Collapsing Confidence: Dynamic Trading with Developing Adverse Selection*

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Abstract

I study a dynamic trading game where a seller and potential buyers start out symmetrically uninformed about the quality of a good, but the seller becomes informed about the quality, so that the asymmetric information between the agents *develops* over time. The introduction of a widening information gap results in several new phenomena. In particular, the interaction between screening and learning generates nonmonotonic price and trading patterns, contrary to the standard models in which asymmetric information is initially given. If the seller's effective learning speed is high, the equilibrium features "collapse-and-recovery" behavior: Both the equilibrium price and the probability of a trade drop at a threshold time and then increase later. The seller's payoff is nonmonotonic in his learning speed, as a slower learning speed can lead to higher payoff for the seller.

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1 Introduction

Akerlof's seminal 1970 paper on adverse selection shows that existence of asymmetric information can lead to inefficient trade outcomes. In the literature following Akerlof's work, many researchers have investigated the dynamic impact of the adverse selection problem. Yet despite this focus, most existing models assume that the asymmetric information exists initially, in the sense that one side of transaction starts with superior information than the other. However, there are many economic environments in which neither agent is perfectly informed in the beginning and one side gradually obtains information, so that the information gap between the agents grows over time. This observation relates to the main innovation of this paper: I consider a dynamic trading situation where the degree of asymmetric information between agents *develops* over time, and analyze its effects on trading patterns and efficiency.

Developing asymmetric information is a general phenomenon that arises in many environments. Consider, for instance, an entrepreneur who wants to sell his start-up firm. When the entrepreneur starts the company, he is not sure about the prospects of his firm or the technology that his firm creates, but over time, he learns about the firm's viability. Trading of a securitized asset (where asset holders are gradually informed about the quality of complex assets, such as collateralized mortgage obligations) and a market for "talent" (where a manager gains an informational advantage regarding the potential of his talent agents) are other environments with asymmetric information. The common theme underlying these examples is the feature of "learning-by-holding." As people hold or use a good, they observe more signals and thereby gain an informational advantage. If an economic environment has the feature of learning-by-holding, the degree of the asymmetric information may increase over time.

To investigate the impact of developing asymmetric information, I study a stylized model of a dynamic trading game between a single seller and a sequence of potential buyers. The seller holds an indivisible unit of a good, the quality of which is either high or low. The potential buyers randomly arrive to be matched with the seller. Upon arrival, the buyer observes how long the good has been up for sale (time-on-the-market) and makes a take-it-or-leave-it offer to the seller. In contrast to existing models, all agents are initially uninformed about the quality of the good and have a common prior belief. Over time, the seller exogenously learns the quality of the good by observing the arrival of a perfectly informative signal. The buyers remain uninformed about the quality of the good; they also do not know whether the seller is informed about it.

The introduction of developing asymmetric information results in several new phenomena. In particular, the interaction between the seller's learning and the buyers' equilibrium behavior generates *nonmonotonic* price and trading patterns, contrary to the standard models in which asymmetric information is initially given. Equilibrium dynamics depend on the effective speed of learning of the seller, which is the ratio of the seller's speed of learning to the arrival rate of the buyers.

In this model, the buyers form two layers of beliefs, the evolution of which works as one of the main driving forces of nonmonotonic equilibrium dynamics. Since the buyers observe neither the quality nor the seller's learning, they form beliefs about the quality of the good *and* about the *seller's belief* about the quality of the good. This belief structure is different from the one in the existing models of dynamic adverse selection in which it is common knowledge that the seller is informed. Specifically, in this model the buyers form beliefs about the seller's status, which fall into one of the following three types: (1), the seller is informed that his good is of high quality; (2), he is informed that his good is of low quality (a "lemon"); or (3), that he is uninformed about the quality of the good.

In the early stage of the game, the buyers believe that the seller is highly likely to be uninformed and that the degree of asymmetric information is small. Therefore, if the buyer arrives early, he targets the uninformed seller by offering a middle-range price. Over time, the seller becomes more informed. If the seller finds that his good is of high quality, then he rejects the middle-range price in hopes of selling at a higher price. But the informed seller with a lemon accepts the middle-range price as waiting is more costly for him. As a result, if the buyer who arrives late targets an uninformed seller by a middle-range price offer, the probability of getting a low-quality good is higher.

If the effective learning speed of the seller is sufficiently high (a fast-learning case), the equilibrium features a "collapse-and-recovery" pattern. If the learning speed is high, the probability that the seller is uninformed rapidly decreases, so buyers become increasingly worried about the quality of the good when targeting an uninformed seller. Therefore, there is a threshold time after which it is no longer optimal for buyers to target an uninformed seller. Therefore, after the threshold time buyers target only the informed seller of a lemon. As a result, both the equilibrium price and the probability of a trade drop at the threshold time. On the other hand, an informed seller with a high-quality good rejects both a middle-range price and a low price, so the overall expected quality of the good increases over time. Therefore, there exists a second threshold time at which the expected quality is high enough that the buyers begin to offer a high price to target all types of sellers. The equilibrium trading price thus jumps at the second threshold time.

If the seller's effective speed of learning is low (a slow-learning case), then the probability that the seller is uninformed remains sufficiently high for a long period, and it is optimal for buyers to offer a middle-range price for that period. Thus the overall expected quality of the good increases over time, because the informed seller with a high-quality good does not trade. Therefore, similar to the fast-learning case, there exists a threshold time at which the buyers begin to offer a high price to target all types of sellers.

On the other hand, the equilibrium price before the threshold time may also be nonmonotonic, because of the seller's value of information. In the early stage of the game, buyers target an uninformed seller. This behavior generates a positive value of information for the seller, since the informed seller can adjust his offer acceptance behavior depending on the information received, and achieve a strictly higher payoff. So the uninformed seller, who expects to be informed later, factors the value of the *future* information into his current reservation price. I show that the change in the value of information may lead to a nonmonotonic reservation price for the seller, leading to a nonmonotonic equilibrium trading price.

After analyzing the equilibrium behavior, I conduct some comparative statics. I show that the threshold time decreases as the learning speed of the seller increases. If the learning speed is arbitrarily small, then the equilibrium of this model converges toward the equilibrium in the model with symmetrically uninformed agents. On the other hand, as the learning speed increases to infinity, the model converges toward the model with initial asymmetric information, and hence the collapse occurs almost immediately after the beginning of the game.

Lastly, I show that the seller's payoff is nonmonotonic with regard to his own learning speed. It is well known that in a situation with initial asymmetric information, the trade surplus is lower (because of the adverse selection problem) and the seller's payoff is higher (because of information rent) compared to an environment with symmetric information. In my model, while the trade surplus decreases as the learning speed increases, the seller may achieve a higher payoff in a case with asymmetric information than in a case where he is initially informed. The higher the seller's learning speed is, the greater division of the surplus the seller obtains. However, if the learning speed is too high, inefficiency caused by asymmetric information becomes too large, leading to a smaller payoff for the seller.

1.1 Related Literature

This paper contributes to the rich literature of dynamic adverse selection. These papers investigate the dynamic impact of asymmetric information in various contexts, such as a dynamic bargaining game with interdependent values (Evans, 1989; Vincent, 1989; Deneckere and Liang, 2006; and Fuchs and Skrzypacz, 2010), a sequential search model (Hörner and Vieille, 2009; Zhu, 2012; Kaya and Kim, 2013; and Lauermann and Wolinsky, 2013), an equilibrium search framework (Moreno and Wooders, 2010; Kim, 2011; Camargo and Lester, 2011; and Guerrieri and Shimer, 2013), and a dynamic signaling model (Janssen and Roy, 2002; Daley and Green, 2012; and Fuchs and Skrzypacz, 2013). All of these papers assume that asymmetric information is initially given, so that from the beginning one side of transaction is perfectly informed about the quality of the good. On the other hand, the present paper considers an environment where asymmetric information increases. Moreover, the richer equilibrium trading dynamics of this paper contribute to the applicability of the literature.

Daley and Green (2012) consider a dynamic setting in which stochastic information (news) about the value of a privately-informed seller's asset is gradually revealed to a market of buyers. So in their model, asymmetric information is initially given and exogenously dissolves over time. In contrast, the present paper considers a case in which agents are initially symmetrically uninformed, and then asymmetric information exogenously increases. Both papers show trading patterns that differ from those in the standard model, but the trading dynamics are different, as is the intuition behind the results.

Plantin (2009) and Bolton *et al.* (2011) consider finite-horizon models in which the seller learns the quality of his asset. In their models, the learning of the seller occurs in a single period. On the other hand, the present paper models the learning process in a full dynamic setting, and finds various equilibrium trading dynamics and underlying belief evolutions. Moreover, the dynamic model in the paper make it possible to conduct comparative statics.

Choi (2013) studies a stationary dynamic equilibrium model of a resale market with adverse selection in which new owners are uninformed and slowly learn the quality of their acquisitions. He characterizes steady-state equilibria of the model and shows that trade efficiency increases as the learning speed of the seller increases. In this paper, I consider a nonstationary environment and analyze the dynamics of trading patterns.

The remainder of the paper is as follows. Section 2 describes the model and shows some preliminary observations. Section 3 presents equilibria under the slow- and fast-learning cases and describes the equilibrium dynamics with the underlying belief evolution. Section 4 presents comparative statics of some important equilibrium values as well as the trade surplus and its division. Section 5 discusses the implications of the results for the recent financial crisis and the role of assumptions of the model. Section 6 concludes. Some of the proofs are presented in the Appendix.

2 Model

Time $t \geq 0$ is continuous. There is a long-lived seller with a countably infinite number of potential buyers. The seller holds an indivisible unit of a good. Buyers arrive at random times which correspond to the jumping times of a Poisson process with constant rate λ. Upon arriving, the buyer observes only how long the the seller has stayed in the game, that is, the calendar time t. In particular, the buyer does not observe the history of past offers.[1] Then the buyer makes a take-it-or-leave-it offer p. If the seller accepts the offer, then the game ends. Otherwise, the buyer leaves and the seller waits for subsequent buyers.[2] The seller discounts future payoffs at a rate $r > 0$.

The quality θ of the good is determined by Nature and is either high (H) or low (L). At time zero, all agents of the game are uninformed, and they form a common prior belief q_0 that the quality of the good is high. Over time, the seller privately receives a series of perfectly informative signals which arrive according to a Poisson process of constant rate ρ. The processes of the arrival of signals and the arrival of the buyers are independent. Since each signal is perfectly informative, upon the first arrival of the signal the seller is perfectly informed about the quality of the good.[3]

The valuation of the good to the buyers is common to all of them and is denoted by v_θ, where $v_H > v_L$. The seller values the good at a discounted proportion of $\alpha < 1$. Therefore, the trading of a quality-θ good yields $(1-\alpha)v_\theta$ of trade surplus.[4]

An outcome of the game is a triple (θ, t, p), with the interpretation that the realized type is θ and that the trade occurs at time t with price p. The case $t = \infty$ (with $p = 0$) corresponds to the outcome in which the trade does not occur. The payoff of the buyer at time t is $v_\theta - p$ if the outcome is (θ, t, p), and zero otherwise. There are two ways to represent the seller's payoff. The first interpretation, which I adopt in the following analysis, assumes that each signal carries a dividend of size $x_\theta = \frac{r}{\rho} v_\theta$. The size of each dividend is precisely determined to ensure that

[1]So the model considers a case in which previous offers are kept hidden to future buyers. To read about the effect of the information available to potential buyers on trading dynamics and efficiency, see Noldeke and van Damme (1990); Swinkels (1999); Hörner and Vieille (2009); Kim (2011); Fuchs *et al.* (2012); and Kaya and Liu (2013).

[2]The assumptions on the arrival process and on the information of the buyers are similar to those of Kim (2011) and Kaya and Kim (2013).

[3]Models with different information processes are discussed in Section 5.

[4]The fact that the trade surplus increases in the quality of the good is not crucial in deriving the equilibrium of the model. Indeed, the result is robust under cases in which the trade surplus is independent or decreasing in the quality of the good, as long as the parameter values satisfy a relevant assumption (counterpart to Assumption 1).

the present expected value of the dividend from quality-θ good is v_θ. Then it is assumed that the seller values each dividend at a rate $\alpha < 1$.[5] Alternate interpretation is that the seller incurs a production cost αv_θ at the time of trade, so the payoff is realized after the trade occurs. It is immediate to verify that this interpretation yields the same incentives of the agents.

The paper analyzes the environment where there is a sufficiently high probability of a low-quality good (lemon). Consider a static bargaining game where the seller knows the quality of his good. In order to attract all types of sellers, the buyer must offer no less than αv_H, the minimum reservation price of the seller with the high-quality good. So the trade outcome is not efficient if offering such a price yields negative payoffs to the buyer, that is,

$$v(q_0) < \alpha v_H,$$

where $v(q_0) = q_0 v_H + (1 - q_0) v_L$ is the ex ante value of the good to the buyers. I call the above inequality the *static lemons condition*. Note that the condition holds if the prior q_0 is sufficiently small. In fact, define q^* such that $q^* v_H + (1 - q^*) v_L = \alpha v_H$. Then the static lemons condition can be equivalently written as

$$q_0 < q^*.$$

I am particularly interested in the case where the seller is sufficiently patient. Specifically, I make the following parametric assumption:

Assumption 1.

$$v(q_0) < \frac{r}{r+\lambda} \alpha v(q_0) + \frac{\lambda}{r+\lambda} \alpha v_H.$$

Assumption 1 ensures that the seller has non-trivial intertemporal incentives. It implies that the buyer's offer targeted to the uninformed seller (which is at most $v(q_0)$) is rejected if the uninformed seller expects that he will receive a non-screening offer (at least αv_H) at the next match. Note that static lemons condition is a necessary condition for Assumption 1. Given the static lemons condition, the assumption is satisfied when the value of r/λ is sufficiently small. Although Assumption 1 is not a necessary condition for the basic economic mechanism I highlight in this paper, it contributes to the analytical tractability of the model.[6]

[5] One interpretation is that the seller is more impatient than the buyers.

[6] If the static lemons condition is not satisfied, then there exists an equilibrium where the first buyer offers a trade-ending price to end the game. If the static lemons condition is satisfied, for a range of parameters that does not satisfy the assumption, there exists an equilibrium whose structure is similar to the one described in the paper. However, in this case it is difficult to get a clear equilibrium characterization result, such as a payoff equivalence result within the set of equilibria of the model.

The information process implies that, at any time $t > 0$ the seller is one of the following three types: 1) one who has received a lump-sum payoff x_H, and so is informed that his good is of high quality; 2) one who is informed that his good is of low quality; and 3) one who has not received a payoff and so is uninformed about the good's quality. I will denote g (good type) for the informed seller with the high-quality good, b (bad type) for the informed seller with the low-quality good, and u (uninformed) for the uninformed seller.

Since the signal is perfectly informative, the good-type (bad-type) seller believes that the quality is high (low) with probability one. The uninformed seller's belief stays the same at the prior q_0. Because the arrival rate of the information is the same for all θ, not receiving any signal does not provide additional information.

