The Perverse Politics of Polarization*

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Abstract

Many policy choices involve gains for some voters at a cost borne by others. When an electorate is asked to select between these policies–either in the context of direct referenda or choosing between candidates whose positions on these policies differ—voters may be uncertain and not all that well-informed about who gains and suffers from these reforms. This paper studies the interplay of distributive politics and private information, and shows that it generates a strategic force of "suspicion": when an uninformed voter contemplates many other voters supporting a policy, she may conclude that she is likely to suffer from it. This force of suspicion induces voters to reject policies that are *ex ante* optimal and that would be selected with high probability were all information public. Our paper characterizes a form of "negative correlation" that is necessary and sufficient for this informational failure.

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1 Introduction

Motivation and Main Results: Many economic policies affect both aggregate welfare and its distribution. Trade policy is a particularly salient example, where the extent of trade liberalization impacts economic growth and accrues gains to some at a loss incurred by others.[1] Similarly, healthcare reform, immigration, and environmental policies have important aggregate and distributional consequences. In budget allocation problems, different members of an organization or legislature may have conflicting interests in how to allocate that budget, so that the gains of some are inextricably intertwined with the losses of others. Our paper studies the role of asymmetric information on voting behavior in the context of distributive politics. We find that asymmetric information fosters and amplifies suspicion in distributive politics, and can lead voters to select perverse policies. In particular, voters may choose policies that are both *ex ante* inferior and would be rejected with significant probability were information public.

Our starting point is that information in distributive politics is scarce. Many of the policy choices mentioned above are those in which voters are not all that well-informed about their consequences. For example, it is genuinely difficult to understand how one's real wages are affected by trade reforms.[2] Many of the other reforms mentioned above also have implications that are uncertain, difficult to predict, and disentangle, thereby making it difficult for a voter to learn which policies benefit her. This information problem is likely to be exacerbated by both a contentious political debate on these polarizing issues (which is likely to generate biased information) and the degree to which voters are not exposed to different sources of information.[3] In effect, voters are forced to rely upon information that is biased, originates from interested parties, and is generally of poor quality. Only a

[1] See Feenstra and Hanson (1999), Goldberg and Pavcnik (2007), Antras, de Gortari, and Itskhoki (2016), and Autor, Dorn, and Hanson (2016).

[2] This point is underscored by how trade models vary in their predictions (and mechanisms) on how trade influences wages and employment. Some mechanisms focus on factor abundance and heterogeneity across industries whereas others highlight both within industry heterogeneity, and the differential impact across low and high wage workers. We thank Gordon Hanson for a helpful discussion on this point.

[3] A rich literature on media markets highlights how the media may have a motive to bias information (e.g. Gentzkow and Shapiro, 2006, 2008), how such information provision may influence voting behavior (DellaVigna and Kaplan, 2007; Martin and Yurukoglu, 2017), and how each voter may consult only a limited number of information sources, thereby concentrating media power (Prat, 2017; Kennedy and Prat, 2017)

small fraction of voters may be relatively well-informed about the outcome being decided.

Consider a voter, Alice, voting on whether her country should lower its trade barriers, either in the context of a direct referendum or in an indirect election where she chooses between candidates whose platforms differ on trade. *Ex ante*, Alice views free trade to be beneficial to her, and obtains no additional information about how she would benefit from such a policy. But she recognizes that others might be supporting this policy because they have learned that they gain from such reforms. For instance, others may have learned that trade will favor their geographical region or economic sector. Alice may reflect upon their good news as being bad news for her: since all regions and all sectors cannot benefit from trade (especially in the absence of compensating transfers), she may fear that their gains imply that she should anticipate a loss of wages or employment opportunities. Analogously, she may view the support as potentially coming from a political and economic elite that is better positioned to capture these gains from trade. Contingencies such as these—when the good news of others—is bad news for Alice are exactly those that we model and describe as *negative correlation*.

This paper studies the electoral implications of negative correlation when voters are asymmetrically informed about the effects of policies. When information is scarce, voters support policies that are *ex ante* inferior, and lead to choices that are informationally inefficient. Our main results—Theorems 1 and 2—offer the following conclusion.

> *If payoffs are negatively correlated, then there is a strict equilibrium that, with high probability, selects the ex ante inferior policy, which would be rejected with high probability if all information were public. If payoffs are not negatively correlated, all equilibria select the ex ante efficient policy, which coincides with that which would be accepted if all information were public.*

Accordingly, our results establish that negative correlation is a necessary and sufficient condition for electoral failures in this setting. Negative correlation may be a feature that is endemic to distributive politics, where in contrast to a common-value setting, information that is "good news" for some voters is "bad news" for others. In such settings, elections can select perverse outcomes.

Our interest is not only in this inefficiency but also the thinking that is at its core. When a voter considers voting in favor of a policy, she realizes that it is chosen only when it commands sufficiently high support from others. But that may be the exact contingency where her prospects for benefiting from that policy are diminished. The condition of negative correlation that we formalize reflects when her *ex interim* expected benefits are sufficiently diminished that she chooses to vote against her *ex ante* favored policy. Alice is "suspicious" of policies that others favor (especially because she cannot access their information), and negative correlation formalizes when such suspicion manifests in distributive politics.

We view this kind of thinking to reflect an *adverse selection* view of politics where an individual recognizes that other individuals are self-interested and favor only those policies where they are beneficiaries. Their support for a policy causes her to worry that if she too supports the policy *when she is uninformed*, she might find herself on the losing end of it. Accordingly, our negative result parallels the force by which asymmetric information causes people to turn down trades that may appear *ex ante* advantageous (Akerlof, 1970; Milgrom and Stokey, 1982; Sebenius and Geanakoplos, 1983), and derives the implications of such thinking in the context of distributive politics.

While we frame our results using the conventional pivotal-voter logic, we believe that the logic of adverse selection applies more broadly. As proof-of-concept, we illustrate in Example 2 how similar behavior emerges in an ethical voter model (Coate and Conlin, 2004; Feddersen and Sandroni, 2006) where each of a continuum of voters in three sectors—namely agriculture, manufacturing, and services—has to decide whether to vote for or against trade liberalization. We show that uninformed voters in each group choose to ethically vote against trade liberalization for fear that if another group is supporting liberalization, it raises the odds that one's sector will face import substitution.

Our perception is this form of suspicion isn't merely a theoretical prospect, but may be an inescapable feature of distributive politics. Given the inability of governments to credibly commit to compensating transfers (Acemoglu, 2003; Jain and Mukand, 2003), voters may have good reasons to be suspicious of policies that others favor. Such suspicion appears to be a recurring theme of recent political discourse, manifesting as distrust of the political and technocratic elite, conflicts between urban and rural voters, and a growing divide between the interests of low and high skill workers. We view our stylized model as formalizing how adverse selection undermines electoral behavior in the context of distributive politics, and how asymmetric information amplifies such suspicion.

When are payoffs negatively correlated? Our results reduce the strategic problem of suspicion to a joint condition on primitives, *negative correlation*, that can be used to rank policies by their tendency to generate suspicion. We first compare policies that involve greater polarization between winners and losers—measured as the ratio of the loss incurred by losers relative to the gains that accrue to winners—and show that such policies are more susceptible to having negatively correlated payoffs. Thus, redistributive policies that transfers gain from winners to offset the costs incurred by losers shall enhance informational efficiency.

We then consider the extent to which a voter's potential to be a winner is "crowded out" when she learns that others are winners; in certain settings, learning that others are winners may be "good news" about the potential number of winners whereas in settings where there is less uncertainty about the number of winners, there may be a substantial crowding out effect. Formally, we characterize an order on probability distributions over the number of winners that measures the resistance to negative correlation from this perspective, and we show that this order is complete and transitive. We represent this order using the *ex interim* perception of the number of winners, and show that for a fixed voting rule, it amounts to a coarsening of the familiar likelihood ratio dominance order.

Finally, we show that the *kind* of information accessed by voters mitigates or exacerbates the issue of adverse selection. If all the information that is provided is of aggregate outcomes—say GDP or economic growth—then payoffs can never be negatively correlated. By contrast, if the information that is provided is purely distributive, then payoffs may be negatively correlated. We view this result to be interesting partly because of how it dovetails with analyses of how competitive information providers may have a motive to provide polarizing information to voters rather than about common valence terms (Perego and Yuksel, 2017).

Structure of the Paper: Section 2 describes three simple examples that illustrate the intuition for our results, relates that intuition to adverse selection and the no-trade theorem, and illustrates how similar results emerge in an ethical voter framework. Section 3 describes our general analysis...COMPLETE LATER.

2 Examples

We illustrate our results using three examples. The first develops the intuition for our main result in a simple example. The second showcases how a similar insight emerges in a three group example in which each of a continuum of voters votes ethically. The final example uses a two-voter model with unanimity rule to connect our results with adverse selection.

Example 1. Suppose that an electorate of 5 voters chooses, using majority-rule, between autarky and free trade. Each voter's payoff from autarky is normalized to 0. Relative to autarky, three voters (the *winners*) each obtain a payoff of 1 from free trade, whereas the others (the *losers*) obtain a payoff of -1. Each permutation of winners and losers is ex ante equally likely.

Were the identity of winners common knowledge, every equilibrium of the election (in weakly undominated strategies) would select free trade because each of these three winners would vote for free trade. At the other extreme, if it were commonly known that every voter is uninformed, free trade wins again: each voter expects to be a winner with probability $\frac{3}{5}$, yielding an ex ante expected gain from free trade of $\frac{1}{5}$. Thus, both with complete and no resolution of uncertainty, free trade defeats autarky in a majority-rule election.

Our interest is in settings where voters may privately learn how they fare under free trade and we show that this form of private information can generate a different outcome. Suppose that a voter learns her payoff from free trade (becomes "informed") with probability $\lambda > 0$, and otherwise remains uninformed, and that this random process is independent across voters. Adhering to our motivation that information is scarce, we study equilibrium behavior when λ is low.

To build intuition, consider the incentives of both informed and uninformed voters. For informed voters, every weakly undominated equilibrium prescribes that an informed loser votes for autarky and an informed winner votes for free trade. The more interesting case is that of uninformed voters.

Consider a strategy profile where all uninformed voters vote in favor of autarky. We establish that this is an equilibrium by fixing the strategy profile and examining the incentives of a single uninformed voter, Alice. Her vote influences her payoff only when it breaks a tie—namely, of the other voters, *exactly* two vote for free trade. Because all uninformed voters are voting for autarky, the two voting for free trade must be informed winners. Because there can be only three winners, this is bad news for Alice: the probability that she is a winner drops from the *ex ante* probability of $\frac{3}{5}$ to a number below $\frac{1}{2}$ so long as $\lambda < \frac{1}{2}$ (see Appendix B.1). Thus, when information is scarce, Alice recognizes that a vote for free trade influences the outcome only when free trade is unfavorable to her, and consequently, votes in favor of autarky. The probability then that autarky wins the election is at least $(1-\lambda)^3$, which is significant when the probability of being informed, λ, is low.

This example illustrates how uninformed people vote for autarky although each views free trade to be ex ante superior: conditioning on others supporting trade, an uninformed voter ascribes sufficiently high probability to being "crowded out" from the benefits of free trade that it is no longer attractive for her. If it's more likely that voters are uninformed, autarky then wins the election with high probability in this equilibrium.

We have so far described a perverse equilibrium of this example. In this example, another equilibrium exists in which all uninformed voters vote for free trade, and conditional on being pivotal, a voter has an even stronger motive to support free trade.

In contrast to this example, a "good" equilibrium need not exist; for certain cases, the perverse equilibrium is the unique pure-strategy and symmetric equilibrium. Even when a good equilibrium exists, we see here a potential instability introduced by distributional considerations—elections may succeed or fail depending on how voters expect others to behave—that contrasts with successful information aggregation results (Feddersen and Pesendorfer, 1996, 1997) that apply across all equilibria.

This distinction is at the core of our results: we show that when preferences across outcomes are *negatively correlated* across voters and information is scarce, then there always exists an equilibrium in which this perverse outcome materializes, but if preferences are not negatively correlated, then every equilibrium is guaranteed to succeed.

Example 2 (Ethical Voters). In this example, we show that the same force can influence behavior in an ethical voter framework where no voter anticipates being pivotal. A continuum of voters of unit mass is divided into three equally sized groups: agriculture (A), manufacturing (M), and services (S). Members of each group obtain a payoff from autarky that is normalized to 0, but trade liberalization has differential effects depending upon which group faces the threat of import substitution. There are three ex ante equally likely states of the world $\{\omega_A, \omega_M, \omega_S\}$, where the state ω_G denotes the state in which group G is threatened and the other groups benefit from trade. In state ω_i, members of group i each obtain a payoff of -1, and members of the other group obtain a payoff of 1. Each voter votes ethically in the spirit of Coate and Conlin (2004) and Feddersen and Sandroni (2006): namely, holding fixed the behavior of members of the other group, members of each group follow the rule that maximizes the payoffs of that group.

As in Example 1, free trade wins both under complete and no resolution of uncertainty. Now suppose that groups are asymmetrically and privately informed. We assume that each group privately learns the true state of the world with probability λ and remains uninformed with complementary probability.[4]

2.1 Related Literature

Our interest is in understanding voting behavior in elections that may change the distribution of wealth and income. Accordingly, our paper fits within the rubric of distributive politics (e.g. Alesina and Rodrik, 1994; Persson and Tabellini, 1994), describing how elections may not serve the common interest. This literature typically assumes that voters are well-informed about the consequences of policy reform, but being that information about policy reforms is often scarce, noisy, and manipulated by interested parties, we

[4]Since voters in each group have common interests, we are implicitly assuming that any information gained by a voter in a group G is freely shared with other members of her group.

consider it important to understand the interplay of distributive politics and information aggregation. Our work shows that when voters are not all that well-informed, electoral politics may lead to perverse outcomes that a median voter would not pursue ex ante or ex post. Private information coupled with distributive conflict exacerbates the failure of elections to pursue measures that improve the outcome for a majority of voters.

Our strategic logic is reminiscent of Fernandez and Rodrik (1991)'s elegant argument for why electorates resist reform. They show how a policy that would win an election ex post—if all voters knew their payoffs from the policy—can fail ex ante. The wedge that they describe is decision-theoretic: the median voter is unwilling to bear the risks of policy reform even if she knows that a strict majority of voters benefit from such reform. We show that private information amplifies this status quo bias. Even if a policy reform is ex ante preferred by all voters (as in Section 2), an uninformed voter favors the status quo because she recognizes that those voting for reform are privately informed about their gains, which makes her pessimistic about her own prospects.[5]

Speaking more broadly, a rich literature on polarization (in various forms) identifies how polarization and conflict lead to political and economic failures (Esteban and Ray, 2006; Padró i Miquel, 2007), and violence (Esteban and Ray, 2011; Mitra and Ray, 2014). Many of these contexts involve non-electoral failures, and we complement this literature by identifying how elections may fail in selecting between polarizing policies.

We also build on the literature that takes an information aggregation approach to elections (e.g. Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1996, 1997), and compare equilibrium outcomes of elections in which voters are privately informed with those in which voters are publicly informed. In much of this literature, voters share common values, or their preferences are aligned with beliefs about the underlying state in the same direction, so that any news that is good for one voter is also good for all others. As we highlight, that property naturally fails for issues of distributive politics, and instead, payoffs may be negatively correlated. Instead of viewing others' good news positively, distributive politics presents a scenario where some voters may view the positive prospects of others with suspicion and skepticism.

The idea that information aggregation fails when voters' interests are not aligned in the same way has been explored in Kim and Fey (2007), Gul and Pesendorfer (2009), Bhattacharya (2013a,b) and Acharya (2016).[6] While the failure of information aggre-

[5]Our findings also resonate with the idea that voters fear losing control, as illustrated by Strulovici (2010). He highlights how electoral mechanisms might underexperiment with risky policies when the gains from experimenting are not perfectly correlated across voters. His analysis focuses on independent private types in a dynamic environment whereas our analysis focuses on negative correlation in a static environment. Thus, his analysis and ours offer complementary lenses to political reforms.

[6]Information aggregation may also fail for reasons apart from those we emphasize, either because

gation across these papers and ours are similar in nature, fundamentally the models are different (from ours and each other), the mathematical results take different forms, and speak to different applications. We view our contribution as developing a simple framework that speaks specifically to questions of distributive politics and polarization, highlighting how even voters who are ex ante identical may face an ex interim conflict that comes from polarization, and stifles information aggregation.

3 Model

There is a finite population of voters, $\mathcal{N} = \{1, \ldots, n\}$, where the population size n can be random. The voters chooses between two policies via a simultaneous election: a status quo Q and an alternative A. Policy A is implemented if it gets strictly more than a proportion $\tau \in (0,1)$ of the votes. Hence, in a population of size n, A is implemented if it receives at least $\lfloor \tau n \rfloor + 1$ votes, where $\lfloor \tau n \rfloor$ is the largest integer such that $\lfloor \tau n \rfloor \leq \tau n$.

Each voter's payoff from Q is normalized to 0. Payoffs from A are uncertain: nature chooses a payoff profile v from $\mathcal{V}^n$, where v_i is voter i's payoff when A is implemented, and $\mathcal{V} \subseteq \mathbb{R}$ is a finite set of possible payoffs. Before casting a vote, each player i obtains private signal s_i that can convey information about the payoff-profile, and is drawn from $\mathcal{S} \equiv \left\{s^0, s^1, ..., s^K\right\}$. The voting environment is therefore described by a probability distribution P on $\Omega = \{(n, v, s) : n \in \mathbb{N}, v \in \mathcal{V}^n, s \in \mathcal{S}^n\}$. We use capital letters to denote random variables on Ω, and lower-case letters to denote their realizations. In particular, for a state $\omega = (n, v, s) \in \Omega$, let $N(\omega) = n$, $S(\omega) = s$, $S_i(\omega) = s_i$, $S_{-i}(\omega) = s_{-i}$, $V(\omega) = v$, and $V_i(\omega) = v_i$ denote the random variables describing, respectively, the population size, signal profile, voter i's signal, the signal-profile of voters other than i, the payoff-profile, and voter i's payoff.[7]

We impose five main assumptions on the primitives of the model (Ω, P, τ). The first assumption is that voters are ex ante symmetric.

Assumption 1. *Voters are ex ante exchangeable: $P(v, s) = P(\tilde{v}, \tilde{s})$ for every permutation $(\tilde{v}, \tilde{s})$ of (v, s).*[8]

voters wish to influence subsequent policy choices (Razin, 2003), information is costly (Martinelli, 2006), there is aggregate uncertainty about the distribution of preferences (Feddersen and Pesendorfer, 1997) or the precision of information (Mandler, 2012), the policymaker cannot commit to a voting rule (Battaglini, 2016), or turnout varies with the underlying state of the world (Ekmekci and Lauermann, 2016a,b).

[7]Given any random variable X on Ω, we denote by $\{x\} \equiv \{\omega : X(\omega) = x\}$ the event where x is realized for X, omitting the brackets when it is clear that x is an event. In particular, s_i^k is the event $\{\omega : S_i(\omega) = s^k\}$.

[8]We say that $(\tilde{v}, \tilde{s})$ is a permutation of (v, s) if there is a one-to-one mapping $\psi : \mathcal{N} \to \mathcal{N}$ such that $(v_i, s_i) = (\tilde{v}_{\psi(i)}, \tilde{s}_{\psi(i)})$ for all $i \in \mathcal{N}$.

For a non-null event $E \subseteq \Omega$, $V_i(E) \equiv \sum_{\omega \in \Omega} V_i(\omega) P(\omega|E)$ is voter i's conditional expected (or ex interim) payoff from A being implemented when voter i knows event E.

Our second assumption distinguishes signal s^0, which we describe as an *uninformative signal*, from the remaining signals $\mathcal{M} \equiv \mathcal{S} \backslash \{s^0\}$, which we describe as *informative*.

Assumption 2. *There is an uninformative signal, and informative signals are sufficient:*

(a) **Uninformative signal:** *For all* $(n, v, s) \in \Omega$ *with* $s_i = s^0$, $P(s_i) > 0$ *and* $P(n, v, s) = P(n, v, s_{-i}) P(s_i)$.

(b) **Informative signals:** *For all* $(n, v, s) \in \Omega$ *with* $s_i \neq s^0$, $V_i(n, s) > 0$ *if and only if* $V_i(s_i) > 0$.

