Costly Search with Adverse Selection: Solicitation Curse vs. Accelerating Blessing
(Preliminary and incomplete)

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Abstract

We study the effects of endogenizing search intensity in sequential search models of trading under adverse selection. Ceteris paribus, the low-type seller obtains more surplus from search and, therefore, searches more intensively than the high-type seller. This has two ramifications for trade. On the one hand, a seller who successfully finds a buyer is more likely to be the low type (solicitation curse). On the other hand, since the low-type seller leaves the market even faster than the high-type seller, a seller who is available is more likely to be the high type (accelerating blessing). We explore the interaction of these two effects in both stationary and non-stationary sequential search environments. In the stationary case, the two effects are balanced, while in the non-stationary case, the relative strengths of the two effects vary over time. We show that reducing search costs can be detrimental to the seller.

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1 Introduction

We study the effects of endogenizing search intensity in sequential search environments with adverse selection. A single seller with an indivisible good faces a sequence of randomly arriving buyers. There are two types of goods, high quality or low quality, but the quality of the good is private information to the seller. Upon arrival, each buyer offers a price to the seller, who then decides whether to accept or reject it. If the seller accepts a price, then the game ends. If not,

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the buyer leaves, and the seller waits for the next buyer. In this canonical environment, we allow the seller to choose her search intensity (i.e., the arrival rate of buyers) and examine its impact on trading outcome.

It is a standard exercise in search theory to endogenize search intensity and evaluate the effects of reducing search costs.[1] The exercise can be quite complicated depending on the context. In addition, it provides important policy implications, because there are various government and private policies that are intended to reduce the opportunity cost of search, thereby encouraging agents' search. For instance, many governments provide unemployed workers with various services, such as counseling, job fairs, and child care subsidies. Nevertheless, basic economic principles apply, and conclusions are fairly straightforward in most environments. The searching agent chooses the intensity level that equates the marginal benefit and the marginal cost of increasing search intensity. In addition, a reduction in search costs is always beneficial to the agent.[2]

Under adverse selection, endogenous search intensity gives rise to a new kind of inference problem on the part of buyers. Ceteris paribus, the low-type seller, due to her lower reservation value, gains more from search and, therefore, chooses a higher search intensity than the high-type seller. This difference in search intensity influences buyers' beliefs about the seller's type in two ways. On the one hand, a seller who successfully finds a buyer is more likely to be the low type. In other words, the fact that a buyer has met a seller is bad news about the quality of the seller's good. Since a buyer's meeting a seller can be interpreted as the seller's soliciting or inviting the buyer, following Lauermann and Wolinsky (2013a), we call this effect the "solicitation curse." On the other hand, since the low-type seller obtains more trading opportunities than the high-type seller for a given length of time, a seller who has not traded yet is more likely to be the high type. In other words, the fact that a unit is still available is good news about its quality. Note that even without endogenous search intensity, the low-type seller has a lower reservation price and, therefore, trades faster than the high-type seller. Endogenous search intensity makes the low type trade even faster than the high type. For this reason, we call this effect the "acceleration blessing." The goal of this paper is to understand how these two effects manifest as well as interact each other in various dynamic environments.

In Section 2, we consider an opaque trading environment where buyers do not receive any information about the seller's trading history. In this environment, all buyers necessarily have the same beliefs about the seller's type and, therefore, would play an identical offer strategy. From the seller's viewpoint, this means that the environment is stationary, as in canonical sequential search models. Since each seller type's optimal search intensity is also stationary, the aforementioned

[1]See Benhabib and Bull (1983) and Mortensen (1986) for some seminal contributions.

[2]This explains why most studies in the literature on endogenous search intensity are empirical work. See, for example, Bloemen (2005), Christensen et al. (2005), and Gautier et al. (2009).

two effects take a simple form. The acceleration blessing can be captured by the extent to which the difference in search intensity increases the probability that the seller who is still playing the game is the high type (which, for convenience, we refer to as buyers' unconditional beliefs). The solicitation curse can be measured by the difference between buyers' unconditional beliefs and conditional beliefs (i.e., the probability that a buyer assigns to the event that the seller is the high type, *conditional* on him actually facing the seller). We quantify these two effects and show that their magnitudes are the same in the stationary environment.

In Section 3, we consider a non-stationary version of the model. Specifically, we consider the case in which buyers observe the seller's time-on-the-market (how long the seller has stayed on the market). The observability assumption allows us to study the effects of endogenous search intensity on non-stationary trading dynamics, in particular, how the seller's optimal search intensity changes over time and how buyers' unconditional and conditional beliefs evolve over time. In this non-stationary model, the solicitation curse and the acceleration blessing take more complex and intriguing forms. The accelerating blessing brings buyers' (both unconditional and conditional) beliefs beyond the level that can be reached with exogenous search intensity. In fact, buyers' beliefs that the seller is the high type converge to 1 with endogenous search intensity, while they always stay below a certain level with exogenous search intensity. As in the stationary model, the solicitation curse brings down buyers' conditional beliefs relative to their unconditional beliefs. Unlike in the stationary model, its strength relative to the acceleration blessing changes over time. In particular, it outweighs the accelerating blessing for a certain length of time and, therefore, leads to the non-monotonicity of buyers' conditional beliefs. We show that unlike buyers' unconditional beliefs that monotonically increase over time, buyers' conditional beliefs first increase, then decrease and stay constant for a while, and finally increase and converge to 1.

Rather surprisingly, in our model the seller does not necessarily benefit from lower search costs. In particular, we show that reducing search costs may not affect or can even strictly decrease the low-type seller's expected payoff. This is because of the solicitation curse and buyers' strategic responses to lower search costs. A decrease in search costs increases the low-type seller's incentive to increase her search intensity, which exacerbates the solicitation curse. Therefore, buyers become more cautious about offering a high price, which negatively affects the seller. In our model, this strategic effect can dominate the direct benefit of lower search costs to the seller, and thus reducing search costs may decrease the seller's expected payoff.

This paper contributes to a growing literature on dynamic adverse selection.[3] To our best knowledge, none of previous work considers the problem of endogenizing search intensity. Our model is closest to those of Kim (2012), Kaya and Kim (2013), and Hwang (2013). Each of

[3] Seminal contributions include Evans (1989), Vincent (1989, 1990), Janssen and Roy (2002), Deneckere and Liang (2006), Hörner and Vieille (2009), Guerrieri, Shimer and Wright (2010), and Moreno and Wooders (2010).

those papers addresses a different economic question under the assumption of exogenous search intensity.

Lauermann and Wolinsky (2013a) consider an auction model in which the seller has private information about the quality of her good and chooses the number of participating bidders. The seller needs to incur a higher cost in order to solicit more bidders. They also identify a solicitation effect, which, as in this paper, stems from the fact that different seller types have different incentives to solicit more bidders. On the contrary, an acceleration effect is absent in their model, because it is a static environment. More importantly, their main economic question is substantially different from ours: they seek the condition on the signal generating process that guarantees information aggregation (meaning that the winning price coincides with the unit's value to buyers), while our main question is the impact of endogenous search intensity on trading outcome (dynamics).

The remainder of the paper is organized as follows. We study an opaque (therefore, stationary) search environment in Section 2 and a non-stationary version of the model in Section 3. In Section 4, we consider various extensions and explain how our insights go beyond the simple environment studied in Sections 2 and 3.

2 Stationary Model

2.1 Environment

We consider a canonical sequential search environment with adverse selection. A seller wishes to sell an indivisible object and sequentially meets buyers. Upon arrival each buyer offers a price, and the seller decides whether to accept the price or not. If an offer is accepted, then the seller and the buyer trade and the game ends. Otherwise, the buyer leaves, and the seller waits for the next buyer.

The good is either of high quality (H) or of low quality (L). For each $a = H, L$, a type-a unit yields utility c_a to the seller and utility v_a to buyers, and a high-quality unit is more valuable to both the seller and buyers (i.e., $c_L < c_H$ and $v_L < v_H$). There are always gains from trade (i.e., $c_a < v_a$ for each $a = L, H$), but the quality of the good is private information to the seller. It is common knowledge that buyers assign probability $\widehat{q}$ to the event that the seller begins the game with a high-quality unit. All agents are risk neutral. If a buyer's offer p is accepted by the type-a seller, then the buyer's utility is $v_a - p$, while that of the seller is $p - c_a$. The seller discounts future payoff at rate $r > 0$.

We focus on the case where adverse selection is severe enough to impede socially desirable trade. Formally, we make use of the following assumption, which is common in the adverse selection literature:

Assumption 1 *(Severe Adverse Selection)*

$$\widehat{q}v_H + (1-\widehat{q})v_L < c_H.$$

This assumption guarantees that it cannot be an equilibrium that the seller trades with probability 1 with the first buyer. In other words, some delay is unavoidable.

Unlike in other models, we allow the seller to increase her search intensity (i.e., the arrival rate of buyers) at a cost. Since signaling through the choice of search intensity is not the main interest of this paper, we assume throughout that the seller's choice of search intensity is not observable to buyers. The search technology is represented by a function $\phi : [\underline{\lambda}, \infty) \to [0, \infty)$ where $\phi(\lambda)$ denotes the flow search cost necessary for the seller to obtain search intensity λ. In other words, if the seller pays constant flow search cost $\phi(\lambda)$, then buyers arrive according to a Poisson process of rate λ. We impose standard restrictions on the cost function $\phi(\cdot)$: it is strictly increasing and strictly convex (i.e., $\phi'(\lambda), \phi''(\lambda) > 0$), $\phi(\underline{\lambda}) = 0$, and $\lim_{\lambda \to \underline{\lambda}} \phi'(\lambda) = 0$. To avoid triviality, we assume that $\underline{\lambda} > 0$. This can be understood as the baseline search intensity the seller obtains for free. The role of this assumption will be clear in the equilibrium analysis. It will also be shown that, while we require $\underline{\lambda} > 0$, $\underline{\lambda}$ can take any arbitrarily small value.

In this section, we consider an *opaque* search environment, in the sense that buyers do not observe any of the sellers' trading histories.[4] This implies that the problem is essentially *stationary*: All buyers necessarily have common beliefs about the seller's type, regardless of their location in the sequence of buyers. Therefore, it is natural to assume that all buyers would play an identical offer strategy. Given this, there also exists a stationary best response by the seller. In order to highlight the most salient aspects of the problem, we focus on the stationary equilibrium in which all buyers play an identical offer strategy and all sellers adopt an identical acceptance strategy.

2.2 Preliminary Observations

As is common in sequential search problems, each seller type's optimal acceptance strategy is a reservation price strategy: Each seller accepts a price if it is above her reservation price and rejects if it is below. Given this, it is straightforward that no buyer offers strictly above c_H and, therefore, the high-type seller's reservation price is always equal to c_H: The high-type seller's reservation price cannot be larger than the highest price offered by buyers. On the other hand, no buyer has an incentive to offer strictly more than the high-type seller's reservation price. These two properties hold only when no buyer offers strictly more than c_H, and the high-type seller's reservation price is equal to her reservation value c_H. From now on, we denote by p^* the reservation price of the

[4]It is well-known that the information structure regarding the sellers' trading histories plays a crucial role in this kind of dynamic games. See, in particular, Hörner and Vieille (2009) and Kim (2012).

low-type seller. The assumption $c_L < c_H$ ensures that $p^* < c_H$.