The buyers' beliefs are represented by a function $\phi : \mathbb{R}_+ \to \Delta\{g,u,b\}$. Let $\phi_z(t) = \phi(t)(z)(z = g,u,b)$ be the belief of the buyer at time t that the seller is type z. Then it is straightforward that $\phi_u(0) = 1$, and that $\phi_g(t) + \phi_u(t) + \phi_b(t) = 1$ for any $t \geq 0$. Let $q(t)$ be the buyer's (unconditional) belief at time t that the quality of the good is high. Then $q(0) = q_0$, and $q(t)$ can be expressed as a function of $\phi_z(t)$:

$$q(t) = \phi_g(t) + \phi_u(t)q_0.$$

The offer strategies of the buyers are represented as a mapping σ_B from $\mathbb{R}_+$ to a set of probability distributions over $\mathbb{R}$, where $\sigma_B(t)$ denotes a probability distribution of the buyer's offer at time t. I denote $\sigma_B(t) = p'$ when $\sigma_B(t)$ is a degenerate distribution at price p'. The acceptance strategy of the seller is represented by a function $\sigma_S : \{g,u,b\} \times \mathbb{R}_+ \times \mathbb{R}_+ \to [0,1]$ where $\sigma_S(z,t,p)$ denotes the probability that a type-z seller accepts price p at time t.

I use the perfect Bayesian equilibrium concept throughout this paper.

Definition 1. A tuple $(\sigma_S, \sigma_B, \phi)$ is a *perfect Bayesian equilibrium (PBE)* if (1) given σ_S and ϕ, for any t, $\sigma_B(t)$ assigns a positive probability to a price p only if p maximizes the expected payoff of the buyer at time t, (2) given σ_S, for any z and t, $\sigma_S(z,t,p) > 0$ only if p is weakly greater than the type-z seller's continuation payoff at time t, and (3) given σ_S and σ_B, ϕ is derived through Bayesian updating.

2.1 Preliminary Observations

I begin by presenting lemmas that help in characterizing the equilibrium structure. The proofs of the lemmas are straightforward, so are omitted. The following lemma states that in any equilibrium of the model, there exists a reservation price function $R_z(t)$ for each type of the seller such that the type-z seller at t accepts $p > R_z(t)$ and rejects $p < R_z(t)$ with probability one.

Lemma 1. *(Reservation Price Strategy) In equilibrium, there exists a function $R_z : \mathbb{R}_+ \to \mathbb{R}$ for each $z = g, u, b$ such that $\sigma_S(z,t,p) = 1$ for any $p > R_z(t)$ and $\sigma_S(z,t,p) = 0$ for any $p < R_z(t)$.*

It is easy to show that $R_z(t)$ equals the type-z seller's continuation payoff if he rejects the buyer's offer at t. This is due to the information structure of the game whereby the current offer is not revealed to future buyers. Note that $R_z(t)$ is continuous in t because the probability that either the buyer or the lump-sum payoff arrives at a given time interval vanishes as the length of the interval shrinks to zero. Moreover, $R_g(t) > R_u(t) > R_b(t)$ for all t because of the heterogeneous expected value of lump-sum payoffs.

Given the seller's reservation price strategy, the buyer's equilibrium offer satisfies the following lemma:

Lemma 2. *In equilibrium, if the buyer's equilibrium offer is accepted with nonzero probability, then it is equal to $R_z(t)$ for some $z = g, u, b$.*

The intuition of the lemma is straightforward: If the offer is above the reservation price of some type of seller, then the buyer can lower his offer slightly and still trade with the same probability. Note that the above lemma does not rule out the case where the buyer's equilibrium offer is rejected with probability one at some t. In that case, the buyer's offer p must be a price between zero and $R_b(t)$.

The seller always has an option to hold the good, which gives lower bounds on the reservation price functions. They are given by

$$\begin{aligned} R_g(t) &\geq \alpha v_H, \\ R_u(t) &\geq \alpha v(q_0), \\ R_b(t) &\geq \alpha v_L. \end{aligned}$$

The following lemma places an upper bound on the buyer's equilibrium offer, and hence provides an upper bound on the reservation price of the good-type seller:

Lemma 3. *In equilibrium, the buyers never offer a price strictly more than αv_H. Therefore, $R_g(t) = \alpha v_H$ for any t.*

The intuition for this lemma is as follows. Suppose not, and let $\bar{p} > \alpha v_H$ be the supremum of the buyer's equilibrium offer. Then there exists $\bar{t}$ such that the buyer at time $\bar{t}$ offers a price arbitrarily close to $\bar{p}$. Then all types of sellers strictly prefer to accept the offer because the seller discount the future payoffs. Now consider a deviation of the buyer at time $\bar{t}$ to lower his offer by sufficiently small $\varepsilon > 0$. Then all types of sellers would still accept the offer as long as the expected cost from discounting is greater than ε. But then offering such price is a profitable deviation of the buyer, leading to a contradiction.

Note that Lemma 3 implies that if the buyer offers αv_H, then the offer is accepted by all types of sellers, so the game ends with probability one. Therefore αv_H serves as the trade-ending offer in this model.

3 Equilibrium

In this section I construct an equilibrium of the model, and present a full characterization result of the equilibria for a range of parameters.

Because of the static lemons condition, offering the trade-ending price αv_H in the early stage yields a negative payoff to the buyer. Then one might expect that the buyer who arrives in the early stage submits a screening offer and targets either the uninformed seller or the bad-type seller. In this case, the expected quality of the good increases gradually over time.

On the other hand, the buyers' beliefs about the seller's type also evolve over time because of the seller's learning. The buyer who arrives in the early stage believes that the seller is likely to be uninformed. So the buyer targets the uninformed seller by offering a middle-range price, which equals to the reservation price of the uninformed seller. But the seller is getting informed over time, hence there is a growing probability that the seller is the bad type. The bad-type seller accepts the middle-range price offer, since it is strictly higher than his reservation price. In this case, the buyer becomes increasingly worried about the possibility of getting a lemon.

It turns out that the seller's speed of learning determines the rate of increase of the probability that the seller is bad type, which in turn affects the equilibrium behavior. Specifically, the equilibrium behavior is qualitatively different depending on the seller's effective speed of learning (ρ/λ).

In the following analysis, I first present the equilibrium when the effective speed of learning is low (the slow-learning case) with the characterization results. After that I turn to the case when the effective speed of learning is high (the fast-learning case).

3.1 Slow-learning Case

In this subsection I consider the case where the seller's effective speed of learning is low. I begin by defining a class of candidate equilibrium strategy profiles.

Definition 2. A strategy profile (σ_S, σ_B) is called a *two-phase strategy profile* if there exists $t^* > 0$ and $\hat{\sigma} \in [0,1]$ such that the profile satisfies the following:

1. Phase I: for any $t < t^*$,
   - $\sigma_B(t) = R_u(t)$;
   - $\sigma_S(g, t, R_u(t)) = 0$; $\sigma_S(z, t, R_u(t)) = 1$ for $z = u, b$.
2. Phase II: for any $t \geq t^*$,
   - $\sigma_B(t)$ assigns a probability $\hat{\sigma}$ to $R_g(t) = \alpha v_H$ and a probability $1 - \hat{\sigma}$ to $p_l \leq R_b(t)$;
   - $\sigma_S(z, t, \alpha v_H) = 1$ and $\sigma_S(z, t, p_l) = 0$ for $z = g, u, b$.

In the two-phase strategy profile, the agents' behavior is divided into two phases by a threshold time $t^* > 0$. In the first phase, the buyer targets the uninformed seller by offering a middle-range price which equals to the reservation price of the uninformed. The uninformed and the bad-type seller accept the offer for sure, while the good-type seller rejects the offer. In the second phase, the buyer randomizes between submitting the trade-ending offer $R_g(t) = \alpha v_H$ and the "losing offer" p_l. The losing offer is any price below or equal to $R_b(t)$ and all types of sellers reject it with probability one. Note that the buyer's randomization probability in the second phase is restricted to be constant over time.

A tuple $(\sigma_S, \sigma_B, \phi)$ is called a *two-phase equilibrium* if it is PBE and (σ_S, σ_B) is a two-phase strategy profile. An outcome of the game is called a two-phase equilibrium outcome as an equilibrium outcome induced by a two-phase equilibrium strategy profile. The following proposition (whose proof is presented in the Appendix) states that if the seller's effective learning speed is smaller than a threshold, then there exists a unique two-phase equilibrium outcome.

Proposition 1. *There exists* $\underline{\eta} > 0$ *such that for* $0 < \rho/\lambda < \underline{\eta}$*, there exists a unique two-phase equilibrium outcome.*

The uniqueness result in Proposition 1 depends on the stationary restriction imposed on the buyer's randomization probability in the second phase. Indeed, one can construct an equilibrium where the randomization probability of the buyers in the second phase follows non-stationary path. However, the threshold time t^* in any such non-stationary equilibrium is the same as one in the two-phase equilibrium, as well as the equilibrium behavior of the agents at any time before t^*. Moreover, the payoff of the buyer at any t and the payoff of each type of seller at any time $t \leq t^*$ is identical. I provide the intuition for the payoff equivalence after I describe the two-phase equilibrium.

The remainder of this subsection is organized as follows. First, I describe the price and belief evolution of the two-phase equilibrium and underlying incentives of the agents. I begin with the equilibrium behavior in the first phase then discuss the behavior in the second phase. Then I present an outline of the proof of the equilibrium construction. Finally, I discuss the multiplicity of the equilibria of the model and present a full characterization result.

First Phase: Price Evolution The upper panel of Figure 1 shows the evolution of the reservation price and equilibrium offer in a two-phase equilibrium in which the price of the losing offer is v_L. The blue lines represent the reservation price of each type of the seller. Note that the reservation price of the good type seller is constant and equals to αv_H. The dark red line represents the equilibrium price offer.

In the first phase, the reservation price of the uninformed seller $R_u(t)$, which is the equilibrium price, must satisfy the recursion

$$R_u(t) = rdt\alpha v(q_0) + (1 - rdt)\left[\rho dt(q_0 \alpha v_H + (1 - q_0)R_b(t + dt)) + (1 - \rho dt)R_u(t + dt)\right].$$

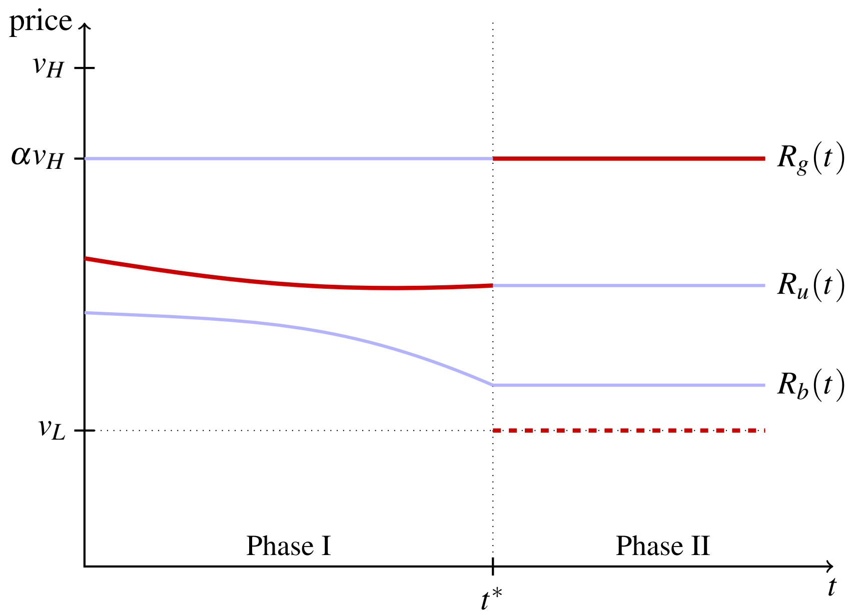
Letting $dt \to 0$ and rearranging yield

$$R_u'(t) = r\underbrace{(R_u(t) - \alpha v(q_0))}_{\text{discounting}} - \rho \underbrace{B_I(t)}_{\text{learning}}, \tag{1}$$

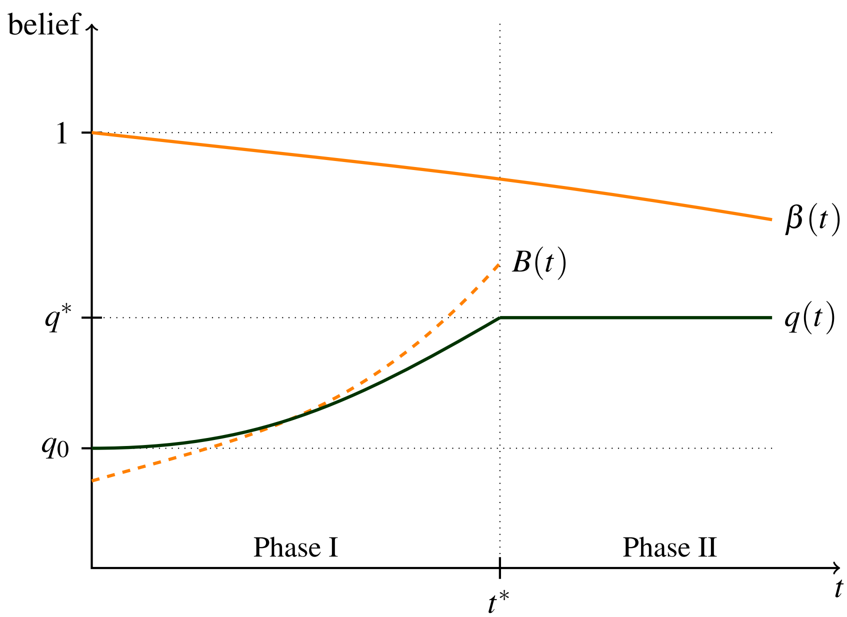
where

$$B_I(t) \equiv q_0 \alpha v_H + (1 - q_0)R_b(t) - R_u(t).$$

The first term on the right-hand side of (1) captures the effect of discounting. Note that its effect on the equilibrium price $R_u(t)$ is nonnegative. In the first phase, the uninformed seller is indifferent between acceptance and rejection, and he discounts future payoffs. Therefore, absent other effects, the buyers who arrive in the future must offer a higher price to attract the



(a) price evolution



(b) belief evolution

Figure 1: Equilibrium behavior of the two-phase equilibrium. The buyers in the first phase target the uninformed seller by offering his reservation price. The second phase begins at t^* when the belief about quality $q(t)$ hits threshold q^*, where the buyers randomize between the trade-ending offer αv_H (solid red line) and the losing offer (dashed red line). The buyers' confidence $\beta(t)$ decreases over time but remains sufficiently high in the first phase, so that it is strictly greater than a threshold $B(t)$.

uninformed. The term $\alpha v(q_0)$ in the first term captures the effect of the expected dividend until the next buyer arrives.

The second term, however, has a negative effect on the equilibrium price. It captures the effect of the uninformed seller's learning. I define $B_I(t)$ as the *value of information* for the uninformed seller, since it measures the difference in the payoff between the informed seller $(q_0 R_g(t) + (1-q_0)R_b(t))$ and the uninformed seller $(R_u(t))$. Under the given profile, $B_I(t)$ is strictly positive in the first phase. The intuition is as follows. Consider the uninformed seller who becomes informed at time t. Then the seller chooses different behavior according to the information: If the information is good $(\theta = H)$, the seller rejects the offer $R_u(t)$ in the first period. If the information is bad $(\theta = L)$, he takes the offer $R_u(t)$, since it is strictly higher than his reservation price. This adjusted behavior gives the seller a strictly higher expected payoff when he is informed.

