EFFICIENT DYNAMIC MECHANISMS IN ENVIRONMENTS WITH INTERDEPENDENT VALUATIONS

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This paper addresses the problem of implementing socially efficient allocations in dynamic environments with interdependent valuations and evolving private information. In the case where the agents' information is correlated across time, we construct efficient and incentive compatible dynamic mechanisms. Unlike the mechanisms with history-independent transfers in the existing literature, these mechanisms feature history-dependent transfers. Moreover, they are reminiscent of the classical VCG mechanism, even though the latter is not incentive compatible with interdependent valuations. In settings where agents' private information evolves independently, we construct the dynamic counterpart of the generalized VCG mechanism in one-dimensional environments. We also provide sufficient conditions for implementability, which are generalizations of the single-crossing conditions in static problems.

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1. INTRODUCTION

In this paper, we study efficient mechanism design in dynamic allocation problems with interdependent valuations. A canonical real-world example of such problems is the following: Periodically, the U.S. government uses auctions to sell licenses for the right to drill for oil in adjacent offshore areas. Bidders in these auctions are oil firms. Presumably, these firms conduct geological surveys to estimate the amount of oil in each area before bidding in each auction, so that the information obtained by one firm is also valuable for the other firms. The efficient allocation of licenses depends on the evolving private information of the firms; so the government should carefully design the auctions to induce truthful revelation by the firms in every period. More abstractly, in the problems of interest, a sequence of decisions needs to be made over time: in each period an allocation is to be made among a group of agents, who have time-varying, payoff-relevant private information. Efficient mechanism design is the question of how to truthfully implement socially efficient allocations, i.e., how to handle the incentive compatibility constraints implied by the evolving private information.

Following the literature, we will restrict ourselves to the case of quasi-linear preferences and private information that follows a general Markov process whose evolution depends on allocations. In this environment, and under the assumption that valuations are private, i.e., not interdependent, Bergemann and Välimäki (2010) and Athey and Segal (2012) have successfully addressed this question, by means of dynamic extensions of the classical VCG and AGV mechanisms. However, with interdependence, it is well known that the VCG mechanism and its dynamic extensions are not incentive compatible without additional strong assumptions. The key insight of the VCG mechanism—making each agent a residual claimant—is not applicable when an agent's information affects others' utilities. In fact, in generic environments with multidimensional and statistically independent private information, Dasgupta and Maskin (2000) and Jehiel and Moldovanu (2001) have shown no efficient mechanism, VCG or not, is Bayesian incentive compatible.[1] On the other hand, with correlated private information, the lottery mechanism of Crémer and McLean (1988) is efficient and Bayesian incentive compatible. Yet in dynamic environments, a period-by-period extension of Crémer and McLean's mechanism is not incentive compatible, because agents have more opportunities to deviate.[2]

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[1]Jehiel, et. al. (2006) further prove that only constant allocation rules are ex post incentive compatible in generic models with multidimensional signals.

[2]See the example in subsection 3.1.

But notice that long-term interactions offer a richer family of transfer schemes compared to the static case, in particular, transfers can be made history-dependent. With such transfers, an agent's current report affects not only her current payoff, but also the entire stream of future transfers. Therefore, one might be able to restore incentive compatibility with a careful choice of inter-temporal trade-offs. We show that this is indeed the case. For the above-mentioned dynamic allocation problems, we construct efficient and incentive compatible dynamic mechanisms, provided that information is correlated over time, as we explain below. In addition, the mechanisms ensure that each agent becomes a residual claimant, as in the VCG mechanism. That is, in each period and regardless of the history, an agent's expected continuation payoff equals the continuation social surplus when all agents truthfully report their private information. In other words, not only do we provide a solution to the dynamic incentive compatibility issue with interdependence, but also the solution shares some of the main features of the VCG mechanism.[3]

The inter-temporal correlation that is required for our results resembles the correlation conditions in Crémer and McLean (1988) when the state space of the Markov process is finite. That is, we require convex or linear independence conditions on the associated transition matrices.[4] In the infinite case, a condition similar to McAfee and Reny's extension (cf. McAfee and Reny (1992)) of Crémer and McLean (1988) allows us to construct an efficient dynamic mechanism that is approximately incentive compatible.

Having established incentive compatible and efficient mechanisms, we proceed to address the issues of balancing the budget and extracting the entire surplus of the agents. Specifically, by modifying the transfers, we construct (i) an average externality mechanism that balances the budget,[5] and (ii) a lottery-augmented mechanism à la Crémer and McLean (1988) and McAfee and Reny (1992) that extracts all the surplus of the agents in the finite case and virtually all the surplus in the infinite case.

Finally, while the main results require inter-temporal correlation, we also study the case where each agent's private information evolves independently. We focus on settings with one-dimensional

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[3]From a practical viewpoint, the constructed history-dependent transfers also point toward a new way of linking information that has been largely ignored in the design of various economic mechanisms.

[4]These conditions are related to, but different from those in Crémer and McLean (1988) for static mechanisms with correlated signals. Specifically, we do not impose any restriction on the information structure within a period.

[5]The mechanism is related to the balanced team mechanism constructed in Athey and Segal (2012), which generalizes the AGV mechanism introduced by Arrow (1979) and d'Aspremont and Gérard-Varet (1979) to dynamic environments with independent private valuations.

private information and construct transfers that are the dynamic counterpart of the generalized VCG mechanism (cf. Crémer and McLean (1985), Jehiel and Moldovanu (2001), Bergemann and Välimäki (2002)). In the private-valuation special case, these transfers reduce to the dynamic pivot mechanism constructed by Bergemann and Välimäki (2010). In the general interdependence case, we identify dynamic single-crossing conditions that ensure incentive compatibility.

1.1. *Related Literature*

Efficient mechanisms with interdependent valuations. In addition to the papers mentioned above, our dynamic mechanisms are also related to the two-stage VCG mechanism in Mezzetti (2004, 2007).[6] Mezzetti provides one way to bypass the above impossibility results, under the assumptions that agents can observe their realized utilities and that transfers can be made based on the reported utilities. From an applied perspective, these are strong assumptions. More importantly, in Mezzetti's mechanism, agents are indifferent among all messages when they report their utilities. If reporting utilities is costly, then agents would rather walk away from the mechanism at this stage. In comparison, we consider direct mechanisms that ask require agents to report their private signals in each period; furthermore, truth-telling constitutes a perfect equilibrium and no agent is indifferent among all messages at any stage.

Dynamic mechanism design. Most of the recent literature on dynamic mechanisms assumes independent private valuations (e.g., Bergemann and Välimäki (2010), Athey and Segal (2012), Said (2012), and Pavan, Segal and Toikka (2013)), with an exception of Gershkov and Moldovanu (2009a). Gershkov and Moldovanu consider a problem of sequential allocations of objects to impatient agents who arrive over time.[7] In their model, time horizon is finite, valuations are private, and signals are one-dimensional. They show that if the distribution of signals is unknown, then interdependence arises endogenously as a result of learning, which may prevent efficient implementation with online mechanisms.[8,9] Since agents are impatient in Gershkov and Moldovanu's model, the incentive problems are static. They identify single-crossing conditions on the underlying uncertainty, which ensure efficient implementation. Related to the history-dependent mechanisms

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[6]See Deb and Mishra (2013) for a related recent study.

[7]See also Gershkov and Moldovanu (2010, 2012) for studies of related questions.

[8]Segal (2003) also emphasizes this feature in a static model.

[9]The term "online mechanism" is mostly used in the algorithmic game theory literature to study allocation problems with arrivals and departures; it requires that allocations and transfers of an agent are made when she is present.

in this paper, they also point out that efficient implementation in their model is possible if all transfers can be delayed to the last period.

In an independent paper, Hörner, Takahashi and Vieille (2013) also study the role of inter-temporal correlation in dynamic Bayesian games with communication. They study truthful equilibria and extend the insight of Crémer and McLean (1988) (and also the static budget-balanced mechanism in Kosenok and Severinov (2008)) to dynamic games. By contrast, we study a general mechanism design problem with transferable utilities and interdependent valuations; and our main results emphasize the VCG feature of history-dependent transfers, which is absent from their game-theoretic analysis.

Full surplus extraction. We also extend the full-surplus-extraction results (cf. Crémer and McLean (1985), Crémer and McLean (1988), and McAfee and Reny (1992)) to dynamic environments with Markov private information. We show that, in dynamic problems, it is critical to exploit the inter-temporal correlation rather than intra-period correlation, as the former is immune to belief manipulation, whereas the latter is not. The dynamic full-surplus-extraction mechanism consists of (1) lottery transfers, which yield stochastic bonuses to agents, and (2) participation fees, which serves to extract the expected flow surplus from agents. These participation fees, which do not appear in Crémer and McLean's lottery mechanism, are needed to address agents' dynamic participation constraints.

2. MODEL

2.1. *The Environment*

We consider a dynamic interdependent valuation environment with N ($N \geq 2$) agents. Time is discrete, indexed by $t \in \{1, 2, \ldots, T\}$, where $T \leq \infty$.[10] In each period t, each agent $i \in \{1, 2, \ldots, N\}$ privately observes a pay-off relevant signal $\theta_t^i \in \Theta_t^i$, where Θ_t^i is a *finite* set. The extension to the infinite signal space case is studied in Section 6. The signal space in period t is $\Theta_t = \prod_{i=1}^N \Theta_t^i$ with a generic element $\theta_t = (\theta_t^1, \ldots, \theta_t^N)$. For each i and t, denote the private information held by agents other than i in period t by $\theta_t^{-i} = (\theta_t^1, \ldots, \theta_t^{i-1}, \theta_t^{i+1}, \ldots, \theta_t^N) \in \prod_{j \neq i} \Theta_t^j$.

In each period t, the flow utility u^i of agent i is determined by the current signal profile θ_t, the current allocation $a_t \in A_t$ and the current monetary transfer $p_t^i \in \mathbb{R}$, where A_t is the *finite* set of social alternatives in period t. The flow utility of each agent is assumed to be quasilinear

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[10]We study both cases where the time horizon is either finite or infinite.

in monetary transfers, and agents have a common discount factor $\delta \in (0, 1)$. Given sequences of signals $\{\theta_t\}_{t=1}^T$, allocations $\{a_t\}_{t=1}^T$ and monetary transfers $\{p_t^1, \ldots, p_t^N\}_{t=1}^T$, the total payoff of each agent i is

$$\sum_{t=1}^T \delta^{t-1} \left[u^i(a_t, \theta_t) - p_t^i \right].$$

Agent's private signals evolve over time following a Markov chain. Specifically, in the initial period, the signal profile θ_1 is drawn from a prior probability $\mu_1 \in \Delta(\Theta_1)$. In each period $t > 1$, the distribution of current signal profile θ_t is determined by the realized signal profile θ_{t-1} and the allocation decision a_{t-1} in the previous period, represented by a transition probability $\mu_t : A_{t-1} \times \Theta_{t-1} \to \Delta(\Theta_t)$. The utility functions u^i, the prior μ_1 and the transition probabilities μ_t are assumed to be common knowledge.

In contrast to previous work that often assumes independent prior and transitions across agents, here we specify a general Markov chain for the evolution of signals, which allows correlation of private information. While in private-valuation environments the existence of efficient mechanisms does not depend on whether correlation is allowed or not, as shown by Athey and Segal (2012), it will be clear in Section 3 how correlation makes a difference in dynamic settings with interdependent valuations.

2.2. *Efficiency and Mechanisms*

A socially efficient allocation rule is a sequence of functions $\{a_t^* : \Theta_t \to A_t\}_{t=1}^T$ that solves the following social program

$$\max_{\{a_t\}_{t=1}^T} \mathbb{E} \left[\sum_{t=1}^T \delta^{t-1} \sum_{i=1}^N u^i(a_t, \theta_t) \right],$$

where the expectation is taken with respect to the processes $\{\theta_t\}$ and $\{a_t\}$. Since the flow utility depends only on current signal profile, which is assumed to be Markov, the social program can also be written in recursive form. Specifically, for each $t \in \{1, 2, \ldots, T\}$

$$(1) \qquad W_t(\theta_t) = \max_{a_t \in A} \sum_{i=1}^N u^i(a_t, \theta_t) + \delta \mathbb{E} \left[W_{t+1}(\theta_{t+1}) | a_t, \theta_t \right],$$

where $W_t(\theta_t)$ is the social surplus starting from period t given the realized signal profile θ_t, and $W_{T+1} \equiv 0$. By the principle of optimality, a_t^* solves the social program if and only if it is a solution to this recursive problem.

We focus on truthful equilibria of direct public mechanisms that implement the socially efficient allocations $\{a_t^*\}_{t=1}^T$. In a direct public mechanism, in each period t, each agent i is asked to make a public report $r_t^i \in \Theta_t^i$ of her current private signal θ_t^i. Then a public allocation decision a_t and a transfer p_t^i for each agent i are made as functions of the current report profile $r_t = (r_t^i)_{i=1}^N$ and the period-t public history h_t.[11] The period-t public history contains all reports and allocations up to period $t - 1$, i.e.,

$$h_t = (r_1, a_1, r_2, a_2, \ldots, r_{t-1}, a_{t-1}).$$

Let H_t denote the set of possible period-t public histories. Formally, an *efficient direct revelation mechanism* $\Gamma = \{\Theta_t, a_t^*, p_t\}_{t=1}^T$ consists of (i) Θ_t as the message space in each period t; (ii) a sequence of allocation rules $a_t^* : \Theta_t \to A$, and (iii) a sequence of monetary transfers $p_t : H_t \times \Theta_t \to \mathbb{R}^N$.

