

Political Bargaining in a Changing World

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December 20, 2013

Abstract

This paper studies negotiations between two parties whose political power changes over time. The model has a unique subgame perfect equilibrium, which becomes very tractable when parties can make offers frequently. This tractability facilitates studying how changes in political power affect implemented policies. An extension of the model analyses how elections influence inter-party negotiations when implemented policies affect the parties' political power. Long periods of gridlock may arise when the time left until the election is short and parties have similar levels of political power.

KEYWORDS: bargaining, political power, elections, delay, gridlock.

*I am indebted to Attila Ambrus, Avidit Acharya, Sylvain Chassang, Sambuddha Ghosh, Edoardo Grillo, Faruk Gul, Wolfgang Pesendorfer, Bart Lipman, Salvatore Nunnari, Federico Weinschelbaum and various seminar and conference participants for helpful comments and suggestions. All remaining errors are my own. Address: Department of Economics, Boston University, 270 Bay State Road, Boston MA 02215. E-mail: jortner@bu.edu.

1 Introduction

This paper studies negotiations between two parties whose political power changes over time. Fluctuations in political power are a common feature in democratic countries. For instance, Gallup's polls show that Barack Obama's approval rate was close to 70 percent when he was sworn in as President in January 2009. By July 2009, his approval rate had dropped to around 55 percent, and in January 2010 it was below 50 percent.¹ These fluctuations in the political climate often reflect changes in the public opinion, and can have a significant impact on the ability of political parties to carry out their legislative agendas. In this paper, I construct a baseline model of bargaining to analyze how time-varying political power affects the outcomes of inter-party negotiations. I then use this model to study how the proximity of elections influences policymaking when the policies that parties implement have an effect on their political power.

The baseline model features two political parties that have to bargain over which policy to implement. The parties' relative political power evolves continuously over time as a diffusion process, but parties can only make offers at times on the grid $\{0, \Delta, 2\Delta, \dots\}$. The constant $\Delta > 0$ measures the time between bargaining rounds. The parties' level of political power determines their relative bargaining strength: the higher a party's political power, the more frequently that party will be making offers. This link between political power and bargaining power reflects situations in which low levels of support among the electorate reduce the degree of unity within a party and weaken its bargaining position.²

This bargaining game has a unique subgame perfect equilibrium (SPE). Parties always reach an agreement at the beginning of the negotiations, and the agreement that they reach depends on the initial level of relative political power. The unique SPE is difficult to analyze for a fixed time period $\Delta > 0$, but I show that it becomes very tractable in the limit as Δ goes to zero. The tractability of the limiting SPE, which is a consequence of the assumption that political power evolves as a diffusion process, allows me to analyze the effect that different features of the environment have on bargaining outcomes. For instance, I find conditions under which a more volatile political climate benefits the party with less political power and leads parties to implement less extreme policies.

I extend this model to study inter-party negotiations in the proximity of elections. As before, two parties bargain over which policy to implement in an environment in which their

¹See www.gallup.com/poll/124922/Presidential-Approval-Center.aspx.

²For instance, individual legislators may choose to vote together with the opposing party on a given issue if their own party's level of support among the electorate is low.

relative political power is changing over time. The two new features of this extension are: (i) there is an upcoming election and the parties' relative political power at the election date determines their chances of winning the vote; and (ii) the policy that parties implement has an effect on their political power, therefore also affecting their chances of winning the election. The model is flexible, allowing for implemented policies to affect the parties' political power in general ways. This flexibility allows me to study the dynamics of bargaining under different assumptions of how policies affect political power.

The proximity of an election has a substantial effect on the outcomes of negotiations. Unlike the baseline model, when there is an election upcoming the unique equilibrium may involve long periods of gridlock; i.e., delay. These delays occur in spite of the fact that implementing a policy immediately is always the efficient outcome. I show that these periods of political inaction can only arise when the time left until the election is short enough. On the other hand, parties are always able to reach a compromise if the election is sufficiently far away. Intuitively, parties cannot uncouple the direct effect of a policy from its indirect effect on the election's outcome. When the election is close enough, this may reduce the scope of trade to the point that there is no policy that both parties are willing to accept.

The equilibrium dynamics when there is an election upcoming depend on the effect that policies have on political power. I use this general model to analyze the dynamics of bargaining under different assumptions of how policies affect political power. The first setting I consider is one in which the party with proposal power sacrifices political power when it implements a policy that is close to its ideal point. This trade-off between ideal policies and political power arises when voters punish parties that implement extreme policies; i.e., policies that are far away from the median voter's ideal point. I show that there will necessarily be gridlock in this setting if parties derive a high enough value to winning the election. Moreover, gridlock is more likely to arise when political power is balanced, with both parties having similar chances of winning the vote. On the other hand, parties are more likely to reach an agreement when one of them has a high level of support among the electorate.

I also study a setting in which the party that obtains a better deal out of the negotiation is able to increase its political power. This link between agreements and political power arises when parties bargain over how to distribute discretionary spending and can use the resources they get from the negotiation to broaden their level of support among the electorate. I show that parties always reach an immediate agreement in this setting. Moreover, an upcoming election leads to more egalitarian agreements relative to the model without elections.³

³I also analyze a setting in which it is always costly in terms of political power for the responder to concede to proposals made by its opponent. I show that there will also be gridlock in this setting if parties

Introducing elections adds a new payoff-relevant state variable to the model: when there is an election upcoming, parties care about both their political power and the time left until the election. This additional state variable introduces a new layer of complexity to the analysis, making it harder to obtain a clean characterization of the equilibrium outcome. I sidestep this difficulty by providing bounds to the parties' equilibrium payoffs. These bounds become tight as the election gets closer and are easy to compute in the limit as the time period goes to zero. I use these bounds on the parties' payoffs to derive sufficient conditions for gridlock to arise, and to study how the likelihood of gridlock depends on the time left until the election and on the parties' level of political power.

Starting with Baron and Ferejohn (1989), there is a large body of literature that uses non-cooperative game theory to analyze political bargaining. Banks and Duggan (2000, 2006) generalize the model in Baron and Ferejohn by allowing legislators to bargain over a multidimensional policy space. A series of papers build on these workhorse models to study the effect that different institutional arrangements have on legislative outcomes.⁴ The current paper adds to this strand of literature by introducing a model of political bargaining in which the parties' political power changes over time. I model time-varying political power as a continuous-time diffusion process. This assumption leads to a tractable characterization of the limiting SPE with frequent offers, allowing me to obtain clean comparative statics results. I use this model to study how the proximity of an election affects bargaining outcomes. This extension highlights the importance of electoral considerations in understanding the dynamics of political bargaining and gives new insights as to when gridlock is more likely to arise.⁵

There are other papers that study settings in which policies affect future political power and electoral outcomes. Besley and Coate (1998) study a two period model in which the policy implemented today may change the identity of the policymaker in the future, and show that this link between policies and future power may lead to inefficient policies in the present. Bai and Lagunoff (2011) construct an infinite horizon model which also features a link between current policies and future political power. They focus on settings in which

attach a sufficiently high value to winning the election.

⁴Winter (1996) and McCarty (2000) analyze models à la Baron-Ferejohn with the presence of veto players. Baron (1998) and Diermeier and Feddersen (1998) study legislatures with vote of confidence procedures. Diermeier and Myerson (1999), Ansolabehere et al. (2003) and Kalandrakis (2004) study legislative bargaining under bicameralism. Snyder et al. (2005) analyze the effects of weighted voting within the Baron-Ferejohn framework. Cardona and Ponsati (2011) analyze the effects of supermajority rules in the model of Banks and Duggan. Diermeier and Vlaicu (2010) construct a legislative bargaining model to study the differences between parliamentarism and presidentialism in terms of their legislative success rate.

⁵There is also a growing literature that studies dynamic political bargaining models with an endogenous status-quo. Papers in this literature include Kalandrakis (2004), Diermeier and Fong (2011), Duggan and Kalandrakis (2011), Dziuda and Loeper (2013), Nunnari (2012) and Bowen, Chen and Eraslan (2013).

the current ruler faces a trade-off between implementing its preferred policy and sacrificing future political power, and characterize the equilibrium dynamics that such a trade-off gives rise to.⁶ The model with elections in the current paper also features a link between policies and future political power. The difference, however, is that policies are implemented through a bargaining process in my model. The results in the current paper show that the link between policies and political power can have a substantial impact on the dynamics of political bargaining when there is an election upcoming.

This paper shares some features with Dixit, Grossman and Gul (2000), who study a model in which two political parties interact repeatedly and in which the parties' political power evolves over time according to a Markov chain. At each period, the party with more political power can unilaterally decide how to allocate a unit surplus. Dixit, Grossman and Gul characterize efficient divisions of the surplus that are self-enforcing over time. The current paper also analyzes a setting with time-varying political power. However, in contrast to Dixit, Grossman and Gul, this paper studies a canonical bargaining model in which parties negotiate over a single policy.⁷

This paper also relates to Simsek and Yildiz (2009), who study a bilateral bargaining game in which the bargaining power of the players evolves stochastically over time. Simsek and Yildiz focus on settings in which players have optimistic beliefs about their future bargaining power. They show that optimism can give rise to costly delays if players expect bargaining power to become more "durable" at a future date. In contrast, there are no differences in beliefs in my model, and delays can arise when there is an election upcoming and when the policies that parties implement before the election affect their chances of winning the vote.

More broadly, this paper relates to the literature on delays and inefficiencies in bargaining. Delays in bargaining can arise when players have private information (Kennan and Wilson, 1993), when players bargain over a stochastic surplus (Merlo and Wilson, 1995, 1998), or when players can build a reputation for being irrational (Abreu and Gul, 2000). Inefficiencies may also arise when players are optimistic about their bargaining power and update their beliefs as time goes by (Yildiz, 2004), or when outside options are history dependent (Compte and Jehiel, 2004). The current paper offers a new rationale for delays in political bargaining and provides novel testable implications as to when these delays are more likely to arise.

⁶Other papers in this literature are Milesi-Ferretti and Spolaore (1994), Bourguignon and Verdier (2000) and Hassler et al. (2003).

⁷Acemoglu, Golosov, Tsyvinski (2011) and Acemoglu, Egorov and Sonin (2013) also study dynamic models of political economy with changing political power.

2 Baseline model

This section introduces the baseline model of political bargaining with time-varying political power. Section 2.1 presents the framework. Section 2.2 proves existence and uniqueness of a SPE and characterizes the parties' limiting SPE payoffs as the time period goes to zero. Section 3 extends this model to study political negotiations in the proximity of elections.

