligned} & z_1 - a_1 E[(y_1-\theta_1)^2] - b_1 E[E[(y_2(s_2,m_1)-\theta_2)^2|s_2]] \\ = \; & z_1 - a_1\frac{1}{12} - b_1\frac{27+q^4}{6(9-q^2)^2}, \end{aligned}$$

using the fact that

$$\begin{aligned} E[(y_1-\theta_1)^2] &= \int_0^1 (E(\theta_1)-\theta_1)^2 f(\theta_1)d\theta_1 \\ &= \int_0^1 (\frac{1}{2}-\theta_1)^2 d\theta_1 = \frac{1}{12}. \end{aligned}$$

This yields the constraint:

$$a_1\frac{1}{36} + b_1\frac{2q^2}{(9-q^2)^2} \geq c.$$

Using the constraints $a_1 \geq 0$ and $b_1 \geq 0$ again, the solution is either $z_1 = a_1 = 36c$, $b_1 = 0$, or $a_1 = 0$, $z_1 = b_1 = \frac{(9-q^2)^2}{2q^2}c$.

The principal's profit is:

$$\begin{aligned} E\pi_{21} &= \max_{a_1,b_1}\left\{2\overline{\pi} - \frac{1}{18} - \frac{3-q^2}{6(9-q^2)} - a_i(1-\frac{1}{18}) - b_i(1-\frac{1}{36}(3-q^2)) - a_2(1-\frac{3-q^2}{6(9-q^2)})c\right\} \\ &= 2\overline{\pi} - \frac{1}{18} - \frac{3-q^2}{6(9-q^2)} - \min\left\{a_1(1-\frac{1}{18}), b_1(1-\frac{3-q^2}{6(9-q^2)})\right\} - 36\frac{9-q^2}{(q^2-3)^2}(1-\frac{3-q^2}{6(9-q^2)})c \end{aligned}$$

simplifying and substituting in the two possible optimal values of a_1 and b_1, we obtain:

$$E\pi_{21} = 2\overline{\pi} - \frac{9-2q^2}{9(9-q^2)} - \min\left\{36(1-\frac{1}{18}), \frac{(9-q^2)^2}{2q^2}(1-\frac{3-q^2}{6(9-q^2)})\right\}c - \frac{6(51-5q^2)}{(3-q^2)^2}c,$$

that equals the formula in the statement, after simplification.

Comparing the two arguments in the minimum, we obtain for $36(1-\frac{1}{18}) < \frac{(9-q^2)^2}{2q^2}(1-\frac{3-q^2}{6(9-q^2)})$, or $q^2 < \frac{252}{5} - \frac{3}{5}\sqrt{3}\sqrt{2267}$, i.e., $q < \sqrt{\frac{(28\sqrt{3}-\sqrt{2267})3\sqrt{3}}{5}} \approx 0.958$, $z_1 = a_1 = 36c$ and $b_1 = 0$. For $q > \sqrt{\frac{(28\sqrt{3}-\sqrt{2267})3\sqrt{3}}{5}}$, we obtain $a_1 = 0$ and $z_1 = b_1 = \frac{(9-q^2)^2}{2q^2}c$.

We determine the optimal linear contract t_1, t_2 to induce both agents $i = 1, 2$ to collect information and neither of them to share it with the other agent. Again, it is the case that $a_i \geq 0$ in the optimal linear contract. Because the agents are not supposed to share their signals, there is no positivity constraint on b_i. The information acquisition constraint is determined by comparing each agent i's equilibrium payoff

$$z_1 - a_1 E[(y_1(s_1)-\theta_1)^2] - b_1 E[(y_2(s_2)-\theta_2)^2] - c = z_1 - a_1\frac{1}{18} - b_1\frac{1}{18} - c,$$

with i's payoff in case he deviates and does not collect information:

$$z_1 - a_1 E[(y_1 - \theta_1)^2] - b_1 E[(y_2(s_2) - \theta_2)^2] = z_1 - a_1 \frac{1}{12} - b_1 \frac{1}{18}.$$

The information acquisition constraint is just $a_i \geq 36c$, so that the optimal linear contract is such that $a_i = 36c$. The parameters z_i and b_i are indetermined, under the constraint that $a_i + b_i = z_i$.

The expected profit by the principal is:

$$E\pi_{20} = 2\overline{\pi} - \frac{1}{9} - 2a_1(1 - \frac{1}{18}) = 2\overline{\pi} - \frac{1}{9} - 2 \cdot 36c(1 - \frac{1}{18}).$$

Q.E.D.

Proof of Proposition 3: First, we calculate the optimal linear contracts t_1, t_2 to induce agent 1 to collect signal s_1 and share with the other agent 2, and agent 2 to not collect information. The optimal contract of agent 2 is, trivially, $t_2(\ell_1, \ell_2) = 0$ for all ℓ_1 and ℓ_2 (the optimal linear contract is such that $z_2 = a_2 = b_2 = 0$).

The equilibrium payoff of agent 1 is:

$$\begin{aligned} & z_1 - a_1 E[(y_1(s_1) - \theta_1)^2] - b_1 E[(y_2(s_1) - \theta_2)^2] - c \\ = \; & z_1 - a_1 \frac{1}{18} - b_1 \frac{3 - q^2}{36} - c \end{aligned}$$

If agent 1 does not collect information, he still sends a message m_1 to agent 2 who mistakenly believes that $m_1 = s_1$. The equilibrium payoff of agent 1 is:

$$\begin{aligned} & z_1 - a_1 E[(y_1 - \theta_1)^2] - b_1 E[(y_2(m_1) - \theta_2)^2] \\ = \; & z_1 - a_1 \frac{1}{12} - b_1 \frac{3 + q^2}{36}, \end{aligned}$$

using:

$$\begin{aligned} E[(y_2(m_1) - \theta_2)^2] &= \int_0^1 [(E[\theta_2 | m = 0] - \theta_2)^2] f(\theta_2) d\theta_2 \\ &= \int_0^1 (\frac{3 - q}{6} - \theta_2)^2 d\theta_2 = \frac{3 + q^2}{36}. \end{aligned}$$

Here, the incentive compatibility constraint is:

$$-a_1 \frac{1}{18} - b_1 \frac{3 - q^2}{36} - c \geq -a_1 \frac{1}{12} - b_1 \frac{3 + q^2}{36}$$

or

$$a_1 \frac{1}{36} + b_1 \frac{1}{18} q^2 \geq c.$$

So, the optimal linear contract is either $a_1 = 0$, $z_1 = b_1 = \frac{18}{q^2}c$ or $z_1 = a_1 = 36c$, $b_1 = 0$.

The principal's profit is:

$$E\pi_{11} = \max_{a_1,b_1} \left\{ 2\overline{\pi} - \frac{1}{18} - \frac{1}{36}(3-q^2) - a_1(1-\frac{1}{18}) - b_1(1-\frac{1}{36}(3-q^2)) \right\},$$

and substituting in the two possible optimal values of a_1 and b_1, we obtain

$$E\pi_{11} = 2\overline{\pi} - \frac{1}{18} - \frac{1}{36}(3-q^2) - \min\left\{ 36(1-\frac{1}{18}), \frac{18}{q^2}(1-\frac{1}{36}(3-q^2)) \right\} c.$$

Equating $36(1-\frac{1}{18}) = \frac{18}{q^2}(1-\frac{1}{36}(3-q^2))$, we obtain the admissible solution $q = \sqrt{\frac{33}{67}} \approx 0.701$. For $q > \sqrt{\frac{33}{67}}$, it is optimal to set $a_1 = 0$, $z_1 = b_1 = \frac{18}{q^2}c$, whereas for $q < \sqrt{\frac{33}{67}}$ it is optimal to set $z_1 = a_1 = 36c$, $b_1 = 0$.

Second, we calculate the optimal linear contracts t_1, t_2 to induce agent 1 to collect signal s_1 but not share it with the other agent 2, and agent 2 to not collect information.

The optimal contract of agent 2 is again such that $z_2 = a_2 = b_2 = 0$.

The equilibrium payoff of agent 1 is:

$$z_1 - a_1\frac{1}{18} - b_1\frac{1}{12} - c,$$

his deviation payoff at the information acquisition stage is:

$$z_1 - a_1\frac{1}{12} - b_1\frac{1}{12},$$

so that the incentive compatibility constraint is:

$$a_1 \geq 36c,$$

and the optimal linear contract is $z_1 = a_1 = 36c$ and $b_1 = 0$.

The principal's expected profit is:

$$E\pi_{10} = 2\overline{\pi} - \frac{1}{18} - \frac{1}{12} - 36(1-\frac{1}{18})c.$$

Q.E.D.

The optimal linear contracts t_1, t_2 in the case that both agents $i = 1, 2$ are not supposed to collect information are such that $z_i = a_i = b_i = 0$. This leads to expected principal's profit:

$$E\pi_{00} = 2\overline{\pi} - 2\frac{1}{12}.$$

Proof of Lemma 3: Subtracting the formulas of $E\pi_{11}(q,c)$ and $E\pi_{10}(q,c)$ and rearranging, we obtain

$$E\pi_{11} - E\pi_{10} = \frac{5}{36} - \frac{1}{36}(5-q^2) + 34c - \min\left\{34, \frac{1}{2}\frac{q^2+33}{q^2}\right\} c,$$

which is obviously strictly positive.
Q.E.D.

Proof of Lemma 4: We first compare $E\pi_{22}(q,c)$ and $E\pi_{21}(q,c)$ and consider

$$\begin{aligned} E\pi_{22}(q,c) - E\pi_{21}(q,c) &= \frac{9-2q^2}{9(9-q^2)} - \frac{3-q^2}{3(9-q^2)} - D_{22-21}(q)c \\ &= \frac{q^2}{9(9-q^2)} - D_{22-21}(q)c \geq -D_{22-21}(q)c. \end{aligned}$$

where

$$D_{22-21}(q) = \min\left\{\frac{12}{(3-q^2)^2}, \frac{(9-q^2)}{6q^2}\right\}(51-5q^2) - \min\left\{34, (51-5q^2)\frac{(9-q^2)}{12q^2}\right\} - 6\frac{51-5q^2}{(3-q^2)^2}.$$

Calculations omitted for brevity show that $D_{22-21}(q) > 0$ for $0 \leq q < q_1 \approx 0.855$ and $D_{22-21}(q) < 0$ for $q_1 < q \leq 1$. We obtain that for $0 \leq q < q_1$, whether $E\pi_{22}(q,c)$ is larger or smaller than $E\pi_{21}(q,c)$ depends on whether c is smaller or larger than a strictly positive threshold $c_{22-21}(q)$ implicitly defined by the equation $E\pi_{22}(q,c) = E\pi_{21}(q,c)$, whereas for $q_1 \leq q \leq 1$ it is the case that $E\pi_{22}(q,c) > E\pi_{21}(q,c)$ for all c.