The period-t private history h_t^i of each agent i contains the period-t public history and the sequence of her realized private signals until period t, i.e.,

$$h_t^i = (r_1, a_1, \theta_1^i, r_2, a_2, \theta_2^i, \ldots, r_{t-1}, a_{t-1}, \theta_{t-1}^i, \theta_t^i).$$

Let H_t^i denote the set of agent i's possible period-t private histories. With a slight abuse of notation, a strategy for agent i is a sequence of mappings $r^i = \{r_t^i\}_{t=1}^T$ where $r_t^i : H_t^i \to \Theta_t^i$ assigning a report to each of her period-t private history. A strategy for agent i is *truthful* if it always reports agent i's private signal θ_t^i truthfully in each period t, regardless of her private history.

Given a mechanism $\Gamma = \{\Theta_t, a_t^*, p_t\}_{t=1}^T$ and a strategy profile $r = \{r^i\}_{i=1}^N$, agent i's expected discounted payoff is

$$\mathbb{E} \sum_{t=1}^T \delta^{t-1} \left[u^i(a_t^*(r_t), \theta_t) - p_t^i(h_t, r_t) \right].$$

The equilibrium concept we adopt is periodic ex post equilibrium defined by Bergemann and Välimäki (2010) and Athey and Segal (2012). We say that the mechanism is *periodic ex post incentive compatible*, or equivalently, the truthful strategy profile is a *periodic ex post equilibrium* if for each agent and in each period, truth-telling is always a best response regardless of the private

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[11]It is well-known that the set of implementable allocations in a dynamic model depends on the information disclosed by the mechanism. Results are the least permissive for public mechanisms as agents can devise their reports contingent on more information.

history and the current signals of other agents, given that other agents adopt truthful strategies. Formally, let $V_t^i(h_t^i)$ be agent i's continuation payoff given period-t private history, given that other agents report truthfully. That is,

$$(2) \qquad V_t^i(h_t^i) = \max_{r_t^i \in \Theta_t^i} \mathbb{E}\left[u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t) - p_t^i(h_t, r_t^i, \theta_t^{-i}) + \delta V_{t+1}^i(h_{t+1}^i)\right],$$

with $V_{T+1}^i \equiv 0$. The efficient mechanism is *periodic ex post incentive compatible* if for each i, t and h_t^i,

$$\theta_t^i \in \arg\max_{r_t^i \in \Theta_t^i} u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t) - p_t^i(h_t, r_t^i, \theta_t^{-i}) + \delta\mathbb{E}\left[V_{t+1}^i(h_{t+1}^i)|a_t^*(r_t^i, \theta_t^{-i}), \theta_t\right],$$

for each $\theta_t \in \Theta_t$.

As suggested by Bergemann and Välimäki (2010), ex post incentive compatibility notions need to be qualified within each period in a dynamic environment, since an agent may wish to change her report in some previous round based on the new information she has received in later periods. Given the fact that interdependent valuations render dominant strategy incentive compatibility impossible, periodic ex post incentive compatibility is the best we can hope for in the current setup.

Finally, we turn to budget-balancedness. The mechanism is *ex ante budget balanced* if

$$\mathbb{E}\left[\sum_{t=1}^{T} \delta^{t-1} \sum_{i=1}^{N} p_t^i\right] \geq 0.$$

The mechanism is *budget balanced* if for each t,

$$\mathbb{E}\left[\sum_{i=1}^{N} p_t^i\right] = 0.$$

The mechanism is *ex post budget balanced* if for each t,

$$\sum_{i=1}^{N} p_t^i \equiv 0.$$

These notions are related to the mechanism designer's financing abilities. When the designer has access to long-term outside financing, an ex ante balanced budget means that the expected present value of all transfers from agents is non-negative. If the financing ability is limited, the relevant notion is budget-balancedness, which says that in each period the designer breaks even on average. Without any outside financing, ex post budget-balancedness requires that agents' transfers sum to zero in each period for any realized signal profile.

3. EFFICIENT MECHANISM DESIGN

3.1. *An Example*

Before presenting the general results, we present a two-period repeated auction example to explain the main ideas.[12] Two firms, 1 and 2, compete for licenses to drill for oil on two adjacent off-shore areas. The two licenses are sold sequentially in two auctions ($t \in \{1, 2\}$) and the allocation in auction t is $a_t \in \{1, 2\}$ where $a_t = i$ means that firm i obtains the license for the corresponding area. Each firm's payoff from obtaining a license depends on its drilling cost and the amount oil s_t in that area:

$$u^1(s_t) = 2s_t - 1, \quad u^2(s_t) = 3s_t - 6.$$

Suppose that there is no discounting and each firm cares about its total profit from both auctions. Each firm i observes a private signal θ_t^i in auction t. Suppose that prior to the auctions each firm can perform a test in one of the areas. In particular, firm 1's private signal $\theta_1^1 \in \{4, 6\}$ indicates the amount of oil in area 1: $\theta_1^1 = s_1$; and firm 2 learns privately from $\theta_2^2 \in \{4, 6\}$ the expected amount of oil in area 2: $\theta_2^2 = s_2$. In addition, we assume that the joint distribution of θ_1^1 and θ_2^2, denoted by $\mu(\theta_1^1, \theta_2^2)$, is

$$\begin{bmatrix} \mu(4, 4) & \mu(4, 6) \\ \mu(6, 4) & \mu(6, 6) \end{bmatrix} = \begin{bmatrix} 3/8 & 1/8 \\ 1/8 & 3/8 \end{bmatrix}$$

so that the conditional distribution of θ_2^2 given θ_1^1, denoted by $\mu(\theta_2^2|\theta_1^1)$, is

$$\begin{bmatrix} \mu(4|4) & \mu(4|6) \\ \mu(6|4) & \mu(6|6) \end{bmatrix} = \begin{bmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{bmatrix}.$$

Finally, we assume that firm 2 does not learn any relevant information in the first auction, nor does firm 1 in the second auction. That is, θ_1^1 and θ_1^2 are independently distributed, so are θ_2^1 and θ_2^2.

First, we notice that efficiency and incentive compatibility are incompatible if only the first auction were conducted. To see this, note that efficiency in the first auction requires firm 1 to give up the license when it is more profitable, i.e.,

$$a_1^* = \begin{cases} 1, & \text{if } \theta_1^1 = 4, \\ 2, & \text{if } \theta_1^1 = 6. \end{cases}$$

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[12]The example is adapted and extended from Dasgupta and Maskin (2000).

This implies that firm 1 needs to be compensated from reporting $r_1^1 = 6$ rather than $r_1^1 = 4$. Specifically, we have the following incentive compatibility conditions:

$$2 \times 4 - 1 - p_1^1(4) \geq 0 - p_1^1(6)$$

$$0 - p_1^1(6) \geq 2 \times 6 - 1 - p_1^1(4).$$

Summing up the two inequalities gives $4 \geq 6$. Thus, no incentive compatible transfer exists. On the other hand, when only the second license is being auctioned, firm 2's incentive constraint matters and it is straightforward to verify that the following generalized VCG transfer for firm 2,

$$p_2^2 = \begin{cases} 0, & \text{if } r_2^2 = 4, \\ 11, & \text{if } r_2^2 = 6, \end{cases}$$

truthfully implements the efficient allocation a_2^* in the second auction, where a_2^* is given by

$$a_2^* = \begin{cases} 1, & \text{if } \theta_2^2 = 4, \\ 2, & \text{if } \theta_2^2 = 6. \end{cases}$$

Now we show that by linking the two auctions, dynamic efficiency is implementable, despite the impossibility for static efficiency. The idea is to use the correlation between θ_1^1 and θ_2^2 and construct a history-dependent transfer for firm 1 in the *second* auction so that firm 1 is willing to report its true signal in the *first* auction. For instance, consider the transfer schedule $p_2^1(a_1, r_2^2)$ given by

$$p_2^1 = \begin{cases} -4, & \text{if } a_1 = 0, \ r_2^2 = 4, \\ -16, & \text{if } a_1 = 0, \ r_2^2 = 6, \\ 0, & \text{otherwise.} \end{cases}$$

We claim that the dynamic mechanism $\Gamma^{link} \equiv \{(a_1^*, a_2^*), (p_2^1, p_2^2)\}$ is ex post incentive compatible. Recall that truth-telling is optimal for firm 2 given p_2^2. Since the transfer p_2^1 has no effect on firm 2's incentive constraints, under $\{p_2^1, p_2^2\}$ firm 2 is still willing to report its true signal in the second auction. Now consider firm 1's incentive constraints. Firm 1, when reporting its signal, takes into account the fact that its future transfer depends on the current allocation a_1 and the opponent's report r_2^2 in the next auction. As a consequence, the incentive compatibility constraints are satisfied given the specified conditional distribution of signals:

$$2 \times 4 - 1 + 0 \geq 0 + \left(\frac{3}{4} \times 4 + \frac{1}{4} \times 16 \right)$$

$$0 + \left(\frac{1}{4} \times 4 + \frac{3}{4} \times 16 \right) \geq 2 \times 6 - 1 + 0.$$

The intuition for this mechanism is as follows. Note that by construction the left-hand-side of the above two inequalities are equal to the social surplus given firm's 1 private signal. By exploiting the inter-temporal correlation between θ_1^1 and θ_2^2, the transfer p_2^1 makes firm 1 a claimant of the social surplus in the first auction (without affecting any firm's incentive constraints in the second auction). Given that firm 2 adheres to truthful strategies, it is optimal for firm 1 to be truthful so as to maximize the social surplus and hence its own profit.

Now let us modify the example to illustrate the role of intertemporal correlation and its superiority over within-period correlation (Crémer and McLean (1988)) in dynamic mechanisms. We remove the assumption that θ_1^1 and θ_1^2 are independent, and suppose that before firms learn their payoff relevant signals, firm 1 has access to some private signal $\theta_0^1 \in \{0, 1\}$ that determines the joint distribution $\mu(\theta_1^1, \theta_1^2 | \theta_0^1)$ of θ_1^1 and θ_1^2:

$$\left[\begin{array}{cc} \mu(4,4|0) & \mu(4,6|0) \\ \mu(6,4|0) & \mu(6,6|0) \end{array} \right] = \left[\begin{array}{cc} 1/8 & 3/8 \\ 3/8 & 1/8 \end{array} \right],$$

$$\left[\begin{array}{cc} \mu(4,4|1) & \mu(4,6|1) \\ \mu(6,4|1) & \mu(6,6|1) \end{array} \right] = \left[\begin{array}{cc} 3/8 & 1/8 \\ 1/8 & 3/8 \end{array} \right].$$

That is, θ_1^1 and θ_1^2 are negatively correlated if $\theta_0^1 = 0$, and positively correlated if $\theta_0^1 = 1$. Finally, the joint distribution of θ_1^1 and θ_2^2 remains the same and is assumed to be independent of θ_0^1.

Suppose that the auctioneer wants to exploit the correlation between θ_1^1 and θ_1^2 to incentivize firm 1. This amounts to constructing lottery transfers for firm 1 based on firm 2's first period report r_1^2. However, for such lotteries to work, the auctioneer needs to know the joint distribution of θ_1^1 and θ_1^2, which is firm 1's private information. Given a lottery scheme in the first auction, firm 1 may have an incentive to misreport its signal θ_0^1. To see this, suppose that the auctioneer believes that firm 1's initial report $r_0^1 \in \{0, 1\}$ is truthful, and thus uses the following transfers $p_1^1(r_1^1, r_1^2; r_0^1)$ for firm 1:

$$\left[\begin{array}{cc} p_1^1(4,4;0) & p_1^1(4,6;0) \\ p_1^1(6,4;0) & p_1^1(6,6;0) \end{array} \right] = \left[\begin{array}{cc} 13 & 5 \\ 0 & 0 \end{array} \right],$$

$$\left[\begin{array}{cc} p_1^1(4,4;1) & p_1^1(4,6;1) \\ p_1^1(6,4;1) & p_1^1(6,6;1) \end{array} \right] = \left[\begin{array}{cc} 5 & 13 \\ 0 & 0 \end{array} \right].$$

Given the joint distributions, it is straightforward to check that under $p_1^1(r_1^1, r_1^2; r_0^1)$, if firm 2 reports its signals truthfully then it is optimal for firm 1 to reveal θ_1^1 and obtain zero surplus in the first auction, *had it reported its initial private signal θ_0^1 truthfully.* However, given $p_1^1(r_1^1, r_1^2; r_0^1)$, firm 1 could benefit from misreporting θ_0^1. For example, when $\theta_0^1 = 0$, the following contingent deviations of firm 1 is profitable: it first reports $r_0^1 = 1$ so that the transfer in the first auction is $p_1^1(r_1^1, r_1^2; 1)$; then after learning θ_1^1, it always reports the opposite $r_1^1 \neq \theta_1^1$. When $\theta_1^1 = 4$, firm 1 reports $r_1^1 = 6$ and loses the first auction with no surplus:

$$0 - \frac{1}{4} \times p_1^1(6, 4; 1) - \frac{3}{4} \times p_1^1(6, 6; 1) = 0;$$

when $\theta_1^1 = 6$, firm 1 wins by reporting $r_1^1 = 4$ and receives a positive surplus:

$$2 \times 6 - 1 - \frac{3}{4} \times p_1^1(4, 4; 1) - \frac{1}{4} \times p_1^1(4, 6; 1) = 4.$$

Similar contingent deviations of firm 1 exist when $\theta_0^1 = 1$.