Equation (1) implies that the positive value of information has a negative effect on the slope of $R_u(t)$. Since the uninformed seller expects the possibility of future learning in the case of rejection, his current reservation price must take into account the value of information. Furthermore, when the seller is sufficiently patient (more precisely, if r/ρ is sufficiently small), the effect of learning on $R'_u(t)$ may be greater than the effect of discounting, so that $R_u(t)$ may decrease over time.

On the other hand, a similar recursive argument for the bad-type seller yields another differential equation for $R_b(t)$ and $R_u(t)$ in the first phase, which is

$$R'_b(t) = r \underbrace{(R_b(t) - \alpha v_L)}_{\text{discounting}} + \lambda \underbrace{(R_b(t) - R_u(t))}_{\text{buyer's offer}}. \tag{2}$$

Similar to (1), the first term on the right-hand side captures the effect of discounting. The second term represents the effect of the buyer's offer of $R_u(t)$, which the bad-type seller accepts for sure. Note that the second term is negative and is proportional to the arrival rate of the buyer. Therefore, similar to $R_u(t)$, $R_b(t)$ may decrease over time in the first phase. Equations (1) and (2) form a system of ordinary differential equations for $R_u(t)$ and $R_b(t)$ in the first phase.

First Phase: Belief Evolution How do the buyers' beliefs evolve over time? Recall that $q(t)$ represents the buyers' beliefs about the quality of the good. But in this paper, the buyers also form beliefs about the seller's belief about the quality. To capture the second-order beliefs of the buyers, define $\beta(t) = \frac{\phi_u(t)}{\phi_u(t)+\phi_b(t)}$ as the buyers' *confidence* at time t. Note that $\beta(t)$ is the probability of buying the uninformed seller's good when the buyer targets the uninformed seller. The buyer's confidence, together with beliefs about quality $q(t)$, plays an important role in determining the equilibrium price.

To understand the role of the buyers' confidence, note that the buyer at time t is better off when he offers $R_u(t)$ than when he offers $R_b(t)$ if and only if

$$\beta(t)(v(q_0)-R_u(t))+(1-\beta(t))(v_L-R_u(t))>(1-\beta(t))(v_L-R_b(t)),$$

which is equivalent to

$$\beta(t)>\frac{R_u(t)-R_b(t)}{v(q_0)-R_b(t)}\equiv B(t). \tag{3}$$

Therefore, the buyer targets the uninformed seller only if his confidence is higher than a threshold $B(t)$. Note that $B(t)$ is a function of reservation prices and hence is determined by the equilibrium price evolution.

The lower panel of Figure 1 describes the belief evolution in the two-phase equilibrium. In the first phase, the buyer's belief about quality $q(t)$ increases over time. The intuition is straightforward: Suppose the buyer submits a losing offer, so there is no trade. Then $q(t)$ does not change as the seller's learning process is a martingale. Then offering $R_u(t)$ increases $q(t)$, since all but the good-type seller accept the offer and leave. However, $q(t)$ is less than the threshold belief q^* throughout the first phase, which makes it suboptimal to make a trade-ending offer.

On the other hand, the buyer's confidence $\beta(t)$ is decreasing over time in the first phase. The buyer's offer $R_u(t)$ does not affect $\beta(t)$, since both the uninformed seller and the low-type seller leave the game at the same rate. But the seller's learning decreases the buyers' confidence, since there is a growing probability that the seller is informed.

However, if the seller's effective speed of learning is slow, the rate of decrease of the buyers' confidence is low. Therefore the buyers remain confident until the expected quality of the good becomes sufficiently high so that submitting the trade-ending offer does not yield negative payoff.

Second Phase The second phase begins as the belief about quality $q(t)$ reaches q^* for the first time. In the second phase, the buyer randomizes between a trade-ending offer $R_g(t)=\alpha v_H$ and a losing offer p_l. The losing offer p_l can be any price below or equal to the bad type's reservation price. Since the all types of sellers reject p_l, the trade occurs only at αv_H. In the upper panel of Figure 1, αv_H is represented as a solid line while the losing offer p_l is represented as a dashed line, illustrating that no trade occurs at p_l.[7]

[7] In Figure 1, losing offer is equal to v_L, but the offer price can be any price less than or equal to $R_b(t)$.

Since the buyer in the second phase purchases a good from all types of sellers or does not buy the good at all, (conditional on the game continues) the buyer's beliefs about quality $q(t)$ is constant and equals q^* in the second phase. Therefore, offering αv_H yields zero payoff, so the buyer in the second phase is indifferent between submitting the trade-ending offer and the losing offer.

The buyers in the second phase randomize their offers in order to satisfy the uninformed seller's intertemporal incentives. Suppose that the buyer in the second phase offers αv_H with probability one. Then the uninformed seller in the first phase would reject the offer in favor of future high offers, leading to the breakdown of the equilibrium structure.

The reservation prices of the bad-type seller and the uninformed seller in the second phase are, respectively,

$$R_b(t) = R_b^* = \underbrace{\frac{r}{r+\lambda\hat{\sigma}}\alpha v_L}_{\text{dividend}} + \underbrace{\frac{\lambda\hat{\sigma}}{r+\lambda\hat{\sigma}}\alpha v_H}_{\text{buyer's offer}}, \tag{4}$$

$$R_u(t) = R_u^* = q_0\alpha v_H + (1-q_0)R_b^*, \tag{5}$$

where $\hat{\sigma}$ is the probability that the buyer offers the trade-ending offer. The bad-type seller's reservation price represented in (4) is a weighted average of the value of holding the asset (αv_L) and the trade-ending offer (αv_H). The reservation price of the uninformed seller (5) is a simple expectation of reservation prices of the good type and the bad type. This is because the value of the seller's information is zero in the second phase. Since the buyers target either all types of the seller or none, becoming informed does not change the seller's strategy, so the information does not provide any value.

The randomization probability $\hat{\sigma}$ is uniquely determined by the indifference condition of the buyer at the threshold time t^*: Targeting the uninformed seller at time t^* must yield zero payoff. The intuition is as follows. Suppose that targeting the uninformed seller at time t^* yields a positive payoff. Then since both $R_u(t)$ and the confidence $\beta(t)$ are continuous over time, there exists $\varepsilon > 0$ such that targeting the uninformed at $t \in (t^*, t^*+\varepsilon)$ yields a positive payoff, violating the optimality condition. Now suppose that targeting the uninformed yields a negative payoff at time t^*. Again the continuity of $R_u(t)$ and $\beta(t)$ implies that for sufficiently small $\varepsilon' > 0$ targeting the uninformed at $t \in (t^*-\varepsilon', t^*)$ is suboptimal, leading to a contradiction. Using the buyer's indifference condition at time t^*, R_u^* is uniquely determined and is given by

$$R_u^* = \beta(t^*)v(q_0) + (1-\beta(t^*))v_L. \tag{6}$$

One can then determine the value of R_b^* from (5). Finally, the randomization probability $\hat{\sigma}$ is determined by (4).

Is the randomizing behavior optimal for the buyers? First, recall that $q(t) = q^*$ implies that the buyer is indifferent between submitting the trade-ending offer and the losing offer. Second, given that the indifference condition (6) is satisfied, then targeting the uninformed seller at any $t > t^*$ yields a strictly negative payoff to the buyer. This is because while $R_u(t) = R_u^*$ is constant, the buyer's confidence $\beta(t)$ decreases because of the seller's learning. Finally, targeting the bad-type seller must yield a nonpositive payoff, so the probability of the trade-ending offer must satisfy

$$R_b^* \geq v_L. \tag{7}$$

Construction Given the above analysis, the two-phase equilibrium is constructed by the following steps:[8]

1. Determine t^* from the condition $t^* = \inf\{t : q(t^*) = q^*\}$.
2. Determine $\beta(t^*)$ from the evolution of the buyer's confidence.
3. Determine $\hat{\sigma}$ by conditions (4)-(6).
4. Check if $\hat{\sigma}$ satisfies (7).
5. Determine $R_u(t)$ and $R_b(t)$ in the first phase, by differential equations (1) and (2) with the boundary conditions at $t = t^*$.

I show in the Appendix that step 4 is satisfied if the seller's learning speed is slow enough relative to the arrival rate of the buyers. Intuitively, a higher learning speed leads to a rapid decrease in the buyer's confidence, which in turn results in lower R_u^* (equation (6)). But if R_u^* is too low, then the correspondingly small $\hat{\sigma}$ may violate the incentive condition (7).

Characterization The two-phase equilibrium described above has a special characteristic: The randomization probability of the buyers in the second phase is constant over time. But there are other equilibria where the probability of the trade-ending price changes over time. In these equilibria, the corresponding $R_u(t)$ and $R_b(t)$ in the second phase are also non-stationary,

[8]The formal proof of the construction result is given in the Appendix (Subsection A.3.1).

but they must satisfy the incentive conditions

$$R_u(t) \geq \beta(t)v(q_0) + (1-\beta(t))v_L,$$
$$R_b(t) \geq v_L,$$

for any $t \geq t^*$. The above incentive conditions imply that there is a continuum of equilibria in this model.

The above argument of equilibrium construction implies that any such non-stationary equilibrium share the main qualitative features with the two-phase equilibrium. As long as the buyers in the first phase target the uninformed seller, the evolution of the belief is identical, hence the value of t^* is the same. Then the indifference condition of the buyer at t^* (equation 6) implies that the boundary of $R_u(t)$ and $R_b(t)$ at t^* is the same, hence it must be that equilibrium behavior before t^* is identical. The only difference between any non-stationary equilibrium and the two-phase equilibrium is the randomization probability of the buyers and the reservation price of the uninformed and the bad-type seller in the second phase.

Moreover, the payoff of the buyer at any t and the payoff of the seller at any $t \leq t^*$ in any non-stationary equilibrium is same as those in the two-phase equilibrium. The discussion in the last paragraph clearly implies that the payoff of all agents in the first phase is the same. In the second phase, while the payoff of the uninformed and the bad-type seller is different, the payoff of the buyers (equals to zero) and the good-type seller (equals to αv_H) is identical across equilibria.

The following proposition (whose proof is presented in the Appendix) states that if the seller is sufficiently patient, there exists no equilibrium of the model other than the class of equilibria discussed above. Since all equilibria are payoff-equivalent, one can conduct the comparative statics in the slow-learning case using the two-phase equilibrium.

Proposition 2. *There exists $\bar{r} > 0$ such that for $r < \bar{r}$ and $0 < \rho/\lambda < \underline{\eta}$ (where $\underline{\eta} > 0$ is the bound from Proposition 1), the equilibrium of the model satisfies the following properties:*

- *in any equilibrium, behavior is divided into two phases, divided by the same threshold time t^*;*
- *the equilibrium behavior of every agent in the first phase is identical across all equilibria;*
- *the payoff of the buyer at each t and the ex ante payoff of the seller is the same across all equilibria.*

3.2 Fast-learning Case

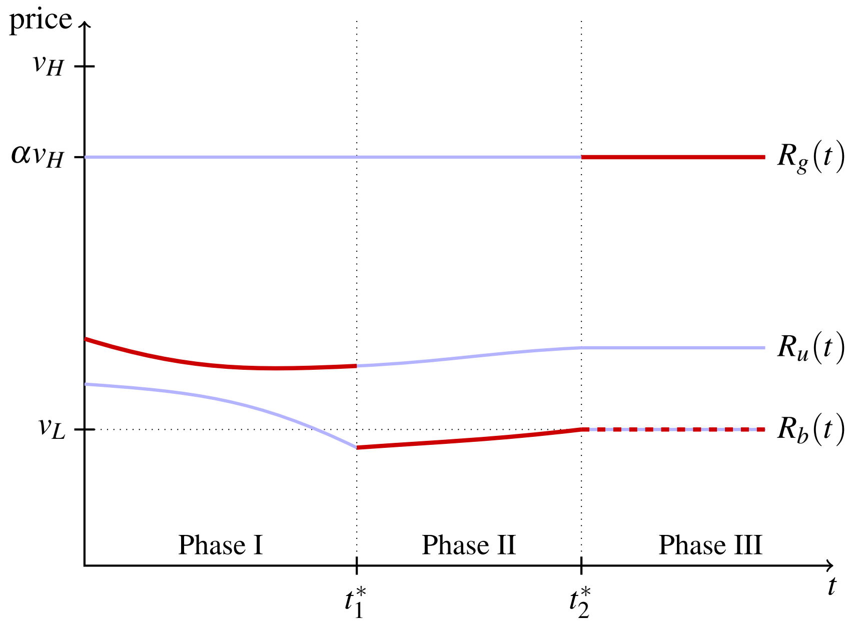
The strategy profile in the previous subsection cannot be supported as an equilibrium when the seller's effective speed of learning (ρ/λ) is high. High learning speed leads to a rapid decrease in the buyer's confidence. Therefore there is a threshold time where the buyers find it suboptimal to target the uninformed seller while the expected quality of the good is still low.

In this case, the equilibrium consists of three phases, divided by two threshold times t_1^* and t_2^*. Similar to the slow-learning case, I define the following class of candidate equilibria:

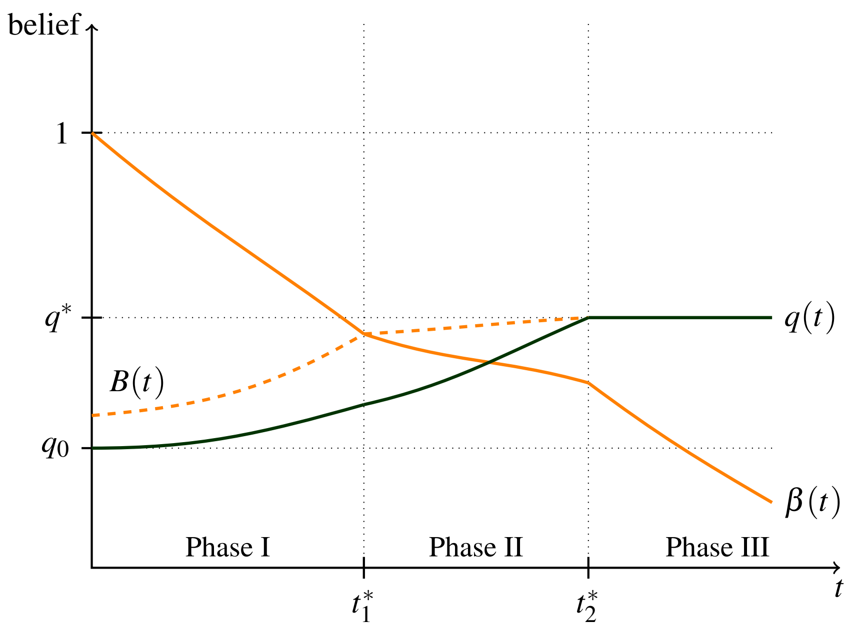
Definition 3. A strategy profile (σ_S, σ_B) is called a *three-phase strategy profile* if there exist t_1^* and t_2^* $(0 < t_1^* < t_2^*)$ and $\hat{\sigma} \in [0,1]$ such that the profile satisfies the following:

1. Phase I: for any $t < t_1^*$,
   - $\sigma_B(t) = R_u(t)$;
   - $\sigma_S(g,t,R_u(t)) = 0$; $\sigma_S(z,t,R_u(t)) = 1$ for $z = u,b$.
2. Phase II: for any $t \in [t_1^*, t_2^*)$,
   - $\sigma_B(t) = R_b(t)$;
   - $\sigma_S(z,t,R_u(t)) = 0$ for $z = g,u$; $\sigma_S(b,t,R_u(t)) = 1$.
3. Phase III: for any $t \geq t_2^*$,
   - $\sigma_B(t)$ assigns a probability $\hat{\sigma}$ to $R_g(t) = \alpha v_H$ and a probability $1-\hat{\sigma}$ to $p_l \leq R_b(t)$;
   - $\sigma_S(z,t,\alpha v_H) = 1$ and $\sigma_S(z,t,p_l) = 0$ for $z = g,u,b$.