The seller's incentive to increase her search intensity comes from her desire to enjoy trade surplus as soon as possible. As explained above, in the current model no buyer offers a price strictly above c_H, and thus the high-type seller cannot obtain a strictly positive expected payoff. It then follows that in equilibrium she always chooses the lowest search intensity $\underline{\lambda}$.[5] On the other hand, the low-type seller may enjoy a positive expected payoff (i.e., $p^* > c_L$) and, therefore, choose a higher search intensity than $\underline{\lambda}$. We denote by λ^* the low-type seller's equilibrium choice of search intensity.

Without loss of generality, we assume that each buyer offers c_H, p^*, or a losing price, and each seller type accepts her reservation price with probability 1. It is clear that no buyer has an incentive to offer strictly more than c_H or between c_H and p^*. It is also straightforward that in equilibrium the high-type seller must accept c_H with probability 1: Otherwise, a buyer would have a strict incentive to offer a price above but arbitrarily close to c_H, which contradicts to the fact that c_H is an optimal price for a buyer. Finally, if there is an equilibrium in which the low-type seller accepts p^* only with probability $\sigma_S \in (0, 1)$, then that portion of the equilibrium can be replaced by a combination of the low-type seller's accepting p^* with probability 1 and buyers' offering p^* with probability σ_S. For each $a = H, L$, we denote by σ_a^* the probability that each buyer offers the reservation price of the a-type seller. Obviously, it must be that $\sigma_H^* + \sigma_L^* \leq 1$

2.3 Buyers' Beliefs

2.3.1 Buyers' Unconditional Beliefs and the Acceleration Blessing

In the stationary model, buyers face two types of uncertainty, one about the seller's type and the other about their position in the sequence of buyers ("contact uncertainty").[6] The combination of these two gives rise to a non-trivial inference problem on the part of buyers. In particular, buyers' beliefs about the seller's type may not coincide with $\widehat{q}$, the probability that the seller is the high type at the beginning of the game. This is because different seller types leave the game at different rates, and thus the probability of the high type changes over time. If buyers could observe the seller's trading history, then their beliefs would begin with $\widehat{q}$ and can be calculated through Bayes' rule for all subsequent points in time. However, in the current environment where buyers receive no information about the seller's trading history, contact uncertainty also must be taken into account in determining their beliefs. There are several, but all equivalent, ways to address this problem. We take probably the simplest approach and directly derive buyers' beliefs.

[5] This stems from our choice of bargaining protocol. In Section 4, we consider an alternative environment in which the high-type seller also obtains a strictly positive expected payoff and, therefore, chooses a search intensity above $\underline{\lambda}$.

[6] The term "contact uncertainty" is due to Zhu (2012).

Denote by q^u the probability that a seller who is still playing the game (i.e., has not traded yet) is the high type. In other words, q^u is the unconditional proportion of the high type among all sellers who play this game. To determine q^u, notice that a high-type seller leaves the game at rate $\underline{\lambda}\sigma_H^*$, while a low-type seller leaves at rate $\lambda^*(\sigma_H^* + \sigma_L^*)$. This is because each seller type accepts any price weakly above her reservation price with probability 1 and the high type (low type) meets buyers at rate $\underline{\lambda}$ (λ^*). Since the expected duration is the inverse of the hazard rate, this means that a high-type seller stays in the game on average for $1/(\underline{\lambda}\sigma_H^*)$ length of time, while a low-type seller stays for $1/(\lambda^*(\sigma_H^* + \sigma_L^*))$. Since the probability that the seller is the high type is equal to $\widehat{q}$ at the beginning of the game, it follows that

$$q^u = \frac{\frac{\widehat{q}}{\underline{\lambda}\sigma_H^*}}{\frac{\widehat{q}}{\underline{\lambda}\sigma_H^*} + \frac{1-\widehat{q}}{\lambda^*(\sigma_H^*+\sigma_L^*)}}. \tag{1}$$

Notice that q^u departs from $\widehat{q}$ for two reasons. The first is familiar in the adverse selection literature and holds true even when both types have the same exogenous search intensity λ. Namely, the high type accepts only c_H, while the low type accepts both p^* and c_H. Since the low type finishes the game faster than the high type, q^u is necessarily higher than $\widehat{q}$. The second effect is due to endogenous search intensity. As explained above, the low type has a stronger incentive to speed up trade, and therefore, chooses a higher search intensity ($\lambda^* \geq \underline{\lambda}$). This means that the low type leaves the game even faster, thereby increasing buyers' beliefs beyond the level induced only by the first effect. This difference in unconditional beliefs due to varied search intensities is what we refer to as the acceleration blessing.[7]

2.3.2 Buyers' Conditional Beliefs and the Solicitation Curse

Endogenous search intensity has one more implication for buyers' beliefs: Buyers have different beliefs about the seller's type, depending on whether they actually face the seller or not. Ceteris paribus, the low-type seller chooses a higher search intensity and, therefore, faces relatively more buyers than the high-type seller. This means that a buyer is more likely to face the low-type seller than the high-type seller, and thus his belief about a particular seller that he has met is necessarily lower than q^u, which is the probability that he (or an outside observer of the game) assigns to the event that the seller is the high type before facing this particular seller (thus, *unconditional*). To be formal, denote by q^* the probability that a buyer assigns to the event that the seller is the high type, *conditional* on the event that he actually met the seller. Given different seller types' choices of search intensity, $\underline{\lambda}$ by the high type and λ^* by the low type, the relationship between q^u and q^*

[7] Note that the arrival rate of buyers helps determine the offer rates, σ_H^* and σ_L^*, even in the exogenous case. To be precise, in the exogenous case, they are functions of common arrival rate λ, while when search is endogenized, they are functions of both $\underline{\lambda}$ and λ^*.

is given by

$$q^* = \frac{q^u \underline{\lambda}}{q^u \underline{\lambda} + (1 - q^u)\lambda^*}. \tag{2}$$

Clearly, q^* is strictly smaller than q^u as long as $\lambda^* > \underline{\lambda}$. This downward adjustment is precisely the manifestation of the solicitation curse.

Combining (1) and (2), it follows that

$$q^* = \frac{q^u \underline{\lambda}}{q^u \underline{\lambda} + (1 - q^u)\lambda^*} = \frac{\frac{\widehat{q}}{\sigma_H^*}}{\frac{\widehat{q}}{\sigma_H^*} + \frac{1-\widehat{q}}{\sigma_H^* + \sigma_L^*}}. \tag{3}$$

Notice that the search intensity parameters, $\underline{\lambda}$ and λ^*, do not appear in this expression. This does not mean that endogenous search intensity has no effect on the market outcome: As shown shortly, all equilibrium objects, including σ_H^* and σ_L^*, are affected by $\underline{\lambda}$ and λ^*. It only means that in our baseline model, the two effects of endogenous search intensity, the acceleration blessing and the solicitation curse, cancel each other out in terms of buyers' conditional beliefs.

2.4 Equilibrium Characterization

A (stationary) equilibrium of our baseline model can be described by a tuple $(p^*, \sigma_H^*, \sigma_L^*, \lambda^*, q^*)$ such that (i) given σ_H^*, p^* is the low-type seller's reservation price and λ^* is her optimal search intensity, (ii) given p^* and q^*, $\sigma_H^* > 0$ ($\sigma_L^* > 0$) only when c_H (p^*) is an optimal price for each buyer, and (iii) given σ_H^* and σ_L^*, q^* is buyers' conditional belief, as derived in (3).

Low-type seller's reservation price and search optimality. Given σ_H^*, the low-type seller's (continuous-time) Bellman equation is given by

$$r(p^* - c_L) = \max_{\lambda} -\phi(\lambda) + \lambda \sigma_H^*(c_H - p^*).$$

As usual, this gives two equilibrium conditions. First, the optimal λ^* must satisfy

$$\phi'(\lambda^*) = \sigma_H^*(c_H - p^*). \tag{4}$$

The strict convexity of $\phi(\cdot)$ ensures the uniqueness of the optimal solution λ^*. Second, λ^* must also satisfy

$$r(p^* - v_L) = -\phi(\lambda^*) + \lambda^* \sigma_H^*(c_H - p^*). \tag{5}$$

It is straightforward that both λ^* and p^* are strictly increasing in σ_H^*.

Buyers' equilibrium offer strategy. As usual, c_H is accepted by both types, while p^* is accepted only by the low type. Therefore, given p^* and q^*, c_H is an optimal price for a buyer to offer if and only if

$$q^*(v_H - c_H) + (1 - q^*)(v_L - c_H) \geq \max\{0, (1 - q^*)(v_L - p^*)\}.$$

The corresponding condition for p^* is

$$(1 - q^*)(v_L - p^*) \geq \max\{0, q^*(v_H - c_H) + (1 - q^*)(v_L - c_H)\}.$$

A useful observation is that in equilibrium buyers must offer both p^* and c_H with a positive probability, which implies that

$$q^*(v_H - c_H) + (1 - q^*)(v_L - c_H) = (1 - q^*)(v_L - p^*) \Leftrightarrow \frac{1 - q^*}{q^*} = \frac{v_H - c_H}{c_H - p^*}. \qquad (6)$$

If buyers never offer c_H (i.e., $\sigma_H^* = 0$) then, by (4) and (5), $p^* = c_L$, while (3) implies that $q^* = 1$. But then, from the above optimality condition, c_H becomes a unique optimal price to buyers, which is a contradiction. If buyers never offer p^* (i.e., $\sigma_L^* = 0$) then, by (3), $q^* = \widehat{q}$. Assumption 1 implies that $q^*(v_H - c_H) + (1 - q^*)(v_L - c_H) < 0$, and thus $\sigma_H^* = 0$ as well. But then, the same contradiction as for the previous case arises.

To fully characterize the equilibrium, we rely on the fact that $\sigma_L^* > 0$ implies $p^* \leq v_L$. This means there are two cases to consider: $p^* < v_L$ or $p^* = v_L$. If $p^* = v_L$, then (3), (4), (5), and (6) provide all the necessary conditions for all other equilibrium variables, σ_H^*, σ_L^*, λ^*, and q^*. If $p^* < v_L$, then no buyer has an incentive to offer a losing price, which implies $\sigma_H^* + \sigma_L^* = 1$. All 5 equilibrium variables can be found from this additional condition and the previous conditions.

The following proposition provides a full characterization of the unique equilibrium of the model. Closed-form solutions for all equilibrium variables are available but rather tedious. We report only the results that are necessary for further discussion, relegating the closed-form expressions as well as the uniqueness proof to the appendix.