Finally, we note that since the inter-temporal correlation cannot be manipulated by either firms, the dynamic mechanism Γ^{link} constructed before remains ex post incentive compatible.

3.2. *Main Results*

In this section, we construct periodic ex post incentive compatible efficient dynamic mechanisms under general transition dynamics. Theorem 3.1 shows that under a generic inter-temporal correlation condition and some restrictions on utility functions and signal spaces in the last period, such a dynamic mechanism always exists.[13] In particular, we show that in each period t the correlation between θ_t^i and θ_{t+1}^{-i} can be used to construct history-dependent transfers such that agent i's incentive is aligned with the social incentive. Moreover, the resulting transfers are reminiscent of both the VCG transfers and the lottery transfers in Crémer and McLean (1988). In Theorem 3.2, we show that a slightly stronger inter-temporal correlation condition ensures dynamic efficiency with a sequence of "VCG-type" transfers.

We make the following assumptions on the utility functions and the evolution of private information.

Assumption 1 (Bounded payoffs) For each agent i,

$$\sup_{(a_t, \theta_t)_{t \geq 1}} \sum_{t=1}^{T} \delta^{t-1} \left| u^i(a_t, \theta_t) \right| < \infty.$$

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[13]For the infinite-horizon case, no such restrictions are imposed.

Assumption 2 (Convex independence) For each $t \geq 1$, $i \in N$, $a_t \in A_t$, and $\theta_t^{-i} \in \Theta_t^{-i}$, there does not exist a $\theta_t^i \in \Theta_t^i$ and a collection $\{\xi^i(\tilde{\theta}_t^i)\}_{\tilde{\theta}_t^i \in \Theta_t^i \setminus \{\theta_t^i\}}$ such that

1. $\xi^i(\tilde{\theta}_t^i) \geq 0$, for all $\tilde{\theta}_t^i \in \Theta_t^i \setminus \{\theta_t^i\}$, and
2. $\mu_{t+1}^{-i}(\theta_{t+1}^{-i}|a_t, \theta_t) = \sum_{\tilde{\theta}_t^i \neq \theta_t^i} \xi^i(\tilde{\theta}_t^i) \mu_{t+1}^{-i}(\theta_{t+1}^{-i}|a_t, \tilde{\theta}_t^i, \theta_t^{-i})$, for all $\theta_{t+1}^{-i} \in \Theta_{t+1}^{-i}$.

Assumption 3 (Spanning condition) For each $t \geq 1$, $i \in N$, $a_t \in A_t$, and $\theta_t^{-i} \in \Theta_t^{-i}$, there does not exist a collection $\{\eta^i(\theta_t^i)\}_{\theta_t^i \in \Theta_t^i}$, not all equal to zero, such that

$$\sum_{\theta_t^i \in \Theta_t^i} \eta^i(\theta_t^i) \mu_{t+1}^{-i}(\theta_{t+1}^{-i}|a_t, \theta_t^i, \theta_t^{-i}) = 0,$$

for all $\theta_{t+1}^{-i} \in \Theta_{t+1}^{-i}$.

Assumption 1 says that the payoff function of each agent is well-defined. This assumption is vacuous in the case where allocation spaces and signal spaces are time-independent. Assumptions 2 and 3 require that transition probabilities exhibit inter-temporal correlation among different agents' signals. In particular, for each agent i and in each period t, conditional on any a_t and θ_t^{-i}, agent i's current private signal θ_t^i is correlated with other agents' signals θ_{t+1}^{-i} in the next period. Independent evolution of private information across agents is ruled out by these assumptions. Moreover, Assumption 3 is stronger than Assumption 2. Crémer and McLean (1988) consider similar conditions in the study of static mechanism design with correlated information.

To motivate the information correlation assumptions, suppose that there is a underlying state of nature ω_t with possible values in a set Ω in each period t. In addition, ω_t follows a hidden Markov process which evolves over time and is not observed by any agent. In each period t, the relationship between the state of nature ω_t and agents' private information θ_t is described by a joint distribution ξ_t over $\Omega \times \Theta_t$. If each agent's private signal θ_t^i provides useful information about ω_t, i.e., the conditional $\xi_t(\omega_t|\theta_t^i)$ varies with θ_t^i, then as long as ω_t is not independently distributed, θ_t^i is correlated with θ_{t+1}^{-i} even conditional on θ_t^{-i} and a_t.

In the finite-horizon case $(T < \infty)$, we also impose the following ex post implementability assumption on the allocation rule a_T^*.

Assumption 4 (Ex post implementability in period T) If $T < \infty$, then the efficient allocation in period T, a_T^*, is ex post implementable.

In our setup, the allocation problem in period T is essentially a static one. Thus, we can adopt a set of sufficient conditions from the existing literature (Bergemann and Välimäki (2002) in par-

ticular) on static mechanism design. The sufficient conditions for ex post efficient implementation in static models are restrictive given the impossibility results in Dasgupta and Maskin (2000) and Jehiel and Moldovanu (2001). In particular, period-T signals have to be one-dimensional, and the utility functions have to satisfy a single-crossing condition.

We also emphasize that no assumption is imposed on the private signals from period 1 to $T-1$. We can think of a situation where agents trade a new asset with each other in multiple periods. Initially, each agent's private information may be multidimensional since there is much uncertainty about many aspects of the asset. As agents trade over time, they gradually learn more information about the asset. In the last period, each agent's signal is simply a real number that represents her estimation of the asset value.

Now we state the main results that generalize the idea of the example in Section 3.1. All proofs of the results in Sections 3 and 4 are relegated to Appendix A.

THEOREM 3.1   *Under Assumptions 1, 2, and 4, there exists a sequence of transfers*

$$p_{t+1}^i : \Theta_{t+1}^{-i} \times \Theta_t^i \times A_t \times \Theta_t^{-i} \to \mathbb{R}, \quad \forall i, t < T,$$

such that the efficient dynamic mechanism $\{a_t^*, p_t\}$ is periodic ex post incentive compatible.

Here we give a heuristic argument. Recall that in the private-valuation case, the following history-independent transfers in the team mechanism (cf. Athey and Segal (2012))

$$(3) \qquad p_t^i(\theta_t) = \sum_{j \neq i} u^j(a_t^*(\theta_t), \theta_t) = \sum_{j \neq i} u^j(a_t^*(\theta_t), \theta_t^j),$$

are incentive compatible. However, with interdependent valuations, transfers in (3) depend directly on agent i's report, which creates incentive for misreporting. To fix this problem, we consider general history-dependent transfers $p_t^i(h_t, \theta_t)$. It turns out that under Assumptions 1, 2, and 4, it is enough to use transfers that depend on the history in the previous round. Specifically, we show that if $T = \infty$, there exist transfers $p_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i})$ under which truthful strategy profile is a periodic ex post equilibrium. These history-dependent transfers work as follows. In each period t, the transfer p_t^i for agent i does not depend on her current report r_t^i, so agent i's incentive in period t is unaffected by p_t^i. Instead, her transfer in the next period p_{t+1}^i depends on r_t^i and a_t, which means that truth-telling incentive in period t is provided through p_{t+1}^i. Under the truth-telling strategy profile, in period $t+1$ agent i receives the sum of period-t flow payoffs of all other agents,

so agent i's continuation payoff in period t is equal to the social surplus from period t onward. Furthermore, the transfer for agent i in period $t + 1$ is such that there will be no expected gain from lying in period t. Therefore, agent i has no incentive to deviate from truth-telling in period t.

The above argument also suggests the necessity of a boundary condition for the incentive problem in the last period (when T is finite). Since the allocation problem in period T is static and there is no available information afterward, Assumption 4 is needed.[14]

The next result shows that under a slightly stronger condition on the transition probabilities, the dynamic efficient allocations are implementable with a sequence of "VCG-type" transfers for each agent in the sense that each agent's report in each period affects her payoff only through the determination of allocation.

THEOREM 3.2   *Under Assumptions 1, 3, and 4, there exists a sequence of transfers*

$$\bar{p}_{t+1}^i : \Theta_{t+1}^{-i} \times A_t \times \Theta_t^{-i} \to \mathbb{R}, \quad \forall i, t < T,$$

such that the efficient dynamic mechanism $\{a_t^*, \bar{p}_t\}$ is periodic ex post incentive compatible.

The efficient mechanism in Theorem 3.2 shares another distinctive feature of the VCG mechanism: each agent's report affects her own transfers only through the impact on allocations. The intuition in this case is even simpler. The transfer $\bar{p}_t^i$ for agent i does not depend on θ_t^i nor θ_{t-1}^i. Instead, incentive for truth-telling in period t is again guaranteed through $\bar{p}_{t+1}^i$: under $\bar{p}_{t+1}^i$, agent i's continuation payoff in period t is equal to the social surplus from period t onward.

REMARK 3.3   If $|\Theta_t^i| \leq |\Theta_{t+1}^{-i}|$ for each i and t, then Assumptions 2 and 3 are generically satisfied even if in each period signals are independently distributed conditional on all the available information. Accordingly, efficient dynamic mechanisms exist in a large class of dynamic environments provided that ex post implementability is achievable in the last period (Assumption 4). Moreover, if the time horizon is infinite then Assumption 4 has no bite. Therefore, instead of creating difficulties for efficient implementation as one would imagine, repeated interactions in fact facilitate the construction of incentive compatible transfers.

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[14]Bayesian implementability of a_T^* is not enough for our result to hold, as agents have the opportunity to manipulate the designer's period-T belief by misreporting in period $T - 1$.

REMARK 3.4   We have considered sufficient conditions for the existence of history-dependent transfers that implement the efficient allocation. There exist other weaker conditions on the transition probabilities. For example, each agent i's period-t signal θ_t^i could be correlated with all future signals θ_s^{-i} $(s > t)$ of other agents. If so, agent i's truth-telling incentive in each period could be provided through all future reports of other agents.

REMARK 3.5   In the finite-horizon case $(T < \infty)$, without imposing the implementability condition in period T (Assumption 4), we can always truthfully implement a constant allocation $a_T \in A_T$, which may not be efficient. Nevertheless, given that period-T reports are truthful, all the efficient allocations up to period $T - 1$, $\{a_1^*, \ldots, a_{T-1}^*\}$, are periodic ex post implementable.

REMARK 3.6   The intuition of the mechanisms—linking information across time to provide incentives—goes beyond the finite-signal case. However, if signal spaces are infinite, the corresponding generalizations of Assumptions 2 or 3 may not be sufficient for implementability in general even if signals are correlated.[15] In this case, we can show that there exist transfers that are almost VCG transfers, under which each agent's incentive is almost aligned with the designer. As a result, the dynamic efficient allocations can be approximately truthfully implemented. See Section 5 for the analysis of the case with infinitely many signals.

3.3. *Budget-Balanced Mechanisms*

In this section, we consider budget-balanced mechanisms when time horizon is infinite $(T = \infty)$.[16] As we mentioned above, one problem with the efficient dynamic mechanisms in Section 3.2 is that they run large deficits subsidizing agents in each period. Budget balance requires these subsidies to be financed by the participants. An important insight from Athey and Segal (2012) is that the problem of contingent deviations needs to be carefully addressed when signals are persistent, since transfers in each period to be calculated based on the conditional distribution of signals in order to balance the budget. However, the conditional distributions are manipulable by agents through their previous reported signals. The balanced team mechanism proposed by Athey and Segal (2012) is not applicable in our settings with interdependent valuations and information correlation.

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[15]See McAfee, McMillan and Reny (1989) and McAfee and Reny (1992) for examples.

[16]For the finite-horizon case, the same approach adopted in this subsection yields a balanced budget in all but the last period.