2.1 Framework

Let $[0, 1]$ be the set of alternatives or policies. Two political parties, $i = 1, 2$, bargain over which policy in $[0, 1]$ to implement. The set of times is a continuum $T = [0, \infty)$, but parties can only make offers at points on the grid $T(\Delta) = \{0, \Delta, 2\Delta, \dots\}$. The constant $\Delta > 0$ measures the time between bargaining rounds. Both parties are expected utility maximizers and have a common discount factor $e^{-r\Delta}$ across periods, where $r > 0$ is the discount rate. Let $z_i \in [0, 1]$ denote party i 's ideal policy and assume that $z_1 > z_2$. Party i 's utility from implementing policy $z \in [0, 1]$ is $u_i(z) = 1 - |z - z_i|$. Throughout the paper I maintain the assumption that the parties' ideal points are at the extremes of the policy space, with $z_1 = 1$ and $z_2 = 0$.⁸ This implies that $u_1(z) = z$ and $u_2(z) = 1 - z$ for all $z \in [0, 1]$. Therefore, this model is equivalent to a setting in which parties bargain over how to divide a unit surplus.

Unlike models of legislative bargaining à la Baron and Ferejohn (1989) and Banks and Duggan (2000, 2006), I assume that bargaining takes place between parties, not individual legislators. This assumption reflects situations in which the leaders of each party bargain over an issue on behalf of their respective parties. The need for parties to negotiate arises when neither party has the ability to implement policies unilaterally. For instance, in the United States parties have to negotiate to implement policies when the two chambers of Congress are controlled by different parties, or when neither party has a filibuster-proof majority in the Senate. The need for parties to negotiate also arises if the president (who has veto power) is from a different party than the majority party in Congress.

The model's key variable is an exogenous and publicly observable stochastic process x_t , which measures the parties' relative political power and which determines the bargaining protocol. Let $B = \{B_t, \mathcal{F}_t : 0 \leq t < \infty\}$ be a one-dimensional Brownian motion on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $\{\mathcal{F}_t : 0 \leq t < \infty\}$ is the filtration generated by the

⁸The assumption that the parties' ideal policies are at the extremes of the policy space is without loss of generality. If the policy space was $[a, b]$ with $a < z_1$ and $b > z_2$, all the alternatives in $[a, z_1) \cup (z_2, b]$ would be strictly Pareto dominated by policies in $[z_1, z_2]$. It is possible to show that adding these Pareto dominated policies would not change the equilibrium outcome.

Brownian motion. The Brownian motion B drives the process x_t . In particular, I assume that x_t evolves as a Brownian motion with constant drift μ and constant volatility $\sigma > 0$, with reflecting boundaries at 0 and 1. That is, while $x_t \in (0, 1)$ this variable evolves as

$$dx_t = \mu dt + \sigma dB_t. \tag{1}$$

When x_t reaches either 0 or 1, it reflects back. The reflecting boundaries guarantee that $x_t \in [0, 1]$ at all times $t \geq 0$.⁹ Note that the process x_t evolves in continuous time, but parties can only make offers at times $t \in T(\Delta)$. This implies that the speed at which the process x_t evolves remains constant as I vary the time period Δ . Moreover, this also implies that the process x_t becomes more persistent across bargaining rounds as the time period shrinks: for smaller values of Δ the distribution of $x_{t+\Delta}$ conditional $x_t = x$ is more concentrated around x than for larger values of Δ . The assumption that parties can only make offers on the grid $T(\Delta)$ makes this a game in discrete time, allowing me to use subgame perfection as a solution concept.

The process x_t measures the parties' relative political power, or their relative level of support among the electorate. I use the convention that high (low) values of x_t represent situations in which party 1 (party 2) has a large level of political power. The parties' relative political power determines the bargaining protocol. In particular, at each bargaining round $t \in T(\Delta)$ the party with more political power has proposal power: party 1 has proposal power if $x_t \geq 1/2$ and party 2 has proposal power if $x_t < 1/2$. The party with proposal power can either make an offer $z \in [0, 1]$ to its opponent or pass. If the other party (i.e., the responder) rejects the offer or if the proposer chooses to pass, then play moves to round $t + \Delta$. Otherwise, if at time $t \in T(\Delta)$ the responder accepts its opponent's proposal to implement policy $z \in [0, 1]$, party $i = 1, 2$ obtains at this date a payoff of $u_i(z)$ and the game ends.

This bargaining protocol implies that changes in the parties' political power translate into changes in their relative bargaining position. Party 1's bargaining position is strong when x_t is large, since a large value of x_t means that party 1 will (on average) be making offers more frequently in the future. Similarly, party 2's bargaining position is strong when x_t is low. This link between political power and bargaining power arises when members in Congress don't want to risk alienating their constituents by opposing the preferences of the more popular party.¹⁰

⁹See Harrison (1985) for a detailed description of diffusion processes with reflecting boundaries.

¹⁰There is empirical evidence showing that legislators respond to the preferences of their constituencies (i.e., Gerber and Lewis, 2004), and that constituents punish legislators that ignore their preferences (i.e., Canes-Wrone et al., 2002).

There are two assumptions in this model that simplify the analysis but are not crucial for the results that follow. The first one is the assumption that only the party with more political power has proposal power. Section 4 shows how the model can be extended to allow for more general bargaining protocols under which at each period $t \in T(\Delta)$ party 1 makes offers with probability $p_1(x_t) \in [0, 1]$ and party 2 makes offers with probability $p_2(x_t) = 1 - p_1(x_t)$. This class of protocols permits modeling asymmetric situations in which one party has, for institutional reasons, more proposal power than its opponent. Moreover, by choosing functions $p_1(\cdot)$ and $p_2(\cdot)$ that don't vary much with x , this class of protocols also permits modeling situations in which changes in political power have a more limited impact on the parties' relative bargaining strength.

The second simplifying assumption is that the process x_t has constant drift μ and constant volatility $\sigma > 0$. Appendix A.7.2 shows how the model can be extended to allow for more general stochastic processes under which the drift and volatility of political power depend at each point in time on the value of x_t . For instance, this generalization allows modeling situations in which the parties' political power has a tendency to revert to its long-run mean.

To illustrate the sequencing of moves in the game suppose that $x_0 \in [1/2, 1]$. In this case party 1 has proposal power from $t = 0$ until the first time x_t goes below $1/2$; i.e., until $\tau_1 = \inf\{t \in T(\Delta) : x_t < 1/2\}$. At each period $t \in T(\Delta)$ until τ_1 party 1 can either make an offer $z \in [0, 1]$ or pass. If party 2 accepts an offer before τ_1 , the bargaining ends and parties collect their payoffs. Otherwise, party 2 becomes proposer between τ_1 and time $\tau_2 = \inf\{t \in T(\Delta), t > \tau_1 : x_t \geq 1/2\}$. Bargaining continues this way, with parties alternating in their right to make proposals according to the realization of x_t , until a party accepts an offer. See Figure 1 for a plot of a sample path of x_t .

Let Γ_Δ denote the bargaining game with time period Δ . I look for the subgame perfect equilibria (SPE) of this game.

2.2 Equilibrium

Let $M_1 := [1/2, 1]$ be the set of values of x at which party 1 has proposal power and let $M_2 := [0, 1/2)$ be the set of values of x at which party 2 has proposal power. For any function $f : [0, 1] \rightarrow \mathbb{R}$ and any $s > t \geq 0$, let $\mathbb{E}[f(x_s)|x_t = x]$ denote the expectation of $f(x_s)$ conditional on $x_t = x$. The following result shows that Γ_Δ has a unique SPE. All proofs are in the appendix.

Theorem 1 *For any $\Delta > 0$, Γ_Δ has a unique SPE. Parties reach an agreement at $t = 0$ in*

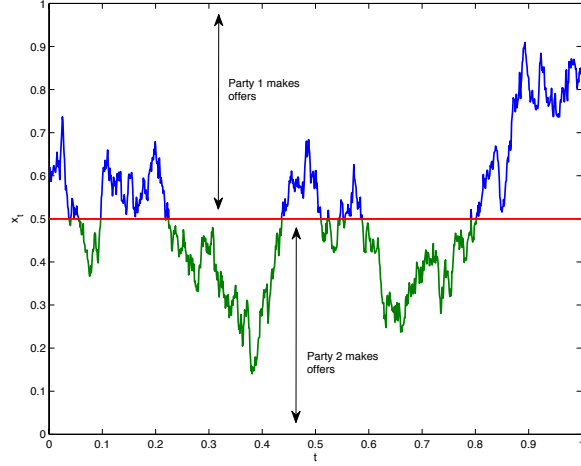


Figure 1: Sample path of x_t .

the unique SPE. For $i = 1, 2$, let $V_i^\Delta(x)$ denote party i 's SPE payoff when relative political power is $x \in [0, 1]$. These payoffs satisfy:

$$V_i^\Delta(x) = \begin{cases} e^{-r\Delta} \mathbb{E} [V_i^\Delta(x_{t+\Delta}) | x_t = x] & \text{if } x \notin M_i, \\ 1 - e^{-r\Delta} \mathbb{E} [V_j^\Delta(x_{t+\Delta}) | x_t = x] & \text{if } x \in M_i. \end{cases}$$

The content of Theorem 1 can be described as follows. In a SPE, the party with less political power accepts any offer giving that party a utility at least as large as its continuation payoff of waiting until the next round. Knowing this, the proposer always makes the lowest offer that its opponent is willing to accept and the game ends with an immediate agreement.

By Theorem 1, for all $x \in M_i$

$$V_i^\Delta(x) = 1 - e^{-r\Delta} \mathbb{E} [V_j^\Delta(x_{t+\Delta}) | x_t = x] = 1 - e^{-r\Delta} + e^{-r\Delta} \mathbb{E} [V_i^\Delta(x_{t+\Delta}) | x_t = x], \quad (2)$$

where the second equality follows since $V_i^\Delta(y) + V_j^\Delta(y) = 1$ for all $y \in [0, 1]$. Combining equation (2) with Theorem 1, it follows that

$$V_i^\Delta(x) = (1 - e^{-r\Delta}) \mathbf{1}_{\{x \in M_i\}} + e^{-r\Delta} \mathbb{E} [V_i^\Delta(x_{t+\Delta}) | x_t = x], \quad (3)$$

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function. Equation (3) shows that party i 's payoff when i has proposal power is equal to $1 - e^{-r\Delta}$ plus its expected continuation value. On the other hand, party i 's payoff when i is the responder is only equal to its expected continuation value. The

term $1 - e^{-r\Delta}$ represents the rent that a party obtains when it has proposal power. As it is standard in bilateral bargaining games, the size of the proposer's rent depends on the rate r at which parties discount payoffs and on the time period Δ .

