Reputation Signals and Market Outcomes

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The importance of reputation signals in markets where product quality is imperfectly observed has been long emphasized. In particular, this is a key consideration for the overall performance of trading platforms and online markets that are becoming increasingly important mechanisms for trade. We consider here reputation signals as imperfect aggregators of trade histories that are correlated with firm quality. This is the case, for example, of quality badges given by some trading platforms that partition sellers into a small number (in many cases two) of groups. This paper considers the impact of such reputation mechanisms on market outcomes (e.g. prices and market shares), the impact of information quality and the question of design of optimal partitions.

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Introduction

The importance of reputation signals in markets where product quality is imperfectly observed has been long emphasized. In particular, this is a key consideration for the overall performance of trading platforms and online markets that are becoming increasingly important mechanisms for trade. We consider here reputation signals as imperfect aggregators of trade histories that are correlated with firm quality. This is the case, for example, of quality badges given by some trading platforms that partition sellers into a small number (in many cases two) of groups. This paper considers the impact of such reputation mechanisms on market outcomes (e.g. prices and market shares), the impact of information quality and the question of design of optimal partitions.

More specifically, we consider a market where suppliers (e.g. retailers in eBay) differ in terms of quality but where consumers observe (coarse) imperfect signals (that are imperfectly correlated with quality.) Such is the case, for example, of reputation signals in many online markets. Sellers are heterogeneous in terms of quality and face costly entry and supply decisions. Consumers differ in preference for quality and the value of outside options. We solve for the Competitive and Cournot equilibria for a given partition of quality signals, determining prices and market shares of different quality segments as well as total welfare. We then consider the impact of changes in reputation signals on prices and market shares and solve for the optimal information partition. We also examine the impact of improvements in information quality on prices and market shares of firms as a function of quality signals.

Reputation signals partition the set of sellers into a discrete set of categories in most cases as a result of some "testing" criteria. For example, in California restaurants are given grades A,B,C or none based on the score obtained in a health evaluation. eBay's high-quality sellers are classified as Top Rated Sellers, Airbnb calls its top quality host a "superhost" badge. From the economic point of view, it is important to distinguish both the size of the partition (i.e. the relative shares of sellers in each group) and their informativeness (the respective conditional average qualities). While raising the bar to qualify as a top seller might contribute to a more selected group, this can come at the cost of reducing its size.

The economic impact of this change cannot be assessed without considering the distribution of preference for quality of consumers and the supply elasticity of sellers. On the demand side, we consider preferences given by random coefficients on the strength of preference for quality and an outside good. In particular, our model nests two extreme cases widely used in the literature: 1) the case where quality is valued equally by all agents but there is heterogeneity in inside/outside options; 2) the case of pure vertical differentiation. On the supply side we consider two alternative market structures, perfect competition and Cournot competition. In the first case, behavior is captured by individual supply functions. For the latter, we assume marginal cost is constant.

Our first results concern a reputation system given by a threshold of quality z^* that partitions perfectly firms in two sets, with qualities below and above this threshold. It is to note that as z^* increases, there are several channels that can impact demand and supply.

1. All things equal, the share and number of firms in the low quality group (high quality group) $F(z^*)$ increases (decreases).

2. Expected quality increases in both partitions.

The impact of the latter effect is ambiguous: depending of the relative changes in expected qualities and how these interact with preferences, it might be that goods in different partitions become more or less substitutable. This makes it hard in general to determine the impact of the change in z^* on equilibrium prices and market shares without of further conditions. While we show that the price of one of the groups must rise, we have examples showing that either of the two prices could decrease. When the gap between average qualities increases, the price of high quality goods is guarantee to rise. More generally, results depend on the joint distribution of the two random coefficients, the distribution of firm qualities and supply functions. We provide conditions under which one or the other or both prices rise. These are conditions that can be easily verified in applications.

Our second set of results concern the design of an optimal partition, restricting attention to the case of constant quality premium (i.e., where all agents have the same preference for quality). These results apply generally to the Cournot model and also to the case of perfect competition with linear supply. We first show that total output is independent of the partition and so is consumer surplus. Hence the optimal partition is the also the one that maximizes firm profits. This in turn is the one that maximizes the average of firm output squared. As an example, for the case where the distribution of qualities is uniform, the optimal partition divides the set of qualities into intervals of equal size.

Our final set of results concerns the impact of improved information. We model this as a mean preserving spread average qualities across partitions, while keeping constant the size of these partitions. This could be the result of a more effective record of sellers that results in less classification errors. We first show that better information always results in an increase in the price of the high quality good. For the two special cases of additive quality premium or pure vertical differentiation, we also show that the price of the low quality good decreases. But while intuition suggests that this should always be the case, we provide an example to the contrary where this price rises.