Proposition 1 *There always exists a unique equilibrium. Let $\tilde{\lambda}$ be the unique value such that $r(v_L - c_L) = \tilde{\lambda}\phi'(\tilde{\lambda}) - \phi(\tilde{\lambda})$. If*

$$\left(\frac{1 - \widehat{q}}{\widehat{q}}\right) \frac{r(v_L - c_L) + \phi(\tilde{\lambda})}{\tilde{\lambda}(v_H - c_H)} \leq 1, \qquad (7)$$

then $p^* = v_L$, $\lambda^* = \tilde{\lambda}$, and $q^* = (c_H - v_L)/(v_H - v_L)$. If (7) *does not hold, then $p^* < v_L$, $\lambda^* = (\phi')^{-1}\left(\widehat{q}(v_H - c_H)/(1 - \widehat{q})\right) < \tilde{\lambda}$, and $q^* < (c_H - v_L)/(v_H - v_L)$.*

Proof. See the appendix. ■

Let us illustrate the proposition with a parametric example where $\phi(\lambda) = b(\lambda - \underline{\lambda})^2$ for some $b > 0$. In this case,

$$\tilde{\lambda}^2 - \underline{\lambda}^2 = \frac{r(v_L - c_L)}{b}.$$

and Condition (7) shrinks to

$$\left(\frac{1-\widehat{q}}{\widehat{q}}\right) \frac{2r(v_L - c_L)}{(\tilde{\lambda} + \underline{\lambda})(v_H - c_H)} \leq 1. \tag{8}$$

It is clear that the inequality holds if and only if $\underline{\lambda}$ is sufficiently large or b is sufficiently small. Both of these are when search frictions are small: In the former case, the seller meets buyers quickly even without any search cost. In the latter case, it is not so costly to increase search intensity. Intuitively, when search frictions are small, the low-type seller has a strong incentive to wait for a high price and her reservation price is also high. Therefore, the low-type seller's reservation price binds at v_L, and all other equilibrium variables are determined subject to this constraint. In the opposite case where $\underline{\lambda}$ is sufficiently small or increasing search intensity is sufficiently costly, the low-type seller's reservation falls short of v_L. This ensures that no buyer offers a losing price ($\sigma_H^* + \sigma_L^* = 1$), and all other equilibrium variables follow from there.

2.5 Effects of Reducing Search Costs

We now study the effects of reducing search costs in the model. Since it is not clear how to measure a change of a function $\phi(\cdot)$, we restrict attention to the parametric case where $\phi(\lambda) = b(\lambda - \underline{\lambda})^2$, where a decrease in search costs can be naturally interpreted as a decrease in b.

The following result is immediate from Proposition 1 and the closed-form solution for the parametric case.

Corollary 1 *Suppose $\phi(\lambda) = b(\lambda - \underline{\lambda})^2$. If* (8) *holds, then a marginal change in b does not affect the low-type seller's expected payoff, while if* (8) *does not hold, then a marginal decrease in b increases the low-type seller's expected payoff.*

Proof. The first part is obvious, because $p^* = v_L$ as long as (8) holds. The second part follows from the following explicit solution for p^*:

$$p^* = c_L + \frac{\underline{\lambda}}{r}\frac{\widehat{q}(v_H - c_H)}{1-\widehat{q}} + \frac{1}{rb}\left(\frac{\widehat{q}(v_H - c_H)}{1-\widehat{q}}\right)^2.$$

This solution can be found by combining (5) with

$$\phi'(\lambda^*) = b(\lambda^* - \underline{\lambda})^2 = \frac{\widehat{q}(v_H - c_H)}{1-\widehat{q}}.$$

∎

The solicitation curse is the underlying reason why the seller does not necessarily benefit from a reduction in search costs. When search costs (measured by b in the example) decrease, the low-type seller obtains a direct benefit of lower costs. However, she also increases her search intensity, which exacerbates the solicitation curse and, therefore, lowers buyers' incentives to offer a high price.[8] When (8) holds, this indirect negative effect exactly offsets the direct positive effect. Therefore, the low-type seller's expected payoff remains unchanged, despite lower search costs. In the next section, we show that in the non-stationary version of our model, this indirect effect could dominate the direct effect, and thus a reduction in search costs could even strictly decrease the low-type seller's expected payoff.

3 Non-Stationary Dynamics

We now study a non-stationary version of the model. This allows us to explore another dimension of costly search: dynamics of endogenous search intensity and its impact on equilibrium trading dynamics.

3.1 Setup

We consider the same physical environment as in Section 2, except for the following change: Now buyers observe how long the seller has stayed on the market (time-on-the-market). In other words, each buyer knows how much time has passed since the seller arrived at the market. This specification fits well into our continuous-time framework and allows us to study non-stationary dynamics in a particularly tractable way.[9] We normalize the time the seller comes to the market to 0.

We use the same notation as in Section 2. We denote by $p(t)$ the low-type seller's reservation price and by $\lambda(t)$ her (expected) search intensity at time t. As in the stationary case, it is not

[8] In the parametric example, this can be explicitly shown from the following closed-form solution for σ_H^*:

$$\sigma_H^* = \frac{2(b^{\frac{1}{2}}(r(v_L - c_L) + b\underline{\lambda}^2)^{\frac{1}{2}} - b\underline{\lambda})}{c_H - v_L}.$$

Differentiating this expression with respect to b,

$$\frac{\partial \sigma_H^*}{\partial b} = \frac{r(v_L - c_L) + 2b\underline{\lambda}^2 - 2\underline{\lambda} b^{\frac{1}{2}}(r(v_L - c_L) + b\underline{\lambda}^2)^{\frac{1}{2}}}{(c_H - v_L)a^{\frac{1}{2}}(r(v_L - c_L) + b\underline{\lambda}^2)^{\frac{1}{2}}} > 0$$

That is, an increase in search costs increases the probability that buyers offer c_H.

[9] The framework was introduced by Kim (2012) and has been adopted to address other substantive questions. See Kaya and Kim (2013) and Hwang (2013).

necessary to separately denote the high-type seller's reservation price and search intensity: No buyer offers a price strictly above c_H. Therefore, the high type's reservation price is always equal to c_H and she never exerts search effort (i.e., always chooses $\underline{\lambda}$). For each $a = L, H$, we let $\sigma_a(t)$ denote the probability that the buyer at time t offers the reservation price of the type-a seller. Finally, we represent by $q^u(t)$ buyers' unconditional beliefs and by $q(t)$ buyers' conditional beliefs at time t.

A collection of functions $(p(\cdot), \lambda(\cdot), \sigma_L(\cdot), \sigma_H(\cdot), q^u(\cdot), q(\cdot))$ is a (weak perfect Bayesian) equilibrium if (i) given $\sigma_H(\cdot)$, $p(t)$ is the low-type seller's reservation price and $\lambda(t)$ is her optimal search intensity at t, (ii) given $p(\cdot)$ and $q(\cdot)$, for each $a = L, H$, $\sigma_a(t) > 0$ only when offering the type-a seller's reservation is optimal for the buyer at time t, and (iii) given $\sigma_L(\cdot)$, $\sigma_H(\cdot)$, and $\lambda(\cdot)$, $q^u(t)$ and $q(u)$ are obtained through Bayes' rule.

We also make one simplification regarding the search technology: we assume that there are only two search intensity levels available.[10] Specifically, we assume that the seller's search intensity at each point in time is either $\underline{\lambda}$ or $\overline{\lambda}$, where $\overline{\lambda} > \underline{\lambda} > 0$. The baseline intensity $\underline{\lambda}$ can be obtained at no cost, while the seller must incur fixed flow cost ϕ to increase her intensity to $\overline{\lambda}$. This specification implies that the low-type seller's expected search intensity $\lambda(t)$ is always restricted to the interval $[\underline{\lambda}, \overline{\lambda}]$ and the probability that the low-type seller chooses $\overline{\lambda}$ is equal to $(\lambda(t) - \underline{\lambda})/(\overline{\lambda} - \underline{\lambda})$.

We focus on the case where market frictions are not prohibitively large that the low-type seller has a non-trivial incentive to increase her search intensity as well as wait for a high price. Precisely, we make use of the following assumption.

Assumption 2

$$\phi < \min\left\{\frac{r(\overline{\lambda} - \underline{\lambda})}{r + \underline{\lambda}}(c_H - c_L), \overline{\lambda}(c_H - v_L) - r(v_L - c_L), (v_L - c_L)\left(\frac{r(\overline{\lambda} - \underline{\lambda})}{\underline{\lambda}}\right)\right\}.$$

To interpret this condition, suppose the low-type seller expects to receive c_H with probability 1 from the next buyer. This is the most optimistic scenario to the low-type seller. Therefore, she has the strongest incentive to increase her search intensity and her reservation price is maximized. If $\overline{\lambda}$ is optimal for her, then her reservation price, denoted by $\overline{p}$, satisfies

$$r(\overline{p} - c_L) = -\phi + \overline{\lambda}(c_H - \overline{p}) \;\Leftrightarrow\; \overline{p} = \frac{-\phi + rc_L + \overline{\lambda}c_H}{r + \overline{\lambda}}. \tag{9}$$

The first part of Assumption 2 claims that the low-type seller strictly prefers $\overline{\lambda}$ to $\underline{\lambda}$ in such a case (i.e., $-\phi + \overline{\lambda}(c_H - \overline{p}) > \underline{\lambda}(c_H - \overline{p})$). The second part states that the low-type seller's reservation

[10]The characterization of the general convex-cost case will be included in the next version of the paper.

price $\overline{p}$ strictly exceeds the value buyers put on a low-quality unit, v_L.

3.2 Equilibrium Structure

We construct an equilibrium of the following structure:[11] there are three increasing time points, t_1^*, t_2^*, and t_3^*, such that

- If $t < t_1^*$, then the buyer offers $p(t)$ with probability 1 ($\sigma_L(t) = 1$), which implies that the low-type seller never chooses $\overline{\lambda}$ (i.e., $\lambda(t) = \underline{\lambda}$).
- If $t \in [t_1^*, t_2^*)$, then the buyer randomizes between $p(t)$ and c_H (i.e., $\sigma_L(t), \sigma_H(t) > 0$), while the low-type seller randomizes between $\overline{\lambda}$ and $\underline{\lambda}$ (i.e., $\lambda(t) \in (\underline{\lambda}, \overline{\lambda})$).
- If $t \in [t_2^*, t_3^*)$, then the buyer randomizes between c_H and a losing price (i.e., $\sigma_H(t) \in (0, 1)$ and $\sigma_L(t) = 0$), while the low-type seller randomizes between $\overline{\lambda}$ and $\underline{\lambda}$ (i.e., $\lambda(t) \in (\underline{\lambda}, \overline{\lambda})$).
- If $t \geq t_3^*$, then the buyer offers c_H with probability 1 (i.e., $\sigma_H(t) = 1$), and the low-type seller always chooses the high search intensity (i.e., $\lambda(t) = \overline{\lambda}$).

The first and the last phases are intuitive. Due to severe adverse selection (Assumption 1), initially there is too much risk of overpaying for a lemon. Therefore, buyers would offer only the reservation price of the low type. On the other hand, the low type is more eager to trade than the high type. Therefore, staying on the market for a sufficiently long time is a good indication of the high quality of the asset. Therefore, the buyers who would come to the market sufficiently late would offer c_H to the seller.

