Information Acquisition, Signaling and Learning in Duopoly

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Abstract

We study firms' information acquisition decisions in the presence of cost uncertainty and signaling incentives. In a duopoly model with differentiated products, firms compete in price in two periods. Before production starts, each firm faces uncertainty on its own cost and can make a costly investment to receive some private information about it. In the first period, firms have incentives to signal their private information through prices in order to manipulate their rivals' beliefs and soften the second-period competition. Although firms benefit from more accurate private information, the "signaling" incentives dampen firms' gain from improved information. That is, compared with myopic firms who do not try to manipulate their rivals' beliefs, strategic firms will acquire more noisy private information.

From the perspective of industry profit, firms acquire too little information because they fail to internalize the positive externality of their improved information on their rivals' profits. When the two goods are close substitutes, firms' improved information exerts a negative impact on consumer surplus. If the degree of substitution between the goods is low, consumers also benefit from more accurate private information possessed by the firms. Overall, form the social planner's point of view, the qualities of firms' information are inefficiently low.

Keywords: information acquisition, signaling, Bertrand competition, product differentiation

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1 Introduction

Firms often face uncertainty over costs when developing a new product. Nevertheless, during the course of production, managers gradually learn their idiosyncratic costs which is unobservable to their rivals and will affect future competition. When a firm's current price conveys its private information about cost, the firm's price will affect rivals' beliefs about the environment for future competition and thus has an impact on the rivals' future prices. This linkage between a firm's current price and future competition provides the firm an incentive to distort its current price from the optimal myopic level in order to manipulate its rivals' beliefs. We refer to the firm's intertemporal incentive as "signaling".

This paper considers a situation in which firms can choose the qualities of their private information in the presence of signaling incentives. To give an example of endogenous private information on cost, consider that a producer of electronics launches a new product with several novel design features. The marketing campaign has a target consumer group and hence determines a price range for the new product. However, the firm may not be fully aware of the cost of these new features before mass production starts and it can spend resources to narrow down the cost uncertainty.

Specifically, we investigate how firms' signaling incentive affects their information acquisition decisions and ask the following questions: 1) How much information will firms acquire anticipating that rivals will learn their private information through their prices? 2) Do firms acquire better information ex-ante when they have signaling incentives as opposed to no signaling incentives? 3) How do firms' incentives to improve the quality of their information differ from the social planner?

In our model, two firms compete in price in three periods. Firms' costs are independent random variables which are initially unknown to both firms. Once a firm's cost is realized, it remains constant. In period 0, firms simultaneously invest in information acquisition about their future idiosyncratic costs. Specifically, each firm can choose the precision a signal to be received in the next period. Firms' choices of signal precisions become public information at the end of the period. In Period 1, firms' costs are realized and each firm receives a private signal about its own cost. Then, firms simultaneously choose their first-period prices and their first-period profits are realized afterwards. Firms' first-period prices are public information but their first-period profits remain their private information. In Period 2, firms compete in price again and their second-period profits are realized.

When a firm chooses the quality of its signal, it considers the impact of a more precise signal on the sum of its expected profits from the two periods. In the first period, each firm's price is only based on the firm's posterior expectation of its own cost. Since firms' costs are idiosyncratic, their first-period prices are independent. As a result, a more precise signal will help a firm better attune its first-period price to its cost and thus is beneficial.

In the second period, firms infer their rivals' private signals from their first-period prices and form posterior beliefs about the rivals' costs. In equilibrium, each firm's second-period price depends on its realized cost, the rival's posterior expectation of its realized cost and its posterior expectation of the rival's realized cost. The impact of more precise signals on firms' second-period profits is more involved because their second-period prices are positively correlated through two channels. First, firms' realized costs are positively correlated with the rivals' posterior expectations of their realized costs. Second, firms' posterior expectations of rivals' costs enter both firms' prices.

In the second period, firms already learn their realized costs from their first-period profits and formulate their second-period prices based on their costs. So, unlike in the first period, a more precise signal does not directly help a firm better attune its second-period price to its cost, but will increase the correlation between the firm's realized cost and its rival's conjecture of the firm's cost. As a result, firms' second-period prices will be more positively correlated, which reduces their second-period expected profits. The intuition is the following: When a firm's cost goes up, it will raise its second-period price accordingly. However, since pass-through is not perfect, the firm's profit margin shrinks and it wishes to sell fewer units. When the firm's signal becomes more precise, due to the positive correlation between prices, its rival is more likely to also charge a high price. This will increase the firm's residual demand and force it to sell more than it wants at the low profit margin. In short, when firms' signals become more precise, their second-period prices are more correlated in an undesirable way which reduce their second-period profits.

When a firm improves the quality of its signal, its first-period profit increases but its second-period profit decreases. Despite the opposing effects, we show that the impact of a more precise signal on the firm's total profit is positive, and the firm's optimal signal precision is uniquely determined by balancing its gain in total profit and the cost of information acquisition.

To answer how firms' information acquisition decisions are affected by their signaling incentives, we first need to understand, how firms' pricing strategies in the presence of signaling incentives differ from their pricing strategies without signaling incentives. If firms do not attempt to manipulate rivals' beliefs, they will choose the first-period prices to maximize their expected first-period profits. In this case, firms choose the optimal myopic prices. By contrast, strategic firms take into account the impact of their first-period prices on second-period competition and try to manipulate rivals' beliefs through their first-period prices. As a result, their equilibrium first-period prices are distorted upward from the optimal myopic first-period prices. This is because by raising the first-period price above the optimal myopic price, a firm signals to its rival that it is likely to have a high cost and hence is likely to charge a high price in the second period. Since pricing strategies are strategic complements, the rival will respond by raising its second-period price which softens the second-period competition. Therefore, an upward distortion in the firm's first-period price will result in a first order gain in its expected second-period profit but a second order loss in its expected first-period profit.

One may think that signaling incentives will induce firms to invest more in the qualities of their information because it is easier for a firm to manipulate its rival's belief when its own information is more reliable. Surprisingly, we show that signaling reduces firms' incentives to improve the quality of its information. That is, strategic firms will acquire more noisy private information than myopic firms. To see this, note that for a fixed pair of signal precisions, firms' expected second-period profits are the same no matter they are strategic or myopic. This is because although strategic firms try to manipulate the rivals' beliefs, in equilibrium their rivals will correctly infer their private information. So, strategic firms adopt the same pricing strategies as myopic firms in the second period. This implies that any difference between strategic and myopic firms' information acquisition decisions is driven by their consideration for the first-period profits.

Both myopic and strategic firms gain in the first period when the precisions of their signals are improved because they can better attune their prices to the costs. However, strategic firms benefit less than myopic firms. This is because strategic firms' first-period prices do not maximize their first-period profits due to the signaling incentives. When a firm's private information becomes more precise, its expected cost conditional on its signal will have a larger variance. Because the firm's first-period price is linear in its conditional expectation of cost, the firm's first-period price will also have a larger variance. This variation in price will dampen the strategic firm's gain from improved signal precision. To see this, recall that Firm's first-period profit is concave in its price. When the price is not at the profit maximizing level, a variation in price will result in a net loss in expected profit due to the concavity of the profit function. By contrast, since myopic firms' first-period prices maximize their first-period profit, they do not bear such a loss when there is a variation in their prices by the envelope theorem.

We evaluate the welfare implications of strategic firms' information acquisition decisions. From the industry's preperceptive, the qualities of firms' information are inefficiently low. This is because firms fail to internalize the positive externality on their rivals' second-period profits. When the degree of substitution between the two goods is low, consumer surplus increases in the qualities of firms' signals. Otherwise, consumer surplus decreases in the qualities of firms' signals. From the social planner's perspective, the qualities of firms' signals are inefficiently low.

The presence of firms' "signaling" incentive when they compete in multiple periods has been documented in Caminal (1990), Mailath (1989) among others. Caminal (1990) studied a two-period Bertrand game with differentiated products, but the uncertainty lies in firms' idiosyncratic demand intercepts, which takes only two possible values. Mailath (1989) considered a more general two-period Bertrand game with idiosyncratic cost uncertainty. Both paper show that in equilibrium, each firm would distort up its first-period price relative to the one-shot game benchmark so as to "signal" the rival that it is of a stronger type hence induces the rival to take a softer action. Like our paper, they all assume firm-specific uncertainty parameter and that firms know perfectly the action profile in the previous period. Contrary to these assumptions, the literature on signal jamming (for example Riordan (1985), Fudenberg and Tirole (1986), Mirman et al. (1993), Bonatti et al. (2015)) typically assumes that firms face the same underlying uncertainty (Bonatti et al. (2015) is an exception in which each firm perfectly knows its own realized cost, but the market is subject to unobservable demand shocks. While trying to signal jam the rivals, firms are able to learn gradually over time the average cost of their rivals.) and that firms' actions in the previous stage is unobservable, namely actions are hidden. Therefore, similar to our paper and Caminal (1990), Mailath (1989), firms have incentive to manipulate their strategies to deliberately jam the rivals' inference about the uncertainty parameter. In comparison our paper goes one step further by analyzing how does the presence of signaling incentive affect firms' incentive to acquire information. To the best of our knowledge, this question has not been addressed in the literature. In addition, we also study the welfare implications of signaling incentives when firms can acquire noisy information.

Information acquisition has been studied in the environment of oligopolistic competition, but most papers only involve single period interaction. Hwang (1993) considered a Cournot duopoly with common demand shock; in Jansen (2008) the common demand shock takes only two possible values and the focus is on the incentive to acquire and disclose private information; Vives (2011) in a homogeneous Cournot market in which firms have idiosyncratic cost shocks and compete in supply schedule; Myatt and Wallace (2015) studied the social value of public and private information in differentiated Cournot market with continuum of products and finite number of firms. In comparison to this strand of literature we study a two-period model that highlights the firms' signaling incentive.

Related to our paper is also the literature on information exchange in oligopoly. For this strand of literature, firms, *prior to* receiving private noisy information about their own uncertain parameters, decide simultaneously whether or not to commit to truthfully share their private information with other firms, for example, through a trade association. It turns out that the source of uncertainty (demand uncertainty or cost uncertainty), the source of information (independent values, private values, common value) and the form of competition (Cournot or Bertrand) will play a role in determining firms' equilibrium sharing strategy. A sample of literature includes Vives (1984), Gal-Or (1985), Gal-Or (1986), Li (1985), Amir et al. (2010). A brief summarization of the results in the following: (Except for Bertrand competition with cost uncertainty), unilaterally reveal all private information is a dominant strategy with independent values, with private values, or with common values and strategic complements; do not reveal any information is a dominant strategy with common value and strategic substitutes. See Section 8.3.1 of Vives (2001) for more details including welfare implications. Raith (1996) presented a general model that synthesises virtually all models along this strand of literature.

There's a growing literature in the recent years on how agents use public and private information. Typically the focus is on a class of quadratic-payoff coordination games with a common uncertain parameter. Each agent observes a private signal and the same public signal concerning the uncertain parameter. For tractability the economy is normally assumed to be populated with continuum of agents. Angeletos and Pavan (2007) adopted a general framework that allows for both strategic complements and substitutes, they show complementarity heightens the sensitivity of equilibrium actions to public information whereas substitutability heightens sensitivity to private information. Particular applications are Cournot and Bertrand games with demand uncertainty: information sharing (which increases precision of public information) is profit enhancing under Bertrand competition, but not necessarily under Cournot. Like our paper, Colombo et al. (2014)'s focus is on agents' ex ante incentive to acquire noisy information. They highlighted that an increase in precision of public information always crowds out acquisition of private information in equilibrium. In addition, they identified 3 channels of inefficiency in information acquisition: externality due to cross-sectional dispersion; discrepancy between equilibrium and the efficient degree of coordination; and discrepancy between the complete information equilibrium actions and the first best allocation. In Myatt and Wallace (2015) finite firms compete in Cournot market with continuum of products, each firms receives a finite number of signals with both common and private noise that may differ in precision and correlation across the agents. They show that from the industry's perspective too much emphasis is placed on the relatively public signals while form consumer's perspective too much emphasis is placed on relatively private signals. If firms can acquire costly information, then firms acquire too much information but use it too little. In our paper, first-period prices can be regarded as public information, each firms also has private information. However, we focus on firms incentive to signal and its impact on information acquisition, which is quite different from this strand of literature.