We first show that ex ante budget balanced mechanisms can be constructed by introducing participation fees to the original efficient dynamic mechanism in the first period. After observing the first period's signal θ_1^i, each agent i pays a proportion of the expected discounted sum of other agents' total subsidies, an amount that is independent of her current signal θ_1^i. In expectation, the total amount of participation fees is equal to the total amount of future subsidies. Specifically, let $\{p_t^i\}$ denote the transfers in efficient dynamic mechanism constructed in Theorem 3.1. Note that for each i, $p_1^i \equiv 0$. For each $\theta_1 \in \Theta_1$, every agent's equilibrium payoff in the efficient mechanism is $W(\theta_1)$. So the expected discounted sum of subsidies for agent i is

$$\mathbb{E}\left[\sum_{t\geq 1}\delta^{t-1}p_t^i\right] = \mathbb{E}\left[W(\theta_1) - \sum_{t\geq 1}\delta^{t-1}u^i(a_t^*(\theta_t),\theta_t)\right],$$

where the expectation is over the entire sequence of signal profiles. For each i and θ_1^i, define

$$\eta^i(\theta_1^i) \triangleq -\mathbb{E}\left[\sum_{t\geq 1}\delta^{t-1}p_t^i \,\Big|\, \theta_1^i\right] = -\mathbb{E}\left[W(\theta_1) - \sum_{t\geq 1}\delta^{t-1}u^i(a_t^*(\theta_t),\theta_t) \,\Big|\, \theta_1^i\right].$$

Then for each agent i, consider the transfers $\{\tilde{p}_t^i\}$ defined as

$$\tilde{p}_1^i(\theta_1) = \frac{1}{N-1}\sum_{j\neq i}\eta^j(\theta_1^j),$$

and $\tilde{p}_t^i = p_t^i$ for $t \geq 2$. Note that $\tilde{p}_1^i$ is independent of agent i's report, so $\{\tilde{p}_t^i\}$ is also periodic ex post incentive compatible. Moreover, by the law of iterated expectations, the expected sum of transfers satisfies

$$\begin{aligned}
\mathbb{E}\left[\sum_{i=1}^{N}\sum_{t\geq 1}\delta^{t-1}\tilde{p}_t^i\right] &= \mathbb{E}\left[\sum_{i=1}^{N}\tilde{p}_1^i + \sum_{i=1}^{N}\sum_{t\geq 2}\delta^{t-1}\tilde{p}_t^i\right]\\
&= \mathbb{E}\left[\sum_{i=1}^{N}\left(\eta^i(\theta_1^i) + \sum_{t\geq 1}\delta^{t-1}p_t^i\right)\right]\\
&= \sum_{i=1}^{N}\mathbb{E}\left[-\mathbb{E}\left[\sum_{t\geq 1}\delta^{t-1}p_t^i \,\Big|\, \theta_1^i\right] + \sum_{t\geq 1}\delta^{t-1}p_t^i\right]\\
&= 0.
\end{aligned}$$

Suppose next that the designer has limited instruments for inter-temporal financing. We now construct a budget balanced mechanism under which the expected sum of transfers in each period

is zero. For each i, t, a_t and θ_t, define

$$\xi^i(a_t, \theta_t) \triangleq u^i(a_t, \theta_t) - \frac{1}{N} \sum_{i=1}^{N} u^i(a_t, \theta_t)$$

to be the deviation of agent i's flow utility from the average flow utility. Since $\xi^i(a_t, \theta_t)$ is bounded, by the argument in the proof of Theorem 3.1, if Assumption 2 holds, there exist transfers $\hat{p}_{t+1}^i :$ $\Theta_{t+1}^{-i} \times \Theta_t^i \times A_t \times \Theta_t^{-i} \to \mathbb{R}$ such that for each a_t, θ_t^{-i} and each pair (θ_t^i, r_t^i), we have

$$\xi^i(a_t, \theta_t) = \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \hat{p}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i}) \mu(\theta_{t+1}|a_t, \theta_t)$$

$$\leq \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \hat{p}_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a_t, \theta_t^{-i}) \mu(\theta_{t+1}|a_t, \theta_t).$$

Set $\hat{p}_1^i \equiv 0$ for each i and consider the dynamic mechanism $\{a_t^*, \hat{p}_t\}$. The expected sum of transfers in period $t + 1$ under the truthful strategies is

$$\sum_{\theta_{t+1} \in \Theta_{t+1}} \sum_{i=1}^{N} \hat{p}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t^*(\theta_t), \theta_t^{-i}) \mu(\theta_{t+1}|a_t^*(\theta_t), \theta_t) = \sum_{i=1}^{N} \xi^i(a_t^*(\theta_t), \theta_t) = 0.$$

Moreover, if Assumption 3 holds, then similar to the logic in Theorem 3.2, there are transfers $\tilde{p}_{t+1}^i : \Theta_{t+1}^{-i} \times A_t \times \Theta_t^{-i} \to \mathbb{R}$ such that for each a_t and θ_t, we have

$$\xi^i(a_t, \theta_t) = \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \tilde{p}_{t+1}^i(\theta_{t+1}^{-i}; a_t, \theta_t^{-i}) \mu(\theta_{t+1}|a_t, \theta_t),$$

and hence a balanced budget

$$\sum_{\theta_{t+1} \in \Theta_{t+1}} \sum_{i=1}^{N} \tilde{p}_{t+1}^i(\theta_{t+1}^{-i}; a_t^*(\theta_t), \theta_t^{-i}) \mu(\theta_{t+1}|a_t^*(\theta_t), \theta_t) = 0.$$

Therefore, we only need to show that either transfer, $\{\hat{p}_t\}$ or $\{\tilde{p}_t\}$, achieves incentive compatibility. The result is summarized in the next proposition.

PROPOSITION 3.7 *Suppose $T = \infty$. Under either Assumption 2 or 3, there exists an efficient dynamic mechanism that is periodic ex post incentive compatible and balances the budget in the truthful equilibrium.*

Note that the above transfers, $\{\hat{p}_t\}$ and $\{\tilde{p}_t\}$, only balance the budget on the equilibrium path. More assumptions on the joint distributions of signals are needed for ex post budget balance along the line of analysis in Kosenok and Severinov (2008) and Hörner, Takahashi and Vieille (2013). Since this question is beyond the scope of the current paper, we leave it for future research.

3.4. *Implementation without Correlation*

If the correlation conditions are violated, the construction in the proof of Theorems 3.1 and 3.2 may not work for some utility functions. In this section, we drop the assumption that signal spaces are finite but restrict our attention to one-dimensional environments and the evolution of private information is independent across agents. We construct a transfer schedule that extends the generalized VCG mechanism to dynamic settings.

We say that a transfer $\{p_t\}_{t=1}^T$ or a mechanism $\{a_t^*, p_t\}_{t=1}^T$ is *history-independent* if for each t and θ_t, and for any two public histories h_t and h_t',

$$p_t(h_t, \theta_t) = p_t(h_t', \theta_t).$$

That is, a history-independent transfer p_t depends only on the reported profile $r_t \in \Theta_t$ in period t. Under a history-independent mechanism, agent i's period-t continuation payoff (2) depends only on her private signal θ_t^i, i.e.,

$$V_t^i(\theta_t^i) = \max_{r_t^i \in \Theta_t^i} \mathbb{E}\left[u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t) - p_t^i(r_t^i, \theta_t^{-i}) + \delta V_{t+1}^i(\theta_{t+1}^i)\right].$$

In this case, we also define $V_t^i(a_t, \theta_t)$ as

$$V_t^i(a_t, \theta_t) = u^i(a_t, \theta_t) + \delta \mathbb{E}\left[V_{t+1}^i(\theta_{t+1}^i)|a_t, \theta_t\right].$$

Assumption 5 (One-dimensional private signals) For each i and each t, $\Theta_t^i = [0, 1]$.

Under Assumption 5, we can generalize the monotonicity condition in static model studied by Bergemann and Välimäki (2002). To save notations, assume that for each t, $A_t = A \equiv \{a^1, \ldots, a^K\}$. For any i, t and θ_t^{-i}. Define the set $\Theta_t^{i,k} \subset \Theta_t^i$ as

$$\Theta_t^{i,k} = \left\{\theta_t^i \in \Theta_t^i \;\middle|\; \begin{array}{l} u^i(a^k, \theta_t) + \delta \mathbb{E}\left[W_{t+1}(\theta_{t+1})|a^k, \theta_t\right] \\ \geq \sum_i u^i(a^l, \theta_t) + \delta \mathbb{E}\left[W_{t+1}(\theta_{t+1})|a^l, \theta_t\right], \end{array} \forall a^l \neq a^k \right\}.$$

We say that the collections of sets $\{\Theta_t^{i,k}\}_{k=1}^K$ satisfies *monotonicity* if for each k, $\theta_t^i, \tilde{\theta}_t^i \in \Theta_t^{i,k}$ implies that for each $\lambda \in [0, 1]$, $\lambda \theta_t^i + (1 - \lambda)\tilde{\theta}_t^i \in \Theta_t^{i,k}$. Under monotonicity, there exists an efficient allocation a_t^* in period t such that after relabeling the social alternatives, Θ_t^i can be partitioned into successive intervals $\{S_t^{i,1}, \ldots, S_t^{i,K}\}$ and each a^k is chosen if and only if $\theta_t^i \in S_t^{i,k}$. Then for each i, t and θ_t^{-i}, there is a linear order $\prec_t^i$ (which also depends on θ_t^{-i}) on A:

$$a^1 \prec_t^i \ldots \prec_t^i a^K.$$

Assumption 6 (Independent transitions) For $t = 1$, $\mu_1 = \prod_{i=1}^N \mu_1^i$, where for each i, $\mu_1^i \in \Delta(\Theta_1^i)$. For each $t > 1$, $\mu_t(\theta_t | a_{t-1}, \theta_{t-1}) = \prod_{i=1}^N \mu_t^i(\theta_t^i | a_{t-1}, \theta_{t-1}^i)$, where for each i, $\mu_t^i : A \times \Theta_{t-1}^i \to \Delta(\Theta_t^i)$ is a transition probability.

Suppose $a_t^*(\theta_t) = a^k$, then consider the following history-independent transfer

$$(4) \qquad p_t^{i*}(\theta_t) = \sum_{\kappa=1}^k \sum_{j \neq i} \left[u^j(a^{\kappa-1}, x_t^i(\kappa, \theta_t^{-i}), \theta_t^{-i}) - u^j(a^\kappa, x_t^i(\kappa, \theta_t^{-i}), \theta_t^{-i}) \right]$$

$$+ \sum_{\kappa=1}^k \delta \mathbb{E} \left[W_{t+1}(\theta_{t+1}) - V_{t+1}^i(\theta_{t+1}) | a^{\kappa-1}, x_t^i(\kappa, \theta_t^{-i}), \theta_t^{-i} \right]$$

$$- \sum_{\kappa=1}^k \delta \mathbb{E} \left[W_{t+1}(\theta_{t+1}) - V_{t+1}^i(\theta_{t+1}) | a^\kappa, x_t^i(\kappa, \theta_t^{-i}), \theta_t^{-i} \right],$$

where $x_t^i(\kappa, \theta_t^{-i}) \triangleq \inf\{\theta_t^i : a_t^*(\theta_t^i, \theta_t^{-i}) = a^\kappa\}$. Note that $p_t^{i*}(\theta_t)$ does not depend directly on θ_t^i under Assumption 6.

Finally, recall that $W_t(\theta_t)$ is the continuation social surplus given period-t signal profile θ_t. For each a_t and θ_t, define $W_t(a_t, \theta_t)$ as

$$W_t(a_t, \theta_t) = \sum_{i=1}^N u^i(a_t, \theta_t) + \delta \mathbb{E}\left[W_{t+1}(\theta_{t+1}) | a_t, \theta_t\right].$$

The next theorem shows that the transfer constructed in (4) is periodic ex post incentive compatible under some restrictions on the primitives. Therefore, it extends of the generalized VCG mechanism to dynamic environments with interdependent valuations.

THEOREM 3.8   *Suppose that Assumptions 5 and 6 hold. There exists a periodic ex post incentive compatible mechanism $\{a_t^*, p_t\}$ with history-independent transfers if for each t, i and θ_t^{-i}, there exists an order on the allocation space A such that*

1. *$W_t(a_t, \theta_t^i, \theta_t^{-i})$ is single-crossing in (a_t, θ_t^i),*
2. *$V_t^i(a_t, \theta_t^i, \theta_t^{-i})$ has increasing difference in (a_t, θ_t^i).*

REMARK 3.9   The transfer schedule (4) can also be viewed a generalization of the dynamic pivot mechanism constructed by Bergemann and Välimäki (2010). To see this, suppose that each utility function u^i does not depend on θ_t^{-i} and that private information is statistically independent across

agents, then (4) can be written as

$$p_t^{i*}(\theta_t) = \sum_{j \neq i} \left[u^j(a_t^*(\underline{\theta}^i, \theta^{-i}), \theta_t^{-i}) - u^j(a_t^*(\theta_t), \theta_t^{-i}) \right]$$
$$+ \delta \mathbb{E} \left[W^{-i}(\theta_{t+1}) | a_t^*(\underline{\theta}^i, \theta^{-i}), \theta_t \right] - \delta \mathbb{E} \left[W^{-i}(\theta_{t+1}) | a_t^*(\theta_t), \theta_t \right],$$

where

$$W^{-i}(\theta_t) = W(\theta_t) - V^i(\theta_t) = \max_{\{a_s\}_{s \geq t}} \mathbb{E} \left[\sum_{s \geq t} \delta^{s-t} \left(u^i(a_s, \underline{\theta}^i) + \sum_{j \neq i} u^j(a_s, \theta_s^j) \right) \right].$$

Therefore each agent i's transfer p_t^{i*} in every period t is the flow externality cost that she imposes on other agents.