The equilibrium behavior in the two interim phases (i.e., the interval $[t_1^*, t_3^*)$) is rather subtle. In the first phase, as the low type accepts $p(t)$ and leaves the market, the probability of the high type increases over time. Once it reaches a certain threshold, buyers begin to offer c_H. But that provides an incentive for the low-type seller to increase her search intensity. Since the seller who successfully meets a buyer is more likely to be the low type, buyers will then be less willing to offer c_H. This then generates the opposite cycle by reducing the low-type seller's incentive to exert search effort. In equilibrium, the low-type seller must choose $\overline{\lambda}$ and buyers must offer c_H with just enough probabilities so that all agents have just right incentives. The main difference between the two interim phases is whether the low-type seller's reservation price $p(t)$ is below or above v_L. As in the stationary model, in equilibrium, the buyer never makes a losing offer in the former case, while he never offers $p(t)$ in the latter case. Interestingly, it turns out that an equilibrium requires both phases. In other words, having only one interim phase is not sufficient to align agents' incentives.

[11]This equilibrium is the unique equilibrium under Assumption 2. A uniqueness proof will be included in the next version of the paper.

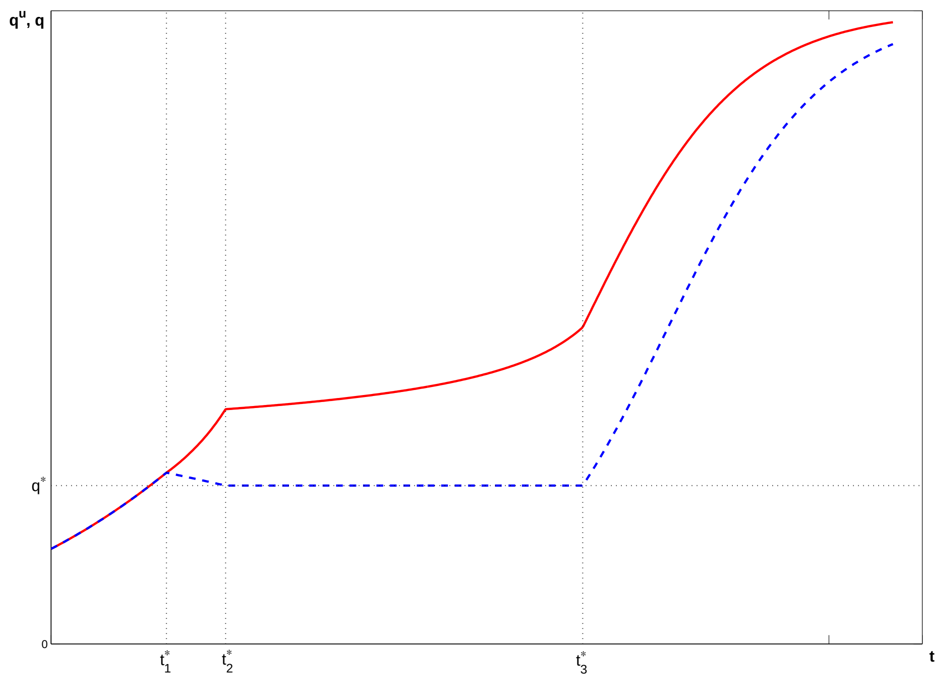

Figure 1: Evolution of buyers' unconditional (solid) and conditional beliefs (dashed).

Figure 1 depicts how buyers' unconditional beliefs $q^u(\cdot)$ and conditional beliefs $q(\cdot)$ evolve over time. $q^u(\cdot)$ always strictly increases over time: If $t < t_1^*$, then only the low type trades. If $t \in [t_1^*, t_2^*)$, then the low type not only accepts $p(t)$, but also chooses a higher search intensity. Finally, if $t \geq t_2^*$, then both types trade only at c_H, but the low type trades faster because she chooses a higher search intensity. The last part best captures the acceleration blessing in the current non-stationary setup. If the seller cannot influence her search intensity, then after t_2^* both types would trade at the same rate and, therefore, buyers' (unconditional) beliefs would stay constant. Endogenous search intensity allows the low type to trade faster than the high type, thereby relaxing future buyers' incentive constraints to offer c_H.

To the contrary, buyers' conditional beliefs $q(\cdot)$ do not necessarily increase over time. In fact, they strictly decrease on the interval $[t_1^*, t_2^*)$ and stay constant on the interval $[t_2^*, t_3^*)$. This is a clear manifestation of the solicitation curse. Although it becomes more likely that the seller is the high type over time (acceleration blessing), the low-type seller also increases her search intensity over time (i.e., $\lambda(t)$ increases), which exacerbates the solicitation curse: it becomes less likely that the matched seller is the high type. Over the interval $[t_1^*, t_3^*)$, the solicitation curse is at least as strong as the acceleration blessing, and thus buyers' conditional beliefs $q(\cdot)$ weakly decrease, even though their unconditional beliefs $q^u(\cdot)$ constantly increase.

3.3 Equilibrium Construction

We construct an equilibrium by moving backward in time.

3.3.1 Last Phase: $t \geq t_3^*$

In the last phase, all buyers offer c_H with probability 1. Under Assumption 2, $\overline{\lambda}$ is optimal for the low-type seller, and her reservation price is given by $p(t) = \overline{p}$.

To determine buyers' unconditional and conditional beliefs, first notice that the buyer at t_3^* must be indifferent between offering c_H and a losing price. This implies that

$$q(t_3^*)(v_H - c_H) + (1 - q(t_3^*))(v_L - c_H) = 0 \Leftrightarrow q(t_3^*) = \frac{c_H - v_L}{v_H - v_L}.$$

Then, $q^u(t_3^*)$ can be recovered from the fact that the high-type seller's search intensity is always equal to $\underline{\lambda}$, while $\lambda(t_3^*) = \overline{\lambda}$:

$$q(t_3^*) = \frac{q^u(t_3^*)\underline{\lambda}}{q^u(t_3^*)\underline{\lambda} + (1 - q^u(t_3^*))\overline{\lambda}} \Rightarrow q^u(t_3^*) = \frac{(c_H - v_L)\overline{\lambda}}{(c_H - v_L)\overline{\lambda} + (v_H - c_H)\underline{\lambda}}.$$

Given $q(t_3^*)$, $q^u(t)$ for any $t > t_3^*$ can be obtained from the fact that both seller types trade whenever they meet a buyer, as the buyer offers c_H with probability 1:

$$q^u(t) = \frac{q^u(t_3^*)e^{-\underline{\lambda}(t-t_3^*)}}{q^u(t_3^*)e^{-\underline{\lambda}(t-t_3^*)} + (1 - q^u(t_3^*))e^{-\overline{\lambda}(t-t_3^*)}} = \frac{(c_H - v_L)\overline{\lambda}e^{-\underline{\lambda}(t-t_3^*)}}{(c_H - v_L)\overline{\lambda}e^{-\underline{\lambda}(t-t_3^*)} + (v_H - c_H)\underline{\lambda}e^{-\overline{\lambda}(t-t_3^*)}}. \tag{10}$$

Finally, since $\lambda(t) = \overline{\lambda}$ for any $t > t_3^*$,

$$q(t) = \frac{q^u(t)\underline{\lambda}}{q^u(t)\underline{\lambda} + (1 - q^u(t))\overline{\lambda}} = \frac{(c_H - v_L)e^{-\underline{\lambda}(t-t_3^*)}}{(c_H - v_L)e^{-\underline{\lambda}(t-t_3^*)} + (v_H - c_H)e^{-\overline{\lambda}(t-t_3^*)}}. \tag{11}$$

3.3.2 Second Interim Phase: $t \in [t_2^*, t_3^*)$

In the two interim phases, the low-type seller is indifferent between $\overline{\lambda}$ and $\underline{\lambda}$. Therefore, given buyers' offer strategies $\sigma_H(\cdot)$, the Bellman equation for the low-type seller's reservation price $p(t)$ is given by

$$\begin{aligned} r(p(t) - c_L) &= -\phi + \overline{\lambda}\sigma_H(t)(c_H - p(t)) + \dot{p}(t) \\ &= \underline{\lambda}\sigma_H(t)(c_H - p(t)) + \dot{p}(t). \end{aligned}$$

It is then straightforward that

$$\sigma_H(t)(c_H - p(t)) = \frac{\phi}{\overline{\lambda} - \underline{\lambda}},$$

and

$$r(p(t) - c_L) = \frac{\phi \underline{\lambda}}{\overline{\lambda} - \underline{\lambda}} + \dot{p}(t).$$

Solving the differential equation, for each $t \in [t_1^*, t_3^*)$,

$$p(t) = c_L + A + e^{r(t-\underline{t})}(p(t_1^*) - c_L - A), \tag{12}$$

where $A \equiv \phi \underline{\lambda} / (r(\overline{\lambda} - \underline{\lambda}))$. From the low-type seller's indifference condition, it also follows that

$$\sigma_H(t) = \left(\frac{\phi}{\overline{\lambda} - \underline{\lambda}} \right) \frac{1}{c_H - c_L - A - e^{r(t-\underline{t})}(p(t_1^*) - c_L - A)}. \tag{13}$$

We first determine the length of the second interim phase, $t_3^* - t_2^*$. It suffices to use the fact that $p(t_2^*) = v_L$ and $p(t_3^*) = \overline{p}$. Applying the values to (12), it is immediate that

$$e^{r(t_3^* - t_2^*)} = \frac{\overline{p} - c_L - A}{v_L - c_L - A}.$$

We now solve for buyers' unconditional beliefs $q^u(\cdot)$ and the low-type seller's equilibrium search intensity $\lambda(\cdot)$ for the second interim phase. In the second interim phase (i.e., $t \in [t_2^*, t_3^*)$), each buyer is indifferent between offering c_H and a losing price. Since $q(t)$ is obtained from $q^u(t)$ and $\lambda(t)$, this implies that

$$\frac{c_H - v_L}{v_H - v_L} = q^* = \frac{q^u(t)\underline{\lambda}}{q^u(t)\underline{\lambda} + (1 - q^u(t))\lambda(t)} \Leftrightarrow \frac{v_H - c_H}{c_H - v_L} = \frac{1 - q^u(t)}{q^u(t)} \frac{\lambda(t)}{\underline{\lambda}}. \tag{14}$$

In addition, since trade occurs only at c_H, given $\sigma_H(\cdot)$ and $\lambda(\cdot)$,

$$q^u(t) = \frac{q^* e^{-\int_{\underline{t}}^t \underline{\lambda} \sigma_H(x) dx}}{q^* e^{-\int_{\underline{t}}^t \underline{\lambda} \sigma_H(x) dx} + (1 - q^*) e^{-\int_{\underline{t}}^t \lambda(t) \sigma_H(x) dx}}.$$

Therefore, $q^u(\cdot)$ increases according to

$$\dot{q}^u(t) = q^u(t)(1 - q^u(t))(\lambda(t) - \underline{\lambda})\sigma_H(t). \tag{15}$$

In what follows, we focus on $q^u(\cdot)$. Given $q^u(t)$, $\lambda(t)$ can be easily recovered through (14).

