Uncertainty-driven Cooperation*

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(PRELIMINARY AND INCOMPLETE: COMMENTS WELCOME)

Abstract

We consider a finite-horizon repeated team production game where team members receive interim feedback which is informative of effort choices as well as an uncertain state of the world. The uncertain state effects the returns to effort. We show that the presence of uncertainty alleviates the inefficiencies due to productive free-ridership. In the unique sequential equilibrium, the agents increase their efforts in order to influence the interim feedback signal, which in turn influences their partners' beliefs about the uncertain state, consequently affecting their future effort choices. We show that the resulting equilibrium effort level is higher than that in the complete information case, and may even exceed its first best level. Our results extend to the continuous-time limit as well as the infinitely repeated version. Last, we study an asymmetric information model in which some of the agents know the true state while the others are uninformed.

Keywords: Team Production, Repeated Games, Uncertainty, Learning

JEL classification: C72, C73, D83

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Contents

1 Introduction

Team production is increasingly prevalent in the modern workplace[1]. Firms may opt to base compensation on team performance measures even when individual measures are available.[2] In other instances, individual performance measures may simply be unavailable.[3] The main disadvantage of using team incentives—i.e. compensation based on team performance measures—is the inclination of team members to free-ride on the efforts of others and thus reduce their own efforts. This disadvantage has been recognized at least since Alchian and Demsetz (1972). Our paper identifies a novel effect that counteracts the effort-reduction impact of free-ridership in environments with uncertainty. We show that in such environments, impact of free-ridership may be completely nullified. In fact, there may be instances of over-provision of effort.

Our result pertains to environments that have the following characteristics: 1) a team of workers engage in joint production over time; 2) team members' individual effort choices are unobservable; and 3) members receive interim feedback. We show that in such environments presence of uncertainty about the returns to effort alleviates the free-ridership problem relative to cases where there is no such uncertainty.

What is the mechanism underlying this result? In general, free-ridership arises because self-maximizing team members fail to internalize the positive externalities of their effort, leading to its under-provision. However, in the presence of uncertainty, and when the interaction is dynamic, there is an additional personal return for the effort which counter-acts the free-ridership effect. This benefit arises because higher current effort increases the likelihood of receiving better interim feedback. A better feedback, in turn, renders team members more optimistic and thus more willing to exert effort in the subsequent phases of joint production. To put it in terms of established terminology, the extra incentives for effort stems from the team members' wish to jam the feedback signal; i.e. to engage in "signal-jamming".

Our main model incorporates the above three crucial characteristics, yet is simple enough to be free

[1] Using survey data, Osterman (1994, 2000) estimates that among private, for-profit establishments that have at least 50 employees, approximately 40% has at least half of their employees organized in teams. Similarly, Lawler, Mohrman, and Benson (2001) reports that 47% of Fortune1000 companies make use of self-managed teams.

[2] E.g. see Hamilton, Nickerson, and Owan (2003) for an empirical analysis of individual versus group incentives in manufacturing and Knez and Simester (2001) for an empirical analysis of team incentives by Continental Airlines.

[3] E.g. see Boning, Ichniowski, and Shaw (2001) (p. 615) for a discussion of U.S. steel minimills where "unlike other production processes where individual output can be accurately measured and individual incentives can be used to raise worker productivity [...], it is only overall output for the entire line [...]that can be measured [...]. As a result, only group incentives are used".

of confounding mechanisms. Specifying such a "minimal" model allows us to focus on the impact of uncertainty on team incentives in a transparent manner. Specifically, in our model, N agents engage in joint production over a finite number of periods. In each period, agents simultaneously choose their effort levels and then receive feedback, which is a noisy signal of effort and the uncertain state of the world. We assume that feedback is additively separable in the state and the total effort, garbled with a Gaussian noise. We assume a quadratic cost function. At the end of the productive relationship, the agents receive their share of the joint output, which is also linear in the total effort with an unknown coefficient. The prior distribution of this unknown coefficient, which we take as the state of the world, is assumed to be Gaussian. Our linear-quadratic-Gaussian specification has various advantages. First, our model has a unique perfect Bayesian equilibrium, which takes a simple and easily interpretable form. Second, it allows us to highlight the mechanism we analyze in this paper by eliminating other factors. In particular, the additive separability of effort and noise implies that all effort levels are equally informative. Therefore, the agents' effort choices are affected only by production externalities among team members and not by informational externalities. Also, it eliminates the "experimentation" motive—namely the inclination to manipulate effort choices to increase the speed of own and others' learning—from the incentives of team members.

The unique equilibrium of our model has a very simple form: the equilibrium effort choice of each agent is linear in his current expectation of the state. Moreover, the coefficient that multiplies the belief—which we term the *belief-sensitivity of effort*—depends only on the calendar time. The larger the belief-sensitivity of effort, the more exaggerates are the equilibrium effort choices. This equilibrium strategy is the same on and off the path of play, which greatly simplifies our analysis. We then compare our result to two benchmarks: the unique equilibrium under complete information; and the socially optimal outcome. In our model, the equilibrium effort levels in both benchmarks are also linear in the expectation of the state. Therefore, comparing our equilibrium outcome to these two benchmarks boils down to comparing the belief-sensitivity of effort. We find that the belief-sensitivity of effort under uncertainty is always larger than that under complete information, except in the last period when they are equal. Moreover, we show that in some periods, the belief-sensitivity of effort may exceed its socially optimal level, implying inefficient overproduction in such periods.

We conduct several comparative statics analysis. First, the belief sensitivity of effort decreases over time. Second, the ex-ante expected total output decreases in the precision of the initial prior and increases in the precision of the feedback technology. To understand these results, note that the agents' incentives to exaggerate their effort increases in the return to jamming the feedback. Then, the first result follows because in earlier periods jamming the feedback will influence the effort choices of others for a longer stretch of time. The second result follows because the belief updating of team members puts a larger weight on the feedback—and therefore, their future efforts become more sensitive to feedback—if the feedback is relatively more precise than the initial prior.

We also analyze an infinite-horizon version of our model. We show the existence of an equilibrium with similar characteristics in the infinitely repeated game. It is well-known that there exists a multiplicity of equilibria in infinitely repeated games. In such equilibria, the agents' intertemporal incentives are provided by "punishment" of the other agents based on the information of its past behavior (e.g. trigger strategies). However, the literature has shown that as the agents' action becomes frequent, the scope of cooperative behavior may be limited (Abreu, Milgrom, and Pearce, 1991; Sannikov and Skrzypacz, 2007). In this paper, we conjecture that as the time between successive periods become arbitrarily small (i.e. as the agents can adjust their effort choices increasingly frequently), our equilibrium gives the maximum attainable equilibrium payoff. That is, even when the standard trigger mechanism is ineffective for incentivizing effort, the presence of uncertainty may provide an incentive to alleviate free-ridership.

We also consider an asymmetric information version of our model in which some team members (*experts*) know the state of the world while the others (*novices*) are uninformed. We characterize the unique symmetric Markov equilibrium that is linear in agents' information. Last, we analyze various implications of our model, such as optimal team composition, optimal information disclosure by a principal and role of the contracts.

1.1 Two-Period Example

In order to demonstrate the underlying mechanism of our main result, let us consider a simple two-period example. Suppose that two agents engage in a team production over two periods. In each period, each agent chooses effort $a_i \in \{L, M, H\}$, which is interpreted as putting zero, one, and two units of effort, respectively. Cost of effort is $c(L) = 0, c(M) = c$, and $c(H) = 2c$. We assume that the effort level is unobservable to each other. A stochastic outcome, either *Success* or *Fail*, is revealed in each period. If *Success* is revealed, each agent receives a payoff of one. In the case of *Fail*, zero payoff is given.

There are two possible states of the world: $\theta \in \{Good, Bad\}$. Probability of *Success* outcome de-

		2		
		H	M	L
1	H	$4p$	$3p$	$2p$
	M	$3p$	$2p$	p
	L	$2p$	p	0

		2		
		H	M	L
1	H	$2p_G+2p$	$2p_G+p$	p_G+p
	M	$2p_G+p$	$2p_G$	p_G
	L	p_G+p	p_G	0

Figure 1: Success probability when $\theta = Bad$ (left) and $Good$ (right)

pends on both the state and the stage action profile. Figure 1 depicts the probability of Success in each state of the world. When the state is *Bad*, each unit of effort increases the success probability by p. When the state is *Good*, the first and the second unit of effort increases the probability by $p_G > p$ and p. Moreover, we assume that $p < c < 2p$ and $p_G > c$. These assumptions guarantee that the free-riding effect exists in both states, but to different degrees: if $\theta = Bad$, L is the dominant strategy of the stage game (since $p < c$) while H is the socially desirable action (since $c < 2p$). On the other hand, if $\theta = Good$, free-riding effect exists only in the second unit of effort: M is the dominant strategy while H is the socially desirable action.

Now let us analyze the equilibrium in the two-period game. First note that in the complete information case, standard backward induction implies that the agents play (M,M) if $\theta = G$ and (L,L) if $\theta = B$ for all $t = 1,2$. Now consider incomplete information about θ: let μ_t be the common period-t belief that $\theta = G$. In the second period, the agents play a myopically optimal strategy. That is, the agents play (M,M) if $\mu_2 > \mu^*$ and (L,L) if $\mu_2 < \mu^*$ where $\mu^* = \frac{c-p}{p_G-p}$. In the first period, however, the agents also need to consider the effect of the current action on the future belief updating.

The following proposition states that uncertainty about the state may enhance the production:

Proposition 1. *Suppose that $p + \delta pc > c$. Then there exist $\underline{\mu} < \mu^* < \bar{\mu}$ such that there is a unique sequential equilibrium for any $\mu_1 \in (\underline{\mu}, \bar{\mu})$. In the equilibrium, the agents play (H,H) in $t = 1$.*[4]

The underlying reasoning is as follows. *Success* in period 1 indicates that the state is likely to be *Good*, which leads the agents to play M in period 2. On the other hand, each agent wants the other to exert more effort regardless of the state. Knowing this, each agent has an incentive to "signal-jam" the other by increasing the success probability.

In the main body of the paper, we build a tractable finite-horizon model in which the same mechanism would induce agents to exert higher effort in the presence of uncertainty. Moreover, tractability of our

[4]The proof of the proposition is in Appendix A.

main model enables us to analyze various applications, such as comparative statics, dynamic information design, and extensions to infinite-horizon and asymmetric information case.

The rest of the paper is organized as follows. The remainder of this section discusses related literature. Section 2 formally describes the model. Section 3 characterizes equilibrium and undertakes comparative statics exercises. Section 4 analyzes infinite-horizon version of the model. Section 5 analyzes a modified model with asymmetric information. Section 6 is concerned with applications. The Appendix collects omitted proofs.

1.2 Literature Review

This paper is related to the literature on agency models with symmetric uncertainty and learning. Holmström (1999) seminal paper on "signal-jamming" shows that a manager who is concerned about the market belief about his ability has an incentive to increase his effort. See also Cisternas (2015) for a generalized continuous-time model.[5] This paper finds the corresponding incentives in the context of team production, and develops novel implications for the team organization such as team size and information structure. Free-riding in teams and partnerships has been studied extensively (Alchian and Demsetz, 1972; Holmström, 1982; Radner, Myerson, and Maskin, 1986). Main theme of these papers is, due to the "free-riding" effect efficient effort can not be sustained as an equilibrium.

Our paper is also related to the literature on dynamic contribution to public-good project (Admati and Perry, 1991; Marx and Matthews, 2000; Yildirim, 2006; Georgiadis, 2014). In these papers, projects are completed when the contributions reach a pre-specified threshold. In similar setups, these papers show that allowing dynamic contribution mitigates the free-riding problem. These models do not future uncertainty about the quality of the project which is the driving force of our results.

The economic forces that operate within our setup has some similarities to the strategic experimentation literature, but there are some crucial differences: that literature typically considers only informational externalities (Bolton and Harris, 1999; Keller, Rady, and Cripps, 2005). In these papers, each player owns and operates his own production technology. However, the production technologies are identical and therefore, observing others' outcomes (with or without observable effort) allows each player to learn about his own technology. Whereas in our model, there is a common production technology that

[5] Dewatripont, Jewitt, and Tirole (1999) and Bonatti and Hörner (2013) also examines the signal jamming model with different pay-off and information structures compared to Holmström (1999).

is operated jointly by all team members, and the externalities we consider are productive, rather than informational.

A notable exception, and perhaps the most closely related to our paper is Bonatti and Hörner (2011) who also consider a joint production environment which entails both productive and informational externalities. [6] Recently Guo and Roesler (2015) extends the Bonatti and Hörner (2011) by allowing private learning in addition to public learning. However, their specification of the model (Poisson bandit) has no free-ridership problem when the state is known, and therefore that model is not suited to address the question we raise in this paper.

Our paper is also related to the repeated games literature. Wiseman (2005, 2012) studies a perfect-monitoring repeated game in which there is a common uncertainty about the underlying state of world and agents learn the state over time. Wiseman (2005) proves a partial Folk theorem in a repeated game in the case agents learn the state by public signals. In Wiseman (2012), in addition to public signal agents receive a private signal. Fudenberg and Yamamoto (2011) allows imperfect monitoring and proves a Folk Theorem under their new notion of perfect public ex-post equilibrium. In this paper, we do not conduct a folk theorem analysis our model is either finite horizon, or infinite horizon with fixed discount factor.

There is a recent literature on how to design an information disclosure policy in economic environments. In that literature closest to our paper is Hörner and Lambert (2015). They study the design of optimal rating system (which maximizes the agent's effort) in the career-concerns framework with a stationary Gaussian environment. Pei (2015) also studies the effect of information disclosure by a third party in the career concerns framework, however, Pei models uncertainty with Poisson-bandits instead of Brownian motion. Smolin (2015) and Orlov (2014) examine how the choice of information disclosure rule effects incentive provision in the principal agent setting. Ely (2014) also examines the information disclosure in principle-agent setting but different then above papers, principal does not allow to use transfers. All of the papers listed above are in the single agent setting. Similar to our paper in the multi-agent settings, Halac, Kartik, and Liu (2014) and Bimpikis, Ehsani, and Mostagir (2014) answer the question how to design a contest in order to maximize the effort of the contestants. Halac et al. (2014) use both information disclosure and the reward structure as the design tool. Also in team problems Moroni (2015) examines the question: How should a principal incentives a team of agents working on a risky project ?

[6] See also Bonatti and Horner (2015).