Assumption 2(a) asserts that there is a strictly positive probability that each voter receives the signal s^0, and that signal conveys no information about the population size, the payoff profile, and the signals received by other voters.[9] We are interested in environments where the probability of being uninformed is significant: for policy reforms at the core of distributive politics, most voters are unlikely to have good information about the extent to which they shall benefit or be hurt by these policies, and have to rely instead on tainted information being provided by interested parties. While it simplifies our analysis to assume that such voters are completely uninformed, it is not crucial to our results; because the equilibrium we study in Theorem 1 is strict, perturbing the model and furnishing these uninformed voters with some additional information would not change our results.

Assumption 2(b) speaks to the informativeness of the other signals, $\mathcal{M} \equiv \left\{s^1, ..., s^K\right\}$: if a voter obtains an informative signal, then her own information is a sufficient statistic for the entire signal profile in determining her ordinal ranking between Q and A. This assumption simplifies our analysis by ensuring that informed voters do not need to consider contingencies when assessing how to vote, and permits us to focus on the behavior of uninformed voters. A special case of Assumption 2 is when each informed voter observes directly her payoff from A, as in the examples from Section 2, and Feddersen and Pesendorfer (1996). More generally, Assumption 2(b) does not imply that informed voters observe or are all that well-informed (in an objective sense) about their payoffs from A, but simply that such individuals are well-informed *relative* to the electorate, insofar as learning others' signals does not change their ex interim ordinal rankings of Q and A.[10]

[9]Most of our analysis corresponds to polarization of interests, not information, but we show in Appendix B.7 that correlating one's opportunity to obtain information with one's payoffs from A—violating Assumption 2(a)—amplifies the prospects for electoral failures.

[10]Indeed, our analysis in Proposition 6 corresponds to a case where no voter is objectively all that well-informed about her *ex post* payoffs.

We partition the set of informative signals, $\mathcal{M}$, into "good" and "bad" news. Signals $\mathcal{G} \equiv \left\{s^k \in \mathcal{M} : V_i\left(s^k\right) \geq 0\right\}$ convey *good-news* about the alternative: a voter who receives signal $s_i \in \mathcal{G}$ expects that she will benefit from a switch to the alternative. Likewise, signals $\mathcal{B} \equiv \left\{s^k \in \mathcal{M} : V_i\left(s^k\right) < 0\right\}$ convey *bad-news* about the alternative. For a signal profile s, $M(s)$ is the number of informed voters, and $G(s)$ is the number of voters who received good news.

Assumption 3. *Ex interim payoffs satisfy non-redundancy and no ties:*

(a) **Non-redundancy***: If $P(n) > 0$, then $P(G = \lfloor \tau n \rfloor + 1|n) > 0$.*

(b) **No ties:** *If $P(E) > 0$, then $V_i(E) \neq 0$.*

Assumption 3(a) and (b) are bookkeeping assumptions that simplify our exposition and analysis without playing a substantive role. Assumption 3(a) guarantees that under public information it would always be possible for the alternative to win; this assumption is not necessary for our results, but the environment would be uninteresting if it fails. Assumption 3(b) holds generically and allows us to avoid tie-breaking rules.

Our fourth assumption describes the marginal distribution over the population size.

Assumption 4. *The population has minimum size $n_0 \geq 2$, with $\lfloor \tau n_0 \rfloor \neq \lfloor \tau(n_0 + 1) \rfloor$, plus a random part z that follows a Poisson distribution with mean $\mu \in \mathbb{R}_+$.*[11]

Following Myerson (2000), it has been common to model population uncertainty in terms of a Poisson distribution, and Assumption 4 is an adjustment to ensure that there is strictly positive minimum population size. The Poisson distribution is a simple distribution over population size with unbounded support. However, our results do not require the specific Poisson functional form and hold for a broad class of marginal distributions on N including, for example, any distribution with a finite support. Lemma 1 and Remark 1 in Section A.1 describe the formal properties of the marginal distribution over population size that we use to establish our results.

Without loss of generality, we assume throughout that A is the ex ante superior policy. Our final assumption strengthens this condition, so that A remains superior for any known population size.

Assumption 5. *When a voter learns only the population size, then A is the superior policy: $V_i(n) > 0$ for all n.*

In particular, Assumption 5 implies that, if it were common knowledge that no voter obtains information, voting A is a weakly dominant strategy, and would remain so if the population size was known.

[11] Hence, $P(n = n_0 + z) = \frac{\mu^z e^{-\mu}}{z!}$ for all $z \in \mathbb{N} \cup \{0\}$. We follow the convention that $0^0 \equiv 1$. Hence, when $\mu = 0$, the population size is n_0 with probability 1.

Strategies and Equilibrium. Our analysis pertains to a *private information* environment: before the election, voter i observes only her private signal s_i and cannot observe the information of others. We consider symmetric Bayes-Nash equilibria in which each voter plays weakly undominated strategies. Henceforth, we refer to these weakly undominated symmetric equilibria simply as *equilibria*.

In an equilibrium, the voting behavior of informed voters is straightforward: if voter i obtains good news ($s_i \in \mathcal{G}$), she votes for A; if she obtains bad news ($s_i \in \mathcal{B}$). Hence, we can focus on the behavior of uninformed voters, who all votes for Q with the same probability. We contrast this setting with a *public information environment*, where the population size and the entire signal profile s is observed by each voter. The following result establishes equilibrium existence in the two environments, and provides a benchmark result for the public information environment when information becomes scarce.

Proposition 1. *The private information environment has an equilibrium. The public information environment has a unique equilibrium, which is strict and symmetric.*

We are particularly interested in equilibrium outcomes when information is scarce, i.e., each voter is uninformed with high likelihood. Assumptions 1 and 2 allow us to formalize this idea, by decomposing P into the probability that a voter obtains the uninformative signal s^0 and a probability distribution over all other primitives (payoffs and informative signal realizations). We use λ to denote the probability that a voter obtains an informative signal. The parameter λ does not need a voter subscript by Assumption 1, and does not need a population subscript by Assumption 2(a). The primitive probability distribution P can then be viewed as a member of a family indexed by $(\tilde{P}, \lambda)_{\lambda \in (0,1)}$, where, for every event $\{n, s\} \subset \Omega$, $\tilde{P}(s) = P(s)\lambda^{-n}$ if $s \in \mathcal{M}^n$, and $\tilde{P}(s) = 0$ otherwise. While λ parameterizes the probability that a voter is informed, $\tilde{P}$ is a joint distribution over $\tilde{\Omega} \equiv \{(n, v, s) \in \Omega : s \in \mathcal{M}^n\}$. The distribution P corresponds to the unique element of the family $(\tilde{P}, \lambda)_{\lambda \in (0,1)}$ where $\lambda = 1 - P(s_i^0)$ (formal details of this decomposition are given in the appendix).

As a benchmark, the following proposition establishes the outcome that would obtain in an environment with public information when information is scarce (i.e., λ is sufficiently small).

Proposition 2. *For every $\varepsilon > 0$, there exists $\lambda_\varepsilon \in (0, 1)$ such that, for all $\lambda < \lambda_\varepsilon$, A wins with probability exceeding $1 - \varepsilon$ in the unique equilibrium of the public information environment.*

Proposition 2 shows that, as information becomes scarce, the ex ante superior policy wins with high likelihood in the unique equilibrium of the public information environment. In the following we contrast this result with an environment where information is private, and establish conditions where the private information can lead to very different equilibrium outcomes.

4 Distributive politics and negative correlation

This section describes our main results: a form of negative correlation is necessary and sufficient for privately informed voters to choose an outcome that differs from that which would be chosen if all information were public. The outcome also differs from one that voters prefer ex ante, when it is commonly known that everyone is uninformed. After presenting the main result and a converse (Section 4.1), we use a tractable sub-class of our general model to highlight three key factors that determine when payoffs satisfy the negative correlation condition (Section 4.2). We also show that ex post redistribution can alleviate the electoral failures induced by negative correlation (Section 4.3), and briefly discusses some extensions of the model (Section 4.4).

4.1 Sufficiency and Necessity of Negative Correlation

We define what it means for payoffs to be negatively correlated relative to the voting threshold τ. For all $\kappa \in [0, \tau]$, let $V^G(\kappa) \equiv V_i(n_0, s_i^0, M = G = \lfloor \kappa n_0 \rfloor)$ be the payoff for voter i in a population of size n_0, when she receives the uninformative signal s^0 and conditions on a proportion κ of other voters being informed, all having received good news.[12]

Definition 1. *Payoffs are τ-**negatively correlated** if $V^G(\tau) < 0$: the conditional expected payoff from the alternative A is strictly negative for an uninformed voter when conditioning on a population size n_0 where there is a proportion τ of informed voters, all of whom received good news.*

Being τ-negatively correlated implies that when an uninformed voter considers the prospect that τ proportion of voters have received good news, and only those voters receive information at all, her expected payoff from A is strictly negative. Clearly, learning that a proportion τ of other voters are informed provides no information, because of the independence condition in Assumption 2(a). However, learning that all of those voters received good news is informative in two opposing directions: (i) on one hand, knowing that a proportion τ of random draws were winners, increases the chances that there are

[12]By Assumption 1, a subscript for a player index is redundant for $V^G(\kappa)$.

more winners; (ii) on the other hand, for any fixed number of winners, knowing that a proportion τ of other voters already know they are winners increases the likelihood that voter i is a loser. Payoffs are τ-negatively correlated if the second effect dominates. The proportion τ plays a special role in the condition because all of these "good-news" voters choose A, and therefore, if the uninformed voter also votes for A, she anticipates that it would win.

Using this definition, we describe our main result.

Theorem 1. *Suppose payoffs are τ-negatively correlated. Then, for every $\varepsilon > 0$, there exists $\lambda_\varepsilon \in (0, 1)$ such that, for all $\lambda < \lambda_\varepsilon$, Q wins with probability exceeding $1 - \varepsilon$ in a strict equilibrium.*

Comparing Proposition 2 and Theorem 1 emphasizes how voting outcomes—when payoffs are τ-negatively correlated—are sensitive to whether information is private or public, especially when information is scarce (i.e., λ is small). When all information is public, then A wins with high probability in the unique equilibrium, whereas when information is private, Q wins with high probability in a strict symmetric equilibrium.

How Q wins highlights the strategic import of negative correlation, and the argument detailed in the Appendix is relatively direct. In the equilibrium of the private information environment, all uninformed players vote for Q. As a result, when an uninformed voter conditions on being pivotal, she recognizes that there is at least a τ proportion of other voters who are informed winners. For low values of λ, she anticipates that, apart from these voters, others are likely to be uninformed. Conditioning on this event, her ex interim payoff from A approximates $V^G(\tau)$, which is strictly negative by Definition 1. Therefore, her strict best response is to vote for Q. As a result, policy Q wins with high chance when it is likely that most voters are uninformed.

Because the equilibrium is strict, it is robust to perturbations of the environment. For example, furnishing uninformed voters with a small amount of additional information—so that their ex interim beliefs diverge (slightly) from their prior beliefs —would not preclude the behavior described in Theorem 1. Moreover, as we discuss in the Supplementary Appendix, augmenting the voting game with the possibility for abstention does not change the result, and under certain additional conditions, the equilibrium described in Theorem 1 is the unique strict equilibrium. We discuss further robustness issues in Section 4.4.

The scarcity of information plays two roles when comparing Proposition 2 and Theorem 1. Because information is scarce, the public information benchmark involves players selecting A with high probability: the chance that public information is revealed that outweighs the ex ante perspective is low. Yet, when information is private, voters may select A with low probability, even though that information is scarce. The equilibrium that exhibits such features in the private information environment may not exist if it were sufficiently likely that other voters are well-informed, and so it is a combination of strategic voting and ignorance that generates this perverse electoral outcome.

A Converse: We prove a converse result to highlight that negative correlation is also necessary for the perverse outcome in Theorem 1, under an additional assumption on ex interim payoffs (not used elsewhere).

Assumption 6. *$V^G(\cdot)$ satisfies positively connectedness: if $V^G(\kappa) > 0$ and $V^G(\kappa') > 0$ for $\kappa' > \kappa$, then $V^G(\kappa'') > 0$ for every κ'' such that $\kappa \leq \kappa'' \leq \kappa'$.*

In the setting for our converse theorem, Assumption 6 imposes a single-crossing property on V^G. Because Assumption 5 implies that $V^G(0) > 0$, Assumption 6 implies that $V^G(\cdot)$ crosses 0 at most once, and from above.[13] We view this condition to be intuitive: if the conditional likelihood favors A when there is a proportion κ of winners other than voter i, and when there is a proportion $\kappa' > \kappa$ of winners other than voter i, then voter i must also prefer A when there is an intermediate proportion of winners. Indeed, all of the examples in the paper satisfy this assumption.[14]

Using Assumption 6, we show that when payoffs are not τ-negatively correlated, then in every equilibrium of the private information environment, the probability that A is chosen is arbitrarily close to the public information environment.

Theorem 2. *Suppose Assumption 6 is satisfied. If payoffs are not τ-negatively correlated, then for all $\varepsilon > 0$, there exists $\lambda_\varepsilon \in (0, 1)$ such that for all $\lambda < \lambda_\varepsilon$, A wins with probability at least $1 - \varepsilon$ in every equilibrium.*

Theorem 2 illustrates how, without negative correlation, one would not anticipate the wedge between the private and public information environments when information is scarce. Taking a sequence $\lambda \to 0$, we see that the presence or absence of negative correlation influences the outcome starkly when information is private: in the former case, there exists an equilibrium in which Q is selected with probability converging to 1, whereas in the latter case, all equilibria select A with probability converging to 1.

Because Theorems 1 and 2 illustrate the importance of negative correlation of payoffs, we regard it as important to understand features of an economic environment that foster or preclude such correlations. We turn to this question in the next section.

[13] More generally, Assumption 6 requires that the set of κ for which $V^G(\kappa)$ is strictly positive is connected; this assumption is weaker than the standard single-crossing condition (Milgrom and Shannon, 1994).

[14] We provide an example in Appendix B.2 to illustrate that this assumption is not redundant, but observe that such an example requires a contrived information structure.

4.2 When are Payoffs Negatively Correlated?

Evidence on trade liberalization (e.g. Autor et al., 2016) suggests that the labor market in all sectors are not uniformly better off from opening trade barriers. While some sectors gain from liberalization, workers in other sectors are forced to cope with periods of unemployment, acquiring new skills, or moving to jobs that involve lower wages. In this context, the question that motivates negative correlation is whether an uninformed voter finds trade liberalization to be more or less favorable to her when she learns that others favor trade.

We address this question using a tractable sub-class of the model that decouples aggregate and distributional uncertainty. This allow us to identify three features of the environment that determine whether payoffs are negatively correlated: (i) *polarization ratios*, (ii) *crowding out*, and (iii) the *nature of information*.

Suppose alternative A generates *winners* and *losers*; each winner obtains $v_w > 0$ and each loser obtains $-v_l < 0$.[15] Uncertainty is about the number and identity of winners: the number of winners is denoted by the random variable η (*aggregate uncertainty*), and the identity of winners is determined by a random vector ρ (*distributional uncertainty*) where ρ_i denotes the priority of voter i in being a winner. Voter i is a winner if and only if $\rho_i \leq \eta$, and so her payoff from the alternative depends on the realization of both aggregate and distribution uncertainty. We denote by W_i the event that voter i is a winner, and by L_i its complement. Individuals can obtain the uninformative signal s^0 or an informative signal $s^k \in \mathcal{M}$. This model can be embedded within the framework of Section 3; where P is now a joint distribution on the number of winners, the priority ranking, and voters' information, and we assume that Assumptions 1–3 are satisfied.[16]

Polarization Ratio: In this setting, A is ex ante optimal when $P(W_i)v_w - (1 - P(W_i))v_l > 0$, where $P(W_i)$ is the ex ante probability that voter i is a winner.[17] This inequality can be re-written as

$$\frac{P(W_i)}{1 - P(W_i)} > \frac{v_l}{v_w}. \tag{1}$$

We describe the RHS of the above inequality as the *polarization ratio*, which specifies the sacrifice borne by losers relative to the gain that accrues to winners. The LHS is the ex ante likelihood ratio of being a winner.

[15] These payoffs may be expected payoffs conditional on being a winner or loser.

[16] Note that Assumption 1 does not imply that ρ and η are independent, and so we do not impose it for our general analysis of this setting.

[17] Necessarily, $P(W_i) = \sum_{\eta=1}^{n} \frac{P(\eta)\eta}{n}$.

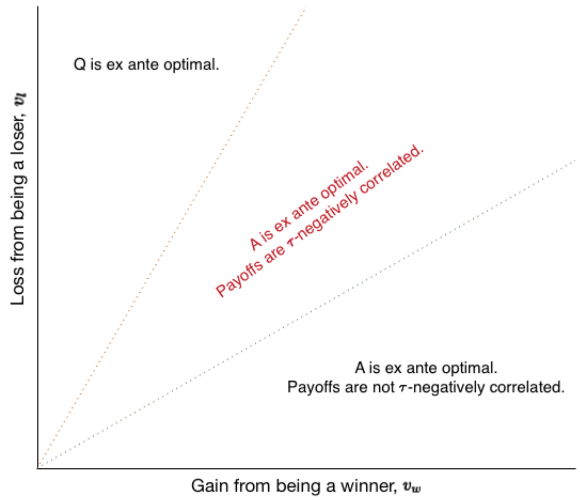

Figure 1: (v_l, v_w) that generate τ-negatively correlated payoffs lie between the two rays.

Our strategic analysis compares the polarization ratio, not with the ex ante likelihood ratio, but with the *ex interim* likelihood ratio for a particular pivotality event. Let $P(W_i|\tau-1)$ denote the probability that i is a winner conditional on herself being uninformed and knowing that there are exactly $\tau-1$ informed voters, all of whom have good news. Payoffs are then τ-negatively correlated if and only if

$$\frac{P(W_i|\tau-1)}{1-P(W_i|\tau-1)} < \frac{v_l}{v_w}, \tag{2}$$

Comparing (1) and (2), we can find payoffs such that (i) the alternative is ex ante optimal, and (ii) payoffs are τ-negatively correlated if and only if

$$P(W_i|\tau-1) < P(W_i). \tag{3}$$

In other words, for a voter i who receives no information, the prospect that exactly $\tau-1$ other voters are informed and received good news decreases her own chances of being a winner relative to her prior belief.

We first classify reforms as follows: a policy is more polarizing if losers have to bear a higher proportional cost for the gains accrued by others, reflected by a higher polarization ratio. We show that polarizing policies in this sense are more conducive to negative correlation.

Figure 1 highlights the parameter region of interest. Holding fixed a probability distribution P satisfying (3), a configuration of payoffs $(v_w, v_l) \in \mathbb{R}^2_{++}$ leads to τ-negatively correlated if the polarization ratio $\frac{v_l}{v_w}$ lies between the slopes of the two rays. When the polarization ratio is low, then not much loss is suffered by losers, and so A is both ex ante optimal and fails to generate τ-negatively correlated payoffs. Increasing the polarization ratio implies that a greater proportional loss is suffered by losers, and so A may be ex ante optimal and payoffs may be τ-negatively correlated. If the polarization ratio is sufficiently high, then Q is ex ante optimal.

This discussion establishes a comparative static on polarization ratios, summarized in the following proposition. Holding fixed a distribution P, increasing the polarization ratio generates the prospect for τ-negatively correlated payoffs (so long as A remains ex ante optimal), and decreasing the ratio can ensure that payoffs are not τ-negatively correlated.

Proposition 3. *The prospect for τ-negatively correlated payoffs is increasing in the polarization ratio: if (P, v_w, v_l) generates τ-negatively correlated payoffs, then for every (v'_w, v'_l) such that $\frac{v'_l}{v'_w} > \frac{v_l}{v_w}$ and (1) is satisfied, (P, v'_w, v'_l) also generates τ-negatively correlated payoffs.*

Crowding Out: The preceding discussion fixed the distribution P and compared polarization ratios. Now, we compare distributions in order to measure the degree of "crowding out" from learning that others are winners. We assume that an informed voter directly observes whether she is a winner or loser. We say that $P \succeq P'$ if, for every (v_w, v_l), whenever (P, v_w, v_l) is τ-negatively correlated, then (P', v_w, v_l) is τ-negatively correlated. Hence, $\succeq$ encodes P's greater resistance to τ-negatively correlated payoffs than P'. We compare only those distributions that agree in an ex ante sense, and parameter regions such that A is ex ante optimal.