The rest of the paper is organized as follows: Section 2 presents the model. Section 3 solves strategic firms' equilibrium choices of precisions and their pricing strategies. Section 4 compares strategic firms' equilibrium choices of signal precisions with myopic firms' choices. Section 5 evaluates the efficiency of firms' information acquisition decisions and Section 6 concludes.

2 The Model

Two risk neutral firms, Firm i and j, produce differentiated products and compete in price in two periods. Firms have constant marginal production costs, denoted by c_i and c_2, which are i.i.d. random variables with $E(c_i) = \mu_c$, $\mathrm{Var}(c_i) = \sigma_c^2$, $i = 1, 2$. Denote by $\tau_c \equiv \frac{1}{\sigma_c^2}$ the precision of the costs.

The representative consumer has a quadratic utility function which takes the form[1]

$$u(q_i, q_j, m) = \eta_0(q_i + q_j) - \frac{1}{2}(\eta_1 q_i^2 + 2\eta_2 q_i q_j + \eta_1 q_j^2) + m, \tag{1}$$

where m is the wealth and $\eta_0 > 0$, $\eta_1 > |\eta_2| \geq 0$. The two goods are substitutes, independent or complements depending on whether $\eta_2 > 0$, $\eta_2 = 0$, or $\eta_2 < 0$. The two goods are perfect substitutes when $\eta_1 = \eta_2$ and perfect complements when $\eta_1 = -\eta_2$. The coefficient η_2/η_1, which ranges from -1 to 1, is therefore a measure of the degree of product differentiation. Given prices p_{it}, p_{jt}, the representative consumer chooses q_{it} and q_{jt} to maximizes her utility, which results in the following linear demand functions in period t:

$$q_{it} = a - bp_{it} + ep_{jt} \tag{2}$$

$$q_{jt} = a - bp_{jt} + ep_{it}, \tag{3}$$

[1]The quadratic utility function is commonly used to generate the linear demand function (see, e.g., Caminal, 1990, Sigh and Vivies, 1984, Caminal and Vivies, 1993).

where a, b, e are positive constants

$$a = \frac{\eta_0}{\eta_1 + \eta_2}, \qquad b = \frac{\eta_1}{\eta_1^2 - \eta_2^2}, \qquad e = \frac{\eta_2}{\eta_1^2 - \eta_2^2}. \tag{4}$$

Firms face initial cost uncertainty which is resolved over time. Prior to the first-period price competition, each firm can make a costly investment to improve the quality of their private information regarding their costs. Specifically, Firm i's investment in information acquisition generates a private signal s_i about c_i, with $s_i = c_i + \epsilon_i$, where $E(\epsilon_i) = 0$, $\text{Var}(\epsilon_i) = \sigma_{\epsilon_i}^2$ (hence $E(s_i|c_i) = c_i$, $\text{Var}(s_i|c_i) = \sigma_{\epsilon_i}^2$). Let $\tau_{\epsilon_i} = \frac{1}{\sigma_{\epsilon_i}^2}$ denote the precision of signal s_i. Denote by $G(s_i|c_i)$ the conditional distribution function of the signal s_i given c_i and by $g(s_i|c_i)$ the corresponding conditional density function. Firm i can choose the precision of its signal τ_{ϵ_i} at a cost $k(\tau_{\epsilon_i})$. We assume $k(\cdot)$ is a strictly increasing and convex C^2 function with $k(0) = 0$ and $\lim_{\tau_{\epsilon_i} \to 0} k'(\tau_{\epsilon_i}) = 0$. These assumptions on $k(\cdot)$ ensure a unique optimal choice for $\tau_{\epsilon_i} \in (0, \infty)$.

The objective of a firm is to maximize the sum of its expected profits from the two periods net of the costs of information acquisition. For simplicity of exposition, we assume that there is no discounting.

We make the following assumptions on the conditional distribution functions to make the model tractable without sacrificing too much in terms of generality.

Assumption 2.1 *Firm i's posterior expectation on its cost upon observing signal s_i is given by*

$$E(c_i|s_i) = \overline{\tau}_i s_i + (1 - \overline{\tau}_i)\mu_c, \quad \textit{where} \;\; \overline{\tau}_i = \frac{\tau_{\epsilon_i}}{\tau_{\epsilon_i} + \tau_c}. \tag{5}$$

Thus, a firm's posterior expectation on its cost upon observing signal s_i is a convex combination of the signal that is observed and the prior expectation on the cost, μ_c. When Firm i's signal is more precise (higher τ_{ϵ_i}), the posterior expectation puts a higher weight on its signal as compared to the prior mean; and similarly, the more dispersed is the prior (small τ_c) the less weight is placed on the prior belief compared to the signal.

Several prior-posterior distribution functions give rise to this linear posterior expectation. For example, when $F(c_i)$ is a gamma distribution and $G(s_i|c_i)$ is a Poisson distribution with unknown mean c_i; when $F(c_i)$ is a beta distribution and $G(s_i|c_i)$ is a negative binomial distribution with unknown mean c_i; or when $F(c_i)$ and $G(s_i|c_i)$ are both normal.[2]

The timing of the game is as follows:

Period 0 Information Acquisition: Nature draws c_i and c_j independently according to $F(\cdot)$ which is common knowledge. The realizations of c_i and c_j are unknown to both firms. Firms choose τ_{ϵ_i} and τ_{ϵ_j} simultaneously and their choices of precisions are publicly known.

Period 1 Price Competition: Each firm receives a private signal about their costs and engage in the first-period price competition. They simultaneously choose p_{i1} and p_{j1} which are publicly observable, but each firm's first-period profit remains its private information.

Period 2 Price Competition: Firms engage in the second-period price competition and choose p_{i2} and p_{j2} simultaneously.

Figure 1 illustrates the sequence of events.

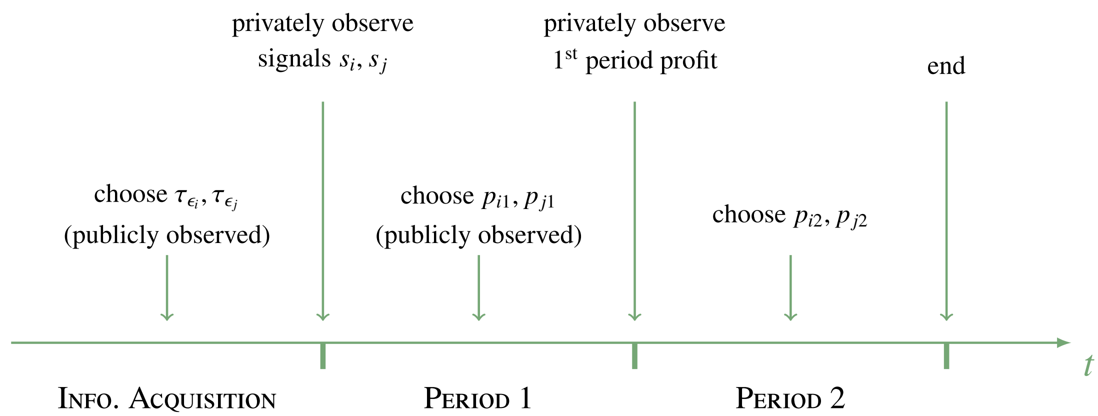

Figure 1: Timeline

Strategies: Firm i's strategy is a triplet $\{\tau_{\epsilon_i}, p_{i1}(\cdot), p_{i2}(\cdot)\}$. In the information acquisition stage, Firm i does not possess any private information and chooses $\tau_{\epsilon_i} \in [0, \infty)$. At the beginning of Period 1, Firm i chooses p_{i1} based on $\{\tau_{\epsilon_i}, \tau_{\epsilon_j}, s_i\}$, where τ_{ϵ_i} and τ_{ϵ_j} are public information and s_i is Firm i's private information. After the first-period competition, p_{i1} and p_{j1} become publicly observable. Moreover, Firm i's realized marginal cost c_i is revealed to Firm i through its realized

[2]It is well known that for the normal family, this property on posterior expectation holds. But the same property also holds for other conjugate families, which allows us more generality. A proof can be found in in Ericson (1969). Also see Gal-Or (1987).

first-period profit. This information, however, is concealed from Firm j because firms cannot observe each other's first-period profits. Hence, the second-period price $p_{i2}(\cdot)$ is a mapping from $\{\tau_{\epsilon_i}, \tau_{\epsilon_j}, s_i, p_{i1}, p_{j1}, c_i\}$ to a nonnegative real number.

Equilibrium: Since our model is a game with imperfect information, the equilibrium concept we use is perfect Bayesian Nash equilibrium. A perfect Bayesian Nash equilibrium consists of the two triples $\{(\tau_{\epsilon_i}, p_{i1}(\cdot), p_{i2}(\cdot)), (\tau_{\epsilon_j}, p_{j1}(\cdot), p_{j2}(\cdot))\}$ and a belief system. The belief system captures firms' beliefs about rivals' private signal, with $\hat{s}_j(p_{j1})$ denoting Firm i's beliefs about Firm j's signal s_j upon observing Firm j's first-period choice of price p_{j1}. In particular, the strategies and beliefs meet the following criteria:

- the triplet $\{\tau_{\epsilon_i}, p_{i1}(\cdot), p_{i2}(\cdot)\}$ maximizes Firm i's expected profit given its belief $\hat{s}_j(p_{j1})$ and Firm j's strategy; and
- Firm i's belief $\hat{s}_j(p_{j1})$ is updated using Bayes' rule whenever possible.

To appreciate why a firm's belief about the rival's signal matters consider the following:

Upon observing its first-period profit, Firm j learns its (time-invariant) cost, which is the basis for determining its second-period price. However, since profits are private information, Firm i does not learn Firm $j's$ cost—and can therefore not fully anticipate Firm $j's$ second-period price.

However, since Firm $j's$ first-period price is based on its private signal which contains information about its underlying cost, Firm i forms beliefs about Firm $j's$ signal in order to glean information about Firm $j's$ cost and subsequent pricing strategy. In short: Firm i infers Firm j's signal from j's first-period price, and uses this inference to update its belief about the distribution of Firm j's actual cost.

Of course, because firms use their rivals' first-period prices to shape posteriors which are then used to determine subsequent prices, firms have an incentive to distort their prices to skew beliefs and influence subsequent competition in the second period. We now show how learning and belief manipulation affect the pricing games and how this feeds back into the incentives of acquiring information in the first place.

3 Strategic Firms

In each period we consider firm pricing strategies that are affine functions of their expected cost. That is, firms prices can be written as

$$p_{it}^{*} = \alpha_{it} E(c_i|\cdot) + \beta_{it} \tag{6}$$

for some constants $\alpha_{it}, \beta_{it} > 0$ that are to be determined.

The use of linear strategies implies a fully revealing equilibrium in that given equilibrium knowledge of α and β allows one to infer $E(c_i|\cdot)$ upon observing the price. It can be shown that the equilibrium in affine strategies is unique among all fully-revealing equilibrium configurations.