The following mathematical results will prove useful for both interim phases.

Lemma 1 *(1) If $\xi(t) = \int_{\underline{t}}^{t} \lambda \sigma_H(x)dx$ where $\sigma_H(\cdot)$ is given as in* (13)*, then*

$$\xi(t) = \frac{A}{c_H - c_L - A} \ln \left(\frac{(c_H - p(\underline{t}))e^{r(t-\underline{t})}}{(c_H - c_L - A) - e^{r(t-\underline{t})}(p(\underline{t}) - c_L - A)} \right).$$

(2) Suppose for some constant B a function $q^u(\cdot)$ satisfies the following ordinary differential equation from $\underline{t}$:

$$\dot{q}^u(t) = q^u(t)\left(Bq^u(t) - 1\right)\underline{\lambda}\sigma_H(t).$$

Then, the unique solution to the differential equation is given by

$$q^u(t) = \frac{e^{-\xi(t)}}{\frac{1}{q^u(\underline{t})} + B(e^{-\xi(t)} - 1)}.$$

Proof. See the appendix. ■

Plugging (14) into (15) and arranging the terms, the system of equations reduces to the following ordinary differential equation:

$$\dot{q}^u(t) = q^u(t)\left(\frac{q^u(t)}{q^*} - 1\right)\underline{\lambda}\sigma_H(t).$$

Notice that this equation takes the same form as in the second part of Lemma 1. It follows that the solution is given by

$$q^u(t) = \frac{e^{-\xi(t)}}{\frac{1}{q^u(t_2^*)} + \frac{e^{-\xi(t)}-1}{q^*}}, \tag{16}$$

where

$$e^{-\xi(t)} = \left(\frac{(c_H - v_L)e^{r(t-t_2^*)}}{(c_H - c_L - A) - e^{r(t-t_2^*)}(v_L - c_L - A)} \right)^{-\frac{A}{c_H - c_L - A}}.$$

There is a terminal condition that the solution must satisfy: $q^u(t_3^*)$ in (16) must coincide with the one derived for the last phase, that is,

$$\frac{(c_H - v_L)\overline{\lambda}}{(c_H - v_L)\overline{\lambda} + (v_H - c_H)\underline{\lambda}} = q^u(t_3^*) = \frac{e^{-\xi(t_3^*)}}{\frac{1}{q^u(t_2^*)} + \frac{e^{-\xi(t_3^*)}-1}{q^*}}.$$

This condition allows us to pin down the unique value of $q^u(t_2^*)$, which is necessary to analyze the first interim phase. Note that $t_3^* - t_2^*$ was derived above.

3.3.3 First Interim Phase: $t \in [t_1^*, t_2^*)$

In the first interim phase, as in the second interim phase, the low-type seller is indifferent between $\overline{\lambda}$ and $\underline{\lambda}$. Therefore, the derivations for $p(\cdot)$ and $\sigma(\cdot)$ are identical to those for the second interim phase. It follows that $p(\cdot)$ and $\sigma_H(\cdot)$ are as given in (12) and (13), respectively.

Unlike in the second interim phase, buyers' conditional beliefs $q(\cdot)$ are not fixed at q^*, but vary over time. In particular, each buyer must be indifferent between offering c_H and $p(t)$. This means that given $p(t)$, $q(t)$ is given by the value that satisfies

$$q(t)(v_H - c_H) + (1 - q(t))(v_L - c_H) = (1 - q(t))(v_L - p(t)) \Leftrightarrow \frac{1 - q(t)}{q(t)} = \frac{v_H - c_H}{c_H - p(t)}. \quad (17)$$

In the first interim phase, $p(t) < v_L$. This implies that no buyer offers a losing price, and thus the low-type seller trades whenever she meets a buyer. Since the low-type seller trades at rate $\lambda(t)$, while the high type at rate $\underline{\lambda}\sigma_H(t)$, it follows that

$$\dot{q}^u(t) = q^u(t)(1 - q^u(t))(\lambda(t) - \underline{\lambda}\sigma_H(t)). \quad (18)$$

Finally, as usual, buyers' conditional and unconditional beliefs are intertwined via $\lambda(t)$:

$$q(t) = \frac{q^u(t)\underline{\lambda}}{q^u(t)\underline{\lambda} + (1 - q^u(t))\lambda(t)} \Leftrightarrow \frac{1 - q(t)}{q(t)} = \frac{1 - q^u(t)}{q^u(t)} \frac{\lambda(t)}{\underline{\lambda}}. \quad (19)$$

Combining (17) and (19) and using the fact that $\sigma_H(t)(c_H - p(t)) = \phi/(\overline{\lambda} - \underline{\lambda})$,

$$\lambda(t) = \frac{\overline{\lambda} - \underline{\lambda}}{\phi}(v_H - c_H)\underline{\lambda}\sigma_H(t)\frac{q^u(t)}{1 - q^u(t)}. \quad (20)$$

Plugging the expression for $\lambda(t)$ into (18) and arranging the terms,

$$\dot{q}^u(t) = q^u(t)\left(\left(\frac{\overline{\lambda} - \underline{\lambda}}{\phi}(v_H - c_H) + 1\right)q^u(t) - 1\right)\underline{\lambda}\sigma_H(t).$$

Notice that this again takes the form in the second part of Lemma 1. Therefore, the solution is given by

$$q^u(t) = \frac{e^{-\xi(t)}}{\frac{1}{q^u(t_1^*)} + \left(\frac{\overline{\lambda} - \underline{\lambda}}{\phi}(v_H - c_H) + 1\right)(e^{-\xi(t)} - 1)}, \quad (21)$$

where

$$e^{-\xi(t)} = \left(\frac{(c_H - p(t_1^*))e^{r(t - t_1^*)}}{(c_H - c_L - A) - e^{r(t - t_1^*)}(p(t_1^*) - c_L - A)}\right)^{-\frac{A}{c_H - c_L - A}}.$$

Given $q^u(t)$, $\lambda(t)$ can be recovered from (20). In addition, $q(t)$ can be recovered either from (17) or from (19).

It remains to determine $t_2^* - t_1^*$ and $q^u(t_1^*)$. The two relevant conditions for them are as follows. First, there is a terminal condition for the solution $q^u(\cdot)$: $q^u(t_2^*)$ from (21) must coincide with the value found in the second interim phase. Formally,

$$q^u(t_2^*) = \frac{e^{-\xi(t_2^*)}}{\frac{1}{q^u(t_1^*)} + \left(\frac{\overline{\lambda}-\underline{\lambda}}{\phi}(v_H - c_H) + 1\right)(e^{-\xi(t_2^*)} - 1)}. \tag{22}$$

Second, it must be that $\lambda(t_1^*) = \underline{\lambda}$: otherwise, buyers right before t_1^* would strictly prefer offering c_H to $p(t)$, because $p(\cdot)$ is always conditions, while $q(\cdot)$ would jump down at t_1^*. Using (17) and (19), this condition is equivalent to

$$\frac{1 - q^u(t_1^*)}{q^u(t_1^*)} = \frac{v_H - c_H}{c_H - p(t_1^*)}. \tag{23}$$

The existence of the solutions to these two conditions follows from the fact that that in (23), the right-hand side is larger than the left-hand side if $t_2^* - t_1^*$ is sufficiently close to 0 (in which case $p(t_1^*)$ is close to v_L, while $q^u(t_1^*)$ is away from q^*), while the opposite is true if $t_2^* - t_1^*$ is sufficiently large (in which case $q^u(t_1^*)$ is close to 0, while the right-hand side is bounded above by $(v_H - c_H)/(c_H - v_L)$). The uniqueness follows from the fact that the right-hand side in (23) is strictly decreasing in $t_2^* - t_1^*$ (because $p(t_1^*)$ is strictly decreasing in $t_2^* - t_1^*$), while the left-hand side is strictly increasing in $t_2^* - t_1^*$: To show the latter, first notice that since $p(t_2^*) = v_L$,

$$e^{-\xi(t_2^*)} = \left(\frac{(c_H - p(t_1^*))e^{r(t_2^*-t_1^*)}}{(c_H - c_L - A) - e^{r(t_2^*-t_1^*)}(p(t_1^*) - c_L - A)}\right)^{-\frac{A}{c_H-c_L-A}} = \left(\frac{c_H - p(t_1^*)}{c_H - v_L}\frac{v_L - c_L - A}{p(t_1^*) - c_L - A}\right)^{-\frac{A}{c_H-c_L-A}}.$$

Therefore, $e^{-\xi(t_2^*)}$ is strictly increasing in $t_2^* - t_1^*$. Applying this to (22), it follows that $q^u(t_1^*)$ is also strictly increasing in $t_2^* - t_1^*$.

3.3.4 First Phase: $t < t_1^*$

In the first phase, buyers offer only $p(t)$. Therefore, the low-type seller always chooses $\underline{\lambda}$. It follows that $p(\cdot)$ increases according to

$$p(t) = c_L + e^{-r(t_1^*-t)}(p(t_1^*) - c_L). \tag{24}$$

It also follows that buyers' unconditional and conditions beliefs coincide (i.e., $q(t) = q^u(t)$ for any $t \in [0, t_1^*)$. Since the low-type seller trades at rate $\underline{\lambda}$, while the high-type seller never trades, $q(\cdot)$

and $q^u(\cdot)$ increase according to

$$q^u(t) = q(t) = \frac{\widehat{q}}{\widehat{q} + (1-\widehat{q})e^{-\underline{\lambda}t}}. \tag{25}$$

Finally, we determine t_1^*. From the characterization of the first interim phase, $q^u(t_1^*)$ is already fixed. Combining the value with (25) gives the unique value of t_1^*. Note that we have found the values of $t_2^* - t_1^*$ and $t_3^* - t_2^*$ before. Therefore, the identification of t_1^* allows us to complete the equilibrium construction.

We summarize the equilibrium construction results in the following proposition.

Proposition 2 *In the discrete case with two search intensity levels, under Assumption 2, there exists a unique equilibrium. In the equilibrium,*

- *The low-type seller's reservation price $p(\cdot)$ increases according to* (24) *if $t < t_1^*$, increases according to* (12) *if $t \in [t_1^*, t_3^*)$, and stays constant at $\overline{p}$ if $t \geq t_3^*$.*
- *Buyers' unconditional beliefs $q^u(\cdot)$ increase according to* (25) *if $t < t_1^*$, according to* (21) *if $t \in [t_1^*, t_2^*)$, according to* (16) *if $t \in [t_2^*, t_3^*)$, and according to* (10) *$t \geq t_3^*$.*
- *Buyers' conditional beliefs $q(\cdot)$ increase according to* (25) *if $t < t_1^*$, decrease according to* (17) *if $t \in [t_1^*, t_2^*)$, stay constant at $(c_H - v_L)/(v_H - v_L)$ if $t \in [t_2^*, t_3^*)$, and increase according to* (11) *$t \geq t_3^*$.*

3.4 Effects of Reducing Search Costs

We now examine the effects of reducing search costs on the low-type seller's expected payoff in the non-stationary model. Due to the complexity of the equilibrium structure, it is quite involved to analyze the effects of a marginal change.[12] Still, it is possible to compare the low-type seller's expected payoff under Assumption 2 to her expected payoff in the model with exogenous search intensity (i.e., only $\underline{\lambda}$ is available), provided that $\underline{\lambda}$ is relatively large. The following result is straightforward from the characterization above and the result in Kim (2012).