Section 1 describes the general model. Section 2 considers the case of perfect competition. Section 3 the case of Cournot competition. Section 4 considers the question of optimal partition (quality reputation signals). Finally Section 5 considers the impact of improved information.

1 General Model

Firms differ along two dimensions, quality z and fixed costs f. Production technology is the same for all firms, given by a strictly increasing supply function $q(p)$, the corresponding variable cost $c(q)$.

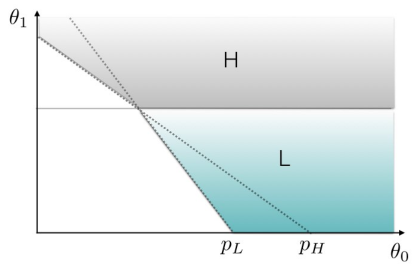
We model the demand side as in a discrete choice model. Letting z denote the quality of the good, the agent's utility is given by

$$U(z, \theta) = \theta_1 z + \theta_0$$

where the outside good's utility (no purchase) is normalized to zero.[1] θ_1 and θ_0 are random coefficients that are jointly distributed $\Gamma(d\theta_0, d\theta_1)$. Goods are only differentiated by quality, so this is a pure vertical differentiation case. Given the linearity in z, we maintain similar expression for lotteries over qualities where z is then interpreted as the corresponding mean. This set up is similar to random coefficient demand function, with two possibly

[1] Alternatively, letting θ_{max} denote the highest θ_0, we can write $U(z, \theta) = \theta_1 z + \theta_{max}$ where the value of the outside good $\tilde{\theta}_0 = \theta_{max} - \theta_0$.

Figure 1: Demand



correlated random coefficients.[2]

All consumers observe a coarse signal related to quality, which we call the reputation signal. We assume this signal has the form of a partition of the set of qualities Z into a finite number of intervals determined by thresholds $z_1 < ... < z_n$. For most of our analysis below we consider the case of a single quality threshold z^*. Let p_H and p_L denote the price for firms above and below the threshold, respectively, and z_H and z_L denote the average quality for firms above and below the threshold, respectively.

As depicted in Figure 1, given prices p_H and p_L, the set of buyers in each group are

$$A_H(z,p) = \{\theta | \theta_1 z_H - p_H \geq \theta_1 z_L - p_L; \theta_1 z_H \geq p_H\}$$

$$A_L(z,p) = \{\theta | \theta_1 z_H - p_H < \theta_1 z_L - p_L; \theta_1 z_L \geq p_L\}$$

and the corresponding demand functions:

$$\begin{aligned} D_H(z,p) &= \Gamma(A_H(z,p)) \\ D_L(z,p) &= \Gamma(A_L(z,p)) \end{aligned}$$

where Γ denotes the probability measure over θ.[3]

To study the effect of changes in z^* on prices, quantity of high- and low-quality sellers, and entry, we consider competitive and Cournot equilibrium separately. Furthermore, the following special cases for the taste preference on quality that we consider in more detail below are:

[2] We are working on the addition of horizontal differentiation that should be included in the final version.

[3] Without loss of generality we normalize the size of the population to one.

1. Additive quality premium: θ_1 the same for all and θ_0 following some distribution $\Phi(\theta_0)$

2. Increasing quality premium: Increasing function $\theta_1(\theta_0)$ and some distribution

3. Pure vertical differentiation: θ_0 fixed and θ_1 following some distribution $\Phi(\theta_1)$

2 Perfect Competition

On the supply side, potential participants differ along two dimensions: quality z and fixed costs f that are independently distributed with cumulative distribution functions $F(z)$ and $G(f)$, respectively. Let $\pi(p)$ denote variable profit of the firms with when faced by price p. Here, we consider the case of a single quality threshold z^*. Letting p_H and p_L denote the price for firms above and below the threshold, respectively, we can define total supply for each group as follows:

$$\begin{aligned} Q_H &= (1 - F(z^*))\, q(p_H)\, n(p_H) \\ Q_L &= F(z^*)\, q(p_L)\, n(p_L) \end{aligned}$$

where $q(\cdot)$ is the supply function of an individual firm and $n(p) = G(\pi(p))$ is the fraction of firms that participate when confronted with price p, i.e. those with fixed cost $f \leq \pi(p)$. Note that for simplicity we have assumed the production technology is independent of quality. This implies that for a given group, the share of output of all firms is the same and thus average quality is independent of price.

Letting $S(p) = q(p)\, n(p)$ we can write[4]

$$\begin{aligned} Q_H(p_H) &= (1 - F(z^*))\, S(p_H) \\ Q_L(p_L) &= F(z^*)\, S(p_L)\,. \end{aligned}$$

A competitive equilibrium for threshold z^* is a vector of prices $p = (p_L, p_H)$ such that $Q_H(p_H) = D_H(z^*, p)$ and $Q_L(p_L) = D_L(z^*, p)$.