The agents' behavior is divided into three phases by two threshold times t_1^* and t_2^*. Same as the two-phase strategy profile, the buyer in the first phase targets the uninformed seller by offering the reservation price of the uninformed. The uninformed and the bad-type seller accept the offer for sure, while the good-type seller rejects the offer. At time t_1^*, the second phase begins where the buyer targets the bad-type seller by offering his reservation price, and only the bad-type seller accepts the offer. Behavior in the third and final phase is similar to that in the second phase of the two-phase strategy profile, where the buyer randomizes between submitting the trade-ending offer and the losing offer. Again, the stationary restriction is imposed on the randomization probability of the buyers.



(a) price evolution



(b) belief evolution

Figure 2: Equilibrium behavior of the three-phase equilibrium. The buyers' confidence $\beta(t)$ rapidly decreases in the first phase. The second phase begins at t_1^* when $\beta(t)$ hits a threshold $B(t)$, and the buyers in the second phase target the bad-type seller. Hence the equilibrium price drops at t_1^*. The third phase begins at t_2^* when the belief about quality $q(t)$ reaches q^*.

A tuple $(\sigma_S, \sigma_B, \phi)$ is called a *three-phase equilibrium* if it is PBE and (σ_S, σ_B) is a three-phase strategy profile. A three-phase equilibrium outcome is defined similar to one of the two-phase equilibrium. The following proposition (whose proof is presented in the Appendix) states that if the seller's effective learning speed is larger than a threshold, then there exists a unique three-phase equilibrium outcome.

Proposition 3. *There exists $\bar{\eta} > 0$ such that for $\rho/\lambda > \bar{\eta}$, there exists a unique three-phase equilibrium outcome.*

The upper panel of Figure 2 shows the evolution of the equilibrium price in the three-phase equilibrium. Same as Figure 1 the blue lines represent the reservation price of each type of the seller, and the dark red line represents the equilibrium price offer. In the first phase, the buyers target the uninformed seller by offering his reservation price. Similar to the two-phase equilibrium, if the seller's effective discount rate (r/ρ) is small, the equilibrium price decreases in the first phase because the seller takes into account the value of future information.

However, the buyers' confidence rapidly decreases in the first phase because the seller's learning speed is high. The evolution of the buyers' beliefs described in the lower panel of Figure 2 shows how the equilibrium behavior is affected by the interaction between the buyers' beliefs about quality and the confidence. Contrary to the two-phase equilibrium in the slow-learning case, the buyers' confidence hits the threshold $B(t)$ *before* the belief about quality $q(t)$ reaches q^*.

So there is a threshold time t_1^* such that the buyers find it no longer optimal to target the uninformed seller, and submitting a trade-ending offer still yields a negative payoff. Therefore, the second phase begins at time t_1^* where the buyers only target the bad-type seller. Therefore, at time t_1^* the equilibrium trading price drops from the reservation price of the uninformed seller to that of the bad-type seller. Moreover, the probability of trade also drops because the uninformed seller begins to reject the buyer's offer.

In the second phase, trade only occurs with the bad-type seller at a price $R_b(t)$. Both $R_b(t)$ and $R_u(t)$ increase in the second phase. Since the bad-type seller receives an offer which is equal to his reservation price, getting an offer does not affect his reservation price. So contrary to $R_b(t)$ in the first phase (2), $R_b(t)$ in the second phase is affected only by the effect of the seller's discounting, and it satisfies the following differential equation:

$$R_b'(t) = r\underbrace{(R_b(t) - \alpha v_L)}_{\text{discount}} > 0. \tag{8}$$

On the other hand, the uninformed seller's reservation value satisfies $R_u(t) = q_0\alpha v_H + (1 - q_0)R_b(t)$. Note that the value of information to the uninformed seller is zero in the second phase. While the seller also has zero value of information in the final phase (as I discussed in the previous subsection), the underlying intuition is different. Contrary to the final phase, the informed seller in the second phase behaves differently according to the quality of his good. But he does not gain higher payoff because the offer the bad-type seller accepts is precisely equal to his reservation value.

The buyers' confidence $\beta(t)$ in the second phase stays below the threshold $B(t)$ so that the buyers find it optimal to target the bad-type seller[9]. On the other hand, throughout the first and second phase the belief about quality $q(t)$ increases over time because the expected quality of the good that is traded is lower than the quality of the remaining good. Therefore there exists a second threshold time, t_2^*, where the belief about the quality $q(t)$ reaches q^*.

The third and final phase begins at t_2^*, and the equilibrium behavior is similar to the final phase of the two-phase equilibrium. The buyers randomize between a trade-ending offer, at which the trade occurs, and a losing offer. Therefore, the equilibrium price at which a trade occurs jumps at t_2^* from the bad type's reservation price to a trade-ending offer. Moreover, trade of the high-quality good resumes at t_2^* as all types of sellers trade.

Figure 3 describes the probability of trade in the three-phase equilibrium. The solid red (dashed blue) line depicts the distribution of the timing of a trade conditional on the good being low- (high-) quality. Note that the probability of trade of the high-quality good is zero in the second phase, because the trade occurs only with the bad-type seller.

In the three-phase equilibrium, the equilibrium behavior is uniquely determined given the threshold times t_1^* and t_2^*. There are two indifference conditions of the buyers which jointly determines two thresholds times: 1) indifference condition between targeting the uninformed and the bad type at t_1^* $(\beta(t_1^*) = B(t_1^*))$, and 2) indifference condition between a trade-ending offer and a losing offer at t_2^* $(q(t_2^*) = q^*)$. The following proposition states that if the effective learning speed of the seller is large enough, then there exists a unique pair of threshold times.

Similar to the slow-learning case, the model has multiplicity of equilibrium in the fast-learning case. The following proposition (whose proof is presented in the Appendix) states that every equilibrium of the model differs only in the randomization probability of the buyers in the final phase, and all equilibria are payoff-equivalent. Note that the characterization result in the fast-learning case does not need additional restriction on the seller's discount rate.

[9]In Section 5, I discuss the case of intermediate learning speed where fast screening behavior may lead to increase in the buyers' confidence more than the threshold.

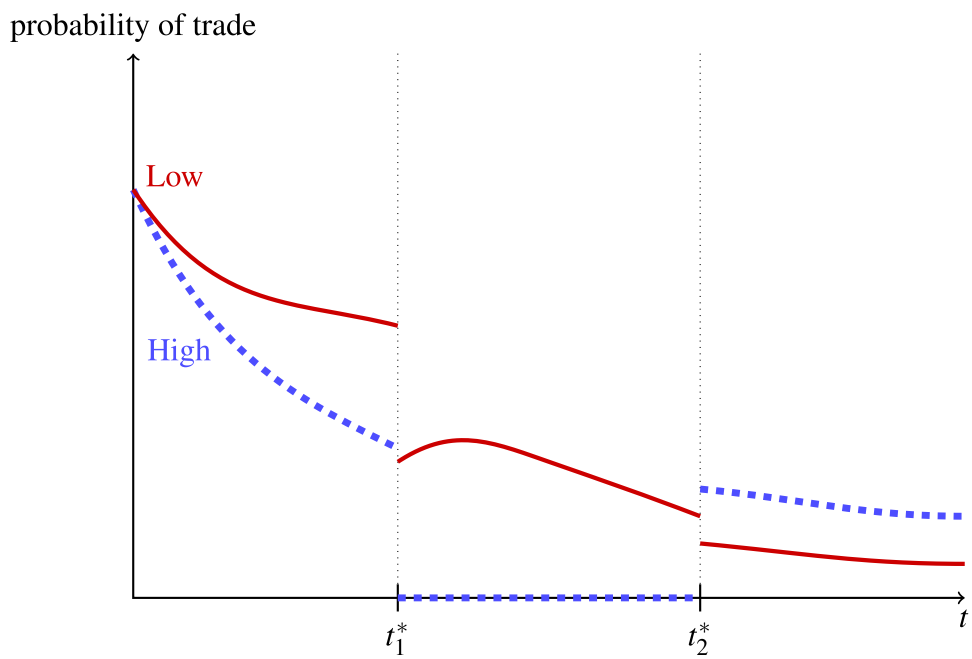


Figure 3: Probability of trade in the three-phase equilibrium.

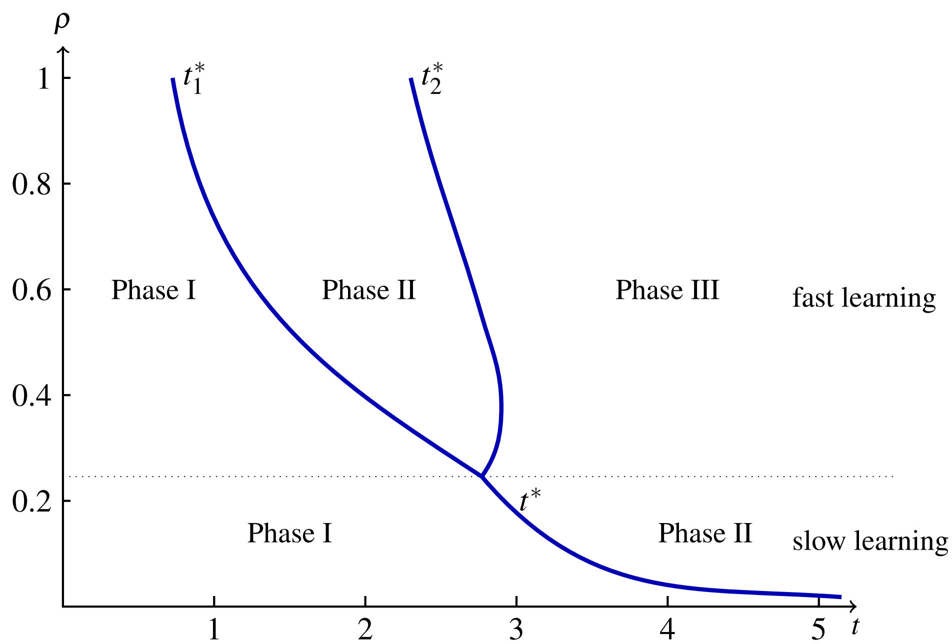
Proposition 4. *Suppose that $\rho/\lambda > \bar{\eta}$ (where $\bar{\eta} > 0$ is the bound from Proposition 3). Then the equilibrium of the model satisfies the following properties:*

- *in any equilibrium, behavior is divided into three phases, divided by the same threshold times t_1^* and t_2^*;*
- *the equilibrium behavior of every agent in the first two phases is identical across all equilibria;*
- *the payoff of the buyer at each t and the ex ante payoff of the seller is the same across all equilibria.*

When the seller's effective learning speed is between $\underline{\eta}$ (the upper bound of the slow-learning case) and $\bar{\eta}$ (the lower bound of the fast-learning case), then there exists an equilibrium where the buyers use a mixed strategy even before the belief about quality $q(t)$ reaches q^*. In Section 5 I discuss the equilibria of the model in this case. The following proposition (whose proof is presented in the Appendix) shows that when the prior q_0 is not too small, there is no such range of parameter.

Proposition 5. *There exists $\underline{q} < q^*$ such that if $q_0 \in (\underline{q}, q^*)$, then $\bar{\eta} = \underline{\eta}$.*

In the following section, I present the results of comparative statics when $q_0 \in (\underline{q}, q^*)$.



(Parameter values: $\lambda = 1, r = 0.1, v_H = 2, v_L = 1, \alpha = 0.8, q_0 = 0.3$)

Figure 4: Threshold times.

4 Comparative Statics

In this section, I present several comparative statics results with respect to the seller's learning speed.

4.1 Threshold Time

As shown in the previous section, the threshold times (t^* in the slow-learning case; t_1^* and t_2^* in the fast-learning case) are important equilibrium values that determine other equilibrium behavior. The following proposition (whose proof is presented in the Appendix) presents comparative statics results of the threshold times with respect to the learning speed of the seller:

Proposition 6.

- *In the two-phase equilibrium, t^* is decreasing in ρ;*
- *In the three-phase equilibrium, t_1^* is decreasing in ρ;*
- $\lim_{\rho \to 0} t^* = \infty$; $\lim_{\rho \to \infty} t^* = 0$.

Figure 4 depicts how the threshold times change with the seller's learning speed. In the slow-learning case, there is one threshold time t^* which decreases in ρ. Note that t^* diverges to infinity as ρ goes to zero. When ρ is arbitrarily close to zero, the environment is close to the one having symmetrically uninformed agents, so the trade occurs at the reservation price of the uninformed seller for an arbitrarily long horizon.

In the fast-learning case there are two threshold times t_1^* and t_2^*. Proposition 6 states that t_1^* decreases in ρ and converges to zero as ρ goes to infinity. The intuition is straightforward, since as ρ goes to infinity the environment converges to one that has initial asymmetric information, so the buyers target the bad type immediately after the beginning of the game. On the other hand, both t_2^* and $t_2^* - t_1^*$ are nonmonotonic under some parameter value.

4.2 Trade Surplus and Division of the Surplus

How do the trade surplus and the division of the surplus change as the learning speed changes? Standard models of adverse selection show that in the presence of initial asymmetric information, 1) the trade surplus is lower because the adverse selection problem leads to inefficient trade outcomes, and 2) the payoff of the informed agent is higher because he has a positive information rent. In this subsection I change the learning speed of the seller from zero (symmetrically uninformed agents) to infinity (initially informed seller) and simulate the value of the trade surplus and its division.