Proposition 4. *Suppose $P(W_i) = P'(W_i)$, then:*

(a) *If there exists (v_w, v_l) such that (P', v_w, v_l) is τ-negatively correlated and (P, v_w, v_l) is not, then $P \succeq P'$.*

(b) *$P \succeq P'$ if and only if $E_P(\eta|M = G = \tau - 1) \geq E_{P'}(\eta|M = G = \tau - 1)$.*

(c) *If P likelihood-ratio dominates P' for every $\eta \geq \tau - 1$, then $P \succeq P'$. In other words, $P \succeq P'$ if, for every $\eta \geq \tau - 1$ and $\eta' > \eta$,*

$$\frac{P'(\eta')}{P'(\eta)} \leq \frac{P(\eta')}{P(\eta)}. \tag{4}$$

Proposition 4(a) establishes that the ranking of distributions P and P' is not sensitive to the choice of v_w and v_l, and so the ordering is well-defined.[18] Proposition 4(b) characterizes the comparative ranking using conditional expectations: the term $E_P(\eta|M = G = \tau - 1)$ is the conditional expectation of the number of winners being that from $\tau - 1$

[18]If P dominates P' for a single (v_w, v_l) (in terms of being less conducive to τ-negative correlation), then it does so for all relevant (v_w, v_l).

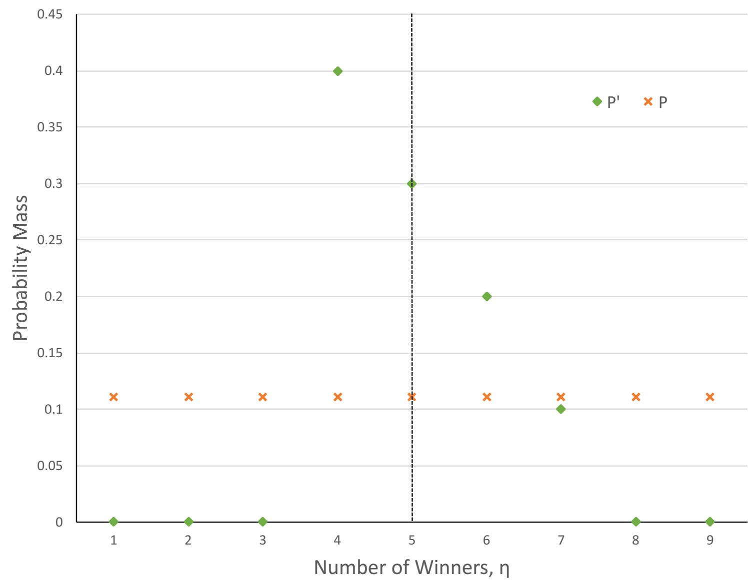

Figure 2: 9 voters use simple-majority rule. We compare distributions on the number of winners: P is uniform on $\{1, ..., 9\}$ while P' has support $\{4, ..., 7\}$. Both distribution offers each voter an ex ante probability of $\frac{5}{9}$ of being a winner but P LR-dominates P' when restricted to $\eta \geq 4$. Conditioning on there being 4 informed voters, all of whom are winners, P' offers an uninformed voter lower odds of being a winner and generates payoffs that are τ-negatively correlated for a large range of (v_w, v_l).

draws, all are winners. A higher value increases the likelihood that an uninformed voter is a winner, whereas a lower value reduces those odds.

We use that characterization to offer a sufficient condition in (c). Inequality (4) is the conventional likelihood ratio inequality used to define the monotone likelihood ratio property, but our definition assumes it only over the range $\{\tau - 1, \ldots, n\}$. When P' is LR-dominated by P, then P' assigns relatively more probability mass to the number of winners being close to the vote threshold τ. This ranking formalizes that elections where the number of winners is likely to be close to τ, are those where the crowding out effect is most pronounced: a voter is more likely to be crowded out under P' than under P, and so learning that there are $\tau - 1$ other winners is less favorable under P'. We illustrate Proposition 4(c) using Figure 2.

Nature of information: Contrast the following kinds of information:

- *Free trade makes people on average better off.*
- *Free trade results in a loss of manufacturing jobs and a gain in corporate profits.*

The former describes implications for the aggregate number of winners without offering information about distributional consequences, whereas the latter conveys information primarily about the priority ranking. Propositions 5 and 6 show that the first kind of information does not foster negative correlation of payoffs but the second does.

We first consider the case in which all information that voters obtain is potentially informative about aggregate payoffs, but is completely uninformative about its distribution. In other words, for every signal profile s, the ex interim expected payoff of voter i conditioning on s, namely $V_i(s)$, is also the same for every other voter j conditioning on the same signal profile. In other words, information does not discriminate across voters. We show that in this case, (3) cannot be satisfied, leading to the following result.

Proposition 5. *If voters' signals are informative only about aggregate consequences, payoffs cannot be τ-negatively correlated.*

In this case, information that is good news for others—e.g., there are many winners—is also good news for an uninformed voter. A special case is **??**, where all voters are winners or losers, and so the prospect of others learning that trade is beneficial improves one's own outlook for opening trade barriers. But this intuition applies more broadly: even if opening trade barriers is polarizing, when voters do not obtain any information about the identities of winners and losers, then payoffs cannot be τ-negatively correlated for any voting rule τ.[19]

The contrasting case is when all information is distributional and not about aggregate consequences, and in this case, the opposite holds.

Proposition 6. *If informative signals reveal a voter's priority but are uninformative about the number of winners, payoffs are τ-negatively correlated for some (v_w, v_l).*

When information is about distributional and not aggregate consequences, then information that is good news for other voters—them having a higher priority—is bad news for oneself. In this case, the inequality in (3) is always satisfied, and so there are always polarization ratios that generate τ-negative correlation. A special case is Example 1, where there is no aggregate uncertainty and good news corresponds to learning if one is a winner. But more broadly, if all that voters learn are which sector is the first to benefit (or suffer) from trade liberalization, but not the overall benefits and costs of free trade, then payoffs are negatively correlated for some polarization ratios.

Indeed, one may combine intuitions of Propositions 5 and 6 to evaluate how adding distributional information changes the implications of having aggregate information. In Example 5 in the Supplementary Appendix, we study a setting where every voter obtains noisy information about the number of winners, and vary the probability that voters

[19]As seen in the proof, Proposition 5 applies for the general framework described in Section 3.

obtain information about priority rankings. We show that when the probability that distributional information is dispersed is 0, information is perfectly aggregated: in a large electorate, A wins with high probability if and only if a majority of voters are winners. By contrast, once voters obtain distributional information with positive probability, there is a prospect for an electoral failure where Q wins with probability 1, regardless of the number of winners. Hence, providing the electorate with more information leads to less information being aggregated.

4.3 Redistribution

Redistributive schemes can alleviate the electoral failures caused by negative correlation. Suppose that if A is implemented, there is perfect redistribution so that every voter gains or loses the same amount relative to the status quo Q. For simplicity, we maintain Assumption 2(b) and therefore assume that for any non-null signal profile s where $s_i, s_j \in \mathcal{M}$, $P(\eta|s_i) = P(\eta|s_j)$ for all $\eta \in \{0, ..., n\}$: all informed voters have the same belief about the aggregate state.

Proposition 7. *With complete redistribution, for all $\varepsilon > 0$, there exists $\tilde{\lambda}$ such that if $\lambda < \tilde{\lambda}$, then in any equilibrium of the private information environment, A wins with probability greater than $1 - \varepsilon$.*

With complete redistribution, voters' ex interim interests are aligned so that ex ante optimal reforms are pursued. Indeed, in many cases—for example if η is known—partial redistribution suffices. Of course, the case that we make for redistribution ignores its costs, and we are definitely not the first to propose that redistribution can enable growth in an electoral context. But the conventional argument for redistribution is that even if trade agreements increase the size of the pie, these gains may not accrue to all citizens, and so transfers are needed to garner the support of the median voter. Our argument complements the conventional case for redistribution by showing how it can avert electoral failures when voters are uncertain and, possibly, privately informed about the payoffs of trade liberalization.

4.4 Extensions and Discussion

We describe briefly how our main results would apply in different settings. For results that require a formal argument, we relegate the proof to the Supplementary Appendix.

Abstention: As alluded in Section 4.1, the electoral failure from Theorem 1 does not depend on the assumption that everyone votes: we show in Appendix B.3 that allowing for abstention does not change the main result.

Comparison of voting rules: We do not undertake a formal comparison of voting rules because under Assumption 6, it is trivial: if payoffs are τ-negatively correlated for voting rule τ, then they are also τ'-negatively correlated under voting rule $\tau' > \tau$. Thus, one way to mitigate the issues identified in Theorem 1 is by leaning the voting rule in favor of the ex ante optimal policy.

Equilibrium selection: Our result highlights how information aggregation might fail in an equilibrium, but we have not established that it does so across all equilibria. The equilibrium that we study is strict, symmetric, and in pure strategies, and thus, is both robust to perturbations and cannot be easily refined away.[20] Moreover, we characterize conditions in the Supplementary Appendix under which this is the unique symmetric pure strategy equilibrium.

Interchanging the status quo and alternative: Our formal results, of course, would apply with the roles of A and Q being reversed, so that even if Q were ex ante optimal, and the likely electoral outcome from public information, A still wins with significant probability. Formally, the results apply once one defines $V^B(\kappa) \equiv V_i(S_i = s^0, M = B = \kappa)$ (as the analogue of $V^G(\kappa)$ defined in Section 4.1) and assumes that $V^B(\cdot)$ satisfies the analogue of Assumption 6 for the converse.

Heterogeneity of voters: In order to develop a simple framework that illustrates how information aggregation fails in the context of distributive politics, we have assumed that voters are ex ante symmetric. For applications of our framework, a natural starting point may be for voters to belong to different groups—say *workers* and *capitalists*—and to have different likelihoods of benefiting from trade liberalization. Extending our results is straightforward: our results only require that payoffs are τ-negatively correlated for sufficiently many voters in either of these groups such that these voters would be willing to reject policy reforms.

[20]Being that winners prefer A, the equilibrium outcome clearly is not dominated at the ex interim stage, and to the extent that coordination may be infeasible at the ex ante stage—either because the ex ante stage is a mere modeling device or coordination is costly—we do not consider ex ante dominance as a selection criterion appropriate for our setting.

Strategic vs. pivotal reasoning: We view the strategic reasoning captured by our model—namely that of *suspicion* of other voters who vote for policy reforms—to be germane to issues of distributive politics, and our interest is in understanding the qualitative implications of such polarization on electoral politics. Necessarily, purely instrumentally motivated voters condition on being pivotal, which may appear to be a controversial (albeit common) modeling choice. Recent empirical work suggests that a large fraction of voters do vote strategically (e.g. Fujiwara, 2011; Kawai and Watanabe, 2013), and there is supporting evidence also from laboratory experiments (e.g. Guarnaschelli, McKelvey, and Palfrey, 2000; Battaglini, Morton, and Palfrey, 2008, 2010).[21] Strategic voting captures how suspicion emerges in the politics of polarization, but the logic of suspicion applies more broadly. For example, suppose that voters came from heterogeneous groups (as discussed above)—e.g., *workers* and *capitalists*—and make decisions on the basis of a group or ethical utility model (e.g. Harsanyi, 1977; Coate and Conlin, 2004; Feddersen and Sandroni, 2006). If information is shared within but not across these groups, all workers may then, on the basis of their ethical decisionmaking, decide to vote against trade liberalization when uninformed, recognizing that that their group is better off by rejecting trade reforms favored by members of the other group.

Polarization of information: We have assumed that a player's information is independent of her interests. But on many occasions, players' information is correlated with their interests. This issue is raised by Caplan (2007), and relevant in the design of policy reforms where the broader electorate may be suspicious of the political and economic elite who support reforms and are privy to their fine details: *What might they know about these policies to support them, and are their interests aligned with those of other voters?* In these settings, there is polarization not only of interests, but also of information. We study this in the Supplementary Appendix and show that electoral failures may now emerge regardless of the probability of being informed, and that increasing the probability of being informed amplifies the potential for electoral failures.

5 Applications

In this Section, we discuss when negative correlation can induce a status quo bias, or garner popular support for ill-advised policy reforms that are ex post regretted and subsequently reversed.

[21] See Palfrey (2009) for a survey of the laboratory evidence, and Battaglini (2016) for further discussion of the evidence in favor of pivotal-voter models.

5.1 Status Quo Biases

In a seminal article, Fernandez and Rodrik (1991) advance an understanding of status quo biases by showing that, even when the alternative would be preferred by a strict majority ex post, the median voter still favors the status quo ex ante. The issue they highlight is decision-theoretic: the lottery of the policy reform has a negative expected payoff for the median voter, even if ex post, a strict majority would prefer the reform.

Our main results show that this force can be amplified by private information and strategic voting: even if voters prefer a reform to the status quo ex ante, but are suspicious of the reasons that others might be voting for the reform, they have additional reasons to reject the reform. This issue is particularly salient in the context of protectionism, where workers are skeptical of the reasons that others (e.g., the elite) might vote in favor of free trade. Policies that involve a more skewed polarization ratio, or where the crowding-out effect is more pronounced, are more likely to fail. Accordingly, our theory offers direct predictions on how some trade liberalization policies may fail, while those that involve less polarization succeed. Being that voters are more likely to be informed about distributional consequences (and because aggregate effects are difficult to predict and measure), our theory illustrates how information amplifies an electorate's resistance to reform.

Such status quo biases also offer a lens to view incumbency advantages in elections (Gelman and King, 1990; Ansolabehere and Snyder Jr, 2002). A challenger may differ not only in her ability to garner more rents for her constituency, but may also have special interests that would benefit some of her constituents at a cost borne by others. Insofar as these alignments (both in terms of the challenger's preferences and her connection with lobby and special interest groups) are difficult to perceive, voters are likely poorly informed about the challenger and better informed about the incumbent. Thus, even if the challenger appears attractive ex ante, an uninformed may vote in favor of the incumbent fearing that the reason others support the challenger is because they have learned that the challenger favors them.[22]

5.2 Ill-Advised Policy Reforms: A Dynamic Model

While we highlight above how our theory may amplify status quo biases, it is also the case that polarization can lead to excessive reforms, some of which are reversed and dismantled in the future. Indeed, we show that voters may anticipate that such reforms

[22]In the context of a one-dimensional spatial model, Gul and Pesendorfer (2009) and Bhattacharya (2013a) discuss how information aggregation failures may lead voters to prefer the candidate they know.

shall be later reversed and yet, lend decisive support to these reforms.

We view issues of suspicion as being germane to populist movements that promise major distributive reforms: these movements often contrast the interests of everyday voters with those of the political elite, highlighting conflicts between the elite and other voters from a distributional perspective. This framing has been discussed in the context of Latin American politics[23] and was a component of President Trump's 2016 campaign:

> *Let me ask America a question: How has the system been working out for you and your family? I, for one, am not interested in defending a system that for decades has served the interest of political parties at the expense of the people. Members of the club—the consultants, the pollsters, the politicians, the pundits and the special interests—grow rich and powerful while the American people grow poorer and more isolated.... The only antidote to decades of ruinous rule by a small handful of elites is a bold infusion of popular will. On every major issue affecting this country, the people are right and the governing elite are wrong. The elites are wrong on taxes, on the size of government, on trade, on immigration, on foreign policy (Trump, 2016).*

Our theory speaks to two aspects of populism. The first is that it offers a plausible rationale for why reforms that make the median voter worse off in ways that are predictable ex ante might nevertheless be popular, and why—as is often the case—such reforms are later regretted, reversed, and dismantled. The second is that by rationalizing how these reforms may be selected in elections, our results suggest that political candidates might gain from framing their platforms in the language of polarization, distribution, and change, especially when competing with candidates who are drawn from the political elite, and represent the status quo.

We consider a two-period model, building on Section 4.2, specialized to a case in which every informed voter perfectly learns her payoffs from A. Recall that each voter is uncertain about the number of winners, η, and whether she is a winner from policy reforms, gaining v_w, or a loser from policy reforms, suffering $-v_l$. In the first period, with probability λ, a voter learns her payoffs and, with probability $1-\lambda$, remains uninformed. Players vote simultaneously between Q and A, and the alternative that commands the support of a simple-majority of voters is selected.[24] If A is implemented in the first period, then every voter realizes (and observes) her payoff from A; by contrast, if Q is selected in the first period, uninformed voters do not learn their payoffs from A. In

[23] See, for example, Acemoglu, Egorov, and Sonin (2013a) and Acemoglu, Robinson, and Torvik (2013b) for both theoretical analyses and case-studies of populist movements in Latin America.

[24] In other words, the threshold τ equals $\frac{n+1}{2}$, where n is odd.

the second period, voters choose again between Q and A. No subsequent information is revealed prior to their vote in period 2, but voters have observed the vote count of the previous period. Voters obtain the sum of their payoffs from the two periods, are fully rational, and forward-looking.

The dynamic model naturally generates an *experimentation* motive: every voter learns about A only when A is chosen in period 1. We consider a parameter region where this motive is not the source of popular support for reforms: if it were commonly known that all voters are uninformed, each voter would prefer Q both in period 1, and in period 2, when Q was chosen period 1. The necessary and sufficient conditions are

$$P(W_i)v_w - P(L_i)v_l < 0, \tag{5}$$

$$\underbrace{P(W_i)v_w - P(L_i)v_l}_{\text{Period 1 payoff from } A} + \underbrace{\sum_{\eta=\frac{n+1}{2}}^{n} \left(\frac{\eta v_w - (n-\eta)v_l}{n}\right) P(\eta)}_{\text{Period 2 payoff from } A \text{ if it is continued.}} < 0. \tag{6}$$

Inequality (5) specifies that Q is ex ante optimal: the condition is relevant in period 2 when Q was chosen in period 1 (so the relevant sub-game is a single-period choice). Inequality (6) is the novel addition, and is relevant for incentives in period 1. The first term is the ex ante expected payoff from A, realized at period 1 when A is selected. The second term describes the payoff from experimentation: if A is selected in period 1, everyone learns their payoff from A; if a majority are winners, A is selected again in period 2 (otherwise the policy is reversed).[25]

Payoffs being τ-negatively correlated implies that the ex interim expected payoff is positive when a voter is herself uninformed and conditions on there being $\tau - 1 = \frac{n-1}{2}$ informed voters, all of whom are losers. We prove the following proposition:

Proposition 8. *Suppose payoffs are τ-negatively correlated and (5)-(6) are satisfied. Then for every $\varepsilon > 0$, there exists $\tilde{\lambda}$ such that if $\lambda < \tilde{\lambda}$,*

A) *A wins with probability at least $1 - \varepsilon$ in period 1 and is reversed in period 2 with probability $P\left(\eta < \frac{1}{2}\right)$ in a sequential equilibrium of the private information environment.*

B) *Q wins with probability at least $1 - \varepsilon$ in both periods in the unique sequential equilibrium of the public information environment.*

[25] This second term need not be positive, corresponding to Fernandez and Rodrik (1991) and the "loser's trap" identified by Strulovici (2010): a voter may fear that she learns that she is a loser but $\frac{n+1}{2}$ voters learn that they are winners, and these voters ensure that A is chosen again at $t = 2$. If $\frac{v_l}{v_w}$ is sufficiently high, the lottery from policy reforms may have a negative expected value.

The above result illustrates how reforms that are inferior ex ante and when all information were public may nevertheless be pursued with probability 1 and then later dismantled with significant probability. We note that the term $P\left(\eta < \frac{1}{2}\right)$ may as high as 1. For example, consider a flipped version of Example 1, where it is ex ante known that there are only two winners. In that case, each uninformed voter may vote for A, while anticipating a policy reversal in period 2; because payoffs are negatively correlated, each anticipates gaining from A in the event that she is pivotal, and is willing to accept those gains even if they are for a single period. The more general intuition, when η is uncertain, is more subtle and is in Appendix A.4.5.

Thus, we see how negatively-correlated payoffs can foster excessive experimentation and reforms, even if subsequent reversals are anticipated, and even though, ex ante, such reforms are unattractive. Private information amplifies the appeal of populist reforms as uninformed voters infer that those opposed to change must benefit from the status quo, which makes the reforms more appealing. The popularity of reforms are driven not only by the prospects of trying something new, but also the knowledge that reforms are opposed by an entrenched elite who benefit from the status quo.