3.1 Second Period

Firms observe their costs directly once production and sales take place. Hence, they have full information about their marginal cost in the second period. Since firms do not observe rival's first-period profits, they remain uncertain about rivals' realized marginal cost. Nevertheless, each firm can infer its rival's private signal from the rival's first-period price.

In particular, using the first-period pricing rule (6) in conjunction with Assumption 2.1, when observing the price p_{j1} Firm i infers that Firm j's signal was

$$\hat{s}_j(p_{j1}) = \frac{p_{j1} - \alpha_{j1}(1 - \overline{\tau}_j)\mu_c - \beta_{j1}}{\alpha_{j1}\overline{\tau}_j}, \tag{7}$$

where the hat denotes that $\hat{s}_j$ is not actually an observation of the signal s_j, but an inference about s_j that is based on the price observation of p_{j1}. Given this belief, Firm i forms the conditional belief about Firm j's cost c_j that is given by $F(c_j|\hat{s}_j)$.

Notice that since beliefs about signals are based upon observable information (namely the first-period prices), $\hat{s}_j$ and $\hat{s}_i$ are common knowledge at the outset of the second period, as are $F(c_j|\hat{s}_j)$ and $F(c_i|\hat{s}_i)$.

Since Firm j is perfectly informed about its marginal cost, its second-period price is a function of c_j which remains unknown to Firm i. Firm i's problem in the second period is thus

$$\max_{p_{i2}} \int_{c_j} \left[a - bp_{i2} + ep_{j2}(c_j)\right](p_{i2} - c_i)dF(c_j|\hat{s}_j); \tag{8}$$

and similarly for Firm j's problem.

The first order conditions yield:

$$p_{i2} = \frac{bc_i + a + eE[p_{j2}(c_j)|\hat{s}_j]}{2b} \tag{9}$$

$$p_{j2} = \frac{bc_j + a + eE[p_{i2}(c_i)|\hat{s}_i]}{2b} \tag{10}$$

In accordance with (6), and given that $\hat{s}_j$ and $\hat{s}_i$ are common knowledge, we characterize the equilibrium in which each firm adopts a pricing strategy that is linear in its c_i (or c_j); namely

$$p_{i2}(c_i; \hat{s}_i, \hat{s}_j) = \alpha_{i2}c_i + \beta_{i2} \tag{11}$$

$$p_{j2}(c_j; \hat{s}_j, \hat{s}_i) = \alpha_{j2}c_j + \beta_{j2}, \tag{12}$$

where $\alpha_{i2}, \beta_{i2}, \alpha_{j2}, \beta_{j2}$ are constants that depend on $\hat{s}_j$ and $\hat{s}_i$.

Using the FOCs (9) and (10) in conjunction with the pricing rules (11) and (12), one can solve for these unknown constants,

$$\alpha_{i2} = \alpha_{j2} = 1/2 \tag{13}$$

$$\beta_{i2} = \frac{2a(2b + e) + 2beE(c_j|\hat{s}_j) + e^2E(c_i|\hat{s}_i)}{2(4b^2 - e^2)} \tag{14}$$

$$\beta_{j2} = \frac{2a(2b + e) + 2beE(c_i|\hat{s}_i) + e^2E(c_j|\hat{s}_j)}{2(4b^2 - e^2)}, \tag{15}$$

which yields the following prices:[3]

$$p_{i2}^{*}(c_i;\hat{s}_i,\hat{s}_j) = \frac{a(2b+e)}{4b^2-e^2} + \underbrace{\frac{c_i}{2}}_{\text{adaptation effect}} + \underbrace{\frac{beE(c_j|\hat{s}_j)}{4b^2-e^2} + \frac{e^2E(c_i|\hat{s}_i)}{2(4b^2-e^2)}}_{\text{strategic complementarity effect}}, \tag{16}$$

$$p_{j2}^{*}(c_j;\hat{s}_j,\hat{s}_i) = \frac{a(2b+e)}{4b^2-e^2} + \frac{c_j}{2} + \frac{beE(c_i|\hat{s}_i)}{4b^2-e^2} + \frac{e^2E(c_j|\hat{s}_j)}{2(4b^2-e^2)} \tag{17}$$

Firms' equilibrium second-period prices depend on their own realized marginal costs and their expectations on rivals' costs. Take Firm i's equilibrium price as an example. The first item reflects how the demand intercept affects Firm $i's$ price, and the second term shows by how much Firm i adapts its second-period price to its costs c_i.

The third and the forth items capture the strategic complementarity between firms' pricing strategies, which depends on both firms' posterior expectations on each other's marginal costs. Specifically, the third item says that Firm i should raise p_{i2} if it expects an increase in Firm j's marginal cost. This is because Firm i anticipates that Firm j will raise p_{j2} due to Firm j's adaptation effect. Since firms' pricing strategies are strategic complements, Firm i should increase p_{i2} as well. The last item in (16) shows that Firm i's price also increases in its rival's posterior expectation on Firm i's marginal cost. Using the same argument for Firm i, Firm j will raise p_{j2} in response to a more optimistic posterior expectation about Firm i's marginal cost. As a consequence, Firm i will also increase p_{i2} due to the strategic complementarity effect.

Using the first order condition (9), Firm i's expected equilibrium profit in the second period can be written as

$$\pi_{i2}^{*}(c_i,\hat{s}_i,\hat{s}_j) = b\left(p_{i2}^{*} - c_i\right)^2, \tag{18}$$

where p_{i2}^{*} is defined by (16).

[3]Note that p_{i2} and p_{j2} cannot be lower than the marginal costs. This condition is satisfied when the demand intercept a is large enough. We assume that a is large enough throughout the paper to focus on interior solutions for the second-period prices.

3.2 First period: signaling and belief manipulation

Now, we consider the price competition in the first period. In this stage, Firm i receives a private signal s_i about its own c_i and updates its belief on c_i to $F(c_i|s_i)$. Firm i expects its rival's first-period price p_{j1} to be a function of the rival's signal s_j which is unobservable to Firm i. Conditional on signal s_i, Firm i's expected first-period profit from charging p_{i1} is given by

$$\begin{aligned} \Pi_{i1}(p_{i1}, p_{j1}|s_i) &\equiv \int_{s_j}\int_{c_i}(a - bp_{i1} + ep_{j1}(s_j))(p_{i1} - c_i)dF(c_i|s_i)dG_j(s_j) \\ &= (a - bp_{i1} + eE(p_{j1}(s_j)))(p_{i1} - E(c_i|s_i)). \end{aligned} \tag{19}$$

Its expected second-period profit from charging p_{i1} conditional on s_i is

$$\begin{aligned} \Pi_{i2}(s_i, \hat{s}_i) &\equiv \int_{c_i}\int_{\hat{s}_j}\pi^*_{i2}(c_i, \hat{s}_i, \hat{s}_j)dG_j(\hat{s}_j)dF(c_i|s_i) \\ &= \int_{c_i}\int_{\hat{s}_j} b(p^*_{i2} - c_i)^2\, dG_j(\hat{s}_j)dF(c_i|s_i), \end{aligned} \tag{20}$$

where the second equality follows from equation (18). Note that $\hat{s}_i$ is a function of p_{i1} by equation (7). In addition, Firm i's future inference about Firm j's private signal, $\hat{s}_j$, is a random variable to Firm i in the first period because $\hat{s}_j$ is a function of p_{j1} which in turn depends on Firm j's private signal s_j.

Firm i's problem in the first period is to choose price p_{i1} to maximize the sum of profits from the two periods:

$$\max_{p_{i1}} \Pi_{i1}(p_{i1}, p_{j1}|s_i) + \Pi_{i2}(s_i, \hat{s}_i). \tag{21}$$

Using (19) and (20), the first order condition is

$$a + bE(c_i|s_i) - 2bp_{i1} + eE(p_{j1}(s_j)) + \frac{\partial \Pi_{i2}(s_i, \hat{s}_i)}{\partial p_{i1}} = 0. \tag{22}$$

Symmetrically for Firm j, we have a similar first order condition

$$a + bE(c_j|s_j) - 2bp_{j1} + eE(p_{i1}(s_i)) + \frac{\partial \Pi_{j2}(s_j, \hat{s}_j)}{\partial p_{j1}} = 0. \tag{23}$$

Note that if $\frac{\partial \Pi_{i2}(s_i,\hat{s}_i)}{\partial p_{i1}} = 0$ in (22), the solution maximizes Firm i's first-period expected profit for a given rival's price p_{j1}. In other words, this is Firm i's myopic first-period best response when it ignores the signaling role of its own price on affecting the Firm j's response in the second period. Hence, $\frac{\partial \Pi_{i2}(c_i,\hat{s}_j|\hat{s}_i,s_i)}{\partial p_{i1}}$ captures Firm i's distortion in first-period price from the optimal myopic level in order to manipulate the rival's belief about the market environment in the second period. Specifically,

$$\begin{aligned}\frac{\partial \Pi_{i2}(s_i,\hat{s}_i)}{\partial p_{i1}} &= \frac{\partial \Pi_{i2}(s_i,\hat{s}_i)}{\partial \hat{s}_i}\frac{\partial \hat{s}_i}{p_{i1}} \\ &= \int_{a_i}\int_{\hat{s}_j} 2b(p_{i2}^* - c_i)\frac{\partial p_{i2}^*}{\partial E(c_i|\hat{s}_i)}\frac{\partial E(c_i|\hat{s}_i)}{\partial \hat{s}_i}\frac{\partial \hat{s}_i}{\partial p_{i1}}\, dG_j(\hat{s}_j)dF(c_i|s_i),\end{aligned} \tag{24}$$

where the second equality follows from (20), (16) and (7). Together with Assumption 2.1, we derive

$$\frac{\partial p_{i2}^*}{\partial E(c_i|\hat{s}_i)} = \frac{e^2}{2(4b^2-e^2)} > 0, \qquad \frac{\partial E(c_i|\hat{s}_i)}{\partial \hat{s}_i} = \overline{\tau}_i > 0, \quad \text{and} \quad \frac{\partial \hat{s}_i}{\partial p_{i1}} = \frac{1}{\alpha_{i1}\overline{\tau}_i} > 0.$$

Given $p_{i2}^* - c_i > 0$, $\frac{\partial \Pi_{i2}(s_i,\hat{s}_i)}{\partial p_{i1}} > 0$. So, Firm i distorts its first-period price above the optimal myopic price. This is because by raising p_{i1} by one unit, Firm i shifts up the rival's posterior expectation of Firm i's marginal cost by

$$\frac{\partial E(c_i|\hat{s}_i)}{\partial p_{i1}} = \frac{\partial E(c_i|\hat{s}_i)}{\partial \hat{s}_i}\frac{\hat{s}_i}{\partial p_{i1}} = \frac{1}{\alpha_{i1}}. \tag{25}$$

As a consequence, Firm j will raise its second-period price by $\frac{be}{4b^2-e^2}\frac{1}{\alpha_{i1}}$ (refer to (17)). Since Firm i's expected second-period profit increases in Firm j's second-period price, Firm i has a first-order gain in its second-period profit and a second-order loss in its first-period profit by distorting its first-period price above the optimal myopic price.