2 Model

Time is discrete. A team of N agents are working on a project with a deadline, T. The payoffs accrue once the complete project is submitted at the end of period T.

Each period, each agent chooses an action $a_{it} \in \mathbb{R}$. [7] The instantaneous cost of choosing effort a_i for agent i is $\frac{1}{2}a_i^2$. Effort choices are not publicly observable and therefore are not contractible.

The total output which is realized at the end of period T and shared equally among agents is given by:

$$\delta^{-T}\left[\theta \times A + \nu\right],$$

where θ is the state of the world, A is given by

$$A = \sum_{t=1}^{T} \delta^{t-1} \sum_{i=1}^{N} a_{it},$$

with $0 < \delta < 1$, and $\nu \sim \mathcal{N}(0, 1/h_\nu)$.

At the end of each period the agents publicly observe "feedback" y_t. This can be the outcome of an internal review done by the members, or a feedback from an employer, or some information about how far the project has progressed. For tractability, we assume that the feedback signal is

$$y_t = \theta + A_t + \varepsilon_t,$$

where $A_t = \sum_{i=1}^{N} a_{it}$, $\varepsilon_t \sim \mathcal{N}(0, 1/h_\varepsilon)$, and ε_t's are independent across t.

At the beginning of period 1, the state of the world is believed to be normally distributed with mean μ_0 and variance $1/h_0$, i.e., $\theta \sim \mathcal{N}(\mu_0, 1/h_0)$.

Histories and strategies Let y^t be a length-t public history of feedbacks and Υ be the set of all histories of all lengths $t = 1, \ldots, T$. Also let a_i^t be the private history of agent $i's$ effort choices in the first t periods, and A_i be the set of all possible private histories. Then, a pure strategy for player i is a map from $\Upsilon \times A_i$ into $\mathbb{R}$.[8]

[7] Alternatively, we can assume noise comes from an eliptic distribution then we can choose action from a compact set. See **?** which uses eliptic distributions

[8] Even though we allow mixed strategies, the unique equilibrium does not feature mixed strategies on or off the path of equilibrium. Therefore, we omit introducing notation for mixed strategies.

Equilibrium We focus on the sequential equilibria of our model. We conclude this section by establishing equilibria in two benchmark cases.

Static setting The effort in the unique equilibrium of the static setting is $a^*_{static} = \mathbb{E}[\theta] = \frac{\mu_0}{N}$, while the socially optimal effort level is $\bar{a} = \mu_0$.

Complete information case ($h_0 = \infty$): The unique equilibrium in the complete information case is $a_{it} = \frac{1}{N}\theta$, while the socially efficient effort is $a_{iT} = \theta$.

3 Equilibrium

Our model admits a unique sequential equilibrium. This unique equilibrium has a particularly simple structure: after any history, whether it involves private deviations or not, the equilibrium strategy prescribes an effort level that is linear in the current expected value of the state of the world, θ. The linear coefficient depends only on the calendar time. We next construct this equilibrium. As a first step to construction, we first discuss belief updating in Section 3.1. We state the uniqueness result and present the equilibrium characterization in Section 3.2.

3.1 Belief Updating

We first remark that, since both the initial distribution of θ and the distribution of noise ε_t are Gaussian, all posteriors are also Gaussian. Then, given equilibrium strategies a^*, a public history y^{t-1} and a private history a_i^{t-1} of player i, his belief is fully determined by the mean $\mu_{it}(y^{t-1}, a_i^{t-1}, a^*)$ and the precision $h_{it}(y^{t-1}, a_i^{t-1}, a^*)$ of the posterior distribution.

Next, we remark that all public histories y^t are on the equilibrium path, and therefore all off-equilibrium beliefs of a deviating player, as well as all equilibrium path beliefs are pinned down by Bayes rule.

We derive the belief updating rule recursively. At the end of period 0, player i updates his belief based on the signal

$$z_0 \equiv y_0 - a_{i0} - \sum_{j \neq i} a^*_{j0} = \theta + \varepsilon_0.$$

Then, by Bayes rule

$$\mu_{1i}(y_0, a_{0i}, a^*) = \frac{h_0\mu_0 + h_\varepsilon z_0}{h_0 + h_\varepsilon}, \quad \text{and} \quad h_{1i}(y_0, a_{0i}, a^*) = h_0 + h_\varepsilon.$$

Similarly, given $\mu_{t-1,i}, h_{t-1,i}, y_{t-1}, a_{t-1,i}, a^*$,

$$\mu_{ti}(y^{t-1}, a_i^{t-1}, a^*) = \frac{h_{t-1}\mu_{t-1} + h_\varepsilon z_{t-1}}{h_{t-1} + h_\varepsilon}, \quad \text{and} \quad h_{ti}(y^{t-1}, a^{t-1,i}, a^*) = h_{t-1} + h_\varepsilon.$$

Iterating the recursive formulation we get,

$$\mu_{ti}(y^{t-1}, a_i^{t-1}, a^*) = \alpha_{t-1}\mu_{t-1} + (1-\alpha_t)z_{t-1}, \quad \text{and} \quad h_{ti}(y_{t-1}, a_{t-1,i}, a^*) = h_0 + th_\varepsilon, \tag{1}$$

where $\alpha_t = h_t/(h_t + h_\varepsilon)$.

Observe that the precision of the posterior belief evolves deterministically, independent of the realization of the feedback y_t and of the actions chosen by players, whether they are equilibrium actions or deviations. Also observe that the mean of the posterior belief of player i also is independent of the specific equilibrium actions or i's own deviations, as when computing the posterior player i discounts the feedback exactly by the total effort. In light of this , in what follows, we drop the arguments and subscript i from the posterior precision, and simply let h_t denote the precision of time t posterior belief. When clear from the context we also drop the arguments for posterior mean and let μ_{ti} denote the mean of player i's posterior belief.

Importantly, even though a deviation of player i leaves his own posterior belief unchanged, it impacts the posterior beliefs of other players, as they would discount the observed feedback by the equilibrium action of player i, rather than his unobserved deviation. This is precisely the channel via which player i is incentivized to increase his effort, as becomes clear in the equilibrium construction of the next section. For future reference, notice that the marginal impact of z_t on belief μ_t is given by h_ε/h_t. Let ρ_t stand for this ratio; i.e. $\rho_t = h_\varepsilon/h_t$.

3.2 Equilibrium

We first state our main result.

Proposition 2. *There exists a unique sequential equilibrium. In this equilibrium, the agents' actions depend linearly on the mean of current belief with time varying coefficients. More specifically, for any player i and any time-t mean belief* μ_{it},

$$a_{it} = \xi_t \mu_{it},$$

where ξ_t *is recursively defined by*

$$\xi_t = \frac{1}{N} + \frac{N-1}{N} \sum_{s=t+1}^{T} \delta^{s-t} \rho_s \xi_s, \quad \xi_T = \frac{1}{N}, \tag{2}$$

and $\rho_s = h_\varepsilon / h_s$.[9]

Before presenting the proof, we comment on the equilibrium structure. Because of the quadratic form of the effort cost, each player's best response involves choosing effort equal to its marginal product. This marginal product must account for not only the current increase in expected output due to increased effort, but also the indirect effect of increased output on the beliefs and therefore future efforts of other players. In the expression for ξ_{it} in (2), $1/N$ is precisely the expected increase in (player i's share) of output in period t. In the second additive term, ρ_s represents the sensitivity of the belief of each player in period $s+1$ to an increase in z_s, while ξ_s, by definition, represents the sensitivity of period s effort of player i to an increase in his belief μ_{is}. Then, ξ_t collects the discounted sum of how future (i.e., from period $t+1$ to T) efforts of all players will be impacted by an increase in today's effort by player i. The scaling factor $(N-1)/N$ accounts for the fact that there are $(N-1)$ *other* players and player i's share is only $1/N$ of the total output. Finally, since the output is θ *times* the total effort, this marginal increase in effort is multiplied by μ_{it}, the current expected value of θ, in order to convert it into marginal increase in payoff.

Finally, notice that the coefficients ξ_t are independent of mean belief μ_{it} even though they do vary with the precision of posterior belief h_t.

Proof. The proof is by induction on the number of periods. It is straightforward to see that when $T = 1$

[9] Recently Iijima and Kasahara (2015) prove an equilibrium uniqueness result in a continuous-time noisy monitoring game. However, our paper has major differences compared to Iijima and Kasahara (2015): i) in our model there is symmetric uncertainty about the state and the agents gradually learn the true state, ii) agents have unbounded action space, iii) we focus on the discrete-time limit as $\Delta \to 0$. We prove equilibrium uniqueness for every given period length Δ and then take the limit as $\Delta \to 0$. Our results does not imply that the continuous-time model has a unique equilibrium. However, we conjecture that continuous-time game has a unique equilibrium. We do not have formal proof of this claim yet.

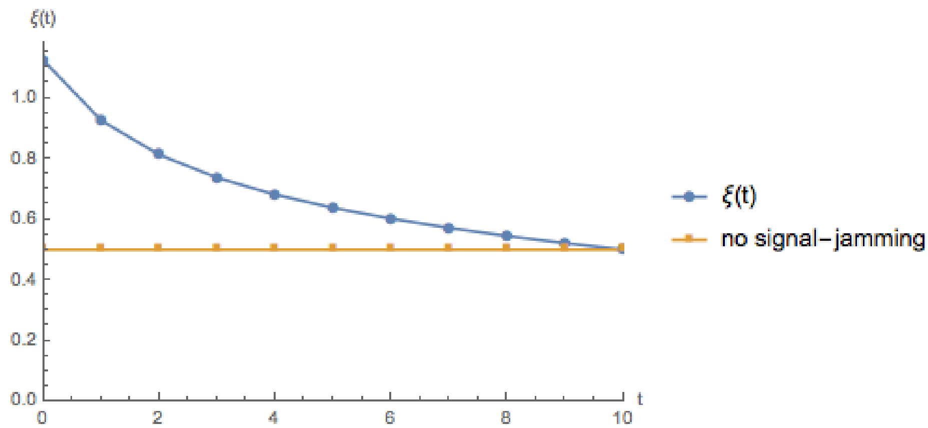


Figure 2: Equilibrium belief-sensitivity ξ_t ($N=2, T=10, \delta=0.95, h_0=h_\varepsilon=1$).

there exists a unique equilibrium with $\xi_T = 1/N$. Assume that for any initial belief specified by (μ_0', h_0') when $T = \tilde{T}-1$ there is a unique equilibrium as described in the proposition. Fix (μ_0, h_0) and consider a game with $T = \tilde{T}$. We show that there is a unique equilibrium as described in the proposition. Take any first period equilibrium effort profile $(a_{01}, \ldots, a_{0N})$ of the $\tilde{T}$-period game. Given (μ_0, h_0), this effort profile leads to a unique common belief (μ_1, h_1) by (1). By the induction hypothesis, the continuation game starting in period 2 with initial belief (μ_1, h_1) has the unique equilibrium described in the proposition. Also, by (1) is independent of $(a_{11}, \ldots, a_{1N})$. Then, letting $\xi_t^{\tilde{T}-1}$ represent the equilibrium coefficients in the $(\tilde{T}-1)$-period game, the marginal gain of player i from increasing his effort in period 0 of the $\tilde{T}$-period game is given by

$$\xi_0 = \frac{1}{N} + \delta \frac{N-1}{N} \sum_{s=0}^{\tilde{T}-1} \delta^s \rho_{s+1} \xi_s^{\tilde{T}-1}.$$

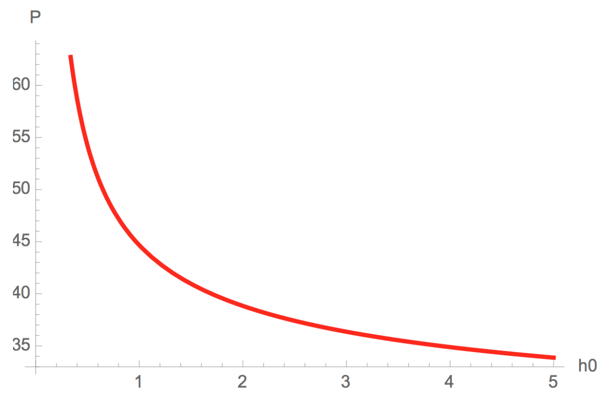
Letting $\xi_t = \xi_{t-1}^{\tilde{T}-1}$, for $t = 1, \ldots, \tilde{T}$, establishes the claim. □

3.3 Comparative Statics

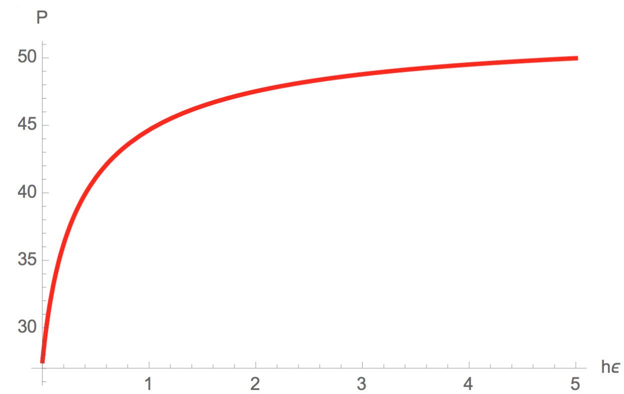
The simple structure of the unique equilibrium enables us to conduct several comparative statics. The following proposition states that the equilibrium effort tends to decrease over time.

Proposition 3. *In the unique sequential equilibrium of the game, ξ_t decreases in t. Furthermore, $\xi_t > 1/N$ for all $t = 0, \ldots, T-1$ and $\xi_T = 1/N$.*

The proof of the Proposition 3 is straightforward from (2), hence is omitted. Figure 2 illustrates the



(a) Total production as a function of h_0



(b) Total production as a function of h_ε

Figure 3: The ex ante expected total production $(N = 2, \delta = 0.95, T = 10, \mu_0 = 2)$

equilibrium belief-sensitivity of effort over time, which decreases over time and is equal to $1/N$ in the last period. The underlying reasoning is clear: The benefit from inducing future effort of other agents decreases as the game approaches to the final period. Note that depending on the signal realization, the equilibrium effort level of the agent may increase.

The next proposition shows the comparative statics with respect to the precision of the information.