4. SURPLUS EXTRACTION

In this section, we consider the problem of full surplus extraction in the infinite-horizon $(T = \infty)$ case. We assume that each agent's utility function is non-negative and normalize each agent's outside option from any period onward to zero. We will show that the designer can always extract all the expect surplus from the agents by exploiting the inter-temporal correlation of private information. We also emphasize that inter-temporal correlation plays a key role in surplus extraction as it does in efficient implementation. In contrast, the attempt of generalizing Crémer and McLean (1988) and McAfee and Reny (1992) based on correlation of intra-period signals fails due to the possibility of belief manipulations by agents, as we have discussed in Section 3.1.

Formally, we say that a dynamic mechanism $\{a_t, p_t\}$ achieves *full-surplus extraction* if

$$\mathbb{E} \left[W(\theta_1) - \sum_{i=1}^N \sum_{t=1}^\infty \delta^{t-1} p_t^i \right] = 0.$$

That is, the expected discounted total transfer collected by the designer is equal to the expected maximal social surplus. The following result shows that a simple modification of the efficient dynamic mechanism in Theorem 3.1 ensures full surplus extraction.

PROPOSITION 4.1 *Suppose $T = \infty$. Under either Assumption 1 or 2, there exists a periodic ex post incentive compatible dynamic mechanism that achieves full surplus extraction.*

In effect, the dynamic surplus extraction mechanism asks each agent to pay a fixed participation fee and choose from a collection of lotteries in each period, followed by announcing her current signal as in the efficient dynamic mechanism in Theorem 3.1. The outcome of each lottery is revealed in the next period, depending on other agents' reports in *both* periods. All lotteries pay bonuses to the agent, thereby ensuring agents to participate in the mechanism in each period. The upfront participation fees, which can be thought as prices of entering any such lotteries, serve to extract the surplus from agents.

REMARK 4.2   An alternative notion of surplus extraction would require that the designer obtains the entire continuation social surplus after *each* history. While in our mechanism each agent collects zero expected surplus from the beginning of their interactions, her continuation payoff after any nontrivial history is in fact positive as she obtains bonuses from the lottery purchased in the previous round. Thus, the mechanism does not satisfy this stronger version of surplus extraction. We conjecture that given agents' interim participation constraints in each period, it is impossible to achieve surplus extraction after each history.

5. INFINITE SIGNAL SPACES AND ε-EX POST INCENTIVE COMPATIBILITY

In section, we extend the main results in section 3 to the case where agents may have infinitely many possible signals in each period. Suppose for each i and t, $\Theta_t^i = [0, 1]$, $A_t = A$, and $u^i(a_t, \cdot)$ is continuous in θ_t for each $a_t \in A$.[17] Also assume for simplicity that $T = \infty$ and that the transition probability $\mu(\theta_{t+1}|a_t, \theta_t)$ is stationary (independent of t) and has a continuous density representation $f(\theta_{t+1}|a_t, \theta_t)$. The marginal density on Θ_{t+1}^{-i} is denoted by $f^{-i}(\theta_{t+1}^{-i}|a_t, \theta_t)$.

We consider a weakening of perfect ex post equilibrium, which requires that after any history, truth-telling is "almost" a best response if all other agents report truthfully. Formally, for any $\varepsilon > 0$, we say that the mechanism $\{a_t^*, p_t\}_{t \geq 1}$ is ε-*periodic ex post incentive compatible* if for each t, i, h_t^i and θ_t^i,

$$u^i(a_t^*(\theta_t^i, \theta_t^{-i}), \theta_t) - p_t^i(h_t, \theta_t^i, \theta_t^{-i}) + \delta \mathbb{E}\left[V^i(h_{t+1}^i)|a_t^*(\theta_t^i, \theta_t^{-i}), \theta_t\right]$$
$$\geq u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t) - p_t^i(h_t, r_t^i, \theta_t^{-i}) + \delta \mathbb{E}\left[V^i(h_{t+1}^i)|a_t^*(r_t^i, \theta_t^{-i}), \theta_t\right] - \varepsilon$$

for any $r_t^i \in \Theta_t^i$, where $V^i(h_{t+1}^i)$ is the continuation payoff of agent i if all agent report truthfully from period $t+1$ onward. The condition implies that after any history, any one-shot deviation from

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[17]The results in the section still hold if each Θ_t^i is a compact and convex subset of an Euclidean space.

truth-telling would yield an agent at most ε improvement in his continuation payoff. Note that because of discounting, if a mechanism is ε-periodic ex post incentive compatible, then truth-telling consists of a (contemporaneous) $\varepsilon(1-\delta)^{-1}$-perfect ex post equilibrium under that mechanism.

In the following two lemmas, we identify conditions on the transition densities $f^{-i}(\theta_{t+1}^{-i}|a_t, \theta_t)$ such that for every $\varepsilon > 0$, there exist transfer schedules p_t that are ε-periodic ex post incentive compatible.

LEMMA 5.1   *Fix any* i, t, a_t *and* θ_t^{-i}. *If for every* θ_t^i *and every* $\mu^i \in \Delta(\Theta_t^i)$

$$(5) \qquad f^{-i}(\cdot|a_t, \theta_t^i, \theta_t^{-i}) = \int_{\Theta_t^i} f^{-i}(\cdot|a_t, \tilde{\theta}_t^i, \theta_t^{-i})\mu^i(d\tilde{\theta}_t^i) \quad \Rightarrow \quad \mu^i\left(\{\theta_t^i\}\right) = 1,$$

then for any $\varepsilon > 0$, *there exist transfers* $p_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i})$ *measurable in* θ_t^i *and continuous in* θ_{t+1}^{-i} *and* θ_t^{-i} *such that*

$$\max_{\theta_t^i \in \Theta_t^i} \left| -\sum_{j \neq i} u^j(a_t, \theta_t) - \delta \int_{\Theta_{t+1}^{-i}} p_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i}) f^{-i}(\theta_{t+1}^{-i}|a_t, \theta_t)d\theta_{t+1}^{-i} \right| \leq \varepsilon,$$

and

$$\int_{\Theta_{t+1}^{-i}} p_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i}) f^{-i}(\theta_{t+1}^{-i}|a_t, \theta_t)d\theta_{t+1}^{-i}$$

$$\leq \int_{\Theta_{t+1}^{-i}} p_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a_t, \theta_t^{-i}) f^{-i}(\theta_{t+1}^{-i}|a_t, \theta_t)d\theta_{t+1}^{-i},$$

for any $r_t^i \in \Theta_t^i$.

LEMMA 5.2   *Fix any* i, t, a_t *and* θ_t^{-i}. *If there does not exist a non-zero signed measure* η^i *on the Borel subsets of* Θ_t^i *such that*

$$(6) \qquad \int_{\Theta_t^i} f^{-i}(\cdot|a_t, \tilde{\theta}_t^i, \theta_t^{-i})\eta^i(d\tilde{\theta}_t^i) = 0,$$

then for any $\varepsilon > 0$, *there exists continuous transfers* $p_{t+1}^i(\theta_{t+1}^{-i}; a_t, \theta_t^{-i})$ *such that*

$$\max_{\theta_t^i \in \Theta_t^i} \left| -\sum_{j \neq i} u^j(a_t, \theta_t) - \delta \int_{\Theta_{t+1}^{-i}} p_{t+1}^i(\theta_{t+1}^{-i}; a_t, \theta_t^{-i}) f^{-i}(\theta_{t+1}^{-i}|a_t, \theta_t)d\theta_{t+1}^{-i} \right| \leq \varepsilon.$$

The proofs of these two lemmas are relegated to Appendix B. Condition (5) in Lemma 5.1 is a generalization of the convex independence condition. Following the proof of Theorem 3.1, it implies that there are ε-periodic ex post incentive compatible transfers of the form $p_{t+1}^i : \Theta_{t+1}^{-i} \times \Theta_t^i \times A_t \times \Theta_t^{-i} \to \mathbb{R}$. Likewise, condition (6) in Lemma 5.2, which generalizes the spanning condition, guarantees ε-periodic ex post incentive compatible transfers of the form $p_{t+1}^i : \Theta_{t+1}^{-i} \times A_t \times \Theta_t^{-i} \to \mathbb{R}$. Similar to the mechanisms presented in Section 3, each agent is *almost* a residual claimant and hence never gains by more than ε from misreporting in any period.

REMARK 5.3 The above formulation for the case with infinitely many signals follows closely that of McAfee and Reny (1992). In particular, we impose topological structures on the signal spaces and continuity assumptions on the utility functions. One could drop the continuity assumptions by considering a weaker notion of ε-incentive compatibility, i.e., in each period and after any history, with probability at least $1 - \varepsilon$, no agent can gain more than ε by deviating from truth-telling.

6. CONCLUDING REMARKS

Dynamic mechanism design features a richer family of history-dependent transfers compared with the static counterpart. This paper has taken a first step toward understanding the implications of such richness on efficient implementations in general environments with interdependent valuations. In particular, we have shown how inter-temporal correlation of private information leads to contingent transfers that resemble dynamic VCG mechanisms. We also emphasize that while the theoretical possibility results in this paper serve as a benchmark in the design of efficient mechanisms, the practicality of contingent transfers may vary with specific economic problems. Finally, we discuss several extensions.

Dynamic Populations. The model can be extended to accommodate the possibility of arrival and departure of potential agents. Several new issues need be addressed. First, with interdependent valuations, agents' arrival and departure would change both the information structure and utility functions, since each active agent holds information that directly affects other agents' payoffs. Second, agents are required to make contingent transfers in the dynamic mechanisms. Thus, transfers to an agent may occur even if she is no longer active. This may be problematic in some situations where monetary transfers have to be made along with the physical allocations. Third, the arrival (or departure) times may also be agents' private information.[18] Moreover, there may be

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[18]See Gershkov, Moldovanu and Strack (2013) and Mierendorff (2011a) for examples.

uncertainty in arrival (or departure) rates, which further complicates the incentive compatibility constraints.

Private Actions. While the baseline model considers only public allocations, one can extend it to incorporate agents' private actions in each period. These actions can be agents' information acquisition or investment decisions. Such an extension is useful to analyze the impact of dynamic free-riding or excessive competition in various economic problems. All results in the present paper carry over to this setting, as long as no agent's private action affects the evolution of signals. In a parallel study (Liu (2013)), we apply this idea to study the design of trade policies when each country possesses private information and can offer unobserved subsidies to domestic firms.

Mechanisms without Transfers. We have focused on quasilinear environments throughout the paper. Without the quasi-linearity assumption, a utilitarian designer may use continuation payoffs to provide incentives, as in the repeated games literature.[19] Alternatively, the designer may apply statistical tests in spirit of the linking mechanism in Jackson and Sonnenschein (2007). Since VCG mechanisms do not extend to non-quasilinear environments, it is unclear whether the main idea of this paper can be generalized to such models. We plan to address this question in future research.

7. APPENDIX A: PROOFS OF RESULTS IN SECTIONS 3 AND 4

Theorems 3.1 and 3.2 consider both the infinite-horizon and the finite-horizon cases. In subsection 7.1, we prove Theorem 3.1 for the infinite-horizon case, using the one-shot deviation principle; In subsection 7.2, we prove Theorem 3.2 for the finite horizon case, using backward induction. The proofs of the other two cases follow similar lines and are therefore omitted.

7.1. *Proof of Theorem 3.1*

Here we prove the infinite-horizon case. The proof consists of two lemmas.

LEMMA 7.1 *Suppose that Assumptions 1 and 2 hold. For each i and t, there exists a transfer function $p_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a_t, \theta_t^{-i})$ such that, for each a_t and θ_t^{-i}, the following two conditions are satisfied:*

1. for each θ_t^i,

$$-\sum_{j \neq i} u^j(a_t, \theta_t^i, \theta_t^{-i}) = \delta \sum_{\theta_{t+1}^{-i} \in \Theta_{t+1}^{-i}} p_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i}) \mu_{t+1}^{-i}(\theta_{t+1}^{-i} | a_t, \theta_t^i, \theta_t^{-i}),$$

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[19]See the comprehensive monograph by Mailath and Samuelson (2006).

2. *for each θ_t^i and r_t^i,*

$$\sum_{\theta_{t+1}^{-i} \in \Theta_{t+1}^{-i}} p_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i}) \mu_{t+1}^{-i}(\theta_{t+1}^{-i}|a_t, \theta_t^i, \theta_t^{-i})$$

$$\leq \sum_{\theta_{t+1}^{-i} \in \Theta_{t+1}^{-i}} p_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a_t, \theta_t^{-i}) \mu_{t+1}^{-i}(\theta_{t+1}^{-i}|a_t, \theta_t^i, \theta_t^{-i}),$$

where $\mu_{t+1}^{-i}(\cdot|a_t, \theta_t)$ is the marginal of $\mu_{t+1}(\cdot|a_t, \theta_t)$ on Θ_{t+1}^{-i}.