Based on the two first order conditions (22), (23) and the linear pricing functions (6), imposing consistency in beliefs (namely, the inferred signals must coincide with the true signals, $\hat{s}_i = s_i$, $\hat{s}_j =$

s_j), we can derive the first-period equilibrium pricing strategies. Define

$$\alpha_1^* = \frac{2b^2 - e^2}{4b^2 - e^2} \tag{26}$$

$$\beta_1^* = \frac{e\mu_c(4b^4 - 3b^2e^2 + e^4) - 4ab^2e^2 + abe^3 + 8ab^4 + ae^4}{(4b^2 - e^2)(2b^2 - e^2)(2b - e)} \tag{27}$$

Proposition 1 *There is a unique pair of linear first-period equilibrium pricing functions*

$$p_{i1}^*(s_i) = \alpha_1^* E(c_i|s_i) + \beta_1^* \tag{28}$$

$$p_{j1}^*(s_j) = \alpha_1^* E(c_j|s_j) + \beta_1^* \tag{29}$$

Proof. We first derive the expression for $\frac{\partial \Pi_{i2}(s_i, \hat{s}_i)}{\partial p_{i1}}$. Using the equilibrium second-period profit (18) and the equilibrium second-period price (16),

$$\begin{aligned}
\frac{\partial \Pi_{i2}(s_i, \hat{s}_i)}{\partial p_{i1}} &= \frac{\partial \Pi_{i2}(s_i, \hat{s}_i)}{\partial \hat{s}_i} \frac{\partial \hat{s}_i}{p_{i1}} \\
&= \int_{a_i} \int_{\hat{s}_j} 2b(p_{i2}^* - c_i) \frac{\partial p_{i2}^*}{\partial E(c_i|\hat{s}_i)} \frac{\partial E(c_i|\hat{s}_i)}{\partial \hat{s}_i} \frac{\partial \hat{s}_i}{\partial p_{i1}} \, dG_j(\hat{s}_j) dF(c_i|s_i) \\
&= \frac{be^2}{(4b^2 - e^2)\alpha_{i1}} \left[\frac{e^2 E(c_i|\hat{s}_i)}{2(4b^2 - e^2)} - \frac{E(c_i|s_i)}{2} + \frac{a(2b + e) + be\mu_c}{4b^2 - e^2} \right].
\end{aligned} \tag{30}$$

The second equality follows from (20) and the last equality is obtained after using p_{i2}^* in (16) and substituting

$$\frac{\partial p_{i2}^*}{\partial E(c_i|\hat{s}_i)} = \frac{e^2}{2(4b^2 - e^2)}, \qquad \frac{\partial E(c_i|\hat{s}_i)}{\partial \hat{s}_i} = \overline{\tau}_i, \quad \text{and} \quad \frac{\partial \hat{s}_i}{\partial p_{i1}} = \frac{1}{\alpha_{i1}\overline{\tau}_i},$$

which are derived from (16), Assumption 2.1 and (7), respectively. After imposing consistent beliefs, namely $\hat{s}_i = s_i, \ \hat{s}_j = s_j$, it follows that

$$\frac{\partial \Pi_{i2}(s_i)}{\partial p_{i1}} = \frac{be^2(e^2 - 2b^2)}{(4b^2 - e^2)^2 \alpha_{i1}} E(c_i|s_i) + \frac{abe^2(2b + e) + b^2e^3\mu_c}{(4b^2 - e^2)^2 \alpha_{i1}}. \tag{31}$$

Letting $\hat{s}_i = s_i$ and $\hat{s}_j = s_j$ in (23) and substituting $E_{s_j}(p_{j1}(s_j)) = \alpha_{j1}\mu_c + \beta_{j1}$ together with (31) into equation (23), the first-period prices p_{i1} and p_{j1} can be written as the following:

$$p_{i1} = \left[\frac{1}{2} + \frac{e^2(e^2-2b^2)}{2(4b^2-e^2)^2\alpha_{i1}}\right] E(c_i|s_i) + \frac{e\mu_c\alpha_{j1} + e\beta_{j1}}{2b} + \frac{be^3\mu_c}{2(4b^2-e^2)^2\alpha_{i1}} + \frac{ae^2(2b+e)}{2(4b^2-e^2)^2\alpha_{i1}} + \frac{a}{2b} \quad (32)$$

$$p_{j1} = \left[\frac{1}{2} + \frac{e^2(e^2-2b^2)}{2(4b^2-e^2)^2\alpha_{i1}}\right] E(c_j|s_j) + \frac{e\mu_c\alpha_{i1} + e\beta_{i1}}{2b} + \frac{be^3\mu_c}{2(4b^2-e^2)^2\alpha_{j1}} + \frac{ae^2(2b+e)}{2(4b^2-e^2)^2\alpha_{j1}} + \frac{a}{2b}. \quad (33)$$

The linear pricing rule (6) together with (32) and (33) implies

$$\alpha_{i1} = \frac{1}{2} + \frac{e^2(e^2-2b^2)}{2(4b^2-e^2)^2\alpha_{i1}} \quad (34)$$

$$\beta_{i1} = \frac{e\mu_c\alpha_{j1} + e\beta_{j1}}{2b} + \frac{be^3\mu_c}{2(4b^2-e^2)^2\alpha_{i1}} + \frac{ae^2(2b+e)}{2(4b^2-e^2)^2\alpha_{i1}} + \frac{a}{2b} \quad (35)$$

$$\alpha_{j1} = \frac{1}{2} + \frac{e^2(e^2-2b^2)}{2(4b^2-e^2)^2\alpha_{j1}} \quad (36)$$

$$\beta_{j1} = \frac{e\mu_c\alpha_{i1} + e\beta_{i1}}{2b} + \frac{be^3\mu_c}{2(4b^2-e^2)^2\alpha_{j1}} + \frac{ae^2(2b+e)}{2(4b^2-e^2)^2\alpha_{j1}} + \frac{a}{2b}. \quad (37)$$

There are two sets of solutions $\{\alpha_{i1}, \beta_{i1}, \alpha_{j1}, \beta_{j1}\}$ for the above system of equations. However, the first order condition is sufficient only at the following solution:

$$\alpha_{i1} = \alpha_{j1} = \frac{2b^2-e^2}{4b^2-e^2} \quad (38)$$

$$\beta_{i1} = \beta_{j1} = \frac{e\mu_c(4b^4-3b^2e^2+e^4) - 4ab^2e^2 + abe^3 + 8ab^4 + ae^4}{(4b^2-e^2)(2b^2-e^2)(2b-e)}. \quad (39)$$

To see this, the second derivative of Firm i's sum of expected profits from the two periods is

$$\begin{aligned}
-2b + \frac{\partial}{\partial p_{i1}}\left(\frac{\partial \Pi_{i2}(s_i, \hat{s}_i)}{\partial p_{i1}}\right) &= -2b + \frac{be^2}{(4b^2-e^2)\alpha_{i1}} \frac{e^2}{2(4b^2-e^2)} \frac{\partial E(c_i|\hat{s}_i)}{\partial p_{i1}}, \\
&= -2b + \frac{be^4}{2(4b^2-e^2)^2} \frac{1}{(\alpha_{i1})^2}, \quad (40)
\end{aligned}$$

where the second equality follows from (30) and the third equality follows from (25). Thus, the second derivative is negative if and only if

$$\frac{e^2}{2(4b^2-e^2)} < \alpha_{i1},$$

which is satisfied at the root $\alpha_{i1} = \frac{2b^2-e^2}{4b^2-e^2}$, given $0 < e < b$, but violated at the other root $\alpha_{i1} = \frac{e^2}{2(4b^2-e^2)}$. ■

To illustrate how signaling incentives distort firms' first-period prices, we derive the optimal myopic first-period price. Let $\frac{\partial \pi_{i2}(s_i,\hat{s}_i)}{\partial p_{i1}} = \frac{\partial \pi_{j2}(s_j,\hat{s}_j)}{\partial p_{j1}} = 0$ in (22) and (23), respectively, and solve for p_{i1} and p_{j1}. It can be verified that the optimal myopic first-period prices are:

$$p_{i1}^M = \frac{1}{2}E(c_i|s_i) + \frac{2a + e\mu_c}{2(2b-e)}, \quad (41)$$

$$p_{j1}^M = \frac{1}{2}E(c_j|s_j) + \frac{2a + e\mu_c}{2(2b-e)}. \quad (42)$$

Given $e < b$, the myopic first-period pricing function is steeper than the strategic first-period pricing function as is illustrated in Figure 2.

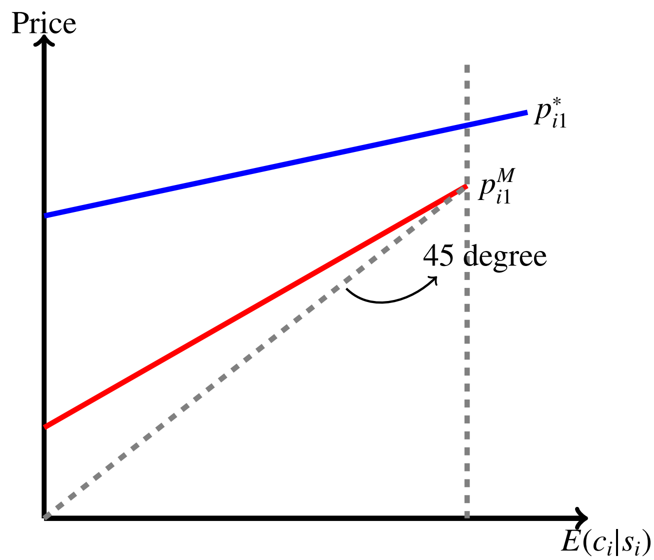

Figure 2: Comparison of First Period Pricing Functions

In this figure, the blue line represents the first-period equilibrium pricing function (28) and the red line represents the optimal myopic first-period pricing function. Once the the red line crosses the 45 degree line Firm i stops selling. The blue line is uniformly higher than the red line which results from firms' signaling incentives. The divergence between the two pricing function decreases in Firm $i's$ expected cost. This implies that firms' signaling incentives weakens when their expected costs increases. Intuitively, firms' prices are bounded above by the demand intercept.

When firms' expected costs go up, the range for feasible prices (between expected cost and demand intercept) becomes narrower and hence there is less room for firms to distort their price upward.