Let S_θ be the trade surplus when the quality of the good is θ. Let $f_\theta(t)$ be the probability distribution of trade of the quality-θ good at time t. Then we have

$$S_\theta = (1-\alpha)v_\theta \int_0^\infty e^{-rt} f_\theta(t)dt.$$

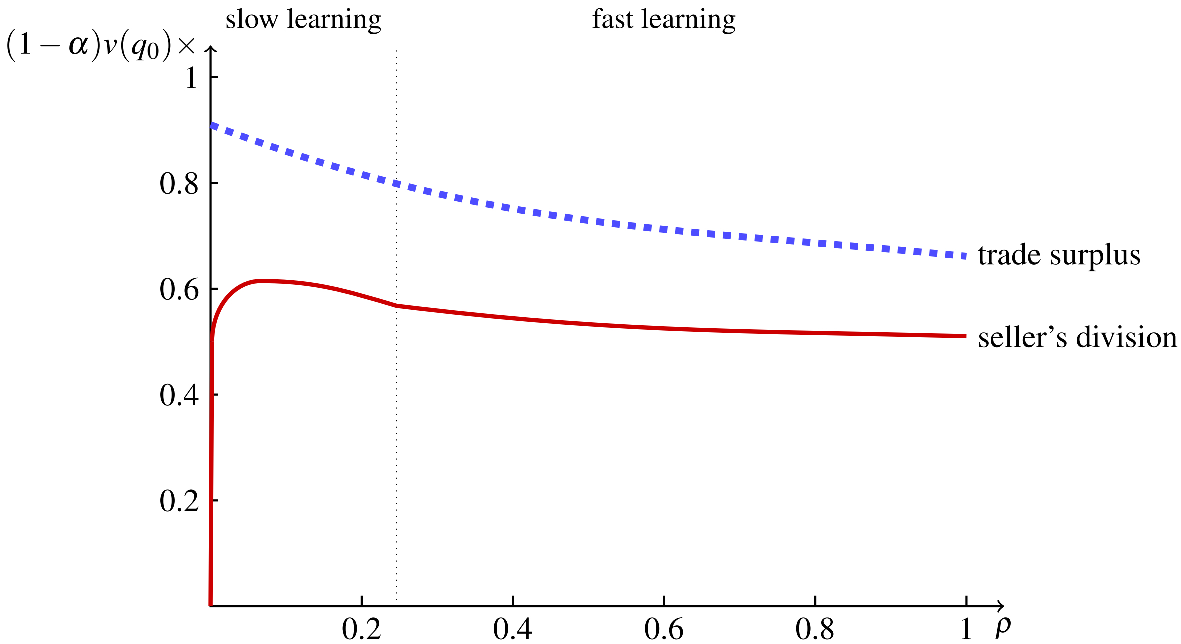
Then the ex ante trade surplus S is given by

$$S = q_0 S_H + (1-q_0)S_L.$$

The ex ante payoff of the seller is $R_u(0)$, because the seller is uninformed at $t=0$ and his reservation price equals the continuation payoff. From the seller's ex ante payoff, his division of trade surplus is calculated.[10]

The solid red line in Figure 5 is the trade surplus as a function of the seller's learning speed ρ. Note that the trade surplus is decreasing in the seller's learning speed. This result is

[10] Details of the calculation are in the Appendix (Subsection A.4).



(Parameter values: $\lambda = 1, r = 0.1, v_H = 2, v_L = 1, \alpha = 0.8, q_0 = 0.3$)

Figure 5: Trade surplus and the seller's division of the surplus.

related to one in Dang *et al.* (2012), who argue that trade is most efficient when the agents are symmetrically uninformed.[11]

On the other hand, the seller's ex ante payoff is nonmonotonic in the seller's speed of learning. The dashed blue line in Figure 5 is the seller's division of the surplus as a function of ρ. Note that the seller's surplus (hence his ex ante payoff) increases when ρ is small, but decreases when ρ is high. This is because there is a trade-off between the value of information and the adverse selection problem. If the degree of asymmetric information is small, then the seller's value of information increases in his learning speed. But if ρ is large, then the buyers' equilibrium behaviors takes into account the effect of seller's asymmetric information. Therefore, the inefficiency caused by severe adverse selection decreases the seller's payoff.

[11]Levin (2001) shows in a static lemon market model that as the quality of seller information increases, trade may decrease *or* increase depending on the information structure. His result implies that the trade surplus in this model can be nonmonotonic in the seller's learning speed under a different learning process of the seller.

5 Discussion

Implication for the Financial Crisis An important feature of the equilibrium in the fast-learning case is the impact of the buyers' second-order beliefs on the equilibrium dynamics. Before the first threshold time t_1^*, trade occurs at a middle-range price $R_u(t)$ and the trading patterns are relatively stable. However, the buyers' confidence $\beta(t)$ rapidly decreases, and eventually hits the threshold level at time t_1^*, leading to drops in both equilibrium price and the probability of a trade.

The results may help to understand what was observed at the beginning of the recent financial crisis. One of the main narratives of the crisis was the collapse of confidence in the market. For example, regarding the timing of the run on the sale and repurchase market (the "repo market") in August 2007, Gorton and Metrick (2012) argue the following:

> ...One large area of securitized banking, the securitization of subprime home mortgages, began to weaken in early 2007 and continued to decline throughout 2007 and 2008 ...The first systemic event occurs in August 2007 ...The reason that this shock occurred in August 2007, as opposed to any other month of 2007, is perhaps unknowable. We hypothesize that the market slowly became aware of the risks associated with the subprime market, which then led to doubts about repo collateral and bank solvency. At some point (August 2007 in this telling) *a critical mass of such fears* led to the first run on repo, with lenders no longer willing to provide short-term finance at historical spreads and haircuts. [Italics added]

Morris and Shin (2012) set up a static model of the adverse selection problem and show that a small amount of adverse selection can lead to the breakdown of "market confidence," defined as the approximate common knowledge of an upper bound on expected losses. In this paper, the dynamic structure of the model can illustrate the evolution of the beliefs and their effect on equilibrium behavior. Investigating the effect of the evolution of the higher-order beliefs in various trading institutions in financial markets is an interesting topic for future potential research.

Intermediate Speed of Learning If the seller's effective learning speed is between $\underline{\eta}$ (upper bound in the slow-learning case) and $\bar{\eta}$ (lower bound in the fast-learning case), then the buyers may not use a pure strategy even before the belief about quality $q(t)$ reaches q^*. Since $\rho/\lambda > \underline{\eta}$, there exists a threshold time where targeting the uninformed is no longer optimal. On the other hand, if $\rho/\lambda < \bar{\eta}$, targeting the bad type increases buyers' confidence so that the buyers' confidence becomes greater than the threshold $B(t)$, so it is suboptimal to target the bad type. In this case, the buyer in the second phase uses a mixed strategy, randomizing between targeting the uninformed and targeting the bad type. Constructing and characterizing the equilibrium in this parameter range is another area of future research.

Pure Good News and Pure Bad News Case One of the assumptions of the model is that the arrival rate of information is same regardless of the quality of the good. If the information arrival rate is quality-dependent, then not receiving a signal would also provide information about the item's quality. An environment with pure good news (bad news) is an example of a quality-dependent arrival rate, where the arrival rate of the information is zero for the low- (high-) quality good. Preliminary results show that for both cases, the equilibrium dynamics are similar to those of either the slow- or fast-learning cases examined in this paper.[12]

6 Conclusion

This paper has introduced a framework with which to study the trading patterns in an environment in which asymmetric information increases over time. In this framework, the interaction between the buyers' screening and the seller's learning generates nonmonotonic pricing and trading patterns, contrary to standard models in which asymmetric information is initially given. If the seller's effective learning speed is high, a rapid decrease of the buyers' confidence leads to drop in the equilibrium price and the probability of a trade. While the trade surplus decreases as the seller's learning speed increases, the seller's payoff is nonmonotonic in his learning speed, as a slower learning speed can lead to higher payoff for the seller.

The findings in this paper have implications for the process of designing optimal interventions for environments with increasing asymmetric information. The nonstationarity of the equilibrium trading pattern implies that the timing of an intervention would be crucial for its effectiveness. Suppose, for instance, that an asset market is hit by a shock which creates symmetric uncertainty about the value of an asset. It may then be the case that the government should not intervene immediately, because at the moment incomplete but symmetric information is not overly harmful to efficiency and only later becomes harmful as the asymmetric information grows worse. Investigating dynamic effects of an intervention and the design of optimal intervention in an environment with increasing asymmetric information are interesting topics for future research.

[12] A partial result for the equilibrium construction and characterization is available upon request.

Appendices

A Preliminaries

In this section I provide basic results which help to prove the results of the paper. First, I state differential equations which describe the dynamics of the buyers' beliefs and the reservation prices of the seller. Then I provide a detailed construction method for the equilibria described in Section 3. Proofs for the propositions of the paper are given in Section B.

A.1 Belief Dynamics

Let $m_z(t)(z=g,u,b)$ be the probability that the seller is type z *and* he is still available at time t. Then the buyers' beliefs about the seller's type $\phi_z(t)$ can be written as $\phi_z(t)=\frac{m_z(t)}{m_g(t)+m_u(t)+m_b(t)}$. Similarly, the beliefs about the quality $q(t)$ and the confidence $\beta(t)$ can be written as functions of $m_z(t)$, which are given by

$$q(t)=\frac{m_g(t)+m_u(t)q_0}{m_g(t)+m_u(t)+m_b(t)},$$
$$\beta(t)=\frac{m_u(t)}{m_u(t)+m_b(t)}.$$

Later it is shown that the evolution of $m_z(t)$ is given by a simple form of differential equations, which makes the equilibrium analysis easier.

By Lemma 2 the equilibrium offer of the buyer at time t is either $R_z(t)(z=g,u,b)$ or a losing offer. Suppose the buyer at time t offers $R_z(t)$ with probability $\sigma_{Bz}(t)$, and submits a losing offer with probability $\sigma_{B\chi}(t)=1-(\sigma_{Bg}(t)+\sigma_{Bu}(t)+\sigma_{Bb}(t))$. Then each $m_z(t)$ satisfies

$$m_g(t+dt)=(m_g(t)+\rho q_o m_u(t)dt)(1-\lambda\sigma_{Bg}(t)dt),$$
$$m_u(t+dt)=m_u(t)(1-\rho dt)(1-\lambda(\sigma_{Bg}(t)+\sigma_{Bu}(t))dt),$$
$$m_b(t+dt)=(m_b(t)+\rho(1-q_o)m_u(t)dt)(1-\lambda(\sigma_{Bg}(t)+\sigma_{Bu}(t)+\sigma_{Bb}(t))dt).$$

Letting $dt\to 0$ and arranging yield

$$m_g'(t)=\rho q_0 m_u(t)-\lambda\sigma_{Bg}(t)m_g(t), \tag{9}$$
$$m_u'(t)=-(\rho+\lambda(\sigma_{Bg}(t)+\sigma_{Bu}(t)))m_u(t), \tag{10}$$
$$m_b'(t)=(1-q_0)\rho m_u(t)-\lambda(\sigma_{Bg}(t)+\sigma_{Bu}(t)+\sigma_{Bb}(t))m_b(t). \tag{11}$$

Solving (9)-(11), combined with boundary conditions $m_u(0)=1$ and $m_g(0)=m_b(0)=0$, gives the value of $m_z(t)$ at each t. Moreover, the evolution of the confidence $\beta(t)$ is given by

$$\begin{aligned}\beta'(t) &= \frac{m_b(t)m_u'(t)-m_u(t)m_b'(t)}{(m_u(t)+m_b(t))^2} \\ &= \beta(t)\cdot\left[-\rho(1-q_0\beta(t))+\lambda\sigma_{Bb}(t)(1-\beta(t))\right]. \end{aligned} \tag{12}$$

A.2 Price Dynamics

Suppose the buyer offers $R_z(t)$ with probability σ_z, and offers p_l with complementary probability. Then $R_u(t)$ and $R_b(t)$ satisfy the following recursions:

$$\begin{aligned} R_u(t) &= rdt\alpha v(q_0)+(1-rdt)[\rho dt(q_0\alpha v_H+(1-q_0)R_b(t+dt))+ \\ &\qquad (1-\rho dt)(\lambda\sigma_{Bg}(t)dt\alpha v_H+(1-\lambda\sigma_{Bg}(t)dt)R_u(t+dt)], \\ R_b(t) &= rdt\alpha v(q_0)+(1-rdt)[\lambda\sigma_{Bg}(t)dt\alpha v_H+\lambda\sigma_{Bu}(t)dtR_u(t+dt)+ \\ &\qquad (1-\lambda(\sigma_{Bg}(t)+\sigma_{Bu}(t))dt)R_b(t+dt)]. \end{aligned}$$

Letting $dt\to 0$ and rearranging yield

$$R_u'(t) = r(R_u(t)-\alpha v(q_0))-\rho B_I(t)-\lambda\sigma_{Bg}(t)(\alpha v_H-R_u(t)), \tag{13}$$

$$R_b'(t) = r(R_b(t)-\alpha v_L)-\lambda\sigma_{Bg}(t)(\alpha v_H-R_b(t))-\lambda\sigma_{Bu}(t)(R_u(t)-R_b(t)), \tag{14}$$

where $B_I(t)=q_0\alpha v_H+(1-q_0)R_b(t)-R_u(t)$ is the seller's value of information. Solving (13) and (14) jointly with the boundary conditions yields the reservation price functions of each type.

Recall that $B(t)=\frac{R_u(t)-R_b(t)}{v(q_0)-R_b(t)}$ is the function used to determine the optimality of the buyer between targeting the uninformed and the bad type (equation 3). Then the evolution of $B(t)$ is given by

$$\begin{aligned} B'(t) &= \frac{v(q_0)(R_u'(t)-R_b'(t))-R_b(t)R_u'(t)+R_u(t)R_b'(t)}{(v(q_0)-R_b(t))^2}, \\ &= B(t)\cdot\left[\rho F_\rho(t)+rF_r(t)+\lambda\sigma_{Bu}(t)(1-B(t))\right], \end{aligned} \tag{15}$$

where $F_\rho(t)=\frac{R_u(t)-q_0\alpha v_H-(1-q_0)R_b(t)}{R_u(t)-R_b(t)}\le 0$ and $F_r(t)=\frac{B(t)(v(q_0)-\alpha v_L)-\alpha q_0(v_H-v_L)}{R_u(t)-R_b(t)}$.

A.3 Equilibrium Construction

In this subsection, I provide a complete description of the equilibrium profile in Section 3. In the Section B I prove the existence of the equilibrium as well as characterization result.

A.3.1 Slow-learning Case

The equilibrium behavior in the second phase is analyzed in the main text (Subsection 3.1).