Corollary 2 *Suppose $r(v_L - c_L) < \underline{\lambda}(c_H - v_L)$. Then, the low-type seller's expected payoff is lower when $\overline{\lambda}$ is available (equivalently, ϕ is relatively small) than when $\overline{\lambda}$ is not available (equivalently, ϕ is prohibitively large).*

Proof. Kim (2012) shows that when the arrival rate of buyers is exogenously given by $\underline{\lambda}$ such that $r(v_L - c_L) < \underline{\lambda}(c_H - v_L)$, the low-type seller's expected payoff is equal to $e^{-rt^*}(v_L - c_L)$ where

[12] More thorough comparative statics results will be included in the next version of this paper.

t^* is the value that satisfies

$$\frac{c_H - v_L}{v_H - v_L} = \frac{\widehat{q}}{\widehat{q} + (1-\widehat{q})e^{-\underline{\lambda} t^*}}.$$

Under Assumption 2, the low-type seller's expected payoff is equal to $e^{-rt_1^*}(p(t_1^*) - c_L)$. The result follows from the fact that $p(t_1^*) < v_L$ and

$$\frac{c_H - v_L}{v_H - v_L} < \frac{\widehat{q}}{\widehat{q} + (1-\widehat{q})e^{-\underline{\lambda} t_1^*}}.$$

■

The result is again due to the solicitation curse. As search costs decrease, the solicitation curse worsens, because the low-type seller has a stronger incentive to increase her search intensity. As in the stationary model, this indirect effect offsets the direct benefit of lower search costs. Unlike in the stationary model, the indirect effect even outweighs the direct effect. This is precisely because of dynamics of endogenous search intensity. As shown above, the solicitation curse is particularly strong in the first interim phase. When search costs decrease and, therefore, the low-type seller has a stronger incentive to increase her search intensity, buyers become more cautious and demand a higher unconditional probability that the seller is the high type. This means that the length of the first phase needs to increase. This decreases the low-type seller's expected payoff, because buyers offer only the reservation price of the low-type seller over the first phase.

4 Discussion

We have focused on a particularly simple environment. In this section, we show that our insights are robust to various changes to the environment. For simplicity, we explain the robustness in the context of the stationary model studied in Section 2.

4.1 Buyer Inspection

There are various models that allow for buyer inspection (i.e., buyers' getting an informative signal about the quality of the good).[13] We first explain how to accommodate buyer inspection within our framework and how our insights extend into such an environment.

Suppose each buyer receives a signal that is identically and independently drawn from the interval $[\underline{s}, \overline{s}]$ according to the distribution function F_a, where a denotes the quality of the good. Assume that each F_a admits a continuous and positive density f_a. For simplicity, also assume that

[13] See, for example, Kaya and Kim (2013), Lauermann and Wolinsky (2013b), and Zhu (2012).

the likelihood ratio $f_H(s)/f_L(s)$ is strictly increasing (MLRP), $f_H(\underline{s})/f_L(\underline{s}) = 0$, and $f_H(\overline{s})/f_L(\overline{s})$ is arbitrarily large. All other specifications of the environment are identical to those in Section 2.1.

Naturally, a buyer's optimal offer strategy is a cutoff strategy: there exists $s^* \in [\underline{s}, \overline{s}]$ such that the buyer offers c_H if and only if his signal is above s^*. For signals below s^*, we denote by $\sigma_L^*(s)$ the probability that each buyer offers p^* when his signal is s.

Given s^* and $\sigma_L^*(\cdot)$, the high-type seller trades at rate $\underline{\lambda}(1 - F_H(s^*))$, while the low type at rate $\lambda^*((1 - F_L(s^*)) + \int_{\underline{s}}^{s^*} \sigma_L(s) dF_L(s))$. Then, as in Section 2.3, buyers' unconditional beliefs are given by

$$q^u = \frac{\frac{\widehat{q}}{\underline{\lambda}(1-F_H(s^*))}}{\frac{\widehat{q}}{\underline{\lambda}(1-F_H(s^*))} + \frac{(1-\widehat{q})}{\lambda^*((1-F_L(s^*))+\int_{\underline{s}}^{s^*} \sigma_L(s) dF_L(s))}}.$$

Unlike in the baseline model, q^u is not necessarily larger than $\widehat{q}$. This is because the high type generates good signals and, therefore, receives c_H more frequently than the low type (i.e., $1 - F_H(s^*) > 1 - F_L(s^*)$). This provides a countervailing force to the usual effect that the high type accepts only c_H, while the low type accepts both c_H and p^*. This does not mean that the acceleration blessing may be absent in this model. It is still present, because without endogenous search intensity, buyers' beliefs would be

$$\frac{\frac{\widehat{q}}{1-F_H(s^*)}}{\frac{\widehat{q}}{1-F_H(s^*)} + \frac{(1-\widehat{q})}{(1-F_L(s^*))+\int_{\underline{s}}^{s^*} \sigma_L(s) dF_L(s)}},$$

which is strictly smaller than q^u.

Given q^u and λ^*, buyers' conditional beliefs are given by

$$q^* = \frac{\widehat{q}\underline{\lambda}}{\widehat{q}\underline{\lambda} + (1-\widehat{q})\lambda^*} = \frac{\frac{\widehat{q}}{1-F_H(s^*)}}{\frac{\widehat{q}}{1-F_H(s^*)} + \frac{(1-\widehat{q})}{(1-F_L(s^*))+\int_{\underline{s}}^{s^*} \sigma_L(s) dF_L(s)}}.$$

As in the baseline model, the difference between q^u and q^* represents the solicitation curse.

4.2 More than Two Types

It is well-known that the equilibrium characterization becomes significantly more complicated once there are more than two types of sellers. Nevertheless, it is relatively easy to show how the two effects of endogenous search intensity arise in the model with more than two types. For simplicity, we consider the case of three types. The generalization into more types is notationally more cumbersome, but conceptually straightforward.

Suppose there are three types of sellers: low type (L), middle type (M), and high type (H). For each $a = L, M, H$, denote by c_a a type-a unit's value to the seller and by v_a its value to buyers, and assume that $c_L < c_M < c_H$ and $v_L < v_M < v_H$. Let $\widehat{q}_a$ be the probability that the seller is of type a at the beginning of the game. The search technology is given as in Section 2.1.

Let p_a^* denote the reservation price of the type-a seller. The assumption $c_L < c_M < c_H$ guarantees that $p_L^* < p_M^* < p_H^*$. This, in turn, guarantees that p_L^* is accepted only by the low type, p_M^* by the low type as well as the middle type, and p_H^* by all three types. In addition, it is straightforward to show that, as in the two-type case, each buyer offers either p_H^*, p_M^*, p_L^*, or a losing price. Denote by σ_a^* the probability that each buyer offers p_a^*. Finally, denote by λ_a^* the type-a seller's optimal search intensity. Since a lower type gains more from search, it is also clear to show that $\underline{\lambda} = \lambda_H^* < \lambda_M^* < \lambda_L^*$.

Let q_a^u represent buyers' unconditional beliefs that the seller is of type a. Following the same steps as in the two-type case,

$$\begin{aligned} q_L^u &= \frac{\frac{\widehat{q}_L}{\lambda_L^*(\sigma_H^*+\sigma_M^*+\sigma_L^*)}}{\frac{\widehat{q}_L}{\lambda_L^*(\sigma_H^*+\sigma_M^*+\sigma_L^*)} + \frac{\widehat{q}_M}{\lambda_M^*(\sigma_H^*+\sigma_M^*)} + \frac{\widehat{q}_H}{\lambda_H^*\sigma_H^*}}, \\ q_M^u &= \frac{\frac{\widehat{q}_M}{\lambda_M^*(\sigma_H^*+\sigma_M^*)}}{\frac{\widehat{q}_L}{\lambda_L^*(\sigma_H^*+\sigma_M^*+\sigma_L^*)} + \frac{\widehat{q}_M}{\lambda_M^*(\sigma_H^*+\sigma_M^*)} + \frac{\widehat{q}_H}{\lambda_H^*\sigma_H^*}}, \\ q_H^u &= \frac{\frac{\widehat{q}_H}{\lambda_H^*\sigma_H^*}}{\frac{\widehat{q}_L}{\lambda_L^*(\sigma_H^*+\sigma_M^*+\sigma_L^*)} + \frac{\widehat{q}_M}{\lambda_M^*(\sigma_H^*+\sigma_M^*)} + \frac{\widehat{q}_H}{\lambda_H^*\sigma_H^*}}. \end{aligned}$$

Since $\lambda_H^* < \lambda_M^* < \lambda_L^*$, endogenous search intensity clearly lowers q_L^u, while increases q_H^u, relative to the exogenous case (which can be interpreted as the case where $\lambda_H^* = \lambda_M^* = \lambda_L^*$). q_M^u can increase or decrease, depending on the values of λ_L^*, λ_M^*, and λ_H^*. Nevertheless, it is easy to show that q_M^*/q_L^* strictly increases, while q_M^*/q_H^* strictly decreases. This shows that the acceleration blessing clearly operates for the case of more than two types.

Let q_a^* denote buyers' conditional beliefs that the seller is of type a. Again, as in the two-type case,

$$\begin{aligned} q_L^* &= \frac{q_L^u\lambda_L^*}{q_L^u\lambda_L^* + q_M^u\lambda_M^* + q_H^u\lambda_H^*} = \frac{\frac{\widehat{q}_L}{\sigma_H^*+\sigma_M^*+\sigma_L^*}}{\frac{\widehat{q}_L}{\sigma_H^*+\sigma_M^*+\sigma_L^*} + \frac{\widehat{q}_M}{\sigma_H^*+\sigma_M^*} + \frac{\widehat{q}_H}{\sigma_H^*}}, \\ q_M^* &= \frac{q_M^u\lambda_M^*}{q_L^u\lambda_L^* + q_M^u\lambda_M^* + q_H^u\lambda_H^*} = \frac{\frac{\widehat{q}_M}{\sigma_H^*+\sigma_M^*}}{\frac{\widehat{q}_L}{\sigma_H^*+\sigma_M^*+\sigma_L^*} + \frac{\widehat{q}_M}{\sigma_H^*+\sigma_M^*} + \frac{\widehat{q}_H}{\sigma_H^*}}, \\ q_H^* &= \frac{q_H^u\lambda_H^*}{q_L^u\lambda_L^* + q_M^u\lambda_M^* + q_H^u\lambda_H^*} = \frac{\frac{\widehat{q}_H}{\sigma_H^*}}{\frac{\widehat{q}_L}{\sigma_H^*+\sigma_M^*+\sigma_L^*} + \frac{\widehat{q}_M}{\sigma_H^*+\sigma_M^*} + \frac{\widehat{q}_H}{\sigma_H^*}}. \end{aligned}$$

Clearly, $q_L^* > q_L^u$ and $q_H^* < q_H^u$. In addition, $q_M^*/q_L^* < q_M^u/q_L^u$, while $q_M^*/q_H^* > q_M^u/q_H^u$. This is how the solicitation curse manifests in the case of more than two types.