3.3 Information Acquisition

When a firm chooses the quality of its signal, the firm takes into account the impact of the information gleaned on profits in both periods. We analyze how the signal precision affect the firm's first and second-period expected profits separately. To begin, we start with firms' expected first-period profit. Recall (19), conditional on the signal s_i, Firm i's expected profit is

$$\begin{aligned}\Pi_{i1}(s_i) &= \left(a - bp_{i1} + eE\left(p_{j1}\left(s_j\right)\right)\right)(p_{i1} - E\left(c_i|s_i\right)) \\ &= -b\left(p_{i1}\right)^2 + bp_{i1}E\left(c_i|s_i\right) + \left(a + eE\left(p_{j1}\left(s_j\right)\right)\right)(p_{i1} - E(c_i|s_i)) . \end{aligned} \tag{43}$$

Taking expectation over s_i, Firm i's ex-ante expected profit is

$$\mathbb{E}\Pi_{i1} = -b\mathbb{E}\left[(p_{i1})^2\right] + b\mathbb{E}\left[p_{i1}E(c_i|s_i)\right] + \left(a + e\mathbb{E}\left[p_{j1}\right]\right)(\mathbb{E}(p_{i1}) - \mu_c) . \tag{44}$$

The third item in (44) does not involve τ_{ϵ_i} and hence is irrelevant for the choice of signal precision. From the linear pricing strategy (6), the second item in (44) can be written as

$$\begin{aligned}b\mathbb{E}\left(p_{i1}E\left(c_i|s_i\right)\right) &= b\mathbb{E}\left(p_{i1}\left(\frac{p_{i1}}{\alpha_{i1}} - \frac{\beta_{i1}}{\alpha_{i1}}\right)\right) \\ &= \frac{b}{\alpha_{i1}}\mathbb{E}\left((p_{i1})^2\right) - \frac{b\beta_{i1}}{\alpha_{i1}}\mathbb{E}\left(p_{i1}\right), \end{aligned} \tag{45}$$

where $\mathbb{E}(p_{i1}) = \alpha_{i1}\mu_c + \beta_{i1}$ and does not depend on τ_{ϵ_i}. Collecting all terms not involving τ_{ϵ_i} into "other," we have

$$\begin{aligned}
\mathbb{E}\Pi_{i1} &= b\left(\frac{1}{\alpha_{i1}} - 1\right)\mathbb{E}(p_{i1})^2 + \text{other} \\
&= b\left(\frac{1}{\alpha_{i1}} - 1\right)\text{Var}(p_{i1}) + \text{other} \\
&= b\alpha_{i1}(1-\alpha_{i1})\text{Var}(E(c_i|s_i)) + \text{other} \\
&= b\alpha_{i1}(1-\alpha_{i1})\frac{\tau_{\epsilon_i}}{(\tau_c + \tau_{\epsilon_i})\tau_c} + \text{other},
\end{aligned} \tag{46}$$

where the third equality is derived using $\text{Var}(p_{i1}) = \text{Var}(\alpha_{i1}E(c_i|s_i) + \beta_{i1}) = (\alpha_{i1})^2\text{Var}(E(c_i|s_i))$, and the last equation follows from Assumption 2.1. The marginal impact of a more precise signal on Firm i's first-period profit is

$$\frac{\partial \mathbb{E}\Pi_{i1}}{\partial \tau_{\epsilon_i}} = b\alpha_{i1}(1-\alpha_{i1})\frac{1}{(\tau_c + \tau_{\epsilon_i})^2}. \tag{47}$$

After substituting Firm i's equilibrium pricing rule (26) in (46) and (47), we obtain:

Lemma 3.1 *Firm i's expected equilibrium first-period profit is:*

$$\mathbb{E}\Pi_{i1}^* = \frac{2b^3(2b^2 - e^2)}{(4b^2 - e^2)^2}\frac{\tau_{\epsilon_i}}{(\tau_c + \tau_{\epsilon_i})\tau_c} + \text{other}; \tag{48}$$

It has a gain in the first period when the quality of its signal is improved, and the marginal gain is

$$\frac{\partial \mathbb{E}\Pi_{i1}^*}{\partial \tau_{\epsilon_i}} = \frac{2b^3(2b^2 - e^2)}{(4b^2 - e^2)^2}\frac{1}{(\tau_c + \tau_{\epsilon_i})^2} > 0. \tag{49}$$

In the first period, each firm's price is only based on the posterior expectation of its own cost. Since firms' costs are idiosyncratic, their first-period prices are independent. As a result, a more precise signal will help Firm i better adjust its first-period price to its cost without changing Firm j's first-period price.

The impact of more precise signals on firms' second-period profits is more involved because their second-period prices are positively correlated through two channels . Recall (16) and (17), each firm's second-period price depends on its realized cost, the rival's posterior expectation of its realized cost and its posterior expectation of the rival's realized cost. First, firms' realized costs are positively correlated with the rivals' posterior expectations of their realized costs. For example, c_i in (16) is positively correlated with $E(c_i|\hat{s}_i)$ in (17). Second, firms' posterior expectations of rivals' costs enter both firms' prices.

In the second period, Firm i already learns its realized cost from its first-period profit and uses it to formulate its second-period price. So, unlike in the first period, a more precise signal s_i does not directly help Firm i to better attune its second-period price to its cost. (It can be seen in (9) that p_{i2} does not depend on s_i, holding the rival's price p_{j2} constant.) But a more precise signal will increase the correlation between Firm i's realized cost c_i and Firm j's conjecture of Firm i's cost $E(c_i|s_i)$. As a result, firms' second-period prices will be more positively correlated when Firm i's information become more accurate, which reduces Firm i's second-period expected profit. To see the intuition, note that an increase in Firm i's cost is not fully passed on to its second-period price. In fact, by (16), p_{i2} increases by half a unit in response to a one unit increase in its cost c_i. Hence, when c_i increases, although Firm i raises it price p_{i2}, its profit margin decreases and it wishes to sell fewer units. However, when Firm i's signal become more precise, a high price p_{i2} is more likely to come with a high p_{j2} which increases Firm i's residual demand and forces Firm i to sell more than it wants at the low profit margin. In short, when firms' signals become more precise, their second-period prices are more correlated in an undesirable way. We summarize this in the following lemma:

Proposition 2 *Each firm's expected second-period profit decreases in its own signal precise. Specifically,*

$$\mathbb{E}\Pi^*_{i2} = -\frac{be^2(8b^2-3e^2)}{4(4b^2-e^2)^2}\frac{\tau_{\epsilon_i}}{(\tau_{\epsilon_i}+\tau_c)\tau_c} + \text{other} \tag{50}$$

$$\frac{\partial \mathbb{E}\Pi^*_{i2}}{\partial \tau_{\epsilon_i}} = -\frac{be^2(8b^2-3e^2)}{4(4b^2-e^2)^2}\frac{1}{(\tau_{\epsilon_i}+\tau_c)^2} < 0. \tag{51}$$

Proof. Firm i's second-period profit is

$$\begin{aligned}\Pi_{i2}^{*} &= (a - bp_{i2}^{*} + ep_{j2}^{*})(p_{i2}^{*} - c_i) \quad (52)\\ &= a(p_{i2}^{*} - c_i) - bp_{i2}^{*}(p_{i2}^{*} - c_i) + ep_{j2}^{*}(p_{i2}^{*} - c_i)\\ \mathbb{E}\Pi_{i2}^{*} &= \mathbb{E}(a(p_{i2}^{*} - c_i)) - b\mathbb{E}(p_{i2}^{*}(p_{i2}^{*} - c_i)) + e\mathbb{E}(p_{j2}^{*}(p_{i2}^{*} - c_i)), \quad (53)\end{aligned}$$

where the expectation is taken over c_i, s_i and s_j. Since p_{i2}^{*} is linear in $E(c_i|s_i)$ and $E(c_j|s_j)$, the first item in (53) does not involve τ_{ϵ_i} or τ_{ϵ_j} and is irrelevant for Firm i's choice of precision. Firm i's signal precision affects $\mathbb{E}(\Pi_{i2}^{*})$ through the second and the third items in (53). Denote

$$\begin{aligned}z_0 &= \frac{a(2b + e)}{4b^2 - e^2}\\ z_1 &= \frac{be}{4b^2 - e^2}\\ z_2 &= \frac{e^2}{2(4b^2 - e^2)}. \quad (54)\end{aligned}$$

Using (16),

$$\begin{aligned}\mathbb{E}[p_{i2}^{*}(p_{i2}^{*} - c_i)] &= \mathbb{E}[(z_2)^2(E(c_i|s_i))^2 + z_1^2(E(c_j|s_j))^2] \quad (55)\\ &= (z_2)^2\mathbb{E}(E(c_i|s_i))^2 + \text{other}\\ &= (z_2)^2\text{Var}(E(c_i|s_i)) + \text{other}, \quad (56)\end{aligned}$$

where items not involving τ_{ϵ_i} are collected in the "other" term. Similarly,

$$\begin{aligned}\mathbb{E}[(p_{i2}^{*} - c_i)p_{j2}^{*}] &= \mathbb{E}[-\frac{z_1}{2}c_iE(c_i|s_i) + z_1z_2(E(c_i|s_i))^2 + \frac{z_1}{2}c_jE(c_j|s_j) + z_1z_2(E(c_j|s_j))^2] \quad (57)\\ &= -\frac{z_1}{2}\mathbb{E}[c_iE(c_i|s_i)] + z_1z_2\mathbb{E}(E(c_i|s_i))^2 + \text{other}\\ &= (-\frac{z_1}{2} + z_1z_2)\mathbb{E}(E(c_i|s_i))^2 + \text{other}\\ &= (-\frac{z_1}{2} + z_1z_2)\text{Var}(E(c_i|s_i)) + \text{other}, \quad (58)\end{aligned}$$

where the third equality is obtained using $\mathbb{E}[c_i E(c_i|s_i)] = \mathbb{E}E[c_i E(c_i|s_i)|s_i] = \mathbb{E}(E(c_i|s_i))^2$ (the law of iterated expectation). Substituting $\mathbb{E}[p_{i2}^*(p_{i2}^* - c_i)]$ and $\mathbb{E}[(p_{i2}^* - c_i)p_{j2}^*]$ in $\mathbb{E}(\Pi_{i2}^*)$, it follows that

$$\begin{aligned}\mathbb{E}\Pi_{i2}^* &= [-b(z_2)^2 - \frac{z_1 e}{2} + z_1 z_2 e]\text{Var}(E(c_i|s_i)) + \text{other} \\ &= -\frac{be^2(8b^2 - 3e^2)}{4(4b^2 - e^2)^2}\frac{\tau_{\epsilon_i}}{(\tau_c + \tau_{\epsilon_i})\tau_c} + \text{other},\end{aligned} \tag{59}$$

where the second equality is obtained after substituting z_0, z_1 and z_2 and using Assumption 2.1. It is easy to verify that

$$\frac{\partial \mathbb{E}\Pi_{i2}^*}{\partial \tau_{\epsilon_i}} = -\frac{be^2(8b^2 - 3e^2)}{4(4b^2 - e^2)^2}\frac{1}{(\tau_{\epsilon_i} + \tau_c)^2}. \tag{60}$$

■

Firm i's objective in the information acquisition stage is to maximize the net expected profits from the two periods, namely

$$\max_{\tau_{\epsilon_i}} \mathbb{E}\Pi_{i1}^*(\tau_{\epsilon_i}) + \mathbb{E}\Pi_{i2}^*(\tau_{\epsilon_i}) - k(\tau_{\epsilon_i}) \tag{61}$$

Given the convexity of $k(\cdot)$, the objective function is strictly concave in τ_{ϵ_i}. We have shown that Firm i gains in the first period but loses in the second period when its signal becomes more accurate. Following (49) and (51), it can be verified that Firm i's total gain from a more precise signal is

$$\frac{\partial \mathbb{E}\Pi_{i1}^*}{\partial \tau_{\epsilon_i}} + \frac{\partial \mathbb{E}\Pi_{i2}^*}{\partial \tau_{\epsilon_i}} = \frac{b(4b^2 - e^2)(4b^2 - 3e^2)}{4(4b^2 - e^2)^2}\frac{1}{(\tau_{\epsilon_i} + \tau_c)^2} > 0. \tag{62}$$

Firm i choose the precision τ_{ϵ_i} to balance its gain in total profit and the information acquisition cost. Its solution is summarized in the following proposition:

Proposition 3 *There exists a unique equilibrium in which firms use linear pricing strategies. In the equilibrium, each firm's optimal signal precision τ_ϵ^* is determined by the unique solution to*

$$\frac{b(4b^2 - e^2)(4b^2 - 3e^2)}{4(4b^2 - e^2)^2}\frac{1}{(\tau_c + \tau_\epsilon)^2} = k'(\tau_\epsilon) \tag{63}$$

and its optimal prices in the two periods are (16) and (28) respectively.