PROOF: First note that Assumption 2 is equivalent to the following condition: for each i, t, a_t, and θ_t^{-i}, and for each $\pi_i : \Theta_t^i \to \Delta(\Theta_t^i)$,

$$(7) \quad \sum_{\tilde{\theta}_t^i \in \Theta_t^i} \pi_i(\tilde{\theta}_t^i|\theta_t^i) \mu_{t+1}^{-i}(\theta_{t+1}^{-i}|a_t, \tilde{\theta}_t^i, \theta_t^{-i}) = \mu_{t+1}^{-i}(\theta_{t+1}^{-i}|a_t, \theta_t^i, \theta_t^{-i}) \quad \Rightarrow \quad \sum_{\tilde{\theta}_t^i \in \Theta_t^i} \pi_i(\theta_t^i|\tilde{\theta}_t^i) = 1, \ \forall \theta_t^i \in \Theta_t^i.$$

Then we show that under Assumption 1, the above condition is equivalent to the existence of a transfer $p_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a_t, \theta_t^{-i})$ satisfying the two requirements in the lemma.[20]

For each i, t, a_t, and θ_t^{-i}, and for each $u^i(a_t, \theta_t^i)$ satisfying Assumption 1, by the Theorem of Alternatives (See Rockafellar (1970), Section 22), there exists a transfer $p_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a_t, \theta_t^{-i})$ such that the following two conditions hold:

1. for each θ_t^i,

$$-\sum_{j \neq i} u^j(a_t, \theta_t^i, \theta_t^{-i}) = \delta \sum_{\theta_{t+1}^{-i} \in \Theta_{t+1}^{-i}} p_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i}) \mu_{t+1}^{-i}(\theta_{t+1}^{-i}|a_t, \theta_t^i, \theta_t^{-i}),$$

2. for each θ_t^i and r_t^i,

$$\sum_{\theta_{t+1}^{-i} \in \Theta_{t+1}^{-i}} p_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i}) \mu_{t+1}^{-i}(\theta_{t+1}^{-i}|a_t, \theta_t^i, \theta_t^{-i})$$

$$\leq \sum_{\theta_{t+1}^{-i} \in \Theta_{t+1}^{-i}} p_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a_t, \theta_t^{-i}) \mu_{t+1}^{-i}(\theta_{t+1}^{-i}|a_t, \theta_t^i, \theta_t^{-i}),$$

if and only if for each $\eta : \Theta_t^i \to \mathbb{R}$ and $\lambda : \Theta_t^i \times \Theta_t^i \to \mathbb{R}_+$, if for each θ_t^i and θ_{t+1}^{-i},

$$(8) \quad \eta(\theta_t^i) \mu_{t+1}^{-i}(\theta_{t+1}^{-i}|a_t, \theta_t^i, \theta_t^{-i}) = \sum_{\tilde{\theta}_t^i \in \Theta_t^i} \left[\lambda(\tilde{\theta}_t^i, \theta_t^i) \mu_{t+1}^{-i}(\theta_{t+1}^{-i}|a_t, \tilde{\theta}_t^i, \theta_t^{-i}) - \lambda(\theta_t^i, \tilde{\theta}_t^i) \mu_{t+1}^{-i}(\theta_{t+1}^{-i}|a_t, \theta_t^i, \theta_t^{-i}) \right],$$

then $\eta(\theta_t^i) \equiv 0$.

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[20]The proof follows closely the argument in Rahman (2010).

Now we show that condition (8) is equivalent to condition (7). Suppose first that for each pair (η, λ) with $\lambda \geq 0$, if for each θ_t^i,

$$(9) \qquad \eta(\theta_t^i)\mu_{t+1}^{-i}(\cdot|a_t, \theta_t^i, \theta_t^{-i}) = \sum_{\tilde{\theta}_t^i \in \Theta_t^i} \left[\lambda(\tilde{\theta}_t^i, \theta_t^i)\mu_{t+1}^{-i}(\cdot|a_t, \tilde{\theta}_t^i, \theta_t^{-i}) - \lambda(\theta_t^i, \tilde{\theta}_t^i)\mu_{t+1}^{-i}(\cdot|a_t, \theta_t^i, \theta_t^{-i}) \right],$$

then $\eta(\theta_t^i) \equiv 0$. Fix any $\pi_i : \Theta_t^i \to \Delta(\Theta_t^i)$ satisfying for each θ_t^i,

$$(10) \qquad \sum_{\tilde{\theta}_t^i \in \Theta_t^i} \pi_i(\tilde{\theta}_t^i|\theta_t^i)\mu_{t+1}^{-i}(\cdot|a_t, \tilde{\theta}_t^i, \theta_t^{-i}) = \mu_{t+1}^{-i}(\cdot|a_t, \theta_t^i, \theta_t^{-i}),$$

we want to show that $\pi_i(\theta_t^i|\theta_t^i) = 1$, for each θ_t^i. Condition (10) implies

$$(11) \qquad \mu_{t+1}^{-i}(\cdot|a_t, \theta_t^i, \theta_t^{-i}) - \sum_{\tilde{\theta}_t^i \in \Theta_t^i} \pi_i(\tilde{\theta}_t^i|\theta_t^i)\mu_{t+1}^{-i}(\cdot|a_t, \tilde{\theta}_t^i, \theta_t^{-i}) = 0.$$

Define $\eta(\theta_t^i) = 1 - \sum_{\tilde{\theta}_t^i} \pi_i(\theta_t^i|\tilde{\theta}_t^i)$. Then condition (11) is equivalent to

$$(12) \qquad \left[\eta(\theta_t^i) + \sum_{\tilde{\theta}_t^i} \pi_i(\theta_t^i|\tilde{\theta}_t^i) \right] \mu_{t+1}^{-i}(\cdot|a_t, \theta_t^i, \theta_t^{-i}) - \sum_{\tilde{\theta}_t^i \in \Theta_t^i} \pi_i(\tilde{\theta}_t^i|\theta_t^i)\mu_{t+1}^{-i}(\cdot|a_t, \tilde{\theta}_t^i, \theta_t^{-i}) = 0.$$

Since π_i is non-negative by definition, it follows from condition (9) that for each θ_t^i, $\eta(\theta_t^i) = 0$, or equivalently, $\sum_{\tilde{\theta}_t^i} \pi_i(\theta_t^i|\tilde{\theta}_t^i) = 1$, which establishes condition (7).

Conversely, suppose that for each $\pi_i : \Theta_t^i \to \Delta(\Theta_t^i)$, if

$$(13) \qquad \sum_{\tilde{\theta}_t^i \in \Theta_t^i} \pi_i(\tilde{\theta}_t^i|\theta_t^i)\mu_{t+1}^{-i}(\theta_{t+1}^{-i}|a_t, \tilde{\theta}_t^i, \theta_t^{-i}) = \mu_{t+1}^{-i}(\theta_{t+1}^{-i}|a_t, \theta_t^i, \theta_t^{-i}),$$

then for each θ_t^i, $\sum_{\tilde{\theta}_t^i \in \Theta_t^i} \pi_i(\theta_t^i|\tilde{\theta}_t^i) = 1$. Fix any pair (η, λ) satisfying $\lambda \geq 0$ and for each θ_t^i,

$$(14) \qquad \eta(\theta_t^i)\mu_{t+1}^{-i}(\cdot|a_t, \theta_t^i, \theta_t^{-i}) = \sum_{\tilde{\theta}_t^i \in \Theta_t^i} \left[\lambda(\tilde{\theta}_t^i, \theta_t^i)\mu_{t+1}^{-i}(\cdot|a_t, \tilde{\theta}_t^i, \theta_t^{-i}) - \lambda(\theta_t^i, \tilde{\theta}_t^i)\mu_{t+1}^{-i}(\cdot|a_t, \theta_t^i, \theta_t^{-i}) \right].$$

We want to show that $\eta(\theta_t^i) \equiv 0$. Condition (14) implies

$$(15) \qquad \left[\eta(\theta_t^i) + \sum_{\tilde{\theta}_t^i \in \Theta_t^i} \lambda(\theta_t^i, \tilde{\theta}_t^i) \right] \mu_{t+1}^{-i}(\cdot|a_t, \theta_t^i, \theta_t^{-i}) = \sum_{\tilde{\theta}_t^i \in \Theta_t^i} \lambda(\tilde{\theta}_t^i, \theta_t^i)\mu_{t+1}^{-i}(\cdot|a_t, \tilde{\theta}_t^i, \theta_t^{-i})$$

and

$$(16) \qquad \eta(\theta_t^i) = \sum_{\tilde{\theta}_t^i \in \Theta_t^i} \left[\lambda(\tilde{\theta}_t^i, \theta_t^i) - \lambda(\theta_t^i, \tilde{\theta}_t^i) \right],$$

where condition (16) follows from integration over θ_{t+1}^{-i} of condition (15). Therefore, we have

$$(17) \qquad \eta(\theta_t^i) + \sum_{\tilde{\theta}_t^i \in \Theta_t^i} \lambda(\theta_t^i, \tilde{\theta}_t^i) = \sum_{\tilde{\theta}_t^i \in \Theta_t^i} \lambda(\tilde{\theta}_t^i, \theta_t^i).$$

Note that $\lambda(\theta_t^i, \theta_t^i) > 0$ can be chosen arbitrarily without affecting condition (14). Therefore, conditions (15) and (17) imply that

$$(18) \qquad \mu_{t+1}^{-i}(\cdot | a_t, \theta_t^i, \theta_t^{-i}) = \frac{\sum_{\tilde{\theta}_t^i \in \Theta_t^i} \lambda(\tilde{\theta}_t^i, \theta_t^i) \mu_{t+1}^{-i}(\cdot | a_t, \tilde{\theta}_t^i, \theta_t^{-i})}{\sum_{\tilde{\theta}_t^i \in \Theta_t^i} \lambda(\tilde{\theta}_t^i, \theta_t^i)}.$$

Moreover, we can set $\lambda(\theta_t^i, \theta_t^i) > 0$ such that for each θ_t^i, $\sum_{\tilde{\theta}_t^i \in \Theta_t^i} \lambda(\tilde{\theta}_t^i, \theta_t^i) = C$, where C is a positive constant. For each pair θ_t^i and $\tilde{\theta}_t^i$, define $\pi_i(\tilde{\theta}_t^i | \theta_t^i) = \lambda(\tilde{\theta}_t^i, \theta_t^i)/C$. Then π_i is a mapping from Θ_t^i to $\Delta(\Theta_t^i)$. It then follows from condition (13) that $\sum_{\tilde{\theta}_t^i} \pi_i(\theta_t^i | \tilde{\theta}_t^i) = 1$ for each θ_t^i. Therefore, we have

$$\sum_{\tilde{\theta}_t^i \in \Theta_t^i} \lambda(\theta_t^i, \tilde{\theta}_t^i) = \sum_{\tilde{\theta}_t^i \in \Theta_t^i} \lambda(\tilde{\theta}_t^i, \theta_t^i)$$

and hence by condition (16), $\eta(\theta_t^i) \equiv 0$. This proves condition (8).

$\square$

LEMMA 7.2   *If for each i and t, there exists a transfer function $p_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a_t, \theta_t^{-i})$ such that, for each a_t and θ_t^{-i}, the following two conditions are satisfied:*

1. *for each θ_t^i,*

$$-\sum_{j \neq i} u^j(a_t, \theta_t^i, \theta_t^{-i}) = \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} p_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i}) \mu(\theta_{t+1} | a_t, \theta_t^i, \theta_t^{-i}),$$

2. *for each θ_t^i and r_t^i,*

$$\sum_{\theta_{t+1} \in \Theta_{t+1}} p_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i}) \mu(\theta_{t+1} | a_t, \theta_t^i, \theta_t^{-i})$$

$$\leq \sum_{\theta_{t+1} \in \Theta_{t+1}} p_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a_t, \theta_t^{-i}) \mu(\theta_{t+1} | a_t, \theta_t^i, \theta_t^{-i}),$$

then the dynamic efficient allocation $\{a_t^*\}$ can be implemented in an ex post equilibrium.

PROOF: Assume all agents other than i report their signals truthfully and focus on agent i's incentive problem. Fix a socially efficient allocation rule a_t^*. By the one-shot deviation principle,

we only need to show that after any public history up to period t, agent i does not benefit from deviating to $r_t^i \neq \theta_t^i$ and $r_s^i = \theta_s^i$ for $s > t$.

If agent i reports truthfully in period t, i.e., $r_t^i = \theta_t^i$, her continuation payoff is

$$
\begin{aligned}
& u^i(a_t^*(\theta_t), \theta_t) - p_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i) \\
& \quad + \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \left[W(\theta_{t+1}) - p_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t^*(\theta_t), \theta_t^{-i}) \right] \mu(\theta_{t+1} | a_t^*(\theta_t), \theta_t) \\
& = u^i(a_t^*(\theta_t), \theta_t) + \sum_{j \neq i} u^j(a_t^*(\theta_t), \theta_t) - p_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i) \\
& \quad + \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} W(\theta_{t+1}) \mu(\theta_{t+1} | a_t^*(\theta_t), \theta_t) \\
& = W(\theta_t) - p_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i).
\end{aligned}
$$

Suppose agent i deviates to a message r_t^i such that $a_t^*(r_t^i, \theta_t^{-i}) = a_t^*(\theta_t)$, then her continuation payoff satisfies

$$
\begin{aligned}
& u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t) - p_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i) + \\
& \quad \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \left[W(\theta_{t+1}) - p_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a_t^*(r_t^i, \theta_t^{-i}), \theta_t^{-i}) \right] \mu(\theta_{t+1} | a_t^*(r_t^i, \theta_t^{-i}), \theta_t) \\
& = u^i(a_t^*(\theta_t), \theta_t) - p_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i) \\
& \quad + \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \left[W(\theta_{t+1}) - p_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a_t^*(\theta_t), \theta_t^{-i}) \right] \mu(\theta_{t+1} | a_t^*(\theta_t), \theta_t) \\
& \leq u^i(a_t^*(\theta_t), \theta_t) - p_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i) \\
& \quad + \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \left[W(\theta_{t+1}) - p_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t^*(\theta_t), \theta_t^{-i}) \right] \mu(\theta_{t+1} | a_t^*(\theta_t), \theta_t) \\
& = W(\theta_t) - p_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i),
\end{aligned}
$$

where the inequality follows from condition 2. Thus, deviating to a message r_t^i without changing the allocation is not profitable.

