Policy Complexity

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Current Version: Feb 23, 2015

Abstract

To understand the origins of complexity in public policy, this paper develops a model of incremental policymaking whereby policy modifications are constrained by backward-dependence: before undoing an older modification, the policymaker must first undo more recent modifications. The analysis focuses on policy outcomes under political conflict. We demonstrate and discuss two mechanisms that result in excessively complex policies. First, complexity may take the form of kludges, i.e., incremental modifications to existing policy that leave fundamental inefficiencies unresolved. Kludges emerge and persist under political conflict between ideologically opposed parties, especially in the presence of frictions that impede policymaking. Second, complexity may be produced by obstructionist behavior, whereby one party deliberately introduces seeming useless policy modifications to impede opponents' attempts to change policy. We describe how the nature of obstructionism, and the persistence of the resultant complexity, depends on the type of opponent that a policymaker faces.

1 Introduction

The complexity of public policy imposes significant costs on society. The United States' Internal Revenue Service has estimated that the various costs of tax compliance exceeded $168 billion in 2010, which was fifteen percent of total tax receipts for that year.[1] In many areas of policy ranging from the tax code to education to healthcare, such complexity is pervasive and persistent.

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[1]This may not be too surprising, in light of the fact that the U.S. tax code contains more than four million words.

This paper develops a model to understand the origins of policy complexity. Our analysis focuses on the dynamics of policymaking under political conflict. We consider a game between two policymakers with conflicting ideological preferences who take turns to make policy. In the model, policymaking is incremental: policy is composed of a sequence of rules that are added or undone, one rule at a time. A policymaker may seek to add or undo rules to achieve his policy ideals. Excessive complexity has a natural interpretation in this setting: a policy is excessively complex if an alternative policy achieves the same ideological outcome using fewer rules.

The novel aspect of our model is that policy is backward-dependent: when undoing policy, the policymaker has to undo recently-added rules before he can undo older rules. Our motivation for modeling backward-dependence is the idea that new rules build upon, and fill gaps in, existing rules. This complementarity implies backward-dependence: later policy modifications rely on features of existing policy, and thus their enactment renders removal of existing policy even more costly and difficult. Consider the Alternative Minimum Tax (AMT) in the U.S. Tax Code. Many observers deem the AMT to be unnecessarily complex, but also believe that it will be difficult to undo or drastically change the AMT because many (more recently enacted) aspects of the federal tax system have come to rely on the AMT.[2] As Teles (2013) points out, "new ideas have to be layered over old programs rather than replace them ... "

In Section 2.3, we use a single-player version of the game to elucidate the fundamental tension that policymakers face in the model: the trade-off between achieving ideological goals and reducing policy complexity. To fix ideas, consider a policymaker whose ideal policy position lies on the left of a left-right ideological spectrum. The policymaker may progress towards his ideal by adding new left-leaning rules to modify existing policy, increasing policy complexity as he does so. Alternatively, the policymaker may choose to start by undoing undesirable existing rules; in doing so, he reduces policy complexity, but may delay the attainment of his policy ideal.

Our subsequent analysis uses the two-player game to model political conflict. It focuses on those cases where each policymaker is ideologically either very

[2]Another example is the U.S. Affordable Care Act (ACA) of 2010, which introduced mechanisms (including mandates, subsidies and insurance exchanges) to fill gaps in the existing patchwork of private and public insurance options. A common view from both proponents and opponents was that the ACA was excessively complex compared to alternatives such as a single-payer healthcare system, and further that it would cement undesirable features of the existing insurance system, thus rendering any future move to a single-payer system even more difficult.

zealous (i.e., prioritizes achieving his policy ideals over reducing policy complexity) or very moderate (i.e., prioritizes reducing complexity over achieving ideals). Keeping this limitation in mind, we highlight a variety of mechanisms that result in the emergence and possible persistence of policy complexity. We will derive sufficient conditions on players' preferences for each mechanism to be active. And, we will show that these mechanisms have different implications for the persistence of excessive complexity.

In Section 3, we show that in conflicts between zealous parties, excessive complexity may emerge in the form of *kludges* – piecemeal attempts to modify existing policy that paper over existing problems rather than resolving them in a fundamental way. In the context of our model, kludges are rules that are added to cancel out the ideological effect of existing rules. In other words, kludges allow policymakers to avoid elaborate policy overhauls, but at the cost of additional policy complexity. Over the long run, conflicting rules set by ideologically opposed policymakers cancel out each others' effect on policy positions, while introducing additional complexity with each additional rule; the result is persistent excessive complexity.

Conversely, we show that the threat of excessive complexity from conflict may lead to what we call *appeasement*: in a situation where a moderate policymaker who is facing a zealous opponent has the opportunity to add new rules towards his policy ideal, he may instead choose to do nothing, so as to avoid an outcome where his opponent adds kludges that increase long-run complexity. To wit, moderate policymakers may choose not to act even when they do not currently face any political constraints.

In Section 4, we describe another source of policy complexity in our model: *obstructionism*, whereby an ideologically zealous policymaker intentionally introduces rules that do not improve his ideological position, but serve to obstruct his opponent's future policy changes (which would obviously be unfavourable to the original policymaker). Interestingly, the form that such obstructionism takes in equilibrium depends on the strength of the opponent's ideological preferences.

Against an ideologically moderate opponent, the policymaker engages in *intentional complexity*: he adds payoff-reducing rules to policy. He does so to protect other existing rules from being undone by his opponent, at least for a time. One way to think about this result is that policymakers, in an attempt to protect their policy gains, may implement policy in a excessively complex fashion that stymies the undoing of said policy. This is consistent with the observation that policymakers often construct complicated bureaucracies (with no apparent ideological purpose) to implement policy (see, e.g., Moe (1989)); our interpretation is that these bureaucracies serve as a moat to protect their policy gains from opponents.

On the other hand, against an ideologically zealous opponent, the policymaker's obstructionism takes the form of *strategic extremism*: he pursues policy outcomes that are even more ideologically extreme than his preferences would naively dictate. This serves to shift the "starting point" of his opponent and thus delays his opponent's future progress. The model thus provides a potential rationale for strategic extremism in policymaking,[3] and predicts that such behavior emerges in conflicts between zealous opponents.

Our analysis highlights the role of kludges in producing persistent complexity. Whereas complexity associated with kludges persists in the long-run, the effects of obstructionism only persist to the extent that they affect subsequent development of kludges. In particular, the effects of intentional complexity are transient; such complexity is introduced by a policymaker only if he anticipates that it will be undone by his opponent in the future. In contrast, a policymaker who engages in strategic extremism induces his opponent to produce more kludges, and thus increases the amount of long-run policy complexity.

In Section 5, we discuss institutional factors that affect policy complexity. First, we argue that political conflict drives the emergence of complexity. In fact, in our model, excessive complexity (either transient or persistent) does not arise in the absence of political conflict. This suggests that countries where political competition is intense (such as the United States) may suffer more policy complexity than countries with little political competition (such as China or Singapore).

Second, we consider the impact of political frictions that make it difficult for policymakers to effect policy changes. Teles (2013), while discussing policy kludges in the context of American public policy, promulgates the common view that the excessive complexity of existing American public policy is driven by the inherent conservatism of American governing institutions, which makes it difficult to create new laws and undo existing laws. Consistent with this claim, our model predicts that kludges are likely to occur in high-friction settings, but not in low-friction settings.

[3] Glaeser, Ponzetto, and Shapiro (2005) present a voting model where politicians may *declare* extreme positions (relative to the voting public) to pander to their base. In contrast, our model purports to explain why politicians may *implement* extreme policies (relative to their own preferences).

Literature Review Ely (2011) studies how excessive complexity, in the form of kludges, may arise and persist in single-player adaptive processes. His focus is on how random shocks to the environment may cause kludges to accumulate over the long run. Our approach differs in two ways. First, we consider a two-player game between rival policymakers. Second, instead of assuming that players behave myopically, we model patient players, but assume that players face exogenous political constraints and can only make small, 'local' changes to policy. This allows us to identify the effect of conflict on complexity, and discuss how strategic motives may lead of excessive complexity in the form of obstructionism.

Related to our results about obstructionism, a number of papers argue that bureaucracy serves to protect existing rules and policies. Moe (1989) discusses the idea that bureaucratic rules are deliberately designed to be burdensome, so as to limit the ability of agents or other parties to subvert the intentions of policymakers. Powell (2014) considers a model of organizational decision-making where communication channels between manager and boss may be deliberately blocked to reduce rent-seeking incentives by the manager. In these models, bureaucracy stymies the ability of agents to take undesirable *actions*; our model offers the perspective that policy complexity affects the future evolution of policy itself, and thus offers novel insights about policy dynamics.

A number of papers from various literatures explore the idea that incremental rule development may be history-dependent. Callander and Hummel (2014) consider a model where successive policymakers with conflicting preferences strategically experiment to find their preferred policy. The first policymaker benefits from a 'surprising' experiment outcome, because it deters experimentation by the second policymaker and thus preserves any policy gains by the first policymaker. Ellison and Holden (2013) study a model of endogenous rule development where there are exogenous constraints on the extent to which new rules may 'overwrite' old rules. Compared to these models, our paper introduces path dependence through a distinct mechanism – backwards dependence – and thus produces very different implications.

2 Model

A *policy* $\phi = \langle d_1, d_2, ..., d_n \rangle$ is a sequence of rules. We say that ϕ and ϕ' are *adjacent* if the two policies differ only by the most recent rule, i.e., $\phi' = \langle d_1, d_2, ..., d_n, d \rangle$ for some rule d or $\phi' = \langle d_1, d_2, ..., d_{n-1} \rangle$. Each rule is characterized by its ideological *direction*: $d \in \{-1, 0, 1\}$. We identify $d = -1$ as a left-sided rule, $d = 1$ as a right-sided rule, and $d = 0$ as a neutral rule. The ideological *position* $\rho\left(\phi\right)$ of policy ϕ is the sum of all rules in ϕ, and the *complexity* $\gamma\left(\phi\right)$ is ϕ's length. For example, if $\phi = \{1, -1, 0, 1\}$, then $\rho\left(\phi\right) = -1$ and $\gamma\left(\phi\right) = 4$ (see Figure 1).

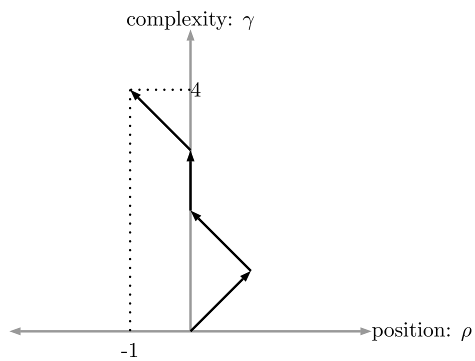

Figure 1: $\phi = \{1, -1, 0, 1\}$

Two players, (L)eft and (R)ight, play a policymaking game in continuous time. Denote the time-t policy as ϕ_t. The game starts at $t = 0$ with the empty policy, which we call the *origin*: $\phi_0 = \langle\rangle$. In any time t, one of the two players I_t is in control. Given the current policy $\phi_t = \langle d_1, ..., d_{n-1}, d_n \rangle$, player I_t chooses from the following options:

1. I_t can attempt to add a rule $d \in \{-1, 0, 1\}$ of his choice. In this case, policy switches from $\phi_t = \langle d_1, ..., d_{n-1}, d_n \rangle$ to $\langle d_1, ..., d_{n-1}, d_n, d \rangle$ at a random time with constant arrival rate p.
2. I_t can attempt to undo the most recent rule d_n. In this case, policy switches from $\phi_t = \langle d_1, ..., d_{n-1}, d_n \rangle$ to $\langle d_1, ..., d_{n-1} \rangle$ at a random time with constant arrival rate q.
3. I_t can *stagnate*, in which case the policy $\phi_t = \langle d_1, ..., d_{n-1}, d_n \rangle$ remains unchanged.

We may think of $1/p$ and $1/q$ as reflecting the magnitude of institutional frictions (e.g. veto powers, supermajority rules) in the policymaking process; the larger $1/p$ $(1/q)$ is, the more time it takes for policymakers to add (undo) rules. We assume that rules are easier to add than to undo: $p > q > 0$.

Change of control from one play to the other is stochastic. Player L is in control at the begining of the game. At each instant that L is in control, he loses control to R at a random time with constant arrival rate $\lambda > 0$. Once R gains control, he is in control forever after. We say that λ is player L's *vulnerability* (R never loses control once he gains it, i.e., he is invulnerable).

Preferences The instantaneous payoff of player $I \in \{L, R\}$ at time t depends on the policy in place:

$$\pi_I\left(\phi_t\right) = -\zeta_I \left|\hat{\rho}_I - \rho\left(\phi_t\right)\right| - \gamma\left(\phi_t\right)$$

where $\rho_I^* \in \mathbb{Z}$ is his *ideal*, and ζ_I is his *ideological zeal*. With this payoff function, players prefer policies that are closer in ideological position to their ideal. We say a policy ϕ is *I-ideal* if its position coincides with player I's ideal, i.e. $\rho\left(\phi\right) = \hat{\rho}_I$. Each player I discounts future payoff at rate r, i.e., his continuation payoff at t is

$$V_{I,t} = E\left[\int_{\tau=t}^{\infty} e^{-r\tau}\pi_I\left(\phi_\tau\right)d\tau\right].$$

Notice that each player cares about policy even when he is not in control.

The two players have conflicting ideological positions: $-\hat{\rho}_L < 0$ and $\hat{\rho}_R > 0$. We restrict attention to $\zeta_L > 1$ and $\zeta_R > 1$, i.e. each player has a sufficiently strong preference over ideological position. This assumption ensures that adding rules is potentially profitable, so that there is a meaningful trade-off between adding and undoing policy: by adding a rule in the direction of his ideal, player I increases his instantaneous payoff by $\zeta_I - 1 > 0$. We describe players with ζ_I close to one as *moderates*, and players with high ζ_I as *zealots*.