To complete the proof we show that, for almost all c and $0 \leq q \leq q_1$ it is either the case that $E\pi_{22}(q,c) > E\pi_{21}(q,c)$ or that $E\pi_{20}(q,c) > E\pi_{21}(q,c)$. We begin by noting that the functions $E\pi_{22}(q,c)$, $E\pi_{21}(q,c)$ and $E\pi_{20}(q,c)$ are all linear in c, and that $E\pi_{22}(q,c) > E\pi_{21}(q,c) > E\pi_{20}(q,c)$ for $c = 0$. As a result, we can proceed by comparing the threshold functions

$$c_{22-21}(q) = \frac{\frac{1}{18} - \frac{3-q^2}{3(9-q^2)} + \frac{3-q^2}{6(9-q^2)}}{\min\left\{\frac{12}{(3-q^2)^2}, \frac{9-q^2}{6q^2}\right\}(51-5q^2) - \min\left\{34, \frac{9-q^2}{12q^2}(51-5q^2)\right\} - 6\frac{51-5q^2}{(q^2-3)^2}}$$

$$c_{21-20}(q) = \frac{\frac{1}{18} - \frac{3-q^2}{6(9-q^2)}}{\min\left\{34, \frac{9-q^2}{12q^2}(51-5q^2)\right\} + 6\frac{51-5q^2}{(q^2-3)^2} - 68}$$

implicitly defined by the equations $E\pi_{22}(q,c) = E\pi_{21}(q,c)$ and $E\pi_{21}(q,c) = E\pi_{20}(q,c)$, respectively. In fact, for any (q,c) such that $c < c_{22-21}(q)$, it is the case that $E\pi_{22}(q,c) > E\pi_{21}(q,c)$, and for any (q,c) such that $c > c_{21-20}(q)$, it is the case that $E\pi_{21}(q,c) < E\pi_{20}(q,c)$.

Calculations omitted for brevity show that $c_{22-21}(q) \geq c_{21-20}(q)$ for all $0 \leq q \leq q_1$. This completes the proof of the Lemma, because it implies that for almost all c and $0 \leq q \leq q_1$, it is either the case that $E\pi_{22}(q,c) > E\pi_{21}(q,c)$ or that $E\pi_{20}(q,c) > E\pi_{21}(q,c)$.
Q.E.D.

Proof of Proposition 4: We need compare the profit functions $E\pi_{22}(q,c)$, $E\pi_{20}(q,c)$, $E\pi_{11}(q,c)$ and $E\pi_{00}(q,c)$. To determine the area in which $E\pi_{22}(q,c)$ is the largest, we note that all the profit functions are linear in c, and that $E\pi_{22}(q,c) > E\pi_{20}(q,c) > E\pi_{11}(q,c) > E\pi_{00}(q,c)$ for $c = 0$ and all q. As a result, we can proceed by comparing the threshold functions

$$c_{22-20}(q) = \frac{\frac{1}{9} - \frac{3-q^2}{3(9-q^2)}}{\min\left\{\frac{12}{(q^2-3)^2}, \frac{9-q^2}{6q^2}\right\}(51-5q^2) - 68}$$

$$c_{22-11}(q) = \frac{\frac{5-q^2}{36} - \frac{3-q^2}{3(9-q^2)}}{\min\left\{\frac{12}{(3-q^2)^2}, \frac{9-q^2}{6q^2}\right\}(51-5q^2) - \min\left\{34, \frac{1}{2}\frac{q^2+33}{q^2}\right\}}$$

$$c_{22-00}(q) = \frac{\frac{1}{6} - \frac{3-q^2}{3(9-q^2)}}{\min\left\{\frac{12}{(3-q^2)^2}, \frac{9-q^2}{6q^2}\right\}(51-5q^2)}$$

implicitly defined by the equations $E\pi_{22}(q,c) = E\pi_{20}(q,c)$, $E\pi_{22}(q,c) = E\pi_{11}(q,c)$ and $E\pi_{22}(q,c) = E\pi_{00}(q,c)$. For any such a threshold function $c_{22-(\cdot)}(q)$, and any value $q \in [0,1]$ for which $c_{22-(\cdot)}(q)$ is positive, it is the case that $E\pi_{22}(q,c) > E\pi_{(\cdot)}(q,c)$ if and only if $c < c_{22-(\cdot)}(q)$. Instead, for all q such that $c_{22-(\cdot)}(q) < 0$, it is the case that $E\pi_{22}(q,c) > E\pi_{(\cdot)}(q,c)$ for all c.

Calculations omitted for brevity prove that $c_{22-20}(q) > 0$ if and only if $q < \sqrt{\frac{252}{5} - \frac{3\sqrt{3}}{5}\sqrt{2267}} \approx 0.958$, and that $c_{22-11}(q) > 0$ and $c_{22-00}(q) > 0$ for all $q \in [0,1]$. Further, comparing $c_{22-11}(q)$ and $c_{22-00}(q)$, omitted calculations show that $c_{22-11}(q) < c_{22-00}(q)$ for all $q \in [0,1]$, and that $c_{22-20}(q) < c_{22-11}(q)$ if and only if $q < \tilde{q} \approx 0.553$ on the relevant range $q \in [0, \sqrt{\frac{(28\sqrt{3}-\sqrt{2267})3\sqrt{3}}{5}}]$. The implication is that $E\pi_{22}(q,c) > \max\{E\pi_{20}(q,c), E\pi_{11}(q,c), E\pi_{00}(q,c)\}$ for every $c < c_{22-20}(q)$ for $q < \tilde{q}$ and for every $c < c_{22-11}(q)$ for $q > \tilde{q}$.

Likewise, to determine the area in which $E\pi_{00}(q,c)$ is larger than $E\pi_{22}(q,c)$, $E\pi_{20}(q,c)$ and $E\pi_{11}(q,c)$, we note that $E\pi_{00}(q,c) > \max\{E\pi_{22}(q,c), E\pi_{20}(q,c), E\pi_{11}(q,c)\}$ for $c \to \infty$ and all q. As a result, we can proceed by comparing the threshold function $c_{22-00}(q)$ reported above with the threshold functions

$$c_{11-00}(q) = \frac{q^2+1}{36\min\left\{34, \frac{1}{2q^2}(q^2+33)\right\}} \text{ and } c_{20-00}(q) = \frac{1}{1224},$$

implicitly defined by the equations $E\pi_{20}(q,c) = E\pi_{00}(q,c)$ and $E\pi_{11}(q,c) = E\pi_{00}(q,c)$. Omitted calculations show that, for all $q \in [0,1]$ all the functions $c_{22-20}(q)$, $c_{22-11}(q)$ and $c_{22-00}(q)$ are strictly positive. Hence, for every $q \in [0,1]$, it is the case that $E\pi_{00}(q,c) > \max\{E\pi_{22}(q,c),\ E\pi_{20}(q,c),\ E\pi_{11}(q,c)\}$ for every $c > \max\{c_{22-00}(q), c_{20-00}(q), c_{11-00}(q)\}$. Comparing $c_{22-00}(q)$, $c_{20-00}(q)$ and $c_{11-00}(q)$, omitted calculations show that $c_{11-00}(q) > c_{22-00}(q)$, and $c_{11-00}(q) > c_{20-00}(q)$ for all $q \in [0,1]$. The implication is that $E\pi_{00}(q,c) > \max\{E\pi_{22}(q,c),\ E\pi_{20}(q,c),\ E\pi_{11}(q,c)\}$ for every q and $c > c_{11-00}(q)$.

For any q and cost c values that are below $c_{11-00}(q)$ and either above $c_{22-20}(q)$, for $q < \tilde{q}$, or above $c_{22-11}(q)$, for $q > \tilde{q}$, it is either the case that $E\pi_{20}(q,c)$ or $E\pi_{11}(q,c)$ is the highest profit function. Because $E\pi_{20}(q,c) > E\pi_{11}(q,c)$ for $c = 0$ and all q, this is once more determined by considering a threshold function:

$$c_{20-11}(q) = \frac{1-q^2}{2448 - 36\min\left\{34, \frac{q^2+33}{2q^2}\right\}},$$

implicitly defined by the equation $E\pi_{20}(q,c) = E\pi_{11}(q,c)$. Because $2448 - 36 \cdot 34 = 1224$, the threshold function $c_{20-11}(q)$ is strictly positive for all $q \in [0,1]$. Hence, for all q it is the case that $E\pi_{20}(q,c) > E\pi_{11}(q,c)$ if and only if $c < c_{20-11}(q)$.

Comparing $c_{20-11}(q)$ with $c_{22-20}(q)$, $c_{22-11}(q)$ and $c_{11-00}(q)$, omitted calculations show that $c_{20-11}(q) = c_{11-00}(q)$ for $q = 0$, that $c_{20-11}(q) < c_{11-00}(q)$ for all $q > 0$, that $c_{20-11}(q) > c_{22-20}(q)$ for $0 \leq q < \tilde{q}$, that $c_{20-11}(q) = c_{22-20}(q) = c_{22-11}(q)$ for $q = \tilde{q}$ and that $c_{20-11}(q) < c_{22-11}(q)$ for $\tilde{q} < q \leq 1$.

This concludes the proof of the Proposition. We have derived the result depicted in Figure 1: For $0 < q < \tilde{q}$ and $c < c(q)_{22-20}$, and for $\tilde{q} < q \leq 1$ and $q < c(q)_{20-11}$, it is the case that $E\pi_{22}(q,c) > \max\{E\pi_{20}(q,c),\ E\pi_{11}(q,c),\ E\pi_{00}(q,c)\}$. For $0 < q < \tilde{q}$ and $c(q)_{22-20} < c < c(q)_{20-11}$, $E\pi_{20}(q,c) > \max\{E\pi_{22}(q,c),\ E\pi_{11}(q,c),\ E\pi_{00(q,c)}\}$. For $c(q)_{22-11} < c < c(q)_{11-00}$, $E\pi_{11}(q,c) > \max\{E\pi_{22}(q,c),\ E\pi_{20}(q,c),\ E\pi_{00}(q,c)\}$. For $c > c(q)_{11-00}$, $E\pi_{00}(q,c) > \max\{E\pi_{22}(q,c),\ E\pi_{20}(q,c),\ E\pi_{11}(q,c)\}$.
Q.E.D.