[4]Note that $S(p)$ contains all the supply-behavior information needed to solve for an equilibrium and can be thus taken as a primitive. In the particular structure given above it corresponds to the envelope of supply functions of firms with different fixed costs. The general underlying assumption as explained above is that the supply function is the same for firms in all quality partitions, scalarly adjusting for the mass of firms in each partition.

Proposition 1. *There exists a unique equilibrium.*

Proof. (Existence can be easily proved from fixed point theorems imposing continuity conditions.) Let (p_L^1, p_L^2) be an equilibrium price vector and suppose towards a contradiction that there is another equilibrium vector (p_L^2, p_H^2). Suppose first that

$$p_H^2 - p_L^2 \geq p_H^1 - p_L^1. \tag{1}$$

Then it must be the case that $p_H^2 \leq p_H^1$ for otherwise supply cannot equal demand in the H market under both equilibrium prices. But then it follows from (1) that $p_L^2 \leq p_L^1$. Moreover, if the inequality were strict demand will exceed suply in the L market under this alternative set of prices. It follows that $p_L^2 = p_L^1$ and then immediately that the same is true in the H market. In the converse case where $p_H^2 - p_L^2 \leq p_H^1 - p_L^1$, a similar argument can be made. □

As consumers strictly prefer high quality goods, it follows that $p_L < p_H$ and thus $q_L < q_H$. Total market shares of both groups depend on the number of firms in each quality partition that obviously varies with z^*.

2.1 Comparative Statics

This section considers the effect of changes in the quality threshold z^* on market prices and total quantities. It is to note that as z^* increases, there are several channels that can impact demand and supply.

1. All things equal, the share and number of firms in the low quality group (high quality group) $F(z^*)$ increases (decreases).

2. Expected quality increases in both partitions.

The impact of the latter effect is ambiguous: depending of the relative changes in expected qualities and how these interact with preferences, it might be that goods in different partitions become more or less substitutable. This makes it hard in general to determine the impact of the change in z^* on equilibrium prices and market shares without of further conditions. This is explored in this section and the following ones.

Proposition 2. *If z^* increases at least one of the prices p_H or p_L must increase.*

Proof. Suppose not. Then total supply cannot increase (and it will strictly decrease for those that switch from H to L) . But if neither price increases, total demand must increase since average quality is greater for both goods. So there must be excess supply for at least one of the groups.

The following corollary provides sufficient conditions for a rise in p_H. □

Corollary 1. *If z^* increases and $z_H - z_L$ increases too, then p_H must increase.*

Proof. Suppose not. Then p_L must increase. But then any consumer that chose H initially must continue doing so, since by the hypothesis $\theta_1 (z'_H - z'_L) \geq \theta_1 (z_H - z_L) \geq (p_H - p_L) \geq (p'_H - p'_L)$. Hence total demand for the H goods increases. However supply strictly decreases as $p'_H \leq p_H$ and there are less firms in the H group. □

While at least one price must increase, it is possible that either of the two prices decrease. Examples are provided in the following section.

Corollary 2. *If p_L increases, Q_L must increase too. If p_H decreases, Q_H must decrease too.*

Consider now a uniform increase in entry cost. With the assumption that the distribution of fixed costs and quality is independent, this will not change the average qualities of both groups. In consequence:

Proposition 3. *An increase in fixed/entry costs increases both prices and reduces Q_L and Q_H. The impact on average quality in the market is ambiguous.*

2.2 Additive Quality Premium

Now to study the effect of the change in information structure in more detail, we assume that buyers preference is in the form of additive quality premium: θ_1 the same for all and θ_0 following some distribution $\Phi(\theta_0)$. We conjecture that for most results the assumption can be weakened to lack of correlation between the two coefficients (this is underway and will be discussed in the final paper.)

The demand side can be thus expressed as follows. There is a baseline demand function $P(Q)$ and an additive quality offset $\bar{z}$ for a good of expected quality $\bar{z}$ so that if total quantity in the market of all goods is Q, the demand price for this good is $P(Q) + \bar{z}$. [5]

For example, in case where individual consumers buy or not a single unit, this assumption implies that there is no correlation between the basic willingness to pay for a good of normalized quality zero and the quality premium: aggregate demand shifts in an upwards parallel way with quality.

A simple extension to Corollary 1 can be proven here.

Corollary 3. *Suppose $z_H(z^*) - z_L(z^*)$ is increasing (resp. decreasing) in z^*. Then an increase in z^* will result in an increase in p_H (resp. p_L).*

Proof. If $z_H - z_L$ increases with z^*, then if p_L increases p_H must also increase. If p_L decreases, then p_H must increase by the previous proposition. Similar proof applies to the converse case. $\square$

It follows immediately that when $z_H - z_L$ does not change with z^*, both prices must increase. This is the case when z is uniformly distributed.