Theorem 1 shows existence and uniqueness of SPE. However, the parties' SPE payoffs are difficult to compute for a fixed time period $\Delta > 0$. This difficulty in computing payoffs limits the possibility of performing comparative statics. To obtain a better understanding of the model, the next result characterizes the parties' limiting SPE payoffs as $\Delta \rightarrow 0$. These limiting payoffs are easy to compute, and provide a very good approximation of the SPE payoffs for settings in which the time between bargaining rounds is short.

Theorem 2 *There exist functions $V_1^*(\cdot)$ and $V_2^*(\cdot)$ such that, for $i = 1, 2$, $V_i^\Delta(\cdot)$ converges uniformly to $V_i^*(\cdot)$ as $\Delta \rightarrow 0$. Moreover, for all $x \neq 1/2$, $V_i^*(\cdot)$ solves*

$$rV_i^*(x) = \begin{cases} \mu(V_i^*)'(x) + \frac{1}{2}\sigma^2(V_i^*)''(x) & \text{if } x \notin M_i, \\ r + \mu(V_i^*)'(x) + \frac{1}{2}\sigma^2(V_i^*)''(x) & \text{if } x \in M_i, \end{cases} \quad (4)$$

with $(V_i^*)'(0) = (V_i^*)'(1) = 0$, $V_i^*(1/2^-) = V_i^*(1/2^+)$ and $(V_i^*)'(1/2^-) = (V_i^*)'(1/2^+)$.

Theorem 2 shows that party i 's limiting payoff as $\Delta \rightarrow 0$ is the solution to the ordinary differential equation (4) with appropriate boundary conditions. The left-hand side of (4) is party i 's limiting payoff measured in flow terms, while the right-hand side of (4) shows the sources of party i 's limiting flow payoff. Party i 's flow payoff when it has proposal power is equal to the flow rent it extracts from being proposer, which in the limit as $\Delta \rightarrow 0$ is equal to r , plus the expected change in its continuation value coming from changes in political power, which is equal to $\mu(V_i^*)'(x) + \frac{1}{2}\sigma^2(V_i^*)''(x)$. On the other hand, party i 's flow payoff when it does not have proposal power is given only by the expected change in its continuation value.

The parties' limiting SPE payoffs satisfy four boundary conditions. The boundary conditions $(V_i^*)'(0) = (V_i^*)'(1) = 0$ are a consequence of the nature of the process x_t : since x_t has reflecting boundaries, party i 's payoff becomes flat as x approaches either 0 or 1. The condition that $V_i^*(1/2^-) = V_i^*(1/2^+)$ implies that party i 's payoff is continuous, while the condition that $(V_i^*)'(1/2^-) = (V_i^*)'(1/2^+)$ implies that party i 's payoff is differentiable.

The solution to the ordinary differential equation in (4) is given by

$$V_i^*(x) = \begin{cases} a_i e^{-\alpha x} + b_i e^{\beta x} & \text{if } x \notin M_i, \\ 1 + c_i e^{-\alpha x} + d_i e^{\beta x} & \text{if } x \in M_i, \end{cases}$$

where $\alpha = (\mu + \sqrt{\mu^2 + 2r\sigma^2})/\sigma^2$, $\beta = (-\mu + \sqrt{\mu^2 + 2r\sigma^2})/\sigma^2$, and where (a_i, b_i, c_i, d_i) are constants determined by the four boundary conditions. Since parties always reach an immediate agreement, the limiting SPE payoffs in Theorem 2 can be used to back out the policy that parties implement as a function of the initial level of political power. Equation (A.4) in Appendix A.2 presents the full expressions for $V_1^*(\cdot)$ and $V_2^*(\cdot)$.

Definition 1 *The political climate is favorable for party 1 (for party 2) if $\mu \geq 0$ (if $\mu \leq 0$).*

I now use the limiting payoffs in Theorem 2 to derive comparative statics results. The first result considers how the parties' payoffs depend on the volatility of political power.

Proposition 1 *Suppose the political climate is favorable for party j . Then, the payoff of party $i \neq j$ is increasing in σ for all $x \in M_j$.*

The intuition behind Proposition 1 is as follows. When the political climate is favorable to the party making offers, an increase in volatility raises the chances that the weaker party will recover political power. This improves the weaker party's bargaining position, and allows it to obtain a better deal in the negotiations. A corollary of Proposition 1 is that higher levels of volatility will lead parties to implement less extreme policies (i.e., policies that are closer to $1/2$) when the political climate is favorable to the party with more political power.¹¹

The political climate is favorable for both parties when $\mu = 0$, so an increase in σ benefits party 1 when $x \in [0, 1/2)$ and benefits party 2 when $x \in (1/2, 1]$. On the other hand, when $\mu \neq 0$ an increase in volatility may decrease the weaker party's payoff if the political climate is favorable for this party. To see the intuition behind this, suppose $\mu > 0$ so that the political climate is favorable for party 1. In this setting, when x_t is slightly below $1/2$ an increase in volatility makes it more likely that party 2 will maintain proposal power for longer. This improves party 2's bargaining position, and thus lowers party 1's payoff. Figure 2 illustrates the results in Proposition 1 by plotting party 1's payoff for different values of σ . The left panel considers a case with $\mu = 0$ and the right panel considers a case with $\mu > 0$.

The next result shows how the parties' payoffs depend on the drift of political power.

Proposition 2 *Party 1's payoff is strictly increasing in μ for all $x \in [0, 1]$, and party 2's payoff is strictly decreasing in μ for all $x \in [0, 1]$.*

¹¹Acemoglu, Golosov, Tsyvinski (2011) study a dynamic production economy in which, at each period, the party in power can decide how to allocate resources. They show that, in this setting, more fluctuations in political power can reduce distortions and lead to more efficient outcomes.

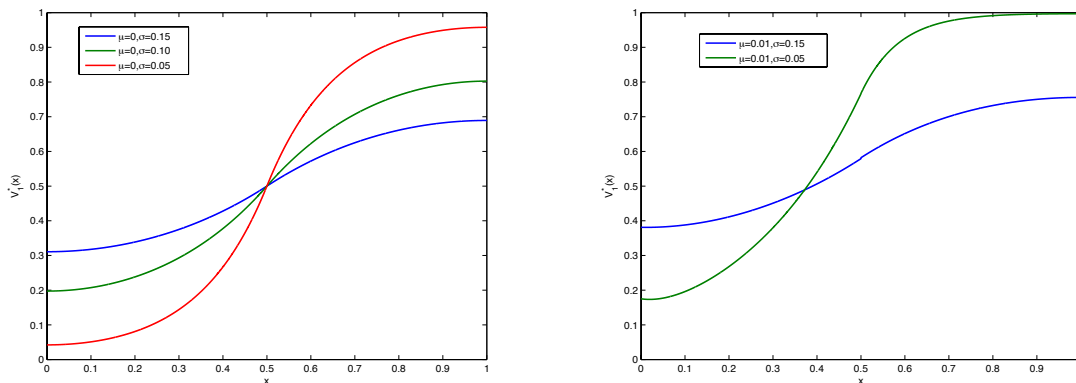


Figure 2: Party 1's limiting payoff. For both figures, $r = 0.05$.

The intuition behind this result is straightforward: a higher μ implies that party 1 will (on average) be making offers more frequently in the future. This improves party 1's bargaining position, and allows it to implement a policy closer to its preferred alternative.

3 Elections and political gridlock

This section extends the model of Section 2 to study political negotiations in the proximity of elections. Section 3.1 presents the extended model. Section 3.2 proves existence and uniqueness of equilibrium payoffs. Section 3.3 provides bounds on payoffs and shows how these bounds can be used to study how the likelihood of gridlock depends on the parties' level of political power and the time left until the election. Section 3.4 studies the dynamics of bargaining under three different specifications of the model. Finally, Section 3.5 discusses empirical implications.

3.1 A model with elections

As in Section 2, parties 1 and 2 bargain over which policy in $[0, 1]$ to implement in a setting in which their relative political power is changing over time. The model has two new features relative to the model in Section 2. First, there will be an election at a future date $t^* > 0$, with $t^* \in T(\Delta)$. The outcome of this election depends on the parties' level of political power at the election date. In particular, the party with more support among the electorate at time t^* wins the election: party 1 wins if $x_{t^*} \geq 1/2$ and party 2 wins if $x_{t^*} < 1/2$. The party that wins the election earns at time t^* a payoff equal to $K > 0$. The constant K measures the

benefit that parties derive from being in office. For simplicity, I focus on the case in which there is a single election at time t^* . Section 4 discusses how the results generalize to settings with multiple elections over time.

The second new feature of this model is that the policy that parties implement affects their political power. From $t = 0$ until the time at which parties reach an agreement, relative political power x_t evolves as a Brownian motion with drift μ and volatility $\sigma > 0$ and with reflecting boundaries at 0 and 1. If at time $t \in T(\Delta)$ parties reach an agreement to implement policy $z \in [0, 1]$, then political power jumps at this date by $h(x_t, z)$; that is, $x_{t^+} = \lim_{s \downarrow t} x_s = x_t + h(x_t, z)$ if parties implement policy z at time t . Then, from time t^+ onwards, the process x_t continues to evolve as a Brownian motion with drift μ and volatility σ and with reflecting boundaries at 0 and 1. The function $h(\cdot, \cdot)$ captures in a reduced form way the effect that policies have on political power. I impose only two restrictions on $h(\cdot, \cdot)$: (i) $x + h(x, z) \in [0, 1]$ for all $x, z \in [0, 1] \times [0, 1]$, and (ii) $h(x, \cdot)$ is continuous for all $x \in [0, 1]$. The first condition guarantees that the parties' relative political power always remains bounded in $[0, 1]$, while the second condition guarantees that there always exist an optimal offer for the party with proposal power.¹²

The bargaining protocol is the same as in the model of Section 2: at each time $t \in T(\Delta)$, party 1 has proposal power if $x_t \geq 1/2$ and party 2 has proposal power if $x_t < 1/2$. The party with proposal power can either make an offer to its opponent or pass. If the responder rejects the offer or if the party with proposal power chooses to pass, then play moves to period $t + \Delta$. Otherwise, if at time t the responder accepts its opponent's proposal to implement policy $z \in [0, 1]$, party 1 obtains at this date a payoff $u_1(z) = z$ and party 2 obtains a payoff $u_2(z) = 1 - z$.