Proof of Proposition 5: Consider the following optimization problem:

$$\min_{(a_i,b_i,d_i)\in\mathbb{R}^3} \ t_i = z_i - a_i(y_i-\theta_i)^2 - b_i(y_j-\theta_j)^2 - d_i(y_i-\theta_i)^2(y_j-\theta_j)^2$$

subject to

$$c \leq a_i\frac{(3-q^2)^2}{36(9-q^2)} + b_i\frac{2q^2}{(9-q^2)^2} + d_i\frac{(3-q^2)(3+q^2)^2}{216(9-q^2)^2},$$
$$t_i \geq 0 \text{ for every profile } (y_1, y_2, \theta_1, \theta_2).$$

The above problem lacks the constraint imposed by the incentives to truthfully communicate an obtained signal. In the following we show that a simultaneous optimization over (a_i, b_i, d_i) in the relaxed problem above does not achieve a better outcome for the principal compared to the optimal linear case. Then, a more constrained problem would not be able to achieve a better outcome compared to the optimal linear case either.

Notice that the transfer increases in the RHS of the first constraint. Therefore, we can assume that the first constraint is satisfied with equality. Limited liability requires a non-negative transfer for every profile $(y_1, y_2\theta_1, \theta_2)$. This requirement allows us to formulate the optimization problem in the following form:

$$\min_{(a_i,b_i,d_i)\in\mathbb{R}^3} \quad t_i = (a_i + b_i)(1 - \frac{3-q^2}{6(9-q^2)}\Big) + d_i\Big[1 - \Big(\frac{3-q^2}{6(9-q^2)}\Big)^2\Big] + z_i'$$

subject to

$$c = a_i\frac{(3-q^2)^2}{36(9-q^2)} + b_i\frac{2q^2}{(9-q^2)^2} + d_i\frac{(3-q^2)(3+q^2)^2}{216(9-q^2)^2} \tag{7}$$

$$z_i' = \begin{cases} 0 & (a_i, b_i, d_i) \geq 0 \\ a_i\mathbb{1}_{a_i<0} + b_i\mathbb{1}_{b_i<0} + d_i\mathbb{1}_{d_i<0} & \text{otherwise,} \end{cases}$$

and where we use $E[-(y_i\theta_i)^2]$ obtained earlier. Notice how z_i' ensures limited liability. If it were optimal for the principal to choose a negative value for one of the variables (a_i, b_i, d_i) and, for example, $(\theta_i = 0, y_i = 1)$ and $\theta_j = y_i$, z' ensures that the transfer is non-negative.

In the next step we express (7) as

$$a = \frac{432bq^2 - 216c\left(q^2-9\right)^2 + d\left(q^2-3\right)\left(q^2+3\right)^2}{6\left(q^2-9\right)\left(q^2-3\right)^2}. \tag{8}$$

Using (8) in t_i we can rewrite

$$t_i = (a_i + b_i)\Big(1 - \frac{3-q^2}{6(9-q^2)}\Big) + d_i\Big(1 - \Big(\frac{3-q^2}{6(9-q^2)}\Big)^2\Big) + z_i' =$$

as

$$\Big(b_i\frac{72q^2}{\left(q^2-9\right)\left(q^2-3\right)^2} + b_i\Big)\Big(1 - \frac{3-q^2}{6(9-q^2)}\Big) +$$

$$\Big(\frac{36c\left(q^2-9\right)^2}{\left(9-q^2\right)\left(q^2-3\right)^2} + d_i\frac{\left(q^2+3\right)^2}{6\left(3-q^2\right)\left(9-q^2\right)}\Big)\Big(1 - \frac{3-q^2}{6(9-q^2)}\Big) + d_i\Big(1 - \Big(\frac{3-q^2}{6(9-q^2)}\Big)^2\Big) + z_i'.$$

Since $b_i\frac{72q^2}{(q^2-9)(q^2-3)^2} + b_i > 0$ for $q < q' \approx 0.8026$, restricting to $q < q'$ implies $z_i' = 0$. Then the principal solves

$$\min_{b_i\geq 0, d_i\geq 0} t_1 = \Big(b_i\frac{72q^2}{(q^2-9)(q^2-3)^2}+b_i\Big)\Big(1-\frac{3-q^2}{6(9-q^2)}\Big)+$$
$$\Big(\frac{36c(q^2-9)^2}{(9-q^2)(q^2-3)^2}+d_i\frac{(q^2+3)^2}{6(3-q^2)(9-q^2)}\Big)\Big(1-\frac{3-q^2}{6(9-q^2)}\Big)+d_i\Big(1-\Big(\frac{3-q^2}{6(9-q^2)}\Big)^2\Big).$$

It turns out that $t_1 = \frac{6(51-5q^2)c}{(3-q^2)^2}$ for $b_i = d_i = 0$, and $t_1 > \frac{6(51-5q^2)c}{(3-q^2)^2}$ if either $b_i > 0$ or $d_i > 0$ or $(b_i, d_i) \gg 0$. Thus, for $q < q'$ and $d_i \geq 0$ the non-linear transfer cannot outperform the best linear transfer.

Consider $q > q'$ while still maintaining $d_i \geq 0$. In this case it is easy to check that the limited liability requirements results in

$$z_i' = -\Big(b_i\frac{72q^2}{(q^2-9)(q^2-3)^2}+b_i\Big)$$

so that the principal solves

$$\min_{b_i\geq 0, d_i\geq 0} t_2 = c\frac{6(51-5q^2)}{(q^2-3)^2}+b_i\frac{(q^6-15q^4+135q^2-81)}{6(3-q^2)(9-q^2)^2}+d_i\frac{(51-5q^2)(2q^4-18q^2+45)}{9(3-q^2)(q^2-9)^2}.$$

It turns out that for $q > q'$and $d_i \geq 0$, $t_2 = \frac{6(51-5q^2)c}{(3-q^2)^2}$ for $b_i = d_i = 0$, and $t_2 > \frac{6(51-5q^2)c}{(3-q^2)^2}$ if either $b_i > 0$ or $d_i > 0$ or $(b_i, d_i) \gg 0$.

Consider, next $d_i < 0$ and $b_i \geq 0$. The restriction $q < q'$ implies that the limited liability requires $z_i' = -\Big(\frac{(q^2+3)^2}{6(3-q^2)(9-q^2)}+d_i\Big)$ such that the principal solves

$$\min_{b_i\geq 0, d_i< 0} t_3 = \frac{6c(51-5q^2)}{(3-q^2)^2}-d_i\frac{(q^4+9)}{18(9-q^2)^2}+b_i\frac{(51-5q^2)(-q^6+15q^4-135q^2+81)}{6(q^4-12q^2+27)^2} \text{ for } q<q'.$$

It turns out that $t_3 < \frac{6(51-5q^2)c}{(3-q^2)^2}$. Allowing for $q > q'$ and $d_i < 0$ and $b_i \geq 0$, limited liability requires that

$$z_i' = -\Big(\frac{(q^2+3)^2}{6(3-q^2)(9-q^2)}+d_i\Big)-\Big(b_i\frac{72q^2}{(q^2-9)(q^2-3)^2}+b_i\Big).$$

The principal solves

$$\min_{b_i\geq 0, d_i\geq 0} t_4 = \frac{6c(51-5q^2)}{(3-q^2)^2}+b_i\frac{q^6-15q^4+135q^2-81}{6(3-q^2)(q^2-9)^2}-d_i\frac{(q^4+9)}{18(9-q^2)^2} \text{ for } q>q'.$$

It turns out that $t_4 < \frac{6(51-5q^2)c}{(3-q^2)^2}$.

Finally, consider $d_i < 0, b_i < 0$. For $q < q'$, and the limited liability constraint, the relevant transfer is t_4 which we have shown to be dominated by the optimal linear contract. Consider $q > q'$. In this case, the limited liability results in a transfer t_3 which we have shown is dominated by the optimal linear contract as well.

Finally, notice that due to the symmetry of the problem, if we would have rearranged (7) for either b_i or d_i, the resulting non-linear transfer would be dominated by the optimal linear transfer as well. We conclude that a non-linear transfer of the form $z_i - a_i(y_i - \theta_i)^2 - b_i(y_j - \theta_j)^2 - d_i(y_i - \theta_i)^2(y_j - \theta_j)^2$ does not yield a strictly higher payoff for the principal compared to the optimal linear contract.
Q.E.D.

Proof of Proposition 6: In the first step we show when an allocation with two signals and one-sided communication dominates the two other allocations with two signals. First, consider the following expected payoffs:

$$E\pi_{22}^{\lambda} = (\lambda_1 + \lambda_2)(\overline{\pi} - \frac{3 - q^2}{6(9 - q^2)}) - (51 - 5q^2)\min\{\frac{12}{(3 - q^2)^2}, \frac{9 - q^2}{6q^2}\}c,$$

$$E\pi_{21}^{\lambda} = \overline{\pi}(\lambda_1 + \lambda_2) - \lambda_1\frac{1}{18} - \lambda_2\frac{3 - q^2}{6(9 - q^2)} - \min\{34, (51 - 5q^2)\frac{(9 - q^2)}{12q^2}\}c - 6\frac{51 - 5q^2}{(3 - q^2)^2}c.$$

$$E\pi_{12}^{\lambda} = \overline{\pi}(\lambda_1 + \lambda_2) - \lambda_2\frac{1}{18} - \lambda_1\frac{3 - q^2}{6(9 - q^2)} - \min\{34, (51 - 5q^2)\frac{(9 - q^2)}{12q^2}\}c - 6\frac{51 - 5q^2}{(3 - q^2)^2}c.$$

such that the corresponding differences are:

$$E\pi_{22}^{\lambda} - E\pi_{21}^{\lambda} = \lambda_1\Big(\frac{1}{18} + \frac{3 - q^2}{6(9 - q^2)}\Big) - D_{22-21}(q)c,$$
$$E\pi_{22}^{\lambda} - E\pi_{12}^{\lambda} = \lambda_2\Big(\frac{1}{18} + \frac{3 - q^2}{6(9 - q^2)}\Big) - D_{22-12}(q)c.$$

where

$$D_{22-21}(q)c = D_{22-12}(q) = (51-5q^2)\min\{\frac{12}{(3 - q^2)^2}, \frac{9 - q^2}{6q^2}\}-\min\{34, (51-5q^2)\frac{(9 - q^2)}{12q^2}\}-6\frac{51 - 5q^2}{(3 - q^2)^2}.$$