A policy ϕ is *excessively complex* if there exists an alternative policy ϕ' that is weakly closer to both ideals ($|\rho(\phi) - \hat{\phi}_L| \geq |\rho(\phi') - \hat{\phi}_L|$ and $|\rho(\phi) - \hat{\phi}_R| \geq |\rho(\phi') - \hat{\phi}_R|$) and has lower complexity ($\gamma(\phi') < \gamma(\phi)$). (See Figure 2 for some examples.) Otherwise, we say that ϕ is *simple*. Notice that simple policies either contain only left-sided rules (L-simple) or contain only right-sided rules (R-simple).

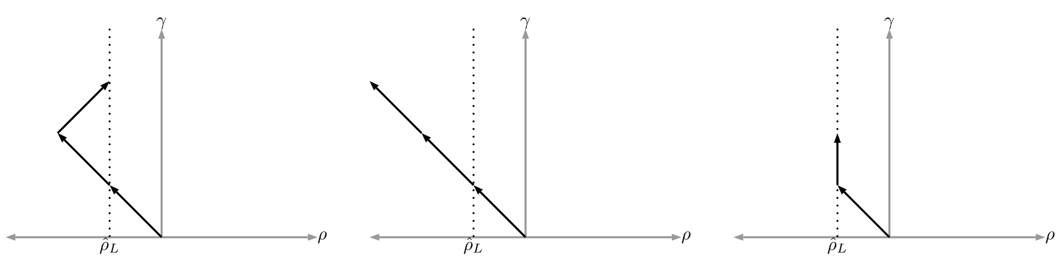

Figure 2: Excessively complex policies

2.1 Discussion

Before proceeding, let us discuss the motivation behind some of our modeling choices.

The assumption that the policymaker may only undo the most recently added rule reflects the premise that there are dependencies between rules: newer rules build upon older rules and rely crucially on the context provided by these older rules, so that undoing older rules would render the newer rules incoherent.[4] We make the extreme assumption that such incoherence incurs effectively infinite costs, so that any older rule cannot be undone without first undoing all newer rules. We conjecture that relaxing this assumption, so that older rules can be undone before newer rules but at a cost, will not change the main insights of the model.

Our model assumes that policymaking is incremental: rules may only be added or undone one at a time. As Levy and Razin (2013) and Teles (2013) point out, political constraints such as resistance by interest groups (see, e.g., Morris and Coate (1999)) force policymakers to focus their efforts on incremental changes rather than complete overhauls.[5] Building on this interpretation, think of delays in adding or undoing rules as being due to resistance from political interest groups (who support or oppose those rules) that has to be overcome before the changes are implemented.

The assumption that $p > q$ captures the premise that there is hysteresis in policymaking, so that rules are easier to add than to undo. This assumption matters for our results: it ensures that players prefer (at least sometimes) to add rules in their favoured direction, rather than undo unfavourable rules. In our setting, there is a natural motivation for this premise: backward dependence implies that only the existing (most recent) rule may be undone, whereas when adding a new rule, there may be multiple potential rules for the policymaker to choose from. This means that the policymaker faces fewer constraints when adding rules than when undoing them. Our model captures this point in reduced form, by assuming that the policymaker can add a new rule more quickly than he can undo the most recent rule. Besides backward dependence, other reasons

[4]Besides public policy, another example of such dependencies is in computer programming: deleting a portion of a program's code that other (more recently added) parts of the program relied upon often causes the entire program to fail to compile.

[5]Besides political constraints, cognitive limitations may introduce uncertainty about the impact of large-scale policy changes and thus force policymakers to focus on making small 'local' changes to policy (see, e.g., Lindblom (1959) and Callander (2011)).

for hysteresis have been extensively discussed and motivated in the literature; for example, Morris and Coate (1999) argue that policies may be easier to enact than undo because, once enacted, interest groups may make policy-specific investments and subsequently fight harder against the removal of these policies.[6]

In the model, the ease with which policy can be modified (as represented by the arrival rates p and q) is independent of the current policy position and of the policymaker's political stance. This assumption is made for tractability; richer models that take into account the political feasibility of potential changes may yield additional insights.

We model the two players' preferences as being diametrically opposed, in the sense that (at least at the origin) a rule that is good for L is bad for R, and vice versa. This assumption is made for parsimony, and in fact makes kludge more difficult to produce in the model: it maximizes each player's motivation to undo rules introduced by his opponent rather than add rules of his own. Accordingly, we expect models with richer player preferences (and a richer space of policies) to preserve our main insights.

2.2 Technical Preliminaries

In this game, the relevant state variable (ϕ_t, I_t) is the combination of the current policy and the identity of the player in control. Given the current state, a pure strategy for the player in control specifies which policy to target next (either by adding a rule, undoing a rule, or doing nothing).

Lemma 1 *A pure-strategy Markov-perfect equilibrium exists.*

We restrict attention to pure-strategy Markov-perfect equilibria. Notice that each player I's pure strategy defines a directed graph on the set of policies, whereby $\phi \to \phi'$ if and only if I's strategy specifies that I target ϕ' when he is at ϕ. We will restrict attention to equilibria where each player's graph is acyclic, i.e., each player never returns to a policy that he previously moved away from.[7]

Lemma 2 *There exists a pure-strategy Markov-perfect equilibrium where both players' graphs are acyclic.*

[6]For more in this vein, see Alesina and Drazen (1991)

[7]This restriction, made for expositional clarity, does not have any substantive impact; equilibria are generically acyclic. It eliminates only knife-edge equilibria where players are indifferent between adjacent policies.

Further, given I's strategy and a policy $\phi_{(0)}$, we define I's *trajectory* to be the (possibly infinite) sequence of policies $\{\phi_{(0)}, \phi_{(1)}, ..., \phi_{(n)}\}$ such that for each $k \geq 0$, I target $\phi_{(k+1)}$ when at $\phi_{(k)}$. In other words, it is the sequence of policies (starting from $\phi_{(0)}$) that I will move along on the equilibrium path while he is in control. (For player L, the sequence may be interrupted if he loses control to R before reaching the last policy in his trajectory.)

2.3 The Ideology-Complexity Trade-off

As a preliminary step, we'll analyze the subgame for the second player R. This analysis is a useful starting point: it allows us to study optimal policymaking in the absence of strategic interactions between players, and build some intuition for the rest of the (strategic) analysis.

We start with some notation, followed by basic observations. $V_I(\phi, J)$ is the equilibrium continuation value for player I when the current policy is ϕ and player J is in control. $V_I(\phi, I; \Phi)$ is the equilibrium continuation value for player I when the current policy is ϕ and I is in control, conditional on I pursuing trajectory Φ.

First, suppose that R's trajectory starting from $\phi_{(0)}$ is $\Phi = \left(\phi_{(0)}, \phi_{(1)}, \phi_{(2)}, ..., \phi_{(n)}\right)$. When the existing policy is at $\phi_{(k)}$, it will jump to $\phi_{(k+1)}$ with arrival rate p if $\phi_{(k+1)}$ is an extension of $\phi_{(k)}$; and with arrival rate q if $\phi_{(k+1)}$ is a truncation of $\phi_{(k)}$. Denote this arrival rate by $w_{(k)}$. For $k < n$ we can write the "asset equation" for R's value function as $(r + \psi_{(k)})V_R(\phi_{(k)}, R; \Phi) = \pi_R(\phi_{(k)}) + w_{(k)}V_R(\phi_{(k+1)}, R; \Phi)$; or equivalently,

$$V_R\left(\phi_{(k)}, R; \Phi\right) = \frac{\pi_R(\phi_{(k)}) + w_{(k)}V_R(\phi_{(k+1)}, R)}{r + w_{(k)}}. \tag{1}$$

Iteratively expanding this expression, defining $\Pi_{i=k}^{k-1} w_{(i)} = 1$ and $w_{(n)} = 0$, we get

$$V_R\left(\phi_{(k)}, R; \Phi\right) = \sum_{j=k}^{n} \frac{\Pi_{i=k}^{j-1} w_{(i)}}{\Pi_{i=k}^{j}\left(r + w_{(i)}\right)} \pi_R\left(\phi_j\right). \tag{2}$$

In other words, R's value function at $\phi_{(k)}$ is the discounted weighted average of his instantaneous payoffs at each of the policies on his trajectory, starting from $\phi_{(k)}$.

Using (2), we may show that if R adds a rule, he always does so towards his ideal. This observation, and the fact that we focus on acyclic strategies, pins down the form of R's optimal strategy:

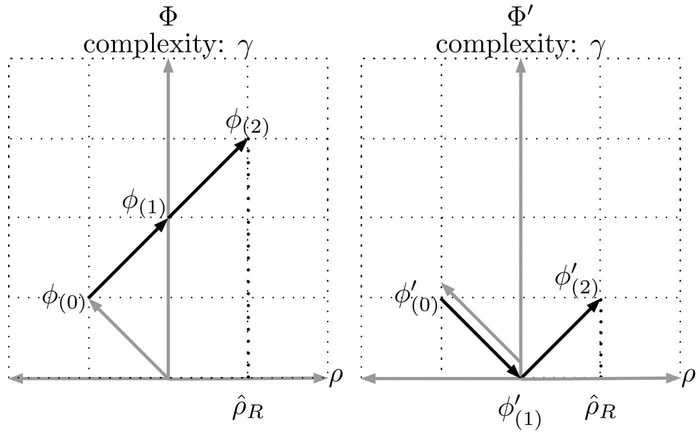

Figure 3: Example 1

Lemma 3 *Fix policy ϕ. Then for some $k \geq 0$ and $k' \geq 0$, R's equilibrium trajectory starting from ϕ is as follows: R will undo k rules from ϕ, then add k' rules in the direction of his ideal until his ideal is attained, and stagnate thereafter.*

The following proposition highlights the trade-off that R faces between attaining his ideal and reducing complexity. It states that a zealous player will add (unless his ideal has been reached), whereas a moderate player will undo any existing rules that are not in his favour.

Proposition 1 *Fix all parameters except for ζ_R. Suppose that the current policy ϕ is not R-simple. Then there exists $\underline{\zeta}_R\left(\phi\right) > 1$ such that, at ϕ, (i) R adds a rule in the direction of his ideal if $\zeta_R > \underline{\zeta}_R\left(\phi\right)$; whereas (ii) R undoes (the most recent rule) if $\zeta_R < \underline{\zeta}_R\left(\phi\right)$.*

Underlying Proposition 1 is the following trade-off. Adding rules is the fastest way for the player to move towards his ideal. In comparison, undoing rules slows the player's progress towards his ideal, but reduces policy complexity. Thus a zealot (who cares greatly about ideological bias relative to complexity) prefers to add, whereas a moderate (who cares more about complexity relative to bias) prefers to undo. The following example clarifies this intuition in a simplified setting.

Example 1 *Suppose that the starting policy $\phi = \{-1\}$, and that $\hat{\rho}_R = 1$ (i.e., R's ideal is one step to the right of the origin). Then there are two candidates for R's optimal trajectory:*

$$\Phi = (\langle -1\rangle, \langle -1, 1\rangle, \langle -1, 1, 1\rangle) = \left(\phi_{(0)}, \phi_{(1)}, \phi_{(2)}\right),$$

where R adds right-sided rules until he achieves his ideal, and

$$\Phi' = (\langle -1\rangle, \langle\rangle, \langle 1\rangle) = \left(\phi_{(0)}, \phi'_{(1)}, \phi'_{(2)}\right),$$

where R undoes the only rule from ϕ to return to the origin, then adds a right-sided rule to reach his ideal. See Figure 3.

Applying (2), the value functions for R given each trajectory are weighted averages of the instantaneous payoffs for the policies on the trajectory:

$$V(\phi, R; \Phi) = \omega_{(0)}\pi_R(\phi_{(0)}) + \omega_{(1)}\pi_R(\phi_{(1)}) + \omega_{(2)}\pi_R\left(\phi_{(2)}\right),$$
$$V(\phi, R; \Phi') = \omega'_{(0)}\pi_R(\phi_{(0)}) + \omega'_{(1)}\pi_R(\phi'_{(1)}) + \omega'_{(2)}\pi_R\left(\phi'_{(2)}\right),$$

where $\omega_{(i)} = \frac{p^i}{(p+r)^{i+1}}$, for $i = 0, 1, 2$, $\omega'_{(0)} = \frac{1}{q+r}$ and $\omega'_{(i)} = \omega'_{(0)} \times \frac{p^{i-1}}{(p+r)^i}$ for $i = 1, 2$.
To compare the two continuation values, note the following facts.

1. *$\pi_R(\phi_{(k)})$ and $\pi_R(\phi'_{(k)})$ are increasing in k: along each trajectory, R's instantaneous payoff increases with each step that he takes.*

2. *for $k = 1, 2$, $\pi_R(\phi'_{(k)}) - \pi_R(\phi_{(k)}) = 2 > 0$: the k-th policy in Φ' has the same bias as, but lower complexity than, the corresponding policy in Φ, and thus is more lucrative.*

3. *$\omega_{(0)} < \omega'_{(0)}$, whereas for $k = 1, 2$, $\omega_{(k)} > \omega'_{(k)}$: compared to $V(\phi, R; \Phi)$, $V(\phi, R; \Phi')$ puts more weight on the (less lucrative) early policy ϕ, and less weight on the (more lucrative) later policies. This is because undoing rules is slower than adding rules ($q < p$), so R is stuck for a longer time at ϕ under Φ' than under Φ.*

To summarize – the advantage of adding (Φ) over undoing (Φ') is that it allows R to move along the trajectory towards more lucrative policies more quickly; this advantage is increasing in ζ_R. The disadvantage is that each policy after ϕ in Φ is more complex and thus less lucrative than the corresponding policy in Φ'. Consequently, adding is optimal for R if ζ_R is high, whereas undoing is optimal if ζ_R is low.

3 Kludges

In the next few sections, we analyze the two-player game. Our main results are limited to cases where each player is either highly zealous or highly moderate. In these cases, the players' equilibrium trajectories have simple and intuitive characterizations. The case where players have intermediate preference intensities is more complicated, and in our view provides less useful insights into the strategic interactions between policymakers; we discuss this case in Appendix A.