Belief dynamics in the first phase is as follows. Since the buyers target the uninformed with probability one for any time between zero and t, each $m_z(t)$ is given by

$$\begin{aligned} m_g(t) &= \frac{\rho}{\lambda+\rho} q_0 (1 - e^{-(\lambda+\rho)t}) \\ m_u(t) &= e^{-(\lambda+\rho)t} \\ m_b(t) &= (1-q_0) e^{-\lambda t} (1 - e^{-\rho t}), \end{aligned}$$

therefore $q(\hat{t})$ and $\beta(\hat{t})$ are

$$q(t) = \frac{\frac{\rho}{\lambda+\rho} q_0 + \frac{\lambda}{\lambda+\rho} q_0 e^{-(\lambda+\rho)t}}{\frac{\rho}{\lambda+\rho} q_0 + \frac{\lambda}{\lambda+\rho} q_0 e^{-(\lambda+\rho)t} + (1-q_0) e^{-\lambda t}}, \tag{16}$$

$$\beta(t) = \frac{e^{-\rho t}}{q_0 e^{-\rho t} + (1-q_0)}. \tag{17}$$

It remains to analyze the price dynamics in the first phase. This can be done by solving (1) and (2) jointly, which yields

$$\begin{pmatrix} R_u(t) \\ R_b(t) \end{pmatrix} = C_1 \begin{pmatrix} D_1 \\ 2\lambda \end{pmatrix} e^{\gamma_1 t} + C_2 \begin{pmatrix} D_2 \\ 2\lambda \end{pmatrix} e^{\gamma_2 t} + \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}, \tag{18}$$

where C_1, C_2 are integration constants, $X = (\lambda+\rho)^2 - 4\lambda\rho q_0$, $\gamma_1 = \frac{2r+\lambda+\rho+\sqrt{X}}{2}$, $\gamma_2 = \gamma_1 - \sqrt{X}$, and

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \frac{1}{r(r+\lambda+\rho)+\rho\lambda q_0} \begin{pmatrix} r(r+\lambda+\rho)\alpha v(q_0) + \rho\lambda q_0 \alpha v_H \\ r(r+\lambda+\rho)\alpha v(q_0) + \rho\lambda q_0 \alpha v_H - r(r+\rho) q_0 \alpha (v_H - v_L) \end{pmatrix}.$$

Note that $\gamma_1 > \gamma_2 > 0$, and $D_1 = \lambda - \rho - \sqrt{X} < 0, D_2 = \lambda - \rho + \sqrt{X} > 0$.

The equilibrium is constructed by the following steps:

1. From the condition $q(t^*) = q^*$, the threshold time t^* is uniquely determined from (16), which is
$$\frac{\rho}{\rho+\lambda} e^{\lambda t^*} + \frac{\lambda}{\rho+\lambda} e^{-\rho t^*} = C, \tag{19}$$
where $C = \frac{q^*}{1-q^*} \cdot \frac{1-q_0}{q_0} > 1$.

2. Calculate $\beta(t^*)$ from equation (17) and calculate $R_u^* = \beta(t^*) v(q_0) + (1-\beta(t^*)) v_L$.

3. Determine unique value of $\hat{\sigma}$ from equations (4) and (5),

$$\hat{\sigma} = \frac{r}{\lambda} \cdot \frac{R_b^* - \alpha v_L}{\alpha v_H - v_L}.$$

4. Determine $R_u(t)$ and $R_b(t)$ in the first phase, by putting boundary conditions

$$R_u(t^*) = R_u^*,\ R_b(t^*) = R_b^*,$$

into (18) to get integration constants, which are given by

$$4\lambda\sqrt{X}\begin{pmatrix} e^{\gamma_1 t^*}C_1 \\ e^{\gamma_2 t^*}C_2 \end{pmatrix} = \begin{pmatrix} -2\lambda \\ 2\lambda \end{pmatrix}(R_u^* - Z_1) + \begin{pmatrix} D_2 \\ -D_1 \end{pmatrix}(R_b^* - Z_2).$$

A.3.2 Fast-Learning Case

Belief evolution in the first phase is same as the slow-learning case, which is summarized by equations (16) and (17). In the second phase, each $m_z(t)$ satisfies

$$\begin{aligned} m_g'(t) &= \rho q_0 m_u(t), \\ m_u'(t) &= -\rho m_u(t), \\ m_b'(t) &= -\lambda m_b(t) + \rho(1-q_0)m_u(t). \end{aligned}$$

Solving with the boundary condition at t_1^*, we have

$$\begin{aligned} m_g(t) &= q_0\left[\frac{\rho}{\lambda+\rho} + \frac{\lambda}{\lambda+\rho}e^{-(\lambda+\rho)t_1^*} - e^{-(\lambda t_1^*+\rho t)}\right], \\ m_u(t) &= e^{-(\lambda t_1^*+\rho t)}, \\ m_b(t) &= (1-q_0)\left[(1-\frac{\lambda}{\lambda-\rho}e^{-\rho t_1^*})e^{-\lambda t} + \frac{\rho}{\lambda-\rho}e^{-(\lambda t_1^*+\rho t)}\right], \end{aligned}$$

hence

$$q(t) = \frac{q_0\left(\frac{\rho}{\lambda+\rho} + \frac{\lambda}{\lambda+\rho}e^{-(\lambda+\rho)t_1^*}\right)}{q_0\left(\frac{\rho}{\lambda+\rho} + \frac{\lambda}{\lambda+\rho}e^{-(\lambda+\rho)t_1^*}\right) + (1-q_0)\left(e^{-\lambda t} + \frac{\lambda}{\lambda-\rho}(e^{-(\lambda t_1^*+\rho t)} - e^{-(\lambda t+\rho t_1^*)})\right)}, \quad (20)$$

and

$$\beta(t) = \frac{e^{-(\lambda t_1^*+\rho t)}}{e^{-(\lambda t_1^*+\rho t)} + (1-q_0)\left((1-\frac{\lambda}{\lambda-\rho}e^{-\rho t_1^*})e^{-\lambda t} + \frac{\rho}{\lambda-\rho}e^{-(\lambda t_1^*+\rho t)}\right)}. \quad (21)$$

Price dynamics are as follows. In the third (and final) phase, the reservation price is determined by (4) and (5), same as the slow-learning case. In the second phase, reservation prices satisfy

$$R_u'(t) = r(R_u(t) - \alpha v(q_0)) + \rho(R_u(t) - q_0 \alpha v_H - (1-q_0)R_b(t)),$$
$$R_b'(t) = r(R_b(t) - \alpha v_L).$$

Solving with the boundary conditions $R_u(t_2^*) = R_u^*, R_b(t_2^*) = R_b^*$ yields

$$R_b(t) = \alpha v_L + (R_b^* - \alpha v_L)e^{r(t-t_2^*)}, \tag{22}$$
$$R_u(t) = q_0 \alpha v_H + (1-q_0)R_b(t). \tag{23}$$

Note that the reservation value of the uninformed is the expectation of those of the good type and the bad type, as the buyer's equilibrium offer $R_b(t)$ gives the seller zero value of information. Last, in the first phase, the reservation values satisfy the same differential equations in the slow-learning case, hence their functional forms are given by (18), but the boundary conditions are different ($R_b(t_1^*)$ and $R_u(t_1^*)$ from the above equations (22) and (23)).

Then the equilibrium is constructed by the following steps:

1. The condition $q(t_2^*) = q^*$, yields

$$\frac{\frac{\rho}{\lambda+\rho} + \frac{\lambda}{\lambda+\rho}e^{-(\lambda+\rho)t_1^*}}{e^{-\lambda t_2^*} + \frac{\lambda}{\lambda-\rho}e^{-(\lambda+\rho)t_1^*}\left(e^{-\rho(t_2^*-t_1^*)} - e^{-\lambda(t_2^*-t_1^*)}\right)} = C, \tag{24}$$

   where $C = \frac{q^*}{1-q^*} \cdot \frac{1-q_0}{q_0} > 1$.

2. From the equations (17), (22) and (23), the buyer's indifference condition at t_1^*, $\beta(t_1^*) = B(t_1^*)$, is given by

$$e^{-r(t_2^*-t_1^*)} = \frac{(1-\alpha)\left\{\frac{q_0}{1-q_0}v_H + v_L\right\} + \alpha(v_H - v_L)q_0(1-e^{\rho t_1^*})}{(1-\alpha)v_L(1+q_0(1-e^{\rho t_1^*}))}. \tag{25}$$

   Equations (24) and (25) jointly give the unique values of t_1^* and t_2^*.

3. From the optimality condition $R_b^* = v_L$, R_u^* and $\hat{\sigma}$ are given by

$$R_u^* = q_0 \alpha v_H + (1-q_0)v_L.$$
$$\hat{\sigma} = \frac{r}{\lambda} \cdot \frac{(1-\alpha)v_L}{\alpha v_H - v_L}.$$

4. Determine $R_u(t)$ and $R_b(t)$ in the second phase, from (22)-(23) and the boundary conditions at t_2^*. They are given by

$$R_b(t) = \alpha v_L + (1-\alpha) v_L e^{r(t-t_2^*)}, \tag{26}$$

$$R_u(t) = q_0 \alpha v_H + (1-q_0) R_b(t). \tag{27}$$

5. Determine $R_u(t)$ and $R_b(t)$ in the first phase, from (18) and the boundary conditions at t_1^*.

A.4 Calculation of the Trade Surplus

Recall that $f_\theta(t)$ is the probability distribution of trade of the quality-θ good over time. Let $F_\theta(t)$ be the cdf of $f_\theta(t)$. Then

$$F_H(t) = 1 - \frac{m_g(t) + q_0 m_u(t)}{q_0},$$
$$F_L(t) = 1 - \frac{(1-q_0) m_u(t) + m_b(t)}{1-q_0},$$

hence

$$f_H(t) = -\frac{m_g'(t) + q_0 m_u'(t)}{q_0},$$
$$f_L(t) = -\frac{(1-q_0) m_u'(t) + m_b'(t)}{1-q_0}.$$

Recall that S_θ is the trade surplus when the quality of the good is θ. Since $S_\theta = (1-\alpha) v_\theta \int_0^\infty e^{-rt} f_\theta(t) dt$, the following can be shown using the results in the previous subsection:

- In the equilibrium under the slow-learning case,

$$\frac{S_H}{(1-\alpha)v_H} = \frac{\lambda}{r+\rho+\lambda}(1-e^{-(r+\rho+\lambda)t^*}) + \frac{\lambda\sigma}{r+\lambda\sigma} e^{-rt^*} \left(\frac{\rho + \lambda e^{-(\rho+\lambda)t^*}}{\rho+\lambda} \right),$$
$$\frac{S_L}{(1-\alpha)v_L} = \frac{\lambda}{r+\lambda}(1-e^{-(r+\lambda)t^*}) + \frac{\lambda\sigma}{r+\lambda\sigma} e^{-(r+\lambda)t^*}.$$

- In the equilibrium under the fast-learning case,

$$
\begin{aligned}
\frac{S_H}{(1-\alpha)v_H} &= \frac{\lambda}{r+\rho+\lambda}(1-e^{-(r+\rho+\lambda)t_1^*}) + \frac{\lambda\sigma}{r+\lambda\sigma}e^{-rt_2^*}\left(\frac{\rho+\lambda e^{-(\rho+\lambda)t_1^*}}{\rho+\lambda}\right), \\
\frac{S_L}{(1-\alpha)v_L} &= \frac{\lambda}{r+\lambda}(1-e^{-(r+\lambda)t_1^*}) \\
&\quad + \lambda\left(1-\frac{\lambda}{\lambda-\rho}e^{-\rho t_1^*}\right)\frac{1}{r+\lambda}\left(e^{-(r+\lambda)t_1^*}-e^{-(r+\lambda)t_2^*}\right) \\
&\quad + \lambda\frac{\rho}{\lambda-\rho}e^{-\lambda t_1^*}\frac{1}{r+\rho}\left(e^{-(r+\rho)t_1^*}-e^{-(r+\rho)t_2^*}\right) \\
&\quad + \frac{\lambda\sigma}{r+\lambda\sigma}e^{-rt_2^*}\left[e^{-\lambda t_2^*}+\frac{\lambda}{\lambda-\rho}\left(e^{-(\lambda t_1^*+\rho t_2^*)}-e^{-(\lambda t_2^*+\rho t_1^*)}\right)\right].
\end{aligned}
$$

B Proofs

B.1 Proof of Propositions 1-4

Here I prove the optimality of the two-phase and three-phase equilibria, and provide the characterization result for both slow- and fast-learning case. I start with the characterization of the final phase, which is common for both cases. Then I analyze characterization for the cases with slow and fast learning, respectively.

B.1.1 Equilibrium Behavior in the Final Phase

Recall that $\beta(t) = \frac{\phi_u(t)}{\phi_u(t)+\phi_b(t)}$ is the buyers' confidence at time t. The following lemma summarizes the results derived in this subsection:

Lemma 4. *In equilibrium, there exists $t^* < \infty$ and $\sigma : \{t : t \geq t^*\} \to [0,1]$ such that the equilibrium behavior after t^* is the following:*

- *the buyer at time t submits the trade-ending offer αv_H with probability $\sigma(t)$ and submits a losing offer $p_\ell \leq R_b(t)$ with probability $1-\sigma(t)$.*

- *the seller's reservation value:*

$$
\begin{aligned}
R_b(t) &= \alpha v_L + \alpha(v_H - v_L)\int_t^\infty e^{-r(\tilde{t}-t)}d(1-e^{-\int_t^{\tilde{t}}\lambda\sigma(s)ds}), \\
R_u(t) &= q_0\alpha v_H + (1-q_0)R_b(t);
\end{aligned}
$$

- *the belief $q(t) = q^*$ for $t \geq t^*$;*

- *the buyer's offer $\sigma(t)$ must satisfy*

$$R_b(t) \geq v_L, \tag{28}$$
$$R_u(t) \geq \beta(t)v(q_0) + (1-\beta(t))v_L, \tag{29}$$

for any $t \geq t^*$; at least one of the above conditions binds at $t = t^*$.

Fix an equilibrium. Let $t^* = \inf\{t : q(t) \geq q^*\}$ be the time when the buyer's unconditional belief reaches q^* for the first time.

Step 1 t^* is finite.

Proof. Suppose not, i.e., $q(t) < q^*$ for any t. Then the buyers never offer αv_H as it yields a negative payoff. Then the similar argument as Lemma 3 shows that the buyers never offer more than $\alpha v(q_0)$, and hence $R_u(t) = \alpha v(q_0)$ for all t.

There must exist finite $\bar{t}$ such that for all $t > \bar{t}$, offering $R_u(t)$ gives negative payoff (if not, there must be a lots of agreement with type-B seller, hence $q(t) > q^*$ for t large). Then after $\bar{t}$, trade is occurred only with type-B seller, so $R_b(t) = \alpha v_L$ for any $t > \bar{t}$. Since this is profitable for the buyer, the trade will occur and eventually $q(t) > q^*$, contradiction. □

Step 2 For any $t \geq t^*$, trade occurs only at $R_g^* = \alpha v_H$. Therefore, $q(t) = q^*$ for any $t \geq t^*$.

Proof. Suppose not, then there exists t_1 and t_2 such that $t^* \leq t_1 < t_2 < \infty$ and the trade happens at $p \neq \alpha v_H$ only if $t \in [t_1, t_2]$.[13]

Then for any $t > t_1$, the buyers' belief $q(t)$ is greater than q^*, so offering αv_H yields positive payoff. Hence the buyer after t_1 never submits a losing offer. That implies the buyer after t_2 offers αv_H for sure. Then $R_u(t)$ and $R_b(t)$ as t approaches to t_2 are given by

$$\lim_{t\to t_2} R_u(t) \to \frac{r}{\lambda+r}\alpha v(q_0) + \frac{\lambda}{\lambda+r}\alpha v_H,$$
$$\lim_{t\to t_2} R_b(t) \to \frac{r}{\lambda+r}\alpha v_L + \frac{\lambda}{\lambda+r}\alpha v_H.$$

[13] It must be the case that there exists finite t_2: suppose not. Then $q(t)$ converges to one as t goes to infinity, since no buyer submits a losing offer after t_1. Furthermore, since the speed of learning $\rho > 0$ is positive, the probability of the good type $\phi_g(t)$ converges to one as $t \to \infty$. However, if $\phi_g(t)$ is sufficiently close to one, expected payoff from targeting the uninformed or the bad type is arbitrarily small because there exist lower bounds for $R_u(t)$ and $R_b(t)$. Therefore, there exists $\hat{t} < \infty$ such that it is strictly optimal to offer $R_g(t) = \alpha v_H$ for $t > \hat{t}$, contradiction.