Proposition 4. *In the unique sequential equilibrium of the game,*

1. *For any $t = 0, \ldots, T-1$, ξ_t decreases in h_0. Furthermore, the ex ante expected total production decreases in h_0.*

2. *For any $t = 0, \ldots, T-1$, ξ_t increases in h_ε. Furthermore, the ex ante expected total production increases in h_ε.*

Proof. See Appendix A. □

Figure 3 illustrates the ex ante total production as functions of h_0 and h_ε. To understand these results note that team members have a greater incentive to increase their effort (that is, higher ξ_t) if the return to jamming the feedback is larger. Then, the result follows because the belief updating of team members puts larger weight on the feedback if 1) the initial prior is less informative; or 2) the feedback is more informative. In this case, their future efforts become more sensitive to feedback, and thus the returns to jamming the feedback is larger.

3.4 Continuous-time Limit

In this section we look at a continuous time limit of the equilibrium behavior. This can be considered a robustness check to demonstrate that the "signal jamming" motivations still have an impact on outcomes when actions can be adjusted more frequently.

Specifically, we fix the real time horizon $\tau > 0$ and allow the players to switch their actions and get feedback increasingly frequently. Formally, we let $\Delta > 0$ to be the length of a period so that the number of periods is $T = \tau/\Delta$. We consider the limit of equilibrium behavior as $\Delta \to 0$.

To do this, define the "flow version" of the parameters of the model in the following sense:

- $r > 0$: discount factor $\Rightarrow \delta = e^{-r\Delta}$
- $\eta > 0$: information disclosure rate $\Rightarrow h_\varepsilon = \eta\Delta$
- per-period cost$= \frac{a^2}{2}\Delta$
- per-period production$= \Delta\theta \sum_i a_{it}$

Also it is convenient to redefine ξ_t as the belief sensitivity in "real-time" t. Let $\chi_t = \xi_t - 1/N$ be the sensitivity parameter net of the impact of the current period output share. Therefore, χ_t captures the impact of the signal-jamming incentives on the sensitivity of effort to belief.

Re-writing (2) in terms of the flow parameters yields the following:

$$\chi_t = \delta\left[\left(1 + \frac{N-1}{N}\frac{h_\varepsilon}{h_t}\right)\chi_{t+\Delta} + \frac{N-1}{N^2}\frac{h_\varepsilon}{h_t}\right].$$

If Δ is small enough, we have

$$\chi_t = (1 - r\Delta)\left[\left(1 + \frac{N-1}{N}\frac{\eta}{h_0 + \eta t}\Delta\right)(\chi_t + \dot{\chi}_t\Delta) + \frac{N-1}{N^2}\frac{\eta}{h_0 + \eta t}\Delta\right].$$

Cancel out Δ^2 terms and rearranging, we have the differential equation

$$\dot{\chi}_t = \left(r - \frac{N-1}{N}\frac{\eta}{h_0 + \eta t}\right)\chi_t - \frac{N-1}{N^2}\frac{\eta}{h_0 + \eta t}$$

with the boundary condition $\chi_\tau = 0$. We demonstrate in the Appendix that the solution to this equation involves $\chi_t > 0$, for all $t < \tau$ and therefore, $\xi_t > 1/N$. This implies, in particular, that signal jamming

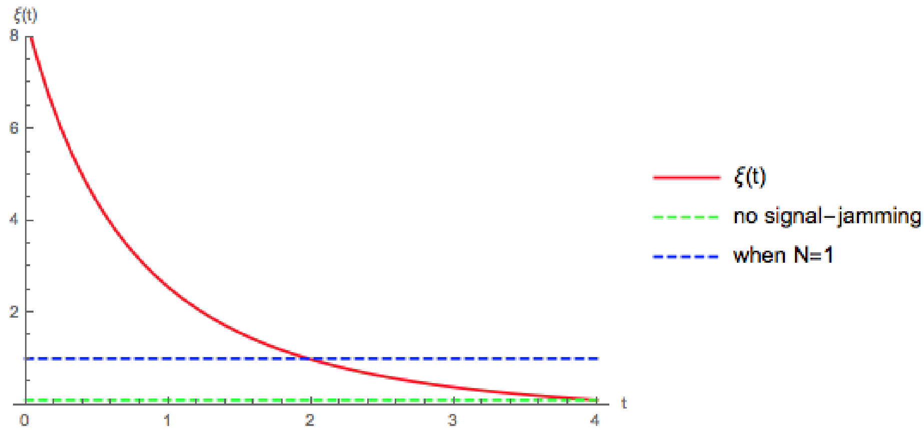


Figure 4: Equilibrium belief-sensitivity $\xi(t)$ ($N = 10, \tau = 4, \eta = 2, h_0 = 3, r = 0.001$).

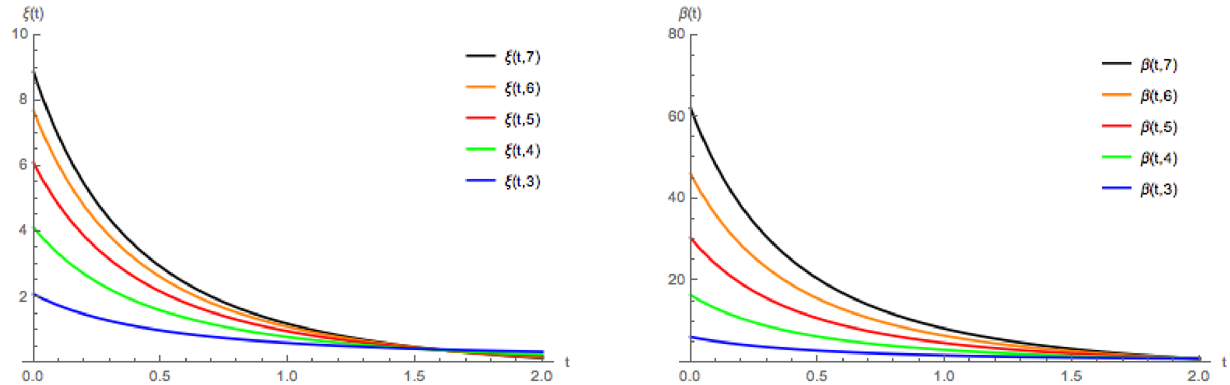
incentives continue to have an impact on the equilibrium outcomes even in the continuous time limit.

Figure 4 illustrates the evolution of belief sensitivity of effort over time. Note that the belief sensitivity of effort (i) always exceeds $1/N$, (ii) declines over time, and (iii) exceeds the value it would take in the absence of free-ridership (the blue dashed line), at least in earlier periods. The first point demonstrates, as discussed above, that the signal jamming incentives are still present and the third point illustrates that they are strong relative to the impact of free-ridership. The behavior of belief sensitivity over time (point (ii)) is intuitive: at earlier times since there is a longer future and therefore, it is possible to influence beliefs and therefore actions of others along a longer horizon, the return on effort is larger. This is one reason why, agents put higher effort in earlier periods, conditional on their belief. A second factor leading to this pattern is that in earlier periods there is larger uncertainty about the underlying state, which implies that agents put a larger weight on the feedback when updating their beliefs. This in turn implies that each agent can potentially have a larger impact on the beliefs of others by jamming the feedback. That is, there is a higher return for effort in such earlier periods.

Point (iii) implies that that depending on the parameter value, the equilibrium production level can even exceed the level without free-ridership, and thus the total production in the team can be larger than one when the agents work individually. Recall that we do not assume any complementarity in the model.

3.4.1 Effect of Team Size

We are going to analyze effect of team-size in the continuous-time limit, which gives us analytical tractability. Let $\beta_t = N\xi_t$ be defined as the sum of the ξ_t.



(a) ξ_t as a function of N (b) β_t as a function of N

Figure 5: Individual and Total Effort over time in small team sizes $(r = 0.003, T = 2, h_0 = 0.8, \eta = 1)$

Proposition 5. *Given the parameters (h_0, h_ε, T) take two team with differet sizes $m, n (m > n)$. Assume r is sufficiently small.*

1. *Then $\exists t^* \in [0, T]$ such that $\forall t \in [t^*, T]$ $\chi_t^n \geq \chi_t^m$ and $\forall t \in [0, t^*]$ $\chi_t^m \geq \chi_t^n$*
2. $\forall t \in [0, T]$ $\beta_t^M \geq \beta_t^N$

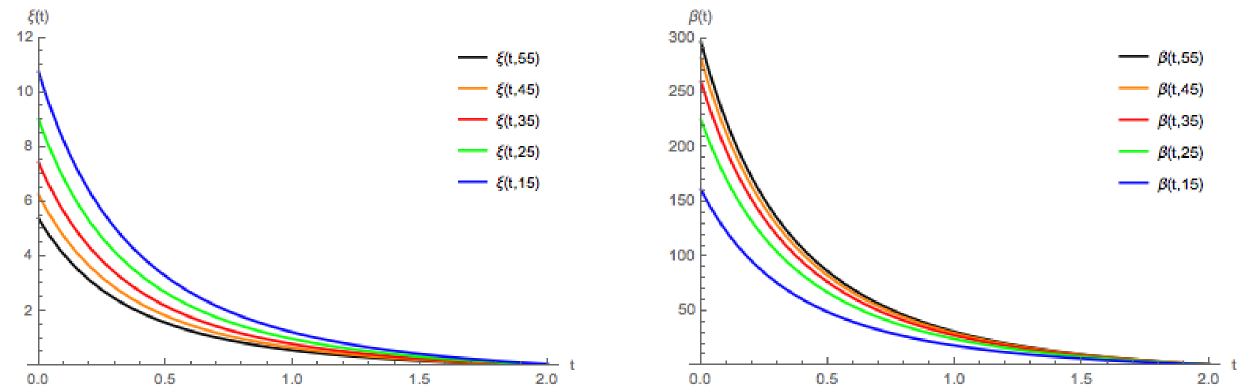
Proof. See Appendix A. □

Figure 5b suggests that in smaller teams adding an extra member increases the total effort. For the individual effort two opposing forces operates: signal jamming and free-riding. In the initial periods signal jamming effect dominates the free-riding effect ,however, due to learning and the deadline signal jamming effect diminishes over time. Therefore, free-riding effect dominates eventually. As a result, in a smaller team individuals work harder near the deadline compared to a larger team.

If the team size is sufficiently big free-riding effect always dominates the signal jamming effect. Similar to small teams, in the large teams increasing the number of member increase the the total-effort. This results suggests, if one wants to maximize the total-effort it is optimal to have two smaller-sized teams instead of one. We will analyze the effect of team decomposition and size in Section 6.

3.5 Role of Imperfect Public Monitoring

The following proposition shows that if the agent's effort level is observable to others, then there is no signal-jamming effect.



(a) ξ_t as a function of N (b) β_t as a function of N

Figure 6: Individual and Total Effort over time in large team sizes $(r = 0.003, T = 2, h_0 = 0.8, \eta = 1)$

Proposition 6. *In the perfect monitoring case, there exists a unique sequential equilibrium where for any* $t = 0, \ldots, T$,

$$a_{it}^* = \frac{1}{N}\mu_t.$$

Proposition 6 implies that imperfect monitoring, in addition to the state uncertainty, is necessary for the signal-jamming effect. In the literature on team production, the inability to monitor the individual effort has been considered as a cost of team production. As Alchian and Demsetz (1972) write:

> ...In team production, marginal products of cooperative team members are not so directly and separably (i.e., cheaply) observable. What a team offers to the market can be taken as the marginal product of the team but not of the team members. The costs of metering or ascertaining the marginal products of the team's members is what calls forth new organizations and procedures.

In this paper, we show the result that in the presence of uncertainty, imperfect monitoring is essential for higher production.

4 The Infinite-horizon Game

4.1 Existence of MPE

The benchmark model of Section 2 can be extended to the infinite-horizon $(t = 0, \ldots)$. In this case, we reinterpret δ as the probability of the project survival: Partnership ends with probability $1 - \delta$ in each period, and each agent receives their share of the output at the end of the partnership.

It is straightforward to show that the equilibrium in Proposition 2 also exists in the infinite-horizon game. Recall that in the unique equilibrium of the finite-horizon case, $a^*_{it} = (\chi_t + \frac{1}{N})\mu_{it}$, where

$$\chi_t = \sum_{s=t+1}^{T} \delta^{s-t} \frac{N-1}{N^2} \frac{h_\varepsilon}{h_s} \left\{ \prod_{k=t+1}^{s-1} \left(1 + \frac{N-1}{N} \frac{h_\varepsilon}{h_k} \right) \right\}$$

and $\chi_T = 0$. If we let T goes to infinity,

$$\chi^*_t = \sum_{s=t+1}^{\infty} \delta^{s-t} \frac{N-1}{N^2} \frac{h_\varepsilon}{h_s} \left\{ \prod_{k=t+1}^{s-1} \left(1 + \frac{N-1}{N} \frac{h_\varepsilon}{h_k} \right) \right\}.$$

Proposition 7. *There exists an equilibrium of the infinite-horizon game where in period t, agent i plays* $a^*_{it} = (\chi^*_t + \frac{1}{N})\mu_{it}$.

Proof. Given that a^*_{it} is finite, it is straightforward that each agent's strategy is the best response to the others' strategy profile. To show that a^*_{it} is finite, note that

$$\begin{aligned} \prod_{k=t+1}^{s-1} \left(1 + \frac{N-1}{N} \frac{h_\varepsilon}{h_k} \right) &< \prod_{k=t+1}^{s-1} \left(1 + \frac{h_\varepsilon}{h_k} \right) \\ &= \prod_{k=t+1}^{s-1} \frac{h_{k+1}}{h_k} = \frac{h_s}{h_{t+1}}. \end{aligned}$$

Therefore, we have

$$\chi^*_t < \sum_{s=t+1}^{\infty} \delta^{s-t} \frac{N-1}{N^2} \frac{h_\varepsilon}{h_{t+1}} = \frac{\delta}{1-\delta} \frac{N-1}{N^2} \frac{h_\varepsilon}{h_{t+1}}$$

so we have proved our desired result. □

4.2 Stochastic State

In this subsection, we consider an infinite-horizon model in which the state of the world changes stochastically over time. Let θ_t be the state of the world at period t. We assume that θ_t follows AR(1) process:

$$\theta_{t+1} = c + \varphi\theta_t + \sigma_t,$$

where $\varphi < 1$ and σ_t is i.i.d. with $\sigma_t \sim \mathcal{N}(0, 1/h_\sigma)$.

In this subsection, the signal structure is slightly generalized that it is a weighted sum of the state and the effort. Specifically, the feedback at the end of each period is given by

$$y_t = \kappa_\theta \theta_t + \kappa_a \sum_{i=1}^{N} a_{it} + \varepsilon_t,$$

where $\kappa_\theta, \kappa_a > 0$ are constants, $\varepsilon_t \sim \mathcal{N}(0, 1/h_\varepsilon)$ and ε_t's are independent across t.