Proof. It is easy to verify that (62) is strictly decreasing and convex in τ_ϵ and that $\lim_{\tau_\epsilon \to \infty} \frac{b(4b^2-e^2)(4b^2-3e^2)}{4(4b^2-e^2)^2} \frac{1}{(\tau_a+\tau_\epsilon)^2} = 0$. On the other hand, we've assumed that $k'(.)$ is strictly increasing and continuous, $k'(0) = 0$ and $\lim_{\tau_\epsilon \to +\infty} k'(\tau_\epsilon) = +\infty$, hence there's a unique solution to (63). ■

4 Comparison with Myopic Firms

In this section, we investigate how signaling incentives affect firms' information acquisition decisions. To answer this question, we use myopic firms' choice of signal precisions as the benchmark and compare it with strategic firms' choice of signal precisions.

In the previous section, we show that strategic firms distort their first-period prices above the optimal myopic prices in order to fool rivals to believe that they have high costs and hence will charge high prices in the second period. Nevertheless, in equilibrium, by consistency, firms' inferences of their rival's signals are correct, hence the second-period prices ((16) and (17)) and the expected second-period profits (18) are the same for strategic and myopic firms. As a result, for a fixed pair of signal precisions, firms' losses in the second period from improved signal precisions are the same when they behave strategically and myopically. This implies that any difference between strategic and myopic firms' choices of signal precisions is driven by the different marginal impacts of improved signal precisions on firms' first-period profits.

We compare strategic and myopic firms' choices of signal precisions in the following proposition:

Proposition 4 *i) Strategic firms will acquire less precise signals than myopic firms. ii) The divergence between strategic and myopic firms' choices of signal precisions increases when the degree of substitution between the two goods increases.*

Proof. Let $\tau_{\epsilon_i}^M$ denote Firm i's optimal signal precision when it is myopic. Consider myopic firms' signal acquisition decisions. Firm i's marginal gain from a more precise s_i is

$$\begin{aligned}\frac{\partial \mathbb{E}\Pi_i^M}{\partial \tau_{\epsilon_i}} &= \frac{\partial \mathbb{E}\Pi_{i1}^M}{\partial \tau_{\epsilon_i}} + \frac{\partial \mathbb{E}\Pi_{i2}^M}{\partial \epsilon_i} \\ &= \frac{b}{4}\frac{1}{(\tau_c + \tau_{\epsilon_i})^2} - \frac{be^2(8b^2 - 3e^2)}{4(4b^2 - e^2)^2}\frac{1}{(\tau_c + \tau_{\epsilon_i})^2} \\ &= \frac{b(4b^2 - 2e^2)^2}{4(4b^2 - e^2)^2}\frac{1}{(\tau_c + \tau_{\epsilon_i})^2} > 0, \end{aligned} \tag{64}$$

where the second equality is obtained by substituting (47) with $\alpha_{i1} = \frac{1}{2}$ for $\frac{\partial \mathbb{E}\Pi_{i1}^M}{\partial \tau_{\epsilon_i}}$ and (51) for $\frac{\partial \mathbb{E}\Pi_{i2}^M}{\partial \epsilon_i}$. Myopic firms' optimal choice of signal precisions τ_ϵ^M is uniquely determined by

$$\frac{b(4b^2 - 2e^2)^2}{4(4b^2 - e^2)^2}\frac{1}{(\tau_c + \tau_\epsilon)^2} = k'(\tau_\epsilon). \tag{65}$$

Note the LHS of (65) is decreasing in τ_ϵ, positive at $\tau_\epsilon = 0$ and approaching 0 as $\tau_\epsilon \to +\infty$. Given the assumptions $k''(\cdot) > 0$, $k'(0) = 0$ and $lim_{\epsilon \to +\infty} k'(\epsilon) = \infty$, there exists a unique solution $\tau_\epsilon^M \in (0, \infty)$ for (65).

Subtracting the LHS of (63) from the LHS of (65),

$$\left(\frac{b(4b^2 - 2e^2)^2}{4(4b^2 - e^2)^2)} - \frac{b(4b^2 - e^2)(4b^2 - 3e^2)}{4(4b^2 - e^2)^2}\right)\frac{1}{(\tau_\epsilon + \tau_c)^2} = \frac{be^4}{4(4b^2 - e^2)}\frac{1}{(\tau_\epsilon + \tau_c)^2} > 0, \tag{66}$$

which implies $\tau_\epsilon^* < \tau_\epsilon^M$. Since (66) increases in e, the degree of substitution between the two goods, the difference in signal precisions $\tau_\epsilon^M - \tau_\epsilon^*$ increases in e. ■

When a firm adopts linear pricing strategies in the first period, the marginal impact of its improved signal precision on the firm's expected first-period profit is proportional to $b\alpha_{i1}(1 - \alpha_{i1})$ as is shown in (47). Recall (41) and (42), the slope of myopic firms' first-period pricing strategies is $\alpha_{i1}^M = \alpha_{j1}^M = \frac{1}{2}$, which maximizes the term $b\alpha_{i1}(1 - \alpha_{i1})$. By Proposition 1, the slope of strategic firms' first-period pricing strategies is $\alpha_{i1}^* = \alpha_{j1}^* = \frac{2b^2 - e^2}{4b^2 - e^2} < \frac{1}{2}$. As a result, strategic firms' marginal gain from improved signal precisions is smaller than myopic firms and therefore strategic firms will acquire less precise signals compared with myopic firms. The relation between τ_ϵ^* and τ_ϵ^M can be illustrated by Figure 3.

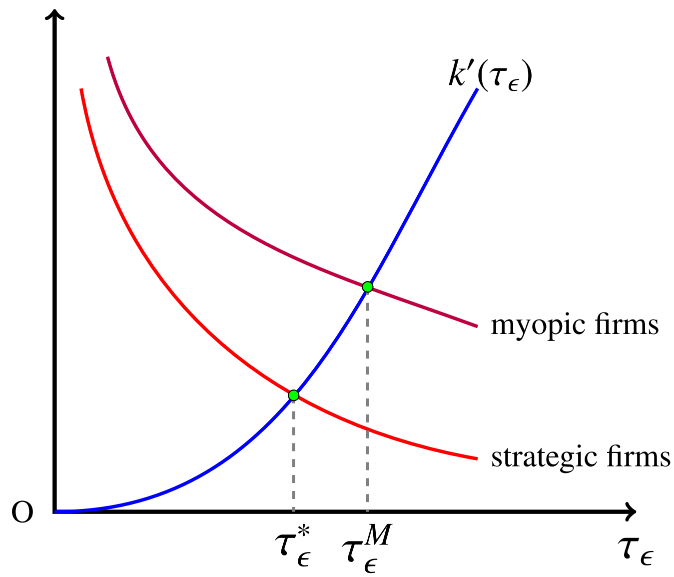

Figure 3: Comparison of τ_ϵ^* and τ_ϵ^M

To understand the intuition for the divergence between strategic and myopic firms' choices of signal precisions, we revisit firms' first-period profits conditional on their own signals. For a given pair of signal precisions, Firm i's myopic equilibrium first-period profit is

$$\Pi_{i1}^M(s_i) = (a - bp_{i1}^M + eE(p_{j1}^M(s_j)))(p_{i1}^M - E(c_i|s_i)), \tag{67}$$

where p_{i1}^M and p_{j1}^M are determined by (41) and (42), respectively. When Firm i increases the precision of its signal, it will affect Π_{i1}^M through $E(c_i|s_i)$. Hence, to evaluate how Firm i's signal precision affect its first-period profit, it is suffice to investigate how its first-period profit changes in $E(c_i|s_i)$. Take the partial derivative

$$\begin{aligned}\frac{\partial \Pi_{i1}^M(s_i)}{\partial E(c_i|s_i)} =& \frac{\partial \Pi_{i1}^M}{\partial E(c_i|s_i)} + \frac{\partial \Pi_{i1}^M}{\partial p_{i1}^M}\frac{\partial p_{i1}^M}{\partial E(c_i|s_i)} \\ =& \frac{\partial \Pi_{i1}^M}{\partial E(c_i|s_i)} \\ =& -\underbrace{(a - bp_{i1}^M + eE(p_{j1}^M(s_j)))}_{q_{i1}^M},\end{aligned} \tag{68}$$

where the second equality follows from the envelope theorem. Because p_{i1}^M maximizes Firm i's first-period profit, a change in $E(c_i|s_i)$ does not have an indirect impact on its profit through p_{i1}^M. Partial derivative (68) says that when Firm i's posterior expected cost $E(c_i|s_i)$ increases, it costs the firm more to produce the quantity q_{i1}^M and thus reduces its profit by q_{i1}^M.

When Firm i increases the precision of its signal, its posterior expectation $E(c_i|s_i)$ will be more correlated with its signal and less correlated with the prior mean μ_c. Hence, higher precision of s_i implies a larger variation in $E(c_i|s_i)$. Now, we investigate whether Firm i benefit from a larger variance in $E(c_i|s_i)$. If Firm i's first-period quantity q_{j1}^M were constant in $E(c_i|s_i)$, it would have a symmetric gain and loss when $E(c_i|s_i)$ changes and therefore does not benefit or lose when $E(c_i|s_i)$ varies. However, recall that p_{i1}^M is strictly increasing in $E(c_i|s_i)$. As a result, Firm i will reduce the quantity q_{i1}^M when $E(c_i|s_i)$ increases and increase the quantity when $E(c_i|s_i)$ decreases. This implies that Firm i has less to lose and more to gain when $E(c_i|s_i)$ varies and therefore benefits from a more precise signal.

We use the same approach to evaluate the impact of more precise signals on strategic firms' first-period profits and draw a comparison with myopic firms. Conditional on s_i, strategic Firm i's equilibrium first-period profit is

$$\Pi_{i1}^*(s_i) = (a - bp_{i1}^* + eE(p_{j1}^*(s_j)))(p_{i1}^* - E(c_i|s_i)), \tag{69}$$

where p_{i1}^* and p_{j1}^* are determined by (28) and (29). Take the partial derivative

$$\begin{aligned} \frac{\partial \Pi_{i1}^*(s_i)}{\partial E(c_i|s_i)} &= \frac{\partial \Pi_{i1}^*}{\partial E(c_i|s_i)} + \frac{\partial \Pi_{i1}^*}{\partial p_{i1}^*}\frac{\partial p_{i1}^*}{\partial E(c_i|s_i)} \\ &= -\underbrace{(a - bp_{i1}^* + eE(p_{j1}^*(s_j)))}_{q_{i1}^*} + \underbrace{\frac{\partial \Pi_{i1}^*}{\partial p_{i1}^*}}_{<0}\alpha_1^*, \end{aligned} \tag{70}$$

where the second item in (70) follows from (28) with α_1^* defined in (26). Different from the case of myopic firms, now a change in $E(c_i|s_i)$ will have a negative indirect impact on $\Pi_{i1}^*(s_i)$ through the price p_{i1}^*. This is because p_{i1}^* is distorted above the optimal myopic first-period price. When $E(c_i|s_i)$ increases, Firm i will raise p_{i1}^*, but this will increase the price distortion and reduce Firm i's profit.

Similar to the case of myopic firms, strategic firms have a direct cost benefit when $E(c_i|s_i)$ varies, which is captured by the first item in (70). Nevertheless, its benefit is dampened by the variation in price p^*_{i1}, which is captured by the second item in (70). Since p^*_{i1} is greater than the optimal myopic price and π^*_{i1} is concave in Firm i's price, when p^*_{i1} varies, Firm i's loss from an increase in p^*_{i1} outweighs its gain from an decrease in p^*_{i1} and thus it suffers from a more precise signal.

In summary, both myopic and strategic firms benefit from improved signal precisions, nevertheless strategic firms benefit less and hence will acquire less precise signals than myopic firms. This is because strategic firms suffers from variation in their own first-period prices whereas myopic firms do not bear this loss.