Finally, if agent i deviates to a message r_t^i such that $a_t^*(r_t^i, \theta_t^{-i}) = a' \neq a_t^*(\theta_t)$, then her continu-

ation payoff satisfies

$$
\begin{aligned}
& u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t) - p_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i) \\
& \quad + \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \left[W(\theta_{t+1}) - p_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a_t^*(r_t^i, \theta_t^{-i}), \theta_t^{-i}) \right] \mu(\theta_{t+1} | a_t^*(r_t^i, \theta_t^{-i}), \theta_t) \\
& = u^i(a', \theta_t) - p_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i) \\
& \quad + \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \left[W(\theta_{t+1}) - p_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a', \theta_t^{-i}) \right] \mu(\theta_{t+1} | a', \theta_t) \\
& \leq u^i(a', \theta_t) - p_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i) \\
& \quad + \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \left[W(\theta_{t+1}) - p_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a', \theta_t^{-i}) \right] \mu(\theta_{t+1} | a', \theta_t) \\
& = u^i(a', \theta_t) + \sum_{j \neq i} u^j(a', \theta_t) - p_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i) \\
& \quad + \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} W(\theta_{t+1}) \mu(\theta_{t+1} | a', \theta_t) \\
& \leq W(\theta_t) - p_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i),
\end{aligned}
$$

where the first inequality is by condition 2, the second inequality is by condition 1, and the second inequality is by the definition of a_t^*. Thus, deviating to a message r_t^i which changes the allocation is not profitable either. Therefore, we conclude that truth-telling consists of an ex post equilibrium.

$\square$

7.2. *Proof of Theorem 3.2*

Here we prove the finite-horizon case. The proof again consists of two lemmas.

LEMMA 7.3   *Under Assumptions 1 and 3, for each i and $t < T$, there exists a transfer function* $\tilde{p}_{t+1}^i : \Theta_{t+1}^{-i} \times A_t \times \Theta_t^{-i} \to \mathbb{R}_+$ *such that*

$$
(19) \qquad -\sum_{j \neq i} u^j(a_t, \theta_t^i, \theta_t^{-i}) = \sum_{\theta_{t+1}^{-i} \in \Theta_{t+1}^{-i}} \tilde{p}_{t+1}^i(\theta_{t+1}^{-i}; a_t, \theta_t^{-i}) \mu_{t+1}^{-i}(\theta_{t+1}^{-i} | a_t, \theta_t),
$$

for every a_t, θ_t^{-i} and $\theta_t^i \in \Theta_t^i$.

PROOF: Fix any a_t and θ_t^{-i}, (19) is a system of linear equations. Since the transition matrix $\mu_{t+1}^{-i}(\theta_{t+1}^{-i} | a_t, \theta_t^i, \theta_t^{-i})$ from θ_t^i to θ_{t+1}^{-i} has full rank under Assumption 2, the system of equations has a solution.

$\square$

LEMMA 7.4 *Under Assumptions 1, 3, and 4, there exists a sequence of transfers* $\bar{p}_t : H_t \times \Theta_t \to \mathbb{R}^N$ *such that the efficient dynamic mechanism* $\{a_t^*, \bar{p}_t\}_{t=1}^T$ *is periodic ex post incentive compatible.*

PROOF: Let $W_t(\theta_t)$ denote the expected period-t continuation social surplus given signal profile θ_t, i.e.,

$$W_t(\theta_t) = \mathbb{E}\left[\sum_{s=t}^T \delta^{s-t} \sum_{i=1}^N u^i(a_t^*(\theta_t), \theta_t)\Big|\theta_t\right].$$

First consider the problem in period T. By Assumption 4, there exists an ex post incentive compatible transfer $p_T : \Theta_T \to \mathbb{R}^N$ that implements the efficient allocation a_T^*. Given (a_T^*, p_T), the payoff V_T^i for each agent i in the truth-telling equilibrium is given by

$$V_T^i(\theta_T) = u^i(a_T^*(\theta_T), \theta_T) - p_T^i(\theta_T),$$

for each θ_T.

Next consider agent i's incentive problem in period $T - 1$ with an arbitrary public history $h_{T-1} = (r_1, a_1, r_2, a_2, \ldots, r_{t-1}, a_{t-1})$. Suppose that agents other than i always report truthfully. For each pair (a_{T-1}, θ_{T-1}), define

$$\pi_{T-1}^i(a_{T-1}, \theta_{T-1}) = \sum_{j \neq i} u^j(a_{T-1}, \theta_{T-1}) + \delta \mathbb{E}\left[W(\theta_T) - V_T^i(\theta_T)|a_{T-1}, \theta_{T-1}\right].$$

By Lemma 7.3 there exists a function $\tilde{p}_T^i(\theta_T^{-i}; a_{T-1}, \theta_{T-1}^{-i})$ such that for every a_{T-1}, θ_{T-1}^{-i} and θ_{T-1}^i,

$$\pi_{T-1}^i(a_{T-1}, \theta_{T-1}) = \delta \sum_{\theta_T \in \Theta_T} \tilde{p}_T^i(\theta_T^{-i}; a_{T-1}, \theta_{T-1}^{-i})\mu_T(\theta_T|a_{T-1}, \theta_{T-1}).$$

Define a new period-T transfer $\bar{p}_T^i : \Theta_{T-1}^{-i} \times A_{T-1} \times \Theta_T \to \mathbb{R}$ for agent i as

$$\bar{p}_T^i(\theta_{T-1}^{-i}; a_{T-1}, \theta_T) = p_T^i(\theta_T) - \tilde{p}_T^i(\theta_T^{-i}; a_{T-1}, \theta_{T-1}^{-i}).$$

Note that $\tilde{p}_T^i$ is independent of θ_T^i, so agent i still finds it optimal to report truthfully in period T under this new transfer $\bar{p}_T^i$. Suppose agent i reports r_{T-1}^i in period $T - 1$, then for any realized signal profile θ_{T-1}, her expected continuation payoff from $T - 1$ on is equal to

$$u^i(a_{T-1}^*(r_{T-1}^i, \theta_{T-1}^{-i}), \theta_{T-1}) + \delta \mathbb{E}\left[V^i(\theta_T)|a_{T-1}^*(r_{T-1}^i, \theta_{T-1}^{-i}), \theta_{T-1}\right]$$
$$+ \pi_{T-1}^i(a_{T-1}^*(r_{T-1}^i, \theta_{T-1}^{-i}), \theta_{T-1})$$
$$= \sum_{i=1}^N u^i(a_{T-1}^*(r_{T-1}^i, \theta_{T-1}^{-i}), \theta_{T-1}) + \delta \mathbb{E}\left[W_T(\theta_T)|a_{T-1}^*(r_{T-1}^i, \theta_{T-1}^{-i}), \theta_{T-1}\right].$$

By definition, the allocation rule $a^*_{T-1} : \Theta_{T-1} \to A_{T-1}$ maximizes the social surplus from period $T - 1$ onward. Given that other agents always report truthfully, it follows that for every realized signal θ^i_{T-1}, it is optimal for agent i to report $r^i_{T-1} = \theta^i_{T-1}$. Also note that for every signal profile θ_{T-1}, agent i's continuation payoff V^i_{T-1} in the truth-telling equilibrium is

$$V^i_{T-1}(\theta_{T-1}) = W_{T-1}(\theta_{T-1}).$$

Now for any $t < T$, suppose that there exist transfer schedules $\{\bar{p}^i_{s+1}\}^{T-1}_{s=t}$ for each agent i such that truth-telling consists of a periodic ex post equilibrium from any period $s = t, \dots, T$ and each agent i's continuation payoff in the truth-telling equilibrium is $V^i_t(\theta_t) = W_t(\theta_t)$ for all θ_t.

We would like to construct a transfer $\bar{p}^i_t : \Theta^{-i}_{t-1} \times A_{t-1} \times \Theta_t \to \mathbb{R}$ for each agent i such that

$$\sum_{j \neq i} u^j(a_{t-1}, \theta_{t-1}) = \delta \sum_{\theta_t \in \Theta_t} \bar{p}^i_t(\theta^{-i}_t; a_{t-1}, \theta^{-i}_{t-1}) \mu_t(\theta_t | a_{t-1}, \theta_{t-1}),$$

for all a_{t-1}, θ^{-i}_{t-1} and θ^i_{t-1}. The existence of $\bar{p}^i_t$ again follows from Lemma 7.3. Since $\bar{p}^i_t$ is independent of θ^i_t, incentive constraints for truth-telling in periods $s = t, \dots, T$ still hold.

For each realized signal profile θ_{t-1}, suppose agent i reports r^i_{t-1}, then her expected continuation payoff from $t - 1$ on is

$$\sum_{i=1}^N u^i(a^*_{t-1}(r^i_{t-1}, \theta^{-i}_{t-1}), \theta_{t-1}) + \delta \mathbb{E}\left[W_t(\theta_t) | a^*_{t-1}(r^i_{t-1}, \theta^{-i}_{t-1}), \theta_{t-1} \right].$$

By the definition of a^*_{t-1}, for each agent i, any report $r^i_{t-1} \in \Theta^i_{t-1}$ in period $t - 1$ other than θ^i_{t-1} is suboptimal under $\hat{p}_{t-1}$ and $\{\bar{p}_s\}^T_{s=t}$. Finally, note that in period $t - 1$, agent i's continuation payoff in the truth-telling equilibrium is

$$V^i_{t-1}(\theta_{t-1}) = W_{t-1}(\theta_{t-1}),$$

for all signal profiles θ_{t-1}.

Inducting on t backwards, we have a sequence of transfers $\{\bar{p}_t\}^T_{t=1}$, where $\bar{p}^i_1 \equiv 0$ for each i. Therefore, truth-telling consists of a periodic ex post equilibrium under the efficient dynamic mechanism $\{a^*_t, \bar{p}_t\}^T_{t=1}$.

$\square$

7.3. *Proof of Proposition 3.7*

PROOF: Since budget balance under either $\{\hat{p}_t\}$ or $\{\tilde{p}_t\}$ is established in the main text, we only need to show that both mechanisms, $\{a^*_t, \hat{p}_t\}$ and $\{a^*_t, \tilde{p}_t\}$, are periodic ex post incentive

compatible. By the one-shot deviation principle, it suffices to prove that truth-telling is incentive compatible for agent i in period t after any history, if all agents report truthfully from period $t+1$ onward. Here we prove the result for $\{a_t^*, \tilde{p}_t\}$. The proof for $\{a_t^*, \hat{p}_t\}$ is similar and hence omitted.

Fix any $h_t = \{h_{t-1}, \theta_{t-1}, a_{t-1}\}$, we need to show that for each i and θ, $r_t^i = \theta_t^i$ is a solution to the following maximization problem

$$(20) \quad \max_{r_t^i} \left\{ u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t) - \tilde{p}_t^i(\theta_t^{-i}; a_{t-1}, \theta_{t-1}^{-i}) \right.$$
$$\left. + \delta \sum_{\theta_{t+1}} \left[\frac{1}{N} W(\theta_{t+1}) - \tilde{p}_{t+1}^i(\theta_{t+1}^{-i}; a_t^*(r_t^i, \theta_t^{-i}), \theta_t^{-i}) \right] \mu(\theta_{t+1} | a_t^*(r_t^i, \theta_t^{-i}), \theta_t) \right\}.$$

By construction, we have

$$\delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \tilde{p}_{t+1}^i(\theta_{t+1}^{-i}; a_t^*(r_t^i, \theta_t^{-i}), \theta_t^{-i}) \mu(\theta_{t+1} | a_t^*(r_t^i, \theta_t^{-i}), \theta_t)$$
$$= u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t) - \frac{1}{N} \sum_{i=1}^{N} u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t).$$

So the problem (20) is equivalent to

$$(21) \quad \max_{r_t^i} \left\{ u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t) - \tilde{p}_t^i(\theta_t^{-i}; a_{t-1}, \theta_{t-1}^{-i}) - u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t) \right.$$
$$\left. + \frac{1}{N} \sum_{i=1}^{N} u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t) + \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \frac{1}{N} W(\theta_{t+1}) \mu(\theta_{t+1} | a_t^*(r_t^i, \theta_t^{-i}), \theta_t) \right\}.$$

Since the second term in the objective function of (21), $\tilde{p}_t^i(\theta_t^{-i}; a_{t-1}, \theta_{t-1}^{-i})$, is independent of r_t^i, solutions to problem (21) are also solutions to the following problem

$$(22) \quad \max_{r_t^i} \left\{ \frac{1}{N} \sum_{i=1}^{N} u^i(a_t^*(r_t^i, \theta_t^{-i}), \theta_t) + \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \frac{1}{N} W(\theta_{t+1}) \mu(\theta_{t+1} | a_t^*(r_t^i, \theta_t^{-i}), \theta_t) \right\}.$$

By definition of a_t^*, $r_t^i = \theta_t^i$ is a solution to (22), which proves the result.