6 Conclusion

This paper develops a framework to evaluate whether elections aggregate information about polarizing policies. We view this question to be of significant importance insofar as voters are often asked to vote on such policies, both in the context of direct democracy, and when selecting candidates who adopt different stances on these policies. We find that policy reforms that generate winners and losers may foster a perverse outcome, where voters opt against reforms that are preferable from an ex ante standpoint, and would have been preferred were all information public.

This perverse outcome is driven by two considerations that we view to be endemic to distributive conflict: first, information is scarce, and so few voters are informed about how they are affected by policy reforms; second, payoffs are negatively correlated, so that conditioning on others obtaining good news about the policy reform, one becomes more pessimistic about one's own fate after those reforms. The central force is that of suspicion: uninformed voters wonder what motivates other voters to support reforms, and whether their interests are aligned with the supporters. Information about the distribution of welfare exacerbates suspicion, whereas information about purely aggregate welfare mitigates it. Our goal here has been to illustrate these forces in a simple model that embeds aggregate and distributional uncertainty, discuss comparative statics predictions on the degree of polarization or crowding-out, and offer measures to mitigate informational efficiency.

We generally abstract from the source and diffusion of information, but we suspect that accounting for it might only exacerbate the forces that we find here. Information comes often from interested parties—politicians, lobbyists, activists, and biased "experts"—and our results suggest that these interested parties benefit from designing information structures in which most voters are uninformed, and any voter that is informed learns about distributional consequences. Of equal concern is that if voters are the ones choosing to acquire information, and can do so flexibly, each voter shall have a greater interest in learning how she fares—*would her own job be outsourced?*—rather than the aggregate effects on the economy. Thus, when information is costly, voters are likely to acquire distributional rather than aggregate information which, as indicated by Proposition 6 and Example 5, may foster electoral failures. Voter ignorance, even though it is individually rational, is collectively costly in this context.

We view these results as offering a bleak perspective on distributive politics, and showcasing how inefficiencies may be exacerbated and amplified by private information. Populist leaders may receive electoral support to pursue agendas that make the electorate collectively worse off and generate policy decisions that would not be chosen were information public. To the extent that political rhetoric and campaigns often focus on the divisiveness of who are winners and losers, these campaigns can increase the prospects for perverse electoral outcomes.

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A Appendix

A.1 Preliminaries

Let $\bar{v} \equiv \max\{|v_i| : v \in \mathcal{V}\})$ be the absolute value of the largest loss/gain from A. We use g and m to denote typical realizations of the random variables G and M, respectively, and Z (with typical realization z) to denote the random part of the population size. For any realization z, denote by $\tau_z \equiv \tau_z$ the minimal number of votes needed to pass Q in a population of size $n_0 + z$.

Poisson distribution: We first provide a lemma that establishes a property of Poisson random variables that we use in the proofs.

Lemma 1. *Let n_0 be a strictly positive integer and let Z follow a Poisson distribution with mean $\mu > 0$. Then the series $\sum_{z=0}^{\infty}(n_0+z)\binom{n_0+z}{\lfloor 0.5(n_0+z)\rfloor}^2 P(z)$ converges absolutely.*

Proof. Let $a(z) \equiv (n_0+z)\binom{n_0+z}{\lfloor 0.5(n_0+z)\rfloor}^2 \frac{\mu^z}{z!}$. By the ratio test for series, it is sufficient to show that $\lim_{z\to\infty}\frac{a(z+1)}{a(z)} < 1$. We consider two cases.

First, suppose $n_0 + z$ is an odd number. Then,

$$a(z+1) = (n_0+z+1)\left(\frac{(n_0+z+1)!}{(0.5(n_0+z+1))!(0.5(n_0+z+1))!}\right)^2 \frac{\mu^{z+1}}{(z+1)!} \quad \text{and}$$
$$a(z) = (n_0+z)\left(\frac{(n_0+z)!}{(0.5(n_0+z+1))!(0.5(n_0+z-1))!}\right)^2 \frac{\mu^z}{z!}.$$

Hence,

$$\begin{aligned}\frac{a(z+1)}{a(z)} &= \left(\frac{n_0+z+1}{n_0+z}\right)\left(\frac{n_0+z+1}{0.5(n_0+z+1)}\right)^2\left(\frac{\mu}{z}\right)\\ &= 4\left(\frac{n_0+z+1}{n_0+z}\right)\left(\frac{\mu}{z}\right).\end{aligned}$$

Now, suppose $n_0 + z$ is an even number. Then,

$$a(z+1) = (n_0+z+1)\left(\frac{(n_0+z+1)!}{(0.5(n_0+z))!(0.5(n_0+z+2))!}\right)^2 \frac{\mu^{z+1}}{(z+1)!} \quad \text{and}$$
$$a(z) = (n_0+z)\left(\frac{(n_0+z)!}{(0.5(n_0+z))!(0.5(n_0+z))!}\right)^2 \frac{\mu^z}{z!}.$$

Hence,

$$\begin{aligned}\frac{a(z+1)}{a(z)} &= \left(\frac{n_0+z+1}{n_0+z}\right)\left(\frac{n_0+z+1}{0.5(n_0+z+2)}\right)^2\left(\frac{\mu}{z}\right)\\ &\leq 4\left(\frac{n_0+z+1}{n_0+z}\right)\left(\frac{\mu}{z}\right).\end{aligned}$$

By the two cases considered above, $\frac{a(z+1)}{a(z)} \leq 4\left(\frac{n_0+z+1}{n_0+z}\right)\left(\frac{\mu}{z}\right)$ for all $z = 0, ..., \infty$. Since $\lim_{z\to\infty} 4\left(\frac{n_0+z+1}{n_0+z}\right)\left(\frac{\mu}{z}\right) = 0$, it follows by the ratio test for series that $\sum_{z=0}^{\infty}(n_0 + z)\binom{n_0+z}{\lfloor 0.5(n_0+z)\rfloor}^2 P(z)$ converges absolutely. □

Remark 1. The proof of Proposition 1 and Theorems 1 and 2 use the fact that when Z follows a Poisson distribution (i) $P(Z = 0) > 0$, and (ii) $\sum_{z=0}^{\infty}(n_0 + z)\binom{n_0+z}{\lfloor 0.5(n_0+z)\rfloor}^2 P(z)$ converges absolutely (Lemma 1). The proofs require no other properties of the Poisson distribution and so our results apply for any random variable Z satisfying properties (i)-(ii). For example, any random variable Z with finite support also satisfies property (ii).

Strategies and equilibrium: Let $\Sigma \equiv \{\sigma : \mathcal{S} \to [0, 1]\}$ be the set of mappings from signals to $[0, 1]$. In the private information environment, a strategy for voter i can be described by $\sigma_i \in \Sigma$, where $\sigma_i(s_i)$ is the probability that i votes for A when receiving signal s_i. With some abuse of notation, a symmetric strategy-profile can also be described in terms of an element $\sigma \in \Sigma$; in particular, we denote by (σ_i, σ_{-i}) a strategy-profile where player i follows strategy $\sigma_i \in \Sigma$, and all players other than i follow the symmetric strategy $\sigma_{-i} \in \Sigma$.

Let $Q_\sigma(\tau|\omega)$ denote the probability that at least $\lfloor \tau N(\omega)\rfloor + 1$ players vote for A when players follow the symmetric strategy profile σ and state ω is realized. Then voter i's expected payoff function for the symmetric strategy-profile σ and signal s_i is defined by $\pi_i(\sigma|s_i) \equiv \sum_{\omega\in\Omega} Q_\sigma(\tau|\omega)V_i(\omega)P(\omega|s_i)$.

Definition 1 (Equilibrium)**.** *A symmetric strategy profile σ is an equilibrium if the following conditions are satisfied for all n and $i \in \{1, ..., n\}$:*

(i) $\pi_i(\sigma|.) \geq \pi_i(\sigma'_i, \sigma_{-i}|.)$ for all $\sigma'_i \in \Sigma$,

(ii) for each $s_i \in \mathcal{S}$: if there exists σ'_i such that $\pi_i(\sigma'_i, \tilde{\sigma}_{-i}|s_i) \geq \pi_i(\tilde{\sigma}|s_i)$ for all $\tilde{\sigma}$, then $\pi_i(\sigma_i, \tilde{\sigma}_{-i}|s_i) \geq \pi_i(\sigma'_i, \tilde{\sigma}_{-i}|s_i)$ for all $\tilde{\sigma}$.

The equilibrium is strict *if the inequality in part (i) is strict for all $s_i \in \mathcal{S}$.*

Condition (i) is the standard requirement of a Bayes Nash equilibrium (BNE): for all possible realizations of their signal, voters play a best response to the strategies of other voters. Condition (ii) states that voters play a weakly undominated strategy. In the public information environment, strategies, payoffs, and equilibrium are defined analogously, except that players condition on the population size n, and the whole signal profile $s \in \mathcal{S}^n$.

Equilibrium characterization: We first provide a simple characterization of equilibrium in the private information environment. For $\alpha \in [0,1]$, define the symmetric strategy profile σ^α as follows:

$$\sigma_i^\alpha(s_i) \equiv \begin{cases} \alpha & \text{if } s_i = s^0 \\ 1 & \text{if } s_i \in \mathcal{G} \\ 0 & \text{if } s_i \in \mathcal{B} \end{cases} .$$

Define the function $\Pi : [0,1] \to \mathbb{R}$ as follows:

$$\begin{aligned} \Pi(\alpha) &\equiv \pi_i(\sigma_i^1, \sigma_{-i}^\alpha | s_i^0) - \pi_i(\sigma_i^0, \sigma_{-i}^\alpha | s_i^0) \\ &= \sum_{z=0}^{\infty} \sum_{m=0}^{n_0+z-1} \sum_{g=0}^{m} V(g,m,z) P_\alpha(piv|g,m,z) P(g|m,z) P(m|z) P(z), \end{aligned}$$

where $V(g,m,z) \equiv V_i(s_i^0, g, m, n_0+z)$ is a voter's expected payoff conditional on receiving the uninformative signal, g voters receiving good news, m voters being informed in a population of size $n_0 + z$ (by Assumption 2, this does not need a voter subscript); $P(piv|g,m,z)$ is the probability that the voter is pivotal given $(G,M,Z) = (g,m,z)$ and that other voters are following the strategy profile σ_{-i}^α, which is

$$P_\alpha(piv|g,m,z) \equiv \begin{cases} f(\alpha,g,m,z) & \text{if } \alpha \in (0,1) \ , \ 0 \le \tau_z - g \le n_0 + z - 1 - m \\ 1 & \text{if } \alpha = 0 \ , \ \tau_z - g = 0 \\ & \text{or } \alpha = 1 \ , \ \tau_z - g = n_0 + z - 1 - m \\ 0 & \text{otherwise} \end{cases} ,$$

where, for $\alpha \in (0,1)$ and $0 \le \tau_z - g \le n_0 + z - 1 - m$,

$$f(\alpha,g,m,z) = \binom{n_0 + z - 1 - m}{\tau_z - g} \alpha^{\tau_z - g} (1-\alpha)^{n_0+z-1-(m-g)-\tau_z};$$

$P(g|m,z) \equiv P(g|s_i^0, m, n_0+z)$ is the probability that g voters receive good news conditional on m voters being informed in a population of size n_0+z; $P(m|z)$ is the probability that m other voters are informed in a population of size $n_0 + z$, which is

$$P(m|z) \equiv \begin{cases} \binom{n_0+m-1}{m}\lambda^m(1-\lambda)^{n_0+z-1-m} & \text{if } 0 \leq m \leq n_0+z-1 \\ 0 & \text{otherwise} \end{cases};$$

and $P(z) \equiv P(Z = z)$ is the probability that the population is of size $n_0 + z$. Hence, $\Pi(\alpha)$ is the difference in the expected payoff for an uninformed voter when they vote for A versus voting for Q, conditional on other voters following the strategy profile σ^α_{-i}, because the payoff-difference is non-zero only when the voter is pivotal.

Lemma 2. *The function* Π *is well-defined and continuous on* $[0, 1]$.

Proof. Define the sequence of functions $\{w_z : [0, 1] \to \mathbb{R}\}$ by

$$w_z(\alpha) = \sum_{m=0}^{n_0+z-1}\sum_{g=0}^{m} V(g, m, z)P_\alpha(piv|g, m, z)P(g|m, z)P(m|z)P(z),$$

for all $\alpha \in [0, 1]$, so that $\Pi(\alpha) = \sum_{z=0}^{\infty} w_z(\alpha)$. Fix some $z \in \mathbb{N} \cup \{0\}$. The argument α enters w_z only in the term $P_\alpha(piv|g, m, z)$. The function $f(\alpha|m, g)$, used to define $P_\alpha(piv|g, m, z)$, is continuous on $(0, 1)$ for all (g, m, z) such that $0 \leq \tau_z - g \leq n_0 + z - 1 - m$. In addition,

$$\lim_{\alpha\to 0} f(\alpha|g, m, z) = \begin{cases} 1 & \text{if } \tau_z - g = 0 \\ 0 & \text{otherwise} \end{cases}, \quad \text{and}$$

$$\lim_{\alpha\to 1} f(\alpha|g, m, z) = \begin{cases} 1 & \text{if } \tau_z - g = n_0 + z - 1 - m \\ 0 & \text{otherwise} \end{cases}.$$

Hence, $P_\alpha(piv|g, m, z)$ is continuous in α on $[0, 1]$, and so $w_z : [0, 1] \to \mathbb{R}$ is continuous. Moreover, for all (g, m, z) and α, $|w_z(\alpha)| \leq \bar{v}P(z)$. Since $\sum_{z=0}^{\infty} \bar{v}P(z) = \bar{v}\sum_{z=0}^{\infty} P(z) = \bar{v}$, it follows by the Weierstrass M-test that the series $\sum_{z=0}^{\infty} w_z(\alpha)$ converges absolutely and uniformly, and so $\Pi(\alpha)$ is well-defined. Since each of the functions w_z are continuous on $[0, 1]$, it then follows by the uniform limit theorem that $\Pi(\alpha)$ is continuous on $[0, 1]$. □

Lemma 3. *Strategy profile* σ^* *is an equilibrium if and only if* $\sigma^* = \sigma^\alpha$ *for some* $\alpha \in [0, 1]$ *and one of the following three conditions is satisfied: (i)* $\alpha = 1$ *and* $\Pi(\alpha) \geq 0$*, (ii)* $\alpha = 0$ *and* $\Pi(\alpha) \leq 0$*, or (iii)* $\alpha \in (0, 1)$ *and* $\Pi(\alpha) = 0$*. Moreover,* σ^α *is a strict equilibrium if and only if either (i')* $\alpha = 1$ *and* $\Pi(\alpha) > 0$*, or (ii')* $\alpha = 0$ *and* $\Pi(\alpha) < 0$*.*

Proof. By standard arguments, the strategy profile σ^α is a BNE if and only if one of the conditions (i)–(iii) is satisfied, and is a strict BNE if and only if either condition (i') or (ii') is satisfied. Moreover, by Assumption 2(b), when σ^α is a BNE, then it is an equilibrium. It therefore remains to show that if σ^* is an equilibrium, then there must be some $\alpha \in [0,1]$ such that $\sigma^* = \sigma^\alpha$.

Suppose σ^* is an equilibrium but $\sigma_i^*(s_i^k) = \beta \neq 1$ for some $s^k \in \mathcal{G}$. Let σ' be the strategy where $\sigma'(s) = \frac{1}{2}$ for all $s \in \mathcal{S}$. When players other than i follow strategy-profile σ'_{-i}, player i is pivotal in a population of size $n_0 + z$ with probability $P(piv|n_0+z, \sigma'_{-i}) = \binom{n_0+z-1}{\tau_z}\left(\frac{1}{2}\right)^{n_0+z-1} > 0$. Moreover, conditioning on being pivotal conveys no information about the information received by other players. By Lemma 1 and the Weierstrass M-test, $\sum_{z=0}^{\infty} P(piv|n_0+z, \sigma'_{-i})V_i(s_i^k, n_0+z)P(z)$ converges absolutely to some $\tilde{c} > 0$ because, for all z, $\binom{n_0+z-1}{\tau_z}\left(\frac{1}{2}\right)^{n_0+z-1} \leq (n_0+z)\binom{n_0+z}{\lfloor 0.5(n_0+z)\rfloor}^2$, and $|V_i(s_i^k, n_0+z)| \leq \bar{v}$.

As a result, $\pi_i(\sigma_i^*, \sigma'_{-i}|s_i^k) = \beta\tilde{c} < \tilde{c} = \pi_i(\sigma_i^1, \sigma_{-i}|s_i^k)$. On the other hand, by Assumption 2(b), $\pi_1(\sigma_i^1, \tilde{\sigma}_{-1}|s_i^k) \geq \pi_1(\tilde{\sigma}|s_i^k)$ for all $\tilde{\sigma}$. Hence, σ^* is not an equilibrium. An analogous argument shows that, in an equilibrium, it must be the case that $\sigma_i^*(s_i^k) = 0$ whenever $s^k \in \mathcal{B}$. Since σ^* is a symmetric strategy-profile, it follows that $\sigma^* = \sigma^\alpha$ for some $\alpha \in [0,1]$. □

A.1.1 Proof of Proposition 1

Proof. First, consider the private information environment. By Lemma 3, if $\Pi(0) \leq 0$ then σ^0 is an equilibrium, and if $\Pi(1) \geq 0$ then σ^1 is an equilibrium. It remains to show that there is an equilibrium when $\Pi(0) > 0$ and $\Pi(1) < 0$. In that case, since Π is continuous on $[0,1]$ by Lemma 1, it follows by the intermediate value theorem that there exists some α^* such that $\Pi(\alpha^*) = 0$ and so σ^{α^*} is an equilibrium.

For the public information environment, define the symmetric strategy-profile σ^{pub} as follows:

$$\sigma^{pub}(n,s) = \begin{cases} 1 & \text{if } V_i(n,s) > 0 \\ 0 & \text{otherwise} \end{cases}.$$

By standard arguments, σ^{pub} is an equilibrium. To show that this is the unique equilibrium, suppose for contradiction that $\sigma^* \neq \sigma^{pub}$ is also an equilibrium. For some (z,s), $\sigma(z,s) \neq \sigma^{pub}(z,s)$ and, by Assumption 3(a), $V_i(s) \neq 0$. Define the strategy σ' as in the proof of Lemma 3. Now consider the case $V_i(s) > 0$, and $\sigma_i(s) < 1$. Then

$$\begin{aligned}\pi_i(\sigma_i^{pub}, \sigma'_{-i}|s, n_0+z) &= V_i(s)\binom{n_0+z-1}{\tau_z}\left(\frac{1}{2}\right)^{n_0+z-1} \\ &> \sigma_i^*(s)V_i(s)\binom{n_0+z-1}{\tau_z}\left(\frac{1}{2}\right)^{n_0+z-1} = \pi_i(\sigma_i^*, \sigma_{-i}|s, n_0+z).\end{aligned}$$

The case where $V_i(s) < 0$ is symmetric. Therefore, σ^* violates condition (ii) in the definition of an equilibrium. □

A.1.2 Decomposition

Consider a distribution P satisfying Assumptions 1 and 2. Let $\lambda = 1 - P(s_i^0)$ and, for all $\{n, s\} \subset \Omega$, let $\tilde{P}(s) = P(s)\lambda^{-n}$ if $s \in \mathcal{M}^n$ for some n and $\tilde{P}(s) = 0$ otherwise. The following claims show that $\tilde{P}$ is a probability distribution on $\tilde{\Omega}$, and that for every $\lambda' \in (0,1)$ there exists a unique P' in the family $(\tilde{P}, \lambda)$ satisfying Assumptions 1 and 2.

Claim 1. *$\tilde{P}$ is a probability distribution.*

Proof. By definition $\tilde{P}$ assigns non-negative weight to each element in $\tilde{\Omega}$. We show that $\sum_{\omega\in\tilde{\Omega}} \tilde{P}(\omega) = 1$. It is sufficient to show that $\sum_{\{\omega\in\tilde{\Omega}:N(\omega)=n\}} \tilde{P}(\omega|n) = 1$ for all n. Fix some n. It follows by integration on $\mathcal{S}^n$ that $\sum_{s\in\mathcal{M}^n} \tilde{P}(s|n) = \lambda^{-n} \sum_{s\in\mathcal{M}^n} P(s|n)$, and so it suffices to establish that $\sum_{s\in\mathcal{M}^n} P(s|n) = \lambda^n$.