5 Welfare

In this section, we investigate the externality of strategic firms' information acquisition decisions on their rivals' profits and on consumer welfare. Since our focus is on the inefficiency of information acquisition not on the inefficiency of firms' pricing strategies, we study how firms' choices of signal precisions differ from a trade association or the social planner, holding firms' pricing strategies constant. We first consider a trade association which chooses τ_{ϵ_i} and τ_{ϵ_j} to maximize firms' joint profit, provided that firms' first-period and second-period prices are ((28), (29)) and ((16), (17)).

5.1 Industry Profit

To gain a clear understanding of the source of inefficiency, if any, we compare the trade association's gain from improved signal precisions with individual firms' gain in each period.

We start with the first period. In the section of "Information Acquisition", equation (46) shows that Firm i's expected first-period profit is independent on Firm j's signal precision τ_{ϵ_j}. The same holds true for Firm j. So, a firm's information acquisition decision does not entail any externality on the rival's first-period profit. Let TP^*_t denote firms' joint equilibrium profits in period t. The above analysis says that

$$\frac{\partial TP_1^*}{\partial \tau_{\epsilon_i}} = \frac{\partial \mathbb{E}\Pi_{i1}^*}{\partial \tau_{\epsilon_i}} \tag{71}$$

where $\frac{\partial \mathbb{E}\Pi_{i1}^*}{\partial \tau_{\epsilon_i}}$ is determined in (49).

We continue the analysis to the second period. When a firm increases its signal precision, it exerts a positive externality on the rival's expected second-period profit. Section "Information Acquisition" has shown that when a firm increases the precision of its signal, it increases the positive correlation between the two firms' second-period prices. This will benefit the rival when their pricing strategies are strategic complements. The comparison between the trade association and the firms' choices of signal precisions is summarized in the following proposition:

Proposition 5 *From the trade association's perspective, the qualities of firms' signals are inefficiently low. This inefficiency is driven by the positive externality of firms' improved signals on the rivals' second-period profits.*

Proof. Trade association's total surplus is $\sum_{t=1}^{2} TP_t^*$, with $TP_t^* = \mathbb{E}\Pi_{it}^* + \mathbb{E}\Pi_{jt}^*$. The trade association chooses τ_{ϵ_i} and τ_{ϵ_j} to maximize $TP_t^* - k(\tau_{\epsilon_i}) - k(\tau_{\epsilon_j})$.

We first derive the impact of τ_{ϵ_i} on TP_2^*. Take the partial derivative

$$\frac{\partial TP_2^*}{\partial \tau_{\epsilon_i}} = \frac{\partial \mathbb{E}\Pi_{i2}^*}{\partial \tau_{\epsilon_i}} + \frac{\partial \mathbb{E}\Pi_{j2}^*}{\partial \tau_{\epsilon_i}}, \tag{72}$$

where $\frac{\partial \mathbb{E}\Pi_{i2}^*}{\partial \tau_{\epsilon_i}}$ is derived in (51). Next, we derive $\frac{\partial \mathbb{E}\Pi_{j2}^*}{\partial \tau_{\epsilon_i}}$. Firm j's expected second-period profit is

$$\begin{aligned}\mathbb{E}\Pi_{j2}^* &= \mathbb{E}\left[(a - bp_{j2}^* + ep_{i2}^*)(p_{j2}^* - c_j)\right] \\ &= \mathbb{E}(a(p_{j2}^* - c_j)) - b\mathbb{E}(p_{j2}^*(p_{j2}^* - c_j)) + e\mathbb{E}(p_{i2}^*(p_{j2}^* - c_j)),\end{aligned} \tag{73}$$

where the first item does not involve precisions. We now derive the second and the third item in

(73). Using (55) and (57) and by symmetry,

$$\begin{aligned}
\mathbb{E}[p_{j2}^*(p_{j2}^* - c_j)] &= \mathbb{E}[(z_2)^2(E(c_j|s_j))^2 + z_1^2(E(c_i|s_i))^2] \\
&= z_1^2\mathbb{E}(E(c_i|s_i))^2 + \text{other} \quad (74)\\
&= z_1^2\text{Var}(E(c_i|s_i)) + \text{other} \quad (75)\\
\mathbb{E}[(p_{j2}^* - c_j)p_{i2}^*] &= \mathbb{E}\left[-\frac{z_1}{2}c_jE(c_j|s_j) + z_1z_2(E(c_j|s_j))^2 + \frac{z_1}{2}c_iE(c_i|s_i) + z_1z_2(E(c_i|s_i))^2\right] \\
&= \frac{z_1}{2}\mathbb{E}(c_iE(c_i|s_i)) + z_1z_2\mathbb{E}(E(c_i|s_i))^2 + \text{other} \\
&= \left(\frac{z_1}{2} + z_1z_2\right)\mathbb{E}(E(c_i|s_i))^2 + \text{other} \quad (76)\\
&= \left(\frac{z_1}{2} + z_1z_2\right)\text{Var}(E(c_i|s_i)) + \text{other}. \quad (77)
\end{aligned}$$

In the above derivation, all the terms not involving τ_{ϵ_i} are relegated to "other". We obtain (76) by applying the law of iterated expectation to $\mathbb{E}[c_iE(c_i|s_i)] = \mathbb{E}E[c_iE(c_i|s_i)|s] = \mathbb{E}\{[E(c_i|s_i)]^2\}$. Substitute (75) and (77) into (73):

$$\begin{aligned}
\mathbb{E}\Pi_{j2}^* &= -bz_1^2\text{Var}(E(c_i|s_i)) + e(\frac{z_1}{2} + z_1z_2)\text{Var}(E(c_i|s_i)) + \text{other} \\
&= \left[-bz_1^2 + e(\frac{z_1}{2} + z_1z_2)\right]\text{Var}(E(c_i|s_i)) + \text{other} \\
&= \left[-bz_1^2 + e(\frac{z_1}{2} + z_1z_2)\right]\frac{\tau_{\epsilon_i}}{(\tau_{\epsilon_i} + \tau_c)\tau_c} + \text{other}. \quad (78)
\end{aligned}$$

Substitute z_1 and z_2 defined in (54), it follows that

$$\mathbb{E}\Pi_{j2}^* = \frac{b^3e^2}{(4b^2 - e^2)^2}\frac{\tau_{\epsilon_i}}{(\tau_{\epsilon_i} + \tau_c)\tau_c} + \text{other}, \quad (79)$$

$$\frac{\partial\mathbb{E}\Pi_{j2}^*}{\partial\tau_{\epsilon_i}} = \frac{b^3e^2}{(4b^2 - e^2)^2}\frac{1}{(\tau_{\epsilon_i} + \tau_c)^2} > 0. \quad (80)$$

The trade association's marginal gain from improved precision is

$$
\begin{aligned}
\frac{\partial \sum_{t=1}^{2} TP_t^*}{\partial \tau_{\epsilon_i}} &= \frac{\partial TP_1^*}{\partial \tau_{\epsilon_i}} + \frac{\partial TP_2^*}{\partial \tau_{\epsilon_i}} \\
&= \frac{\partial \Pi_{i1}^*}{\partial \tau_{\epsilon_i}} + \frac{\partial \mathbb{E}\Pi_{i2}^*}{\partial \tau_{\epsilon_i}} + \frac{\partial \mathbb{E}\Pi_{j2}^*}{\partial \tau_{\epsilon_i}} \\
&> \frac{\partial \Pi_{i1}^*}{\partial \tau_{\epsilon_i}} + \frac{\partial \mathbb{E}\Pi_{i2}^*}{\partial \tau_{\epsilon_i}},
\end{aligned}
\tag{81}
$$

where the second equality follows from (71) and the last equality follows from (80). Substitute (62) and (80), the trade association's marginal gain from improved τ_{ϵ_i} is

$$
\frac{\partial \sum_{t=1}^{2} TP_t^*}{\partial \tau_{\epsilon_i}} = \frac{b[(4b^2 - e^2)(4b^2 - 3e^2) + 4b^2e^2]}{4(4b^2 - e^2)^2} \frac{1}{(\tau_{\epsilon_i} + \tau_c)^2}, \tag{82}
$$

and its optimal choice of τ_{ϵ_i} is uniquely determined by

$$
\frac{b[(4b^2 - e^2)(4b^2 - 3e^2) + 4b^2e^2]}{4(4b^2 - e^2)^2} \frac{1}{(\tau_{\epsilon_i} + \tau_c)^2} = k'(\tau_{\epsilon_i}). \tag{83}
$$

■

5.2 Consumer Surplus

We now consider how the qualities of firms' private information affect consumer surplus. Let CS_t denote the consumer surplus in period t, $t = 1, 2$. Using consumers' utility function (1), expected consumer surplus in period t is

$$
\begin{aligned}
\mathbb{E}(CS_t) &= \mathbb{E}\left\{u(q_{it}, q_{jt}) - p_{it}q_{it} - p_{jt}q_{jt}\right\} \\
&= \underbrace{\mathbb{E}\left[\eta_0(q_{it} + q_{jt})\right]}_{\text{Part I}} \underbrace{-\frac{1}{2}\mathbb{E}\left(\eta_1 q_{it}^2 + 2\eta_2 q_{it}q_{jt} + \eta_1 q_{jt}^2\right)}_{\text{Part II}} \underbrace{-\mathbb{E}\left(p_{it}q_{it} + p_{jt}q_{jt}\right)}_{\text{Part III}}.
\end{aligned}
\tag{84}
$$

Part I in (84) does not involve precisions and we focus on Parts II and III. Since it is easier to express expected consumer surplus in terms of quantity, we substitute quantities for prices in (84).

Using (2) and (4), inverse demand functions are derived as follows:

$$p_{it} = \eta_0 - \eta_1 q_{it} - \eta_2 q_{jt}$$
$$p_{jt} = \eta_0 - \eta_1 q_{jt} - \eta_2 q_{it}. \tag{85}$$

Substitute p_{it} and p_{jt} into $\mathbb{E}(CS_t)$,

$$\begin{aligned}\mathbb{E}(CS_t) &= -\frac{1}{2}\mathbb{E}\left(\eta_1 q_{it}^2 + 2\eta_2 q_{it} q_{jt} + \eta_1 q_{jt}^2\right) + \eta_1\left(\mathbb{E}(q_{it}^2) + \mathbb{E}(q_{jt}^2)\right) + 2\eta_2 \mathbb{E}(q_{it} q_{jt}) + other \\ &= \frac{\eta_1}{2}\left(E(q_{it}^2) + E(q_{jt}^2)\right) + \eta_2 \mathbb{E}(q_{it} q_{jt}) + other. \end{aligned} \tag{86}$$

Given $\eta_1 > 0$, consumer surplus in period t increases in the variance of each firm's quantity. Recall that firms' first-period prices only depend on the expectations of their own costs conditional on their private signals. Given that signals are independent, firms' first-period prices are independent. The linear demand functions imply that the variance of Firm i's quantity is proportional to the variance of its own price, which is shown in the previous section to be increasing in Firm i's signal precisions. Hence, increased signal precisions have a positive impact on consumer welfare through increased variance of each firm's quantity. On the other hand, firms' first-period quantities are negatively correlated when the goods are substitutes and the magnitude of the covariance increases in firms' signal precisions. For example, holding p_{j1} constant, when p_{i1} increases, q_{i1} decreases but q_{j1} increases. When Firm i increases its signal precision, it increases the magnitude of the negative covariance between firms' quantities and hence reduce consumer welfare. Despite the opposing effects, consumer welfare in the first period increases in each firm's signal precision.