$\square$

7.4. *Proof of Theorem 3.8*

PROOF: The proof is by backward induction on t. For each t, the argument follows the same lines as the proof of Proposition 3 in Bergemann and Välimäki (2002) (pages 1029–1030) with the transfers defined in (4).

□

7.5. *Proof of Proposition 4.1*

PROOF: The proof is similar to that of Theorem 3.1, with an additional adjustment of the transfers which takes into account agents' participation constraints.

For each t and i, agent i's current signal θ_t^i is correlated with other agents' signals θ_{t+1}^{-i} in the next period as in Assumption 1, there exists a function $\tilde{q}_{t+1}^i : \Theta_{t+1}^{-i} \times \Theta_t^i \times A_t \times \Theta_t^{-i} \to \mathbb{R}$ such that for each a_t, θ_t^{-i} and each pair (θ_t^i, r_t^i),

$$
u^i(a_t, \theta_t) = \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \tilde{q}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i}) \mu_{t+1}(\theta_{t+1}|a_t, \theta_t)
$$

$$
\leq \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \tilde{q}_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a_t, \theta_t^{-i}) \mu_{t+1}(\theta_{t+1}|a_t, \theta_t).
$$

For each a_t, θ_t^{-i}, let $K_t^i(a_t, \theta_t^{-i}) \in \mathbb{R}$ be an upper bound of $|q_{t+1}^i|$, i.e.,

$$
K_t^i(a_t, \theta_t^{-i}) > \sup_{\theta_{t+1}^{-i}, \theta_t^i} \left| \tilde{q}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i}) \right|.
$$

Finally, set $\tilde{q}_1^i \equiv 0$ for all i.

We first show that the dynamic mechanism $\{a_t^*, \tilde{q}_t\}$ is periodic ex post incentive compatible. Again assume all agents other than agent i report truthfully. If agent i reports truthfully in period t, i.e., $r_t^i = \theta_t^i$, her continuation payoff is

$$
u^i(a_t^*(\theta_t), \theta_t) - \tilde{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i)
$$

$$
- \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \tilde{q}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t^*(\theta_t), \theta_t^{-i}) \mu(\theta_{t+1}|a_t^*(\theta_t), \theta_t)
$$

$$
= -\tilde{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i).
$$

Suppose agent i deviates to a message r_t^i such that $a_t^*(r_t^i, \theta_t^{-i}) = a_t^*(\theta_t)$, then her continuation

payoff satisfies

$$u^i(a_t^*(\theta_t), \theta_t) - \tilde{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i)$$
$$- \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \tilde{q}_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a_t^*(\theta_t), \theta_t^{-i}) \mu(\theta_{t+1} | a_t^*(\theta_t), \theta_t)$$
$$\leq u^i(a_t^*(\theta_t), \theta_t) - \tilde{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i)$$
$$- \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \tilde{q}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t^*(\theta_t), \theta_t^{-i}) \mu(\theta_{t+1} | a_t^*(\theta_t), \theta_t)$$
$$= - \tilde{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i).$$

Suppose agent i deviates to a message r_t^i such that $a_t^*(r_t^i, \theta_t^{-i}) = a' \neq a_t^*(\theta_t)$, then her continuation payoff satisfies

$$u^i(a', \theta_t) - \tilde{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i) - \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \tilde{q}_{t+1}^i(\theta_{t+1}^{-i}, r_t^i; a', \theta_t^{-i}) \mu(\theta_{t+1} | a', \theta_t)$$
$$\leq u^i(a', \theta_t) - \tilde{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i) - \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \tilde{q}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a', \theta_t^{-i}) \mu(\theta_{t+1} | a', \theta_t)$$
$$= - \tilde{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i).$$

Thus, after any history h_t, truth-telling is optimal for agent i provided that other agents also report their signals truthfully. The transfers $\tilde{q}_{t+1}^i$ can be viewed as lottery *payments* in period $t+1$ that agent i commits to fulfill in period t. Since each agent in every period on average pays her flow utility in the previous period, it is straightforward to verify that the designer extracts all surplus with the mechanism $\{a_t^*, \tilde{q}_t\}$.

Although agent i's ex ante participation constraint is satisfied under the mechanism $\{a_t^*, \tilde{q}_t\}$ as we have $\tilde{q}_1^i \equiv 0$, agent i's participation constraints in any subsequent period could be violated. To see this, note that the above reasoning also shows that agent i's equilibrium continuation payoff after history h_t is $-\tilde{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i)$, which may be less attractive than her outside option from period t onward.

This problem can be resolved by replacing the transfers $\tilde{q}_{t+1}^i$ in period $t+1$ with an upfront charge in period t and lottery *bonuses* in period $t + 1$. Recall that for each a_t, θ_t^{-i}, $K_t^i(a_t, \theta_t^{-i})$ is a bound of $q_{t+1}^i(\cdot, \cdot; a_t, \theta_t^{-i})$. For each t and i, define a new transfer function $\hat{q}_{t+1}^i : \Theta_{t+1}^{-i} \times \Theta_t^i \times A_t \times \Theta_t^{-i} \to \mathbb{R}$ by

$$\hat{q}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i}) \equiv \tilde{q}_{t+1,l}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i}) - K_t^i(a_t, \theta_t^{-i}).$$

Also define $\hat{q}_1^i \equiv 0$. Note that by construction $q_{t+1}^i \leq 0$ for each t and i. Thus, $\hat{q}_{t+1}^i$ can be viewed as lottery bonuses for agent i. Set $c_t^i(a_t, \theta_t^{-i}) = \delta K_t^i(a_t, \theta_t^{-i})$ to be the entrance fee or "price" of the lottery $\{\hat{q}_{t+1}^i\}$ that agent i pays in period t.

Finally, for each agent i, define a sequence of transfers $\hat{p}_t^i$ as follows: (a) in the first period, agent i pays an entrance fee $\hat{p}_1^i(\theta_1) = c_1^i(a_1^*(\theta_1^i, \theta_1^{-i}), \theta_1^{-i})$; (b) in each subsequent periods, agent i collects the lottery bonus and pays another entrance fee, i.e., $\forall t \geq 1$,

$$\hat{p}_{t+1}^i(h_t, \theta_{t+1}) = \hat{q}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t, \theta_t^{-i}) + c_{t+1}^i(a_{t+1}^*(\theta_{t+1}^i, \theta_{t+1}^{-i}), \theta_{t+1}^{-i}).$$

Under the mechanism $\{a_t^*, \hat{p}_t\}$, after any history h_t agent i's continuation payoff from truthtelling is

$$u^i(a_t^*(\theta_t), \theta_t) - \hat{p}_t^i(h_t, \theta_t) - \delta \sum_{\theta_{t+1} \in \Theta_{t+1}} \hat{q}_{t+1}^i(\theta_{t+1}^{-i}, \theta_t^i; a_t^*(\theta_t), \theta_t^{-i}) \mu(\theta_{t+1} | a_t^*(\theta_t), \theta_t)$$

$$= -\hat{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i) \geq 0.$$

On the other hand, agent i's continuation payoff from lying in period t is no greater than $-\hat{q}_t^i(\theta_t^{-i}, r_{t-1}^i; a_{t-1}, \theta_{t-1}^i)$. Since the expected discounted sum of transfers satisfies

$$\mathbb{E}\left[\sum_t \delta^{t-1} \hat{p}_t^i\right] = \mathbb{E}\left[\sum_t \delta^{t-1} \tilde{q}_t^i\right],$$

it follows that the mechanism $\{a_t^*, \hat{p}_t\}$ is periodic ex post incentive compatible and achieves full surplus extraction.

$\square$

8. APPENDIX B: PROOFS OF RESULTS IN SECTION 5

In this section, we prove two lemmas, which are the infinite versions of the convex independence (Lemma 8.1) and linear independence (Lemma 8.2) conditions. Applying the measurable "measurable choice" theorem in Mertens (2003) to establish measurability of the transfers, Lemma 5.1 then follows from Lemmas 8.1 and Lemma 5.2 follows from Lemma 8.2.[21]

Let $C[0, 1]$ denote the set of continuous functions on $[0, 1]$. Let $f(s|t)$ be a continuous conditional density function of $s \in [0, 1]$, given $t \in [0, 1]$. Define the following sets

$$C(f) = \left\{ \pi : \exists p : [0,1]^2 \to \mathbb{R} \text{ s.t. } \begin{array}{ll} 1) & \forall t, \; p(\cdot, t) \in C[0, 1], \forall s, p(s, \cdot) \text{ is Borel measurable,} \\ 2) & \forall t, t', \pi(t) = \int_0^1 p(s, t) f(s|t) ds \leq \int_0^1 p(s, t') f(s|t) ds \end{array} \right\},$$

---

[21]See also Barelli and Duggan (2013) for an application of Merten's theorem in stochastic games.

and

$$S(f) = \left\{ \pi : \ \exists p(s) \in C[0,1] \text{ s.t. } \forall t \in [0,1], \ \pi(t) = \int_0^1 p(s) f(s|t) ds \right\}.$$

Note that $C(f)$ and $S(f)$ are linear subspaces of $C[0,1]$. We consider the supnorm $\|\pi\| = \max_{t \in [0,1]} |\pi(t)|$, and denote the closure of $C(f)$ under this norm by $\bar{C}(f)$. Similarly, $\bar{S}(f)$ is the closure of $S(f)$ under the same norm. In the next two lemmas, we identify conditions on the conditional density $f(s|t)$ such that either $\bar{C}(f) = C[0,1]$ or $\bar{S}(f) = C[0,1]$.

LEMMA 8.1 $\bar{C}(f) = C[0,1]$ if and only if the following condition holds: for each $t \in [0,1]$ and each $\eta \in \Delta([0,1])$,

$$(23) \qquad f(\cdot|t) = \int_0^1 f(\cdot|\tilde{t}) \eta(d\tilde{t}) \quad \Rightarrow \quad \eta(t) = 1.$$

PROOF: This follows directly from Theorem 2 in McAfee and Reny (1992) (pages 404–406). $\square$

LEMMA 8.2 $\bar{S}(f) = C[0,1]$ if and only if the following condition holds: there does not exist a regular, non-zero signed measure ξ on the Borel sets of $[0,1]$ such that

$$(24) \qquad \int_0^1 f(\cdot|t) \xi(dt) = 0.$$

PROOF: For the only if part, suppose by contradiction that there is a regular, non-zero signed measure ξ on the Borel sets of $[0,1]$ such that $\int_0^1 f(\cdot|t) \xi(dt) = 0$. Since $\bar{S}(f) = C[0,1]$, for any $\varepsilon > 0$ and any $\pi \in C[0,1]$, there exists a $\tilde{\pi} \in S(f)$ such that $\|\pi - \tilde{\pi}\| < \varepsilon$. Then we

$$\int_0^1 \tilde{\pi}(t) \xi(dt) = \int_0^1 \left[\int_0^1 p(s) f(s|t) ds \right] \xi(dt),$$

for some $p(s) \in C[0,1]$ by the definition of $S(f)$. By Fubini's theorem,

$$\int_0^1 \left[\int_0^1 p(s) f(s|t) ds \right] \xi(dt) = \int_0^1 p(s) \left[\int_0^1 f(s|t) \xi(dt) \right] ds = 0.$$

That is, $\int_0^1 \tilde{\pi}(t) \xi(dt) = 0$. Hence, $\xi = 0$, which is a contradiction.

For the if part, suppose by contradiction that $\bar{S}(f) \neq C[0,1]$. Then there exists $\bar{\pi} \in C[0,1]$ such that $\bar{\pi} \notin \bar{S}(f)$. Since $\bar{S}(f)$ is closed and convex, by the separating hyperplane theorem (see Aliprantis and Border (2006), Theorem 5.79, page 207), there is a nonzero continuous linear functional on $C[0,1]$ separating $\bar{S}(f)$ and $\bar{\pi}$. Since $\bar{S}(f)$ is a linear subspace of $C[0,1]$, it follows

from the Riesz representation theorem (see Aliprantis and Border (2006), Corollary 14.15, page 498) that there exist a regular, nonzero signed measure ξ on the Borel sets of $[0, 1]$ such that for each $\pi \in \bar{S}(f)$,

$$\int_0^1 \pi(t)\xi(dt) = 0.$$

By the definition of $S(f)$, we then have

$$\int_0^1 \left[\int_0^1 p(s)f(s|t)ds \right] \xi(dt) = 0,$$

for each $p(s) \in C[0, 1]$. It then follows from Fubini's theorem that

$$\int_0^1 p(s) \left[\int_0^1 f(s|t)\xi(dt) \right] ds = 0,$$

for each $p(s) \in C[0, 1]$. Therefore, $\int_0^1 f(\cdot|t)\xi(dt) = 0$, which is a contradiction. $\square$

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