This section discusses the case where both policymakers are zealous. It demonstrates how conflict between zealous opponents leads to the emergence and persistence of excessive complexity in the form of what we call *kludges*.

Definition 1 *A rule $d_k \neq 0$ in a policy $\phi = \langle d_1, ..., d_n \rangle$ is a kludge if it has opposite sign with an earlier rule, i.e., $d_k = -d_{k'}$ for some $k' < k$. A policy containing at least one kludge is said to be kludged.*

Any kludged policy is excessively complex. Further, any kludges added by player R are never subsequently undone (Lemma 3), so kludges created by player R introduce persistent excessive complexity to policy.[8]

The next result states that kludges are produced in the course of conflict between zealous players. Define $\bar{\zeta}_R \equiv 1 + \frac{2q}{(p-q)\left(\frac{r}{p+r}\right)}$ and $\bar{\zeta}_L \equiv 1 + \frac{2\lambda p^2}{r\left(r+p\right)^2 + \lambda\left(\left(r+p\right)^2 - p^2\right)}$.

Proposition 2 *Suppose both players are sufficiently zealous: $\zeta_L > \bar{\zeta}_L$ and $\zeta_R > \bar{\zeta}_R$. Then starting from the origin, L adds left-sided rules only. His trajectory includes (and possibly overshoots) the simple L-ideal policy. Once R takes control, R adds right-sided rules until he reaches his ideal, then stagnates there. Consequently, the long-run policy $\lim_{t\to\infty} \phi_t$ is kludged with positive probability.*

When both players are zealous, they add rules in opposite directions. These conflicting rules cancel out in terms of policy bias, but add up in terms of complexity. The result is kludges – i.e., persistent excessive complexity.

The argument for this result is simplified by the strong assumption that R is so zealous ($\zeta_R > \bar{\zeta}_R$) that he adds rules towards his ideal at every non-ideal policy. Such single-minded behavior by R simplifies dramatically the strategic considerations for L. In this case, L's choice of trajectory has no effect on R's behavior, so L simply chooses between adding left-sided rules (to improve his ideological position in the short-run) and stagnating (to reduce policy complexity in the long-run). With this trade-off in mind, a zealous L thus chooses to add

[8]Player L may also produce kludges, i.e., add rules that are opposite to *his own* previously added rules. However, in these cases, L's kludges turn out to be a form of intentional complexity (see Section 4.1) and are transient, i.e., they are subsequently undone by player R. See Appendix A for more details.

left-sided rules rather than stagnate because he puts greater weight on ideology over complexity. And whenever L succeeds in adding at least one left-sided rule before he loses control to R (which occurs with positive probability), R's subsequent right-sided rules will qualify as kludges.

Proposition 2 specifies that kludges are produced if both players are sufficiently zealous.[9] While we do not derive necessary *and* sufficient conditions, the next two propositions demonstrate a partial converse. First, with a moderate R, kludges are avoided because R chooses to undo the rules added by L on the equilibrium path. Define $\underline{\zeta}_R \equiv 1 + \frac{q}{p\left(1-\left(\frac{p}{p+r}\right)^{\hat{\rho}_R-\hat{\rho}_L}\right)} < \bar{\zeta}_R$.

Proposition 3 *If player R is sufficiently moderate, ($\zeta_R < \underline{\zeta}_R$), then the long-run policy $\lim_{t\to\infty} \phi_t$ is simple. Specifically, on the equilibrium path, R will undo any rules that L added until he returns to the origin, then add right-sided rules until he attains his ideal.*

What if L is moderate? This case is more interesting; it turns out that kludge is avoided in this case, even if R is zealous. Here, L engages in we call *appeasement*: at the origin, L will stagnate rather than add left-sided rules. He does so to avoid the production of kludge by R.

Proposition 4 *There exists $\underline{\zeta}_L^A < \bar{\zeta}_L$ such that if L is sufficiently moderate ($\zeta_L < \underline{\zeta}_L^A$) and R is sufficiently zealous ($\zeta_R > \bar{\zeta}_R$), then L will stagnate at the origin until he loses control to R. Consequently, the long-run policy $\lim_{t\to\infty} \phi_t$ is simple.*

Although L would increase his payoff by adding left-sided rules starting from the origin (and in fact would do so in the absence of political competition), the prospect of being succeeded by a zealot induces him to avoid adding any rules at all. L anticipates that R, upon taking control, will not undo any existing rules but instead will add right-sided rules until he reaches his ideal. Thus, by adding rules while he is in control, L shifts the policy position towards his ideal in the short-run; however, in the long-run, this benefit dissipates and instead L suffers due to higher complexity. A sufficiently moderate L thus chooses not to add any rules at all, effectively conceding the policymaking process entirely to his opponent.

[9]In fact, as hinted above, the bound on ζ_R specified in Proposition 2 is stronger than necessary; weaker bounds may be found where the equilibrium path remains the same, but the off-path computations become more complicated. The same caveat, that we choose stricter bounds than necessary to preserve tractability, holds for most of our other propositions as well.

4 Obstructionism

Intentional complexity – in the form of designs or rules that are intentionally made to be excessively complex or confusing – appears in fields ranging from patent law to software development. Meanwhile, a number of literatures have discussed the phenomenon of obstructionism, whereby agents take actions that do not improve their current position, but instead make it more difficult for their opponents to make progress.[10] In this section, we show how obstructionism manifests in our model: policymakers deliberately implement inefficiently complex policies to obstruct their opponents. A key insight is that the form of optimal obstructionism depends on whether the opponent is a moderate (Proposition 5) or a zealot (Proposition 6).

Two observations before diving in. First, in our model, only the first player L engages in obstructionism, if at all: the second player R moves last, and thus has no reason to engage in strategic behavior. Second, only sufficiently zealous players will engage in obstructionism: they prioritize ideological position over policy complexity, and thus they are willing to tolerate the increased complexity that arises from obstructionist behavior. So, we focus on the case where player L is zealous.

4.1 Intentional Complexity

In this section, we show that obstructionism against a moderate opponent optimally takes a form we term *intentional complexity*. Specifically, the following proposition shows that a zealous first player L will, after reaching his ideal policy, add neutral rules that increase policy complexity without improving ideological position.

Proposition 5 *Fix all parameters except ζ_L and ζ_R. There exists $\underline{\zeta}_L^I$ such that if L is sufficiently zealous ($\zeta_L > \underline{\zeta}_L^I$) and R is sufficiently moderate ($\zeta_R < \underline{\zeta}_R$), then along L's trajectory, he adds left-sided rules until he reaches his ideal, after which he adds one or more neutral rules.*

Further, the long-run policy is simple. Specifically, after R takes control, he removes all rules that L added to return to the origin, then adds right-sided rules until he attains his ideal.

[10] See, for e.g., the corporate finance literature on poison-pill takeovers, summarized in Jensen (1988).

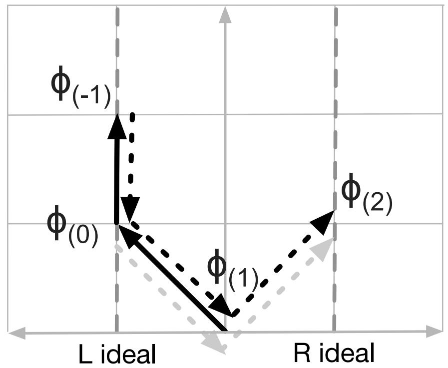

Figure 4: Example 2

At first glance, this result may seem surprising: by adding neutral rules, L is lowering his instantaneous payoffs. To understand L's strategy, remember that R is moderate, and thus will undo any and all rules that L had added. Thus L can delay R's rightward movement simply by adding more rules that R is compelled to undo. In other words, these ostensibly useless rules that L adds serve as a bulwark against the future advance of R.

Example 2 *(Figure 4) Suppose that $\hat{\rho}_L = -1$ and that $\hat{\rho}_R = 1$ (i.e., L's and R's ideals are each one step from the origin). Suppose that L is zealous and R is moderate. Suppose also that L is in control, and is at the simple L-ideal policy: $\phi_{(0)} = \langle -1 \rangle$. We ask whether L prefers to stagnate at $\phi_{(0)}$, or to add a neutral rule and attain $\phi_{(-1)} = \langle -1, 0 \rangle$. To do so, compare L's value functions from stagnating at $\phi_{(0)}$ versus stagnating at $\phi_{(-1)}$. R, being moderate, will undo any rules that L adds before adding his own. Applying (2), the corresponding value functions for L are:*

$$
\begin{aligned}
V(\phi_{(0)}, L) =& \omega_{(0)}\pi_L(\phi_{(0)}) + \omega_{(1)}\pi_L(\phi_{(1)}) + \omega_{(2)}\pi_L(\phi_{(2)}) \text{ where} \\
\omega_{(0)} =& \frac{1}{r+\lambda} + \frac{\lambda}{(r+\lambda)(r+q)}, \omega_{(1)} = \frac{\lambda q}{(r+\lambda)(r+p)(r+q)}, \omega_{(2)} = \frac{\lambda q p}{r(r+\lambda)(r+p)(r+q)}, \\
V(\phi_{(-1)}, L) =& w'_{(-1)}\pi_L(\phi_{(-1)}) + w'_{(0)}\pi_L(\phi_{(0)}) + w'_{(1)}\pi_L(\phi_{(1)}) + w'_{(2)}\pi_L(\phi_{(2)}) \text{ where} \\
\omega'_{(-1)} =& \frac{1}{r+\lambda} + \frac{\lambda}{(r+\lambda)(r+q)}, \\
\omega'_{(0)} =& \frac{\lambda q}{(r+\lambda)(r+q)^2}, \omega'_{(1)} = \frac{\lambda q^2}{(r+\lambda)(r+p)(r+q)^2}, \omega'_{(2)} = \frac{\lambda q^2 p}{r(r+\lambda)(r+p)(r+q)^2}.
\end{aligned}
$$

To rank the two continuation values, we combine the following observations.

1. $\pi_L(\phi_{(0)}) > \pi_L(\phi_{(-1)}) > \pi_L(\phi_{(1)}) > \pi_L(\phi_{(2)})$*: along the trajectory, the two L-ideal*

policies ($\phi_{(-1)}$ and $\phi_{(0)}$) are more profitable for L than the others (remember that L is zealous, thus prioritizes ideological position over policy complexity).

2. $\omega'_{(-1)} + \omega'_{(0)} > \omega_{(0)}$*: compared to* $V(\phi_{(0)}, L)$*,* $V(\phi_{(-1)}, L)$ *puts more weight on the L-ideal portion of the trajectory (*$\phi_{(-1)}$ *and* $\phi_{(0)}$*).*

3. *For* $i = 1, 2$*,* $\omega'_{(i)} \equiv \frac{q}{r+q}\omega_{(i)} < \omega_{(i)}$*: compared to* $V(\phi_{(0)}, L)$*,* $V(\phi_{(-1)}, L)$ *puts less weight on the trajectory's non-L-ideal policies (*$\omega'_{(1)}$ *and* $\omega'_{(2)}$*).*

To summarize: the advantage of stagnating at $\phi_{(-1)}$ *instead of* $\phi_{(0)}$ *is that doing so induces R (upon taking control) to undo along the L-ideal (*$\phi_{(-1)} \to \phi_{(0)}$*) before moving rightward (*$\phi_{(0)} \to \phi_{(1)}$ *and* $\phi_{(1)} \to \phi_{(2)}$*). This delays the transition from L-ideal policies (*$\phi_{(-1)}$ *and* $\phi_{(0)}$*) towards non-ideal policies, which is a profitable trade-off if L is zealous. Thus a zealous L prefers to add from* $\phi_{(0)}$ *to* $\phi_{(-1)}$ *rather than stagnate at* $\phi_{(0)}$*.*

Notice that intentional complexity produces excessively complex policies, but that such excessive complexity is transient: under the conditions of Proposition 5, the long-run policy is R-simple. This is because L only introduces intentional complexity if he anticipates that R will undo said complexity later.

4.2 Strategic Extremism

When facing a zealous player R, player L may engage in a different form of obstructionism that we term *strategic extremism*. Specifically, L may add leftward even after attaining his ideal policy, resulting in 'extremist' policies that lie left of L's ideal.

Proposition 6 *Fix all parameters except* $\lambda, r, \zeta_L, \zeta_R$*. Suppose the first player L is vulnerable (*$\lambda > \frac{p}{\hat{\rho}_R - \hat{\rho}_L - 1}$*). Then there exist* $\underline{r} > 0, \bar{r} > \underline{r}, \bar{\zeta}^S_L > 1, \bar{\zeta}^S_R > 1$ *such that if both players are zealous (*$\zeta_L > \bar{\zeta}^S_L$ *and* $\zeta_R > \bar{\zeta}^S_R$*) and players are intermediate in patience (*$\underline{r} < r < \bar{r}$*), then L will (starting from the origin) add left-sided rules until he attains his ideal, after which he adds at least one more left-sided rule.*

Proposition 6 tells us that when facing a zealous opponent, L may 'overshoot' by adding left-sided rules even after attaining his ideal. This differs from how L obstructs a moderate opponent in Proposition 5, where L adds neutral rules after attaining his ideal. To understand the difference, recall that when R is a moderate, L can slow his progress by adding neutral rules that R is compelled to undo before starting to move rightward. Such a strategy fails when R is a zealot, because R does not undo any of the rules (neutral or otherwise) added

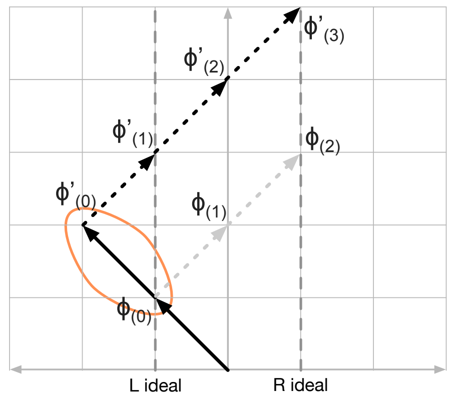

Figure 5: Example 3

by L; consequently, intentional complexity does nothing to impede R's progress. Instead, anticipating that R will add rightward, L can delay R by adding leftward, so that R's starting point (when he takes control) is further left. By doing so instead of stagnating at his ideal, L ensures that R has to travel further to reach the same ideological position. In other words, by engaging in strategic extremism, L profitably delays the reduction in his own payoffs that occurs as R moves rightward.