4.3 Alternative Bargaining Protocol

One undesirable property of the baseline model is that the high-type seller has no incentive to increase her search intensity and always chooses $\underline{\lambda}$. Although it simplifies the analysis, it seems to prevent a full exploration of the effects of endogenous search intensity. The property is a consequence of the bargaining protocol we adopt in the baseline model, in which the Diamond paradox always applies to the highest t5ype. There are several alternative bargaining protocols that allow us to overcome the problem.[14] In this section, we consider a bargaining protocol, which is particularly simple but has been widely adopted in the literature,[15] and study the effects of non-trivially endogenizing both types' search intensities.

4.3.1 Setup

The physical environment is as in Section 2.1. It is also the same that each buyer offers a price and the seller decides whether to accept it or not. But now, each buyer is restricted to offer only one of two exogenously given prices, p_L and p_H. In order to avoid triviality, we assume that $c_L < p_L < v_L$ and $c_H < p_H < v_H$. In other words, a buyer is willing to offer p_a if he believes that it would be accepted by the type-a seller. Of course, p_H would be accepted not only by the high type, but also by the low type, which creates an adverse selection problem. Notice that the high-type seller now obtains a strictly positive expected payoff, as long as p_H is offered by buyers with a positive probability. This implies that the high-type seller also has an incentive to increase her search intensity. Assumption 1 turns out to be more stringent than necessary. We now make use of the following assumption:

Assumption 3

$$\widehat{q}v_H + (1 - \widehat{q})v_L < p_H.$$

We denote by p_a^* the type-a seller's reservation price, by λ_a^* her equilibrium search intensity, and by σ_a^* the probability that each buyer offers p_a. Since p_H is the highest price that can be ever offered, it is clear that $c_H \leq p_H^* < p_H$ and the high type seller accepts p_H with probability 1. For the low-type seller, we denote by σ_S^* the probability that she accepts p_L.[16] Obviously, $\sigma_S^* = 1$ if

[14] For example, introducing simultaneous competition as in Vincent (1990), allowing the informed player to make offers as in Gerardi, Hörner and Maestri (2013), and randomly generating offers as in Lauermann and Wolinsky (2013b).

[15] See, for example, Wolinsky (1990), Blouin and Serrano (2001), and Camargo and Lester (2013).

[16] Note that we retain the stationarity of the problem by requiring the low-type seller to play a stationary acceptance strategy.

$p_L^* < p_L$, while $\sigma_S^* = 0$ if $p_L^* > p_L$. For the case where $p_L^* = p_L$, we no longer require that p_L^* should be accepted by the low type with probability 1. This is necessary for the existence of an equilibrium.

4.3.2 Buyers' Beliefs

Buyers' unconditional and conditional beliefs can be derived as in Section 2.3. Given σ_H^* and σ_S^*, buyers' unconditional beliefs are given by

$$q^u = \frac{\frac{\widehat{q}}{\lambda_H^* \sigma_H^*}}{\frac{\widehat{q}}{\lambda_H^* \sigma_H^*} + \frac{1-\widehat{q}}{\lambda_L^*(\sigma_H^* + (1-\sigma_H^*)\sigma_S^*)}}.$$

The difference from the baseline model is that the high-type seller no longer chooses $\underline{\lambda}$. Still, as shown shortly, the low-type seller gains more from search than the high-type seller, and thus $\lambda_H^* < \lambda_L^*$. This implies that the acceleration blessing still increases buyers' unconditional beliefs.[17] As usual, buyers' conditional beliefs are given by

$$q^* = \frac{q^u \lambda_H^*}{q^u \lambda_H^* + (1-q^u)\lambda_L^*} = \frac{\frac{\widehat{q}}{\sigma_H^*}}{\frac{\widehat{q}}{\sigma_H^*} + \frac{1-\widehat{q}}{\sigma_H^* + (1-\sigma_H^*)\sigma_S^*}}. \tag{26}$$

4.3.3 Equilibrium Characterization

Let σ_S^* denote the probability that the low-type seller accepts an offer of p_L. An equilibrium can be described by a tuple $(p_L^*, p_H^*, \lambda_L^*, \lambda_H^*, \sigma_H^*, \sigma_S^*, q^*)$ such that (i) given σ_H^*, p_a^* is the type-a seller's reservation price and λ_a^* is her optimal search intensity for each $a = L, H$, (ii) $\sigma_S^* > 0$ only when $p_L^* \geq p_L$, and (iii) given λ_L^*, λ_H^*, σ_H^*, and σ_S^*, q^* is derived as in (26).

We begin with an observation that the low-type seller's reservation price cannot strictly exceed p_L, that is, it is necessary that $p_L^* \leq p_L$. Suppose $p_L^* > p_L$. Then, both types accept only p_H. But this implies that $q^* = \widehat{q}$ and, under Assumption 3, a buyer would obtain a strictly negative payoff if he offers p_H. Therefore, all buyers would offer p_L with probability 1. This brings down p_L^* below p_L, which is a contradiction.

Low type's reservation price and search optimality. Similarly to the baseline model, the Bellman equation for the low-type seller is given by

$$r(p_L^* - c_L) = \max_{\lambda} -\phi(\lambda) + \lambda(\sigma_H^*(p_H - p_L^*) + (1-\sigma_H^*)(p_L - p_L^*)).$$

[17] In Section **??**, we show that with the introduction of buyer inspection, it is possible to have that $\lambda_L^* < \lambda_H^*$, and thus endogenous search intensity may create deceleration curse for buyers' unconditional beliefs.

Notice that it is assumed that the low-type seller would accept p_L with probability 1. This incurs no loss of generality, because, as explained above, $p_L^* \leq p_L$, and thus the low-type seller must weakly prefer accepting p_L to rejecting it. It follows that λ_L^* must satisfy

$$\phi'(\lambda_L^*) = \sigma_H^*(p_H - p_L^*) + (1 - \sigma_H^*)(p_L - p_L^*), \tag{27}$$

and

$$r(p_L^* - c_L) = -\phi(\lambda_L^*) + \lambda_L^*(\sigma_H^*(p_H - p_L^*) + (1 - \sigma_H^*)(p_L - p_L^*)). \tag{28}$$

High type's reservation price and search optimality. Unlike in the baseline model, the high-type seller now faces a non-trivial optimization problem. Since she accepts only p_H, the Bellman equation is given by

$$r(p_H^* - c_H) = \max_{\lambda} -\phi(\lambda) + \lambda\sigma_H^*(p_H - p_H^*).$$

Therefore, her optimal search intensity λ_H^* must satisfy

$$\phi'(\lambda_H^*) = \sigma_H^*(p_H - p_H^*), \tag{29}$$

and

$$r(p_H^* - c_H) = -\phi(\lambda_H^*) + \lambda_H^*\sigma_H^*(p_H - p_L^*). \tag{30}$$

From (30), we see that the high-type seller obtains a strictly positive expected payoff (i.e., $p_H^* > c_H$), as long as $\sigma_H^* > 0$ (which is shown to be the case shortly). Then, from (29), $\lambda_H^* > \underline{\lambda}$. On the other hand, comparing (27) and (29), it is clear that $\lambda_H^* < \lambda_L^*$. This shows that the fundamental ingredients regarding endogenous search intensity are preserved in the model with exogenous prices.

Buyers' equilibrium offer strategies. For the same reason as in the baseline model, in equilibrium buyers must randomize between p_H and p_L. Given q^* and σ_S^*, this implies that

$$q^*(v_H - p_H) + (1 - q^*)(v_L - p_H) = (1 - q^*)\sigma_S^*(v_L - p_L). \tag{31}$$

There are 7 equilibrium variables, but only 6 equilibrium conditions, (26)-(31). As in the baseline model, an additional condition comes from the fact that $p_L^* \leq p_L$. If $p_L^* = p_L$, then the other 6 variables solve the 6 equations. If $p_L^* < p_L$, then the low-type seller strictly prefers accepting p_L to rejecting it, and thus $\sigma_S^* = 1$. Then, again, we have 6 variables and 6 equations.

The following proposition characterizes the equilibrium in which $p_L^* = p_L$. The detailed derivations of the equilibrium variables are relegated to the appendix. The equilibrium in which $p_L^* < p_L$

is omitted for brevity. It can be derived in an analogous manner.

Proposition 3 *Let $\tilde{\lambda}$ be the unique value such that $r(p_L - c_L) = -\phi(\tilde{\lambda}) + \tilde{\lambda}\phi'(\tilde{\lambda})$. If $\phi'(\tilde{\lambda}) \leq \widehat{q}(v_H - p_H)/(1-\widehat{q})$, then there exists an equilibrium in which $p_L^* = p_L$. In the equilibrium, $\lambda_L^* = \tilde{\lambda}$ and $\sigma_H^* = \phi'(\lambda_L^*)/(p_H - p_L)$. Given λ_L^* and σ_H^*, σ_S^*, q^*, p_H^*, and λ_H^* can be obtained from* (26), (29), (30), *and* (31).

Proof. See the appendix. ∎

4.3.4 Discussion

An intriguing possibility arises if buyer inspection is combined with exogenous prices: the solicitation effect can be a blessing, while the acceleration effect can be a curse. We illustrate this in the simplest model where each buyer receives a perfect signal about the quality of the good. It is fairly straightforward to generalize the result into the case where buyers' signals are noisy but sufficiently informative.

With perfect signals, buyers' optimal offer strategies are straightforward: each buyer offers p_H with a perfectly good signal and p_L with a perfectly bad signal. Given this, the type-a seller's reservation price p_a^* and optimal search intensity λ_a^* satisfy

$$r(p_a^* - c_a) = -\phi(\lambda_a^*) + \lambda_a^*(p_a - p_a^*),$$

and

$$\phi'(\lambda_a^*) = p_a - p_a^*.$$

Combining the two conditions,

$$\phi'(\lambda_a^*) = \frac{\phi(\lambda_a^*) + r(p_a - c_a)}{r + \lambda_a^*} \Leftrightarrow (r + \lambda_a^*)\phi'(\lambda_a^*) - \phi(\lambda_a^*) = r(p_a - c_a).$$

Since $(r + \lambda)\phi'(\lambda) - \phi(\lambda)$ is strictly increasing in λ, there exists a unique value of λ_a^* for each $a = H, L$.

Notice that λ_a^* is an increasing function of $p_a - c_a$. Therefore, if $p_H - c_H > p_L - c_L$, then $\lambda_H^* > \lambda_L^*$. This immediately implies that endogenous search intensity has the effect of decreasing buyers' unconditional beliefs (deceleration curse), while make their conditional beliefs higher than their unconditional beliefs (solicitation blessing). Formally, since $\lambda_H^* > \lambda_L^*$ and each type trades with the first buyer they meet,

$$q^u = \frac{\frac{\widehat{q}}{\lambda_H^*}}{\frac{\widehat{q}}{\lambda_H^*} + \frac{1-\widehat{q}}{\lambda_L^*}} < q^* = \widehat{q}.$$

Appendix: Omitted Proofs

Proof of Proposition 1.