Same as the benchmark model, the posterior belief about the state in any period follows a normal distribution. let μ_t and h_t be the mean and the precision of belief about θ_t in the beginning of period t. Given the equilibrium strategy a^*_{it}, the agents use the signal $z_t = y_t - \kappa_a \sum_{i=1}^{N} a^*_{it}$ to update the belief. Note that similar to the benchmark model, $z_t \sim \mathcal{N}(\kappa_\theta \theta_t + \kappa_a \alpha, 1/h_\varepsilon)$ when agent i plays $a_{it} = a^*_{it} + \alpha$. Let $\hat{\mu}_t$ and $\hat{h}_t$ be the mean and the precision of belief about θ_t *after* the feedback y_t is realized. Then we have

$$\hat{\mu}_t = \frac{h_t \mu_t + h_\varepsilon \kappa_\theta z_t}{h_t + h_\varepsilon \kappa_\theta^2},$$

$$\hat{h}_t = h_t + h_\varepsilon \kappa_\theta^2.$$

Then μ_{t+1} and h_{t+1} are given by

$$\mu_{t+1} = c + \varphi \hat{\mu}_t = c + \varphi \frac{h_t \mu_t + h_\varepsilon \kappa_\theta z_t}{h_t + h_\varepsilon \kappa_\theta^2} \tag{3}$$

$$\frac{1}{h_{t+1}} = \frac{\varphi^2}{\hat{h}_t} + \frac{1}{h_\sigma} \Longrightarrow h_{t+1} = \frac{(h_t + h_\varepsilon \kappa_\theta^2) h_\sigma}{h_t + h_\varepsilon \kappa_\theta^2 + \varphi^2 h_\sigma} \tag{4}$$

We consider a case where the belief precision is stationary over time, that is, $h_t = h^*$ for all t. Then by (4), h^* satisfies

$$h^*(h^* + h_\varepsilon \kappa_\theta^2 + \varphi^2 h_\sigma) = (h^* + h_\varepsilon \kappa_\theta^2) h_\sigma.$$

The above equation has unique positive solution

$$h^* = \frac{h_\varepsilon \kappa_\theta^2 - (1-\varphi^2) h_\sigma}{2} \left(-1 + \sqrt{1 + \frac{4 h_\sigma h_\varepsilon \kappa_\theta^2}{(h_\varepsilon \kappa_\theta^2 - (1-\varphi^2) h_\sigma)^2}} \right).$$

If $h_t = h^*$ for all t, then the equilibrium sensitivity level is also stationary, that is, $\xi_t = \xi^*$ for all t. Define $\rho^*_k = \partial \mu_{t+k} / \partial z_t$ as the rate at which the period-t signal affects the period-$(t+k)$ belief (which is same for all t in the stationary case). Then from (3), we have

$$\rho_k^* = \left(\frac{\partial \mu_{t+1}}{\partial \mu_t}\right)^{k-1} \frac{\partial \mu_{t+1}}{\partial z_t} = (\varphi H^*)^k \frac{h_\varepsilon \kappa_\theta}{h^*},$$

where $H^* = \frac{h^*}{h^* + h_\varepsilon \kappa_\theta^2}$.

Then the equilibrium action in period t is given by

$$a_{it}^* = \frac{\mu_{it}}{N}\left(1 + (N-1)\kappa_a \sum_{s=t+1}^{\infty} \delta^{t-s} (\varphi H^*)^{t-s} \frac{h_\varepsilon \kappa_\theta}{h^*} \xi_s\right).$$

Therefore, if the equilibrium sensitivity of belief is stationary, its level is given by

$$\xi^* = \frac{1}{N - (N-1)X^*}, \tag{5}$$

where

$$X^* = \frac{\delta\varphi(1-H^*)}{1-\delta\varphi H^*}\frac{\kappa_a}{\kappa_\theta}.$$

There are several observations: 1) the stationary effort level is increasing in δ and decreasing in h_σ / h_ε; 2) for any parameter values δ, h_ε, and h_σ, there exists a value of κ_θ and κ_a such that the stationary effort level is first-best efficient.

4.2.1 Dynamic Programming

In the case of stochastic state, the same MPE can be derived by solving a dynamic programming. Assume that $\kappa_a = \kappa_\theta = 1$, and consider a symmetric Markov perfect profile

$$a_i(h^t) = \hat{a}(\mu_{it}) \qquad \text{for all } i.$$

Let μ be the common posterior mean of the state (which is only a function of history of public signal). Then agent i's value function can be written as a function of μ_i and μ, and the dynamic programming problem can be written as

$$V(\mu_i, \mu) = \max_a (1-\delta)\left\{\frac{\mu_i}{N}((N-1)\hat{a}(\mu) + a) - \frac{a^2}{2}\right\} + \delta \mathbb{E}[V(\mu_i', \mu')|a]$$

subject to

$$
\begin{aligned}
\mu_i' &= c + \varphi \frac{h^* \mu_i + h_\varepsilon z_i}{h^* + h_\varepsilon}, \\
\mu' &= c + \varphi \frac{h^* \mu_{-i} + h_\varepsilon z}{h^* + h_\varepsilon}, \\
z_i &= y - (N-1)\hat{a}(\mu) - a = \theta_t + \varepsilon_t, \\
z &= y - N\hat{a}(\mu) = \theta_t + (a - \hat{a}(\mu_{-i})) + \varepsilon_t.
\end{aligned}
$$

The first-order condition with respect to a is given by

$$
(1-\delta)\left\{\frac{\mu_i}{N} - a^*(\mu_i,\mu)\right\} + \delta\varphi \frac{h_\varepsilon}{h^* + h_\varepsilon} \mathbb{E}\left[\frac{\partial V(\mu_i',\mu')}{\partial \mu'} | a^*(\mu_i,\mu)\right] = 0. \tag{6}
$$

On the other hand, the envelope theorem gives

$$
\frac{\partial V(\mu_i,\mu)}{\partial \mu} = (1-\delta)\frac{\mu_i}{N}(N-1)\hat{a}'(\mu) + \delta\varphi \frac{h^*}{h^* + h_\varepsilon} \mathbb{E}\left[\frac{\partial V(\mu_i',\mu')}{\partial \mu'} | a^*(\mu_i,\mu)\right]. \tag{7}
$$

Combining (6) and (7), we have

$$
\begin{aligned}
\frac{\partial V(\mu_i,\mu)}{\partial \mu} =& (1-\delta)\frac{\mu_i}{N}(N-1)\hat{a}'(\mu) - (1-\delta)\frac{h^*}{h_\varepsilon}\left\{\frac{\mu_i}{N} - a^*(\mu_i,\mu)\right\} \\
=& \frac{1-\delta}{N}\left\{\mu_i(N-1)\hat{a}'(\mu) - \frac{h^*}{h_\varepsilon}(\mu_i - Na^*(\mu_i,\mu))\right\}.
\end{aligned} \tag{8}
$$

Plugging (8) back into (6), we have

$$
a^*(\mu_i,\mu) = \frac{\mu_i}{N} + \delta\varphi \frac{h_\varepsilon}{h^* + h_\varepsilon}\frac{1}{N} \mathbb{E}\left[\mu_i(N-1)\hat{a}'(\mu) - \frac{h^*}{h_\varepsilon}(\mu_i' - Na^*(\mu_i',\mu')) | a^*(\mu_i,\mu)\right]. \tag{9}
$$

On the equilibrium path, $\mu_i = \mu$. Assume that the optimal action is linear in the individual belief, that is, $a^*(\mu) = \hat{a}(\mu) = \xi^*\mu$ for all i. Then we have

$$
N\xi^*\mu = \mu + \delta\varphi\left[(1-H^*)(N-1)\xi^* - H^*(1-N\xi^*)\right\}\mathbb{E}[\mu'],
$$

where $H^* = \frac{h^*}{h^*+h_\varepsilon}$. Since $E[\mu'] = \mu$, we have

$$\xi^* = \frac{1}{N-(N-1)\frac{\delta\varphi(1-H^*)}{1-\delta\varphi H^*}},$$

which is the same as (5).

4.3 Comparison to Trigger Strategies

In the infinite-horizon game, there would be many other equilibria that relies on grim-trigger strategy. Then we conjecture that when the length of a period vanishes—so that the agents can change their action more frequently according to the feedback—such trigger equilibria would vanish while our equilibrium survives. Formally, we conjecture that when the length of a period vanishes, the equilibrium payoff of the game is bounded above by the payoff of our equilibrium. The underlying reasoning is similar to Sannikov and Skrzypacz (2007): When the information becomes noisy, then the trigger strategy profile must punish the agents based on the noisy information, which increases the probability of type I error. Therefore, the cost of making type I error outweighs the benefit from future cooperation.

Note that our equilibrium does not rely on trigger strategies, yet the agents cooperate in the equilibrium. Our result shows that in the presence of uncertainty in partnership game, we are able to induce cooperation without using the trigger mechanism. The comparison between our mechanism and trigger mechanism under various settings, and analysis for the possible combination of the two would be topics for future research.

5 Asymmetric Information

In this section, we look an alternative model in which agents are heterogeneous in their private information about the state.

We consider a team production game played in continuous time ($t \in [0,T]$).[10] There are N agents in the team. Each agent can be either an *expert* or a *novice*: The expert has perfect information about the state θ, while the novice has the prior same as the benchmark model ($\theta \sim \mathcal{N}(\mu_0, 1/h_0)$). Denote N^e

[10]In this section instead of looking for the limit of discrete-time model, we conduct the analyses directly in continuous-time. We argue that the limit of equilibrium of the discrete-time game ($\Delta \to 0$) and the continuous-time model would agree.

and N^n as the number of the experts and the novices, respectively. We assume number of the experts in the team is commonly known.[11]

At time t, agent i chooses the effort level a_{it}. The signal $\{Y_t\}$ follows a stochastic process

$$dY_t = (\theta + A_t)dt + \frac{1}{\sqrt{\eta}}dW_t,$$

where $A_t = \sum_{i=1}^{N} a_{it}$.

We focus on the linear symmetric equilibrium where the effort level of the expert and novice are affine functions of the θ and μ_t, respectively. Let a_t^e and a_t^n be the effort level of the expert and the novice at time t. An equilibrium is called a *symmetric linear Markov perfect equilibrium* [12] if there exist functions $\{\gamma_t, \psi_t, \xi_t\}$ such that $a_t^e = \gamma_t\theta + \psi_t$ and $a_t^n = \xi_t\mu_t$. As a regularity condition, we require strategies to be admissible. Strategy a is admissible if $\mathbb{E}\left[\int_0^t (a_{it})^2\right] dt < \infty$. Since we are focusing on linear strategies necessary and sufficient condition of admissibility becomes$(\gamma_t, \psi_t, \xi_t)$ being in L^2.

5.1 Belief Updating

Given the strategy of the expert $a_t^e = \gamma_t\theta + \psi_t$ and $a_t^n = \xi_t\mu_t$, the novices use the following process Z_t to update their beliefs:

$$\begin{aligned} dZ_t &= dY_t - (N^e\psi_t + N^n\xi_t\mu_t)dt \\ &= (1 + N^e\gamma_t)\theta dt + \frac{1}{\sqrt{\eta}}dW_t. \end{aligned}$$

On the other hand, If expert i deviates to play a, then the process becomes

$$\begin{aligned} dZ_t &= (a + \theta + (N^e - 1)(\gamma_t\theta + \psi_t) + N^n\xi_t\mu_t)dt - (N^e\psi_t + N^n\xi_t\mu_t)dt \\ &= (a - \psi_t + (1 + (N^e - 1)\gamma_t)\theta)dt + \frac{1}{\sqrt{\eta}}dW_t. \end{aligned}$$

[11] It is an interesting extension in which each team members has an prior about the other team members type. Another possible extension is that each team member has partial information about the state. To be precise, assume aggreagate state is defined as $\Theta = \sum_i \theta_i$ and θ_i is the private information of each agent. In this case each agent also cares about the private information of the other agents.

[12] Linear strategies is well known for being tractable in the Gaussian setup of insider trading Kyle (1985), Back (1992). For a recent application in a Duopoly game see Bonatti, Cisternas, and Toikka (2015)

Then by Liptser and Shiryaev (2013) (Theorem 12.1), the novices' belief μ_t follows

$$d\mu_t = \frac{\eta(1+N^e\gamma_t)}{h_t}(dZ_t - (1+N^e\gamma_t)\mu_t dt),$$

where

$$\dot{h}_t = \eta(1+N^e\gamma_t)^2.$$

5.2 Equilibrium

Proposition 8. *There exists a symmetric linear Markov perfect equilibrium, in which the coefficients of the strategies are obtained from the solution to following system of differential equations*

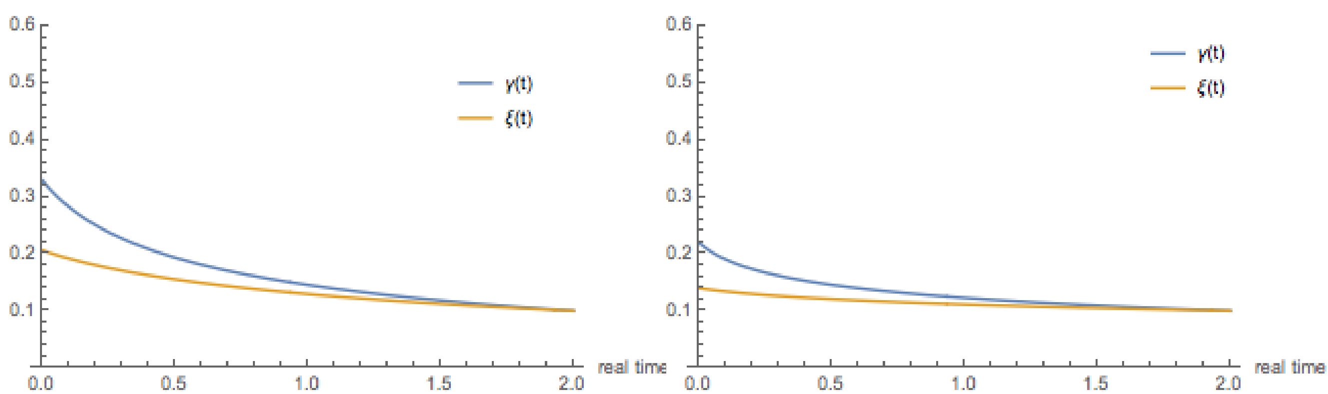
$$\dot{\gamma}_t = \frac{(1+N^e\gamma_t)(\gamma_t - \frac{1}{N})rh_t - \eta(1+\gamma_t N^e)^2 \frac{N^n}{N}\xi_t}{h_t(1+\frac{N^e}{N})}$$

$$\dot{\xi}_t = \left(r - \frac{N-N^e-1}{N}\frac{\eta}{h_t}\right)\left(\xi_t - \frac{1}{N}\right) - \frac{N-N^e-1}{N^2}\frac{\eta}{h_t}$$

$$\dot{h}_t = \eta(1+N^e\gamma_t)^2$$

with boundary conditions $\gamma_T = \xi_T = 1/N$.