Example 3 *(Figure 5) Suppose that $\hat{\rho}_L = -1$ and $\hat{\rho}_R = 1$ (i.e., L's and R's ideals are each one step from the origin). Suppose that L is zealous and R is moderate. Suppose also that L is in control, and is at the L-simple, L-ideal policy: $\phi_{(0)} = \langle -1 \rangle$. We ask whether L prefers to stagnate at $\phi_{(0)}$, or to engage in strategic extremism, i.e. add a left-sided rule and attain $\phi'_{(0)} = \langle -1, -1 \rangle$. To do so, compare L's value functions from stagnating at $\phi_{(0)}$ versus stagnating at $\phi'_{(0)}$. R, being zealous, will simply add right-leaning rules until he attains his ideal. Applying (2), the corresponding value functions for L are:*

$$\begin{aligned} V(\phi_{(0)}, L) =& \omega_{(0)}\pi_L(\phi_{(0)}) + \omega_{(1)}\pi_L(\phi_{(1)}) + (\omega_{(2)} + \omega_{(3)})\pi_L(\phi_{(2)}), \\ V(\phi'_{(0)}, L) =& \omega'_{(0)}\pi_L(\phi'_{(0)}) + \omega'_{(1)}\pi_L(\phi'_{(1)}) + \omega'_{(2)}\pi_L(\phi'_{(2)}) + \omega'_{(3)}\pi_L(\phi'_{(3)}). \end{aligned}$$

To compare the two continuation values, combine the following observations.

1. *$\omega_{(k)} \equiv \omega'_{(k)}$: $V(\phi_{(0)}, L)$ and $V(\phi'_{(0)}, L)$ put the same weight on each step of their respective trajectories.*

2. *$\pi_L(\phi'_{(0)}) - \pi_L(\phi_{(0)}) = -\zeta_L - 1 < 0$: by stagnating at $\phi'_{(0)}$ instead of $\phi_{(0)}$, L starts off left of his ideal rather than at his ideal, and thus lowers his short-run payoff (on the first step of the trajectory).*

3. $\pi_L(\phi'_{(i)}) - \pi_L(\phi_{(i)}) = \zeta_L - 1 > 0$ *for* $i = 1, 2$*: by stagnating at* $\phi'_{(0)}$ *instead of* $\phi_{(0)}$*, L delays R's rightward movement along the trajectory, and thus increases his medium-run payoff (on the second and third steps of the trajectory).*

4. $\pi_L(\phi'_{(3)}) - \pi_L(\phi_{(2)}) = -2 < 0$*: by stagnating at* $\phi'_{(0)}$ *instead of* $\phi_{(0)}$*, L ends up at a more complex policy, and thus lowers his long-run payoff (on the last step of the trajectory).*

Notice that stagnating at $\phi'_{(0)}$ *instead of* $\phi_{(0)}$ *produces benefits for L only in the medium-run, and incurs costs at other times. Further, these medium-term benefits are relatively significant only if* ζ_L *is large. Finally, the short-run costs are large if* λ *is small (because L stays at* $\phi_{(0)}$ *for a long time). Thus, engaging in strategic extremism (i.e., moving from* $\phi_{(0)}$ *to* $\phi'_{(0)}$*) is profitable only if L is zealous and vulnerable, and is neither too patient nor too impatient.*

In contrast to intentional complexity (which produces transient effects), the complexity introduced by strategic extremism is persistent; this is illustrated in Example 3, where by engaging in strategic extremism, L increases long-run policy complexity. One way to view this point is that strategic extremism increases the potential for policy kludges: by overshooting his own ideal and thus moving policy away from the R-ideal, L induces R to implement more kludges (and thus add more complexity to policy) to reach the R-ideal.

5 Other Factors

So far, we have analyzed how players' ideological preferences affect equilibrium outcomes (and thus the extent and persistence of policy complexity). In this section, we consider two other factors. Section 5.1 considers the effect of political competition, and highlights the interaction with policymakers' preferences. Section 5.2 considers the effect of political frictions that slow down the policymaking process.

5.1 Political Competition and Ideological Heterogeneity

We analyze the effect of political competition on policy complexity by comparing the equilibrium outcomes of the one-player game versus the two-player game. We may interpret the number of players as a measure of of the degree of political competition: the one-player game corresponds to an politically uncompetitive setting (e.g. an autocracy) whereas the two-player game corresponds to a competitive setting (e.g. a democracy).

To highlight the role of political conflict in producing kludge, consider the outcome of the one-player game where L is in control for all time, starting from $t = 0$. The following Proposition is isomorphic to Lemma 3.

Proposition 7 *On the equilibrium path of the one-player game, starting from the origin, L adds left-sided rules until he attains his ideal, then stagnates. Consequently, every policy on the equilibrium trajectory is unkludged.*

Proposition 7 shows that kludge never arises in the absence of political competition, whereas Proposition 2 shows that kludge may arises from competition between zealous players. This comparison has two implications.

First, it suggests that an increase in political competition may come at the cost of increased long-run policy complexity.[11] Thus, a patient social planner who abhors policy complexity may prefer an autocratic political system over a democracy.

Second, even with political competition, persistent complexity can be avoided when policymakers are not too zealous. This suggests that in societies where political views are relatively homogenous, so that competing political parties have moderate policy preferences relative to the median voter, political competition may not have a severe impact on policy complexity. On the other hand, culturally heterogenous societies where relatively strong ideological preferences proliferate may be better off with autocratic institutions, so as to avoid the emergence of policy kludge.

5.2 Frictions

Now, consider the effect of frictions in policymaking that constrain the ability of policymakers to add or undo rules to policy. For example, in the U.S. political system, there are a multitude of veto points in the legislative process, and it is difficult for the party in power to successfully shepherd legislative proposals through these veto points; with the result that attempts to pass or undo legislation take longer to succeed. This corresponds to a high-friction political system.

[11]Note that an increase in policymaking frictions involves a similar tradeoff: increased friction results in slower movement towards extreme policies, but makes policymakers more inclined to implement kludged policies.

These frictions are often mooted as a positive feature of U.S. democracy: by making it difficult to change policy, shifts in policy position may be avoided, thus reducing the prevalence of extreme policy outcomes.

The following proposition points out that such frictions come at a cost: high-friction political systems may induce players to prioritize adding over undoing rules, and thus may result in the emergence and persistence of excessive complexity. To model frictions, let $p = \hat{p}/\chi$ and $q = \hat{q}/\chi$; so χ represents the degree of friction in the political system. Specifically, fixing $\hat{p}$ and $\hat{q}$, higher χ means that adding and undoing rules occurs (proportionally) more slowly.

Proposition 8 *Consider the two-player game, and fix all parameters except the degree of friction χ. Suppose $\zeta_R > \frac{\hat{p}+\hat{q}}{\hat{p}-\hat{q}}$.*

- *If frictions are low, then the long-run policy is simple. (Specifically, there exists $\underline{\chi}^Z > 0$ such that for $\chi < \underline{\chi}^Z$, on the equilibrium path, the first player L adds left-sided rules until he stops at the simple L-ideal policy; whereas the second player R undoes any rules that L added, then adds right-sided rules until he stops at the simple R-ideal policy.)*
- *If frictions are high, then the long-run policy is kludged with positive probability. (Specifically, there exists $\overline{\chi}^Z > 0$ such that for $\chi > \overline{\chi}^Z$, each player I adds rules in his favoured direction until his stops at his ideal.)*

On the other hand, if $\zeta_R < \frac{\hat{p}+\hat{q}}{\hat{p}-\hat{q}}$, then the long-run policy is simple for both high and low frictions. (Specifically, there exist $\underline{\chi}^M$ and $\overline{\chi}^M$ such that for $\chi < \underline{\chi}^M$ or $\chi > \overline{\chi}^M$, R undoes any rules that L added, and the long-run policy is the simple R-ideal policy.)

To discuss Proposition 8, start with the case where R is not too moderate ($\zeta_R > \frac{\hat{p}+\hat{q}}{\hat{p}-\hat{q}}$). When frictions are high, changes in policy are relatively slow in arriving. The policymaker anticipates that it is unlikely for many policy changes to occur within his relevant time horizon, and thus focuses on maximizing the payoff from the next policy change that he is attempting to make, i.e., "short-range" outcomes. Given that player R is not too moderate, each of them is better off adding rather than undoing (so as to get to a more favourable ideological position more quickly). Kludge thus arises with positive probability because the second player R will not attempt to undo any rules added by the first player.

On the other hand, when frictions are low, changes in policy arrive rapidly. Each policymaker anticipates that he is likely to achieve any sequence of policy changes he wishes to make in a relatively short period of time; thus, he focuses on "long-range" outcomes, chooses the trajectory that achieves the payoff-maximizing terminal policy. (He does so by undoing any existing rules, then adding in his favoured direction).

Now consider the the case where player R is relatively moderate ($\zeta_R < \frac{\hat{p}+\hat{q}}{\hat{p}-\hat{q}}$). In this case, for player R, both short-range and long-range concerns favour undoing over adding. Thus, for both high and low frictions, kludges are not produced.[12]

Note that in all cases (i.e., low and high frictions) covered by Proposition 8, player L does not engage in obstructionism: he adds rules until he attains his ideal, then stops there. When frictions are high, L seeks to maximize his immediate payoffs; thus he avoids obstructionism, which entails a short-run cost. When frictions are low, L avoids obstructionist actions because he anticipates that any such moves will not be effective at delaying R's (rapid) progress. The upshot is that obstructionism occurs only for intermediate levels of friction.

Although Proposition 8 emphasizes that kludge emerges and persists only in sufficiently high-friction systems, the impact of friction on the expected amount of long-run complexity, $E\left[\lim_{t\to\infty} \gamma(\phi_t)\right]$, is not strictly increasing. In fact, as the degree of friction χ goes to infinity, the probability that the long-run policy is kludged converges to zero. Remember that policy becomes kludged, and remains so in the long-run, only if each player successfully adds rules in his favoured direction. In a high friction environment, L is unlikely to successfully add (left-sided) rules before he loses control to R control; so policy is unlikely to become kludged. In other words, high frictions increase policymakers' *incentives* to add rather than undo rules (thus producing kludges), but also reduce their *ability* to add rules. The result of this tension is that the most kludges are produced for intermediate levels of frictions. That said, we view arbitrarily high frictions as being unrealistic in practice, because of the need for policy to be adapted readily to shocks in the political environment (e.g. economic, cultural or technological changes).[13] So a more careful interpretation of Proposition 8 is that, at least within an intermediate range, an increase in policymaking frictions may induce an increase in policy kludge.

[12] In fact, in the case $\zeta_R < \frac{\hat{p}+\hat{q}}{\hat{p}}$, we can produce a stronger result: the long-run policy is simple for all χ.

[13] To capture this point, a richer model could incorporate random shocks to either preferences or payoffs that require policy to adapt in response.

6 Conclusion

In this paper, we have worked out a simple model of incremental policymaking. The key assumption, backward-dependence, is technological: when undoing existing policy, newer rules have to be undone before older rules. The analysis focuses on the effect of political conflict between policymakers. Our key contribution is to highlight a number of interesting phenomena that arise from strategic interactions in this setting: how kludges emerge when zealots conflict, and distinct forms of obstructionary behavior. We also discuss the effect of institutional factors such as frictions and political competition. The analysis generates a taxonomy of the factors that tend to favor the emergence and persistence of policy complexity.

Throughout the paper, we have emphasized the application of our model to public policy. However, we believe that our model may also be relevant to other settings, such as the politics of organizational policy-making. The insights we derive in the model can be straightforwardly reinterpreted for an organizational context; for example, our results on long-run kludge suggest that political conflict between different factions within an organization may give rise to persistently inefficient bureaucratic routines and procedures within the organization.

A Appendix: Intermediate Preference Intensities

So far, we have highlighted the variety of phenomena that may arise in our model by focusing on the cases where each player is either very zealous or very moderate. The case where one or both players have intermediate preference intensities is subtle, and we are unable to provide a full characterization of the equilibrium outcome. We will discuss this case here with some numerical calculations, and offer some conjectures. In particular, we will argue that to the extent that equilibrium outcomes in the case of intermediate preference intensities differ from the cases of very zealous / very moderate policymakers that we have discussed so far, these outcomes correspond to unorthodox forms of intentional complexity.

To simplify the discussion, we restrict attention to the case where both ideals are exactly one step from the origin: $\hat{\rho}_L = -1, \hat{\rho}_R = 1$. We will present two example that illustrate, qualitatively, the equilibrium outcome as a function of ζ_L and ζ_R.