As in the baseline model, $D_{22-21}(q) < 0$ for $q > q_1 \approx 0.855$ which implies that in this case $E\pi_{22}^{\lambda} > E\pi_{21}^{\lambda}$. For $q < q_1$, $D_{22-21}(q) > 0$, so it could be that $E\pi_{22}^{\lambda} < E\pi_{21}^{\lambda}$. Similar to the approach in the proof of Lemma 4, define the following cost thresholds:

$$c^{\lambda}_{22-21}(q) = \frac{\lambda_1\left(\frac{1}{18} - \frac{3-q^2}{6(9-q^2)}\right)}{(51-5q^2)\min\{\frac{12}{(3-q^2)^2}, \frac{9-q^2}{6q^2}\} - \min\{34, (51-5q^2)\frac{(9-q^2)}{12q^2}\} - 6\frac{51-5q^2}{(3-q^2)^2}}$$

$$c^{\lambda}_{22-12}(q) = \frac{\lambda_2\left(\frac{1}{18} - \frac{3-q^2}{6(9-q^2)}\right)}{(51-5q^2)\min\{\frac{12}{(3-q^2)^2}, \frac{9-q^2}{6q^2}\} - \min\{34, (51-5q^2)\frac{(9-q^2)}{12q^2}\} - 6\frac{51-5q^2}{(3-q^2)^2}}$$

$$c^{\lambda}_{21-20}(q) = \frac{\lambda_2\left(\frac{1}{18} - \frac{3-q^2}{6(9-q^2)}\right)}{\min\{34, (51-5q^2)\frac{(9-q^2)}{12q^2}\} + 6\frac{51-5q^2}{(3-q^2)^2} - 68}$$

$$c^{\lambda}_{12-20}(q) = \frac{\lambda_1\left(\frac{1}{18} - \frac{3-q^2}{6(9-q^2)}\right)}{\min\{34, (51-5q^2)\frac{(9-q^2)}{12q^2}\} + 6\frac{51-5q^2}{(3-q^2)^2} - 68}$$

such that $E\pi^{\lambda}_{22} > E\pi^{\lambda}_{21}$ for all $c < c^{\lambda}_{22-21}(q)$ and $E\pi^{\lambda}_{20} > E\pi^{\lambda}_{21}$ for all q and $c > c^{\lambda}_{22-21}(q)$.

Consider the following cases depending on q:

- First, suppose that $q \leq q' \approx 0.803 < q_1 \approx 0.855$. Thus the relevant thresholds are:

$$c^{\lambda}_{22-21}(q) = \frac{\lambda_1\left(\frac{1}{18} - \frac{3-q^2}{6(9-q^2)}\right)}{(51-5q^2)\frac{12}{(3-q^2)^2} - 34 - 6\frac{51-5q^2}{(3-q^2)^2}} = \lambda_1\frac{(3-q^2)^2}{18(9-q^2)(87-17q^2)}$$

$$c^{\lambda}_{22-12}(q) = \frac{\lambda_2\left(\frac{1}{18} - \frac{3-q^2}{6(9-q^2)}\right)}{(51-5q^2)\frac{12}{(3-q^2)^2} - 34 - 6\frac{51-5q^2}{(3-q^2)^2}} = \lambda_2\frac{(3-q^2)^2}{18(9-q^2)(87-17q^2)}$$

$$c^{\lambda}_{21-20}(q) = \frac{\lambda_2\left(\frac{1}{18} - \frac{3-q^2}{6(9-q^2)}\right)}{34 + 6\frac{51-5q^2}{(3-q^2)^2} - 68} = \lambda_2\frac{(3-q^2)^2}{18(9-q^2)(87-17q^2)}$$

$$c^{\lambda}_{12-20}(q) = \frac{\lambda_1\left(\frac{1}{18} - \frac{3-q^2}{6(9-q^2)}\right)}{34 + 6\frac{51-5q^2}{(3-q^2)^2} - 68} = \lambda_1\frac{(3-q^2)^2}{18(9-q^2)(87-17q^2)}.$$

  To see when an asymmetric allocation can be optimal, consider the case $\lambda_1 > \lambda_2$ such that $c^{\lambda}_{22-21}(q) > c^{\lambda}_{22-12}(q)$. We have the following cases:

  – For $c < c^{\lambda}_{22-12}(q)$, $E\pi^{\lambda}_{22} > \max\{E\pi^{\lambda}_{12}, E\pi^{\lambda}_{21}\}$.

– If $c \in (c^{\lambda}_{21-20}(q), c^{\lambda}_{12-20}(q)]$, we have $E\pi^{\lambda}_{22} > E\pi^{\lambda}_{21}$ and $E\pi^{\lambda}_{20} > E\pi^{\lambda}_{21}$ so $E\pi^{\lambda}_{21}$ cannot be optimal. However, $E\pi^{\lambda}_{12} > E\pi^{\lambda}_{22}$ and at the same time $E\pi^{\lambda}_{12} > E\pi^{\lambda}_{20}$.

Since the case for $\lambda_2 < \lambda_1$ is symmetric, we conclude as follows

1. Suppose $\lambda_1 > \lambda_2$. Then, for $c \in (c^{\lambda}_{21-20}(q), c^{\lambda}_{12-20}(q)]$, $E\pi^{\lambda}_{12} > \max\{E\pi^{\lambda}_{20}, E\pi^{\lambda}_{22}\}$.
2. Suppose $\lambda_2 > \lambda_1$. Then, for $c \in (c^{\lambda}_{12-20}(q), c^{\lambda}_{21-20}(q)]$, $E\pi^{\lambda}_{21} > \max\{E\pi^{\lambda}_{20}, E\pi^{\lambda}_{22}\}$.

- Second, suppose that $q \in (q', q_1)$. Then, the relevant cost thresholds are:

$$c^{\lambda}_{22-21}(q) = \frac{\lambda_1\left(\frac{1}{18} - \frac{3-q^2}{6(9-q^2)}\right)}{(51-5q^2)\frac{9-q^2}{6q^2} - 34 - 6\frac{51-5q^2}{(3-q^2)^2}} =$$

$$\lambda_1 \frac{2q^4\left(q^2-3\right)^2}{3\left(9-q^2\right)\left(5q^8 - 330q^6 + 2484q^4 - 7290q^2 + 4131\right)}$$

$$c^{\lambda}_{22-12}(q) = \frac{\lambda_2\left(\frac{1}{18} - \frac{3-q^2}{6(9-q^2)}\right)}{(51-5q^2)\frac{9-q^2}{6q^2} - 34 - 6\frac{51-5q^2}{(3-q^2)^2}} =$$

$$\lambda_2 \frac{2q^4\left(q^2-3\right)^2}{3\left(9-q^2\right)\left(5q^8 - 330q^6 + 2484q^4 - 7290q^2 + 4131\right)}$$

$$c^{\lambda}_{21-20}(q) = \frac{\lambda_2\left(\frac{1}{18} - \frac{3-q^2}{6(9-q^2)}\right)}{34 + 6\frac{51-5q^2}{(3-q^2)^2} - 68} = \lambda_2 \frac{(3-q^2)^2}{18(9-q^2)(87-17q^2)}$$

$$c^{\lambda}_{12-20}(q) = \frac{\lambda_1\left(\frac{1}{18} - \frac{3-q^2}{6(9-q^2)}\right)}{34 + 6\frac{51-5q^2}{(3-q^2)^2} - 68} = \lambda_1 \frac{(3-q^2)^2}{18(9-q^2)(87-17q^2)}.$$

Notice that for the relevant range of q:

$$\frac{2q^4\left(q^2-3\right)^2}{3\left(9-q^2\right)\left(5q^8 - 330q^6 + 2484q^4 - 7290q^2 + 4131\right)} > \frac{(3-q^2)^2}{18(9-q^2)(87-17q^2)}.$$

and therefore for $\lambda_1 > 0,\ \lambda_2 > 0$, $c^{\lambda}_{22-21}(q) > c^{\lambda}_{12-20}(q)$ and $c^{\lambda}_{22-12}(q) > c^{\lambda}_{21-20}(q)$. To see, when an asymmetric allocation can be optimal, consider the case $\lambda_1 > \lambda_2$ and therefore $c^{\lambda}_{22-21}(q) > c^{\lambda}_{22-12}(q)$. We have the following cases:

– For $c < c^{\lambda}_{22-12}(q)$, $E\pi^{\lambda}_{22} > \max\{E\pi^{\lambda}_{12}, E\pi^{\lambda}_{21}\}$.

– If $c^\lambda_{12-20}(q) > c^\lambda_{22-12}(q)$, then for $c \in (c^\lambda_{12-20}(q), c^\lambda_{22-21}(q)]$, $E\pi^\lambda_{20} > \max\{E\pi^\lambda_{21}, E\pi^\lambda_{12}\}$. However, for $c \in [c^\lambda_{22-12}(q), c^\lambda_{12-20}(q))$, $E\pi^\lambda_{21} < E\pi^\lambda_{20} < E\pi^\lambda_{12}$. Thus, in this region $E\pi^\lambda_{12} > \max\{E\pi^\lambda_{20}, E\pi^\lambda_{22}\}\}$.

Since the case for $\lambda_2 < \lambda_1$ is symmetric, we conclude as follows

1. Suppose $\lambda_1 > \lambda_2$. Then, for $c \in [c^\lambda_{22-12}(q), c^\lambda_{12-20}(q))$, $E\pi^\lambda_{12} > \max\{E\pi^\lambda_{20}, E\pi^\lambda_{22}\}\}$.
2. Suppose $\lambda_2 > \lambda_1$. Then, for $c \in [c^\lambda_{22-21}(q), c^\lambda_{21-20}(q))$ $E\pi^\lambda_{21} > \max\{E\pi^\lambda_{20}, E\pi^\lambda_{22}\}\}$.