Proof. See Appendix A.[13] □



(a) when $N^e = 1$ and $N^n = 9$ (b) when $N^e = 4$ and $N^n = 6$

Figure 7: The equilibrium behavior of the experts and the novices $(N = 10, T = 2)$

[13] Sufficient conditions for existence of a solution to be added.

In Figure 7 we plot the experts and novices equilibrium strategies given the team composition. Increasing the number of experts in the team has two effects. First, when the number of novices decreases each novice has smaller incentive for "signal jamming". Second, "signal jamming" incentives are increased due to the fact that signals become more precise. In the above example first effect dominate the second one.

5.3 Team Composition

In our main model, the agents are homogeneous. In the presence of heterogeneous agents, a relevant question would be how to group these agents into teams to work on projects. A particularly interesting dimension of heterogeneity is with respect to agents' expertise in the project, as captured by the asymmetric information model discussed in Section 5 above. In this section, we take up a much simplified special case of that model and address within the context of examples, two related questions: (1) if a principal has a fixed number of experts and a fixed number of novices that work for him, how would he allocate these workers among fixed number of projects? (2) what is the optimal number of experts in a fixed-sized team? We assume, in each application, that the principal wants to maximize the agent's effort levels. The simple model we adopt goes back to the discrete time and limits the number of periods to two. The next proposition characterizes the equilibrium effort level in this model:

Proposition 9. *Consider a discrete-time version of the asymmetric information model. Assume that $T = 2$ and the team consists of N_E experts and N_N novices. Then, there exists an equilibrium in which the first period effort level of an expert is given by*

$$\gamma\theta,$$

where γ solves

$$\gamma = \frac{1}{N} + \delta\frac{N-N_I}{N}\frac{h_\varepsilon(1+N_I\gamma)}{h_0+h_\varepsilon(1+N_I\gamma)^2} = \frac{1}{N} + \delta\frac{N-N_I}{N}\frac{1}{\frac{h_0}{h_\varepsilon(1+N_I\gamma)^2}+1} \times \frac{1}{1+N_I\gamma}.$$

The first period effort level of a novice is

$$\mu_0\left[\frac{1}{N}+\delta\frac{N-N_I-1}{N}\frac{1}{\frac{h_0}{h_\varepsilon(1+N_I\gamma)^2}+1}\times\frac{1}{1+N_I\gamma}.\right].$$

The second period effort level of an expert is θ/N, while that of a novice is μ_2/N.

5.3.1 Allocating experts among teams

Assume that a principal has two experts and two novices working for him. He has to form two teams, each team consisting of two agents, to work on two separate projects. If the principal allocates two novices in one team and two experts in another–i.e. if he forms "segregated teams"—, then in the team of experts, the effort choices contain no signaling or signal jamming incentives. The team of novices have the signal jamming incentives as in our main model. Therefore, in this configuration, there are two agents whose efforts are augmented due to "signal jamming incentives". If, conversely, the principal allocates one expert and one novice to each team, then the novices have no incentive to signal jam, since their partners are informed. On the other hand, each expert has an incentive to "signal" his information to his partner. Therefore, in this configuration, two agents' efforts are augmented due to the "signaling incentives". The following proposition establishes cases in which either configuration is optimal.

Proposition 10. *There exists $\bar{\rho}>\underline{\rho}$ such that if $h_0/h_\varepsilon>\bar{\rho}$ then the principal optimally allocates one expert and one novice in each team; and if $\underline{\rho}>h_0/h_\varepsilon$, then the principal optimally forms one team of experts and one team of novices.*

Proof. Proof is by simple algebra.

$$\frac{1}{\frac{h_0/h_\varepsilon}{(1+\gamma)^2}+1}\times\frac{1}{1+\gamma}\quad\text{vs}\quad\frac{1}{\frac{h_0}{h_\varepsilon}+1}$$

$$\left(\frac{h_0/h_\varepsilon}{(1+\gamma)^2}+1\right)\times(1+\gamma)\quad\text{vs}\quad\frac{h_0}{h_\varepsilon}+1$$

$$\frac{h_0/h_\varepsilon}{(1+\gamma)^2}+\gamma\left(\frac{h_0/h_\varepsilon}{(1+\gamma)^2}+1\right)-\frac{h_0}{h_\varepsilon}=\frac{h_0}{h_\varepsilon}\left(\frac{1}{1+\gamma}-1\right)+\gamma$$

Then argue γ remains bounded away from 0 and away from infinity as h_0/h_ε varies. □

The proposition states that when h_0/h_ε is fairly large, effort is maximized in the mixed teams configuration, and in the opposite case, effort is maximized in the segregated team configuration. Intuitively, when the teams are mixed, the uninformed agents understand that the feedback signal they are receiving is less noisy. For this reason, their second period effort tend to be more sensitive to the first period signal, which gives the experts a larger incentive to exert effort. To counter this effect, the novices in each team also understand that the feedback signal is "exaggerated" by the increased effort of the experts, and discount it accordingly. The result follows because the former effect is particularly strong if the initial precision of the state distribution h_0 is large relative to the precision of ε, since that is the case when the feedback signal is relatively imprecise and an increase in this precision has a large impact. Consequently, mixed teams are more desirable in this case.

5.3.2 Optimal number of experts in a team

Next, consider a principal who must decide how many experts to include in a team of fixed size N. Again, the principal seeks to maximize total effort in the team. Figure 8 below represents numerical calculations showing total effort as a function of number of experts in a team for three different values of h_0/h_ε.

The figure and the underlying numerical calculations suggest that optimal number of experts is zero when h_0/h_ε is small but is increasing as this ratio increases. The intuition is similar to above: When there are more experts, the feedback signal is more precise, and therefore the effort choices of novices are more sensitive to feedback. This incentivizes the experts (and novices) to increase their effort. On the other hand, understanding the exaggeration in the feedback, the novices discount it more when there are more experts. And once again, the former effect is strong when h_0/h_ε is large. When there are multiple novices, there is an additional factor that was absent in the previous application: when the number of experts increases by one, the number of novices decreases by one. Therefore, there are fewer teammates whose effort will be impacted by feedback, and therefore less incentive to signal on the part of the experts, and to signal-jam on the part of the novices. A combination of these three factors leads to the non-monotonicity of total effort in the number of experts in a team.

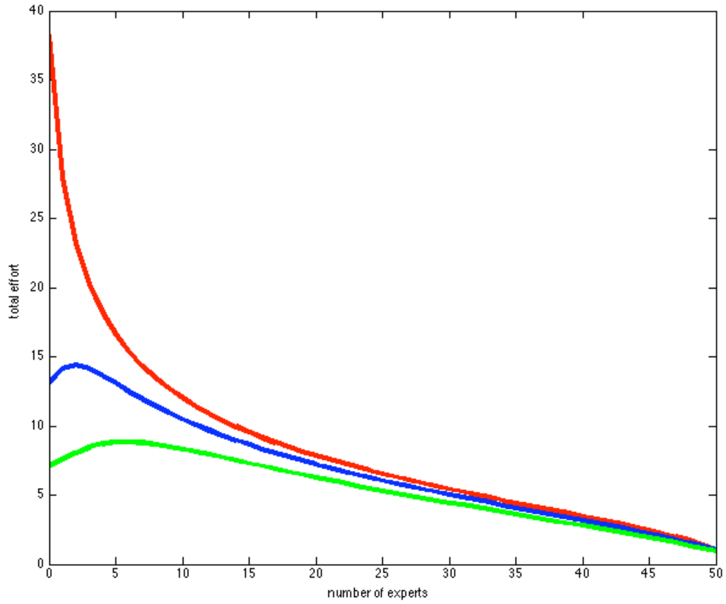


Figure 8: Total effort as a function of the number of experts in a team. ($N = 50, T = 2$).

5.3.3 Private Learning

In a ongoing work, we are studying the model in which each agent observes a private signal about the true state. To be precise, look the following monitoring structure

$$dY_{it} = (\theta + A_t)dt + dW_{it}$$

where W_i's are independent Brownian Motions.[14] In this model each agents cares not only about his private belief, he cares also about the public belief. Similar to asymetric information case, we will be focus on the linear strategies in which action at t is an affine function of public and the private belief.

6 Applications

Given the tractability of the model, we are able to explore several applications of the model. This part is work in progress, so for some applications we have conjectures and for others we present the ideas in

[14]In the repeated game-literautre this monitoring is called as conditionally indepedent private monitoring. In general we can assume any correlation structure among the shocks.

simple examples.

6.1 Dynamic Information Disclosure Design

Consider the following information design problem: Suppose that there is a principal who hires a group of agents who work together. The principal can control the provision of information, and he wants to maximize the ex-ante total production (he doesn't care about the cost).

Formally, given the uncertainty about the marginal productivity $\theta \sim \mathcal{N}(\mu_\theta, \frac{1}{h_\theta})$, the principal can choose the value of

- $h_\eta \geq 0$: the precision of the additional prior information $x_0 = \theta + \eta$, where $\eta \sim \mathcal{N}(0, \frac{1}{h_\eta})$; and
- $h_\varepsilon(t) \geq 0$: the precision of the feedback y_t (so $\varepsilon_t \sim \mathcal{N}(0, \frac{1}{h_\varepsilon(t)})$).

Since the precision of each feedback can be heterogeneous, we need to slightly extend our result before presenting the principal's problem.

Belief Updating Let μ_t and h_t $(t = 0, \ldots T)$ be the mean and the precision of the posterior at the beginning of period t. Then

$$h_t = h_\theta + h_\eta + \sum_{s=0}^{t-1} h_\varepsilon(s), \qquad \mu_t = \frac{h_\theta \mu_0 + h_\eta z_\eta + \sum_{s=0}^{t-1} h_\varepsilon(s) z_s}{h_t},$$

where $z_\eta = x_0 - \theta$. Define $\rho_{s,t} (s < t)$ be the rate at which increase in a_{is} affects μ_t. Then from the above equation, $\rho_{s,t} = \frac{h_\varepsilon(s)}{h_t}$.

Equilibrium and Ex Ante Production By the backward induction argument, we show that there exists unique sequential equilibrium in which the players choose $a_{it}^* = \xi_t \mu_t$, where

$$\begin{aligned}
\xi_T &= \frac{1}{N} \\
\xi_t &= \frac{1}{N}\left(1 + (N-1)\sum_{s=t+1}^{T} \delta^{s-t}\rho_{t,s}\xi_s\right) \\
&= \frac{1}{N} + \frac{\delta}{N}\left\{(N-1)\rho_{t,t+1}\xi_{t+1} + (N-1)\sum_{s=t+2}^{T}\delta^{s-t-1}\rho_{t,s}\xi_s\right\}, \\
&= \frac{1}{N} + \delta\left\{A(t)\xi_{t+1} + B(t)\left(\xi_{t+1} - \frac{1}{N}\right)\right\},
\end{aligned}$$

where $A(t) = \frac{N-1}{N}\frac{h_\varepsilon(t)}{h_{t+1}}$ and $B(t) = \frac{h_\varepsilon(t)}{h_\varepsilon(t+1)}$. Note that $\delta A(t)\xi_{t+1}$ captures the effect of a_{it} to $a_{-i,t+1}$ and $\delta B(t)\left(\xi_{t+1} - \frac{1}{N}\right)$ captures the effect of a_{it} to $a_{-i,s}$ $(s \geq t+2)$.

To calculate the ex ante expected payoff, we need to compute the distribution of posterior belief (which is captured by μ_t) from the perspective of period 0. Since $z_t = \theta + \varepsilon_t$ and ε_t are independent across time,

$$\sum_{s=0}^{t-1} z_s = t\theta + \sum_{s=0}^{t-1}\varepsilon_s \sim \mathcal{N}\left(t\mu_0, \frac{t^2}{h_0} + \frac{t}{h_\varepsilon}\right).$$

Since $\mu_t = \frac{h_0}{h_t}\mu_0 + \frac{h_\varepsilon}{h_t}\sum_{s=0}^{t-1} z_s$, the distribution of the posterior μ_t is

$$\mu_t \sim \mathcal{N}\left(\mu_0, \left(\frac{h_\varepsilon}{h_t}\right)^2\left(\frac{t^2}{h_0} + \frac{t}{h_\varepsilon}\right)\right) = \mathcal{N}\left(\mu_0, \frac{1}{h_0} - \frac{1}{h_t}\right).$$

Since the period-t individual production when the posterior mean is μ_t is $\xi_t\mu_t^2$, the ex-ante total production is given by

$$N\mathbb{E}_0[P] = N\sum_{t=0}^{T}\delta^t\xi_t\left(\mu_0^2 + \left(\frac{1}{h_0} - \frac{1}{h_t}\right)\right).$$

Information Disclosure Design Problem Given the above analysis, we define the principal's information disclosure design problem as follows:

$$\max_{\left(h_\eta, \{h_\varepsilon(t)\}_{t=0}^{T}\right)} \sum_{t=0}^{T}\delta^t\xi_t\left(\mu_0^2 + \left(\frac{1}{h_0} - \frac{1}{h_t}\right)\right)$$

subject to

$$\xi_T = \frac{1}{N}$$
$$\xi_t = \frac{1}{N} + \delta\left\{A(t)\xi_{t+1} + B(t)\left(\xi_{t+1} - \frac{1}{N}\right)\right\}$$
$$h_\eta \geq 0, h_\varepsilon(t) \geq 0 \text{ for all } t = 0, \ldots, T.$$

Conjecture 1. In the optimal information design scheme,

$$h_\varepsilon(t-1) < h_\varepsilon(t) \qquad \text{for all } t = 1, \ldots T-1.$$

Conjecture 1 implies that the principal would like to "backload" information in the optimal design scheme. The intuition is as follows. Naturally, higher precision of period-t signal induces higher incentive in period t and lower incentive for all other periods. At the same time, however, incentive is accumulated in backward direction: Higher sensitivity in the future induces more incentive to effort now. This second effect makes the back-loading information scheme more optimal.