The election is decided at date t^* , with its outcome depending on the value of x_{t^*} . The party that wins the election obtains at time t^* a payoff of K , and the other party obtains a payoff of 0. If parties had reached an agreement before time t^* , then the game ends immediately after the election. Otherwise, if parties have not reached an agreement by time t^* , the party with more political power can either make a proposal immediately after the election (i.e., still at date t^*) or can pass. Bargaining then continues, with parties alternating in their right to make offers according to the realization of x_t , until parties reach an agreement.

This model allows for general ways in which policies can affect the parties' political power: not only do different policies may have a different effect on the level of political power (i.e.,

¹²This specification implies that implemented policies have an instantaneous effect on the parties' political power. Section 4 discusses how the results in this paper would generalize if implemented policies affected the parties' political power in alternative ways.

for a fixed x , $h(x, z)$ may vary with z), but also the same policy may have a different effect on political power depending on the current level of x (i.e., for a fixed z , $h(x, z)$ may vary with x). This general model can accommodate a variety of settings. For instance, this model can accommodate settings in which the party with proposal power loses political power if it implements policies that are extreme (i.e., policies that are far from the median voter's preferred alternative). This framework can also accommodate settings in which the party that obtains a better deal out of the negotiation is able to increase its political power.

There are four assumptions in this model with elections that are made for simplicity but are not crucial for the analysis that follows. First, as in the baseline model of Section 2, this model with elections can be extended to allow for more general bargaining protocols under which at each time $t \in T(\Delta)$ party $i = 1, 2$ makes offers with probability $p_i(x_t)$. Second, as I already mentioned above, this model can also be extended to allow for multiple elections over time. Third, this model can be extended to allow for more general stochastic processes for the evolution of relative political power. Section 4 describes the first two extensions, and Appendix A.7.2 describes the third one.

Finally, this model assumes that parties are purely office motivated: they obtain a private benefit from just winning the election. Section 4 extends the model to a setting in which parties also have policy motivations for winning elections. The extension has two new features relative to the model in this section. First, the outcome of the election affects the bargaining protocol from time t^* onwards; in particular, a party is more likely to make offers after time t^* if it wins the election than if it loses it. Second, after the election parties bargain over a new issue. In this setting, parties want to win the election in order to have a larger influence in the policies that get implemented after time t^* .

Let $\Gamma_\Delta(t^*)$ denote the game with time period $\Delta > 0$ and election date $t^* > 0$. I look for the SPE of this game. To guarantee uniqueness of equilibrium payoffs, I focus on SPE in which the responder always accepts offers that leave her indifferent between accepting and rejecting, and in which the party with proposal power always makes an acceptable offer to its opponent whenever it is indifferent between making the acceptable offer that maximizes its payoff or passing. The first condition rules out multiplicities arising in knife-edge cases in which all acceptable offers by the responder leave this party just indifferent between accepting or rejecting, while the second condition rules out multiplicities arising in knife-edge cases in which the proposer is indifferent between making the acceptable offer that is best for it or passing. From now on I use the word equilibrium to refer to an SPE that satisfies these properties.

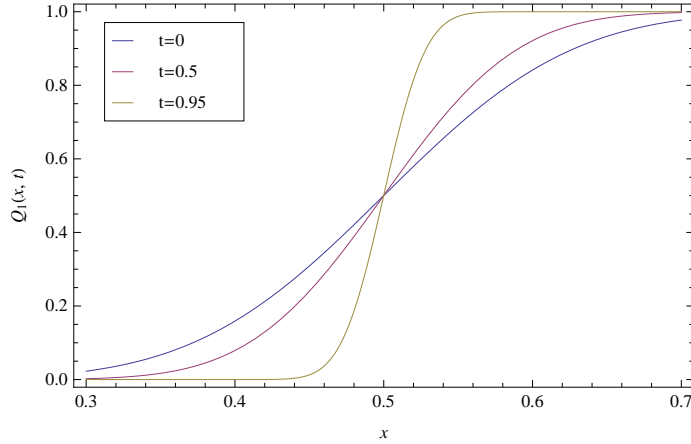


Figure 3: Parameters: $\mu = 0$, $\sigma = 0.1$, $t^* = 1$.

3.2 Equilibrium

For any function $f : [0, 1] \rightarrow \mathbb{R}$ and any $s > t \geq 0$, let $\mathbb{E}_{NA}[f(x_s)|x_t = x]$ denote the expectation of $f(x_s)$ conditional on $x_t = x$ assuming that parties don't reach an agreement between times t and s ; i.e., assuming that between t and s relative political power evolves as a Brownian motion with drift μ and volatility σ and with reflecting boundaries at 0 and 1.¹³

For all $x \in [0, 1]$ and all $t < t^*$, let $Q_i(x, t) := \mathbb{E}_{NA}[\mathbf{1}_{\{x_{t^*} \in M_i\}}|x_t = x]$ be the probability with which at time t party i is expected to win the election when $x_t = x$ if parties don't reach an agreement between t and t^* . If parties reach an agreement to implement policy z at $t < t^*$, the probability that party i wins the election is $Q_i^z(x_t, t) := Q_i(x_t + h(x_t, z), t)$. Figure 3 plots $Q_1(\cdot, t)$ for different values of t . Note that $Q_1(\cdot, t)$ is steep when parties have similar levels of political power, and it becomes flatter as x goes to 0 or 1. Intuitively, the likelihood that the weaker party recovers political power before the election is small when its opponent has a large political advantage. Therefore, as x approaches 0 or 1, further increments in the stronger party's political power have a limited effect on the parties' electoral chances. Note also that $Q_1(\cdot, t)$ becomes steeper around 1/2 as $t \rightarrow t^*$, since the chances that the weaker party recovers political power before time t^* become smaller as the election gets closer.

For $i = 1, 2$ and for any $t < t^*$, let $U_i(z, x, t) := u_i(z) + e^{-r(t^*-t)}KQ_i^z(x, t)$ be the expected payoff that party i would obtain if parties reached an agreement to implement policy $z \in [0, 1]$ at time $t < t^*$ with $x_t = x$: if parties implement policy z at time $t < t^*$, party i earns a payoff

¹³Since x_t evolves as a Brownian motion with drift μ and volatility σ and with reflecting boundaries at 0 and 1 while parties don't reach an agreement, the expectation operator $\mathbb{E}_{NA}[f(x_s)|x_t = x]$ is equal to the expectation operator $\mathbb{E}[f(x_s)|x_t = x]$ from Section 2.

$u_i(z)$ and it expects to win the election at time t^* with probability $Q_i^z(x, t)$. The following result establishes that this game has unique equilibrium payoffs.

Theorem 3 *For any $\Delta > 0$, $\Gamma_\Delta(t^*)$ has unique equilibrium payoffs. For $i = 1, 2$, let $W_i^\Delta(x, t)$ be party i 's equilibrium payoff at time $t \in T(\Delta)$ when $x_t = x$. For all $t \in T(\Delta)$ and all $x \in [0, 1]$, $W_i^\Delta(x, t)$ satisfies:*

- (i) if $t > t^*$, $W_i^\Delta(x, t) = V_i^\Delta(x)$,
- (ii) if $t = t^*$, $W_i^\Delta(x, t) = K\mathbf{1}_{\{x \in M_i\}} + V_i^\Delta(x)$,
- (iii) if $t < t^*$,

$$W_i^\Delta(x, t) = \begin{cases} e^{-r\Delta} \mathbb{E}_{NA} [W_i^\Delta(x_{t+\Delta}, t + \Delta) | x_t = x] & \text{if } A^\Delta(x, t) = \emptyset, \\ U_i(z^\Delta(x, t), x, t) & \text{if } A^\Delta(x, t) \neq \emptyset, \end{cases}$$

where $A^\Delta(x, t) = \{z \in [0, 1] : U_i(z, x, t) \geq e^{-r\Delta} \mathbb{E}_{NA} [W_i^\Delta(x_{t+\Delta}, t + \Delta) | x_t = x]\}$ for $i = 1, 2$ and, for all (x, t) such that $A^\Delta(x, t) \neq \emptyset$,

$$z^\Delta(x, t) \in \begin{cases} \arg \max_{z \in A^\Delta(x, t)} U_1(z, x, t) & \text{if } x \in M_1, \\ \arg \max_{z \in A^\Delta(x, t)} U_2(z, x, t) & \text{if } x \in M_2. \end{cases}$$

Parties always reach an agreement at times $t \geq t^$. Moreover, parties reach an agreement at times $t < t^*$ if and only if $A^\Delta(x, t) \neq \emptyset$.*

Theorem 3 can be summarized as follows. Part (i) shows that the parties' payoffs at times $t > t^*$ are equal to their payoffs in the game without elections. Part (ii), on the other hand, shows that the parties' payoffs at time $t = t^*$ are equal to their payoffs from the election plus their payoffs in the game without elections. Intuitively, for all $t \geq t^*$ the game ends immediately after parties reach an agreement. Therefore, the subgame that starts at any $t \geq t^*$ if parties have failed to reach an agreement before this date is strategically identical to the game in Section 2. This implies that for all such dates the outcome of the bargaining will be identical to the outcome of the game in Section 2: parties will reach an agreement at time $t \geq t^*$ if they have failed to do so before, and party $i = 1, 2$ will get a payoff of $V_i^\Delta(x_t)$ from this agreement.

Finally, part (iii) of Theorem 3 shows that at each time $t < t^*$ parties will reach an agreement only if there is a policy $z \in [0, 1]$ that, if implemented, would leave both parties weakly better-off than waiting until the next period and getting their continuation payoffs;

i.e., only if $A^\Delta(x, t) \neq \emptyset$. In this case, the policy $z^\Delta(x, t)$ that parties implement is the best policy for the party with proposal power among those policies that both parties are willing to accept.¹⁴ Otherwise, parties delay an agreement at time t if there is no policy that they are both willing accept.

Theorem 3 establishes uniqueness of equilibrium payoffs and leaves open the possibility of delay. The next result shows that, if there is delay, then this delay will only occur when the time left until the election is short enough.