More generally, without making further assumption one cannot rule that either price decreases. Our first example gives conditions under which p_L decreases.

Example 1. Suppose that there are only 3 types of firms with qualities $z_1 < z_2 < z_3$. Under the original threshold z^* the partitions are $\{z_1\}, \{z_2, z_3\}$ while under z_2^* it is $\{z_1, z_2\}, \{z_3\}$. Suppose that $z_2 \approx z_1$ so that the average quality of the L group remains basically unchanged. Assume that $S(p)$ is strictly convex. Suppose towards a contradiction that total output remain unchanged (or where to decrease), then this assumption implies that the sum of the output of firms in groups 2 and 3 would rise and as the average quality of the low group would remain unchanged, the output of firms in group 1 would remain the same or increase. But then total output increases. This proves that total output must rise and consequently p_L decreases.

Our second example shows that p_H can decrease when z^* rises.

Example 2. Consider a similar setting as above, but now assume that $z_2 \approx z_3$ and that $S(p)$ is strictly concave. The first assumption implies

[5] One can replace z with an increasing function $\Delta(z)$ which denotes the quality premiums rather than quality levels, all the proofs will go through and the only thing that needs to be changing is the distribution of qualities with that distribution transformed by this function.

that the quality of the H group remains unchanged when z^* rises. Now assume towards a contradiction that Q remains unchanged or decreases. By strict concavity of $S(p)$ it follows that the total output of firms of type 1 and 2 rises after the increase in z^*. So if p^H were to increase, total output would increase. This proves that total output must rise and consequently p_H decreases.

The following Proposition gives alternate sufficient conditions for either of the prices to rise monotonically with z^*.

Proposition 4. *1) If supply function $S(p)$ is concave then an increase in z^* will result in an increase in p_L. 2) If supply function $S(p)$ is convex then an increase in z^* will result in an increase in p_H. 3) If supply function is linear then an increase in z^* will result in an increase in both P_L and p_H.*

Proof. See Appendix. □

As a result of an increase in the quality threshold z^*, there will be a set of firms that switch from the first to the second partition. While our previous results apply to the price changes faced by those firms that remain in their partitions, a natural question is what happens to

those firms that switch. The following proposition shows that the price faced by these firms will go down which in turn implies that their quantity supplied decreases.

Proposition 5. $p'_L < p_H$.

Proof. Suppose towards a contradiction that $p'_L \geq p_H$. Since $p'_H > p'_L$ it follows that both prices have increased. Hence total output must increase too, i.e. $Q' > Q$. Then $p'_L = P(Q') + z'_L < P(Q) + z_H = p_H$ which is a contradiction. □

Additional properties can be obtained in the special case where the aggregate supply function is linear.

Proposition 6. *If Supply $S(p)$ is linear, then total output Q is independent of z^*.*

Corollary 4. *If Supply $S(p)$ is linear market shares for those that stay in the original groups increases while those that transition decreases.*

3 Cournot Model

We restrict here the analysis to a fixed set of firms, without considering explicitly the effect of changes in z^* on entry. There is a total of N firms and given signal z^* a fraction $F(z^*)$ in the first group and $(1 - F(z^*))$ in the second. Demand structure is the same as in the competitive case considered above. Assume firms face a constant marginal cost c regardless of their type. Equilibrium conditions are:

$$
\begin{aligned}
MR_H &= P'(Q)\, q_h + P(Q) + z_H = c &\quad (2)\\
MR_L &= P'(Q)\, q_l + P(Q) + z_L = c &\quad (3)
\end{aligned}
$$

Multiplying each equation by the number of firms in the respective group and adding up we get:

$$p'(Q)\, Q + Np(Q) + N\bar{z} = Nc$$

where $\bar{z}$ is the mean quality for the N firms. Interestingly, this equation determines Q independently of the signal threshold z^*.

Proposition 7. *In the Cournot model, total quantity Q is independent of z^*.*

While total quantity does not change, the shares of both groups do. In particular, as $z_L(z^*)$ and $z_H(z^*)$ increase, the output of individual firms q_L and q_H also increase. This is compensated by some of the H firms becoming now L firms and lowering output. It follows then that the output share of those firms that remain low and high increases, while that of the firms that shift category decreases.

Corollary 5. *Suppose marginal revenue is decreasing in Q. Then an increase in z^* increases the share of those firms that remain in the L and H groups and decreases for those that transition. The total share of the L group increases while that of the H group decreases.*

Another implication of the invariance of total output, is that consumer surplus does not change with z^*. This occurs because price increases capture exactly the change in average quality in each group.

Corollary 6. *Consumer surplus is independent of z^*.*

The following Assumption is used to derive further properties.