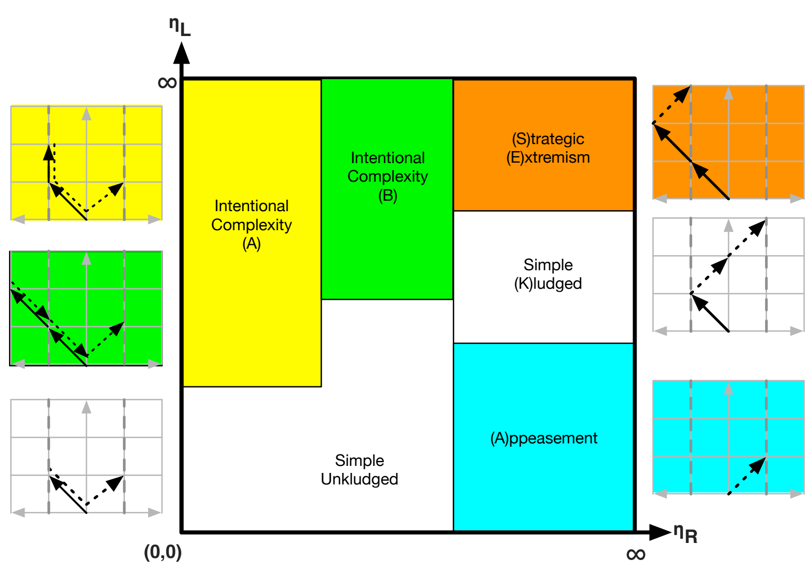

Figure 6: *Equilibrium outcomes with $\lambda = 100, p = 1, q = 0.5, r = 0.3$*

Example 4 *Choose $\lambda = 100, p = 1, q = 0.5, r = 0.3$. The equilibrium outcome, as a function of ζ_L, ζ_R, is illustrated in Figure 6. Notice that the equilibrium outcomes at*

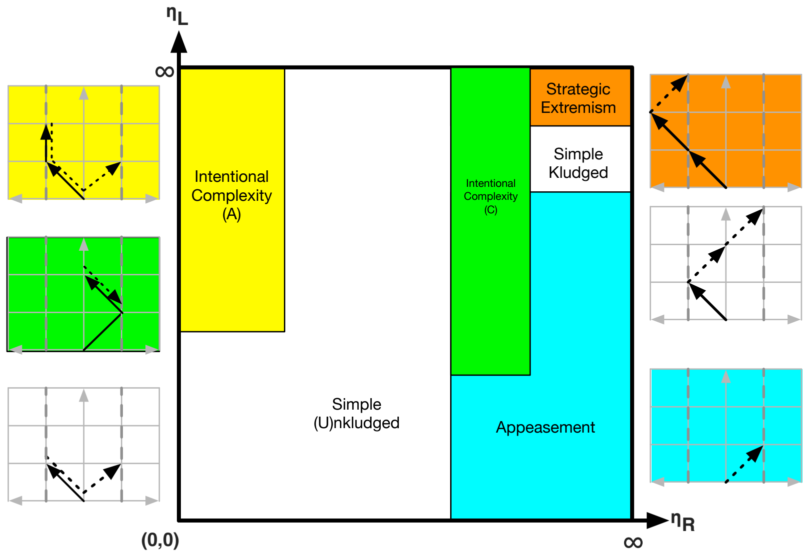

Figure 7: *Equilibrium outcomes with* $\lambda = 100000, p = 1000, q = 0.1, r = 0.3$

each corner of the diagram correspond to the outcomes we have discussed so far. In the top-right corner, where both L and R are zealous (i.e. ζ_L, ζ_R large), kludged policies are produced. Further, if ζ_L is very large, then L engages in strategic extremism. Away from the top-right corner, long-run policies are unkludged. In the bottom-right, where L is moderate and R is zealous, L engages in appeasement and does not add from the origin. In the top-left corner, where L is zealous and R is moderate, L engages in intentional complexity and adds neutral rules after attaining his ideal.

A heretofore undiscussed form of intentional complexity arises when L is zealous while R is somewhat, but not very, moderate. In this case, L adds left-sided rules towards and then beyond his ideal to $\langle -1, -1 \rangle$. However, unlike the case of strategic extremism, R undoes rather than adds from $\langle -1, -1 \rangle$ when he takes over. In fact, L is engaging in intentional complexity, but has to adjust his trajectory to ensure that R will behave 'appropriately' (i.e., undo rather than add). R, being insufficiently moderate, would be unwilling to undo $\langle -1 \rangle$, but willing to undo $\phi = \langle -1, -1 \rangle$.

Example 5 *Figure 7 highlights yet another form that intentional complexity takes. Here, we choose $\lambda = 100000, p = 1000, q = 0.1, r = 0.3$. When L is zealous and R is somewhat, but not very, moderate, L adds* rightward *before subsequently moving leftward, stopping at $\phi = \langle 1, -1 \rangle$. The reason for such behavior is subtle: it relies on the fact that, ceteris paribus, R is more willing to undo from a policy that is closer to his ideal. In a situation such as this where adding rules occurs rapidly and undoing rules* *occurs slowly (large p and small q), L has a strong incentive to induce R to undo, so as to slow R's advance; he does so by initially adding rightward towards R's ideal, so that undoing becomes sufficiently attractive for R to undertake.*

Figures 6 and 7 highlight the point that for intermediate preference intensities, unusual forms of intentional complexity may emerge. These examples are not exhaustive, and other forms of intentional complexity emerge for different parameter values. However, there is a common underlying logic: when R is neither very moderate nor very zealous, he is willing to undo some, but not all, policies. Consequently, L has to 'tailor' his trajectory to suit R's preferences, to ensure that R is willing to undo along that trajectory.[14]

One final observation: in Figures 6 and 7, the occurrence of long-run excessive complexity is monotone in both players' zealousness: kludge emerges (and persists) if *and only if* both ζ_L and ζ_R are large. We conjecture (but have been unable to prove) that this result holds more generally, for $\hat{\phi}_L \leq -1$ and $\hat{\phi}_R \geq -1$. This conjecture, if true, would strengthen the results of Propositions 2 and 3.

B Appendix: Proofs

Proof of Lemma 1 Lippman (1976), Theorem 7; notice that we apply the more general assumptions from Lippman (1975). These more general assumptions do not affect the argument. ■

Proof of Lemma 2 First, we will show that in any equilibrium, R's strategy is acyclic. We start by introducing a few notations. Let $V_R\left(\phi\right)$ be a continuation payoff of player R when the current policy is ϕ.

Now suppose that R's strategy is cyclic by way of contradiction. Then, without loss of generality, we can assume that there exist t and $t' > t$, such that $\phi_t = \phi_{t'}$, and a policy ϕ_τ adjacent to ϕ_t and $\phi_{t'}$ for all $\tau \in (t, t')$. It must be the case that R would receive the exact same continuation values at ϕ_t, ϕ_τ, and $\phi_{t'}$, that is, $V_R\left(\phi_t\right) = V_R\left(\phi_\tau\right) = V_R\left(\phi_{t'}\right)$. Moreover,

$$V_R\left(\phi_t\right) = \frac{\pi_R\left(\phi_t\right) + w_1 V_R\left(\phi_\tau\right)}{r + w_1} \text{ and } V_R\left(\phi_\tau\right) = \frac{\pi_R\left(\phi_\tau\right) + w_{-1} V_R\left(\phi_{t'}\right)}{r + w_{-1}},$$

[14] That said, for such forms of intentional complexity to be optimal, L must have precise knowledge of R's preferences (so that L can tailor his optimal trajectory precisely to R's preferences). In our view, this limits the applicability of these examples to reality, where policymakers likely have imperfect knowledge of their opponent's preferences.

where

$$w_1 = \begin{cases} p & \text{if } \phi_\tau \text{ is an extension of } \phi_t \\ q & \text{if } \phi_\tau \text{ is a truncation of } \phi_t \end{cases} \quad \text{and } w_{-1} = \begin{cases} p & \text{if } w_1 = q \\ q & \text{if } w_1 = p \end{cases}.$$

Rearranging, we obtain $rV_R\left(\phi_t\right) = \pi_R\left(\phi_t\right) = \pi_R\left(\phi_\tau\right)$. However, since $\eta_R > 1$, adjacent policies produce different instantaneous payoffs. That is, $\pi_R(\phi_t) \neq \pi_R(\phi_\tau)$. Therefore, we have $V_R\left(\phi_t\right) \neq V_R\left(\phi_\tau\right)$, which is a contradiction.

Next, let σ be L's equilibrium strategy that generates a cyclic graph, and suppose that ϕ and ϕ' are policies that are part of cycle. Since L is indifferent between ϕ and ϕ', the continuation payoffs at ϕ and ϕ' under strategy σ are identical. Then, L would receive the exact same continuation payoff at ϕ' under the strategy where L stagnates once the graph reaches ϕ'. Thus, we have the pure strategy equilibrium where both players graphs are acyclic.■

Proof of Proposition 1 To facilitate the proof, define $\pi_R^\rho\left(\phi\right) \equiv -\left|\hat{\rho}_R - \rho\left(\phi\right)\right|$ and $\pi_R^\gamma\left(\phi\right) \equiv -\gamma\left(\phi\right)$ so that $\pi_R\left(\phi\right) = \zeta_R \pi_R^\rho\left(\phi\right) + \pi_R^\gamma\left(\phi\right)$. We start with the case where the existing policy ϕ is neither R-ideal nor R-simple. Let σ_k be the strategy of undoing k rules from ϕ and then adding R-favoured rules until R's ideal is reached, and $\Phi^k = (\phi^k_{(0)}, \phi^k_{(1)}, ..., \phi^k_{(n_k)})$ be the associated trajectory of $n_k + 1$ steps, where $\phi^k_{(0)} = \phi$. Lemma 3 ensures that we can restrict attention to strategies σ_k for $k \in \left\{0, 1, \cdots, \gamma\left(\phi\right)\right\}$ such that $\pi_R^\rho\left(\phi^k_{(m)}\right) = \pi_R^\rho\left(\phi^k_{(m')}\right)$ for $m' > m$ implies $\pi_R^\gamma\left(\phi^k_{(m)}\right) > \pi_R^\gamma\left(\phi^k_{(m')}\right)$.

With a slight abuse of notation, let $V_R\left(\tilde{\phi}, ; \Phi^k\right)$ be R's continuation payoff of taking strategy σ_k at an on-trajectory policy $\tilde{\phi}$. Define $\omega^k_{(i)} = \frac{\Pi_{j=m}^{i-1} w^k_{(j)}}{\Pi_{j=m}^{i}\left(r + w^k_{(j)}\right)}$, where $w^k_{(m)} = p$ if $\phi^k_{(m+1)}$ is added from $\phi^k_{(m)}$, $w^k_{(m)} = q$ if $\phi^k_{(m+1)}$ is undone from $\phi^k_{(m)}$, and $w^k_{(m)} = 0$ if $m = n_k$.Then $V_R\left(\phi^k_{(m)}; \Phi^k\right) = \sum_{i=m}^{n_k} \omega^k_{(i)} \pi_R\left(\phi^k_{(i)}\right)$, which further can be decomposed into $\zeta_R V^\rho\left(\phi; \Phi^k\right) + V^\gamma\left(\phi; \Phi^k\right)$, where $V_R^\rho\left(\phi^k_{(m)}; \Phi^k\right) = \sum_{i=m}^{n_k} \omega^k_{(i)} \pi_R^\rho\left(\phi^k_{(i)}\right)$ and $V_R^\gamma\left(\phi^k_{(m)}; \Phi^k\right) = \sum_{i=m}^{n_k} \omega^k_{(i)} \pi_R^\gamma\left(\phi^k_{(i)}\right)$. This proves that $V_R\left(\phi; \Phi^k\right)$ is linear in ζ_R.

Therefore, we are done if we show that, for all k, (a) $V_R^\rho\left(\phi; \Phi^k\right) + V_R^\gamma\left(\phi; \Phi^k\right) > V_R^\rho\left(\phi; \Phi^0\right) + V_R\left(\phi; \Phi^k\right)$ and (b) $V^\rho\left(\phi; \Phi^0\right) > V^\rho\left(\phi; \Phi^k\right)$. This is because (a), and (b) together imply that for any k, there exists $\underline{\zeta}_R^k(\phi) \in (1, \infty)$ such that $V_R\left(\phi; \Phi^0\right) > V_R\left(\phi; \Phi^k\right)$ if and only if $\zeta_R > \underline{\zeta}_R^k(\phi)$. Thus, if we define $\underline{\zeta}_R(\phi) \equiv \max_{k \in \{1, \cdots, \gamma(\phi)\}} \left\{\underline{\zeta}_R^k(\phi)\right\} > 1$, then we can conclude that R will optimally play σ_0 (i.e. add at ϕ) if and only if $\zeta_R > \underline{\zeta}_R(\phi)$; otherwise he will prefer some σ_k, and undo at ϕ instead.

Now, we prove (a). To see this, note that along the trajectory corresponding to σ_0, we have $\pi_R^\rho\left(\phi_{(m)}^0\right)+\pi_R^\gamma\left(\phi_{(m)}^0\right)=\pi_R\left(\phi\right)$, for all $m\in\{0,1,\cdots,n_0\}$. Therefore, $V_R^\rho\left(\phi;\Phi^0\right)+V_R\left(\phi;\Phi^k\right)=\pi_R\left(\phi\right)/r$. Similarly, along the trajectory corresponding to σ_k, $\pi_R^\rho\left(\phi_{(m)}^k\right)+\pi_R^\gamma\left(\phi_{(m)}^k\right)\geq\pi_R\left(\phi\right)$. Moreover, since ϕ is not R-simple, there exists an $\bar{m}$ such that $\pi_R^\rho\left(\phi_{(m)}^k\right)+\pi_R^\gamma\left(\phi_{(m)}^k\right)\geq\pi_R^\rho\left(\phi\right)+\pi_R^\gamma\left(\phi\right)+1$ for all $m\geq\bar{m}$. Therefore, $V_R^\rho\left(\phi;\Phi^k\right)+V_R^\gamma\left(\phi;\Phi^k\right)\geq\frac{\pi_R(\phi)}{r}+\sum_{i=\bar{m}}^{n_k}\omega_{(i)}^k$.