Proposition 3 *There exists $s > 0$ such that, for any $h(\cdot, \cdot)$, parties always reach an agreement at times $t \in T(\Delta)$ with $t^* - t > s$. The cutoff s is strictly increasing in K .*

The intuition behind Proposition 3 is as follows. The discounted benefit $e^{-r(t^*-t)}K$ of winning the election is small when the election is far away. This limits the effect that implementing a policy has on the parties' payoffs from the election, making it easier for them to reach a compromise. The cutoff $s > 0$ in Proposition 3 is increasing on the benefit K that parties obtain from being in office. Therefore, gridlock may arise when the election is further away if parties attach a higher value to being in office.

3.3 Bounds on payoffs

The election at date $t^* > 0$ introduces an additional state variable to the model: when there is an election upcoming, parties care both about the level of relative political power and about the time left until the election. With this additional state variable, it is no longer possible to obtain a tractable characterization of the parties' payoffs. In this subsection, I sidestep this difficulty by providing bounds on payoffs. These bounds become tight as the election gets closer, and are easy to compute in the limit as $\Delta \rightarrow 0$. Moreover, I show how these bounds can be used to derive sufficient conditions for gridlock to arise in equilibrium, and to analyze how the likelihood of gridlock depends on the time left until the election and on the parties' level of political power.

For all $t \in T(\Delta)$, $t < t^*$, for all $x \in [0, 1]$ and for $i = 1, 2$, let

$$\underline{W}_i^\Delta(x, t) := \mathbb{E}_{NA} [e^{-r(t^*-t)}V_i^\Delta(x_{t^*})|x_t = x] + Ke^{-r(t^*-t)}Q_i(x, t).$$

¹⁴When $A^\Delta(x, t) \neq \emptyset$, there may be more than one policy in $A^\Delta(x, t)$ that maximizes the proposer's payoffs. Clearly, the proposer would obtain the same payoff by implementing any such policy. Note that for all $t < t^*$ and all $x \in [0, 1]$, $U_1(z, x, t) + U_2(z, x, t) = 1 + Ke^{-r(t^*-t)}$ for all $z \in [0, 1]$. Therefore, implementing any policy in $A^\Delta(x, t)$ that maximizes the proposer's payoff would also give the responder the same payoff.

Note that $\underline{W}_i^\Delta(x, t)$ is the expected payoff that party i would obtain if parties delayed an agreement until the election. Let $\overline{W}_i^\Delta(x, t) := \underline{W}_i^\Delta(x, t) + 1 - e^{-r(t^* - t)}$.

Lemma 1 *For all $t \in T(\Delta), t < t^*$, for all $x \in [0, 1]$ and for $i = 1, 2$, $W_i^\Delta(x, t) \in [\underline{W}_i^\Delta(x, t), \overline{W}_i^\Delta(x, t)]$.*

Lemma 1 derives bounds on the parties' payoffs prior to the election. Note that these bounds become tight as the election gets closer: $\overline{W}_i^\Delta(x, t) - \underline{W}_i^\Delta(x, t) = 1 - e^{-r(t^* - t)} \rightarrow 0$ as $t \rightarrow t^*$. Moreover, these bounds don't depend on the way in which policies affect the parties' political power; i.e., they don't depend on $h(x, z)$.

For fixed values of $\Delta > 0$ it is difficult to calculate the bounds $\underline{W}_i^\Delta(x, t)$ and $\overline{W}_i^\Delta(x, t)$. The reason for this is that these bounds depend on the parties' payoffs in the game without elections, and these payoffs are difficult to compute for fixed values of $\Delta > 0$. However, since $V_i^\Delta(\cdot)$ converges uniformly to $V_i^*(\cdot)$ as $\Delta \rightarrow 0$ (Theorem 2), it follows that

$$\underline{W}_i^\Delta(x, t) \rightarrow \underline{W}_i^*(x, t) := \mathbb{E}_{NA} [e^{-r(t^* - t)} V_i^*(x_{t^*}) | x_t = x] + K e^{-r(t^* - t)} Q_i(x, t) \text{ as } \Delta \rightarrow 0.$$

Moreover, this convergence is uniform.¹⁵ Lemma A4 in Appendix A.5 shows that $\underline{W}_i^*(x, t)$ solves a partial differential equation (PDE). This characterization of $\underline{W}_i^*(x, t)$ as a PDE allows for simple numerical computations of the bounds on payoffs in the limit as $\Delta \rightarrow 0$.

The next result uses the bounds in Lemma 1 to derive conditions under which there will be delay or agreement at states $(x, t) \in [0, 1] \times T(\Delta)$ with $t < t^*$.

Proposition 4 *For any time $t \in T(\Delta), t < t^*$ and any $x \in [0, 1]$,*

- (i) *if there exists $i \in \{1, 2\}$ such that $U_i(z, x, t) < \underline{W}_i^\Delta(x, t)$ for all $z \in [0, 1]$, then parties delay an agreement at time t if $x_t = x$;*
- (ii) *if there exists $i \in \{1, 2\}$ and $z', z'' \in [0, 1]$ such that $U_i(z', x, t) \leq \underline{W}_i^\Delta(x, t)$ and $\overline{W}_i^\Delta(x, t) \leq U_i(z'', x, t)$, then parties reach an agreement at time t if $x_t = x$.*

The intuition behind Proposition 4 is as follows. In this model with elections the range of possible payoffs that party i can obtain from implementing a policy at time $t < t^*$ is $[\min_z U_i(z, x_t, t), \max_z U_i(z, x_t, t)]$ (recall that $U_i(z, x_t, t) = u_i(z) + e^{-r(t^* - t)} Q_i^z(x_t, t)$). This range of payoffs is too small for parties to reach an agreement when $\max_z U_i(z, x_t, t) <$

¹⁵To see that this convergence is uniform, note that $|\underline{W}_i^\Delta(x, t) - \underline{W}_i^*(x, t)| \leq \mathbb{E}_{NA} [e^{-r(t^* - t)} (|V_i^\Delta(x_{t^*}) - V_i^*(x_{t^*})|) | x_t = x]$. Since $V_i^\Delta(x)$ converges uniformly to $V_i^*(x)$, for every $\eta > 0$ there exists $\overline{\Delta} > 0$ such that, for all $(x, t) \in [0, 1] \times [0, t^*]$, $\mathbb{E}_{NA} [e^{-r(t^* - t)} (|V_i^\Delta(x_{t^*}) - V_i^*(x_{t^*})|) | x_t = x] < e^{-r(t^* - t)} \eta \leq \eta$ whenever $\Delta < \overline{\Delta}$.

$\underline{W}_i^\Delta(x, t)$ for some party i , since party i would never be willing to implement a policy that gives itself a payoff lower than $\underline{W}_i^\Delta(x, t)$. On the other hand, parties are always able to reach an agreement when $\min_z U_i(z, x, t) \leq \underline{W}_i^\Delta(x, t)$ and $\max_z U_i(z, x, t) \geq \underline{W}_i^\Delta(x, t)$. Intuitively, the range $[\min_{z \in [0,1]} U_i(z, x, t), \max_{z \in [0,1]} U_i(z, x, t)]$ of attainable payoffs is large in this case, so parties are always able to find a compromise policy that they are both willing to accept.

Note that there is a gap between the conditions in the two parts of Proposition 4: there might exist states (x, t) at which the parties' payoffs satisfy neither the conditions in part (i) of Proposition 4 nor those in part (ii). Proposition 4 is silent about whether parties will be able or not to reach an agreement at those states. This gap in Proposition 4 arises because I work with bounds on the parties' payoffs. For each $t \in T(\Delta), t < t^*$, let $I(t) \subset [0, 1]$ be the set of values of x such that the parties' payoffs at (x, t) satisfy neither conditions in Proposition 4. Since the bounds on payoffs become tight as the election gets closer, the (Lebesgue) measure of $I(t)$ converges to 0 as $t \rightarrow t^*$.

Proposition 4 can be used to study the equilibrium dynamics of this model with elections: for each state $(x, t) \in [0, 1] \times T(\Delta)$, I can use the results in Proposition 4 to check whether parties will be able to reach an agreement or not when the state of the game is (x, t) . The next subsection illustrates this by studying how the proximity of elections influences bargaining outcomes under different assumptions on how policies affect political power.

For fixed values of $\Delta > 0$ it is difficult to check the conditions in Proposition 4, since it is difficult to compute the bounds on payoffs. Recall that $\underline{W}_i^\Delta(x, t)$ converges uniformly to $\underline{W}_i^*(x, t)$ as $\Delta \rightarrow 0$. Letting $\overline{W}_i^*(x, t) := 1 - e^{-r(t^*-t)} + \underline{W}_i^*(x, t)$, it follows that $\overline{W}_i^\Delta(x, t)$ converges uniformly to $\overline{W}_i^*(x, t)$ as $\Delta \rightarrow 0$. This observation, together with Proposition 4, leads to the following corollary.

Corollary 1 *For any time $t < t^*$ and any $x \in [0, 1]$,*

- (i) *if there exists $i \in \{1, 2\}$ such that $U_i(z, x, t) < \underline{W}_i^*(x, t)$ for all $z \in [0, 1]$, then there exists $\overline{\Delta} > 0$ such that parties delay an agreement at time t if $x_t = x$ and $\Delta < \overline{\Delta}$.*
- (ii) *if there exists $i \in \{1, 2\}$ and $z', z'' \in [0, 1]$ such that $U_i(z', x, t) < \underline{W}_i^*(x, t)$ and $\overline{W}_i^*(x, t) < U_i(z'', x, t)$, then there exists $\overline{\Delta} > 0$ such that parties reach an agreement at time t if $x_t = x$ and $\Delta < \overline{\Delta}$.*

Corollary 1 provides conditions for there to be delay or agreement at states (x, t) when the time between bargaining rounds is small. The conditions in Corollary 1 are easy to check numerically, since $\underline{W}_i^*(x, t)$ solves a PDE.¹⁶

¹⁶Moreover, the functions $U_i(z, x, t) = u_i(z) + e^{-r(t^*-t)}KQ_i^z(x, t)$ are also easy to compute numerically,

3.4 Bargaining and gridlock in the shadow of elections

The equilibrium dynamics in this model with elections will in general depend on the way in which the policies that parties implement affect their political power; i.e., on the function $h(x, z)$. I now explore different ways in which policies affect political power. The goal is to study how the proximity of elections affects the dynamics of bargaining under these settings.