Assumption 1. *P is log concave.*

The following Proposition considers the effect of an increase in z^*.

Proposition 8. *Under Assumption 1, an increase in z^* results in an increase in both prices and quantities q_L and q_H supplied by firms in each group.*

Proof. As seen in the previous section, total output Q remains unchanged. Also as z^* increases, both z_L and z_H increase. At the original output levels of firms this results in higher marginal revenue and as a consequence both q_L and q_H must increase. $\square$

We consider now the effect of an increase in the number of firms. This is a first step for analyzing equilibrium entry decisions that will be considered in the future draft.

Proposition 9. *Under Assumption 1, an increase in the number of firms keeping constant the distribution of qualities results in an increase in total output, decrease in prices and decrease in q_H and q_L.*

Suppose, towards a contradiction, that total output does not increase. Then the assumption implies that marginal revenue for each type does not increase at the original level of output, so each firm's output cannot decrease. But since there are more firms, total output increases, a contradiction. The increase in Q reduces marginal revenue and as a consequence decreases the equilibrium output of firms in both groups.

4 Optimal Partition

We consider here the optimal determination of the threshold z^*. Our analysis is restricted to the case of additive quality premium. We first consider the case of perfect competition and then the Cournot equilibrium. The last section considers the impact of improved information.

4.1 Perfect Competition

In analyzing this it is convenient to formulate first a planners problem that gives the competitive equilibrium allocations.

4.1.1 A planner's problem

We explore here the connection between equilibrium and a restricted notion of optimality. The results of this section will be used below when considering the problem of an optimal partition. Let

$$\begin{aligned} C(y) &= \min_{q,f} c(q)\, n + \int^{f} x dG(dx) \\ & \quad \text{subject } qn = y \end{aligned}$$

where $n = G(f)$. This is the minimal cost (through intensive and extensive margins) of producing quantity y. It is easy to show that the solution satisfies:

$$c'(q) = \frac{c(q) + f}{q} = C'(y), \tag{4}$$

that is marginal costs equal average costs of the marginal firm and this is also equal to $C'(y)$.

4.1.2 The optimal partition

For a fixed z^* the optimal program solved by the planner is:

$$\begin{aligned} U(z^*) &= \max_{y_L, y_H} \int_0^Q P(x)\, dx + \Delta_L(z^*) F(z^*) y_L + \Delta_H(z^*)(1 - F(z^*)) y_H \\ & \quad - F(z^*) C(y_L) - (1 - F(z^*)) C(y_H) \end{aligned} \tag{5}$$

where $Q = F(z^*) y_L + (1 - F(z^*)) y_H$.

First order conditions for this problem imply:

$$\begin{aligned} P(Q) + \Delta_L(z^*) &= C'(y_L) \\ P(Q) + \Delta_H(z^*) &= C'(y_H) \end{aligned}$$

and together with (4) the conditions for a competitive equilibrium.

Consider now the effect of an in increase in z^* on welfare. By the envelope theorem, it is sufficient to consider the direct effect of z^* in (5) without changing y_L and y_H. Note that $Q = F(z^*) y_L + (1 - F(z^*)) y_H$ so $\partial Q / \partial z^* = -f(z^*)(y_H - y_L)$. We can then calculate:

$$\begin{aligned} \partial U(z^*) / \partial z^* &= f(z^*)(p_L y_L - p_H y_H - C(y_L) + C(y_H)) \\ & \quad + F(z^*) \Delta_L'(z^*) y_L + (1 - F(z^*)) \Delta_H'(z^*) y_H \\ &= -f(z^*)(\pi_H - \pi_L) + Q_L \Delta_L'(z^*) + Q_H \Delta_H'(z^*) \end{aligned}$$

where $\pi_L = p_L y_L - C(y_L)$ and $\pi_H = p_H y_H - C(y_H)$. Observing more closely the last line above, this is the effect of z^* on total profits since there is really no direct effect of the change of z^* on consumer surplus. This can be seen more directly considering

$$\begin{aligned} CS(z^*) &= \int^Q P(x)\,dx + \Delta_L(z^*) Q_L + \Delta_H(z^*) Q_H \\ &\quad - p_L Q_L - p_H Q_H \end{aligned} \tag{6}$$

where $Q_L = \Delta_L(z^*) F(z^*) y_L$ and $Q_H = \Delta_H(z^*)(1 - F(z^*)) y_H$. Noting that $p_L = P(Q) + \Delta_L(z^*)$ and $p_H = P(Q) + \Delta_H(z^*)$ equation (6) simplifies to:

$$CS = \int^Q P(x)\,dx - P(Q)$$

so all the quality premium is captured by firms. Moreover, starting from an equilibrium (efficient allocation) the direct effect of total output on consumer surplus is zero, explaining the above result.