(1) The equilibrium in which $p^* = v_L$.

It is straightforward that $q^* = (c_H - v_L)/(v_H - v_L)$. The other three equilibrium conditions are

$$\begin{aligned} \frac{\sigma_H^* + \sigma_L^*}{\sigma_H^*} &= \frac{1-\widehat{q}}{\widehat{q}} \frac{c_H - v_L}{v_H - c_H}, \\ r(v_L - c_L) &= -\phi(\lambda^*) + \lambda^* \sigma_H^* (c_H - v_L), \\ \phi'(\lambda^*) &= \sigma_H^*(c_H - v_L). \end{aligned}$$

From the last two conditions,

$$r(v_L - c_L) = -\phi(\lambda^*) + \lambda^* \phi'(\lambda^*).$$

This implies that $\lambda^* = \tilde{\lambda}$. Given λ^*, it follows that

$$\sigma_H^* = \frac{r(v_L - c_L) + \phi(\lambda^*)}{\lambda^*(c_H - v_L)}.$$

Finally, from the first condition,

$$\sigma_L^* = \left(\frac{1-\widehat{q}}{\widehat{q}} \frac{c_H - v_L}{v_H - c_H} - 1 \right) \sigma_H^* = \left(\frac{1-\widehat{q}}{\widehat{q}} \frac{c_H - v_L}{v_H - c_H} - 1 \right) \frac{r(v_L - c_L) + \phi(\lambda^*)}{\lambda^*(c_H - v_L)}.$$

This equilibrium is well-defined if and only if

$$\sigma_H^* + \sigma_L^* = \frac{1-\widehat{q}}{\widehat{q}} \left(\frac{r(v_L - c_L) + \phi(\lambda^*)}{\lambda^*(v_H - c_H)} \right) \leq 1.$$

(2) The equilibrium in which $p^* < v_L$.

As explained in the main body, $\sigma_H^* + \sigma_L^* = 1$. Therefore, we have the following four equilibrium conditions:

$$\begin{aligned} q^* &= \frac{\widehat{q}}{\widehat{q} + (1-\widehat{q})\sigma_H^*}, \\ \frac{1}{\sigma_H^*} &= \frac{1-\widehat{q}}{\widehat{q}} \frac{c_H - p^*}{v_H - c_H}, \\ r(p^* - c_L) &= -\phi(\lambda^*) + \lambda^* \sigma_H^* (c_H - p^*), \\ \phi'(\lambda^*) &= \sigma_H^*(c_H - p^*). \end{aligned}$$

From the second and last conditions,

$$\phi'(\lambda^*) = \frac{\widehat{q}(v_H - c_H)}{1 - \widehat{q}}.$$

For this, it is straightforward to calculate the following:

$$\begin{aligned} p^* &= c_L + \frac{\lambda^* \phi'(\lambda^*) - \phi(\lambda^*)}{r}, \\ \sigma_H^* &= \frac{\widehat{q}}{1 - \widehat{q}} \frac{v_H - c_H}{c_H - p^*}, \\ q^* &= \frac{\widehat{q}}{\widehat{q} + (1 - \widehat{q})\sigma_H^*}. \end{aligned}$$

This equilibrium is well-defined if and only if $p^* < v_L$. We show that this condition holds whenever (7) is violated. Note that

$$\begin{aligned} p^* < v_L \quad &\Leftrightarrow \quad r(p^* - c_L) = \lambda^* \phi'(\lambda^*) - \phi(\lambda^*) < r(v_L - c_L) \\ &\Leftrightarrow \quad \phi'(\lambda^*) < \frac{r(v_L - c_L) + \phi(\lambda^*)}{\lambda^*} \\ &\Leftrightarrow \quad 1 < \frac{1 - \widehat{q}}{\widehat{q}} \frac{r(v_L - c_L) + \phi(\lambda^*)}{\lambda^*(v_H - c_H)}. \end{aligned}$$

To see that this inequality is implied whenever (7) does not hold, define

$$F(\lambda) \equiv \frac{1 - \widehat{q}}{\widehat{q}} \frac{r(v_L - c_L) + \phi(\lambda)}{\lambda(v_H - c_H)}.$$

By its definition, it suffices to prove that $F(\tilde{\lambda}) > 1$ implies $F(\lambda^*) > 1$. Notice that

$$F'(\lambda) = \frac{1 - \widehat{q}}{\widehat{q}} \frac{\lambda \phi'(\lambda) - \phi(\lambda) - r(v_L - c_L)}{\lambda^2(v_H - c_H)}.$$

Since $\lambda\phi'(\lambda) - \phi(\lambda)$ is strictly increasing and $F'(\tilde{\lambda}) = 0$, $F(\cdot)$ strictly decreases until $\tilde{\lambda}$ and then strictly increases. The result immediately follows from this property of $F(\cdot)$.

(3) Uniqueness.

It suffices to show that if (7) holds, then there does not exist an equilibrium in which $p^* < v_L$. Suppose such an equilibrium exists. From the equilibrium conditions,

$$\lambda^* \phi'(\lambda^*) - \phi(\lambda^*) = r(p^* - c_L) < r(v_L - c_L) = \tilde{\lambda}\phi'(\tilde{\lambda}).$$

Since $\lambda\phi'(\lambda) - \phi(\lambda)$ is strictly increasing in λ, $\lambda^* < \tilde{\lambda}$. On the other hand, if (7) holds, then

$$\phi'(\tilde{\lambda}) = \frac{r(v_L - c_L) + \phi(\tilde{\lambda})}{\tilde{\lambda}} \leq \frac{\widehat{q}(v_H - c_H)}{1 - \widehat{q}} = \phi'(\lambda^*).$$

The convexity of $\phi(\cdot)$ implies $\tilde{\lambda} < \lambda^*$, which is a contradiction to the previous conclusion. ■

Proof of Proposition 3. Combining (27) and (28) and imposing $p_L^* = p_L$,

$$r(p_L - c_L) = -\phi(\lambda_L^*) + \lambda_L^*\phi'(\lambda_L^*).$$

Since $\lambda\phi'(\lambda) - \phi(\lambda)$ is strictly increasing, there exists a unique solution λ_L^* to this equation. Given λ_L^*, from (27),

$$\sigma_H^* = \frac{\phi'(\lambda_L^*)}{p_H - p_L}.$$

From (26) and (31),

$$\frac{1 - \widehat{q}}{\widehat{q}} \frac{\sigma_H^*}{\sigma_H^* + (1 - \sigma_H^*)\sigma_S^*} = \frac{1 - q^*}{q^*} = \frac{v_H - p_H}{p_H - v_L + \sigma_S^*(v_L - p_L)}.$$

If $\sigma_S^* = 0$ then, by Assumption 3, the left-hand side is strictly larger than the right-hand side. Therefore, a necessary and sufficient condition for σ_S^* to be well-defined is

$$\frac{1 - \widehat{q}}{\widehat{q}}\sigma_H^* \leq \frac{v_H - p_H}{p_H - p_L},$$

which is equivalent to

$$\phi'(\lambda_L^*) \leq \frac{\widehat{q}(v_H - p_H)}{1 - \widehat{q}}.$$

This is the condition given in the proposition.

Finally, rewriting (29),

$$p_H^* = \frac{-\phi(\lambda_H^*) + rc_H + \lambda_H^*\sigma_H^* p_H}{r + \lambda_H^*\sigma_H^*}.$$

Plugging this into (30),

$$\phi'(\lambda_H^*) = \sigma_H^*(p_H - p_H^*) = \sigma_H^* \frac{\phi(\lambda_H^*) + r(p_H - c_H)}{r + \lambda_H^*\sigma_H^*},$$

which is equivalent to

$$(r + \lambda_H^*\sigma_H^*)\phi'(\lambda_H^*) - \sigma_H^*\phi(\lambda_H^*) = \sigma_H^* r(p_H - c_H).$$

The left-hand side is smaller than the right-hand side if $\lambda_H^* = 0$. On the other hand, the left-hand side is larger if $\lambda_H^* = \lambda_L^*$, because

$$(r + \lambda_L^* \sigma_H^*)\phi'(\lambda_L^*) - \phi(\lambda_L^*)\sigma_H^* > r\phi'(\lambda_L^*) = \sigma_H^* r(p_H - p_L) > \sigma_H^* r(p_H - c_H).$$

Since the left-hand side is strictly increasing in λ_H^*, while the right-hand side is independent of it, there exists a unique value of λ_H^* that satisfies the equation. ■

Proof of Lemma 1. (1) $\xi(t)$ can be explicitly calculated as follows:

$$\begin{aligned}
\xi(t) &= \int_{\underline{t}}^{t} \frac{\phi\underline{\lambda}}{\overline{\lambda} - \underline{\lambda}} \frac{1}{(c_H - c_L - A) - e^{r(x-\underline{t})}(v_L - c_L - A)} dx \\
&= -A \int_{c_H - v_L}^{(c_H - c_L - A) - e^{r(t-\underline{t})}(v_L - c_L - A)} \frac{1}{y((c_H - c_L - A) - y)} dy \\
&= -\frac{A}{c_H - c_L - A} \int_{c_H - v_L}^{(c_H - c_L - A) - e^{r(t-\underline{t})}(v_L - c_L - A)} \left(\frac{1}{y} + \frac{1}{(c_H - c_L - A) - y} \right) dy \\
&= -\frac{A}{c_H - c_L - A} \left(\ln \left(\frac{(c_H - c_L - A) - e^{r(t-\underline{t})}(v_L - c_L - A)}{c_H - v_L} \frac{v_L - c_L - A}{e^{r(t-\underline{t})}(v_L - c_L - A)} \right) \right) \\
&= \frac{A}{c_H - c_L - A} \ln \left(\frac{(c_H - v_L)e^{r(t-\underline{t})}}{(c_H - c_L - A) - e^{r(t-\underline{t})}(v_L - c_L - A)} \right).
\end{aligned}$$

(2) Let $\omega(t) = \ln(q^u(t)) + \xi(t)$. Then, the differential equation is equivalent to

$$\omega'(t) = Be^{\omega(t) - \xi(t)} \underline{\lambda} \sigma_H(t) \Leftrightarrow (-e^{-\omega(t)})' = B(-e^{-\xi(t)})'.$$

This implies that

$$e^{-\omega(t)} = e^{-\omega(\underline{t})} + B(e^{-\xi(t)} - e^{-\xi(t_2^*)}) = \frac{1}{q^u(\underline{t})} + B(e^{-\xi(t)} - 1).$$

Combining this with $q^u(t) = e^{\omega(t) - \xi(t)}$,

$$q^u(t) = \frac{e^{-\xi(t)}}{\frac{1}{q^u(\underline{t})} + B(e^{-\xi(t)} - 1)}.$$

■

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