Lastly, we prove (b). We prove that $V_R^\rho\left(\phi_{(m)}^0;\Phi^0\right)>V_R^\rho\left(\phi_{(m)}^k;\Phi^k\right)$ for all $m\in\{0,1,\cdots,n_0\}$ by induction. For $m=n_0$, note that $\pi_R^\rho\left(\phi_{n_0}^0\right)=\pi_R^\rho\left(\phi_{n_k}^k\right)$, and $\pi_R^\rho\left(\phi_{n_k}^k\right)>\pi_R^\rho\left(\phi_{m'}^k\right)$ for all $m'\in\{n_0,n_1,\cdots,n_{k-1}\}$. Therefore, $V_R^\rho\left(\phi_{(n_0)}^0\right)=V_R^\rho\left(\phi_{(n_k)}^k\right)>V_R^\rho\left(\phi_{(n_0)}^k\right)$. Next, suppose that $V_R^\rho\left(\phi_{(m)}^0\right)>V_R^\rho\left(\phi_{(m)}^k\right)$ for all $m\in\{\bar{m}+1,\bar{m}+2,\cdots,n_0\}$ for some $\bar{m}\in\{0,1,\cdots,n_0-1\}$. Note that $V_R^\rho\left(\phi_{(\bar{m})}^0;\Phi^0\right)=\frac{r}{r+w_{(\bar{m})}^0}\frac{\pi_R\left(\phi_{(\bar{m})}^0\right)}{r}+\frac{w_{(\bar{m})}^0}{r+w_{(m)}^0}V^\rho\left(\phi_{(\bar{m}+1)}^0;\Phi^0\right)$. Rearranging, we obtain

$$
\begin{aligned}
V_R^\rho\left(\phi_{(\bar{m})}^0;\Phi^0\right)=&\frac{r}{r+w_{(\bar{m})}^k}\frac{\pi_R\left(\phi_{(\bar{m})}^0\right)}{r}+\frac{w_{(\bar{m})}^k}{r+w_{(m)}^k}V_R^\rho\left(\phi_{(\bar{m}+1)}^0;\Phi^0\right)\\
&+\left(\frac{r}{r+w_{(\bar{m})}^k}-\frac{r}{r+w_{(\bar{m})}^0}\right)\left(V^\rho\left(\phi_{(\bar{m}+1)}^0;\Phi^0\right)-\frac{\pi_R\left(\phi_{(\bar{m})}^0\right)}{r}\right)
\end{aligned}
$$

Since $w_{(m)}^0\equiv p\geq w_{(m)}^k$ for all $m\in\{0,1,\cdots,n_0\}$, we have $\frac{r}{r+w_{(\bar{m})}^k}\geq\frac{r}{r+w_{(\bar{m})}^0}$. Also, note that $V_R^\rho\left(\phi_{(\bar{m}+1)}^0;\Phi^0\right)>\pi_R\left(\phi_{(\bar{m})}^0\right)/r$ because $\pi_R^\rho\left(\phi_{(m)}^0\right)$ is strictly increasing in $m\in\{0,1,\cdots,n_0\}$, and $V_R^\rho\left(\phi_{(n_0)}^0\right)=\pi_R^\rho\left(\phi_{(n_0)}^0\right)/n$. That is, $V_R^\rho\left(\phi_{(\bar{m})}^0;\Phi^0\right)\geq\frac{r}{r+w_{(\bar{m})}^k}\frac{\pi_R\left(\phi_{(\bar{m})}^0\right)}{r}+\frac{w_{(\bar{m})}^k}{r+w_{(m)}^k}V^\rho\left(\phi_{(\bar{m}+1)}^0;\Phi^0\right)$. Then, $\pi_R\left(\phi_{(m)}^0\right)\geq\pi_R\left(\phi_{(m)}^k\right)$ and the induction hypothesis that $V^\rho\left(\phi_{(\bar{m}+1)}^0;\Phi^0\right)>V^\rho\left(\phi_{(\bar{m}+1)}^k;\Phi^k\right)$ together imply

$$
\begin{aligned}
V^\rho\left(\phi_{(\bar{m})}^0;\Phi^0\right)>&\frac{r}{r+w_{(\bar{m})}^k}\frac{\pi_R\left(\phi_{(\bar{m})}^k\right)}{r}+\frac{w_{(m)}^k}{r+w_{(m)}^k}V^\rho\left(\phi_{(\bar{m}+1)}^k;\Phi^k\right)\\
=&V^\rho\left(\phi_{(\bar{m}+1)}^k;\Phi^k\right)>V^\rho\left(\phi_{(\bar{m})}^k;\Phi^k\right)
\end{aligned}
$$

The case where ϕ is R-ideal but not R-simple is completely analogous. ■

Proof of Proposition 2 We start with the first part of Proposition 2. For notational simplicity, let $d_{R,\phi}$ be R's favoured direction at ϕ, that is, $d_{R,\phi}=1$ if

$\rho\left(\phi\right) < \hat{\rho}_R$ and $d_{R,\phi} = -1$ if $\rho\left(\phi\right) > \hat{\rho}_R$.

Lemma 4 *Fix a non R-ideal policy $\phi = \langle d_1, ..., d_k\rangle$ and define $n \equiv \left|\hat{\rho}_R - \rho\left(\phi\right)\right| > 0$. Then, R prefers to add in his favoured direction at ϕ rather than undo exactly once at ϕ and then to add in his favoured direction afterwords if and only if (i) $\zeta_R \geq \bar{\zeta}_R\left(n; -d_{R,\phi}\right) \equiv 1 + \frac{2q}{(p-q)\left(1-\left(\frac{p}{p+r}\right)^n\right)}$ if $d_k = -d_{R,\phi}$ and (ii) $\zeta_R \geq \bar{\zeta}_R(n;0) \equiv 1 + \frac{q}{p\left(1-\left(\frac{p}{p+r}\right)^n\right)}$ if $d_k = 0$. Consequently, R adds in his favoured direction at any policy ϕ that is not R-ideal if $\zeta_R > \bar{\zeta}_R \equiv 1 + \frac{2q}{(p-q)\left(\frac{r}{p+r}\right)}$.*

Proof. Let Φ_0 and Φ_1 be the trajectories associated with strategies whereby R adds in his favour direction at ϕ and exactly undo once at ϕ and then to add in his favoured direction afterwords, respectively.

We start with the case where $d_k = -d_{R,\phi}$. Then, $V_R\left(\phi, R; \Phi_0\right) = \sum_{i=0}^{n} \omega_{(i)}\left(-\zeta_R(n-i) - (\gamma(\phi)+i)\right)$ where $\omega_{(i)} = \frac{1}{r+p}\left(\frac{p}{r+p}\right)^i$ for $i = 1, ..., n-1$ and $\omega_{(n)} = \frac{1}{r}\left(\frac{p}{r+p}\right)^n$. Similarly, $V_R\left(\phi, R; \Phi_1\right) = \frac{1}{r+q}\left(-\zeta_R n - \gamma(\phi)\right) + \sum_{m=1}^{n} \tilde{\omega}_{(m)}\left(-\zeta_R(n-m) - (\gamma(\phi)+m-2)\right)$, where $\tilde{\omega}_{(i)} = \frac{qp^{i-1}}{(r+q)(r+p)^i}$ for $i = \{1, ..., n-1\}$, and $\tilde{\omega}_{(n)} = \frac{qp^{n-1}}{r(r+q)(r+p)^{n-1}}$. Thus,

$$V_R\left(\phi, R; \Phi_0\right) - V_R\left(\phi; R; \Phi_1\right) = \frac{-2q + (\zeta_R - 1)(p-q)\left(1 - \left(\frac{p}{p+r}\right)^n\right)}{r(q+r)}.$$

Therefore, R prefers to add rather than undo if $\zeta_R \geq \bar{\zeta}_R\left(n; -d_{R,\phi}\right)$.

The case $d_k = 0$ is very similar: we get $V\left(\phi, R; \Phi_0\right) - V\left(\phi; R; \Phi_1\right) = \frac{-q + (\zeta_R - 1)p\left(1-\left(\frac{p}{p+r}\right)^n\right)}{r(q+r)}$. Thus R prefers to add rather than undo if $\zeta_R \geq 1 + \frac{q}{p\left(1-\left(\frac{p}{p+r}\right)^n\right)}$.

To see the last part, note that $\bar{\zeta}_R\left(n; -d_{R,\phi}\right) > \bar{\zeta}_R(n;0)$ for any n; $\bar{\zeta}_R\left(n; -d_{R,\phi}\right)$ is strictly decreasing in n and $\bar{\zeta}_R\left(1; -d_{R,\phi}\right) = \bar{\zeta}_R \equiv 1 + \frac{2q}{(p-q)\left(\frac{r}{p+r}\right)}$. That is $\zeta_R > \bar{\zeta}_R$ implies $\zeta_R > \bar{\zeta}_R\left(n; -d_{R,\phi}\right)$ and $\zeta_R > \bar{\zeta}_R(n;0)$. ■

To prove the second part of Proposition 2, we prove the following lemma.

Lemma 5 *If $\zeta_R > \bar{\zeta}_R$, then every policy on L's trajectory, starting from the origin, adds a left-leaning rule to the previous policy; equivalently, every policy on L's trajectory consists solely of left-leaning rules.*

Proof. We start with a claim that we can restrict attention to a finite trajectory of L, $\Phi_L = \left(\phi_{(0)}, \phi_{(1)}, ..., \phi_{(n)}\right)$, $n < \tilde{k} \equiv \zeta_L(\hat{\rho}_L + \hat{\rho}_R) + \hat{\rho}_R$ starting from the origin. To see this, suppose that L's trajectory Φ_L is not finite. Then, note that by definition, $V_L\left(\phi_{(m)}, L; \Phi_L\right) \geq V_L\left(\phi_{(0)}, L; \Phi_L\right)$ for any $m \geq 1$; and $V_L\left(\phi_{(0)}, L; \Phi_L\right) \geq$

$V_L\left(\phi_{(0)}, L; \tilde{\Phi}_L\right)$ for any other trajectory $\tilde{\Phi}_L$. Consider a trivial trajectory $\tilde{\Phi}_L = (\langle\rangle)$, whereby L stagnates at the origin. Then, only R-simple policies are attained on the continuation path. Since $-\tilde{k}$ is L's instantaneous payoff at R-ideal policy, we have $V_L\left(\phi_{(0)}, L; \tilde{\Phi}_L\right) > -\tilde{k}/r$. However, L's instantaneous payoff from $\phi_{(m)}$, $m \geq \tilde{k}$ is bounded from above by $-\tilde{k}$. That is, for $m \geq \tilde{k}$, $\pi\left(\phi_{(m)}\right) < -\tilde{k}$. Moreover, for any policy ϕ that is on R's trajectory starting from $\phi_{(m)}$, we have $\pi\left(\phi\right) < -\tilde{k}$. This leads to a contradiction because $V_L\left(\phi_{(m)}, L; \Phi_L\right) < -\tilde{k}/r < V_L\left(\phi_{(0)}, L; \tilde{\Phi}_L\right)$.

Next, we show that L's finite trajectory consists Φ_L solely of left-leaning rules. Consider a sequence of rules $(d_{(1)}, d_{(2)}, ...d_{(n)})$ such that $\phi_{(k)} = \left\langle d_{(1)}, ..., d_{(k)}\right\rangle$ induced by L's trajectory. By way of contradiction, suppose that there exists at least one neutral or right-biased rule in the sequence. We consider two cases: (1) there exist m and $m' > m$ such that $\rho(\phi_{(m')}) = \rho(\phi_{(m)})$; and (2) there is no m such that $d_{(1)} = -1$.

We will start with Case (1). By definition, $\phi_{(m')}$is more complex than $\phi_{(m)}$, that is, $\hat{\gamma} \equiv \gamma(\phi_{(m)}) - \gamma(\phi_{(m')}) > 0$. We show that Lstrictly prefers the trajectory Φ'_L defined by a sequence of rules that is identical to the equilibrium rule sequence for the first $m$rules and the last $n - m'$rules, but omits all rules in between: $\left\langle d_{(1)}, d_{(2)}, \cdots, d_{(m)}, d_{(m'+1)}, \cdots, d_{(n)}\right\rangle$. That is, define

$$d'_{(k)} \equiv \begin{cases} d_{(k)} & \text{if } k = 1, \cdots, m \\ d_{(k+(m'-m))} & \text{if } k = m+1, \cdots, n-(m'-m) \end{cases},$$

and $\phi'_{(k)} \equiv \left\langle d'_{(1)}, ..., d'_{(k)}\right\rangle$. Then, $\Phi'_L = \left(\phi'_{(0)}, \phi'_{(1)}, ..., \phi'_{(n-(m'-m))}\right)$.

We now claim that $V_L\left(\phi'_{(k)}, L, \Phi'_L\right) > V_L\left(\phi_{(k+(m'-m))}, L, \Phi_L\right)$for all $k \geq m+1$. Now, recall that $\rho(\phi'_{(k)}) = \rho(\phi_{(k+(m'-m))})$and $\gamma(\phi'_{(k)}) - \gamma(\phi_{(k+(m'-m))}) = -\hat{\gamma} < 0$. i.e., $\pi_L\left(\phi'_{(k)}\right) = \pi_L\left(\phi_{(k+(m'-m))}\right) + \hat{\gamma}$for all $k \geq m+1$. Furthermore, observe that Lemma 4 implies R's trajectories starting at $\phi'_{(k)}$and $\phi_{(k+(m'-m))}$induce the same sequence of rules $\langle d^k_{(1)}, d^k_{(2)}, \cdots, d^k_{(n)}\rangle$ for any $k \geq m+1$. Therefore, for $k \geq m+1$, $V_L\left(\phi'_{(k)}, R, \Phi_R\left(\phi'_{(k)}\right)\right) = V_L\left(\phi_{(k+(m'-m))}, R, \Phi_R\left(\phi_{(k+(m'-m))}\right)\right) + \hat{\gamma}/r$.

Next, we show that $V_L\left(\phi'_{(k)}, L, \Phi'\right) > V_L\left(\phi_{(k)}, L, \Phi\right)$for all for all $k = 0, \cdots, m$. Observe that for all $k = 0, \cdots, m$, $\pi_L\left(\phi_{(k)}\right) = \pi_L\left(\phi'_{(k)}\right)$and $V_L\left(\phi_{(k)}, R, \Phi_R\left(\phi_{(k)}\right)\right) = V_L\left(\phi'_{(k)}, R, \Phi_R\left(\phi'_{(k)}\right)\right)$. We thus have

$$V_L\left(\phi'_{(k)}, L, \Phi'_L\right) - V_L\left(\phi_{(k)}, L, \Phi_L\right) = \frac{p\left(V_L\left(\phi'_{(k+1)}, L, \Phi'_L\right) - V_L\left(\phi_{(k+1)}, L, \Phi_L\right)\right)}{r+p+\lambda}.$$

Furthermore, since $V_L\left(\phi_{(k)}, L, \Phi_L\right)$is weakly increasing in k,

$$V_L\left(\phi'_{(m+1)}, L, \Phi'_L\right) - V_L\left(\phi_{(m+1)}, L, \Phi_L\right) \geq V_L\left(\phi'_{(m+1)}, L, \Phi'_L\right) - V_L\left(\phi_{(m'+1)}, L, \Phi_L\right)$$
$$> 0.$$

We thus have $V_L\left(\phi'_{(k)}, L, \Phi'\right) > V_L\left(\phi_{(k)}, L, \Phi\right)$ for all for all $k = 0, \cdots, m$.