In the next step we compare an allocation with two signals and one-sided communication to all other allocation with at most one acquired signal. First, notice that the non–informative allocation is dominated by an allocation with two acquired and not shared signals for

$$-(\lambda_1+\lambda_2)\frac{1}{18} - 68c \geq -(\lambda_1+\lambda_2)\frac{1}{12},$$

which implies $c \leq \frac{\lambda_1+\lambda_2}{2488}$. Further, the non-informative allocation is dominated by an allocation with one acquired and communicated signal (by agent i) for

$$-\lambda_i\frac{1}{18} - \lambda_j\frac{3-q^2}{36} - \min\{34, \frac{q^2+33}{2q^2}\}c > -(\lambda_i+\lambda_j)\frac{1}{12}.$$

Since the LHS of the above inequality increases in q, the inequality is satisfied if the following is true

$$-\lambda_i\frac{1}{18} - 34c > -\lambda_i\frac{1}{12},\ i = 1, 2,$$

which is true for $c \leq \frac{\lambda_i}{1224}$. In the following we assume that the above constraints are true. In the following we focus on the case of $E\pi^\lambda_{21}$ as the case of $E\pi^\lambda_{12}$ is symmetric and can be shown in the same way. In the following we show when $E\pi^\lambda_{21}$ is strictly larger than $E\pi^\lambda_{11}$. The condition is:

$$E\pi^\lambda_{21} = \overline{\pi}(\lambda_1+\lambda_2) - \lambda_1\frac{1}{18} - \lambda_2\frac{3-q^2}{6(9-q^2)} - 34c - 6\frac{51-5q^2}{(3-q^2)^2}c >$$

$$E\pi^\lambda_{11} = \overline{\pi}(\lambda_1+\lambda_2) - \lambda_1\frac{1}{18} - \lambda_2\frac{3-q^2}{36} - \min\{34, \frac{q^2+33}{2q^2}\}c,$$

which implies:

$$\lambda_2\Big(\frac{3-q^2}{36} - \frac{3-q^2}{6(9-q^2)}\Big) + \min\{34, \frac{q^2+33}{2q^2}\}c - 34c - 6\frac{51-5q^2}{(3-q^2)^2}c > 0.$$

We distinguish between two cases:

- $q < q'' = \sqrt{\frac{33}{67}} \approx 0.701$. Then the inequality becomes

$$\lambda_2\Big(\frac{3-q^2}{36} - \frac{3-q^2}{6(9-q^2)}\Big) - 6\frac{51-5q^2}{(3-q^2)^2}c > 0$$

which implies

$$c < \frac{\lambda_2(3-q^2)^4}{216(9-q^2)(51-5q^2)}.$$

- Suppose, $q \in [q'', q_1)$. Then the inequality becomes

$$\lambda_2\Big(\frac{3-q^2}{36} - \frac{3-q^2}{6(9-q^2)}\Big) + \frac{q^2+33}{2q^2}c - 34c - 6\frac{51-5q^2}{(3-q^2)^2}c > 0.$$

which implies

$$c \leq \frac{\lambda_2 q^2\left(3-q^2\right)^4}{18\left(9-q^2\right)\left(67q^6-495q^4+1413q^2-297\right)}$$

which is strictly positive for the above range of parameters.

Now, we consolidating the obtained conditions:

Given the discussion at the start, we restrict attention to costs such that $c \leq \frac{3-(\lambda_1+\lambda_2)}{1244}$ and $c \leq \frac{6-(2\lambda_1+3\lambda_2)}{1224}$.

1. $q < q'' \approx 0.701$:

   It turns out that $\frac{(3-q^2)^4}{216(9-q^2)(51-5q^2)} > \frac{(3-q^2)^2}{18(9-q^2)(87-17q^2)}$ for

   $$q < \frac{1}{\sqrt{\frac{17}{63-\frac{242}{\sqrt[3]{6822-17\sqrt{154907}}}-2\sqrt[3]{6822-17\sqrt{154907}}}}} \equiv \hat{q} \approx 0.553$$

   so that for $q < \hat{q}$ the asymmetric contract leading to $E\pi_{21}^{\lambda}$ is a global optimum for $c < \frac{\lambda_2(3-q^2)^4}{216(9-q^2)(51-5q^2)}$. If $\lambda_2 > \lambda_1$, then $E\pi_{21}^{\lambda}$ is a global optimum for

   $$c \in \Big(\lambda_1\frac{(3-q^2)^2}{18(9-q^2)(87-17q^2)}, \lambda_2\frac{(3-q^2)^4}{18(9-q^2)(87-17q^2)}\Big].$$

2. $q'' \leq q < q' \approx 0.803$.

   It turns out that for the above range of parameters

$$\frac{q^2\left(3-q^2\right)^4}{18\left(9-q^2\right)\left(67q^6-495q^4+1413q^2-297\right)} > \frac{(3-q^2)^2}{18(9-q^2)(87-17q^2)}$$

and so $E\pi_{21}^{\lambda}$ is a global optimum for

$$c \in \left(\lambda_1 \frac{(3-q^2)^2}{18(9-q^2)(87-17q^2)}, \lambda_2 \frac{(3-q^2)^2}{18(9-q^2)(87-17q^2)}\right].$$

3. Finally, $q' < q < q_1 \approx 0.855$.

   Given the previous point, we conclude that the asymmetric contract yielding $E\pi_{21}^{\lambda}$ is a global optimum for

$$c \in \left(\lambda_1 \frac{2q^4\left(q^2-3\right)^2}{3\left(9-q^2\right)\left(5q^8-330q^6+2484q^4-7290q^2+4131\right)}, \lambda_2 \frac{(3-q^2)^2}{18(9-q^2)(87-17q^2)}\right].$$

Q.E.D.

Appendix not for publication

Costly decision implementation. Suppose that making the decision y_i precise costs effort $c_0 > 0$ to agent i. If she does not pay c_0, the choice y_i is a random draw from a uniform distribution on $[0, 1]$. Even in the case agent i has no information, precise implementation is valuable, because the loss $L_i(y_i) = \int_0^1 (\frac{1}{2} - \theta)^2 d\theta = \frac{1}{12}$ determined by $y_i = E[\theta] = 1/2$ is smaller than the loss induced by a random y_i: $\bar{L}_i = \int_0^1 \int_0^1 (y - \theta)^2 d\theta dy = \frac{1}{6}$.

We first show that, despite the implementation cost c_0, the principal assigns contracts that require both agents to implement y_i precisely in equilibrium, for every value q of the correlation between the states θ_1 and θ_2 and of the research cost c. Together with the different courses of decision considered in the paper (both agents collect and share signals, both collect a signal and one shares it, both collect signals without sharing, one agent collects and shares a signal and the other does not, only one agent collects a signal and does not share it, and neither agent collects a signal) in which agents chose decisions precisely, we consider also two additional possibilities: only one agent collects and does not share a signal and the other agent makes a random decision, and neither agent collects information with either or both making imprecise decisions. The other possibilities make no sense. The principal would never remunerate an agent to collect information to then ask him to act imprecisely, nor she would remunerate an agent to communicate with the other, to then ask the latter to act imprecisely.

Let us consider the simplest course of decision: neither agent collects information and either or both make imprecise decisions. In this case, both agent i's optimal contract is such that $z_i = a_i = b_i = 0$. The optimal contract to induce an agent i to make a precise decision $y_i = E[\theta]$, while still not requiring either agent to collect information is such that $b_i = 0$ and $z_i = a_i = 12c_0$, because a_i is pinned down by the precision obedience constraint:

$$a_i[\bar{L}_i - L_i(y_i)] = a_i \frac{1}{12} \geq c_0,$$

The principal's payoff for asking i to be precise is

$$E\pi_i = -\frac{1}{12} - (1 - a_i)\frac{1}{12}.$$

Plugging in $a_i = 12c_0$, this quantity is shown to be larger than $\bar{L}_i = \frac{1}{6}$, the payoff when i makes a random decision:

$$-\frac{1}{12} - (1 - 12c_0)\frac{1}{12} = -\frac{1}{6} + c_0 \geq -\frac{1}{6}, \quad \text{for all } c_0 \geq 0.$$

As a result, inducing neither agent to collect information but both to implement y_i precisely yields a higher payoff than letting agents neither collect information, nor implementing y_i

precisely. The same argument shows that the course of decision in which only one agent collects and does not share a signal and the other agent makes a random decision yields a lower profit than optimally inducing one agent to collect a signal and not share it, and the other agent to not collect information, but implement y_i precisely to the best of his knowledge.

In the remainder, we assume that $c_0 < c$: implementation requires less effort than research and information acquisition.

Let us calculate the optimal linear contract to induce both agents collect and share signals and to chose decisions precisely. Using the calculations in the paper, agent 1's equilibrium payoff is:

$$u_1(s_i) = z_1 - (a_1 + b_1)\frac{3-q^2}{6(9-q^2)} - c - c_0.$$

His payoff in case she does not choose y_1 precisely is:

$$u_1(s_1) = z_1 - a_1\frac{1}{6} - b_1\frac{3-q^2}{6(9-q^2)} - c,$$

and his payoff if not collecting s_1 is:

$$u_i^o = z_1 - a_1\frac{3-q^2}{36} - b_1\frac{27+q^4}{6(9-q^2)^2} - c_0.$$

Hence, the information acquisition constraint (5) is unchanged, where precise implementation entails the additional constraint:

$$a_1\frac{1}{(9-q^2)} \geq c_0,$$

or, $a_1 \geq (9-q^2)c_0$.

The minimization of $(a_1 + b_1)[1 - \frac{3-q^2}{6(9-q^2)}]$ subject to $a_1 \geq (9-q^2)c_0$, $b_1 \geq 0$ and the information acquisition constraint () yields the following solution. If $q < q' \approx 0.803$, because $c_0 < c$, the precise implementation constraint is slack and the solution is $z_1 = a_1 = 36\frac{9-q^2}{(3-q^2)^2}c$ and $b_1 = 0$, as in the case without costly decision precise implementation. The expected profit $E\pi$ is also unchanged.

If $q > q'$, then $z_1 = a_1 + b_1$, $a_1 = (9-q^2)c_0$ and b_1 solves the information acquisition constraint

$$(9-q^2)c_0\frac{(3-q^2)^2}{36(9-q^2)} + b_1\frac{2q^2}{(9-q^2)^2} = c,$$

so that $b_1 = \frac{(9-q^2)^2}{2q^2}(c - \frac{(3-q^2)^2}{36}c_0)$. (Recall that without costly precise implementation, the optimal solution is $a_1 = 0$ and $z_1 = b_1 = \frac{(9-q^2)^2}{2q^2}c$.) The expected profit is

$$\begin{aligned} E\pi &= 2\overline{\pi} - \frac{3-q^2}{3(9-q^2)} - 2(a_1+b_1)[1 - \frac{3-q^2}{6(9-q^2)}] \\ &= 2\overline{\pi} - \frac{3-q^2}{3(9-q^2)} - (51-5q^2)[\frac{9-q^2}{6q^2}c + \frac{1}{3}(1 - \frac{(9-q^2)(3-q^2)^2}{72q^2})c_0], \end{aligned}$$

smaller than in the case without costly precise implementation because $1 + \frac{(9-q^2)(3-q^2)^2}{72q^2}$ if and only if $q > q'$.