Fix a positive integer q that is strictly less than n, and consider an event $\{n, s_1, \ldots, s_q\} \equiv \{n, s^q\}$ where $s_i \in \mathcal{M}$ for every $i = 1, \ldots, q$. Then, by Assumption 3(a),

$$\begin{aligned}
P(s^q|n) &= P(s^q|n)P(s_{q+1} = s^0|n) + \sum_{s_{q+1}\in\mathcal{M}} P(s^q, s_{q+1}|n) \\
&= P(s^q|n)(1-\lambda) + \sum_{s_{q+1}\in\mathcal{M}} P(s^q, s_{q+1}|n) = \frac{1}{\lambda} \sum_{s_{q+1}\in\mathcal{M}} P(s^q, s_{q+1}|n).
\end{aligned} \tag{7}$$

Proceeding by induction, $P(s^q|n) = \left(\frac{1}{\lambda}\right)^{n-q} \sum_{j=q+1}^{n} \sum_{s_j\in\mathcal{M}} P(s_1, \ldots, s_n|n)$. Substituting $q = 1$, and adding across all $s_1 \in \mathcal{M}$ yields

$$\sum_{s_1\in\mathcal{M}} P(s_1|n) = \sum_{s_1\in\mathcal{M}} \left(\frac{1}{\lambda}\right)^{n-1} \sum_{j=2}^{n} \sum_{s_j\in\mathcal{M}} P(s_1, \ldots, s_n|n) = \left(\frac{1}{\lambda}\right)^{n-1} \sum_{s\in\mathcal{M}^n} P(s|n). \tag{8}$$

By the same reasoning leading to (7), $P(s_2, \ldots, s_n|n) = \frac{1}{\lambda} \sum_{s_1\in\mathcal{M}} P(s_1, s_2, \ldots, s_n|n)$ for each $(s_2, \ldots, s_n) \in \mathcal{S}^{n-1}$. As $\sum_{(s_2,\ldots,s_n)\in\mathcal{S}^{n-1}} P(s_2, \ldots, s_n|n) = 1$, it follows that

$$1 = \sum_{(s_2,\ldots,s_n)\in\mathcal{S}^{n-1}} \frac{1}{\lambda} \sum_{s_1\in\mathcal{M}} P(s_1, s_2, \ldots, s_n|n) = \frac{1}{\lambda} \sum_{s_1\in\mathcal{M}} P(s_1|n),$$

implying that $\sum_{s_1\in\mathcal{M}} P(s_1|n) = \lambda$. Using (8), we conclude that $\sum_{s\in\mathcal{M}^n} P(s|n) = \lambda^n$. □

The following claim shows that, for each $\lambda' \in (0,1)$, there exists a unique distribution P' in the family $(\tilde{P}, \lambda)_{\lambda\in(0,1)}$ that satisfies Assumptions 1–4. For each n and $s \in \mathcal{S}^n$, let $\tilde{E}(n, s) = \{\tilde{s} \in \mathcal{M}^n : \tilde{s}_j = s_j \text{ whenever } s_j \neq s^0\}$.

Claim 2. *For each* $\lambda' \in (0,1)$*, there exists a unique distribution* P' *in the family* $(\tilde{P}, \lambda)_{\lambda\in(0,1)}$ *that satisfies Assumptions 1 and 2. In particular, for any* $\omega = (n, v, s) \in \Omega$*,*

$$P'(\omega) = \lambda^{M(\omega)} (1-\lambda)^{n-M(\omega)} \sum_{s' \in \tilde{E}(n,s)} \tilde{P}(n, v, s'). \tag{9}$$

Proof. We fix n and consider $\omega \in \Omega$ such that $N(\omega) = n$, proceeding by induction. First, suppose $M(\omega) = n$. Then (9) follows from our construction in the text. Now suppose (9) is true for any ω where $M(\omega) = m + 1$ for some $m < n$. We establish below that this is true for any ω' where $M(\omega') = m$.

Consider any $\omega' = (n, v, s)$ where $M(\omega') = m$, and suppose $s_i = s^0$. Observe that,

$$\begin{aligned} P(v, s_{-i}|n) &= \sum_{s_i' \in \mathcal{S}} P(v, s_{-i}, s_i'|n) = P(v, s_{-i}|n)P(s_i' = s^0|n) + \sum_{s_i' \in \mathcal{M}} P(v, s_{-i}, s_i'|n) \\ &= P(v, s_{-i}|n)(1-\lambda) + \sum_{s_i' \in \mathcal{M}} P(v, s_{-i}, s_i'|n) = \frac{1}{\lambda} \sum_{s_i' \in \mathcal{M}} P(v, s_{-i}, s_i'|n). \end{aligned} \tag{10}$$

where the first equality is by definition, the second equality follows from Assumption 3(a), the third equality follows from $P(s_i' = s^0|n) = (1-\lambda)$, and the fourth equality follows from simplification. Using Assumption 3(a), $s_i = s^0$, and $P(s_i|n) = (1-\lambda)$, it follows that $P(\omega'|n) = (1-\lambda)P(v, s_{-i}|n)$. Substituting (10) for $P(v, s_{-i}|n)$ yields

$$P(v, s|n) = \left(\frac{1-\lambda}{\lambda}\right) \sum_{s_i' \in \mathcal{M}} P(v, s_{-i}, s_i'|n) = \lambda^{M(\omega')} (1-\lambda)^{n-M(\omega')} \sum_{s' \in \tilde{E}(n,s)} \tilde{P}(v, s'|n),$$

where the second equality follows from the induction hypothesis and simplification. □

A.1.3 Proof of Proposition 2

In the public information environment, the unique equilibrium is σ^{pub} from the proof of Proposition 1. Given this strategy profile, if every player receives the uninformative signal, then $V(g = m = 0|n) > 0$ for all n implies that A wins the election in the unique equilibrium. The probability that all voters receive the uninformative signal is $(1-\lambda)^{n_0+z}$ in a population of size $n_0 + z$. Hence, the probability that A wins the election is at least $\sum_{z=0}^{\infty}(1-\lambda)^{n_0+z}P(z)$. For any fixed $\bar{z}$, this is greater than $(1-\lambda)^{n_0+\bar{z}}P(z \leq \bar{z})$. Since $P(z \leq \bar{z})$ converges to 1, we can choose $\bar{z}$ so that $P(z \leq \bar{z}) \geq \sqrt{(1-\varepsilon)}$. Now fix $\bar{\lambda}' \in (0,1)$ such that $(1-\bar{\lambda}')^{n_0+\bar{z}} \geq \sqrt{1-\varepsilon}$. Then, for all $\lambda \in (0, \bar{\lambda}')$, $\sum_{z=0}^{\infty}(1-\lambda)^{n_0+z}P(z) \geq (\sqrt{1-\varepsilon})^2 = 1-\varepsilon$, and so A wins with probability exceeding $1-\varepsilon$.

A.2 Proof of Theorem 1

Suppose payoffs are τ-negatively correlated (i.e., $V^G(\tau) < 0$). Recall that σ^0 is the strategy profile where uninformed voters choose Q, and informed voters choose A when they receive good news and Q when they receive bad news. We first show that there exists $\bar{\lambda} \in (0,1)$ such that for all $\lambda \in (0, \bar{\lambda})$, σ^0 is a strict equilibrium.

By Lemma 3, the strategy profile σ^0 is a strict equilibrium if and only if the following is strictly negative

$$\Pi(0) = \pi_i\left(\sigma_i^1, \sigma_{-i}^0 | s_i^0\right) - \pi_i\left(\sigma_i^0, \sigma_{-i}^0 | s_i^0\right)$$
$$= \sum_{z=0}^{\infty} \sum_{m=\tau_z}^{n_0+z-1} \binom{n_0+z-1}{m} \lambda^m (1-\lambda)^{n_0+z-1-m} P(g=\tau_z|m,z) V(\tau_z,m,z) P(z), \quad (11)$$

Showing that (11) is strictly negative is equivalent to showing that the following term is strictly negative,

$$\sum_{z=0}^{\infty} \sum_{m=\tau_z}^{n_0+z-1} \binom{n_0+z-1}{m} \lambda^{m-\tau_0} (1-\lambda)^{n_0+z-1-m} P(g=\tau_z|m,z) V(\tau_z,m,z) P(z). \quad (12)$$

We first consider the summand in (12) where $z = 0$ (i.e., $n = n_0$):

$$\sum_{m=\tau_0}^{n_0-1} \binom{n_0-1}{m} \lambda^{m-\tau_0} (1-\lambda)^{n_0-1-m} P(g=\tau_z|m,n_0) V(\tau_z,m,n_0) P(z=0).$$

Since $\lambda < 1$, $V(\tau_z, m, n_0) \leq \bar{v}$, and $P(\tau_z|m,z) \leq 1$, the above is bounded above by

$$\binom{n_0-1}{\tau_0} P(g=\tau_0|m=\tau_0,n_0) P(z=0) V^G(\tau) + \lambda \bar{v} P(z=0) \sum_{m=\tau_0+1}^{n_0-1} \binom{n_0-1}{m}.$$

We now consider the remaining terms in the series (12). Since $V(\tau_z, m, z) \leq \bar{v}$, $P(g|m,z) \leq 1$, $(1-\lambda)^{n_0+z-1-m} \leq 1$, $\lambda^{m-\tau_0} \leq \lambda$, and $\binom{n_0+z-1}{m} \leq \binom{n_0+z-1}{\lfloor 0.5(n_0+z-1) \rfloor}$, this remaining series can be bounded above by

$$\lambda \left(\sum_{z=1}^{\infty} \sum_{m=\tau_z}^{n_0+z-1} \binom{n_0+z-1}{m} \bar{v} P(z) \right) \leq \lambda \bar{v} \left(\sum_{z=1}^{\infty} \tau_z \binom{n_0+z}{\lfloor 0.5(n_0+z) \rfloor} P(z) \right). \quad (13)$$

Series (13) converges absolutely by Lemma 1 and the Weierstrass M-test because $\tau_z \binom{n_0+z}{\lfloor 0.5(n_0+z) \rfloor} \leq (n_0+z) \binom{n_0+z}{\lfloor 0.5(n_0+z) \rfloor}^2$ for all $z = 1, ..., \infty$. Hence, there exists $\bar{c}$, such that the series in (13) is bounded above by $\lambda\bar{c}$. As a result, $\Pi_i(\lambda)$ is bounded above by

$$\binom{n_0-1}{\tau_0}P(g=\tau_0|m=\tau_0,n_0)P(z=0)V^G(\tau)+\lambda\left(\bar{c}+\sum_{m=\tau_0+1}^{n_0-1}\binom{n_0-1}{m}\bar{v}P(z=0)\right).$$

The first term does not depend on λ and is strictly negative because payoffs are τ-negatively correlated (i.e., $P(g=\tau_0|m=\tau_0,n_0)>0$ and $V^G(\tau)<0$) and $P(z=0)>0$. In the second term, the bracket is some finite positive number that does not depend on λ. Hence, there exists $\bar{\lambda}\in(0,1)$ such that, for all $\lambda<\bar{\lambda}$, $\Pi_i(\lambda)<0$ and so σ^0 is an equilibrium.

Now suppose that the players follow the strategy profile σ^0. If all players are uninformed, Q wins the election. Hence, following the argument in the proof of Proposition 2, there exists $\bar{\lambda}'$ such that for all $\lambda\in(0,\bar{\lambda}')$, Q wins with probability exceeding $(1-\varepsilon)$ in the equilibrium strategy σ^0.

A.3 Proof of Theorem 2

Suppose payoffs are not τ-negatively correlated and, without loss of generality, let $\lambda\leq\frac{1}{2}$. Let $v^*=\min_{\kappa\in\{0,\dots,\tau_0\}}V^G(\kappa)P(g=\kappa|m=\kappa,n=n_0)$. By Assumption 5, $v^*>0$ because payoffs are not τ-negatively correlated and $V^G(g=m=0|n_0)>0$. Recall that, for $\alpha\in[0,1]$, σ^α is the (private) strategy profile where uninformed voters chose A with probability α and choose Q with probability $(1-\alpha)$, and informed voters choose A when they receive good news and Q when they receive bad news. The following Lemma establishes the key step in the proof.

Lemma 4. *For every $\bar{\alpha}\in(0,1)$, there exists $\lambda_{\bar{\alpha}}\in(0,1)$ such that, if $\alpha\in(0,\bar{\alpha})$ and $\lambda\in(0,\lambda_{\bar{\alpha}})$, then $\Pi(\alpha)>0$ (i.e., σ^α is not an equilibrium).*

Proof. Fix some $\bar{\alpha}\in(0,1)$ and let $\alpha\in(0,\bar{\alpha})$. For $\omega=(n,v,s)\in\Omega$, an uninformed voter i is pivotal if and only if τ_z vote for A. If $G(n,v,s)=g$ this requires $q\equiv\tau_z-g$ uninformed voters to choose A. Define $\Theta\equiv\{(q,m,z)\in\mathbb{N}^3:0\leq\tau_z-q\leq m\leq n_0+z-1\}$. We want to show that, for λ sufficiently small,

$$\Pi(\alpha)=\sum_{z=0}^{\infty}\sum_{m=0}^{n_0+z-1}\sum_{g=0}^{m}V(g,m,z)P(piv|g,m,z)P(g|m,z)P(m|z)P(z)\equiv\sum_{\theta\in\Theta}c(\theta)>0,$$

where for $\theta = (q, m, z) \in \Theta$,

$$\begin{aligned}
c(\theta) &\equiv \alpha^q \lambda^m \tilde{V}(q,m,z) \tilde{P}(q|m,z) B(\theta) A(\theta) P(z), \\
\tilde{V}(q,m,z) &\equiv V(g = \tau_z - q, m, z), \\
\tilde{P}(q|m,z) &\equiv P(g = \tau_z - q | m, z), \\
B(\theta) &\equiv \binom{n_0 + z - 1 - m}{q} \binom{n_0 + z - 1}{m}, \\
A(\theta) &\equiv (1-\alpha)^{n_0 + z - 1 - m - q} (1-\lambda)^{n_0 + z - 1 - m}.
\end{aligned}$$

We first provide a lower bound for $\Pi(\alpha)$ by giving a lower bound $\underline{c}(\theta)$ of $c(\theta)$ for each $\theta \in \Theta$. We partition Θ into four sets.

(1) $\Theta_1 = \{(q,m,z) \in \Theta : z = 0, m = \tau_0 - q\}$. Then, $\tilde{V}(q,m,z)\tilde{P}(q|m,z) \geq v^* > 0$ and $P(z) > 0$. Hence, for all $(q,m,z) \in \Theta_1$,

$$c(q,m,z) \geq \underline{c}(q,m,z) \equiv \alpha^q \lambda^{\tau_0 - q} v^* \left(\frac{1-\bar{\alpha}}{2}\right)^{n_0} P(z=0),$$

because $B(q,m,z) \geq 1$, $A(q,m,z) \geq \left(\frac{1-\bar{\alpha}}{2}\right)^{n_0}$. As a result,

$$\sum_{\theta \in \Theta_1} \underline{c}(\theta) = \sum_{q=0}^{\tau_0} \alpha^q \lambda^{\tau_0 - q} v^* \left(\frac{1-\bar{\alpha}}{2}\right)^{n_0} P(z=0).$$

(2) $\Theta_2 = \{(q,m,z) \in \Theta : z > 0, m = 0\}$. Then, $\tilde{V}(q,m,z) = \tilde{V}(q,0,z) \equiv \tilde{V}(0|z) > 0$. Hence, for all $(q,m,z) \in \Theta_2$,

$$c(q,m,z) \geq \underline{c}(q,m,z) \equiv 0$$

As a result, $\sum_{\theta \in \Theta_2} \underline{c}(\theta) = 0$.

(3) $\Theta_3 = \{(q,m,z) \in \Theta : z = 0, m > \tau_0 - q\}$. In that case, $\tilde{V}(q,m,z) \geq -\bar{v}$ which is negative. Hence, for all $(q,m,z) \in \Theta_3$,

$$c(q,m,z) \geq \underline{c}(q,m,z) \equiv -\alpha^q \lambda^{\tau_0 - q + 1} \bar{v} \binom{n_0}{\lfloor 0.5 n_0 \rfloor}^2,$$

because $m \geq \tau_0 - q + 1$, and so $\lambda^m \leq \lambda^{\tau_0 - q + 1}$, $\tilde{P}(q|m,z) \leq 1$, $B(q,m,z) \leq \binom{n_0}{\lfloor 0.5 n_0 \rfloor}^2$,

$A(q,m,z) \leq 1$, and $P(z=0) \leq 1$. As a result,

$$\sum_{\theta\in\Theta_2} \underline{c}(\theta) = -\sum_{q=0}^{\tau_0} \sum_{m=\tau_0-q+1}^{n_0-q-1} \alpha^q \lambda^{\tau_0-q+1} \bar{v} \binom{n_0}{\lfloor 0.5n_0 \rfloor}^2$$
$$\geq -\sum_{q=0}^{\tau_0} n_0 \alpha^q \lambda^{\tau_0-q+1} \bar{v} \binom{n_0}{\lfloor 0.5n_0 \rfloor}^2.$$

(4) $\Theta_4 = \{(q,m,z) \in \Theta : z > 0, m > 0\}$. In that case, $\tilde{V}(q,m,z) \geq -\bar{v}$ which is negative. Hence, for all $(q,m,z) \in \Theta_4$,

$$c(q,m,z) \geq \underline{c}(q,m,z) \equiv \begin{cases} -\alpha^q \lambda^{\tau_0-q+1} \bar{v} \binom{n_0+z}{\lfloor 0.5(n_0+z) \rfloor}^2 P(z) & \text{if } q \leq \tau_0, \\ -\alpha^q \lambda \bar{v} \binom{n_0+z}{\lfloor 0.5(n_0+z) \rfloor}^2 P(z) & \text{if } q > \tau_0, \end{cases}$$

because $m \geq \tau_z - q$, and so $\lambda^m \leq \lambda^{\tau_0-q+1}$ when $q \leq \tau_0$, and $\lambda^m \leq \lambda$ when $q > \tau_0$, $\tilde{P}(q|m,z) \leq 1$, $B(q,m,z) \leq \binom{n_0+z}{\lfloor 0.5(n_0+z) \rfloor}^2$, and $A(q,m,z) \leq 1$. Since $\underline{c}(\theta) < 0$ for every $\theta \in \Theta_4$,

$$\sum_{\theta\in\Theta_4} \underline{c}(\theta) \geq -\sum_{q=0}^{\tau_0} \sum_{z=0}^{\infty} \sum_{m=\tau_z-q}^{n_0+z-1-q} \alpha^q \lambda^{\tau_0-q+1} \bar{v} \binom{n_0+z}{\lfloor 0.5(n_0+z) \rfloor}^2 P(z)$$
$$- \sum_{q=\tau_0+1}^{\infty} \sum_{z=0}^{\infty} \sum_{m=\max\{0,\tau_z-q\}}^{\max\{0,n_0+z-1-q\}} \alpha^q \lambda \bar{v} \binom{n_0+z}{\lfloor 0.5(n_0+z) \rfloor}^2 P(z)$$
$$\geq -\sum_{q=0}^{\tau_0} \alpha^q \lambda^{\tau_0-q+1} \bar{v} \sum_{z=0}^{\infty} (n_0+z) \binom{n_0+z}{\lfloor 0.5(n_0+z) \rfloor}^2 P(z)$$
$$- \sum_{q=\tau_0+1}^{\infty} \alpha^q \lambda \bar{v} \sum_{z=0}^{\infty} (n_0+z) \binom{n_0+z}{\lfloor 0.5(n_0+z) \rfloor}^2 P(z)$$

By Lemma 1, the series $\sum_{z=0}^{\infty} (n_0+z) \binom{n_0+z}{\lfloor 0.5(n_0+z) \rfloor}^2 P(z)$ converges absolutely to some $\bar{c}$. As a result,

$$\sum_{\theta\in\Theta_4} \underline{c}(\theta) \geq -\lambda \bar{v} \bar{c} \sum_{q=0}^{\tau_0} \alpha^q \lambda^{\tau_0-q} - \lambda \bar{v} \bar{c} \sum_{q=\tau_0+1} \alpha^q.$$

Since $(\Theta_1, \Theta_2, \Theta_3, \Theta_4)$ is a partition of Θ, and $n_0 \binom{n_0}{\lfloor 0.5 n_0 \rfloor}^2 P(z=0) \leq \bar{c}$,

$$\begin{aligned}
\Pi(\alpha) &\geq \sum_{\theta \in \Theta_1} \underline{c}(\theta) + \sum_{\theta \in \Theta_2} \underline{c}(\theta) + \sum_{\theta \in \Theta_3} \underline{c}(\theta) + \sum_{\theta \in \Theta_4} \underline{c}(\theta) \\
&\geq \left(v^* \left(\frac{1-\bar{\alpha}}{2} \right)^{n_0} P(z=0) - 2\lambda \bar{v}\bar{c} \right) \sum_{q=0}^{\tau_0} \alpha^q \lambda^{\tau_0 - q} - \lambda \bar{v}\bar{c} \sum_{q=\tau_0+1}^{\infty} \alpha^q \\
&= \left(v^* \left(\frac{1-\bar{\alpha}}{2} \right)^{n_0} P(z=0) - 2\lambda \bar{v}\bar{c} \right) \sum_{q=0}^{\tau_0} \alpha^q \lambda^{\tau_0 - q} - \lambda \bar{v}\bar{c} \alpha^{\tau_0+1} \frac{1}{1-\alpha} \\
&\geq \left(v^* \left(\frac{1-\bar{\alpha}}{2} \right)^{n_0} P(z=0) - 2\lambda \bar{v}\bar{c} \right) \sum_{q=0}^{\tau_0} \alpha^q \lambda^{\tau_0 - q} - \lambda \bar{v}\bar{c} \alpha^{\tau_0} \frac{1}{1-\bar{\alpha}} \\
&= \alpha^{\tau_0} \left(v^* \left(\frac{1-\bar{\alpha}}{2} \right)^{n_0} P(z=0) - \lambda \bar{v}\bar{c} \left(2 + \frac{1}{1-\bar{\alpha}} \right) \right) \\
&\quad + \left(v^* \left(\frac{1-\bar{\alpha}}{2} \right)^{n_0} P(z=0) - 2\lambda \bar{v}\bar{c} \right) \sum_{q=0}^{\tau_0 - 1} \alpha^q \lambda^{\tau_0 - q}.
\end{aligned}$$

There exists some $\bar{\lambda} \in (0,1)$ such that, for all $\lambda < \bar{\lambda}$, $v^* \left(\frac{1-\bar{\alpha}}{2} \right)^{n_0} P(z=0) > \lambda \bar{v}\bar{c} \left(2 + \frac{1}{1-\bar{\alpha}} \right)$. Hence, for $\lambda < \bar{\lambda}$, $\Pi_i(\alpha) > 0$. □

We now use Lemma 4 to complete the proof. Fix $\varepsilon \in (0,1)$. We want to show that there exists λ_ε such that in every equilibrium of the private information environment, A wins with probability exceeding $1-\varepsilon$. (The proof for the public information environment follows from Proposition 1).