3.4.1 Electoral trade-off

I start by considering a setting in which the party with proposal power faces the following trade-off: implementing policies that are close to its ideal point lowers its level of political power, while implementing moderate policies allows it to maintain its political advantage. This trade-off arises when voters punish parties that implement policies that are too extreme; i.e., policies that are far away from the median voter's ideal point.

To model this trade-off, I assume that for all $(x, z) \in [0, 1] \times [0, 1]$,

$$h(x, z) = \begin{cases} -\lambda|z - \frac{1}{2}| & \text{if } x \geq 1/2, \\ \lambda|z - \frac{1}{2}| & \text{if } x < 1/2, \end{cases} \quad (5)$$

where $\lambda \in (0, 1]$ measures the effect that implemented policies have on the parties' political power. The assumption that $\lambda \in (0, 1]$ guarantees that $x + h(x, z) \in [0, 1]$ for all $(x, z) \in [0, 1] \times [0, 1]$. This functional form of $h(x, z)$ captures the trade-off mentioned above, since the stronger party sacrifices political power when it implements a policy that is close to its preferred alternative.

Definition 2 *There is gridlock if there are states $(x, t) \in [0, 1] \times T(\Delta)$ at which parties don't reach an agreement. There is no gridlock if parties reach an agreement at all states $(x, t) \in [0, 1] \times T(\Delta)$.*

The following result shows that, in this setting, there will be gridlock whenever parties derive a sufficiently high value from winning the election.

Proposition 5 *Suppose $h(x, z)$ is given by equation (5). Then, there exists $\bar{K} > 0$ such that there is gridlock whenever $K > \bar{K}$.*

Proposition 5 shows that voters may promote gridlock if they punish parties that implement extreme policies. The intuition for this result is as follows. In this model with elections,

since $Q_i(x, t)$ also solves a PDE; see the proof of Lemma A4 in Appendix A.5.

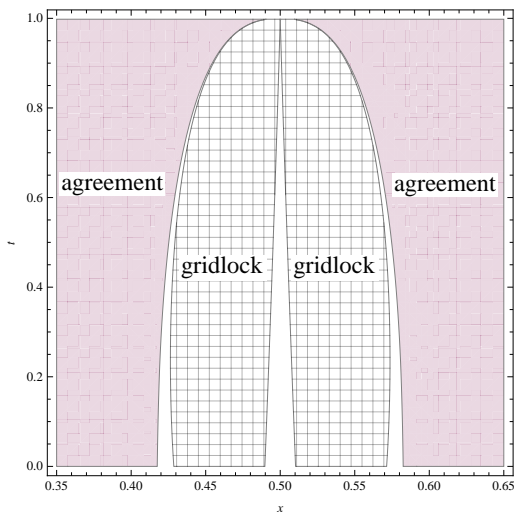


Figure 4: Parameters: $\mu = 0$, $\sigma = 0.05$, $r = 0.05$, $t^* = 1$, $K = 1.5$ and $\lambda = 0.1$.

implementing a policy at $t < t^*$ has two effects on the parties' payoffs: a direct effect, since parties derive utility from the policy that they implement, and an indirect effect, since the policy they implement affects their electoral chances. These two effects run in opposite directions for the party making offers when $h(x, z)$ satisfies equation (5), since implementing a policy closer to its ideal point increases the proposer's current payoff but it reduces its electoral chances. When K is large, these opposing forces reduce the set of policies that the proposer is willing to implement, making it harder for parties to reach a compromise.

Figure 4 considers a setting with $K > \bar{K}$ and illustrates the typical patterns of gridlock when $h(x, z)$ satisfies equation (5). The squared areas in the figure are the values of (x, t) at which parties will delay an agreement if Δ is small enough; i.e., states that satisfy the conditions in part (i) of Corollary 1. Therefore, if parties have not reached an agreement by time $t < t^*$ and the value of x_t is such that (x_t, t) is inside the squared region in Figure 4, then parties won't reach an agreement at time t either (provided Δ is small enough). Moreover, at times $s \in (t, t^*)$ parties will continue to delay an agreement as long as (x_s, s) remains inside the squared area in Figure 4. On the other hand, the shaded areas in Figure 4 are values of (x, t) at which parties will reach an agreement if Δ is small enough, i.e., states that satisfy the conditions in part (ii) of Corollary 1. The white areas are the values of (x, t) that are not covered by either parts of Corollary 1.

Figure 4 shows that parties will delay an agreement when one side has a small political advantage, and that they will reach a compromise when one party has a very strong bargaining position. To see the intuition for this, consider first states at which the party with

proposal power has a small political advantage. Note that this party has a lot to lose by implementing a policy close to its preferred alternative at such states, since implementing such a policy would have a large negative impact on its electoral chances (see Figure 3). If K is large, at such states the proposer will prefer to delay an agreement until the election than to implement a policy close to its preferred alternative and lose its electoral advantage. Moreover, at such states the party with proposal power doesn't want to implement a policy close to $1/2$ either: since it has a small political advantage, by delaying an agreement until the election date this party would very likely be able to implement a policy that lies closer to its ideal point than $1/2$. This implies that at such states any policy $z \in [0, 1]$ would give the proposer a lower payoff than what it could get by delaying an agreement until the election. Thus, there must be delay at such states.

Consider next states at which the party with proposal power has a very strong political advantage. At such states, the probability that the stronger party wins the election will still be very large even after implementing a policy that lies relatively close its ideal point. Therefore, the party with proposal power would be willing to implement such a policy at these states. Moreover, at these states the weaker party would also be willing to implement policies that are relatively close to its opponent's ideal point, since doing this would increase (at least marginally) its chances of winning the election. Thus, at these states parties are able to find a compromise policy that they are both willing to accept.

Proposition 5 shows that gridlock may arise when the party with proposal power faces an electoral trade-off. These delays are inefficient. To see this, suppose that parties have not reached an agreement by time $t < t^*$. The sum of their payoffs from implementing any policy $z \in [0, 1]$ at time t is $U_1(z, x, t) + U_2(z, x, t) = 1 + Ke^{-r(t^*-t)}$, while the sum of their payoffs from implementing any policy at $s > t$ is (from the perspective of time t) $e^{-r(s-t)} + Ke^{-r(t^*-t)} < 1 + Ke^{-r(t^*-t)}$.

Finally, for states (x, t) at which there is agreement I can obtain bounds on the policies that parties will agree on using the bounds on their payoffs: since party i 's payoff is bounded by $\underline{W}_i^\Delta(x, t)$ and $\overline{W}_i^\Delta(x, t)$, the policy $z^\Delta(x, t)$ that parties agree on at state (x, t) must be such that $U_i(z^\Delta(x, t), x, t) \in [\underline{W}_i^\Delta(x, t), \overline{W}_i^\Delta(x, t)]$. These bounds on policies are easy to compute numerically in the limit as $\Delta \rightarrow 0$. Moreover, since $\lim_{t \rightarrow t^*} \underline{W}_i^\Delta(x, t) - \overline{W}_i^\Delta(x, t) = 0$, these bounds become tight as the election gets closer.

3.4.2 Costly concessions

I now consider a setting in which the party with proposal power always benefits when a policy is implemented. This specification of the model is motivated by empirical evidence showing that voters usually hold the stronger party in the legislature accountable for the job performance of Congress. That is, voters usually reward or punish the party that controls the legislature depending on the performance that Congress has had; i.e., Jones and McDermott (2004) and Jones (2010). As journalist Ezra Klein wrote in an article for *The New Yorker*: “...it is typically not in the minority party’s interest to compromise with the majority party on big bills – elections are a zero-sum game, where the majority wins if the public thinks it has been doing a good job.”¹⁷

I model this setting by assuming that the stronger party’s level of political power jumps up discretely if parties reach an agreement to implement a policy. That is, for all $z \in [0, 1]$,

$$h(x, z) = \begin{cases} \min\{g, 1 - x\} & \text{if } x \geq 1/2, \\ -\min\{g, x\} & \text{if } x < 1/2, \end{cases} \quad (6)$$

where $g > 0$ is a constant. Note that in this setting it is always costly for the weaker party to concede to a policy put forward by its opponent: conceding to a policy lowers its political power by g , leading to a decrease in its electoral chances.

The next result shows there will also be gridlock in this setting if the payoff that parties obtain from winning the election is large enough.

Proposition 6 *Suppose $h(x, z)$ is given by equation (6). Then, there exists \bar{K} such that there is gridlock if $K > \bar{K}$.*

The intuition behind Proposition 6 is as follows. The weaker party incurs an electoral cost if it accepts an offer by its opponent prior to the election, since accepting an offer will negatively affect its electoral chances. When parties attach a high value to winning the election, there are states at which no offer $z \in [0, 1]$ compensates the weaker party for this electoral cost. There is no possible compromise at such states, since the weaker party strictly prefers to delay an agreement than to implement a policy.

Figure 5 considers a setting with $K > \bar{K}$ and illustrates the typical patterns of gridlock when $h(x, z)$ satisfies equation (6). The squared areas in the figure are values of (x, t) at which parties will delay an agreement if Δ is small; i.e., states that satisfy the conditions in part (i) of Corollary 1. On the other hand, the shaded areas in Figure 5 are values of (x, t)

¹⁷“Unpopular Mandate. Why do politicians reverse their positions?,” *The New Yorker*, June 25, 2012.

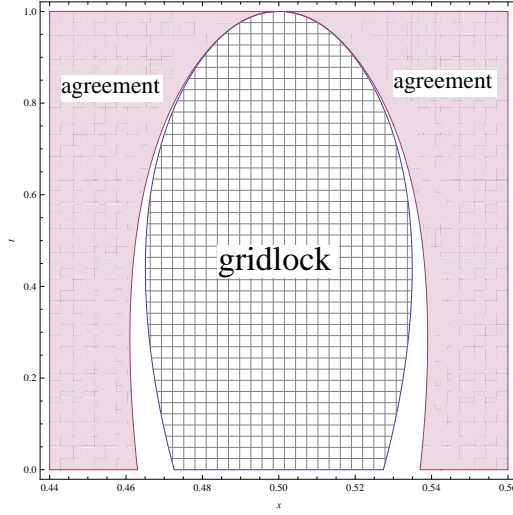


Figure 5: Parameters: $\mu = 0$, $\sigma = 0.1$, $r = 0.05$, $t^* = 1$, $K = 2$ and $g = 0.1$.

at which parties will reach an agreement if Δ is small; i.e., states that satisfy the conditions in part (ii) of Corollary 1. The white areas are the values of (x, t) that are not covered by either parts of Corollary 1.