Wrapping up the two cases, the expected profit is:

$$E\pi_{22} = 2\overline{\pi} - \frac{3-q^2}{3(9-q^2)} - (51-5q^2)\min\left\{\frac{12}{(3-q^2)^2}, \frac{9-q^2}{6q^2}c + \frac{1}{3}(1 - \frac{(9-q^2)(3-q^2)^2}{72q^2})c_0\right\}.$$

Proceeding in the same fashion, the optimal linear contracts t_1, t_2 to induce both agents $i = 1, 2$ to collect information and only one of them, say agent 1, to transmit it to the other agent j is such that $z_2 = a_2 = 36\frac{9-q^2}{(q^2-3)^2}c$, $b_2 = 0$, and that $z_1 = a_1 = 36c$, $b_1 = 0$ for $q < \sqrt{\frac{(28\sqrt{3}-\sqrt{2267})3\sqrt{3}}{5}} \approx 0.958$, like in the case without implementation costs. For $q > \sqrt{\frac{(28\sqrt{3}-\sqrt{2267})3\sqrt{3}}{5}}$, the implementation constraint binds, so that $a_1 = 9c_0$ and the information acquisition constraint becomes:

$$9c_0\frac{1}{36} + b_1\frac{2q^2}{(9-q^2)^2} = c,$$

thus yielding $b_1 = \frac{(9-q^2)^2}{2q^2}(c - \frac{1}{4}c_0)$ and $z_1 = a_1 + b_1$. The resulting principal's payoff is:

$$E\pi_{21} = 2\overline{\pi} - \frac{1}{9}\frac{9-2q^2}{9-q^2} - \min\left\{34c, (51-5q^2)(c\frac{9-q^2}{12q^2} + (\frac{9}{9-q^2} - \frac{9-q^2}{8q^2})\frac{1}{6}c_0)\right\} - 6\frac{51-5q^2}{(3-q^2)^2}c.$$

Likewise, the optimal linear contracts t_1, t_2 to induce both agents $i = 1, 2$ to collect information and not to transmit it to the other agent j are such that $z_i = a_i = 36c$, $b_i = 0$, and yield expected profit:

$$E\pi_{20} = 2\overline{\pi} - \frac{1}{9} - 68c.$$

Further, the optimal linear contracts t_1, t_2 to induce one agent, say agent 1, to collect information and transmit it to the other agent 2, and agent 2 to not collect information, are such that $z_2 = a_2 = 9c_0$, $b_2 = 0$, and that $z_1 = a_1 = 36c$, $b_1 = 0$ for $q < \sqrt{\frac{33}{67}} \approx 0.701$, and $a_1 = 9c_0$, $b_1 = \frac{18}{q^2}(c - \frac{1}{4}c_0)$, $z_1 = a_1 + b_1$ for $q > \sqrt{\frac{33}{67}}$. They yield the principal's expected profit:

$$E\pi_{11} = 2\overline{\pi} - \frac{1}{36}(5-q^2) - \min\left\{34c, \frac{1}{2}\frac{q^2+33}{q^2}(\frac{1}{4}c_0(2q^2-1)+c)\right\}.$$

Optimal linear contracts t_1, t_2 to induce only one agents, say agent 1 to collect signal s_1, but not to transmit to agent 2 are such that $z_1 = a_1 = 36c$, $b_1 = 0$ and $z_2 = a_2 = b_2 = 0$. They yield expected payoff:

$$E\pi_{10} = 2\overline{\pi} - \frac{5}{36} - 34c.$$

The optimal linear contracts t_1, t_2 in the case that both agents $i = 1, 2$ are not supposed to collect information are such that $z_i = a_i = 12c_0$ and $b_i = 0$. This leads to expected principal's profit:

$$\begin{aligned} E\pi_{00} &= 2\overline{\pi} - 2\frac{1}{12} - 2 \cdot 12c_0(1 - \frac{1}{12}) \\ &= 2\overline{\pi} - \frac{1}{6} - 22c_0. \end{aligned}$$

Comparing $E\pi_{11}$ and $E\pi_{10}$, it is still the case that $E\pi_{11} > E\pi_{10}$ for all c, q, c_0 as in the case without costly implementation. Comparing $E\pi_{22}$, $E\pi_{21}$ and $E\pi_{20}$, we see that both $E\pi_{20}(q, c, c_0) - E\pi_{21}(q, c, c_0)$ and $E\pi_{22}(q, c, c_0) - E\pi_{21}(q, c, c_0)$ increase in c_0.

Further, the results about profit maximizing allocation achieved through optimal linear contracts with $c_0 = 0$ extend qualitatively (in the sense that the thresholds and threshold functions in the statements below are functions of the implementation cost c_0):

1. For correlation values $q < \tilde{q}(c_0)$ and information acquisition cost values $c < c(q)_{22-20}$, and for $q > \tilde{q}$ and $c < c(q)_{22-11}$, both agents $i = 1, 2$ collect signal s_i and share it with the other agent j.
2. When $q < \tilde{q}$ and $c(q)_{22-20} < q < c(q)_{20-11}$, both agents $i = 1, 2$ collect signal s_i but do not share it with j.
3. When $c(q)_{22-11} < c < c(q)_{11-00}$, for all q, only one agent i collects signal s_i and shares it with j.
4. When $c > c(q)_{11-00}$, for all q, neither agent i collects signal s_i.

Most importantly, we obtain generalization of our main result, which determines the optimal linear contracts ℓ_1, ℓ_2 that induce agents to collect and share information by making the payment depend on the precision of the other agent j.

Corollary: The profit maximizing linear contracts t_1, t_2 are as follows.

1. If the states are sufficiently correlated ($q > q' \approx 0.803$) and signal acquisition cheap ($c < c(q)_{22-11}$), then $a_i = 0$ and $z_i = b_i > 0$ for both $i = 1, 2$. Each agent $i = 1, 2$ is induced to collect signal s_i and share it with the other agent j with a reward based on the other agent j's performance.
2. For sufficient correlation ($q > \sqrt{\frac{33}{67}} \approx 0.701$) and intermediate signal costs ($c(q)_{22-11} < c < c(q)_{11-00}$), only one agent i is induced to collect s_i and share with j with a reward based j's performance. (The other agent receives a flat payment.)

3. For all other values of q and c, each agent i is induced to collect s_i and possibly share s_i with j only with rewards based on i's own performance.

The only difference with Corollary 1 in the paper is that the formulas $c(q)_{22-11}$ and $c(q)_{11-00}$ depend on c_0, and it is interesting that the thresholds q' and $\sqrt{\frac{33}{67}}$ do not.

Calculations omitted from the proof of Lemma 4. To show that $D_{22-21}(q) > 0$ for $0 \leq q < q_1 \approx 0.855$ and $D_{22-21}(q) < 0$ for $q_1 < q \leq 1$, we distinguish three cases.

For $0 \leq q \leq q' \approx 0.803$, it is the case that $\min\left\{\frac{12}{(3-q^2)^2}, \frac{9-q^2}{6q^2}\right\} = \frac{12}{(3-q^2)^2}$ and $\min\left\{34, \frac{9-q^2}{12q^2}(51-5q^2)\right\} = 34$. As a result, simplifying notation,

$$D(q) = \frac{12}{(3-q^2)^2}(51-5q^2) - 34 - 6\frac{51-5q^2}{(3-q^2)^2} = 2q^2\frac{87-17q^2}{(q^2-3)^2} > 0.$$

Because $E\pi_{22}(q,c) - E\pi_{21}(q,c)$ is linear in c and $E\pi_{22}(q,c) > E\pi_{21}(q,c)$ for $c = 0$, whether $E\pi_{22}(q,c)$ is larger or smaller than $E\pi_{21}(q,c)$ depends on whether c is smaller or larger than a threshold $c_{22-21}(q)$ implicitly defined by the equation $E\pi_{22}(q,c) = E\pi_{21}(q,c)$.

For $q' \leq q \leq \sqrt{\frac{(28\sqrt{3}-\sqrt{2267})3\sqrt{3}}{5}} \approx 0.958$, we have that $\min\left\{\frac{12}{(3-q^2)^2}, \frac{9-q^2}{6q^2}\right\} = \frac{1}{6q^2}(9-q^2)$ and $\min\left\{34, \frac{9-q^2}{12q^2}(51-5q^2)\right\} = 34$. As a result,

$$D(q) = \frac{9-q^2}{6q^2}(51-5q^2) - 34 - 6\frac{51-5q^2}{(3-q^2)^2}.$$

This function is strictly decreasing for $q \in [0,1]$, and crosses zero at $q_1 \approx 0.855$.

For $\sqrt{\frac{(28\sqrt{3}-\sqrt{2267})3\sqrt{3}}{5}} \leq q \leq 1$, because $\min\left\{\frac{12}{(3-q^2)^2}, \frac{9-q^2}{6q^2}\right\} = \frac{1}{6q^2}(9-q^2)$ and $\min\left\{34, \frac{9-q^2}{12q^2}(51-5q^2)\right\} = \frac{9-q^2}{12q^2}(51-5q^2)$, it is the case that

$$D(q) = \tilde{f}(q) = \frac{9-q^2}{6q^2}(51-5q^2) - \frac{9-q^2}{12q^2}(51-5q^2) - 6\frac{51-5q^2}{(3-q^2)^2}.$$

This function is strictly negative $q > \sqrt{\frac{(28\sqrt{3}-\sqrt{2267})3\sqrt{3}}{5}}$, so that $E\pi_{22}(q,c) > E\pi_{21}(q,c)$ for all c.

To show that $c_{22-21}(q) \geq c_{21-20}(q)$ for all $q \leq q_1$, we distinguish two cases.

For $0 \leq q \leq q'$, because $\min\left\{\frac{12}{(3-q^2)^2}, \frac{9-q^2}{6q^2}\right\} = \frac{12}{(3-q^2)^2}$ and $\min\left\{34, \frac{9-q^2}{12q^2}(51-5q^2)\right\} = 34$, the formulas for $c(q)_{22-21}$ and $c(q)_{21-20}$ become:

$$c_{22-21}(q) = \frac{\frac{1}{18} - \frac{3-q^2}{3(9-q^2)} + \frac{3-q^2}{6(9-q^2)}}{12\frac{51-5q^2}{(3-q^2)^2} - 34 - 6\frac{51-5q^2}{(3-q^2)^2}} = \frac{\frac{1}{18} - \frac{3-q^2}{6(9-q^2)}}{34 + 6\frac{51-5q^2}{(q^2-3)^2} - 68} = c(q)_{21-20}.$$

We conclude that $c_{22-21}(q) = c_{21-20}(q)$ for all $0 \leq q \leq q'$.