6.2 Contracting

Let the game start with a share structure s which specifies what fraction of the output owned by each agent. Share structure s is a vector in $[0,1]^N$ such that $\sum_i s_i = 1$. We are looking for the share structure which maximizes the total output given h_0 and h_ε.

We can recursively define ξ_t as follows

$$\xi_{i,t} = s_i\left[1 + \sum_{l=t+1}^{T} \delta^{l-t}\rho_l \sum_{j\neq i} \xi_{l,t}\right] \quad \xi_{i,T} = s_i$$

Given the ξ_t expected equilibrium output becomes;

$$\max_{\{s_i\}_{i=1}^n} \sum_{t=0}^{T} \delta^t \sum_i \xi_{i,t}\left(\mu_0^2 + \left(\frac{1}{h_0} - \frac{1}{h_t}\right)\right)$$

Conjecture 2. Total output of the team is maximized when $s = \left(\frac{1}{N}, \ldots.., \frac{1}{N}\right)$

Here is an argument for equal share structure. Each agent's action at time t has two component: i) myopic part (s_i) ii) signal jamming part (ξ_t). Given that we want to maximize the total sum of effort the distribution of the shares does not matter for the myopic part. Therefore, it is enough to consider maximizing the signal jamming part. For the signal jamming part notice that for each agent i, $(\xi_{j,t})$ consists of s_i and s_j. Then to maximize $s_i s_j$ it is optimal to choose $s_i = s_j$. To finish the argument observe that this is true for every i, j pair.

Appendices

A Omitted Proofs

A.1 Proof of Proposition 1

Stage Payoff Let $\pi^{\theta}(a_1a_2)$ be the probability of *Success* when the state is θ and the agents play a_1a_2. Those probabilities are described in Figure 16. Note that $\pi^{\theta}(a_1a_2) = \pi_1^{\theta}(a_1) + \pi_2^{\theta}(a_2)$, where

$$\pi_i^G(a_i) = \begin{cases} p_G + p & \text{if } a_i = H \\ p_G & \text{if } a_i = M \\ 0 & \text{if } a_i = L, \end{cases} \qquad \pi_i^B(a_i) = \begin{cases} 2p & \text{if } a_i = H \\ p & \text{if } a_i = M \\ 0 & \text{if } a_i = L. \end{cases}$$

Let $\pi(a_1a_2;\mu) = \mu\pi^G(a_1a_2) + (1-\mu)\pi^B(a_1a_2)$. Define $\hat{p}(\mu) = p + \mu(p_G - p)$ be the marginal productivity of the first unit of effort given the belief μ. Then, for any $i = 1,2$, the stage payoff $u_i(a_ia_j;\mu)$ can be written as $u_i(a_ia_j;\mu) = \pi(a_ia_j;\mu) - c_i(a_i) = \pi_i(a_i;\mu) + \pi_j(a_j;\mu) - c_i(a_i)$, where

$$\pi_i(a_i;\mu) = \begin{cases} \hat{p}(\mu) + p & \text{if } a_i = H \\ \hat{p}(\mu) & \text{if } a_i = M \\ 0 & \text{if } a_i = L, \end{cases} \qquad c_i(a_i) = \begin{cases} 2c & \text{if } a_i = H \\ c & \text{if } a_i = M \\ 0 & \text{if } a_i = L. \end{cases}$$

Note that $\pi_i(a_i;\mu)$ is the productivity contributed by agent i.

We can also rewrite the stage payoff as $u_i(a_ia_j;\mu) = \alpha_i(a_i;\mu) + \pi_j(a_j;\mu)$, where $\alpha_i(a_i,\mu) = \pi_i(a_i;\mu) - c_i(a_i)$ captures the self-contribution part of agent i's stage payoff. Note that $\alpha_1(H;\mu) < \alpha_1(M;\mu)$ for any μ, and that $\alpha_1(M;\mu) > \alpha_1(L;\mu)$ if and only if

$$\mu \geq \mu^* \equiv \frac{c-p}{p_G - p}.$$

On the other hand, $\pi_j(H;\mu) > \pi_j(M;\mu) > \pi_j(L;\mu)$ for any μ, while implies that inducing the other guy to work is always better.

Belief Updating Let $\mu^S(\mu;a_1a_2)$ and $\mu^F(\mu;a_1a_2)$ be the posterior belief when the prior is μ, the agents play a_1a_2 and observe a signal of *Success* and *Fail*, respectively. Then for any $a_1a_2 \neq LL$, we have

$$\frac{\mu^S(\mu;a_1a_2)}{1-\mu^S(\mu;a_1a_2)} = \frac{\mu}{1-\mu}\lambda^S(a_1a_2), \qquad \frac{\mu^F(\mu;a_1a_2)}{1-\mu^F(\mu;a_1a_2)} = \frac{\mu}{1-\mu}\lambda^F(a_1a_2),$$

where $\lambda^S(a_1a_2) = \frac{\pi^G(a_1a_2)}{\pi^B(a_1a_2)}$ and $\lambda^F(a) = \frac{1-\pi^G(a_1a_2)}{1-\pi^B(a_1a_2)}$ are likelihood functions for the *Success* and *Fail* signal, respectively.

If $a_1a_2 = LL$, Bayes' rule does not apply when *Success* is observed. However, the belief consistency condition implies that $\mu^S(\mu;LL) > \mu$ in any sequential equilibrium. In the case of *Fail*, Bayes' rule implies that $\mu^F(\mu;LL) = \mu$.

For any $a_1a_2 \neq LL$, define $\bar{\mu}(a_1a_2)$ and $\underline{\mu}(a_1a_2)$ be such that

$$\frac{\bar{\mu}(a_1a_2)}{1-\bar{\mu}(a_1a_2)} = \frac{\mu^*}{1-\mu^*}\cdot\frac{1}{\lambda^S(a_1a_2)}, \qquad \frac{\underline{\mu}(a_1a_2)}{1-\underline{\mu}(a_1a_2)} = \frac{\mu^*}{1-\mu^*}\cdot\frac{1}{\lambda^F(a_1a_2)},$$

Then if the agents play a_1a_2 in the first period, $\mu^S(\mu;a_1a_2) > \mu^*$ if and only if $\mu > \underline{\mu}(a_1a_2)$ and $\mu^F(\mu;a_1a_2) > \mu^*$ if and only if $\mu > \bar{\mu}(a)$. It is easy to check that $\underline{\mu}(a_1a_2) < \mu^* < \bar{\mu}(a_1a_2)$ for all $a_1a_2 \neq LL$.

Equilibrium Analysis Let μ_{it} be agent i's belief in period t. Note that $\mu_{11} = \mu_{21} \equiv \mu_1$ by the common prior assumption, and $\mu_{12} = \mu_{22}$ on the equilibrium path.

In the second period, the agents chooses the action that maximizes his stage payoff. Since the payoff is additively separable, regardless of the other agent's action, the agent's payoff difference between playing H and M is $p-c<0$ and difference between playing M and L is $\hat{p}(\mu)-c$. Therefore, agent i plays M if $\mu_{i2} > \mu^*$ and plays L if $\mu_{i2} < \mu^*$.

Now let us analyze the behavior in the first period. Define $\underline{\mu}^* = \max_{a_1a_2\neq LL}\underline{\mu}(a_1a_2)$ and $\bar{\mu}^* = \min_{a_1a_2\neq LL}\bar{\mu}(a_1a_2)$. Then for any prior $\mu_1 \in (\underline{\mu}^*, \bar{\mu}^*)$, unless the action profile is LL in the first period, then $\mu_2 > \mu^*$ in the case of *Success* and $\mu_2 < \mu^*$ in the case of *Fail*.

Suppose the agents play $a_1a_2 \neq LL$ in the first period. Then agent 1's expected payoff is

$$\begin{aligned} U_1(a_1a_2;\mu_1) &= [\alpha_1(a_1;\mu_1)+\beta_1(a_2;\mu_1)] + \delta\pi(a_1a_2;\mu_1)\left[2\hat{p}(\mu^S(\mu_1;a_1a_2)) - c\right] \\ &= [\alpha_1(a_1;\mu_1)+\beta_1(a_2;\mu_1)] + \delta\pi(a_1a_2;\mu_1)\left[2\{p+\mu^S(\mu_1;a_1a_2)(p_G-p)\} - c\right] \\ &= [\alpha_1(a_1;\mu_1)+\beta_1(a_2;\mu_1)] + \delta\left[\pi(a_1a_2;\mu_1)(2p-c) + 2\mu_1\pi^G(a_1a_2)(p_G-p)\right]. \end{aligned}$$

Note that the above payoff is same when a_1a_2 is on the equilibrium path and when a_1a_2 is off the equilibrium path. Since $\mu_1 \in (\underline{\mu}^*, \bar{\mu}^*)$, the agents' action in period 2 is same regardless of the deviation from the equilibrium profile.

Let $a_1^*a_2^*$ be the equilibrium strategy profile in the first period, and consider the incentive for agent 1 to deviate to play a_1'. Then if $a_1^*a_2^* \neq LL$ and $a_1'a_2^* \neq LL$, the diffence in agent 1's expected payoff is

$$\begin{aligned} U_1(a_1'a_2^*;\mu_1) - U_1(a_1^*a_2^*;\mu_1) = &\underbrace{\alpha_1(a_1';\mu_1) - \alpha_1(a_1^*;\mu_1)}_{\text{myopic benefit of deviation}} \\ &+ \underbrace{\delta\left[(\pi_1(a_1';\mu_1) - \pi_1(a_1^*;\mu_1))(2p-c) + 2\mu_1(\pi_1^G(a_1') - \pi_1^G(a_1^*))(p_G-p)\right]}_{\text{future cost of deviation}}. \end{aligned}$$

Note that agent 1's expected payoff difference does not depend on agent 2's action in the first period.

Now we are ready to prove the uniqueness of the sequential equilibrium. We devide into the analysis into the following cases:

1. $a_1^*a_2^* = HH$: We first show the existence of a sequential equilibrium with $a_1^*a_2^* = HH$. Let us first consider the deviation to M. The payoff difference when playing H and M is

$$U_1(Ma_2^*;\mu_1) - U_1(Ha_2^*;\mu_1) = (c-p) - \delta p[2p - c + 2\mu_1(p_G - p)]. \tag{10}$$

   Note that the difference is decreasing in μ_1, and is equal to $c - p - \delta pc$ when $\mu_1 = \mu^* = \frac{c-p}{p_G-p}$. Since we assume that $c - p - \delta pc < 0$, there exists $\tilde{\mu} \in [0, \mu^*)$ such that $U_1(Ma_2^*;\mu_1) - U_1(Ha_2^*;\mu_1) \leq$

0 whenever $\mu_1 \geq \tilde{\mu}$. Now consider the deviation to L. Observe that

$$U_1(La_2^*;\mu_1) - U_1(Ma_2^*;\mu_1) = (c - \hat{p}(\mu_1)) - \delta[\hat{p}(\mu_1)(2p - c) + 2\mu_1 p_G(p_G - p)] \tag{11}$$
$$< U_1(Ma_2^*;\mu_1) - U_1(Ha_2^*;\mu_1),$$

Thus $U_1(La_2^*;\mu_1) - U_1(Ha_2^*;\mu_1) < 0$ whenever $\mu_1 \geq \tilde{\mu}$. Define $\hat{\mu} = \max\{\underline{\mu}^*, \tilde{\mu}\}$, then above analysis implies that there exists a sequential equilibrium with $a_1^* a_2^* = HH$ if $\mu_1 \in (\hat{\mu}, \bar{\mu}^*)$.

2. $a_1^* a_2^* \neq HH$ and $a_1^* a_2^* \neq LL$: Next, we show that there is no other sequential equilibrium. Equation (16) implies that $U_1(Ha_2^*;\mu_1) - U_1(Ma_2^*;\mu_1) > 0$ whenever $\mu_1 > \tilde{\mu}$, and thus the strategy profiles in which at least one agent plays M in $t = 1$(such as HM, MH, MM, ML, LM) are not an equilibrium. Similarly, (17) implies that $U_1(Ma_2^*;\mu_1) - U_1(La_2^*;\mu_1) > 0$ whenever $\mu_1 > \tilde{\mu}$, and thus the strategy profiles in which $a_1^* a_2^* = LH$ or HL are not an equilibrium.

3. $a_1^* a_2^* = LL$: Last, it remains to show that there is no equilibrium with $a_1^* a_2^* = LL$. Recall that $\mu^F(\mu_1; LL) = \mu_1$) and $\mu^S(\mu_1; LL) > \mu_1$ in any sequential equilibrium.

   Consider the case where $\mu_1 < \mu^*$. Then each agent's expected payoff is zero if they follow the profile. Now suppose agent 1 deviates to play M in the first period. Consider the worst-case scenario where $\mu^S(\mu_1; LL) < \mu^*$, then agent 1's expected payoff by deviating to M is

$$(\hat{p}(\mu_1) - c) + \delta[\hat{p}(\mu_1)(p - c) + \mu_1 p_G(p_G - p)].$$

   Note that the expected payoff is strictly greater than zero when $\mu_1 = \mu^*$. Therefore, there exists $\mu^\dagger \in [0, \mu^*)$ such that the profile with $a_1^* a_2^* = LL$ is not an equilibrium for $\mu_1 \in (\mu^\dagger, \mu^*)$.
   Finally, consider the case where $\mu_1 \geq \mu^*$. By following the prescribed action, agent 1 receives

$$\delta(2\hat{p}(\mu_1) - c).$$

   Suppose agent 1 deviates to M. Since $\mu^S(\mu_1; LL) > \mu_1$, agent 2 plays M after *Success* is observed. Suppose that $\mu_1 \in [\mu^*, \bar{\mu}(ML))$, then the agent 1 chooses different action depending on the signal, and thus will receive positive value of information. In this case, agent 1's expected payoff is

$$(\hat{p}(\mu_1)-c)+\delta[\pi(ML;\mu_1)(\hat{p}(\mu^S(\mu_1;ML))-c)+\hat{p}(\mu_1)].$$

Note that $\hat{p}(\mu_1)-c\geq 0$ since $\mu_1\geq\mu^*$, and $\pi(ML;\mu_1)(\hat{p}(\mu^S(\mu_1;ML))-c)>\hat{p}(\mu_1)-c$ since value of information is positive. Therefore, such deviation is profitable.

Define $\underline{\mu}=\max\{\underline{\mu}^*,\tilde{\mu},\mu^\dagger\}$ and $\bar{\mu}=\bar{\mu}^*$. Then the above analysis proves Proposition 1.