However, submitting either offer is suboptimal for the buyer, since under Assumption 1,

$$v_L - \left(\frac{r}{\lambda+r}\alpha v_L + \frac{\lambda}{\lambda+r}\alpha v_H\right) < 0,$$

$$\phi_u(t)\left[v(q_0) - \left(\frac{r}{\lambda+r}\alpha v(q_0) + \frac{\lambda}{\lambda+r}\alpha v_H\right)\right] + \phi_b(t)\left[v_L - \left(\frac{r}{\lambda+r}\alpha v(q_0) + \frac{\lambda}{\lambda+r}\alpha v_H\right)\right] < 0.$$

So if t is arbitrarily close to t_2, trade must happen only at αv_H, which contradicts to the definition of t_2. □

Step 3 $R_u(t)$ and $R_b(t)$ satisfy (28) and (29) for any $t \geq t^*$; at least one of the conditions binds at $t = t^*$.

Proof. By step 2, the buyers arrive at $t \geq t^*$ receives zero payoff. If either (28) or (29) is violated, then the buyer has a profitable deviation to target the low type or the uninformed, respectively.

Suppose that both (28) and (29) are strict at $t = t^*$. Then since $R_z(t)$ and $\phi_z(t)$ are continuous in t, there exists $\varepsilon > 0$ such that offering $R_b(t)$ or $R_u(t)$ yields negative payoff to the buyer for all $t \in (t^* - \varepsilon, t^*)$. But it contradicts to the definition of t^*. □

B.1.2 Equilibrium Before the Final Phase: Preliminary Observations

By the definition of t^*, offering $p = \alpha v_H$ at any $t < t^*$ yields negative payoff to the buyer, hence it is suboptimal. So the buyer either offers $R_u(t)$ to target the uninformed or offers $R_b(t)$ to target the bad type. Recall that the buyer receives more payoff by offering $R_u(t)$ than $R_b(t)$ if and only if

$$\beta(t) > B(t),$$

where $B(t) = \frac{R_u(t) - R_b(t)}{v(q_0) - R_b(t)}$.

The result in Step 3 implies that there are three cases at $t = t^*$:

1. (29) is binding, but (28) is not: if this is the case, then $R_u(t^*) = \beta(t^*)v(q_0) + (1 - \beta(t^*))v_L$ and $R_b(t^*) > v_L$. Moreover, $R_u(t^*) = q_0\alpha v_H + (1-q_0)R_b(t^*)$ since the value of information is zero in the final phase. Hence

$$\begin{aligned}\beta(t^*) &= \frac{R_u(t^*) - v_L}{v(q_0) - v_L} \\ &= \frac{q_0\alpha v_H + (1-q_0)R_b(t^*) - v_L}{v(q_0) - v_L} > q^*.\end{aligned}$$

   Similar calculation shows that $B(t^*) > q^*$. On the other hand, $B(t^*) < \beta(t^*)$ since targeting the bad type is worse than targeting the uninformed. As a result, $\beta(t^*) > B(t^*) > q^*$.

2. both (28) and (29) are binding: then $R_u(t^*) = \beta(t^*)v(q_0) + (1-\beta(t^*))v_L$ and $R_b(t^*) = v_L$. Similar calculation shows that $\beta(t^*) = B(t^*) = q^*$.

3. (28) is binding, but (29) is not: in this case, we have $\beta(t^*) < B(t^*) = q^*$.

B.1.3 Slow-learning Case: Proof of Propositions 1 and 2

Lemma 5. *Fix an equilibrium. Suppose at time t^*, (29) is binding, but (28) is not. Then the buyers at any $t < t^*$ offer $R_u(t)$ for sure.*

Proof. First I show that the buyers at any $t < t^*$ do not target the bad type, that is $\sigma_{Bb}(t) = 0$ for any $t < t^*$. Suppose to the contrary that $\sigma_{Bb}(t) > 0$ for some $t < t^*$. Let $t^\dagger = \sup\{t < t^* : \sigma_{Bb}(t) > 0\}$. Then $t^\dagger < t^*$ because (28) does not bind at t^*. Since $R_b(t^\dagger) \leq v_L$ and $R_u(t) \leq q_0\alpha v_H + (1-q_0)R_b(t)$ for all t,

$$B(t^\dagger) = \frac{R_u(t) - R_b(t)}{v(q_0) - R_b(t)} \leq \frac{q_0(\alpha v_H - R_b(t))}{v(q_0) - R_b(t)} \leq q^*,$$

so it must be the case that $\beta(t^\dagger) \leq q^*$. On the other hand, $\beta(t^*) > q^*$ since (29) binds at t^* while (28) does not bind. Moreover, since $\sigma_{Bb}(t) = 0$ for $t \in (t^\dagger, t^*]$, (12) implies that $\beta(t)$ is decreasing for $t \in (t^\dagger, t^*]$, leading to a contradiction.

Now it remains to show that any buyer at $t < t^*$ has no incentive to submit a losing offer. Define $\tilde{p}(t) = \beta(t)v(q_0) + (1-\beta(t))v_L$ be a expected value of traded good to the buyer then he targets the uninformed seller . Then submitting a losing offer is no worse than targeting the uninformed at time t if and only if $R_u(t) \geq \tilde{p}(t)$. I claim that $R_u(t) < \tilde{p}(t)$ for any $t < t^*$. From the previous argument,

$$R_u(t^*) = \tilde{p}(t^*) \geq q_0\alpha v_H + (1-q_0)v_L = q^*v(q_0) + (1-q^*)v_L.$$

Since $\beta'(t) = -\rho\beta(t)(1-q_0\beta(t))$ and $\beta(t^*) \geq q^*$ from the above equation,

$$\begin{aligned}\tilde{p}'(t) &= -\rho\beta(t)(1-q_0\beta(t)) \cdot q_0(v_H - v_L) \\ &\leq -\rho q_0(v_H - v_L)\min\{1-q_0, q^*(1-q_0q^*)\},\end{aligned}$$

and $\tilde{p}(t) > q^*v(q_0) + (1-q^*)v_L$. On the other hand, since $R_u'(t) = -\rho(q_0\alpha v_H + (1-q_0)R_b(t) - R_u(t))$, So either $R_u(t) \leq q^*v(q_0) + (1-q^*)v_L$ or

$$\begin{aligned}R_u'(t) &\geq -\rho q_0(\alpha v_H - R_u(t)) \\ &> -\rho q_0(1-q_0)(\alpha v_H - v_L) = -\rho q_0(v_H - v_L)q^*(1-q_0).\end{aligned}$$

Therefore, whenever $R_u(t) \geq q^* v(q_0) + (1-q^*)v_L$ it must be that $R_u'(t) > \tilde{p}'(t)$, leading to the desired result. □

Let $z(t) = e^{-\rho t}$ and $z^* = z(t^*)$. Let $\kappa = \frac{\lambda}{\rho} = \frac{1}{\eta}$ be the inverse of the seller's effective learning speed. Consider a strategy profile in which the buyers target the uninformed for any $t < t^*$. Then the condition (19) can be rewritten as

$$z^* \kappa + (z^*)^{-\kappa} = C(1+\kappa), \tag{30}$$

where $C = \frac{q^*}{1-q^*}\frac{1-q_0}{q_0} > 1$. By the implicit function theorem, $\frac{\partial z^*}{\partial \kappa} > 0$.[14] Moreover, it can be shown that $\lim_{\kappa \to 0} z^* = 0$ and $\lim_{\kappa \to \infty} z^* = 1$.

On the other hand, equation (17) can be rewritten as

$$\beta(t) = \frac{z(t)}{q_0 z(t) + (1-q_0)}, \tag{31}$$

so $\beta(t^*) = \frac{z^*}{q_0 z^* + (1-q_0)} \equiv \beta^*$. Since β^* is increasing in z^*, there exists $\bar{\kappa}$ such that $\kappa \geq \bar{\kappa}$ if and only if $\beta(t^*) \geq q^*$.

Lemma 6. *(1) $\kappa \geq \bar{\kappa}$ if and only if there exists an equilibrium of the game where the buyers at $t < t^*$ target the uninformed for sure, and hence t^* is uniquely determined by equation (30).*

(2) There exists $\bar{r} > 0$ such that if $\kappa \geq \bar{\kappa}$ and $r < \bar{r}$, then in any equilibrium of the game, buyers at $t < t^*$ target the uninformed for sure, and hence t^* is determined by equation (30).

Proof. (1) Consider a strategy profile in which the buyers at any $t < t^*$ target the uninformed seller. Then (29) must binds at t^*, that is,

$$R_u(t^*) = \beta(t^*)v(q_0) + (1-\beta(t^*))v_L.$$

[14] Let $F(z^*, \kappa) = z^*\kappa + (z^*)^{-\kappa} - C(1+\kappa)$. Then

$$\begin{aligned}\frac{\partial z^*}{\partial \kappa} &= -\frac{\frac{\partial F}{\partial \kappa}}{\frac{\partial F}{\partial z^*}} \\ &= \frac{z^* + (z^*)^{-\kappa}\log\frac{1}{z^*} - C}{\kappa((z^*)^{-\kappa-1} - 1)} \\ &= \frac{z * \kappa + (z^*)^{-\kappa}\log(z^*)^{-\kappa} - C\kappa}{\kappa^2((z^*)^{-\kappa-1} - 1)} \\ &= \frac{-(z^*)^{-\kappa}(1 - \log(z^*)^{-\kappa}) + C}{\kappa^2((z^*)^{-\kappa-1} - 1)}\end{aligned}$$

Since $x(1-\log x) < 1$ if $x \in (0,1)$, $\frac{\partial z^*}{\partial \kappa} > 0$.

On the other hand, by Lemma 4, $R_u(t^*) = q_0\alpha v_H + (1-q_0)R_b(t^*)$. Then a simple calculation shows that $R_b(t^*) \geq v_L$ if and only if $\beta(t^*) \geq q^*$. Therefore, the incentive constraint for the bad type (28) is satisfied if and only if $\kappa \geq \bar{\kappa}$.

(2) Suppose to the contrary that there exists an equilibrium where $\sigma_{Bu}(t) < 1$ for some $t < t^*$. Then Step 3 in Subsection B.1.1 and Lemma 5 imply that (28) must bind at $t = t^*$, and hence $B(t^*) = q^*$. Moreover, proof of Lemma 5 implies that $\sigma_{Bb}(t) > 0$ for $t < t^*$. Let $\hat{t} = \inf\{t < t^* : \sigma_{Bb}(t) > 0\}$. Then since $R_b(\hat{t}) \leq v_L$ and $R_u(t) \leq q_0\alpha v_H + (1-q_0)R_b(t)$ for all t,

$$B(\hat{t}) = \frac{R_u(\hat{t}) - R_b(\hat{t})}{v(q_0) - R_b(\hat{t})} \leq \frac{q_0(\alpha v_H - R_b(\hat{t}))}{v(q_0) - R_b(\hat{t})} \leq q^*,$$

and hence $\beta(\hat{t}) \leq B(\hat{t}) \leq q^*$. Furthermore, $\beta(\hat{t}) = B(\hat{t})$ by the following argument: Suppose to the contrary that $\beta(\hat{t}) < B(\hat{t})$. Then there exists $\varepsilon > 0$ such that $\sigma_{B\chi}(t) = 1$ for any $t \in [\hat{t}-\varepsilon, \hat{t})$. However, then from (14) $R_b(t)$ is strictly increasing $t \in [\hat{t}-\varepsilon, \hat{t})$, so the buyer at $\hat{t}-\varepsilon$ has a profitable deviation to offer $R_b(t)$, contradiction.

Therefore, it must be that there exists a time before $\hat{t}$ where the buyer submits a losing offer with positive probability, that is, $\sigma_{B\chi}(t) > 0$ for some $t < \hat{t}$ (if not, $q(\hat{t}) > q^*$ because $\kappa \geq \bar{\kappa}$, so it contradicts to the definition of t^*). Let $\tilde{t} = \sup\{t \leq \hat{t} : \sigma_{B\chi}(t) > 0\}$. Then the buyer at time $\tilde{t}$ must be indifferent between submitting a losing offer and targeting the uninformed seller, that is, $R_u(\tilde{t}) = \tilde{p}(\tilde{t}) = \beta(t)v(q_0) + (1-\beta(t))v_L$[15]. Moreover, the definition of $\tilde{t}$ implies that $\tilde{p}'(t) \geq R_u'(t)$. From the equation (12),

$$\begin{aligned}\tilde{p}'(\tilde{t}) &= q_o(v_H - v_L)\beta'(\tilde{t}) \\ &= -\rho q_0(v_H - v_L)\beta(\tilde{t})(1 - q_0\beta(\tilde{t})).\end{aligned}$$

On the other hand, using equation (13) and the condition $R_u(\tilde{t}) = \tilde{p}(\tilde{t})$, lower bound on $R_u'(\tilde{t})$ is given by

$$\begin{aligned}R_u'(\tilde{t}) &> -\rho(q_0\alpha v_H + (1-q_0)R_b(\tilde{t}) - R_u(\tilde{t})) \\ &> -\rho q_0(\alpha v_H - R_u(\tilde{t})) \\ &= -\rho q_0(\alpha v_H - \tilde{p}(\tilde{t})) \\ &= -\rho q_0(v_H - v_L)(q^* - \beta(\tilde{t})q_0).\end{aligned}$$

[15] Suppose the contrary that $R_u(\tilde{t}) < \tilde{p}(\tilde{t})$. Then at $\tilde{t}$ the buyer must be indifferent between submitting a losing offer and targeting the bad type, that is, $R_b(\tilde{t}) = v_L$. Moreover, it must be that $\sigma_{B\chi}(t) = 1$ at $t \in (\tilde{t}-\varepsilon, \tilde{t})$ for sufficiently small $\varepsilon > 0$. But then the price dynamics described in Subsection A.2 implies that $R_b(t) < R_b(\tilde{t}) = v_L$ for $t \in (\tilde{t}-\varepsilon, \tilde{t})$, so the buyers at $t \in (\tilde{t}-\varepsilon, \tilde{t})$ are better off by targeting the bad type, a contradiction.

Simple calculation gives that $\tilde{p}'(\tilde{t}) \geq R_u'(\tilde{t})$ only if

$$\beta(\tilde{t}) \leq \beta^\dagger \equiv \frac{1+q_0-\sqrt{(1+q_0)^2-4q^*q_0}}{2q_0} \in (0,q^*).$$

Note that $\beta^\dagger$ is independent of the seller's discount rate r. Since $\beta(t)$ is decreasing for $t \in [0,\hat{t})$, it follows that $B(\hat{t}) = \beta(\hat{t}) \leq \beta^\dagger$. However, the price dynamics described in Subsection A.2 implies that there exists $\bar{r} > 0$ such that if $r < \bar{r}$, then the value of $\hat{t}-t^*$ must be sufficiently large to satisfy $B(\hat{t}) \leq \beta^\dagger$, so that the condition $q(t^*) = q^*$ is violated, leading to a contradiction. □

B.1.4 Fast-learning Case: Proof of Propositions 3 and 4

Now consider the case in which $\kappa \leq \bar{\kappa}$.