First, fix $\bar{z}_\varepsilon$ such that $P(z \leq \bar{z}_\varepsilon) \geq (1-\varepsilon)^{\frac{1}{3}}$. Such $\bar{z}_\varepsilon$ exists because $P(z)$ is countably additive, and so $\lim_{z' \to \infty} P(z \leq z') = 1$.

For a given λ, if players follow a strategy profile σ^α, and a population size $n_0 + z$ is realized, the probability that A wins exceeds $(1-\lambda)^{n_0+z} \alpha^{n_0+z}$, which describes the probability that all players are uninformed and vote for A in a population $n_0 + z$ given the strategy profile σ^α. Hence, the ex-ante probability that A wins exceeds $(1 - \lambda)^{n_0+\bar{z}_\varepsilon} \alpha^{n_0+\bar{z}_\varepsilon} P(z \leq \bar{z}) \geq (1-\lambda)^{n_0+\bar{z}_\varepsilon} \alpha^{n_0+\bar{z}_\varepsilon} (1-\varepsilon)^{\frac{1}{3}}$.

Now let $\bar{\lambda}_\varepsilon = 1 - (1-\varepsilon)^{\frac{1}{3(n_0+\bar{z}_\varepsilon)}}$, and let $\bar{\alpha}_\varepsilon = (1-\varepsilon)^{\frac{1}{3(n_0+\bar{z}_\varepsilon)}}$. Then, $\bar{\lambda}_\varepsilon \in (0,1)$ and $\bar{\alpha}_\varepsilon \in (0,1)$. Moreover, if $\lambda < \bar{\lambda}_\varepsilon$ and $\alpha > \bar{\alpha}_\varepsilon$, then the probability that A wins when players follow the strategy profile σ^α exceeds $(1-\varepsilon)^{\frac{n_0+\bar{z}}{3(n_0+\bar{z})}} (1-\varepsilon)^{\frac{n_0+\bar{z}}{3(n_0+\bar{z})}} (1-\varepsilon)^{\frac{1}{3}} = (1-\varepsilon)$.

Finally, by Lemma 4, there exists $\bar{\lambda}'_\varepsilon \in (0, \frac{1}{2})$ such that, if $\lambda < \bar{\lambda}'_\varepsilon$ and $\alpha \leq \bar{\alpha}_\varepsilon$, then σ^α is not an equilibrium in the private information environment. Let $\lambda_\varepsilon = \min\{\bar{\lambda}'_\varepsilon, \bar{\lambda}_\varepsilon\}$. Then for all $\lambda < \lambda_\varepsilon$, σ^α is an equilibrium only if A wins with probability exceeding $(1-\varepsilon)$.

To complete the proof, it only remains to show that, when $\lambda < \lambda_\varepsilon$, then σ^0 is not an equilibrium, but this argument follows closely Theorem 1 (using the assumption that payoffs are not τ-negatively correlated to show that a lower bound on $\Pi(0)$ is strictly positive for sufficiently small λ).

A.4 Proof of Propositions

A.4.1 Proof of Proposition 4 on p. 17

Part (a): Since $P(W_i) = P'(W_i)$, $P \succeq P'$ if and only if $\frac{P(W_i|\tau-1)}{1-P(W_i|\tau-1)} \geq \frac{P'(W_i|\tau-1)}{1-P'(W_i|\tau-1)}$, from which the result directly follows.

Part (b): The result follows directly from the subsequent calculations:

$$\begin{aligned}
P(W_i|\tau-1) &= \sum_{\hat{\eta}=0}^{n} P(W_i|S_i = s^0, M = G = \tau-1, \hat{\eta})P(\hat{\eta}|S_i = s^0, M = G = \tau-1) \\
&= \sum_{\hat{\eta}=\tau-1}^{n} \left(\frac{\hat{\eta}-(\tau-1)}{n-(\tau-1)}\right) P(\hat{\eta}|M = G = \tau-1) \\
&= \left(\frac{1}{n-(\tau-1)}\right)\left(-(\tau-1) + \sum_{\hat{\eta}=\tau-1}^{n} \hat{\eta}P(\hat{\eta}|M = G = \tau-1)\right) \\
&= \left(\frac{1}{n-(\tau-1)}\right)\left(-(\tau-1) + E_P(\eta|M = G = \tau-1)\right),
\end{aligned}$$

where we can drop the conditioning on $S_i = s^0$ because of Assumptions 1 and 2.

Part (c): We use the following lemma:

Lemma 5. *For every $m \geq 1$, and vectors $a, q, r \in \mathbb{R}^m_+$ such that $a \cdot q \neq 0$, $q_1 \leq ... \leq q_m$, and $r_1 \leq ... \leq r_m$,*

$$\frac{\sum_{i=1}^m r_i a_i q_i}{\sum_{i=1}^m a_i q_i} \geq \frac{\sum_{i=1}^m r_i a_i}{\sum_{i=1}^m a_i}. \tag{14}$$

Proof. Because all the denominators are non-negative, (14) is equivalent to

$$\sum_{i,j=1}^m r_i q_i a_i a_j \geq \sum_{i,j=1}^m r_i q_j a_i a_j.$$

which is obtained by cross-multiplying and re-grouping terms. Each $a_i a_j$, which is non-negative, is multiplied by $r_i q_i + r_j q_j$ on the LHS and $r_i q_j + r_j q_i$ on the RHS. Therefore, this inequality is satisfied if for each i and j

$$r_i q_i + r_j q_j \geq r_i q_j + r_j q_i. \tag{15}$$

For $i \geq j$, (15) is equivalent to $(r_i - r_j)q_i \geq (r_i - r_j)q_j$, which is true since $r_i - r_j \geq 0$ and $q_i \geq q_j$. Therefore (14) is satisfied. $\square$

We use Lemma 5 to prove our result. To distinguish random variables from their realizations, we use η to denote the random variable representing the number of winners in each state, and $\hat{\eta}$ as a particular realization of η. Observe that

$$\begin{aligned}P(\hat{\eta}|M = G = \tau - 1) =& \frac{P(G = \tau - 1|\hat{\eta}, M = \tau - 1)P(\hat{\eta})}{\sum_{\tilde{\eta}=\tau-1}^{n} P(G = \tau - 1|\tilde{\eta}, M = \tau - 1)P(\tilde{\eta})} \\ =& \frac{\frac{\hat{\eta}!}{(\hat{\eta}-(\tau-1))!}P(\hat{\eta})}{\sum_{\tilde{\eta}=\tau-1}^{n} \frac{\tilde{\eta}!}{(\tilde{\eta}-(\tau-1))!}P(\tilde{\eta})},\end{aligned}$$

where we use Assumption 2(a) to derive that $P(\tilde{\eta}, M = \tau - 1|\tilde{\eta})$ is independent of $\tilde{\eta}$ and

$$P(G = \tau - 1|\tilde{\eta}, M = \tau - 1) = \frac{\binom{\tilde{\eta}}{\tau-1}\binom{n-\tilde{\eta}}{0}}{\binom{n}{\tau-1}} = \frac{\tilde{\eta}!}{(\tilde{\eta} - (\tau - 1))!}\frac{(n - (\tau - 1))!}{n!}.$$

So, by the second equation of the proof of Proposition 4(b), $P \succeq P'$ if and only if

$$\frac{\sum_{\hat{\eta}=\tau-1}^{n} (\hat{\eta} - (\tau - 1))\frac{\eta!}{(\eta-(\tau-1))!}P(\hat{\eta})}{\sum_{\hat{\eta}=\tau-1}^{n} \frac{\hat{\eta}!}{(\hat{\eta}-(\tau-1))!}P(\hat{\eta})} \geq \frac{\sum_{\hat{\eta}=\tau-1}^{n} (\hat{\eta} - (\tau - 1))\frac{\eta!}{(\eta-(\tau-1))!}P'(\hat{\eta})}{\sum_{\hat{\eta}=\tau-1}^{n} \frac{\hat{\eta}!}{(\hat{\eta}-(\tau-1))!}P'(\hat{\eta})}. \tag{16}$$

Consider a transformation of variables from $\hat{\eta}$ to i such that $i = \hat{\eta} - (\tau - 2)$. With this new index, define the vectors r, a, q such that $r_i = i - 1$, $a_i = \frac{(i+\tau-2)!}{(i-1)!}P'(i + \tau - 2)$ and $q_i = \frac{P(i+\tau-2)}{P'(i+\tau-2)}$. Inequality (16) is then re-written as

$$\frac{\sum_{i=1}^{n-(\tau-2)} r_i a_i q_i}{\sum_{i=1}^{n-(\tau-2)} a_i q_i} \geq \frac{\sum_{i=1}^{n-(\tau-2)} r_i a_i}{\sum_{i=1}^{n-(\tau-2)} a_i}.$$

Because r_i is non-decreasing in i and $a_i \geq 0$, it follows from Lemma 5 that the above inequality is satisfied if q_i is non-decreasing in i. Therefore, a sufficient condition for (16) is that for every $\hat{\eta} \geq \tau - 1$, $\frac{P(\hat{\eta})}{P'(\hat{\eta})}$ is non-decreasing in $\hat{\eta}$, generating Inequality (4).

A.4.2 Proof of Proposition 5 on p. 19

Signals convey no distributional information when for every signal profile $s \in \mathcal{S}$ and for every pair of players i and j, $V_i(s) = V_j(s)$. We proceed by showing that $V^G(\tau - 1) > 0$ in two steps.

Step 1: We show that $P(s_i \in \mathcal{G}, s_j \in \mathcal{B}) = 0$. Towards a contradiction, consider a non-null signal profile $s \in \mathcal{S}$ where $s_i \in \mathcal{G}$ and $s_j \in \mathcal{B}$. Then, $V_i(s_i) > 0$ and $V_j(s_j) < 0$, and by Assumption 2(b), the sign of $V_i(s)$ is that of $V_i(s_i)$ and the sign of $V_j(s)$ is that of $V_j(s_j)$. But since signals convey no distributional information, $V_i(s) = V_j(s)$, generating a contradiction.

Step 2: Consider the event $E \equiv \{s \in \mathcal{S} : s_i = s^0, M(s) = G(s) = \tau - 1\}$. Observe that $V^G(\tau - 1) = V_i(E) = \sum_{s \in E} V_i(s)P(s|E)$. Consider a particular $s \in E$. Because $s_i = s^0$, by Assumption 2(a), $V_i(s) = V_i(s_{-i})$. By Bayes Rule,

$$\begin{aligned} V_i(s_{-i}) &= (1-\lambda)V_i(s_{-i}) + \sum_{s' \in \mathcal{G}} V_i(s', s_{-i})P(s'|s_{-i}) + \sum_{s' \in \mathcal{B}} V_i(s', s_{-i})P(s'|s_{-i}) \\ &= (1-\lambda)V_i(s_{-i}) + \sum_{s' \in \mathcal{G}} V_i(s', s_{-i})P(s'|s_{-i}) \\ &= \frac{1}{\lambda} \sum_{s' \in \mathcal{G}} V_i(s', s_{-i})P(s'|s_{-i}), \end{aligned}$$

where the second equality follows from Step 1, and the third equality follows from rearranging terms. By definition of $\mathcal{G}$, $V_i(S_i = s') > 0$ for every $s' \in \mathcal{G}$, and therefore, it follows from Assumption 2(b) that $V_i(s', s_{-i}) > 0$. Therefore, the above expression confirms that for every $s \in E$, $V_i(s) > 0$, and therefore, $V^G(\tau - 1) = V_i(E) > 0$.

A.4.3 Proof of Proposition 6 on p. 19

We assume that signals convey no aggregate information (i.e., $P(\eta|s) = P(\eta)$ for all η and s) and a voter with an informative signal learns her priority. This implies that priorities (ρ) and the number of winners (η) must be independent and, by Assumption 1, that the prior distribution over priorities is uniform. As a result,

$$P(W_i) = P(\rho_i \le \eta) = \sum_{\hat{\rho}=1}^{n} P(\rho_i = \hat{\rho}, \eta \ge \rho) = \sum_{\hat{\rho}=1}^{n} P(\rho_i = \hat{\rho})P(\eta \ge \hat{\rho}) = \sum_{\hat{\rho}=1}^{n} \frac{1}{n} P(\eta \ge \hat{\rho}).$$

We want to show that $P(W_i|\tau - 1) < P(W_i)$, where $P(W_i|\tau - 1)$ is the probability that an uninformed voter i is a winner conditional on $M = G = \tau - 1$. First note that there is a priority $\rho^* \in \{0, ..., n\}$ such that $V_j(\rho_j) > 0$ if and only if $\rho_j \le \rho^*$ (i.e., all priorities less than or equal to ρ^* are good news, and higher priorities are bad news). Now consider an uniformed voter i in the pivotal event where $M = G = \tau - 1$ (for simplicity we denote this event by $\tau - 1$). Observe that

$$\begin{aligned}
P(W_i|\tau - 1) =& P(\rho_i \leq \eta|\tau - 1) = \sum_{\hat{\rho}=1}^{n} P(\rho_i = \hat{\rho}, \eta \geq \hat{\rho}|\tau - 1) \\
=& \sum_{\hat{\rho}=1}^{n} P(\rho_i = \hat{\rho}|\tau - 1)P(\eta \geq \hat{\rho}) \\
=& \sum_{\hat{\rho}=1}^{\rho^*} P(\rho_i = \hat{\rho}|\tau - 1)P(\eta \geq \hat{\rho}) + \sum_{\hat{\rho}=\rho^*+1}^{n} P(\rho_i = \hat{\rho}|\tau - 1)P(\eta \geq \hat{\rho}) \\
=& \sum_{\hat{\rho}=1}^{\rho^*} P(\rho_i \leq \rho^*|\tau - 1)P(\rho_i = \hat{\rho}|\rho_i \leq \rho^*, \tau - 1)P(\eta \geq \hat{\rho}) \\
&+ \sum_{\hat{\rho}=\rho^*+1}^{n} P(\rho_i > \rho^*|\tau - 1)P(\rho_i = \hat{\rho}|\rho_i > \rho^*, \tau - 1)P(\eta \geq \hat{\rho}),
\end{aligned}$$

where the first and second equalities follow by definition; the third equality follows because η is independent of ρ; the fourth equality follows by definition; and the fifth equality follows by Bayes rule. Then, because the marginal distribution over ρ is uniform,

$$\begin{aligned}
P(W_i|\tau - 1) =& \sum_{\hat{\rho}=1}^{\rho^*} \left(\frac{\rho^* - (\tau - 1)}{n - (\tau - 1)}\right)\left(\frac{1}{\rho^*}\right) P(\eta \geq \hat{\rho}) \\
&+ \sum_{\hat{\rho}=\rho^*+1}^{n} \left(\frac{n - \rho^*}{n - (\tau - 1)}\right)\left(\frac{1}{n - \rho^*}\right) P(\eta \geq \hat{\rho}),
\end{aligned}$$

As a result,

$$P(W_i|\tau - 1) - P(W_i) = \frac{(\tau - 1)(\rho^* - n)}{n\rho^*(n - \tau + 1)} \sum_{\hat{\rho}=1}^{\rho^*} P(\eta \geq \hat{\rho}) + \frac{(\tau - 1)}{n(n - \tau + 1)} \sum_{\hat{\rho}=\rho^*+1}^{n} P(\eta \geq \hat{\rho}).$$

The sign of the right hand side in the above equation is negative if and only if

$$\frac{(\rho^* - n)}{\rho^*} \sum_{\hat{\rho}=1}^{\rho^*} P(\eta \geq \hat{\rho}) + \sum_{\hat{\rho}=\rho^*+1}^{n} P(\eta \geq \hat{\rho}) < 0.$$

The preceding inequality is equivalent to $\frac{1}{\rho^*}\sum_{\hat{\rho}=1}^{\rho^*} P(\eta \geq \hat{\rho}) > \frac{1}{n}\sum_{\hat{\rho}=1}^{n} P(\eta \geq \hat{\rho})$, which, by further manipulation, is equivalent to $\frac{1}{\rho^*}\sum_{\hat{\rho}=1}^{\rho^*} P(\eta < \hat{\rho}) < \frac{1}{n}\sum_{\hat{\rho}=1}^{n} P(\eta < \hat{\rho})$. As $P(\eta < \hat{\rho}) \geq P(\eta < \rho^*)$ for all $\hat{\rho} > \rho^*$ (with a strict inequality for some $\hat{\rho}$), it follows that the right hand side of the above inequality is strictly greater than $\frac{1}{n}\sum_{\hat{\rho}=1}^{\rho^*} P(\eta <$

$\hat{\rho}) + \left(\frac{n-\rho^*}{n}\right) P(\eta < \rho^*)$. It is therefore sufficient to show that

$$\frac{1}{\rho^*}\sum_{\hat{\rho}=1}^{\rho^*} P(\eta < \hat{\rho}) \leq \frac{1}{n}\sum_{\hat{\rho}=1}^{\rho^*} P(\eta < \hat{\rho}) + \left(\frac{n-\rho^*}{n}\right) P(\eta < \rho^*)$$

$$\iff \left(\frac{n-\rho^*}{n\rho^*}\right)\sum_{\hat{\rho}=1}^{\rho^*} P(\eta < \hat{\rho}) \leq \left(\frac{n-\rho^*}{n}\right) P(\eta < \rho^*),$$

and, since $P(\eta < \hat{\rho}) \leq P(\eta < \rho^*)$ for all $\hat{\rho} = 1, ..., \rho^*$, the result follows.