Turning to $q' < q \leq q_1$, because $\min\left\{\frac{12}{(3-q^2)^2}, \frac{9-q^2}{6q^2}\right\} = \frac{9-q^2}{6q^2} < \frac{12}{(3-q^2)^2}$ and $\min\left\{34, \frac{9-q^2}{12q^2}(51-5q^2)\right\} = 34$, the formula of $c(q)_{21-20}$ is unchanged, and the formula for $c(q)_{22-21}$ becomes

$$c_{22-21}(q) = \frac{\frac{1}{18} - \frac{3-q^2}{3(9-q^2)} + \frac{3-q^2}{6(9-q^2)}}{\frac{9-q^2}{6q^2}(51-5q^2) - 34 - 6\frac{51-5q^2}{(3-q^2)^2}} > \frac{\frac{1}{18} - \frac{3-q^2}{3(9-q^2)} + \frac{3-q^2}{6(9-q^2)}}{\frac{12}{(3-q^2)^2}(51-5q^2) - 34 - 6\frac{51-5q^2}{(3-q^2)^2}} = c(q)_{21-20}.$$

We obtain that $c_{22-21}(q) > c_{21-20}(q)$ for all $q' < q < q_1$.

Calculations omitted from the Proof of Proposition 4. We first prove that $c_{22-20}(q) > 0$ if and only if $q < \sqrt{\frac{(28\sqrt{3}-\sqrt{2267})3\sqrt{3}}{5}} \approx 0.958$. Instead, it is the case that $c_{22-11}(q) > 0$ and $c_{22-00}(q) > 0$ for all $q \in [0, 1]$.

To show $c_{22-20}(q) > 0$ if and only if $q < \sqrt{\frac{(28\sqrt{3}-\sqrt{2267})3\sqrt{3}}{5}}$, we consider two cases.

For $0 \leq q < q' \approx 0.803$, $\frac{12}{(q^2-3)^2} < \frac{9-q^2}{6q^2}$ and so,

$$c_{22-20}(q) = \frac{\frac{1}{9} - \frac{3-q^2}{3(9-q^2)}}{\frac{12}{(q^2-3)^2}(51-5q^2) - 68} = \frac{1}{18}\frac{(q^2-3)^2}{(87-17q^2)(9-q^2)} > 0,$$

whereas for $q' < q \leq 1$,

$$c_{22-20}(q) = \frac{\frac{1}{9} - \frac{3-q^2}{3(9-q^2)}}{\frac{9-q^2}{6q^2}(51-5q^2) - 68} = \frac{4}{3}\frac{q^4}{(9-q^2)(459-504q^2+5q^4)} \propto \frac{1}{459-504q^2+5q^4} > (<)0$$

if and only if $q < (>)\sqrt{\frac{(28\sqrt{3}-\sqrt{2267})3\sqrt{3}}{5}}$ on the admissible range $q \in [0, 1]$.

To show $c_{22-11}(q) > 0$ for all $q \in [0, 1]$, we consider three cases.

For $0 \leq q < \sqrt{\frac{33}{67}} \approx 0.701$, $34 < \frac{1}{2}\frac{q^2+33}{q^2}$ and $\frac{12}{(q^2-3)^2} < \frac{9-q^2}{6q^2}$ and so,

$$c_{22-11}(q) = \frac{\frac{5-q^2}{36} - \frac{3-q^2}{3(9-q^2)}}{\frac{12}{(3-q^2)^2}(51-5q^2) - 34} = \frac{(3-q^2)^2(9-2q^2+q^4)}{72(9-q^2)(153+72q^2-17q^4)} > 0.$$

For $\sqrt{\frac{33}{67}} < q < q' \approx 0.803$, $34 > \frac{1}{2}\frac{q^2+33}{q^2}$ and $\frac{12}{(q^2-3)^2} < \frac{9-q^2}{6q^2}$ and so,

$$\begin{aligned} c_{22-11}(q) &= \frac{\frac{5-q^2}{36} - \frac{3-q^2}{3(9-q^2)}}{\frac{12}{(3-q^2)^2}(51-5q^2) - \frac{1}{2}\frac{q^2+33}{q^2}} = -\frac{\frac{1}{18}q^2(q^2-3)^2(9-2q^2+q^4)}{(297-1413q^2+147q^4+q^6)(9-q^2)} \\ &\propto -\frac{1}{297-1413q^2+147q^4+q^6} > 0 \end{aligned}$$

if $q > q_2 \approx 0.464$.

For $q' < q \leq 1$,

$$c_{22-11}(q) = \frac{\frac{5-q^2}{36} - \frac{3-q^2}{3(9-q^2)}}{\frac{9-q^2}{6q^2}(51-5q^2) - \frac{1}{2}\frac{q^2+33}{q^2}} = \frac{1}{6}\frac{q^2(9-2q^2+q^4)}{(24-5q^2)(15-q^2)(9-q^2)} > 0$$

for all $q \in [0,1]$.

We now compare $c_{22-11}(q)$ and $c_{22-00}(q)$, and prove that $c_{22-11}(q) < c_{22-00}(q)$ for all $q \in [0,1]$, and that $c_{22-20}(q) < c_{22-11}(q)$ if and only if $q < \tilde{q} \approx 0.553$ on the relevant range $q \in [0, \sqrt{\frac{(28\sqrt{3}-\sqrt{2267})3\sqrt{3}}{5}}]$.

To show $c_{22-11}(q) < c_{22-00}(q)$ for all $q \in [0,1]$, we consider three cases.

For $0 \leq q < \sqrt{\frac{33}{67}} \approx 0.701$,

$$\begin{aligned} c_{22-11}(q) - c_{22-00}(q) &= \frac{\frac{5-q^2}{36} - \frac{3-q^2}{3(9-q^2)}}{\frac{12}{(3-q^2)^2}(51-5q^2) - 34} - \frac{\frac{1}{6} - \frac{3-q^2}{3(9-q^2)}}{\frac{12}{(3-q^2)^2}(51-5q^2)} \\ &= -\frac{1}{18}\frac{(3-q^2)^2 q^2(129-10q^2-3q^4)}{(9-q^2)(51-5q^2)(153+72q^2-17q^4)} < 0. \end{aligned}$$

For $\sqrt{\frac{33}{67}} < q < q'$,

$$\begin{aligned} c_{22-11}(q) - c_{22-00}(q) &= \frac{\frac{5-q^2}{36} - \frac{3-q^2}{3(9-q^2)}}{\frac{12}{(3-q^2)^2}(51-5q^2) - \frac{1}{2}\frac{q^2+33}{q^2}} - \frac{\frac{1}{6} - \frac{3-q^2}{3(9-q^2)}}{\frac{12}{(3-q^2)^2}(51-5q^2)} \\ &= -\frac{(3-q^2)^2(891-2106q^2-1560q^4+394q^6-19q^8)}{72(9-q^2)(51-5q^2)(297-1413q^2+147q^4+q^6)} < 0, \end{aligned}$$

because $891 - 2106q^2 - 1560q^4 + 394q^6 - 19q^8 < 0$ for $q > q_3 \approx 0.586$ and $297 - 1413q^2 + 147q^4 + q^6 < 0$ for $q > q_2 \approx 0.464$.

For $q' < q \leq 1$,

$$\begin{aligned} c_{22-11}(q) - c_{22-00}(q) &= \frac{\frac{5-q^2}{36} - \frac{3-q^2}{3(9-q^2)}}{\frac{9-q^2}{6q^2}(51-5q^2) - \frac{1}{2}\frac{q^2+33}{q^2}} - \frac{\frac{1}{6} - \frac{3-q^2}{3(9-q^2)}}{\frac{9-q^2}{6q^2}(51-5q^2)} \\ &= -\frac{1}{6}q^2\frac{2349+2160q^2-1200q^4+136q^6-5q^8}{(15-q^2)(24-5q^2)(51-5q^2)(9-q^2)^2} < 0. \end{aligned}$$

To show that $c_{22-20}(q) < c_{22-11}(q)$ if and only if $q < \tilde{q}$ on the relevant range $q \in [0, \sqrt{\frac{(28\sqrt{3}-\sqrt{2267})3\sqrt{3}}{5}}]$, we consider two cases.

For $0 \leq q < \sqrt{\frac{33}{67}}$,

$$c_{22-20}(q) - c_{22-11}(q) = \frac{\frac{1}{9} - \frac{3-q^2}{3(9-q^2)}}{\frac{12}{(q^2-3)^2}(51-5q^2) - 68} - \frac{\frac{5-q^2}{36} - \frac{3-q^2}{3(9-q^2)}}{\frac{12}{(3-q^2)^2}(51-5q^2) - 34}$$

$$= \frac{1}{72}\frac{(615q^2-189q^4+17q^6-171)(3-q^2)^2}{(9-q^2)(87-17q^2)(153+72q^2-17q^4)} \propto 615q^2 - 189q^4 + 17q^6 - 171 < 0$$

if and only if

$$q < \tilde{q} = \sqrt{\frac{63}{17} - \sqrt[3]{\frac{54\,576}{4913} - \frac{1}{289}\sqrt{9914\,048}} - \frac{484}{289\sqrt[3]{\frac{54\,576}{4913} - \frac{1}{289}\sqrt{9914\,048}}}} \approx 0.553.$$

For $\sqrt{\frac{33}{67}} < q < \sqrt{\frac{(28\sqrt{3}-\sqrt{2267})3\sqrt{3}}{5}} \approx 0.958$,

$$\begin{aligned} c_{22-20}(q) - c_{22-11}(q) &= \frac{\frac{1}{9} - \frac{3-q^2}{3(9-q^2)}}{\frac{12}{(q^2-3)^2}(51-5q^2) - 68} - \frac{\frac{5-q^2}{36} - \frac{3-q^2}{3(9-q^2)}}{\frac{12}{(3-q^2)^2}(51-5q^2) - \frac{1}{2}\frac{q^2+33}{q^2}} \\ &= \frac{1}{18}\frac{(3-q^2)^2(297 - 630q^2 - 180q^4 + 122q^6 - 17q^8)}{(9-q^2)(87-17q^2)(297 - 1413q^2 + 147q^4 + q^6)} \\ &\propto \frac{297 - 630q^2 - 180q^4 + 122q^6 - 17q^8}{297 - 1413q^2 + 147q^4 + q^6} > 0 \end{aligned}$$

for all $q > \sqrt{\frac{33}{67}}$, because $297 - 630q^2 - 180q^4 + 122q^6 - 17q^8 > 0$ for all $q > q_4 \approx 0.658$ and $297 - 1413q^2 + 147q^4 + q^6 > 0$ for all $q > q_2$.