A.2 Proof of Proposition 4

To calculate the ex ante expected total production, we need to compute the distribution of posterior belief (which is captured by μ_t) from the perspective of period 0. Since $z_t=\theta+\varepsilon_t$, and ε_t are independent across time, we have

$$\sum_{s=0}^{t-1} z_s = t\theta+\sum_{s=0}^{t-1}\varepsilon_s\sim\mathcal{N}\left(t\mu_0,\frac{t^2}{h_0}+\frac{t}{h_\varepsilon}\right).$$

Since $\mu_t=\frac{h_0}{h_t}\mu_0+\frac{h_\varepsilon}{h_t}\sum_{s=0}^{t-1}z_s$, the distribution of the posterior μ_t is[15]

$$\mu_t\sim\mathcal{N}\left(\mu_0,\left(\frac{h_\varepsilon}{h_t}\right)^2\left(\frac{t^2}{h_0}+\frac{t}{h_\varepsilon}\right)\right)=\mathcal{N}\left(\mu_0,\frac{1}{h_0}-\frac{1}{h_t}\right).$$

Therefore, the ex ante total production is given by

$$\mathbb{E}[TP]=\sum_{t=0}^{T}\delta^t\xi_t\left(\mu_0^2+\left(\frac{1}{h_0}-\frac{1}{h_t}\right)\right)$$

subject to

$$\xi_t=\frac{1}{N}+\frac{N-1}{N}\sum_{s=t+1}^{T}\delta^{s-t}\frac{h_\varepsilon}{h_s}\xi_s$$

[15] It is easy to check that the variance of posterior μ_t increases in t, starts from zero (when $t=0$), and converges to $1/h_0$ as $t\to\infty$.

and $\xi_T = \frac{1}{N}$. Note that

$$\xi_t = \frac{1}{N} + \frac{\delta}{N}\left\{(N-1)\rho_{t+1}\xi_{t+1} + (N-1)\sum_{s=t+2}^{T}\delta^{s-t-1}\rho_s\xi_s\right\},$$
$$= \frac{1}{N} + \delta\left\{\frac{N-1}{N}\rho_{t+1}\xi_{t+1} + \left(\xi_{t+1} - \frac{1}{N}\right)\right\}. \tag{12}$$

Given this result, we conduct the following comparative statics:

1. Effect of h_0: Taking a derivative of ξ_t with respect to h_0 from (18), we get

$$\frac{\partial \xi_t}{\partial h_0} = \delta\left[-\frac{N-1}{N}\frac{h_\varepsilon}{h_{t+1}^2}\xi_{t+1} + \left(1 + \frac{N-1}{N}\rho_{t+1}\right)\frac{\partial \xi_{t+1}}{\partial h_0}\right]. \tag{13}$$

It is straightforward that $\partial\xi_T/\partial h_0 = 0$ and $\partial\xi_{T-1}/\partial h_0 < 0$. Since $\xi_t > 0$ for any t, it follows from (19) that $\partial\xi_t/\partial h_0 < 0$ for all $t = 0, \ldots, T-1$. On the other hand, we have

$$\frac{\partial}{\partial h_0}\left(\frac{1}{h_0} - \frac{1}{h_t}\right) = -\frac{1}{h_0^2} + \frac{1}{h_t^2} < 0,$$

thus $\frac{\partial}{\partial h_0}(\mathbb{E}[TP]) < 0$.

2. Effect of h_ε: Taking a derivative of ξ_t with respect to h_0 from (18), we get

$$\frac{\partial \xi_t}{\partial h_\varepsilon} = \delta\left[\frac{N-1}{N}\frac{h_0}{h_{t+1}^2}\xi_{t+1} + \left(1 + \frac{N-1}{N}\rho_{t+1}\right)\frac{\partial \xi_{t+1}}{\partial h_\varepsilon}\right]. \tag{14}$$

It is straightforward that $\partial\xi_T/\partial h_\varepsilon = 0$ and $\partial\xi_{T-1}/\partial h_\varepsilon > 0$. Since $\xi_t > 0$ for any t, it follows from (19) that $\partial\xi_t/\partial h_\varepsilon > 0$ for all $t = 0, \ldots, T-1$. Moreover, since

$$\frac{\partial}{\partial h_\varepsilon}\left(\frac{1}{h_0} - \frac{1}{h_t}\right) = \frac{t}{h_t^2} > 0,$$

we have $\frac{\partial}{\partial h_\varepsilon}(\mathbb{E}[TP]) > 0$.

3. Effect of N: Taking a partial derivative with respect to N, we have

$$\frac{\partial \xi_t}{\partial N} = -(1-\delta)\frac{1}{N^2} + \delta\left[\frac{1}{N^2}\rho_{t+1}\xi_{t+1} + \left(1+\frac{N-1}{N}\rho_{t+1}\right)\frac{\partial \xi_{t+1}}{\partial N}\right].$$

A.3 Solution of the Differential Equation in Subsection 3.4

In Subsection 3.4, we derive the following ordinary differential equation

$$\dot{\chi}_t = \left(r - \frac{N-1}{N}\frac{\eta}{h_0+\eta t}\right)\chi_t - \frac{N-1}{N^2}\frac{\eta}{h_0+\eta t} \tag{15}$$

with the boundary condition $\chi_\tau = 0$, as describing the limit of equilibrium behavior in the continuous time limit. In this section, we present and verify the solution to this equation.

Claim: The following is a solution to (21): [16]

$$\begin{aligned}\chi_t &= e^{\frac{r}{\eta}(h_0+\eta t)}\left[(h_0+\eta t)^{-\frac{N-1}{N}}C_1 + \frac{N-1}{N^2}\eta\left(\frac{r}{\eta}\right)^{\frac{1}{N}}(h_0+\eta t)^{-\frac{N-1}{N}}\Gamma(-\frac{N-1}{N},\frac{r}{\eta}(h_0+\eta t))\right] \\ &= e^{\frac{r}{\eta}(h_0+\eta t)}(h_0+\eta t)^{-\frac{N-1}{N}}\left[C_1 + \frac{N-1}{N^2}\eta\left(\frac{r}{\eta}\right)^{\frac{1}{N}}\Gamma(-\frac{N-1}{N},\frac{r}{\eta}(h_0+\eta t))\right]\end{aligned}$$

Proof: We verify that the above solution satisfies (21). First note that

$$\frac{\partial \Gamma(s,x)}{\partial x} = -x^{s-1}e^{-x},$$

and

$$\frac{\partial\left\{e^{\frac{r}{\eta}(h_0+\eta t)}(h_0+\eta t)^{-\frac{N-1}{N}}\right\}}{\partial t} = e^{\frac{r}{\eta}(h_0+\eta t)}(h_0+\eta t)^{-\frac{N-1}{N}}\left(r - \frac{\frac{N-1}{N}\eta}{h_0+\eta t}\right).$$

[16] $\Gamma(\cdot,\cdot)$ denotes the upper incomplete gamma function which has the following representation $\Gamma(s,x) = \int_x^\infty t^{s-1}e^{-t}dt$.

we have

$$
\begin{aligned}
\dot{\chi}_t &= e^{\frac{r}{\eta}(h_0+\eta t)}(h_0+\eta t)^{-\frac{N-1}{N}}\left(r-\frac{\frac{N-1}{N}\eta}{h_0+\eta t}\right)\left[C_1+\frac{N-1}{N^2}\eta\left(\frac{r}{\eta}\right)^{\frac{1}{N}}\Gamma(\frac{N-1}{N},\frac{r}{\eta}(h_0+\eta t))\right] \\
&\quad - e^{\frac{r}{\eta}(h_0+\eta t)}(h_0+\eta t)^{-\frac{N-1}{N}}\frac{N-1}{N^2}\eta\left(\frac{r}{\eta}\right)^{\frac{1}{N}}\left\{\frac{r}{\eta}(h_0+\eta t)\right\}^{-\frac{1}{N}}e^{-\frac{r}{\eta}(h_0+\eta t)} \\
&= e^{\frac{r}{\eta}(h_0+\eta t)}(h_0+\eta t)^{-\frac{N-1}{N}}\left(r-\frac{\frac{N-1}{N}\eta}{h_0+\eta t}\right)\left[C_1+\frac{N-1}{N^2}\eta\left(\frac{r}{\eta}\right)^{\frac{1}{N}}\Gamma(\frac{N-1}{N},\frac{r}{\eta}(h_0+\eta t))\right]-\frac{\frac{N-1}{N^2}\eta}{h_0+\eta t},
\end{aligned}
$$

Comparing with (21) verifies that χ_t is indeed a solution of it, as claimed.

Finally, using the boundary condition $\chi_T = 0$ to compute C_1 and substituting in the solution, we get

$$
\chi_t = \frac{N-1}{N^2}\eta\left(\frac{r}{\eta}\right)^{\frac{1}{N}}e^{\frac{r}{\eta}(h_0+\eta t)}(h_0+\eta t)^{-\frac{N-1}{N}}\left[\Gamma(-\frac{N-1}{N},\frac{r}{\eta}(h_0+\eta t))-\Gamma(-\frac{N-1}{N},\frac{r}{\eta}(h_0+\eta T))\right]
$$

and $\xi_t = \chi_t + 1/N$.

Ex Ante Production The ex ante production is given by

$$
\int_0^T e^{-rt}\left(\xi_t-\frac{\xi_t^2}{2}\right)\left(\mu_0^2+\left(\frac{1}{h_0}-\frac{1}{h_0+\eta t}\right)\right)dt.
$$

Belief Updating Belief evolution in the continuous time limit also has similar characteristics to the discrete time case. We re-write the equilibrium belief updating equation (1) in terms of the flow parameters, and take the limit as $\Delta \to 0$ to get:

$$
d\mu_t = \frac{\eta(z_t-\mu_t)}{h_t} = \frac{\sqrt{\eta}}{h_t}dW_t
$$

where W_t is a Brownian motion. Finally, the conditional variance of z_t follows

$$
\dot{h}_t = \eta.
$$

A.4 Proof of Proposition 5

First, we will conduct comparative statics for individual effort, let's look two teams one with size M and other with size N. Wlog assume $M > N$. When $r = 0$

$$\dot{\chi}_t = -\frac{(N-1)}{N}\frac{\eta}{h_0+\eta t}\chi_t - \frac{N-1}{N^2}\frac{\eta}{h_0+\eta t}$$

Notice that $\dot{\chi_t}^N$ and $\dot{\chi_t}^M$ are convex function and both of them equal to 0 at time T. Given the form of $\dot{\chi}_t$ there exists a open neighborhood (B_ρ) of T such that for all $t \in B_\rho, \dot{\chi_t}^N > \dot{\chi_t}^M$. Also this implies in that neighborhood $\chi_t^N > \chi_t^M$. Due to convexity of these function either they will cross at some point (continuity guarantess existence of such point) or they will never cross. Suppose they cross and t^* is the point they cross. $\dot{\chi_t}^M$ approaches $\dot{\chi_t}^N$ from below. Therefore at t^* $\chi_{t^*}^N > \chi_{t^*}^M$ but notice that $\forall t < t^*, \dot{\chi_t}^M > \dot{\chi_t}^N$ then this implies $\exists t^{**}$ such that $\chi_t^M = \chi_t^N$ and $\forall t < t^{**}, \chi_t^M > \chi_t^N$. The previous results follows from definition χ_t and $\dot{\chi}_t$. Then if such $t^{**} \in [0,T]$ exists we know that $\forall t \in [0,t^{**}]$ $\chi_t^M > \chi_t^N$ afterwards $\chi_t^N > \chi_t^M$ if $t \in (t^{**},T]$. If such $t^{**} \in [0,T]$ does not exist then $\forall t \in [0,T]$ $\chi_t^N > \chi_t^M$.

Let β_t^N denotes the total effort by a team with N members at time t. By definition $\beta_t^N = N\xi_t = N\chi_t + 1$. Therefore

$$\dot{\beta_t^N} = -(N-1)\frac{\eta}{h_0+\eta t}\chi_t^N - \frac{N-1}{N}\frac{\eta}{h_0+\eta t}$$

Given these functional form it is easy to see that in a open neighborhood around $T, \dot{\beta_t^M} > \dot{\beta_t^N}$. Since both $\dot{\beta_t^N}$ and $\dot{\beta_t^M}$ are convex either they will cross or one will stay below of the other. [To be completed.]

A.5 Proof of Proposition 8

It is straightforward to check that the novices' problem is identical to the benchmark model (Subsection 3.4). The pure signal-jamming part of the belief sensitivity, $\chi_t = \xi_t - \frac{1}{N}$, solves the following differential equation

$$\dot{\chi}_t = \left(r - \frac{N-N^e-1}{N}\frac{\eta}{h_0+h(t)}\right)\chi_t - \frac{N-N^e-1}{N^2}\frac{\eta}{h_0+h(t)}.$$

Let $F(\mu,t)$ be the value function of the expert at time t when the (public) belief of the novice is μ. Then we have a Hamilton-Jacobi-Bellman equation for $F(\mu,t)$, which is given by

$$\begin{aligned} rF(\mu,t) &= \sup_{a\in\mathbb{R}} \frac{\theta}{N}\left(a+(N^e-1)(\gamma_t\theta+\psi_t)+N^n\xi_t\mu\right)-\frac{1}{2}a^2 \\ &\quad +\frac{\eta(1+N^e\gamma_t)}{h_t}\left(a-\psi_t-\gamma_t\theta+(1+N^e\gamma_t)(\theta-\mu)\right)F_\mu+F_t+\frac{\left(\eta(1+N^e\gamma_t)\right)^2}{2h_t^2}F_{\mu\mu} \end{aligned}$$

Then the first-order condition yields

$$a(t)=\frac{\theta}{N}+\frac{\eta(1+N^e\gamma_t)}{h_t}F_\mu$$

Conjecture the value function has the following linear-quadratic form:

$$F(\mu,t)=v_0(t)+v_1(t)\theta+v_2(t)\theta^2+v_3(t)\mu\theta$$

Then by matching the coefficients, we have

$$\begin{aligned} \psi_t &= 0, \\ \gamma_t &= \frac{1}{N}+\frac{\eta(1+N^e\gamma_t)}{h_t}v_3(t). \end{aligned}$$

We solve for $v_3(t)$ to have

$$\begin{aligned} v_3(t) &= \frac{(\gamma_t-\frac{1}{N})h_t}{\eta(1+N^e\gamma_t)} \\ \dot{v}_3(t) &= (1+N^e\gamma_t)(\gamma_t-\frac{1}{N})+\frac{1+\frac{N^e}{N}}{\eta(1+N^e\gamma_t)^2}h_t\dot{\gamma}_t \end{aligned}$$

On the other hand, applying the envelope theorem on HJB equation and using $F_{\mu\mu}=0$, we have

$$rv_3(t)\theta=\frac{\theta}{N}N^n\xi_t+\dot{v}_3(t)\theta-\frac{\eta(1+\gamma_tN^e)^2}{h_t}v_3(t)\theta$$

by plugging the values of $(v_3(t), \dot{v}_3(t))$ and cancelling out θ,

$$\dot{\gamma}_t = \frac{(1+N^e\gamma_t)(\gamma_t - \frac{1}{N})rh_t - \eta(1+\gamma_t N^e)^2 \frac{N^n}{N}\xi_t}{h_t(1+\frac{N^e}{N})}.$$

Verification argument to be added.