A.4.4 Proof of Proposition 7 on p. 20

It is sufficient to show that for all $\bar{\alpha} \in [0, 1)$ there exists $\lambda_{\bar{\alpha}}$ such that for all $\lambda < \lambda_{\bar{\alpha}}$, σ^α is not an equilibrium when $\alpha < \bar{\alpha}$. Note that σ^α is an equilibrium if and only if

$$\Pi_i(\alpha) = \sum_{m=0}^{n-1}\sum_{g=0}^{m} \left[V^R(m,g)P^R(g|m)\,P(m)\right] P^R(piv|m,g) = 0,$$

where $V^R(m, g)$ (resp. $P^R(.)$) is understood as the conditional expected payoff (resp. probability) given that we have a full redistribution.

Observe that, with the redistribution players face a common value problem. As a result, $V^R(g, g)$ is strictly positive for all $g > 0$. Moreover, $V^R(0, 0)$ is strictly positive by the assumption that the alternative is ex ante optimal. Then, following similar arguments as in the proof of **??**, there exists $\lambda_{\bar{\alpha}}$ such that for $\lambda < \lambda_{\bar{\alpha}}$, σ^α is not an equilibrium for all $\alpha < \bar{\alpha}$. The remainder of the proof follows the arguments in the proof of Theorem 2.

A.4.5 Sketch of Proposition 8 on p. 25

For intuition, we offer a sketch for $\lambda \approx 0$. Proving the full result requires more details, accounting for off-path beliefs in the dynamic game, and uses arguments from Theorem 1 (see Supplementary Appendix for more details). Consider a strategy profile where:

- **t = 1**: every uninformed voter and informed winner votes for A, and only informed losers vote for Q;
- **t = 2**: if A was selected at $t = 1$, then each voter votes for her privately preferred outcome;
- **t = 2**: if Q was selected at $t = 1$, then each voter votes for her ex interim preferred choice.[26]

Consider the incentives of an uninformed voter i at $t = 1$. Being pivotal implies that $\frac{n-1}{2}$ other voters are voting for Q, all of which must be informed losers. In this contingency,

[26] In other words, for an uninformed voter, if Q has κ votes, then she votes for A if $V_i\left(S_i = s^0, M = B = \kappa\right) > 0$ and for Q otherwise.

for $\lambda \approx 0$, voter i's expected payoff from A is $V^B(n-\tau)$, which by negative correlation, is strictly positive. Thus, conditioning on being pivotal, her expected period 1 payoff from voting for A is strictly positive.

How about her period 2 payoff? In the event that policy reforms are reversed, then her period 2 payoff is 0, so she suffers no loss from A having been selected at $t = 1$. However, if A is selected again at $t = 2$, then voter i learns that there must be exactly $\frac{n+1}{2}$ winners at $t = 2$, which means she must be a winner (since she is pivotal at $t = 1$ only when there are $\frac{n-1}{2}$ informed losers other than her). Therefore, conditioning on being pivotal, her period 2 payoff from A is $P\left(\eta = \frac{n+1}{2}\right) v_w$, which is also strictly positive.

B Supplementary Appendix

B.1 Calculations for Example 1

It follows from direct calculation that

$$\begin{aligned}P(L_i|s_i=s^0,G=2)=&\sum_{m=2}^{4}P(L_i|s_i=s^0,g=2,M=m)P(M=m|s_i=s^0,g=2)\\=&\sum_{m=2}^{4}\left(\frac{4-m}{5-m}\right)\left(\frac{\frac{m(m-1)(5-m)}{20}\binom{4}{m}\lambda^m(1-\lambda)^{4-m}}{\frac{12}{20}\lambda^2\left(3(1-\lambda)^2+4\lambda(1-\lambda)+\lambda^2\right)}\right)\\=&\frac{2(1-\lambda)}{3-2\lambda}.\end{aligned}$$

B.2 A violation of single-crossing

All examples in the paper satisfy Assumption 6. The following is an example of an environment that satisfies Assumptions 1–3 but violates the single-crossing property.

Example 3. Let $(n,v_w,v_l)=(5,1,1)$, $P(\eta=2)=\gamma=\frac{3}{4}$, $P(\eta=5)=1-\gamma=\frac{1}{4}$, $\mathcal{M}=\{s^1,s^2,s^3,s^4\}$, and

$$P\left(s_i=s^1|\eta,\rho\right)=\begin{cases}\beta & \text{if } \eta=2\\ 0 & \text{otherwise}\end{cases},\quad P\left(s_i=s^3|\eta,\rho\right)=\begin{cases}1-\beta & \text{if } \rho_i\leq 3\\ 0 & \text{otherwise}\end{cases},$$

$$P\left(s_i=s^2|\eta,\rho\right)=\begin{cases}\beta & \text{if } \eta=5\\ 0 & \text{otherwise}\end{cases},\quad P\left(s_i=s^3|\eta,\rho\right)=\begin{cases}1-\beta & \text{if } \rho_i\geq 3\\ 0 & \text{otherwise}\end{cases},$$

where $\beta=\frac{1}{22}$, and $P\left(s_i\in\{s^1,s^2\},s_j\in\{s^3,s^4\}\right)=0$ for all $i\neq j$.

This example satisfies all our assumptions except the single-crossing condition. To note this single-crossing failure, observe that, $P\left(W_i\right)=\frac{2}{5}\gamma+1-\gamma=\frac{11}{20}$, $P\left(t_i=W|s^1\right)=\frac{2}{5}$, $P\left(t_i=W|s^2\right)=1$, $P\left(t_i=W|s^3\right)=\frac{2}{3}\gamma+1-\gamma=\frac{9}{12}$, and $P\left(t_i=W|s^4\right)=1-\gamma=\frac{1}{4}$. Hence, players vote for Q if they receive signals s^1 or s^4, and they vote for A if they receive signals s^2 or s^3. Then, $V^G(0)=V^G(\Omega)=\frac{1}{10}$, and $V^G(4)=2P\left(t_i=W|M=G=4,s_i=\emptyset\right)-1=1$.

Let E denote the event in which all informative signals are in $\{s^1, s^2\}$. Therefore,

$$
\begin{aligned}
V^G(3) &= 2P\left(t_i = W | M = G = 3, s_i = \emptyset\right) - 1 \\
&= 2P\left(t_i = W | M = G = 3, s_i = \emptyset, E\right) P\left(E | M = G = 3, s_i = \emptyset\right) \\
&\quad + 2P\left(t_i = W | M = G = 3, s_i = \emptyset, E^C\right) P\left(E^C | M = G = 3, s_i = \emptyset\right) - 1 \\
&= 2P\left(E | M = G = 3, s_i = \emptyset\right) + 2P\left(E^C | M = G = 3, s_i = \emptyset\right)(1-\gamma) - 1 \\
&= 1 - \frac{3}{2} P\left(E^C | M = G = 3, s_i = \emptyset\right) \\
&= 1 - \frac{3}{2} \frac{P\left(M = G = 3 | E^C, s_i = \emptyset\right) P\left(E^C | s_i = \emptyset\right)}{P\left(M = G = 3 | s_i = \emptyset\right)} \\
&\leq 1 - \frac{3}{2}\left(\frac{\frac{1-\beta}{10}}{\frac{1-\beta}{10} + \beta}\right) = -\frac{1}{62}
\end{aligned}
$$

Hence, our single-crossing property from Assumption 6 is not satisfied.

B.3 Abstention

In this section, we analyze the environment in which voters can abstain. Suppose that A is implemented if and only if a strict majority of the votes cast choose A. For simplicity, let n be an odd number.

Proposition 9. *In a private information environment with abstention, if payoffs are $\frac{n+1}{2}$-negatively correlated, then for every $\varepsilon > 0$, there exists $\tilde{\lambda}$ such that if $\lambda < \tilde{\lambda}$, Q wins with probability exceeding $1 - \varepsilon$ in a strict equilibrium of the private information environment.*

Proof. In the private information environment, suppose payoffs are $\left(\frac{n+1}{2}\right)$-negatively correlated. Consider the strategy profile where uninformed players vote for Q, and informed players vote according to their signals. Then, an uninformed voter i is pivotal if and only if $G = \frac{n-1}{2}$ and so the argument follows the proof of Theorem 1. □

B.4 Uniqueness of Equilibrium

Here, we offer further conditions under which the equilibrium described in Theorem 1 is unique within the class of strict (and hence pure-strategy) equilibria.

Proposition 10. *Suppose payoffs are τ-negatively correlated and $V^B(n-\tau) < 0$. Then for all $\varepsilon > 0$, there exists $\tilde{\lambda}$ such that when $\lambda < \tilde{\lambda}$, Q wins with probability greater than $1 - \varepsilon$ in the unique strict equilibrium of the private information environment.*

Proof. If σ is a strict equilibrium, it must be that case that $\sigma \in \{\sigma^0, \sigma^1\}$ (from the proof of Proposition 1).

First, consider the strategy profile σ^0. By Theorem 1, τ-negative correlation implies there exists $\tilde{\lambda}_1$ such that, for all $\lambda < \lambda_1$, σ^0 is a strict equilibrium.

Second, consider the strategy profile σ^1. Under the strategy profile σ^1, an uninformed voter i is pivotal if and only if $B = n - \tau$. Hence, σ^1 is an equilibrium if and only if

$$\sum_{m=n-\tau}^{n-1} V_i\left(s_i = s^0, M = m, B = n-\tau\right) P\left(B = n-\tau | M = m\right) P\left(M = m\right) \geq 0.$$

As a result, following similar arguments as in the proof of Theorem 1, σ^1 is not an equlibrium if

$$\left(\frac{\lambda}{1-\lambda}\right) \bar{v} \sum_{m=n-\tau+1}^{n-1} \binom{n-1}{m} < -V^B(n-\tau) P(n-\tau) \binom{n-1}{n-\tau}.$$

Since the RHS is strictly positive and does not depend on λ, there is a $\tilde{\lambda}_2$ such that, for all $\lambda < \tilde{\lambda}_2$, the strict inequality holds and σ^1 is not an equilibrium.

Now, fix $\varepsilon > 0$. There exists $\tilde{\lambda}_3$ such that for all $\lambda < \tilde{\lambda}_3$, Q is implemented with probability greater than $1 - \varepsilon$ in the strategy profile σ^0. As a result, the proof follows by letting $\tilde{\lambda} = \min\left\{\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\lambda}_3\right\}$. □

The following example illustrates that it is possible for payoffs to be τ-negatively correlated and $V^B(n-\tau) < 0$.

Example 4. Let $(n, \tau, v_w, v_l) = (11, 6, \frac{6}{5}, 1)$, $P(\eta = \hat{\eta}) = \frac{1}{12}$ for all $\hat{\eta} \in \{0, ..., 11\}$, $\mathcal{M} = \{s^1, s^2, s^3\}$, and

$$P\left(s_i = s^1 | \eta, \rho\right) = \begin{cases} 1 & \text{if } \eta = 0 \\ 0 & \text{otherwise} \end{cases},$$

$$P\left(s_i = s^2 | \eta, \rho\right) = \begin{cases} 1 & \text{if } \eta \geq 1 \text{ and } \rho_i \geq 8 \\ 0 & \text{otherwise} \end{cases},$$

$$P\left(s_i = s^3 | \eta, \rho\right) = \begin{cases} 1 & \text{if } \eta \geq 1 \text{ and } \rho_i \leq 7 \\ 0 & \text{otherwise} \end{cases}.$$

For any λ, these conditions define a unique distribution P that satisfies Assumptions 1–3. The *ex ante* best policy is A, payoffs are τ-negatively correlated, and $V^B(n-\tau) < 0$.

Therefore, when λ is sufficiently small, every strict equilibrium chooses Q with high probability.

B.5 Proof of Proposition 8 on p. 25

For every number $\kappa \in \{1, \ldots, n\}$, define the number $\bar{\kappa}(\kappa)$ to be the maximal element of $\{\kappa' \in \{0, \ldots, \kappa\} : P(B = \kappa') > 0\}$, which is non-empty because $P(B = 0) \geq (1 - \lambda)^n$.

Define the strategy profile $\tilde{\sigma}$, as follows. In period 1, uninformed players and informed winners vote for A, and informed losers vote for Q. In period 2, informed players vote for their privately preferred policy. Finally, for an uninformed voter i in period 2 when Q received $\kappa \geq \frac{n+1}{2}$ votes in period 1:

1. if i voted A in period 1, then she votes A in period 2 if and only if $V_i(S_i = s^0, B = \bar{\kappa}(\kappa)) > 0$, where $\bar{\kappa}(\kappa)$ is defined above.

2. if i voted Q in period 1, then she votes A in period 2 if and only if $V_i(S_i = s^0, B = \bar{\kappa}(\kappa - 1)) > 0$.

We show that strategy profile $\tilde{\sigma}$ constitutes a sequential equilibrium (Kreps and Wilson, 1982). In this setting, a system of beliefs must specify, for any voter, and conditional on any $\kappa \in \{0, ..., n\}$ votes for Q in period 1, the posterior of the voter over the number of informed winners and losers, and how other players voted. This system of beliefs is derived by Bayes Rule on every on-path history, and on every off-path history, we have the flexibility to assign beliefs that are consistent, i.e., the limits of beliefs generated by strict mixed strategies that converge to the equilibrium strategies. There are two forms of off-path histories, both of which involve other players voting for Q:

1. Suppose that player i voted for A at $t = 1$, observes κ votes for Q, and $P(B = \kappa) = 0$. In other words, she cannot attribute all of these votes for Q as emerging from losers, and the strategy profile we have described puts probability 0 on this history. In this contingency, she assigns probability 1 to event that the number of informed losers who voted for Q is $\bar{\kappa}(\kappa)$.

2. Suppose that player i voted for Q at $t = 1$, observes κ votes for Q, and $P(B = \kappa - 1) = 0$. In other words, she cannot attribute all of the other votes for Q as emerging from losers, and the strategy profile we have described puts probability 0 on this history. In this contingency, she assigns probability 1 to event that the number of informed losers who voted for Q is $\bar{\kappa}(\kappa - 1)$.

Notice that both beliefs emerge as the limits of beliefs from a perturbed model in which uninformed voters vote for Q with probability ε, taking $\varepsilon \to 0$, and therefore, these off-path beliefs are consistent.

We show that $\tilde{\sigma}$ is sequentially rational given the beliefs. Since all voters who learn whether they are winners or losers are voting for their privately preferred alternatives in both periods, we only need to consider voters who remain uninformed at period 2 (if Q was chosen in period 1) or those who are uninformed in period 1.

First, suppose that player i voted for A in period 1 and Q received $\kappa \geq \frac{n+1}{2}$ votes. The strategy-profile $\tilde{\sigma}$ prescribes i to vote for A in period 2 if and only if $V_i(s_i = s^0, B = \bar{\kappa}(\kappa)) > 0$, which is a best-response.

Second, suppose that player i voted for Q in period 1 and Q received $\kappa \geq \frac{n+1}{2}$ votes. The strategy-profile prescribes i to vote for A in period 2 if and only if $V_i(s_i = s^0, B = \bar{\kappa}(\kappa - 1)) > 0$, which is a best-response.

Finally, we consider the initial information set in period 1. Given the strategy profile $\tilde{\sigma}$, voter i is pivotal in period 1 in events where $M \geq \frac{n-1}{2}$ and $B = \frac{n-1}{2}$. Note that $V^B(n-\tau) = V^B(\frac{n-1}{2}) > 0$ implies that $P(B = \frac{n-1}{2}) \neq 0$. Moreover, for each of these events, if A is implemented in period 1, player i's beliefs assigns probability 1 to the event that A is also implemented in period 2 if and only if player i is a winner (since A is implemented in period 2 according to $\tilde{\sigma}$ if and only if $\eta \geq \frac{n+1}{2}$, and $B = \frac{n-1}{2}$). Hence, the difference between the expected payoff of voting for A and voting for Q in period 1 is

$$\sum_{m=\frac{n-1}{2}}^{n-1} V\left(m, \frac{n-1}{2}\right) P\left(B = \frac{n-1}{2}\Big| M = m\right) P(M = m) + P\left(\eta \geq \frac{n+1}{2}\right) v_w$$

$$= \sum_{m=\frac{n-1}{2}}^{n-1} V\left(m, \frac{n-1}{2}\right) P\left(B = \frac{n-1}{2}\Big| M = m\right) \left(\frac{\lambda}{1-\lambda}\right)^m (1-\lambda)^{n-1} + P\left(\eta \geq \frac{n+1}{2}\right) v_w$$

where $V(m, \frac{n-1}{2}) \equiv V_i(S_i = s^0, M = m, B = \frac{n-1}{2})$. The term $P\left(\eta \geq \frac{n+1}{2}\right) v_w$ represents the expected payoff in period 2 and it is strictly positive. The first sum is the payoff in period 1. In particular, the summand for $m = \frac{n-1}{2}$ is strictly positive because $V(\frac{n-1}{2}, \frac{n-1}{2}) = V^B(\frac{n-1}{2}) > 0$. Hence, following the same arguments as in the proof of Theorem 1, there exists a $\tilde{\lambda}_1$ such that, for all $\lambda < \tilde{\lambda}_1$, the payoff difference is strictly positive, and it is a best-response for player i to vote for A in period 1. As a result, for $\lambda < \tilde{\lambda}_1$, there is a PBE in which no voter plays a weakly dominated strategy.

To complete the proof, fix some small $\varepsilon > 0$. Let $\tilde{\lambda}_2 = 1-(1-\varepsilon)^{\frac{1}{n}}$. Now observe that, for any $\lambda < \min\{\tilde{\lambda}_1, \tilde{\lambda}_2\}$, we have the following: (i) the probability that no voter receives a signal in $\mathcal{M}$ in period 1 exceeds $1-\varepsilon$, (ii) in the private information environment, there is a PBE in which no voter plays a weakly dominated strategy; (iii) in the strategy profile $\tilde{\sigma}$, A is implemented in period 1 if no voter is informed, and is then reversed in period 2 if there are strictly less than $\frac{n+1}{2}$ winners; (iv) in the public information environment, if no voter receives a signal in $\mathcal{M}$, by (6) it is a dominant strategy for players to vote Q.

B.6 Combining distributional and aggregate information

Example 5. We consider a sequence of games indexed by n, where $n > 2$ is an odd number, and the voting rule τ is simple majority-rule. We set $v_l = v_w = 1$ and assume that the number of winners is independent of the priority rankings. The marginal distribution over the number of winners is

$$P(\eta) = \begin{cases} \frac{1-\gamma}{n+1} & \text{if } \eta \neq \frac{n+1}{2}, \\ \gamma + \frac{1-\gamma}{n+1} & \text{otherwise.} \end{cases}$$

When $\gamma > 0$, A is ex ante optimal.

A voter's signal is an element of $\{s^0, ...s^n\} \times \{s^B, s^G\}$, where the first component relates to the ranking ρ_i (distributional) and the second relates to the number of winners η (aggregate). All voters obtain aggregate information: conditional on η, each voter receives an independent signal governed by the distribution:

$$P\left(s_i = s^G | \eta\right) = \begin{cases} \frac{1}{2} + \delta & \text{if } \eta \geq \frac{n+1}{2} \\ \frac{1}{2} - \delta & \text{otherwise} \end{cases},$$

$$P\left(s_i = s^B | \eta\right) = \begin{cases} \frac{1}{2} - \delta & \text{if } \eta \geq \frac{n+1}{2} \\ \frac{1}{2} + \delta & \text{otherwise} \end{cases}.$$

Thus, s^G and s^B are good and bad news, respectively, about the number of winners.

Voters may or may not receive informative signals about their ranking: with probability λ, the first component for player i's signal is drawn from $\mathcal{M} = \{s^1, ...s^n\}$; if voter i obtains signal s^j, then she learns that her rank is j. With complementary probability, voter i obtains signal s^0 that is uninformative about her rank.

Different values of δ and λ relate this setting to prior environments: $\delta = 0$ and $\lambda > 0$ is a special case of Section 4.2, and $\delta > 0$ and $\lambda = 0$ is a common-values election in which voters are uncertain about the number of winners. The case of $\delta > 0$ and $\lambda > 0$ extends our previous model by allowing i.i.d. information about aggregate consequences. We show that the strategic logic of Theorem 1 nevertheless applies where the prospect of distributional information destroys that of successful information aggregation.