Comparing $c_{22-00}(q)$, $c_{20-00}(q)$ and $c_{11-00}(q)$, we show that $c_{11-00}(q) > c_{22-00}(q)$, and $c_{11-00}(q) > c_{20-00}(q)$ for all $q \in [0,1]$.

To show $c_{11-00}(q) > c_{22-00}(q)$, we distinguish three cases.

For $0 \leq q < \sqrt{\frac{33}{67}} \approx 0.701$,

$$c_{11-00}(q) - c_{22-00}(q) = \frac{q^2+1}{36 \cdot 34} - \frac{\frac{1}{6} - \frac{q^2-3}{3q^2-27}}{\frac{12}{(q^2-3)^2}(51-5q^2)} = \frac{q^2(129 - 10q^2 - 3q^4)}{306(9-q^2)(51-5q^2)} > 0.$$

For $\sqrt{\frac{33}{67}} < q < q' \approx 0.803$,

$$\begin{aligned} c_{11-00}(q) - c_{22-00}(q) &= \frac{q^2+1}{36\frac{1}{2q^2}(q^2+33)} - \frac{\frac{1}{6} - \frac{q^2-3}{3q^2-27}}{\frac{12}{(q^2-3)^2}(51-5q^2)} \\ &= \frac{1}{72}\frac{2106q^2 + 1560q^4 - 394q^6 + 19q^8 - 891}{(q-3)(q+3)(q^2+33)(5q^2-51)} \\ &\propto 2106q^2 + 1560q^4 - 394q^6 + 19q^8 - 891 > 0 \end{aligned}$$

for $q > q_3 \approx 0.586$.

For $q' \approx 0.803 < q \leq 1$,

$$\begin{aligned} c_{11-00}(q) - c_{22-00}(q) &= \frac{q^2+1}{36\frac{1}{2q^2}(q^2+33)} - \frac{\frac{1}{6} - \frac{q^2-3}{3q^2-27}}{\frac{9-q^2}{6q^2}(51-5q^2)} \\ &= \frac{1}{18}q^2\frac{-2160q^2 + 1200q^4 - 136q^6 + 5q^8 - 2349}{(q^2+33)(5q^2-51)(q-3)^2(q+3)^2} > 0. \end{aligned}$$

To show $c_{11-00}(q) > c_{20-00}(q)$, we distinguish two cases.

For $0 \leq q < \sqrt{\frac{33}{67}}$,

$$c_{11-00}(q) - c_{20-00}(q) = \frac{q^2+1}{36 \cdot 34} - \frac{1}{1224} = \frac{1}{1224} q^2 > 0.$$

For $\frac{1}{67}\sqrt{33}\sqrt{67} \approx 0.701 < q \leq 1$,

$$\begin{aligned} c_{11-00}(q) - c_{20-00}(q) &= \frac{q^2+1}{36\frac{1}{2q^2}(q^2+33)} - \frac{1}{1224} = \frac{1}{1224}\frac{67q^2+68q^4-33}{q^2+33} \\ &\propto -33+67q^2+68q^4 > 0 \end{aligned}$$

for $q > \sqrt{\frac{1}{34}\sqrt{13\,465} - \frac{67}{34}} \approx 0.600$.

We conclude by comparing $c_{20-11}(q)$ with $c_{22-20}(q)$, $c_{22-11}(q)$ and $c_{11-00}(q)$, and prove that $c_{20-11}(q) = c_{11-00}(q)$ for $q = 0$, that $c_{20-11}(q) < c_{11-00}(q)$ for all $q > 0$, that $c_{20-11}(q) > c_{22-20}(q)$ for $0 \leq q < \tilde{q} \approx 0.553$, that $c_{20-11}(q) = c_{22-20}(q) = c_{22-11}(q)$ for $q = \tilde{q}$ and that $c_{20-11}(q) < c_{22-11}(q)$ for $\tilde{q} < q \leq 1$.

First, we note

$$c_{20-11}(0) = \frac{1}{2448 - 36 \cdot 34} = \frac{1}{1224} = c_{11-00}(q) = \frac{1}{36 \cdot 34},$$

and that

$$\begin{aligned} & c_{20-11}(\tilde{q}) = \frac{1-q^2}{2448-36\cdot 34} = c_{22-20}(\tilde{q}) = \frac{\frac{1}{9} - \frac{3-q^2}{3(9-q^2)}}{\frac{12}{(3-q^2)^2}(51-5q^2) - 68} \\ = \;& c_{22-11}(\tilde{q}) = \frac{\frac{5-q^2}{36} - \frac{3-q^2}{3(9-q^2)}}{\frac{12}{(3-q^2)^2}(51-5q^2) - 34} \\ = \;& \frac{1}{353\,736} \frac{484 - 782\sqrt[3]{-\frac{8}{289}\sqrt{154\,907} + \frac{54\,576}{4913}} + 289\sqrt[3]{-\frac{8}{289}\sqrt{154\,907} + \frac{54\,576}{4913}}^{\,2}}{\sqrt[3]{-\frac{8}{289}\sqrt{154\,907} + \frac{54\,576}{4913}}} \approx 5.67 \times 10^{-4}. \end{aligned}$$

To show $c_{20-11}(q) < c_{11-00}(q)$ for all $q > 0$, we distinguish two cases.

For $0 \leq q < \sqrt{\frac{33}{67}}$,

$$c_{20-11}(q) - c_{11-00}(q) = \frac{1-q^2}{2448 - 36 \cdot 34} - \frac{1+q^2}{36 \cdot 34} = -\frac{1}{612} q^2 < 0.$$

For $\sqrt{\frac{33}{67}} < q \leq 1$,

$$\begin{aligned} c_{20-11}(q) - c_{11-00}(q) &= \frac{1-q^2}{2448 - 36\frac{q^2+33}{2q^2}} - \frac{1+q^2}{36\frac{1}{2}\frac{q^2+33}{q^2}} \\ &= -\frac{1}{3942} q^2 \frac{3597 - 6008q^2 - 4933q^4}{(q^2+33)(11-45q^2)} \propto -\frac{3597 - 6008q^2 - 4933q^4}{(q^2+33)(11-45q^2)} < 0 \end{aligned}$$

for $q > \sqrt{\frac{1}{4933}\sqrt{26\,768\,017} - \frac{3004}{4933}} \approx 0.663$. (Specifically, $3597 - 6008q^2 - 4933q^4 > 0$, for $q > \sqrt{\frac{1}{4933}\sqrt{26\,768\,017} - \frac{3004}{4933}}$ and $11 - 45q^2 > 0$, for $q > \frac{1}{15}\sqrt{5}\sqrt{11} \approx 0.494$).

To show $c_{20-11}(q) > c_{22-20}(q)$ for $0 \leq q < \tilde{q}$, we note that

$$\begin{aligned} c_{22-20}(q) - c_{20-11}(q) &= \frac{\frac{1}{9} - \frac{3-q^2}{3(9-q^2)}}{\frac{12}{(3-q^2)^2}(51-5q^2) - 68} - \frac{1-q^2}{2448 - 36 \cdot 34} \\ &= \frac{1}{1224}\frac{-171 + 615q^2 - 189q^4 + 17q^6}{(9-q^2)(87-17q^2)} \propto -171 + 615q^2 - 189q^4 + 17q^6 < 0 \end{aligned}$$

for $q < \tilde{q}$.

To show $c_{20-11}(q) < c_{22-11}(q)$ for $\tilde{q} < q \leq 1$, we distinguish three cases.

For $0 \leq q < \sqrt{\frac{33}{67}}$,

$$\begin{aligned} c_{22-11}(q) - c_{20-11}(q) &= \frac{\frac{5-q^2}{36} - \frac{3-q^2}{3(9-q^2)}}{\frac{12}{(3-q^2)^2}(51-5q^2) - 34} - \frac{1-q^2}{2448 - 36 \cdot 34} \\ &= \frac{1}{612}q^2\frac{-171 + 615q^2 - 189q^4 + 17q^6}{(9-q^2)(153 + 72q^2 - 17q^4)} \propto -171 + 615q^2 - 189q^4 + 17q^6 > 0 \end{aligned}$$

for $q > \tilde{q}$.

For $\sqrt{\frac{33}{67}} < q < q'$,

$$\begin{aligned} c_{22-11}(q) - c_{20-11}(q) &= \frac{\frac{5-q^2}{36} - \frac{3-q^2}{3(9-q^2)}}{\frac{12}{(3-q^2)^2}(51-5q^2) - \frac{1}{2}\frac{q^2+33}{q^2}} - \frac{1-q^2}{2448 - 36\frac{q^2+33}{2q^2}} \\ &= \frac{4}{27}\frac{q^4(-297 + 630q^2 + 180q^4 - 122q^6 + 17q^8)}{(9-q^2)(11-45q^2)(297 - 1413q^2 + 147q^4 + q^6)} \\ &\propto \frac{-297 + 630q^2 + 180q^4 - 122q^6 + 17q^8}{297 - 1413q^2 + 147q^4 + q^6} > 0, \end{aligned}$$

for $q > q_4 \approx 0.658$. (Specifically, $297 - 1413q^2 + 147q^4 + q^6 > 0$ for $q > q_2 \approx 0.464$ and $-297 + 630q^2 + 180q^4 - 122q^6 + 17q^8 > 0$ for $q > q_4 \approx 0.658$.)

For $q' < q \leq 1$,

$$\begin{aligned} c_{22-11}(q) - c_{20-11}(q) &= \frac{\frac{5-q^2}{36} - \frac{3-q^2}{3(9-q^2)}}{\frac{9-q^2}{6q^2}(51-5q^2) - \frac{1}{2}\frac{q^2+33}{q^2}} - \frac{1-q^2}{2448 - 36\frac{q^2+33}{2q^2}} \\ &= \frac{1}{54}\frac{q^2(4131 - 8334q^2 + 2304q^4 - 554q^6 + 5q^8)}{(9-q^2)(15-q^2)(24-5q^2)(11-45q^2)} \\ &\propto \frac{4131 - 8334q^2 + 2304q^4 - 554q^6 + 5q^8}{11 - 45q^2} > 0. \end{aligned}$$

(Specifically, $4131 - 8334q^2 + 2304q^4 - 554q^6 + 5q^8 < 0$ for $q > q_5 \approx 0.758$ and $11 - 45q^2 < 0$ for $q > \frac{1}{15}\sqrt{5}\sqrt{11} \approx 0.494$.)