4.1.3 Special Case: Linear Supply

Consider again the case of a linear supply function $S(p) = Ap$. In this case total profits are:

$$\begin{aligned} \Pi(z^*) &= p_L Q_L + p_H Q_H - F(z^*) \int_0^{p_L} S(p)\,dp - (1 - F(z^*)) \int_0^{p_H} S(p)\,dp\,(1 - F(z^*)) \\ &= F(z^*) A p_L^2 + (1 - F(z^*)) A p_H^2 - F(z^*) A p_L^2/2 - (1 - F(z^*)) A p_H^2/2 \\ &= \frac{1}{2}\left[P(Q)^2 + 2P(Q) Ez + F(z^*) \Delta_L(z^*)^2 + (1 - F(z^*)) \Delta_H(z^*)^2\right] \end{aligned}$$

Given that as seen above in the linear case Q is constant, the first two terms do not depend on z^*. Thus the optimal z^* is the one that maximizes

$$F(z^*) \Delta_L(z^*)^2 + (1 - F(z^*)) \Delta_H(z^*)^2 .$$

We prove results for the case of additive quality premium, both with linear supply and in the Cournot case.

Proposition 10. *Both in the competitive model with linear supply and the Cournot model, the optimal partition maximizes:* $N_L(z^*) \Delta_L(z^*)^2 + N_H(z^*) \Delta_H(z^*)^2 .$

Corollary 7. *Both in the competitive model with linear supply and the Cournot model, the optimal partition maximizes:* $N_L(z^*) q_L^2(z^*) + N_H(z^*) q_H^2(z^*) .$

4.2 Cournot Model

Since as observed before consumer surplus is invariant to z^*, the socially optimal value maximizes firm profits. This is given by:

$$\begin{aligned}\Pi(z^*) &= (P(Q)-c)Q + N_L(z^*)\,q_L(z^*)\,\Delta_L(z^*) \\ &\quad + N_H(z^*)\,q_H(z^*)\,\Delta_H(z^*)\end{aligned}$$

Using equations (2) and (3), it follows that q_L and q_H and be written as

$$q_L = a(Q) + b(Q)\,\Delta_L(z^*) \tag{7}$$

$$q_H = a(Q) + b(Q)\,\Delta_H(z^*) \tag{8}$$

Substituting in the above equation

$$\begin{aligned}\Pi(z^*) &= (P(Q)-c)Q + N_L(z^*)\left[a(Q)+b(Q)\,\Delta_L(z^*)\right]\Delta_L(z^*) \\ &\quad + N_H(z^*)\left[a(Q)+b(Q)\,\Delta_H(z^*)\right]\Delta_H(z^*) \\ &= (P(Q)-c)Q + Na(Q)\,\Delta\bar{z} \\ &\quad + b(Q)\left[N_L(z^*)\,\Delta_L(z^*)^2 + N_H(z^*)\,\Delta_H(z^*)^2\right].\end{aligned}$$

Since the only term that depends on z^* is the last term in brackets, this proves:

Proposition 11. *In the Cournot model, the optimal z^* is the one that maximizes $N_L(z^*)\,\Delta_L(z^*)^2 + N_H(z^*)\,\Delta_H(z^*)^2$.*

Note that this is the same result that obtains in the case of perfect competition with linear supply functions. In terms of observables, a more convenient representation can be obtained. Using (7) and (8) we can write $\Delta_H = \frac{q_H - a(Q)}{b(Q)}$ and $\Delta_L = \frac{q_L - a(Q)}{b(Q)}$ so

$$\begin{aligned}& N_L(z^*)\,\Delta_L(z^*)^2 + N_H(z^*)\,\Delta_H(z^*)^2 \\ =\ & N_L(z^*)\left(\frac{q_L - a(Q)}{b(Q)}\right)^2 + N_H(z^*)\left(\frac{q_H - a(Q)}{b(Q)}\right)^2 \\ =\ & \frac{2a(Q)\left[N_L(z^*)\,q_L + N_H(z^*)\,q_H\right] + \left[N_L(z^*)\,q_L^2 + N_H(z^*)\,q_H^2\right]}{b(Q)^2} \\ =\ & \frac{2a(Q)\,Q + \left[N_L(z^*)\,q_L^2 + N_H(z^*)\,q_H^2\right]}{b(Q)^2}\end{aligned}$$

and since Q is independent of z^*, maximizing $U(z^*)$ is equivalent to maximizing $N_L(z^*)\,q_L^2 + N_H(z^*)\,q_H^2$. This proves:

Proposition 12. *In the Cournot model the optimal z^* is the one that maximizes $N_L(z^*)\, q_L^2(z^*) + N_H(z^*)\, q_H^2(z^*)$.*

This proposition provides a more convenient way for checking optimality.