Proposition 11. *Let $0 < 2\delta < \gamma < 1$ and fix some $\varepsilon > 0$. There exists $\tilde{\lambda} > 0$ and $\tilde{N} \in \mathbb{N}$ such that, for all $n \geq \tilde{N}$:*

A) *If $\lambda = 0$, then in every strict equilibrium, A wins with probability exceeding $1-\varepsilon$ when there is a majority of winners, and with probability less than ε otherwise.*

B) *If $\lambda \in (0, \tilde{\lambda})$, then there is a strict equilibrium where Q wins with probability exceeding $1-\varepsilon$, regardless of the number of winners.*

The first part highlights how, with only aggregate information (i.e., $\lambda = 0$), A wins with high likelihood whenever a majority of voters are better off with A. Thus, with aggregate information, the outcome that benefits the majority of voters succeeds. The slight prospects for distributional information—when $\lambda > 0$—destroys this prospect even though each voter is better informed. Once voters obtain distributional information, each uninformed voter views with suspicion the motives of other voters; such suspicions would not emerge in the absence of distributional information.[27]

Proof. **Part A:** Let $\lambda = 0$ and consider the strategy profile where player i votes for the alternative if and only if she receives signal s^G. Then, voter i is pivotal if and only if $\frac{n-1}{2}$ other voters receive the signal s^B; denote this event piv. Then,

$$P\left(W_i | piv, s^B\right) = \sum_{\eta=0}^{n} P\left(W_i | piv, s^B, \eta\right) P\left(\eta | piv, s^B\right) = \sum_{\eta=0}^{n} \left(\frac{\eta}{n}\right) \frac{P\left(piv, s^B | \eta\right) P(\eta)}{P\left(piv, s^B\right)}$$

where

$$P\left(piv, s^B | \eta\right) = \begin{cases} \binom{n-1}{\frac{n-1}{2}} \left(\frac{1}{2}-\delta\right)^{\frac{n-1}{2}} \left(\frac{1}{2}+\delta\right)^{\frac{n-1}{2}} \left(\frac{1}{2}+\delta\right) & \text{if } \eta \leq \frac{n-1}{2} \\ \binom{n-1}{\frac{n-1}{2}} \left(\frac{1}{2}-\delta\right)^{\frac{n-1}{2}} \left(\frac{1}{2}+\delta\right)^{\frac{n-1}{2}} \left(\frac{1}{2}-\delta\right) & \text{if } \eta \geq \frac{n+1}{2} \end{cases},$$

and $P\left(piv, s^B\right) = \binom{n-1}{\frac{n-1}{2}} \left(\left(\frac{1}{2}-\delta\right)\left(\frac{1}{2}+\delta\right)\right)^{\frac{n-1}{2}} \left(\frac{1}{2}-\gamma\delta\right)$. Hence,

$$P\left(W_i | piv, s^B\right) = \left(\frac{1}{1-2\gamma\delta}\right)\left(\frac{1}{2n}\right)\left(n + \gamma - \delta - n\delta - \gamma\delta - n\gamma\delta\right),$$

which is strictly less than $1/2$ for n sufficiently large. Likewise, it can be shown that $P\left(W_i | piv, s^G\right) > \frac{1}{2}$ for all n. As a result, this strategy profile is a strict equilibrium for n sufficiently large. Moreover, for sufficiently large n, it follows from the Law of Large Numbers that A wins with probability greater than $1-\varepsilon$ when there is a majority of

[27] One feature of Proposition 11 is that this failure of information aggregation holds regardless of population size.

winners, and Q wins with probability greater than $1 - \varepsilon$ when there is a majority of losers.

There is only one other possible strategy profile that could be a strict equilibrium, where players with signals s^G vote for Q, and players with signals s^B vote for A. Then, voter i is pivotal in the same event as above and so our previous calculations show that, for n sufficiently large, this is not an equilibrium.

Part B: Let $0 < \lambda < \tilde{\lambda}$, and consider the following strategy profile: all voters ignore the second component of their signal; in terms of their first component, players vote for Q if and only if they receive signals s^j, where $j > \frac{n+1}{2}$ or $j = 0$. We first show that informed voters are playing a strict best response. In particular this follows because independence of ρ and η imply,

$$\begin{aligned} P\left(W_i \,\middle|\, s^B, \rho_i = \frac{n+1}{2}\right) &= \sum_{\eta=0}^{n} P\left(W_i \,\middle|\, s^B, \rho_i = \frac{n+1}{2}, \eta\right) P\left(\eta \,\middle|\, s^B, \rho_i = \frac{n+1}{2}\right) \\ &= \sum_{\eta=0}^{n} P\left(W_i \,\middle|\, \rho_i = \frac{n+1}{2}, \eta\right) P\left(\eta | s^B\right) = \sum_{\eta=\frac{n+1}{2}}^{n} P\left(\eta | s^B\right) \\ &= \left(\frac{\frac{1}{2} - \delta}{\frac{1}{2} - \gamma\delta}\right)\left(\frac{n\gamma + 2 + n - 1 + \gamma}{2(n+1)}\right), \end{aligned}$$

which is strictly greater than 1/2 for n sufficiently large if $\gamma > 2\delta$; and from

$$P\left(W_i \,\middle|\, s^G, \rho_i = \frac{n+3}{2}\right) = \sum_{\eta=\frac{n+3}{2}}^{n} P\left(\eta | s^G\right) = \left(\frac{n-1}{2}\right)\left(\frac{\left(\frac{1}{2} + \delta\right)\left(\frac{1-\gamma}{n+1}\right)}{\frac{1}{2} + \gamma\delta}\right)$$

which is strictly lower than 1/2 for n sufficiently large for $\gamma > 2\delta$.

It remains to show that uninformed voters are also playing a strict best response. Given the strategy profile, an uninformed voter is pivotal if and only if $\frac{n-1}{2}$ other voters receive signals informing them that their rank is weakly below $\frac{n+1}{2}$; denote this event by piv. We need to show that the expected payoff of voting for A is lower than the expected payoff of voting for Q for an uninformed agent with signal s^G. Thus, we need to show that,

$$\begin{aligned} \pi(A|s^G) - \pi(Q|s^G) &= P\left(W_i, piv \,\middle|\, s^G\right) - P\left(L_i, piv \,\middle|\, s^G\right) \\ &= \sum_{m=\frac{n-1}{2}}^{n-1} V\left(m, piv, s^G\right) P\left(piv \,\middle|\, s^G, m\right) P\left(m | s^G\right) < 0, \end{aligned}$$

where $V\left(m, piv, s^G\right) \equiv 2P\left(W_i | s^G, m, piv\right) - 1$. We will use a similar argument as in the proof of Theorem 1 to show that there exists a sufficiently small λ such that the negative summands in the above summation dominate all positive terms. Note that

$V\left(m, piv, s^G\right) < 0$ if and only if $P\left(W_i | s^G, m,\right) < 1/2$, and

$$
\begin{aligned}
P\left(W_i \,\middle|\, s^G, m, piv\right) &= \sum_{\eta=0}^{n} P\left(W_i \,\middle|\, s^G, m, piv, \eta\right) P\left(\eta | s^G\right) \\
&\leq \sum_{\eta=0}^{n} P\left(W_i \,\middle|\, s^G, m, piv, \eta, first\right) P\left(\eta | s^G\right) \\
&= \frac{1}{n-m} + \left(\frac{1}{4}\right)\left(\frac{\left(\frac{1}{2}-\delta\right)(1-\gamma)}{\left(\frac{1}{2}-\gamma\delta\right)}\right),
\end{aligned}
$$

where "$first$" is the event in which no voter received a signal indicating she has the first priority in the ranking,

$$
P\left(W_i \,\middle|\, s^G, m, piv, \eta, first\right) = \begin{cases} \frac{1}{n-m} & \text{if } \eta \leq \frac{n+1}{2} \\ \frac{1}{n-m} + \frac{\eta - \frac{n+1}{2}}{\frac{n-1}{2}} & \text{if } \eta > \frac{n+1}{2}, \end{cases}
$$

and

$$
P\left(\eta | s^G\right) = \frac{P\left(s^G | \eta\right) P(\eta)}{P\left(s^G\right)} = \begin{cases} \frac{\left(\frac{1}{2}-\delta\right)\left(\frac{1-\gamma}{n+1}\right)}{\frac{1}{2}+\gamma\delta} & \text{if } \eta \leq \frac{n-1}{2} \\ \frac{\left(\frac{1}{2}+\delta\right)\left(\gamma+\frac{1-\gamma}{n+1}\right)}{\frac{1}{2}+\gamma\delta} & \text{if } \eta = \frac{n+1}{2} \\ \frac{\left(\frac{1}{2}+\delta\right)\left(\frac{1-\gamma}{n+1}\right)}{\frac{1}{2}+\gamma\delta} & \text{if } \eta \geq \frac{n+3}{2}. \end{cases}
$$

Therefore, voting Q is a strict best response for player i if and only if

$$
\sum_{m=\frac{n-1}{2}}^{n-1} \binom{n-1}{m} \lambda^m (1-\lambda)^{n-1-m} P\left(piv \,\middle|\, m\right) V\left(m, \frac{n-1}{2}, s^G\right) < 0. \tag{17}
$$

Assume that $n > 10$; we show that the inequality (17) is satisfied when $\lambda < 1/768$. Since $V\left(m, \frac{n-1}{2}, s^G\right) < 0$ for $n - m > 4$, when $\lambda < 1/2$ it is sufficient to show

$$
\sum_{m=\frac{n-1}{2}}^{n-5} -V\left(m, piv, s^G\right) P\left(piv \,\middle|\, m\right)\binom{n-1}{m} > \left(\frac{\lambda}{1-\lambda}\right) \sum_{m=n-4}^{n-1} V\left(m, piv, s^G\right) P\left(piv \,\middle|\, m\right)\binom{n-1}{m}.
$$

Since $P(piv|m) = \frac{\binom{\frac{n+1}{2}}{\frac{n-1}{2}}\binom{\frac{n-1}{2}}{m-\frac{n-1}{2}}}{\binom{n}{m}}$, and $1 \geq V\left(m, \frac{n-1}{2}, s^G\right)$, it is sufficient to show

$$
\begin{aligned}
&\sum_{m=\frac{n-1}{2}}^{n-5} -V\left(m, piv, s^G\right)\left(\frac{(n+1)\left(\frac{n-1}{2}\right)!}{n}\right)\left(\frac{(n-m)}{(n-1-m)!\left(m-\frac{n-1}{2}\right)!}\right) \\
&> \left(\frac{\lambda}{1-\lambda}\right) \sum_{m=n-4}^{n-1} v_w \left(\frac{(n+1)\left(\frac{n-1}{2}\right)!}{n}\right)\left(\frac{(n-m)}{(n-1-m)!\left(m-\frac{n-1}{2}\right)!}\right).
\end{aligned}
$$

in each summand on the LHS, $-V\left(m, \frac{n-1}{2}, s^G\right) \leq V\left(n-5, \frac{n-1}{2}, s^G\right)$, $(n-m) \geq 5$,

$(n-1-m) \geq 4$, and $m - \frac{n-1}{2} \leq n - 4 - \frac{n-1}{2}$. In each summand on the RHS, $4 \geq n-m$, $(n-1-m)! \geq 1$, and $m - \frac{n-1}{2} \geq n - 4 - \frac{n-1}{2}$. Hence, it is sufficient to show that

$$-V\left(n-5, \frac{n-1}{2}, s^G\right)\left(\frac{5}{4!\left(n-4-\frac{n-1}{2}\right)!}\right) > \left(\frac{\lambda}{1-\lambda}\right)\sum_{m=n-4}^{n-1}\left(\frac{4}{\left(n-4-\frac{n-1}{2}\right)!}\right),$$

which holds if

$$-V\left(n-5, \frac{n-1}{2}, s^G\right) > \frac{384}{5}\left(\frac{\lambda}{1-\lambda}\right).$$

This inequality is satisfied for $\lambda < \frac{1}{768}$ since $-V\left(n-5, \frac{n-1}{2}, s^G\right) = \frac{6}{10} - \frac{\left(\frac{1}{2}-\delta\right)(1-\gamma)}{2\left(\frac{1}{2}-\gamma\delta\right)}$.

Moreover it is also a strict best response to vote for Q when voter i receives the signal s^B. As a result, the strategy profile is a strict equilibrium and, for λ sufficiently small, Q wins with probability greater than $1-\varepsilon$. □

B.7 Polarization of Information: An Amplification

Consider a stylized extension of Section 4.2, relaxing Assumption 2(a) (whereby an uninformed voter learns nothing from being uninformed). Instead, we now assume that any voter who is informed is someone who anticipates gaining from trade reforms, A (an "elite" voter) and therefore, any voter who is uninformed is more likely to be a loser from trade liberalization. Formally, each voter's signal is an element of $\{s^0, s^1\}$ where (i) conditioning on the event that voter i is a loser (L_i), $P(s_i = s^0 | L_i) = 1$, and (ii) conditioning on being a winner (W_i), voter i obtains signal s^1 with probability $\lambda \in (0,1)$ and signal s^0 otherwise. Thus, obtaining signal s^1 confirms that one is a winner, whereas obtaining the "no information" signal s^0 depresses one's beliefs of being a winner. As before, we assume that the prior probability of being a winner is sufficiently high that A is ex ante optimal (as described by inequality (1) on p. 15).

The condition for payoffs being τ-negatively correlated remains as in inequality (2) (on p. 16). However, the probability of being a winner conditional on being pivotal and signal s^0 now depends on λ. In this context, we show that information has an adverse effect on electoral failures: the more likely it is that winners are informed, the more an uninformed voter has to gain from voting against reforms.

Proposition 12. *We establish three facts about how the polarization of information amplifies electoral failures.*

a) *There exists a strict symmetric equilibrium in which all voters who obtain signal s^0 vote for Q if and only if payoffs are τ-negatively correlated.*

b) *If payoffs are τ-negatively correlated for λ, then payoffs are τ-negatively correlated for all $\lambda' \geq \lambda$.*

c) *Payoffs are τ-negatively correlated for every $\lambda \in (0,1)$ if*

$$\frac{\sum_{\eta=\tau-1,\ldots,n}(\eta-(\tau-1))\binom{\eta}{\tau-1}P(\eta)}{\sum_{\eta=\tau-1,\ldots,n}(n-\eta)\binom{\eta}{\tau-1}P(\eta)} < \frac{v_l}{v_w}. \tag{18}$$

Once information may be polarized, then perverse electoral outcomes can occur even if information isn't scarce; the fact that information is being released only to subsets of the population that gain from policy reforms cements the behavior of other voters. That information exacerbates polarization implies that a sufficient condition for payoffs to be τ-negatively correlated for every λ is for it to be true when $\lambda \approx 0$, a property used in (c).

Proof. We proceed in the order of results.
(a) Consider a strategy profile in which any voter with a signal s^1 votes for A and any voter with a signal s^0 votes for Q. Voting for A with signal s^1 is a strict best-response. With a signal s^0, the payoff difference between voting for Q and A is $P(G=\tau-1)V^G(\tau-1)$, which is strictly negative when payoffs are τ-negatively correlated (by (2)).

(b) We begin by re-writing the probability of being a winner, conditional on being pivotal and uninformed. To make it clear as to which environment we are referring to, we write $P^{\tilde{\lambda}}(W_i|\tau-1)$ to represent $P(W_i|\tau-1)$ when the probability of receiving signal s^1 conditional on being a winner is $\tilde{\lambda}$. Observe that:

$$\begin{aligned}
P^{\lambda}(W_i|\tau-1) &= \sum_{\eta=0}^{n} P(W_i|S_i=s^0, G=\tau-1,\eta)P(\eta|S_i=s^0,G=\tau-1) \\
&= \sum_{\eta=\tau-1}^{n} \frac{\eta-(\tau-1)}{n-(\tau-1)} P(\eta|S_i=s^0,G=\tau-1) \\
&= \sum_{\eta=\tau-1}^{n} \frac{\eta-(\tau-1)}{n-(\tau-1)} P(\eta|G=\tau-1) \\
&= \left(\frac{1}{n-(\tau-1)}\right) \frac{\sum_{\eta=\tau-1}^{n}(\eta-(\tau-1))P(\eta)\binom{\eta}{\tau-1}\lambda^{\tau-1}(1-\lambda)^{\eta-(\tau-1)}}{\sum_{\eta=\tau-1}^{n} P(\eta)\binom{\eta}{\tau-1}\lambda^{\tau-1}(1-\lambda)^{\eta-(\tau-1)}} \\
&= \left(\frac{1}{n-(\tau-1)}\right) \frac{\sum_{\eta=\tau-1}^{n}(\eta-(\tau-1))P(\eta)\binom{\eta}{\tau-1}(1-\lambda)^{\eta-(\tau-1)}}{\sum_{\eta=\tau-1}^{n} P(\eta)\binom{\eta}{\tau-1}(1-\lambda)^{\eta-(\tau-1)}}
\end{aligned}$$

where the first equality is by definition and Bayes Rule, the second equality uses the exchangeability of voters, the third equality uses the exchangeability of voters to highlight that $S_i = s^0$ is redundant information given $G=\tau-1$, the fourth equality uses Bayes

Rule, and the fifth equality cancels $\lambda^{\tau-1}$ from the numerator and denominator.

Now consider a fixed $\lambda \in (0,1)$ and $\lambda' > \lambda$. Observe that $P^{\lambda}(W_i|\tau-1) \geq P^{\lambda'}(W_i|\tau-1)$ is equivalent to

$$\frac{\sum_{\eta=\tau-1}^{n}(\eta-\tau+1)P(\eta)\binom{\eta}{\tau-1}(1-\lambda)^{\eta-\tau+1}}{\sum_{\eta=\tau-1}^{n} P(\eta)\binom{\eta}{\tau-1}(1-\lambda)^{\eta-\tau+1}} \geq \frac{\sum_{\eta=\tau-1}^{n}(\eta-\tau+1)P(\eta)\binom{\eta}{\tau-1}(1-\lambda')^{\eta-\tau+1}}{\sum_{\eta=\tau-1}^{n} P(\eta)\binom{\eta}{\tau-1}(1-\lambda')^{\eta-\tau+1}}.$$

We prove the claim using Lemma 5. Let $\zeta = \frac{1-\lambda}{1-\lambda'} > 1$, and re-index by $i = \eta - (\tau-2)$. Define the vectors $r_i = i-1$, $a_i = P(i+\tau-2)\binom{i+\tau-2}{\tau-1}(1-\lambda')^{i-1}$, and $q_i = \zeta^{i-1}$. Then observe that the above inequality is equivalent to

$$\frac{\sum_{i=1}^{n-(\tau-2)} r_i a_i q_i}{\sum_{i=1}^{n-(\tau-2)} a_i q_i} \geq \frac{\sum_{i=1}^{n-(\tau-2)} r_i a_i}{\sum_{i=1}^{n-(\tau-2)} a_i},$$

which, by Lemma 5, is satisfied since both q and r are increasing in their index.

Finally, we note that payoffs are τ-negatively correlated for λ implies that $\frac{P^{\lambda}(W_i|\tau-1)}{1-P^{\lambda}(W_i|\tau-1)} < \frac{v_l}{v_w}$, which implies that $\frac{P^{\lambda'}(W_i|\tau-1)}{1-P^{\lambda'}(W_i|\tau-1)} < \frac{v_l}{v_w}$ because $P^{\lambda}(W_i|\tau-1) \geq P^{\lambda'}(W_i|\tau-1)$.

(c) We consider $\lim_{\lambda\to 0} \frac{P^{\lambda}(W_i|\tau-1)}{P^{\lambda}(L_i|\tau-1)}$. Observe that

$$\frac{P^{\lambda}(W_i|\tau-1)}{P^{\lambda}(L_i|\tau-1)} = \frac{\sum_{\eta=\tau-1}^{n}(\eta-(\tau-1))P(\eta)\binom{\eta}{\tau-1}(1-\lambda)^{\eta-(\tau-1)}}{\sum_{\eta=\tau-1}^{n}(n-\eta)P(\eta)\binom{\eta}{\tau-1}(1-\lambda)^{\eta-(\tau-1)}},$$

where we use similar steps to calculate the denominator as we did for the numerator. Taking the limit as $\lambda \to 0$ generates the term in (18), and using (b), it follows that payoffs are τ-negatively correlated for every $\lambda \in (0,1)$. □