Now we will turn to Case (2). Then $\pi_L(\phi_{(n)}) < \pi_L(\phi_{(n-1)})$ and $V_L(\phi_{(n)}, R; \Phi_R\left(\phi_{(n)}\right)) < V_L(\phi_{(n-1)}, R; \Phi_R\left(\phi_{(n-1)}\right))$. Therefore,

$$\begin{aligned}
V_L(\phi_{(n-1)}, L; \Phi_L) &= \frac{\pi_L(\phi_{(n-1)}) + pV_L\left(\phi_{(n)}, L; \Phi_L\right) + \lambda V_L\left(\phi_{(n-1)}, R; \Phi_R\left(\phi_{(n-1)}\right)\right)}{r + \lambda + p} \\
&= \frac{\pi_L(\phi_{(n-1)}) + p\left(\frac{\pi_L(\phi_{(n)})}{r+\lambda} + \frac{\lambda V_L(\phi_{(n)}, R; \Phi_R(\phi_{(n)}))}{r+\lambda}\right) + \lambda V_L\left(\phi_{(n-1)}, R; \Phi_R\left(\phi_{(n-1)}\right)\right)}{\lambda + p + r} \\
&> \frac{\pi_L(\phi_{(n)}) + p\left(\frac{\pi_L(\phi_{(n)})}{r+\lambda} + \frac{\lambda V_L(\phi_{(n)}, R; \Phi_R(\phi_{(n)}))}{r+\lambda}\right) + \lambda V_L(\phi_{(n)}, R; \Phi_R\left(\phi_{(n)}\right))}{\lambda + p + r} \\
&= \frac{\pi_L\left(\phi_{(n)}\right) + \lambda V_L(\phi_{(n)}, R; \Phi_R\left(\phi_{(n)}\right)}{r + \lambda} = V_L(\phi_{(n)}, L; \Phi_L),
\end{aligned}$$

This contradicts the fact that L's continuation value must be weakly increasing as he moves along his trajectory: $V_L(\phi_{(n)}, L; \Phi_L) \geq V_L(\phi_{(n-1)}, L; \Phi_L)$. Our claim thus holds in this case. ■

Lemma 6 *If $\zeta_R > \bar{\zeta}_R$, then L's trajectory either includes L-ideal policy, or solely consists of origin.*

Proof. Lemma 5 states that L either stagnates or adds leftward at the origin. Let Φ_L^n be L's trajectory associated with a strategy where L adds leftward n times at the origin. We are done if we show that L's continuation payoffs from Φ_L^n and Φ_L^{n-1} at $\phi_{(n-1)}$ satisfy $V_L(\phi_{(n-1)}, L; \Phi_L^n) > V_L(\phi_{(n-1)}, L; \Phi_L^{n-1})$ for any $n \leq -\hat{\rho}_L$. Note that

$$\begin{aligned}
V_L(\phi_{(n-1)}, L; \Phi_L^n) &= \frac{\pi_L(\phi_{(n-1)}) + pV_L\left(\phi_{(n)}, L; \Phi_L^n\right) + \lambda V_L\left(\phi_{(n-1)}, R; \Phi_R\left(\phi_{(n-1)}\right)\right)}{r + \lambda + p}, \\
V_L(\phi_{(n-1)}, L; \Phi_L^{n-1}) &= \frac{\pi_L(\phi_{(n-1)}) + \lambda V_L\left(\phi_{(n-1)}, R; \Phi_R\left(\phi_{(n-1)}\right)\right)}{r + \lambda}.
\end{aligned}$$

Therefore, using $V_L\left(\phi_{(n)}, L; \Phi_L^n\right) = \frac{\pi_L(\phi_{(n)}) + \lambda V_L(\phi_{(n)}, R; \Phi_R(\phi_{(n)}))}{r+\lambda}$, we have

$$
\begin{aligned}
&V_L(\phi_{(n-1)}, L; \Phi_L^n) - V_L(\phi_{(n-1)}, L; \Phi_L^{n-1}) \\
&= \frac{p}{(r+\lambda)(r+p+r)} \left\{ \left(\pi_L(\phi_{(n)}) - \pi_L(\phi_{(n-1)})\right) + \lambda \left(V_L(\phi_{(n)}, R; \Phi_R\left(\phi_{(n)}\right)\right) - V_L\left(\phi_{(n-1)}, R; \Phi_R\left(\phi_{(n-1)}\right)\right)\right) \right\} \\
&= \frac{p}{(r+\lambda)(r+p+r)} \left\{ (\zeta_L - 1) + \lambda \left(V_L(\phi_{(n)}, R; \Phi_R\left(\phi_{(n)}\right) - V_L\left(\phi_{(n-1)}, R; \Phi_R\left(\phi_{(n-1)}\right)\right)\right) \right\} \\
&> \frac{p}{(r+\lambda)(r+p+r)} (\zeta_L - 1).
\end{aligned}
$$

■

We now complete the proof of the second part of Proposition 2. In order to do so, first note that Lemma 6 implies that player L's trajectory (i) contains L's ideal policy if $V_L\left(\phi_{(0)}, L, \Phi_L^1\right) > V_L\left(\phi_{(0)}, L, \Phi_L^0\right)$, and

$$
\begin{aligned}
&V_L\left(\phi_{(0)}, L, \Phi_L^1\right) - V_L\left(\phi_{(0)}, L, \Phi_L^0\right) \\
&= \frac{p}{(r+\lambda)(r+p+\lambda)} \left\{ (\zeta_L - 1) + \lambda \Delta_1(\hat{\rho}_R) \right\},
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta_1(\hat{\rho}_R) &= V_L(\phi_{(1)}, R; \Phi_R\left(\phi_{(1)}\right) - V_L\left(\phi_{(0)}, R; \Phi_R\left(\phi_{(0)}\right)\right) \\
&= \frac{1}{r}\left(1 - \left(\frac{p}{p+r}\right)^{\hat{\rho}_R+1}\right)\zeta_L - \frac{1}{r}\left(1 + \left(\frac{p}{p+r}\right)^{\hat{\rho}_R+1}\right).
\end{aligned}
$$

Since $\Delta_1(\hat{\rho}_R)$ is increasing and linear in ζ_L,there exists a unique $\bar{\zeta}_L$ such that $\zeta_L - 1 + \lambda\Delta_1(\hat{\rho}_R) \geq 0$ if and only if $\zeta_L \geq \bar{\zeta}_L$. Since $\Delta_1(\hat{\rho}_R)$ is increasing in $\hat{\rho}_R$, $\bar{\zeta}_L(\hat{\rho}_R)$ is decreasing in $\hat{\rho}_R$. Therefore, we obtain $\bar{\zeta}_L(\hat{\rho}_R) \leq \bar{\zeta}_L(1) = 1 + \frac{2\lambda p^2}{r(r+p)^2 + \lambda\left((r+p)^2 - p^2\right)}$ ■

Proof of Proposition 3 Define $\phi^*_{(n)} = \left\langle d^*_1, \cdots, d^*_n \right\rangle$, where $d^*_k = -1$ if $k = 0, \cdots, -\hat{\rho}_L$, and $d^*_k = 0$ if $k \geq -\hat{\rho}_L + 1$. By Lemma 4, R will undo at any $\phi^*_{(n)}$ if $\zeta_R < 1 + \min\left\{ \frac{q}{p\left(1-\left(\frac{p}{p+r}\right)^{\hat{\rho}_R - \hat{\rho}_L}\right)}, \frac{2q}{(p-q)\left(1-\left(\frac{p}{p+r}\right)^{\hat{\rho}_R-\hat{\rho}_L}\right)} \right\} = \underline{\zeta}_R$. Below, we show that L's trajectory is $\Phi_L^{(m)} = \left(\phi^*_{(0)}, \cdots, \phi^*_{(m)}\right)$ for some $m \geq -\hat{\rho}_L$ and hence the equilibrium trajectory is unkludged. We prove this by showing that, for any m and arbitrary trajectory $\tilde{\Phi}_L^{(m)} = \left(\tilde{\phi}_{(0)}, \cdots, \tilde{\phi}_{(m)}\right)$,

$$
V_L\left(\phi^*_{(k)}, L; \Phi_L^{(m)}\right) \geq V_L\left(\tilde{\phi}_{(k)}, L; \tilde{\Phi}_L^{(m)}\right) \text{ for any } k \in \{0, \cdots, m\}. \tag{3}
$$

Note that $\pi_L\left(\phi^*_{(n)}\right) \geq \pi_L\left(\phi\right)$ for any ϕ such that $\gamma\left(\phi\right) \geq n$. Therefore, (3) follows when $V_L\left(\phi^*_{(k)}, R; \Phi_R\left(\phi^*_{(k)}\right)\right) \geq V_L\left(\tilde{\phi}_{(k)}, R; \Phi_R\left(\tilde{\phi}_{(k)}\right)\right)$ for all k.

Let $\Phi_R\left(\phi^*_{(k)}\right) = \left(\phi^R_{(0)}, \cdots, \phi^R_{(k+\hat{\rho}_L)}\right)$ be R's trajectory starting at $\phi^*_{(k)}$. Then for any k, $V_L\left(\phi^*_{(k)}, R; \Phi_R\left(\phi^*_{(k)}\right)\right) = \sum_{j=0}^{k+\hat{\rho}_R} \omega_{(j)}\pi_L\left(\phi^R_{(j)}\right)$, where $\omega_{(j)} = \frac{\Pi_{i=0}^{j-1} w_{(i)}}{\Pi_{i=0}^{j}\left(r+w_{(i)}\right)}$and

$$w_{(i)} = \begin{cases} q & \text{if } i = 0, \cdots, k \\ p & \text{if } i = k+1, \cdots, k+\hat{\rho}_R - 1 \\ 0 & \text{if } i = k+\hat{\rho}_R \end{cases}.$$

Similarly, let $\Phi_R\left(\tilde{\phi}_{(k)}\right) = \left(\tilde{\phi}^R_{(0)}, \cdots, \tilde{\phi}^R_{(l)}\right)$ be R's trajectory starting at $\tilde{\phi}_{(k)}$, where R undoes $\tilde{k}$ rules. Notice that $l \leq k + \hat{\rho}_R$. Therefore if we define $\tilde{\phi}^R_{(i)} = \tilde{\phi}^R_{(l)}$ for $j = l+1, \cdots, k+\hat{\rho}_R$, we have $V_L\left(\tilde{\phi}_{(k)}, R; \Phi_R\left(\tilde{\phi}_{(k)}\right)\right) = \sum_{j=0}^{l} \tilde{\omega}_{(j)}\pi_L\left(\tilde{\phi}^R_{(j)}\right)$, where $\tilde{\omega}_{(j)} = \frac{\Pi_{i=0}^{j-1} \tilde{w}_{(i)}}{\Pi_{i=0}^{j}\left(r+\tilde{w}_{(i)}\right)}$ and

$$\tilde{w}_{(i)} = \begin{cases} q & \text{if } i = 0, \cdots, \tilde{k} \\ p & \text{if } i = \tilde{k}+1, \cdots, k+\hat{\rho}_R - 1 \\ 0 & \text{if } i = k+\hat{\rho}_R \end{cases}.$$

Note that $\pi_L\left(\tilde{\phi}^R_{(j)}\right)$ is decreasing in $j \geq \tilde{k}$, and $\omega_{(j)} \leq \tilde{\omega}_{(j)}$ for all j.

$$\begin{aligned} & V_L\left(\phi^*_{(k)}, R; \Phi_R\left(\phi^*_{(k)}\right)\right) - V_L\left(\tilde{\phi}_{(k)}, R; \Phi_R\left(\phi_{(k)}\right)\right) \\ & \geq \sum_{j=0}^{k+\hat{\rho}_R} \omega_{(j)}\left(\pi_L\left(\phi^R_{(j)}\right) - \pi_L\left(\tilde{\phi}^R_{(j)}\right)\right) > 0. \end{aligned}$$

This completes the proof of Proposition 3.■

Proof of Proposition 4 We use the same notation as in the proof of Proposition 2. First, observe that there exists an $\bar{n} \geq -\hat{\rho}_L$ such that for all $n \geq \bar{n}$, $V_L\left(\phi_{(n)}, L, \Phi_L^{(n)}\right) < V_L\left(\phi_{(-\hat{\rho}_L)}, L, \Phi_L^{(-\hat{\rho}_L)}\right)$. Moreover, for any $n < \bar{n}$,

$$\begin{aligned} & V_L\left(\phi_{(0)}, L, \Phi_L^n\right) - V_L\left(\phi_{(0)}, L, \Phi_L^0\right) \\ & = \begin{cases} \sum_{i=1}^{n} \omega_{(i)}\left(\lambda\Delta_i + (\zeta_L - 1)\right) & \text{if } n \leq -\hat{\rho}_L \\ \sum_{i=1}^{-\hat{\rho}_L} \omega_{(i)}\left(\lambda\Delta_i + (\zeta_L - 1)\right) + \sum_{i=-\hat{\rho}_L+1}^{n} \omega_{(i)}\left(\lambda\Delta_i - (\zeta_L - 1)\right) & \text{otherwise} \end{cases}, \end{aligned}$$

where $\omega_{(i)} = \frac{p^i}{(r+p+\lambda)^i(r+\lambda)}$, and

$$\Delta_i = \frac{1}{r}\left(\left(1-\left(\frac{p}{p+r}\right)^{\hat{\rho}_R+i}\right)\zeta_L - \left(1+\left(\frac{p}{p+r}\right)^{\hat{\rho}_R+i}\right)\right)$$