A.6 Stationary h_t

Now assume h_σ has the precision $\frac{l}{\Delta}$. Then we can write,

$$h_{t+\Delta} = \frac{\left(h_t + \eta\Delta\kappa_\theta^2\right)\frac{l}{\Delta}}{h_t + \eta\Delta\kappa_\theta^2 + \frac{l}{\Delta}}$$

then as $\Delta \to 0$ after arrenging

$$\dot{h_t} = \eta\kappa_\theta^2 - \frac{h_t^2}{l}$$

then it has the positive stationary solution which is $h_t = \sqrt{l\eta}\kappa_\theta$.

B Value of Uncertainty

Let us calculate the value of information by comparing the ex ante expected payoff of equilibrium under incomplete information and one under complete information. Define u_t and $\hat{u}_t$ be the expected period-t equilibrium payoff under incomplete information and under complete information, respectively. Also define U and $\hat{U}$ be the present-discounted ex ante payoff under imcomplete information and under complete information.

Then the *value of information* V is defined by

$$V = \hat{U} - U.$$

Under the complete information, the players play $a^*(\theta) = \theta/N$ in every period. Then the total payoff is

$$\text{(total output)-(total cost)} = \theta \cdot N \cdot \frac{\theta}{N} - N \cdot \frac{1}{2}\left(\frac{\theta}{N}\right)^2$$
$$= \left(1 - \frac{1}{2N}\right)\theta^2.$$

Since $\theta \sim \mathcal{N}(\mu_0, 1/h_0)$, the expected total payoff is given by

$$\hat{u}_t = \left(1 - \frac{1}{2N}\right)\left(\mu_0^2 + \frac{1}{h_0}\right).$$

- Note that while the total payoff is increasing in N, per-player payoff decreases as N increases.

Using the formula of the distribution of the posterior belief (Subsection A.2), we write the present-discounted ex ante payoff as

$$\hat{U} = \sum_{t=0}^{T} \delta^t \left(1 - \frac{1}{2N}\right)\left(\mu_0^2 + \frac{1}{h_0}\right)$$
$$= \frac{1 - \delta^{T+1}}{1 - \delta}\left(1 - \frac{1}{2N}\right)\left(\mu_0^2 + \frac{1}{h_0}\right).$$

Now let us look at the incomplete information case. Since in the equilibrium $a_{it}^* = \xi_t \mu_t$, we have

$$\text{(total output)-(total cost)} = \theta \cdot N \cdot \xi_t \mu_t - N \cdot \frac{1}{2}\left(\xi_t \mu_t\right)^2$$

Therefore, the expected total payoff is

$$u_t = N\left(\xi_t - \frac{\xi_t^2}{2}\right)\mu_t^2.$$

Therefore, the ex ante payoff under incomplete information is

$$
\begin{aligned}
U &= \mathbb{E}_0\left[\sum_{t=0}^{T}\delta^t u_t\right] \\
&= \sum_{t=0}^{T}\delta^t N\left(\xi_t - \frac{\xi_t^2}{2}\right)\mathbb{E}_0[\mu_t^2] \\
&= \sum_{t=0}^{T}\delta^t N\left(\xi_t - \frac{\xi_t^2}{2}\right)\left(\mu_0^2 + \left(\frac{1}{h_0} - \frac{1}{h_t}\right)\right).
\end{aligned}
$$

Finally, the ex ante value of information is given by

$$
\begin{aligned}
V &= \hat{U} - U \\
&= \sum_{t=0}^{T}\delta^t\left[\left(1-\frac{1}{2N}\right)\left(\mu_0^2+\frac{1}{h_0}\right) - N\left(\xi_t - \frac{\xi_t^2}{2}\right)\left(\mu_0^2 + \left(\frac{1}{h_0} - \frac{1}{h_t}\right)\right)\right].
\end{aligned}
$$

C Information Disclosure Design: Comparative Statics

Let's conduct some comparative statics with respect to the precisions h_η and $h_\varepsilon(t)$. The first-order derivative of the ex-ante production P is given by

$$
\begin{aligned}
\frac{\partial P}{\partial h_\eta} &= \sum_{t=0}^{T-1}\delta^t\left(\mu_0^2 + \left(\frac{1}{h_0} - \frac{1}{h_t}\right)\right)\frac{\partial \xi_t}{\partial h_\eta} + \sum_{t=0}^{T}\delta^t \xi_t\left(\frac{1}{h_t^2} - \frac{1}{h_0^2}\right) \\
\frac{\partial P}{\partial h_\varepsilon(s)} &= \sum_{t=0}^{T-1}\delta^t\left(\mu_0^2 + \left(\frac{1}{h_0} - \frac{1}{h_t}\right)\right)\frac{\partial \xi_t}{\partial h_\varepsilon(s)} + \sum_{t=s+1}^{T}\delta^t \xi_t \frac{1}{h_t^2}
\end{aligned}
$$

What is the derivative of ξ_t with respect to the precisions h_η and $h_\varepsilon(t)$? Let $h_\eta = h_\varepsilon(-1)$ and $\xi'_{t,s} = \partial\xi_t/\partial h_\varepsilon(s)$ for ease of notation. And define

$$\begin{aligned}
X_t =& \delta\frac{\partial A(t)}{\partial h_\varepsilon(s)} = -\delta\frac{N-1}{N}\frac{h_\varepsilon(t)}{h_{t+1}^2} < 0\\
\hat{X}_t =& \delta\frac{\partial A(t)}{\partial h_\varepsilon(t)} = \delta\frac{N-1}{N}\frac{h_t}{h_{t+1}^2} > 0\\
Y_t =& \delta\frac{\partial B(t)}{\partial h_\varepsilon(t)} = \delta\frac{1}{h_\varepsilon(t+1)} > 0\\
\hat{Y}_t =& \delta\frac{\partial B(t)}{\partial h_\varepsilon(t+1)} = -\delta\frac{h_\varepsilon(t)}{h_\varepsilon(t+1)^2} < 0\\
W_t =& \delta(A(t)+B(t)) = \delta\left(\frac{N-1}{N}\frac{h_\varepsilon(t)}{h_{t+1}} + \frac{h_\varepsilon(t)}{h_\varepsilon(t+1)}\right) > 0,
\end{aligned}$$

Then we have a recursive representation of $\xi'_{t,s}$:

$$\frac{\partial\xi_t}{\partial h_\varepsilon(s)} = \xi'_{t,s} = \begin{cases} 0 & \text{if } t = T\\ W_t\xi'_{t+1,s} + \underbrace{X_t\xi_{t+1}}_{<0} & \text{if } t \geq s+1\\ W_t\xi'_{t+1,s} + \underbrace{\hat{X}_t\xi_{t+1} + Y_t\left(\xi_{t+1} - \frac{1}{N}\right)}_{>0} & \text{if } t = s\\ W_t\xi'_{t+1,s} + \underbrace{\hat{Y}_t\left(\xi_{t+1} - \frac{1}{N}\right)}_{\leq 0} & \text{if } t = s-1\\ W_t\xi'_{t+1,s} & \text{if } t \leq s-2 \end{cases}$$

For example, the following is the table for $T = 3$ (four-period) case:

$\frac{\partial \xi_t}{\partial h}$	ξ_0	ξ_1	ξ_2	$\xi_3 = \frac{1}{N}$
h_η	$W_0 \xi'_{1,\eta} + \underbrace{X_0 \xi_1}_{<0}$	$W_1 \xi'_{2,\eta} + \underbrace{X_1 \xi_2}_{<0}$	$\underbrace{X_2 \xi_3}_{<0}$	0
$h_\varepsilon(0)$	$W_0 \xi'_{1,0} + \underbrace{\hat{X}_0 \xi_1 + Y_0 \left(\xi_1 - \frac{1}{N} \right)}_{>0}$	$W_1 \xi'_{2,0} + \underbrace{X_1 \xi_2}_{<0}$	$\underbrace{X_2 \xi_3}_{<0}$	0
$h_\varepsilon(1)$	$W_0 \xi'_{1,1} + \underbrace{\hat{Y}_0 \left(\xi_1 - \frac{1}{N} \right)}_{<0}$	$W_1 \xi'_{2,1} + \underbrace{\hat{X}_1 \xi_2 + Y_1 \left(\xi_2 - \frac{1}{N} \right)}_{>0}$	$\underbrace{X_2 \xi_3}_{<0}$	0
$h_\varepsilon(2)$	$W_0 \xi'_{1,2}$	$W_1 \xi'_{2,2} + \underbrace{\hat{Y}_1 \left(\xi_2 - \frac{1}{N} \right)}_{<0}$	$\underbrace{\hat{X}_2 \xi_3 + Y_2 \left(\xi_3 - \frac{1}{N} \right)}_{>0}$	0
$h_\varepsilon(3)$	0	0	$\hat{Y}_2 \left(\xi_3 - \frac{1}{N} \right) = 0$	0

Observations:

- If we increase $h_\varepsilon(t)$, then it will make the period-t signal z_t more important, while making the other signals $z_s (s \neq t)$ less important.

  1. $X_t \xi_{t+1} < 0$: Effect of $h_\varepsilon(s)$ on future response level $(\xi'_{t,s}, t \geq s+1)$
  2. $\hat{X}_t \xi_{t+1} + Y_t \left(\xi_{t+1} - \frac{1}{N} \right) > 0$: Effect of $h_\varepsilon(s)$ on the present response level $(\xi'_{t,s}, t = s)$
  3. $\hat{Y}_t \left(\xi_{t+1} - \frac{1}{N} \right) \leq 0$: Effect of $h_\varepsilon(s)$ on the past response $(\xi'_{t,s}, t = s-1)$

- But at the same time, the effect is accumulated backwards.

  - $Z_t > 0$: If the future effort level increases (decreases), then the marginal benefit of the current effort increases (decreases).

Given these observations, we conjecture that the optimal information design backloads the information, that is, the precision increases in t.

D Public Good Provision Case

Consider the case in which each agent receives the payoff

$$\theta \sum_{i=1}^{N} a_{it}$$

in each period. That is, compared to the benchmark model, we multiply the total payoff by N. Note that in this setting, the equilibrium effort under perfect information is θ, while the socially optimal effort level is $N\theta$.

- In period T: there is no future effect of my effort, so every player chooses the static optimal effort level. Since the expected marginal benefit of effort is now μ_T, we have

$$a_{iT}^* = \mu_T.$$

  Define ξ_t be the rate at which each player increase his effort when μ_t increases. Then above equation shows that $\xi_T = 1$.

- In period $T-1$: not only there is static marginal benefit μ_{T-1}, but also there is benefit from changing $\tilde{\mu}_T$, thus affecting $a_{-i,T}$. Therefore, the marginal benefit is given by

$$\begin{aligned} a_{i,T-1}^* &= \mu_{T-1} + \delta\mu_{T-1} \cdot \underbrace{(N-1)\rho_T\xi_T}_{\text{increase in future effort}} \\ &= \mu_{T-1}(1 + \delta(N-1)\rho_T\xi_T). \end{aligned}$$

  Therefore, $\xi_{T-1} = 1 + (N-1)\delta\rho_T\xi_T$.

- In period t: by continuing the backward induction, we can show that

$$a_{it}^* = \xi_t\mu_t,$$

  where ξ_t is recursively defined by

$$\begin{aligned} \xi_t &= 1 + (N-1)\sum_{s=t+1}^{T} \delta^{s-t}\rho_s\xi_s \\ &= (1-\delta) + \delta(1 + (N-1)\rho_{t+1})\xi_{t+1}. \end{aligned}$$

- From the last equation, we can also represent ξ_t as a function of exogenous variables:

$$\xi_t = (1-\delta)\sum_{s=t}^{T-1}\delta^{s-t}\left(\prod_{k=t+1}^{s}(1+(N-1)\rho_k)\right)+\delta^{T-t}\left(\prod_{k=t+1}^{T}(1+(N-1)\rho_k)\right)$$

- a_{it}^* is socially efficient if and only if $\xi_t = N$.

D.1 Value of Information

Again we define u_t and $\hat{u}_t$ be the expected period-t equilibrium payoff under incomplete information and under complete information, respectively.

Under the complete information, the players play $a^*(\theta)=\theta$ in every period. Then the total payoff is

$$\begin{aligned}\text{(total output)-(total cost)} &= N\cdot\theta\cdot N\theta - N\cdot\frac{1}{2}\theta^2 \\ &= \left(N^2-\frac{N}{2}\right)\theta^2.\end{aligned}$$

Since $\theta\sim\mathscr{N}(\mu_0,1/h_0)$, the expected total payoff is given by

$$\hat{u}_t = \left(N^2-\frac{N}{2}\right)\left(\mu_0^2+\frac{1}{h_0}\right).$$

Then the present-discounted ex ante payoff is given by

$$\begin{aligned}\hat{U} &= \sum_{t=0}^{T}\delta^t\left(1-\frac{1}{2N}\right)\left(\mu_0^2+\frac{1}{h_0}\right) \\ &= \frac{1-\delta^{T+1}}{1-\delta}\left(1-\frac{1}{2N}\right)\left(\mu_0^2+\frac{1}{h_0}\right).\end{aligned}$$

Now let's look at the incomplete information case. Since in the equilibrium $a_{it}^*=\xi_t\mu_t$, we have

$$\text{(total output)-(total cost)} = N\cdot\theta\cdot N\cdot\xi_t\mu_t - N\cdot\frac{1}{2}(\xi_t\mu_t)^2$$

Therefore, the expected total payoff is

$$u_t = \left(N^2\xi_t-\frac{N}{2}\xi_t^2\right)\mu_t^2.$$

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