Example 3. Suppose z is uniform between zero and one. Assuming N is large, take the approximation $N_L = F(z^*)\, N$ and $N_H = (1 - F(z^*))\, N$. Maximizing profits is equivalent to maximizing M as given by:

$$\begin{aligned} M &= (1 - F(z^*))\, \Delta_h(z^*)^2 + F(z^*)\, \Delta_l(z^*)^2 \\ &= (1 - z^*) \left(\frac{1 + z^*}{2}\right)^2 + z^* \left(\frac{z^*}{2}\right)^2 \\ &= \frac{1}{4}\left(1 + z^* - (z^*)^2\right) \end{aligned}$$

This is maximized when $z^* = 1/2$.

5 Improved Information

Given a partition in two groups L and H, we consider better information as increasing the likelihood that higher quality firms are included in the H group and conversely for lower quality firms. For example, for two levels of quality $z_2 > z_1$, one possible definition is that the likelihood ratio $P(H|z_2)/P(H|z_1)$ is higher when the informational content of reputation signals (L, H) is better. A better classification system results in a mean preserving spread of z_L and z_H, i.e. increases z_H and decreases z_L, preserving the average.

5.1 Comparative Statics

We examine here the effect of better information on prices.

Proposition 13. *Better information always leads to higher p_H.*

Proof. If p_H weakly decreases and so does p_L, total demand will increase (non-generically) so it will exceed supply. If p_H weakly increases and p_L increases, then total demand for H group will strictly increase (generically) so demand for this group will exceed supply. □

Example 4. It is possible for p_L to increase as this example shows. Suppose that consumers are distributed uniformly on the following sets: $[(0,x)\,|0\le x\le A]\cup[(x,0)\,|0\le x\le B]$ and $A>B$. Suppose the total measure of consumers is $A+B$, so there is a mass B on the θ_1-axis and A on the θ_0-axis. Suppose there are two types of firms of total measure one, half of them of quality zero and half of quality 2. Individual supply function $S(p)=p$ for each firm. Initially there is no signal so only one group of average quality 1. Given price p, demand will then be $B-p+A-p=A+b-2p=S(p)=p$. This solves for $p=\frac{A+B}{3}$ and the supply of each group equal to half of this.

Now suppose that the scenario with a high quality signal is perfectly separating. We construct the equilibrium so the high quality firms serve the quality sensitive consumers and the low quality firm those that don't care about quality. Demand for the low quality group is thus $A-p$ and supply $\frac{1}{2}p$, so equilibrium price $p_L=\frac{2}{3}A>\frac{A+B}{3}$, the price in the original scenario. Demand for the high quality group is $B-\frac{p}{2}$ and since supply is $\frac{1}{2}p$ it follows that $p_H=B$. For $p_H>p_L$ we would need $B>\frac{A+B}{3}$ or equally that $B>A/2$. So this example seems to work provided that $A>B>A/2$.

So we need some extra conditions on the demand side or on supply to get p_L to decrease. In our example the gains from sorting are so large that they support an increase of both prices. So limiting the gains from sorting or the "spillover" to the L group should make it. One extreme case is when there is constant quality premium.

Proposition 14. *For the case of additive quality premium, an improvement in the quality of information lowers p_L.*

Proof. If p_L increases, then total demand must decrease. But since both prices have increased, supply increases exceeding total demand. □

The second case considered is increasing quality premium, or the case of pure vertical differentiation. This is the case where all the mass of the distribution lies on a unique value of θ_0, without loss of generality equal to one. Let G denote the cdf for θ_1. Given prices p_L and p_H and assuming that both groups have positive supply, total demand is given by $1-G(p_L/q_L)$. Better information implies a decrease in q_L and increase in q_H. If $p_L$does note decrease, then from the above total demand decreases. But since p_H must increase too, then total supply would strictly increase and we get a contradiction.

Proposition 15. *For the standard case of vertical differentiation, better information results in higher p_H but lower p_L.*

5.2 Welfare effect

In this section we restrict attention to the case of constant quality premium for either perfect competition with linear supply or Cournot competition, as considered in Section 4Our results in that Section show that total surplus can be expressed as a function that is monotonically increasing in Ez_i^2. By Jensen's inequality this increases with a mean preserving spread. This proves the following:

Proposition 16. *A better information system, defined as a mean preserving spread in quality signals z_L and z_H, increases total surplus.*

As noted previously, the change in total surplus will be the same as the change in total profits while consumer surplus remains unchanged..