When the goods are substitutes, an increase in the firms' signal precisions will reduce the variance of each firm's quantity and thus have a negative impact on consumer surplus in the second period. To see this, recall that firms' second-period prices are positively correlated and the correlation increases in each firm's precision. So, when Firm i increases p_{it} in response to a high cost, it is likely that Firm j will also increase p_{j2}, which dampen the quantity reduction by Firm i. Because firms' second-period prices are positively correlated, their second-period quantities are also positively correlated. Although an increase in firms' signal precisions will increase the covariance between quantities and hence benefit consumer surplus, the overall impact of an increase in signal precisions in the second-period consumer welfare is negative.

The next proposition shows how consumer surplus is affected by the signal precision of each firm.

Proposition 6 *Consumer surplus in the first period is strictly increasing in both firms' signal precisions while consumer surplus in the second period is strictly decreasing in signal precisions. Sum of the expected consumer surplus in the two periods is*

$$\sum_{t=1}^{2} \mathbb{E}(CS_t) = \frac{b(16b^4 - 20b^2e^2 + 3e^4)}{8(4b^2 - e^2)^2} \left[\frac{\tau_{\epsilon_i}}{(\tau_{\epsilon_i} + \tau_c)\tau_c} + \frac{\tau_{\epsilon_j}}{(\tau_{\epsilon_j} + \tau_c)\tau_c} \right] + \text{constant}, \tag{87}$$

which is strictly decreasing in both τ_{ϵ_i} and τ_{ϵ_j} when e is sufficiently close to b (namely when the two goods are close substitutes); Otherwise, the total consumer surplus is strictly increasing in τ_{ϵ_i} and τ_{ϵ_j}.

Proof. We start with expected consumer surplus in the first period. The market demand function for Firm i in period 1 is $q_{i1} = a - bp_{i1} + ep_{j1}$. Recall firms' first-period pricing function $p_{i1} = \beta_1 + \alpha_1 E(c_i|s_i),\ i = 1, 2$, where $\alpha_1,\ \beta_1$ are given by (26) (27) and they are known constants that don't involve signal precisions. Hence

$$q_{i1} = [a - (b-e)\beta_1] - \alpha_1[bE(c_i|s_i) - eE(c_j|s_j)],\ \ q_{j1} = [a - (b-e)\beta_1] - \alpha_1[bE(c_j|s_j) - eE(c_i|s_i)] \tag{88}$$

Part I in (84) is a constant doesn't involve $\tau_{\epsilon_i},\ \tau_{\epsilon_j}$, we only need to focus on Part II and Part III.

Using (88) we can show

$$\begin{aligned}
\mathbb{E}(\text{Part II}) &= -\frac{\eta_1}{2}\left[\alpha_1^2 b^2 \text{Var}(E(c_i|s_i)) + \alpha_1^2 e^2 \text{Var}(E(c_j|s_j))\right] - \frac{\eta_1}{2}\left[\alpha_1^2 b^2 \text{Var}(E(c_j|s_j)) + \alpha_1^2 e^2 \text{Var}(E(c_i|s_i))\right] \\
&\quad + \eta_2\left[eb\alpha_1^2 \text{Var}(E(c_i|s_i)) + eb\alpha_1^2 \text{Var}(E(c_j|s_j))\right] + \text{constant} \\
&= \alpha_1^2\left[-\frac{\eta_1}{2}(b^2+e^2) + \eta_2 eb\right][\text{Var}(E(c_i|s_i)) + \text{Var}(E(c_j|s_j))] + \text{constant} \qquad (89) \\
\mathbb{E}(\text{Part III}) &= -[\beta_1 + \alpha_1 E(c_i|s_i)]\left\{[a-(b-e)\beta_1] - \alpha_1[bE(c_i|s_i) - eE(c_j|s_j)]\right\} \\
&\quad - [\beta_1 + \alpha_1 E(c_j|s_j)]\left\{[a-(b-e)\beta_1] - \alpha_1[bE(c_j|s_j) - eE(c_i|s_i)]\right\} \\
&= b\alpha_1^2[\text{Var}(E(c_i|s_i)) + \text{Var}(E(c_j|s_j))] + \text{constant} \qquad (90)
\end{aligned}$$

where the term “constant” doesn’t involve τ_{ϵ_i} and τ_{ϵ_j}. Using (4), we derive the following:

$$\eta_0 = \frac{a}{b-e}, \quad \eta_1 = \frac{b}{b^2-e^2}, \quad \eta_2 = \frac{e}{b^2-e^2}. \qquad (91)$$

Substitute (90) and (89) with η_1, η_2 defined above, we obtain

$$\begin{aligned}
\mathbb{E}(CS_1) &= \frac{b}{2}\alpha_1^2[\text{Var}(E(c_i|s_i)) + \text{Var}(E(c_j|s_j))] + \text{constant} \qquad (92) \\
&= \frac{b(2b^2-e^2)^2}{2(4b^2-e^2)^2}[\text{Var}(E(c_i|s_i)) + \text{Var}(E(c_j|s_j))] + \text{constant} \qquad (93)
\end{aligned}$$

where $\alpha_1 = \frac{2b^2-e^2}{4b^2-e^2} > 0$. Hence $\mathbb{E}(CS_1)$ is strictly increasing in τ_{ϵ_i} and τ_{ϵ_j} since $\text{Var}(E(c_i|s_i))$, $\text{Var}(E(c_j|s_j))$ are strictly increasing in τ_{ϵ_i} and τ_{ϵ_j} respectively.

Next we derive consumer surplus in the second period. The firms’ second-period equilibrium pricing functions (16) (17) can be expressed as:

$$\begin{aligned}
p_{i2} &= z_0 + \frac{c_i}{2} + z_1 E(c_j|\hat{s}_j) + z_2 E(c_i|\hat{s}_i) \qquad (94) \\
p_{j2} &= z_0 + \frac{c_j}{2} + z_1 E(c_i|\hat{s}_i) + z_2 E(c_j|\hat{s}_j), \qquad (95)
\end{aligned}$$

where z_0, z_1 and z_2 are defined in (54). Note $\hat{s}_i$ is Firm j’s conjecture on Firm i’s signal s_i, similarly for $\hat{s}_j$. In equilibrium conjectures are correct, namely $\hat{s}_i = s_i$, $\hat{s}_j = s_j$. Substitute (94) and (95) into the demand functions $q_{i2},\ q_{j2}$, we obtain

$$q_{i2} = [a - z_0(b-e)] - \frac{b}{2}c_i + \frac{e}{2}c_j + E(c_j|s_j)(-bz_1 + ez_2) + E(c_i|s_i)(-bz_2 + ez_1) \tag{96}$$

$$q_{j2} = [a - z_0(b-e)] - \frac{b}{2}c_j + \frac{e}{2}c_i + E(c_i|s_i)(-bz_1 + ez_2) + E(c_j|s_j)(-bz_2 + ez_1). \tag{97}$$

Again for the second period Part I in (84) doesn't involve τ_{ϵ_i} or τ_{ϵ_j}. We only need to calculate Part II and Part III in (84). It can be verified that

$$\begin{aligned}\mathbb{E}(\text{Part II}) = \text{constant} + \Big\{&-\frac{\eta_1}{2}[(-bz_2+ez_1)(-bz_2+ez_1-b) + (-bz_1+ez_2)(-bz_1+ez_2+e)] \\ &-\eta_2[\frac{e}{2}(-bz_2+ez_1) - \frac{b}{2}(-bz_1+ez_2) + (-bz_2+ez_1)(-bz_1+ez_2)]\Big\}[\text{Var}(E(c_i|s_i)) + \text{Var}(E(c_j|s_j))] \\ = \frac{1}{8}&\frac{(4b^2+e^2)be^2}{(4b^2-e^2)^2}[\text{Var}(E(c_i|s_i)) + \text{Var}(E(c_j|s_j))] + \text{constant}\end{aligned} \tag{98}$$

$$\begin{aligned}\mathbb{E}(\text{Part III}) &= -[(-bz_2+ez_1) + z_2(-bz_2+ez_1) + z_1(-bz_1+ez_2)][\text{Var}(E(c_i|s_i)) + \text{Var}(E(c_j|s_j))] + \text{constant} \\ &= -\frac{1}{4}\frac{(4b^2+e^2)be^2}{(4b^2-e^2)^2}[\text{Var}(E(c_i|s_i)) + \text{Var}(E(c_j|s_j))] + \text{constant}.\end{aligned} \tag{99}$$

Substitute (98) and (99) with $\eta_1,\ \eta_2$ defined in (91), we obtain

$$\mathbb{E}(CS_2) = -\frac{1}{8}\frac{(4b^2+e^2)be^2}{(4b^2-e^2)^2}[\text{Var}(E(c_i|s_i)) + \text{Var}(E(c_j|s_j))] + \text{constant}. \tag{100}$$

Since $\text{Var}(E(c_i|s_i))$ is strictly increasing in τ_{ϵ_i}, (100) indicates that that the expected second-period consumer surplus is strictly decreasing in both firms' signal precisions.

The aggregate consumer surplus of the two periods is obtained by simply adding (93) and (100)

$$\begin{aligned}\sum_{t=1}^{2}\mathbb{E}(CS_t) &= \frac{b(16b^4 - 20b^2e^2 + 3e^4)}{8(4b^2-e^2)^2}\Big[\text{Var}(E(c_i|s_i)) + \text{Var}(E(c_j|s_j))\Big] + \text{constant} \\ &= \frac{b(16b^4 - 20b^2e^2 + 3e^4)}{8(4b^2-e^2)^2}\left[\frac{\tau_{\epsilon_i}}{(\tau_{\epsilon_i}+\tau_c)\tau_c} + \frac{\tau_{\epsilon_j}}{(\tau_{\epsilon_j}+\tau_c)\tau_c}\right] + \text{constant},\end{aligned} \tag{101}$$

which is decreasing in $\tau_{\epsilon_i},\ \tau_{\epsilon_j}$ when e is sufficiently close to b. ■

5.3 Social Planer

Having analyzed consumer surplus and industry profits, we are ready to consider social planer's optimal signal precisions and compare them with firms' optimal choices of precisions. We have shown that the qualities of firms' signals are inefficiently low from the trade association's perspective, and that total consumer surplus increases in the qualities of firms' signals. This observation suggests that when a firm chooses the precision of its signals, it fails to incorporate positive externalizes on the rival and on consumer. Hence, the quality of the firm's signal is inefficiently low from the social planner's perspective. We characterize the social planner's optimal signal precisions in the following proposition:

Proposition 7 *From the social planner's perspective, the qualities of firms' signals are inefficiently low. The social planner's optimal signal precisions are uniquely determined by*

$$\frac{b(48b^4 - 44b^2e^2 + 9e^4)}{8(4b^2 - e^2)^2}\frac{1}{(\tau_\epsilon + \tau_c)^2} = k'(\tau_\epsilon).^4 \tag{102}$$

6 Conclusion

We study firms' incentive to acquire private information on costs anticipating that they can signal this information to the rivals through their prices. Overall, firms benefit when the qualities of their private information are improved. However, "signaling" reduces firms' incentives to acquire more accurate information. Compared with myopic firms, strategic firms will acquire less precise signals.

From the perspective of the industry, the qualities of firms' information are inefficiently low, which is driven by the positive externality of firms' improved information on their rivals' second-period profits. When firms acquire more accurate information, consumers benefit in the first period,

[4] If we subtract from the coefficient of (102) the coefficient of total consumer surplus (87, we obtains the coeffiient of total industrial profit (83))

but will suffer a loss in the second period. The overall, firms' more accurate information has a positive externality on consumers when the degree of substitution between the goods is not too high. From the social planner's perspective, the the qualities of firms' private information are also inefficiently low.

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