Lemma 7. *In equilibrium, there exists $t < t^*$ such that $\sigma_{Bb}(t) > 0$.*

Proof. Suppose not; that is, $\sigma_{Bb}(t) = 0$ for any $t < t^*$. Then from (12), $\beta(t)$ is given by

$$\beta(t) = \frac{z(t)}{q_0 z(t) + (1-q_0)}.$$

Then by the definition of $\bar{\kappa}$, $\beta(t^*) < q^*$. Since $R_u(t^*) = \beta(t^*)v(q_0) + (1-\beta(t^*))v_L = q_0\alpha v_H + (1-q_0)R_b(t^*)$, it must be that $R_b(t^*) < v_L$, which contradicts to Step 3. □

Let $\hat{t} < t^*$ be the first time in which the buyer offers $R_b(\hat{t})$ with positive probability, that is, $\hat{t} = \inf\{t < t^* : \sigma_{Bb}(t) > 0\}$. Then the proof of Lemma 6 implies that $\beta(\hat{t}) = B(\hat{t}) \leq q^*$, and hence that $\hat{t} > 0$.

Lemma 8. *There exists $\underline{\kappa} > 0$ such that if $\kappa < \underline{\kappa}$, for any fixed $x > 0$, if a strategy profile with $t^* - \hat{t} = x$ is an equilibrium, then*

(1) the buyers at $t \in (\hat{t}, t^*)$ offer $R_b(t)$ with probability one and the buyers at $t \in [0, \hat{t})$ offer $R_u(t)$ with probability one;

(3) the value of $\hat{t}$ (hence t^*) is uniquely determined.

Proof. (1) Let $\tilde{q} = q_0 \frac{\alpha v_H - \alpha v_L}{v(q_0) - \alpha v_L} \in (q_0, q^*)$. I claim that if $\kappa < \frac{1-\tilde{q}q_0}{1-\tilde{q}}$, the buyers at $t \in (\hat{t}, t^*)$ offer $R_b(t)$ with probability one. Suppose to the contrary that $\sigma_{Bb}(t) < 1$ for some $t \in (\hat{t}, t^*)$. Let $\tilde{t} = \sup\{t : \sigma_{Bb}(t) < 1\}$. To derive contradiction, it is sufficient to show that $\beta'(\tilde{t}-) < B'(\tilde{t}-)$.

From (12),

$$\beta'(\tilde{t}-) \leq -\rho\beta(\tilde{t}) \cdot [(1-q_0\beta(\tilde{t})) - \kappa(1-\beta(\tilde{t}))].$$

Since the value of information at $\tilde{t}$ is zero (that is, $R_u(\tilde{t}) = q_0\alpha v_H + (1-q_0)R_b(\tilde{t})$), and $R_b(\tilde{t}) \geq \alpha v_L$, hence $B(\tilde{t}) \geq \tilde{q}$. Since $\kappa < \frac{1-\tilde{q}q_0}{1-\tilde{q}}$, it is easy to verify that $\beta'(\tilde{t}-) < 0$. On the other hand, from (15),

$$B'(\tilde{t}-) \geq B(\tilde{t}) \cdot [\rho F_\rho(\tilde{t}) + rF_r(\tilde{t})].$$

Since $R_u(\tilde{t}) = q_0\alpha v_H + (1-q_0)R_b(\tilde{t})$, $F_\rho(\tilde{t}) = 0$ and $F_r(\tilde{t}) \geq 0$. Therefore, $B'(\tilde{t}-) \geq 0 > \beta'(\tilde{t}-)$, leading to the contradiction.

On the other hand, Since the buyers offer $R_b(t)$ with probability one for all $t \in (\hat{t}, t^*)$, from Subsection A.3.2 the reservation prices of the uninformed and the bad type at $\hat{t}$ are given by (since $t^* - \hat{t} = x$)

$$R_b(\hat{t}) = \alpha v_L + (v_L - \alpha v_L)e^{-rx},$$
$$R_u(\hat{t}) = q_0\alpha v_H + (1-q_0)R_b(\hat{t}).$$

From equations (18) with the above boundary conditions at time $\hat{t}$, it is easy to show that there exists $\kappa^\dagger > 0$ such that for any $\kappa < \kappa^\dagger$, $R_u(t) < \tilde{p}(t)$ for any $t < \hat{t}$. Then defining $\underline{\kappa} = \min\{\frac{1-\tilde{q}q_0}{1-\tilde{q}}, \kappa^\dagger\}$ leads to the desired result.

(2) Since the buyer at time $\hat{t}$ is indifferent between targeting the uninformed and targeting the bad type, it must be that $B(\hat{t}) = \beta(\hat{t})$. The value of $R_u(\hat{t})$ and $R_b(\hat{t})$ calculated above imply that $B(\hat{t}) = \beta(\hat{t}) = q_0\frac{\alpha v_H - (\alpha v_L + (v_L - \alpha v_L)e^{-rx})}{v(q_0) - (\alpha v_L + (v_L - \alpha v_L)e^{-rx})}$. Then from the belief evolution equation (17), the value of $\hat{t}$ is uniquely determined. Note that $B(\hat{t}) = \beta(\hat{t})$ is decreasing in the value of x, so $\hat{t}$ is increasing in x. □

Lemma 9. *Suppose $\kappa < \underline{\kappa}$ where $\underline{\kappa}$ is determined in Lemma 8. Then there exists unique x such that the strategy profile in the previous lemma with $t^* - \hat{t} = x$ is an equilibrium.*

Proof. Let $\tilde{q}(t_1, t_\Delta)$ be the value of $q(t_1 + t_\Delta)$ under the strategy profile in which the buyers at any $t \in [0, t_1)$ offer $R_u(t)$ for sure and the buyers at any $t \in [t_1, t_1 + t_\Delta)$ offer $R_b(t)$ for sure. Then it is sufficient to show that $\tilde{q}(t_1, t_\Delta)$ is strictly increasing in t_1 and t_Δ. From equation (20),

$$\frac{\tilde{q}(t_1,t_\Delta)}{1-\tilde{q}(t_1,t_\Delta)} = \frac{q_0}{1-q_0} \cdot \frac{\frac{\rho}{\lambda+\rho}e^{(\lambda+\rho)t_1} + \frac{\lambda}{\lambda+\rho}}{e^{\rho t_1 - \lambda t_\Delta} + \frac{\lambda}{\lambda-\rho}(e^{-\rho t_\Delta} - e^{-\lambda t_\Delta})}$$
$$= \frac{q_0}{1-q_0} \cdot \frac{e^{(\lambda+\rho)t_1} + \frac{\lambda}{\rho}}{e^{\rho t_1} + \frac{\lambda}{\lambda-\rho}(e^{(\lambda-\rho)t_\Delta} - 1)} \cdot \frac{\frac{\rho}{\lambda+\rho}}{e^{-\lambda t_\Delta}}.$$

Since

$$\frac{\partial\left(\frac{e^{(\lambda+\rho)t_1} + \frac{\lambda}{\rho}}{e^{\rho t_1} + \frac{\lambda}{\lambda-\rho}(e^{(\lambda-\rho)t_\Delta} - 1)}\right)}{\partial t_1} = \frac{\lambda\left\{(e^{(\lambda+\rho)t_1} - 1) + (\lambda+\rho)e^{(\lambda+\rho)t_1}\frac{e^{(\lambda-\rho)t_\Delta}-1}{\lambda-\rho}\right\}}{\left\{e^{\rho t_1} + \frac{\lambda}{\lambda-\rho}(e^{(\lambda-\rho)t_\Delta} - 1)\right\}^2} > 0,$$

and
$$\frac{\partial\left(\left\{e^{\rho t_1}+\frac{\lambda}{\lambda-\rho}\left(e^{(\lambda-\rho)t_\Delta}-1\right)\right\}e^{-\lambda t_\Delta}\right)}{\partial t_1}=-\frac{\lambda\rho}{\lambda-\rho}\left(e^{-\rho t_\Delta}-e^{-\lambda t_\Delta}\right)<0,$$

$\frac{\partial\tilde{q}(t_1,t_\Delta)}{\partial t_1}>0$ and $\frac{\partial\tilde{q}(t_1,t_\Delta)}{\partial t_\Delta}>0$. □

B.2 Proof of Proposition 5

It is sufficient to show that if q_0 is close to q^*, $z^*=e^{-\rho t^*}$ calculated when $\kappa=\frac{1-\tilde{q}q_0}{1-\tilde{q}}$ satisfies $\frac{z^*}{q_0z^*+(1-q_0)}>q^*$, or $z^*>\frac{q^*-q^*q_0}{1-q^*q_0}$. It can be shown from equation (30) that z^* is increasing in q_0, and that z^* converges to one as q_0 converges to q^*. Moreover, $\frac{q^*-q^*q_0}{1-q^*q_0}$ is decreasing in q_0, and converges to $\frac{q^*}{1+q^*}<1$ as q_0 converges to q^*, completing the proof.

B.3 Proof of Proposition 6

Suppose $\kappa>\bar{\kappa}$. Then by equation (16), t^* is determined by

$$\rho e^{\lambda t^*}+\lambda e^{-\rho t^*}=C(\rho+\lambda), \tag{32}$$

where $C=\frac{q^*}{1-q^*}\cdot\frac{1-q_0}{q_0}>1$. Let $Y(\rho,t^*)=\rho e^{\lambda t^*}+\lambda e^{-\rho t^*}-C(\rho+\lambda)$, then

$$\begin{aligned}\frac{\partial t^*}{\partial\rho}&=-\frac{\frac{\partial Y}{\partial\rho}}{\frac{\partial Y}{\partial t^*}}\\&=\frac{1}{\lambda\rho}\cdot\frac{C+t^*\lambda e^{-\rho t^*}-e^{-\lambda t^*}}{e^{\lambda t^*}-e^{-\rho t^*}}.\end{aligned}$$

Since $e^{\lambda t^*}-e^{-\rho t^*}>0$, it remains to show that $\Phi\equiv C+t^*\lambda e^{-\rho t^*}-e^{-\lambda t^*}<0$. Let $w^*=\rho t^*$. Then by (19),

$$\begin{aligned}\Phi&=C+\kappa w^*e^{-w^*}-C(1+\kappa)+\kappa e^{-w^*}\\&=\kappa(e^{-w^*}(1+w^*)-C).\end{aligned}$$

Since $(1-a)e^{-a}<1<C$ for any $a>0$, we have $\Phi<0$.

Now suppose that $\kappa<\bar{\kappa}$. Let $t^*_\Delta=t^*_2-t^*_1$, and define

$$g_1(t^*_1,t^*_\Delta;\rho)\equiv\frac{\frac{\rho}{\lambda+\rho}e^{(\lambda+\rho)t^*_1}+\frac{\lambda}{\lambda+\rho}}{e^{\rho t^*_1-\lambda t^*_\Delta}+\frac{\lambda}{\lambda-\rho}\left(e^{-\rho t^*_\Delta}-e^{-\lambda t^*_\Delta}\right)}-C,$$

$$g_2(t^*_1,t^*_\Delta;\rho)\equiv e^{-rt^*_\Delta}-\frac{(1-\alpha)\left\{\frac{q_0}{1-q_0}v_H+v_L\right\}+\alpha q_0(v_H-v_L)(1-e^{\rho t^*_1})}{(1-\alpha)v_L(1+q_0-q_0e^{\rho t^*_1})},$$

Then by the implicit function theorem,

$$\frac{\partial t_1^*}{\partial \rho} = \frac{-A_{22}B_1 + A_{12}B_2}{A_{11}A_{22} - A_{12}A_{21}},$$
$$\frac{\partial t_\Delta^*}{\partial \rho} = \frac{A_{21}B_1 - A_{11}B_2}{A_{11}A_{22} - A_{12}A_{21}},$$

where

$$A_{11} \equiv \frac{\partial g_1(t_1^*, t_\Delta^*; \rho)}{\partial t_1^*} = \frac{1}{\Lambda^2} \cdot e^{-\lambda t_\Delta^*} \frac{\lambda\rho}{\lambda+\rho} \left\{ (e^{(\lambda+\rho)t_1^*} - 1) + (\lambda+\rho) e^{(\lambda+\rho)t_1^*} \frac{e^{(\lambda-\rho)t_\Delta^*} - 1}{\lambda - \rho} \right\} > 0,$$
$$A_{12} \equiv \frac{\partial g_1(t_1^*, t_\Delta^*; \rho)}{\partial t_\Delta^*} = \frac{1}{\Lambda^2} \cdot \left\{ \frac{\rho}{\lambda+\rho} e^{(\lambda+\rho)t_1^*} + \frac{\lambda}{\lambda+\rho} \right\} \frac{\lambda\rho}{\lambda-\rho} (e^{-\rho t_\Delta^*} - e^{-\lambda t_\Delta^*}) > 0,$$
$$B_1 \equiv \frac{\partial g_1(t_1^*, t_\Delta^*; \rho)}{\partial \rho} = \frac{1}{\Lambda^2} \cdot \left\{ e^{\rho t_1^* - \lambda t_\Delta^*} + \frac{\lambda}{\lambda-\rho} (e^{-\rho t_\Delta^*} - e^{-\lambda t_\Delta^*}) \right\} \cdot \frac{(\lambda+\rho) t_1^* \rho e^{(\lambda+\rho)t_1^*} + \lambda (e^{(\lambda+\rho)t_1^*} - 1)}{(\lambda+\rho)^2}$$
$$- \frac{1}{\Lambda^2} \cdot \left\{ t_1^* e^{\rho t_1^* - \lambda t_\Delta^*} + \frac{\lambda}{(\lambda-\rho)^2} (-(\lambda-\rho) t_\Delta^* e^{-\rho t_\Delta^*} + \lambda (e^{-\rho t_\Delta^*} - e^{-\lambda t_\Delta^*})) \right\} \cdot \frac{\rho e^{(\lambda+\rho)t_1^*} + \lambda}{\lambda+\rho} > 0,$$

$$A_{21} \equiv \frac{\partial g_2(t_1^*, t_\Delta^*; \rho)}{\partial t_1^*} = \frac{\rho q_0 (\alpha v_H - v(q_0)) e^{\rho t_1^*}}{(1-q_0)(1-\alpha) v_L (1 + q_0 - q_0 e^{\rho t_1^*})^2} > 0,$$
$$A_{22} \equiv \frac{\partial g_2(t_1^*, t_\Delta^*; \rho)}{\partial t_\Delta^*} = -r e^{-r t_\Delta^*} < 0,$$
$$B_2 \equiv \frac{\partial g_2(t_1^*, t_\Delta^*; \rho)}{\partial \rho} = A_{21} \cdot \frac{t_1^*}{\rho} > 0,$$

and $\Lambda = e^{\rho t_1^* - \lambda t_\Delta^*} + \frac{\lambda}{\lambda-\rho}(e^{-\rho t_\Delta^*} - e^{-\lambda t_\Delta^*})$. Then it is easy to check that $\frac{\partial t_1^*}{\partial \rho} < 0$.

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