6 Appendix

Proof of Proposition 4

Let $z_2^* > z_1^*$. Denote Δ_{1L} and Δ_{2L} the quality premium for the L group in each of the two scenarios. Define similarly, $\Delta_{1H}, \Delta_{2H}, q_{1L}, q_{2L}, q_{1H}, q_{2H}, p_{1L}, p_{2L}, p_{1H}, p_{2H}$. Suppose, towards a contradiction, that $p_{2L} \leq p_{1L}$. Since $\Delta_{2L} > \Delta_{1L}$ this implies that $Q_2 > Q_1$. Note that

$$\begin{aligned}
p_{1H} &= P(Q_1) + \Delta_{1H} = P(Q_1) + E(z \geq z_1^*) \\
&= \frac{1 - F(z_2^*)}{1 - F(z_1^*)} \left(P(Q_1) + E(z \geq z_2^*) \right) \\
&+ \frac{F(z_2^*) - F(z_1^*)}{1 - F(z_1^*)} \left(P(Q_1) + E(z_1^* \leq z \leq z_2^*) \right)
\end{aligned}$$

So it follows that:

$$\begin{aligned}
Q_{1H} &= (1 - F(z_1^*)) S(p_{1H}) \\
&\geq (1 - F(z_2^*)) S(P(Q_1) + E(z \geq z_2^*)) + [F(z_2^*) - F(z_1^*)] S(P(Q_1) + E(z_1^* \leq z \leq z_2^*)) \\
&> (1 - F(z_2^*)) S(P(Q_2) + E(z \geq z_2^*)) + [F(z_2^*) - F(z_1^*)] S(P(Q_2) + E(z_1^* \leq z \leq z_2^*)) \\
&\geq (1 - F(z_2^*)) q_{2H} + [F(z_2^*) - F(z_1^*)] q_{2L}
\end{aligned}$$

$$\begin{aligned}
p_{2L} &= P(Q_2) + \Delta_{2L} \leq P(Q_2) + E(z | z_1^* \leq z \leq z_2^*) \\
p_{1H} &= P(Q_1) + E(z | z > z_1^*)
\end{aligned}$$

By the contradiction hypothesis, $q_{2L} = S(p_{2L}) \leq S(p_{1L}) = q_{1L}$. It then follows that:

$$\begin{aligned}
Q_1 &= Q_{1H} + F(z_1^*) q_{1L} \\
&> (1 - F(z_2^*)) q_{2H} + [F(z_2^*) - F(z_1^*)] q_{2L} + F(z_1^*) q_{2L} \\
&= Q_{2}.
\end{aligned}$$

But then $p_{2L} > p_{1L}$, thus completing the proof of the first part.

Now to prove the case for convex supply function. Let $z_2^* > z_1^*$. Denote Δ_{1L} and Δ_{2L} the quality premium for the L group in each of the two scenarios. Define similarly, $\Delta_{1H}, \Delta_{2H}, q_{1L}, q_{2L}, q_{1H}, q_{2H}, p_{1L}, p_{2L}, p_{1H}, p_{2H}$. Suppose, towards a contradiction, that $p_{2H} \leq p_{1H}$. Since $\Delta_{2H} > \Delta_{1H}$ this implies that $Q_2 > Q_1$. Note that

$$\begin{aligned} p_{2L} &= P(Q_2) + \Delta_{2L} = P(Q_2) + E(z \leq z_2^*) \\ &= \frac{F(z_1^*)}{F(z_2^*)} \left(P(Q_2) + E(z \leq z_1^*)\right) \\ &+ \frac{F(z_2^*) - F(z_1^*)}{F(z_2^*)} \left(P(Q_2) + E(z_1^* \leq z \leq z_2^*)\right) \end{aligned}$$

So it follows that:

$$\begin{aligned} Q_{2L} &= F(z_2^*) S(p_{2L}) \\ &\leq F(z_1^*) S(P(Q_2) + E(z \leq z_1^*)) + [F(z_2^*) - F(z_1^*)] S(P(Q_2) + E(z_1^* \leq z \leq z_2^*)) \\ &< F(z_1^*) S(P(Q_1) + E(z \leq z_1^*)) + [F(z_2^*) - F(z_1^*)] S(P(Q_1) + E(z_1^* \leq z \leq z_2^*)) \\ &\leq F(z_1^*) q_{1L} + [F(z_2^*) - F(z_1^*)] q_{1H} \end{aligned}$$

By the contradiction hypothesis, $q_{2H} = S(p_{2H}) \leq S(p_{1H}) = q_{1H}$. It then follows that:

$$\begin{aligned} Q_2 &= Q_{2L} + (1 - F(z_2^*)) q_{2H} \\ &< (1 - F(z_1^*)) q_{1L} + [F(z_2^*) - F(z_1^*)] q_{1H} + (1 - F(z_2^*)) q_{2H} \\ &\leq (1 - F(z_1^*)) q_{1L} + [F(z_2^*) - F(z_1^*)] q_{1H} + (1 - F(z_2^*)) q_{1H} \\ &= Q_{1.} \end{aligned}$$

But then $p_{2H} > p_{1H}$, thus completing the proof.