Since Δ_i is linear in ζ_L, so is $V_L\left(\phi_{(0)}, L, \Phi_L^i\right) - V_L\left(\phi_{(0)}, L, \Phi_L^0\right)$. Further, $\lim_{\zeta_L\to 1}\Delta_i = -2\left(\frac{p}{p+r}\right)^{\hat{\rho}_R+i} < 0$. Therefore, $\lim_{\zeta_L\to 1}\left(V_L\left(\phi_{(0)}, L, \Phi_L^n\right) - V_L\left(\phi_{(0)}, L, \Phi_L^0\right)\right) < 0$. Therefore, for each $n < \bar{n}$, there exists an $\underline{\zeta}_L^n > 1$ such that $\zeta_L < \underline{\zeta}_L^n$ implies $V_L\left(\phi_{(0)}, L, \Phi_L^n\right) < V_L\left(\phi_{(0)}, L, \Phi_L^0\right)$. Since Δ_n is increasing in i, if we define $\underline{\zeta}_L^A = \min_{i\in\{-\hat{\rho}_L,\cdots,\bar{n}-1\}}\left\{\underline{\zeta}_L^i\right\}$, then we have the required result. ■

Proof of Proposition 5 By the proof of Proposition 3, we know L's trajectory is $\Phi_L^{(k)} = \left(\phi^*_{(0)}, \cdots, \phi^*_{(k)}\right)$ for some $k \geq -\hat{\rho}_L$ where $\phi^*_{(n)} = \left\langle d_1^*, \cdots, d_n^*\right\rangle$; and $d_k^* = -1$ if $k = 0, \cdots, -\hat{\rho}_L$, and $d_k^* = 0$ if $k \geq -\hat{\rho}_L + 1$. Thus, we have the required result if $V_L\left(\phi^*_{(-\hat{\rho}_L+1)}, L; \Phi_L^{(-\hat{\rho}_L+1)}\right) > V_L\left(\phi^*_{(-\hat{\rho}_L)}, L; \Phi_L^{(-\hat{\rho}_L)}\right)$.

Note that

$$\begin{aligned} & V_L\left(\phi^*_{(-\hat{\rho}_L+1)}, L; \Phi_L^{(-\hat{\rho}_L+1)}\right) - V_L\left(\phi^*_{(-\hat{\rho}_L)}, L; \Phi_L^{(-\hat{\rho}_L)}\right) \\ &= \frac{\lambda\left(\hat{\rho}_L - rV_L\left(\phi_{(-\hat{\rho}_L)}; R; \Phi_R\left(\phi_{(-\hat{\rho}_L)}\right)\right)\right) - (r+q+\lambda)}{(r+q)(r+\lambda)}; \end{aligned}$$

and $V_L\left(\phi_{(-\hat{\rho}_L)}; R; \Phi_R\left(\phi_{(-\hat{\rho}_L)}\right)\right) = \sum_{j=0}^{-\hat{\rho}_L+\hat{\rho}_R}\omega_{(i)}\pi_L\left(\phi^R_{(j)}\right)$, where

$$\omega_{(i)} = \frac{\Pi_{i=0}^{i-1}w_{(j)}}{\Pi_{i=0}^{i}\left(r+w_{(j)}\right)}, w_{(i)} = \begin{cases} q & \text{if } i = 0, \cdots, -\hat{\rho}_L - 1 \\ p & \text{if } i = -\hat{\rho}_L, \cdots, -\hat{\rho}_L + \hat{\rho}_R \end{cases},$$

$$\pi_L\left(\phi^R_{(j)}\right) = \begin{cases} -\zeta_L i + \hat{\rho}_L - i & \text{if } i = 0, \cdots, -\hat{\rho}_L - 1 \\ -\zeta_L i - (i+\hat{\rho}_L) & \text{if } i = -\hat{\rho}_L, \cdots, -\hat{\rho}_L + \hat{\rho}_R - 1 \end{cases}.$$

Since $V_L\left(\phi_{(-\hat{\rho}_L)}; R; \Phi_R\left(\phi_{(-\hat{\rho}_L)}\right)\right)$ is linear and increasing in ζ_L, there exists a $\underline{\zeta}_L^I$ such that $\zeta_L > \underline{\zeta}_L^I$ if and only if $V_L\left(\phi^*_{(-\hat{\rho}_L+1)}, L; \Phi_L^{(-\hat{\rho}_L+1)}\right) > V_L\left(\phi^*_{(-\hat{\rho}_L)}, L; \Phi_L^{(-\hat{\rho}_L)}\right)$.
■

Proof of Proposition 6 Take an $\underline{r}$ such that $r < \bar{r}$ implies $\Psi(r) > 0$, where

$$\Psi(r) \equiv \lambda r \sum_{m=0}^{-\hat{\rho}_L+\hat{\rho}_R} \omega_{(i)} m - (p + r + \lambda),$$

where $\omega_{(i)} = \frac{1}{r+p}\left(\frac{p}{r+p}\right)^m$ for $i = 0, \cdots, -\hat{\rho}_L + \hat{\rho}_R - 1$ and $\omega_{(-\hat{\rho}_L+\hat{\rho}_R)} = \frac{1}{r}\left(\frac{p}{r+p}\right)^{-\hat{\rho}_L+\hat{\rho}_R}$.

There exists such an $\bar{r}$ because

$$\begin{aligned}\lim_{r\to 0} \Psi(r) &= \lambda(-\hat{\rho}_L + \hat{\rho}_R) - (p + \lambda) \\ &= \lambda(\hat{\rho}_R - \hat{\rho}_L - 1) - p > 0.\end{aligned}$$

Define $\bar{\zeta}_R(r) = 1 + \frac{2q}{(p-q)\left(\frac{r}{p+r}\right)}$ and $\bar{\zeta}_L(r) = 1 + \frac{2\lambda p^2}{r(r+p)^2+\lambda\left((r+p)^2-p^2\right)}$. Now take an arbitrary $\underline{r} < \bar{r}$. We show that there exists a $\zeta_L^I \geq \bar{\zeta}_L(\underline{r})$ such that if $\zeta_R > \bar{\zeta}_R(\underline{r})$ and $\zeta_L > \zeta_L^I$, then for any $r \in (\underline{r}, \bar{r})$, L will (starting from the origin) add left-sided rules until he attains his ideal, after which he adds at least one more left-sided rule.

First, notice that both $\bar{\zeta}_R(r)$ and $\bar{\zeta}_L(r)$ are decreasing in r. Therefore if $\zeta_R > \bar{\zeta}_R(\underline{r})$ and $\zeta_L \geq \bar{\zeta}_L(\underline{r})$, then by Lemma 4 and Lemma 6, for any $r \in (\underline{r}, \bar{r})$, L's trajectory takes the following form: $\Phi_L^{(k)} = \left(\phi_{(0)}, \phi_{(1)}, \cdots, \phi_{(k)}\right)$ for some $k \geq -\hat{\rho}_L$, where $\phi_{(i)} = \left(d_{(1)}, d_{(2)}, \cdots, d_{(i)}\right)$ and $d_{(j)} = -1$ for all j; and R adds in his favoured direction at any policy of L's trajectory.

To see that there exists a ζ_L^I such that $V_L\left(\phi_{(-\hat{\rho}_L+1)}; L; \Phi_L^{(-\hat{\rho}_L+1)}\right) > V_L\left(\phi_{(-\hat{\rho}_L+1)}; L; \Phi_L^{(-\hat{\rho}_L+1)}\right)$ for all ζ_L and $r \in (\underline{r}, \bar{r})$, note that

$$V_L\left(\phi_{(-\hat{\rho}_L)}; R; \Phi_R\left(\phi_{(-\hat{\rho}_L)}\right)\right) = -\sum_{m=0}^{-\hat{\rho}_L+\hat{\rho}_R} \omega_{(i)}(\zeta_L m - \hat{\rho}_L + m).$$

Therefore, $V_L\left(\phi_{(-\hat{\rho}_L+1)}; L; \Phi_L^{(-\hat{\rho}_L+1)}\right) - V_L\left(\phi_{(-\hat{\rho}_L+1)}; L; \Phi_L^{(-\hat{\rho}_L+1)}\right) = \frac{\Xi(r,\zeta_L)}{(p+r)(r+\lambda)}$, where $\Xi(r, \zeta_L) \equiv \lambda\left(\hat{\rho}_L + r\sum_{m=0}^{-\hat{\rho}_L+\hat{\rho}_R} \omega_{(i)}(\zeta_L m - \hat{\rho}_L + m)\right) - (p + r + \lambda)(\zeta_L + 1) - \frac{2\lambda p}{r}$. Since $\Xi(r, \zeta_L)$ is linear in ζ_L and $\frac{\partial \Xi(r,\zeta_L)}{\partial \zeta_L} = \Psi(r) > 0$, there exists a $\tilde{\zeta}_L(r)$ such that $V_L\left(\phi_{(-\hat{\rho}_L+1)}; L; \Phi_L^{(-\hat{\rho}_L+1)}\right) > V_L\left(\phi_{(-\hat{\rho}_L+1)}; L; \Phi_L^{(-\hat{\rho}_L+1)}\right)$ if and only if $\zeta_L > \tilde{\zeta}_L(r)$. Thus the required result follows if we define $\zeta_L^I \equiv \max\left\{\max_{r\in[\underline{r},\bar{r}]}\left\{\tilde{\zeta}_L(r)\right\}, \bar{\zeta}_L(\underline{r})\right\}$.

■

Proof of Proposition 8 We start with the case where $\zeta_R < \frac{\hat{p}+\hat{q}}{\hat{p}-\hat{q}}$ and show the existence of $\bar{\chi}^M$ with the required property. Let $V_L\left(\phi, L, \Phi\left(\phi\right)\right)$ be L's continuation payoff from the strategy in which L stagnates at policy ϕ. Also, define $\phi_L^* = \langle d_1, ..., d_{-\hat{\rho}_L}\rangle$ where $d_i = -1$ for all i.

Notice that for any ϕ, $\lim_{\chi\to\infty} V_L\left(\phi, L, \Phi\left(\phi\right)\right) = \pi_L\left(\phi\right)$. Further, L's equilibrium trajectory Φ_L has length no longer than $\bar{n} \equiv \zeta_L(\hat{\rho}_L + \hat{\rho}_R) + \hat{\rho}_R$. Therefore, there exists an $\bar{\chi}_1$ such that $\chi > \bar{\chi}_1$ implies $V_L\left(\phi_L^*, L, \Phi\left(\phi_L^*\right)\right) > V_L\left(\phi, L, \Phi\left(\phi\right)\right)$ for all ϕ such that $\gamma\left(\phi\right) \leq \bar{n}$.

Further, by Lemma 4, R undoes all rules in ϕ_L^* if $\zeta_R < \bar{\zeta}_R\left(\chi\right) = \frac{2q}{(p-q)\left(1-\left(\frac{p}{p+\chi r}\right)^{-\hat{\rho}_L+\hat{\rho}_R}\right)}$. Since $\bar{\zeta}_R\left(\chi\right)$ is decreasing in χ and $\lim_{\chi\to\infty}\bar{\zeta}_R\left(\chi\right) = \frac{\hat{p}+\hat{q}}{\hat{p}-\hat{q}}$, there exists a $\bar{\chi}_2$ such that $\chi > \bar{\chi}_2$ implies $\zeta_R < \bar{\zeta}_R\left(\chi\right)$. Thus, there will be no long-run kludge if $\chi > \bar{\chi}^M \equiv \max\{\bar{\chi}_1, \bar{\chi}_2\}$.

We now construct $\underline{\chi}^M$. Note that $\underline{\zeta}_R = 1 + \frac{q}{p\left(1-\left(\frac{p}{p+r}\right)^{\hat{\rho}_R-\hat{\rho}_L}\right)}$ is decreasing in χ. Moreover, $\lim_{\chi\to 0}\underline{\zeta}_R = \infty$. Therefore, there exists $\underline{\chi}^M$ such that $\zeta_R \leq \underline{\zeta}_R$ if and only if $\chi \leq \underline{\chi}^M$. Therefore, if $\chi < \underline{\chi}^M$, then there is no long-run kludge by the proof of Proposition 3.

The construction of $\underline{\chi}^Z$ is identical to $\underline{\chi}^M$. That is, if $\chi < \underline{\chi}^Z = \underline{\chi}^M$, then there is no long-run kludge.

Lastly, we show the existence of $\bar{\chi}^Z$. Fix $\zeta_R > \frac{\hat{p}+\hat{q}}{\hat{p}-\hat{q}}$ and ζ_L. Note that $\bar{\zeta}_R = 1 + \frac{2q}{(p-q)\left(\frac{r}{p+r}\right)}$ is decreasing in χ and $\lim_{\chi\to\infty}\bar{\zeta}_R = \frac{\hat{p}+\hat{q}}{\hat{p}-\hat{q}}$. Therefore, there exists an $\bar{\chi}_R$ such that $\zeta_R > \bar{\zeta}_R$ if and only if $\chi > \bar{\chi}_R$. Furthermore, note that $\bar{\zeta}_L = 1 + \frac{2\lambda p^2}{r(r+p)^2 + \lambda\left((r+p)^2 - p^2\right)}$ is decreasing in χ, and $\lim_{\chi\to\infty}\left(1 + \frac{2\lambda p^2}{r(r+p)^2+\lambda\left((r+p)^2-p^2\right)}\right) = 1$. Thus, there exists a $\bar{\chi}_L$ such that $\chi > \bar{\chi}_L$ implies $\zeta_L > \bar{\zeta}_L$. Therefore, if $\chi > \bar{\chi}^Z \equiv \max\{\bar{\chi}_L, \bar{\chi}_R\}$, then the long-run kludge occurs by Proposition 2.■

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