Figure 5 shows that parties will delay an agreement when their level of political power is relative balanced, and will reach an agreement when one party has a strong political advantage. Intuitively, the cost that the weaker party incurs when it accepts a proposal is larger when political power is balanced, since in this case implementing a policy has a larger negative impact on the weaker parties' electoral chances (see Figure 3). If K is large, at these states there is no policy $z \in [0, 1]$ that compensates this party for its lower electoral chances, and so gridlock arises. As in the model in Section 3.4.1, the delay at these states is inefficient. On the other hand, the cost that the weaker party incurs by accepting an offer is lower when its opponent has a strong political advantage, since in this case its opponent will likely win the election even if parties don't reach an agreement. Therefore, at such states its easier for parties to reach a compromise.

Finally, for those values of (x, t) at which parties reach an agreement, I can again obtain bounds on the policy that parties will implement using the bounds on payoffs in Lemma 1: for such states (x, t) , the policy $z^\Delta(x, t)$ that parties agree on must be such that $U_i(z^\Delta(x, t), x, t) \in [\underline{W}_i^\Delta(x, t), \overline{W}_i^\Delta(x, t)]$.

3.4.3 Success begets success

I now consider a setting in which the party that obtains a better deal out of the negotiation is able to increase its level of political support. For instance, this link between agreements and political power arises when parties bargain over how to distribute discretionary spending and can use the resources they get out of the negotiation to broaden their level of support among the electorate.

To model a situation in which a better deal translates into more political power, I assume that for all $x \in [0, 1]$ the function $h(x, \cdot)$ is continuous and increasing. I also assume that $h(x, 0) \leq 0 \leq h(x, 1)$ for all $x \in [0, 1]$; that is, party i 's political power is weakly larger after the agreement if parties implement i 's preferred policy (recall that $z_1 = 1$ and $z_2 = 0$). The following result shows that parties will always reach an immediate agreement in this setting.

Proposition 7 *Suppose that, for all $x \in [0, 1]$, $h(x, \cdot)$ is continuous and increasing, with $h(x, 0) \leq 0 \leq h(x, 1)$. Then, there is no gridlock. Moreover, for all $t \in T(\Delta)$, $t < t^*$, for all $x \in [0, 1]$ and for $i = 1, 2$,*

$$W_i^\Delta(x, t) = V_i^\Delta(x) + e^{-r(t^*-t)} K Q_i(x, t). \quad (7)$$

The intuition behind Proposition 7 is as follows. Recall that policies have two effects on the parties' payoffs: a direct effect, since parties derive utility from the policies they implement, and an indirect effect, since the policies they implement affect their electoral chances. These two effects run in the same direction when $h(\cdot, \cdot)$ satisfies the assumptions in Proposition 7. As a result of this, the range of payoffs $[\min_z U_i(z, x, t), \max_z U_i(z, x, t)]$ that party $i = 1, 2$ can obtain from implementing a policy is always large in this setting.¹⁸ Given this large range of attainable payoffs, the party with proposal power is always able to find an offer that leaves its opponent indifferent between accepting and rejecting. The responder always accepts such an offer in equilibrium, and parties always reach an immediate agreement. In contrast, in the models of Sections 3.4.1 and 3.4.2 there might be states (x, t) at which the range of attainable payoffs is small, with $\max_z U_i(z, x, t) < \underline{W}_i^\Delta(x, t)$ for some party $i = 1, 2$. There is no policy that both parties are willing to accept at such states, so there is gridlock.

Equation (7) in Proposition 7 characterizes the parties' equilibrium payoffs in this setting. To see why the parties' payoffs satisfy equation (7), consider states $(x, t^* - \Delta)$ at which party i is proposer. At such states party $j \neq i$ accepts any offer giving it a payoff at least as large

¹⁸The proof of Proposition 7 shows that, in this setting, $\min_z U_i(z, x, t) \leq \underline{W}_i^\Delta(x, t)$ and $\max_z U_i(z, x, t) \geq \overline{W}_i^\Delta(x, t)$ for all states (x, t) with $t < t^*$.

as $e^{-r\Delta}\mathbb{E}_{NA}[W_j^\Delta(x_{t^*}, t^*)|x_{t^*-\Delta} = x] = e^{-r\Delta}\mathbb{E}_{NA}[V_j^\Delta(x_{t^*})|x_{t^*-\Delta} = x] + e^{-r\Delta}KQ_j(x, t^* - \Delta)$. In equilibrium, party i makes an offer that gives party j a payoff exactly equal to this quantity.¹⁹ Therefore, party j 's payoff at state $(x, t^* - \Delta)$ is

$$\begin{aligned} W_j^\Delta(x, t^* - \Delta) &= e^{-r\Delta}\mathbb{E}_{NA}[V_j^\Delta(x_{t^*})|x_{t^*-\Delta} = x] + e^{-r\Delta}KQ_j(x, t^* - \Delta) \\ &= V_j^\Delta(x) + e^{-r\Delta}KQ_j(x, t^* - \Delta), \end{aligned} \quad (8)$$

where the last equality follows since $V_j^\Delta(x) = e^{-r\Delta}\mathbb{E}_{NA}[V_j^\Delta(x_{t+\Delta})|x_t = x]$ for all x at which party j is responder.²⁰ Moreover, since parties reach an agreement at $t = t^* - \Delta$, the sum of their payoffs at state $(x, t^* - \Delta)$ must be equal to $1 + e^{-r\Delta}K$. Therefore, party i 's payoff at $(x, t^* - \Delta)$ satisfies

$$W_i^\Delta(x, t^* - \Delta) = 1 + e^{-r\Delta}K - W_j^\Delta(x, t^* - \Delta) = V_i^\Delta(x) + e^{-r\Delta}KQ_i(x, t^* - \Delta), \quad (9)$$

where the first equality follows since $W_i^\Delta(x, t^* - \Delta) + W_j^\Delta(x, t^* - \Delta) = 1 + e^{-r\Delta}K$ and the second equality follows from equation (8) and from the fact that $V_i^\Delta(x) + V_j^\Delta(x) = 1$ and $Q_i(x, t) + Q_j(x, t) = 1$ for all $t < t^*$. Equations (8) and (9) show that the parties' payoffs satisfy equation (7) at $t = t^* - \Delta$. The proof of Proposition 7 shows by induction that equation (7) also holds for all $t < t^*$. Finally, note that a crucial step in deriving equations (8) and (9) is that the proposer can always find an offer that leaves its opponent indifferent between accepting and rejecting. Such an offer always exists when $h(\cdot, \cdot)$ satisfies the assumptions in Proposition 7, but it might not exist in other settings (as in Sections 3.4.1 and 3.4.2).

Equation (7) can be used to back out the agreement that parties reach in this setting. Let $z^\Delta(x, t) \in [0, 1]$ be the agreement that parties reach at state (x, t) . Party 1's payoff from this agreement is $U_1(z^\Delta(x, t), x, t) = z^\Delta(x, t) + e^{-r(t^*-t)}KQ_1(x + h(x, z^\Delta(x, t)), t)$. On the other hand, by Proposition 7 party 1's payoff at state (x, t) is $W_1^\Delta(x, t) = V_1^\Delta(x) + e^{-r(t^*-t)}KQ_1(x, t)$. Therefore, $z^\Delta(x, t)$ must be such that

$$z^\Delta(x, t) - V_1^\Delta(x) = e^{-r(t^*-t)}K [Q_1(x, t) - Q_1(x + h(x, z^\Delta(x, t)), t)]. \quad (10)$$

Equation (10) can be used to study the effect that an upcoming election has on the agreement that parties reach. For instance, suppose that x is such that $V_1^\Delta(x) > 1/2$, so in the game

¹⁹The proof of Proposition 7 shows that such an offer always exists when $h(\cdot, \cdot)$ satisfies the assumptions in Proposition 7.

²⁰This follows from Theorem 1 and from the fact that the expectation operator $\mathbb{E}_{NA}[f(x_s)|x_t = x]$ is equal to the expectation operator $\mathbb{E}[f(x_s)|x_t = x]$ from Section 2; see footnote 13.

without elections the implemented policy is closer to party 1's ideal point when $x_t = x$. Suppose further that the function $h(\cdot, \cdot)$ is such that $h(x, 1/2) = 0$ for all x , so that the parties' relative political power remains unchanged if they implement policy $z = 1/2$. In this case, the agreement $z^\Delta(x, t)$ that parties reach at state (x, t) must be such that $z^\Delta(x, t) \in [1/2, V_1^\Delta(x)]$. To see this, note that the left-hand side of (10) would be strictly positive if $z^\Delta(x, t) > V_1^\Delta(x)$, while the right-hand side would be negative (since $Q_1(\cdot, t)$ and $h(x, \cdot)$ are increasing and since $h(x, 1/2) = 0$). On the other hand, if $z^\Delta(x, t) < 1/2$ then the left-hand side of (10) would be strictly negative and the right-hand side would be positive. Hence, it must be that $z^\Delta(x, t) \in [1/2, V_1^\Delta(x)]$. A symmetric argument establishes that $z^\Delta(x, t) \in [V_1^\Delta(x), 1/2]$ for all x such that $V_1^\Delta(x) < 1/2$. Thus, in this setting an upcoming election leads to more moderate policies compared to the model without elections. Intuitively, in this setting the policy that parties implement must compensate the weaker party (i.e., the party that gets a worse deal) for its lower electoral chances.

3.5 Implications of the model

The model in this section illustrates how electoral considerations can affect the dynamics of inter-party negotiations, leading to long periods of political gridlock. The model predicts that there may be gridlock at times prior to an election, but that parties will always reach an agreement after the election. Importantly, this result does not depend on there being only one election; see Section 4 below for a discussion of how this result generalizes to settings with multiple elections. An implication of this result is that we should expect to see higher levels of legislative productivity in periods immediately after elections. This implication of the model is consistent with the so-called *honeymoon* effect: the empirical finding that presidents usually enjoy higher levels of legislative success during their first months in office; see, for instance, Dominguez (2005).

In the models of Sections 3.4.1 and 3.4.2 parties are less likely to reach an agreement when political power is balanced, with both of them having similar chances of winning the vote; see Figures 4 and 5 above. These models therefore predict that elections will have a larger negative impact on legislative productivity in years in which the election's outcome is expected to be close. This is a novel prediction, which (to the extent of my knowledge) has so far not been empirically investigated.

The results in Section 3.4 show that, with an upcoming election, the type of issue over which parties are bargaining might be an important determinant of whether there will be gridlock or not. In particular, Section 3.4.3 shows that parties will be able to reach an

agreement quickly when the bargaining over how to distribute discretionary spending. This is another implication of the model which would be interesting to investigate empirically.

Finally, Proposition 3 shows that gridlock will only occur when the election is close enough; that is, when the time left until the election is smaller than s . The cutoff s is strictly increasing in the value K that parties attach to winning the election. This result can be used to obtain an estimate on the value that parties derive from winning an election based on observable patterns of gridlock: if Congress becomes gridlocked t days before an election, we can use the results in Proposition 3 to obtain a lower bound on K .

4 Extensions

General bargaining protocols. The models in Sections 2 and 3 assume that only the party with more political power can make offers. I now show how this assumption can be relaxed to allow for a broader class of bargaining protocols.

Consider first the model without elections of Section 2, but with the following bargaining protocol: at each time $t \in T(\Delta)$ party 1 has proposal power with probability $p_1(x_t) \in [0, 1]$ and party 2 has proposal power with probability $p_2(x_t) = 1 - p_1(x_t)$. Suppose that $p_1(\cdot)$ is continuous and increasing. Note that $p_1(\cdot)$ increasing captures the idea that party 1's bargaining power is increasing in x and party 2's bargaining power is decreasing in x .²¹

The same arguments as in the proof of Theorem 1 can be used to show that this game has a unique SPE. Let $\hat{V}_i^\Delta(\cdot)$ denote party i 's SPE payoffs of the game with time period $\Delta > 0$. These payoffs are again difficult to compute for a fixed time period, but they become tractable in the limit as $\Delta \rightarrow 0$: Appendix A.7.1 shows that there exist functions \hat{V}_1^* and \hat{V}_2^* such that \hat{V}_i^Δ converges uniformly to \hat{V}_i^* as $\Delta \rightarrow 0$. Moreover, for $i = 1, 2$, \hat{V}_i^* solves

$$r\hat{V}_i^*(x) = rp_i(x) + \mu(\hat{V}_i^*)'(x) + \frac{1}{2}\sigma^2(\hat{V}_i^*)''(x) \text{ for } x \in [0, 1], \quad (11)$$

with boundary conditions $(\hat{V}_i^*)'(0) = (\hat{V}_i^*)'(1) = 0$. Equation (11) has the same interpretation as equation (4): the left-hand side represents party i 's limiting payoff measured in flow terms, while the right-hand side shows the sources of this flow payoff. In this setting, party i 's flow payoff when $x_t = x$ is equal to the expected flow rent $rp_i(x)$ it extracts from making offers plus the expected change in its continuation payoff due to changes in political power

²¹With this specification, the bargaining protocol of Sections 2 and 3 can be approximated by a sequence of continuous functions $\{p_1^n(x)\}$ converging to the step function $p_1(x) = \mathbf{1}_{\{x \in M_1\}}$.

$$\mu(\hat{V}_i^*)'(x) + \frac{1}{2}\sigma^2(\hat{V}_i^*)''(x).$$

Consider next the model with an election at date $t^* > 0$. By the same arguments as in Lemma 1, for all $t < t^*$ and all $x \in [0, 1]$ party i 's equilibrium payoff at state (x, t) is bounded below by $\mathbb{E}_{NA}[e^{-r(t^*-t)}\hat{V}_i^\Delta(x_{t^*})|x_t = x] + e^{-r(t^*-t)}KQ_i(x, t)$ and is bounded above by $1 - e^{-r(t^*-t)} + \mathbb{E}_{NA}[e^{-r(t^*-t)}\hat{V}_i^\Delta(x_{t^*})|x_t = x] + e^{-r(t^*-t)}KQ_i(x, t)$. These bounds on payoffs also become tight as the election approaches, and are easy to compute numerically in the limit as $\Delta \rightarrow 0$. By the same arguments as in Section 3.3, these bounds can be used to study how the likelihood of gridlock depends on the time left until the election and on the parties' level of political power.

Multiple elections. The model in Section 3 assumes that there is only one election at time t^* . This assumption implies that any subgame starting at time $t \geq t^*$ in which parties haven't yet reached an agreement is strategically equivalent to the game without elections. Therefore, by Theorem 1 parties will always reach an agreement immediately after the election if they haven't done so before.

The model can be extended to allow for multiple elections over time. To see this, suppose that there is a second election scheduled for time $t^{**} > t^*$. Suppose further that the time between elections is large, with $t^{**} - t^* > s$ (where s is the threshold in Proposition 3). It then follows from Proposition 3 that parties will reach an agreement immediately after the first election if they haven't done so before. Therefore, a model with multiple elections that are sufficiently apart in time would deliver a similar equilibrium dynamics than the model with a single election: in this setting gridlock would only arise when the next election is close, and parties would always be able to reach an agreement as soon as they pass an election.²²

Implemented policies and political power. The model with elections in Section 3 assumes that implemented policies have an instantaneous effect on the parties' political power: if parties implement policy z at time t , then the parties' relative political power jumps by $h(x_t, z)$ immediately after the agreement. An alternative (and more general) specification would be to assume that implemented policies affect the law of motion of the process x_t . This specification would allow to model situations in which implementing policy $z \in [0, 1]$ at time t affects the drift and/or volatility of the process that drives the parties' political power

²²Moreover, it can be shown that if the elections are sufficiently apart in time, the parties' payoffs after the first election will be close to their payoffs $V_i^\Delta(\cdot)$ of the game without elections. The proof of this result is available upon request. Therefore, in this case the parties' equilibrium payoffs at times $t < t^*$ would be bounded by $\underline{W}_i^\Delta(x, t) - \eta$ and $\overline{W}_i^\Delta(x, t) + \eta$, where η is a positive constant that depends on the time $t^{**} - t^*$ between elections such that $\lim_{t^{**} - t^* \rightarrow \infty} \eta = 0$. Applying the arguments in Section 3.3, these bounds on payoffs can be used to study the equilibrium dynamics prior to the first election.

going forward. The results in this paper can also be used to study the equilibrium dynamics of the model with elections under this alternative specification.

To see this, assume that from time $t = 0$ until the time at which parties reach an agreement, x_t evolves as a Brownian motion with constant drift μ and constant volatility $\sigma > 0$. If at time t parties reach an agreement to implement policy $z \in [0, 1]$, this agreement affects the law of motion of x_t from time t onwards. For example, if parties agree to implement policy z at time t , then from time t onwards political power evolves as a Brownian motion with drift $\mu(z)$ and volatility $\sigma(z) > 0$. In this setting, party i can still guarantee itself a payoff of $\underline{W}_i^\Delta(x, t)$ by delaying an agreement until the election. Therefore, by the same arguments as in Lemma 1, the parties' payoffs at times $t < t^*$ are bounded by $\underline{W}_i^\Delta(x, t)$ and $\overline{W}_i^\Delta(x, t)$. These bounds on payoffs can again be used to study how the likelihood of gridlock depends on the time until the election and on the level of political power.

Elections, proposal power and multiple issues. The model in Section 3 assumes that parties are purely office motivated: they obtain a private benefit K from winning the election. I now extend the model to a setting in which parties also have policy motivations to win the election. The extension has two additional features relative to the model in Section 3: (i) the outcome of the election affects the bargaining protocol from time t^* onwards; and (ii) after the election parties bargain over a second issue.

Suppose that the bargaining protocol at times $t \geq t^*$ depends on the identity of the party that wins the election: if party $i \in \{1, 2\}$ wins the election, then at each time $t \in T(\Delta), t \geq t^*$ party 1 has proposal power with probability $p_1^i(x_t) \in [0, 1]$ and party 2 has proposal power with probability $p_2^i(x_t) = 1 - p_1^i(x_t)$. Suppose that $p_1^i(\cdot)$ is continuous and increasing in x for $i = 1, 2$, so that party 1's (party 2's) bargaining power after the election is increasing in x (decreasing in x). Assume also that $p_1^1(x) \geq p_1^2(x)$ for all $x \in [0, 1]$; this condition implies that a party is more likely to have proposal power after time t^* if it wins the election than if it loses. For simplicity, assume that the bargaining protocol prior to the election is the same as in Sections 2 and 3.

At times $t < t^*$, parties know that immediately after the election they will bargain over a second issue. When bargaining over the second issue, parties again have to decide which policy in $[0, 1]$ to implement. Party i 's utility from implementing policy $z \in [0, 1]$ for this second issue is $v_i(z) = \gamma \times (1 - |z - z_i|)$, where $z_1 = 1$ and $z_2 = 0$ are the ideal policies of parties 1 and 2, respectively, and where $\gamma > 0$ is a constant that measures how important the second issue is relative to the first one. The parameter γ can also be thought of as representing in a reduced form way situations in which parties expect to bargain over multiple issues after

the election (with γ measuring the number of issues after the election). If parties have not reached an agreement over the first issue by the election date, then from time t^* onwards they bargain over the two issues simultaneously.

Appendix A.7.3 derives bounds on the parties' equilibrium payoffs for this setting. As in the model in Section 3, these bounds also become tight as the election approaches and are easy to compute in the limit as $\Delta \rightarrow 0$. By the same arguments as in Proposition 4, these bounds on payoffs can be used to analyze the dynamics of bargaining in this model.

Note that in this setting the payoff that parties derive from winning an election depends on three things: (a) how office motivated parties are (measured by K), (b) the relative importance of the second issue (measured by γ), and (c) the extent to which the party that wins the election has more bargaining power (measured by how different the bargaining protocols $p_i^1(x)$ and $p_i^2(x)$ are). Suppose that policies affect political power as in Sections 3.4.1 or 3.4.2. In either case, it can be shown that there will be gridlock if the combined payoff that parties get from winning the election is large enough. Moreover, if the last two effects are sufficiently large, then gridlock will arise even if parties are purely policy motivated (i.e., even if $K = 0$).

5 Conclusion

The first part of this paper constructs a model of bargaining to study how changes in political power affect the outcomes of inter-party negotiations. At an abstract level, this model generalizes standard bilateral bargaining games à la Rubinstein (1982) to settings in which bargaining power varies over time. The model has a unique SPE, in which parties always reach an immediate agreement. The unique SPE becomes very tractable in the limit as $\Delta \rightarrow 0$. This tractability facilitates studying how changes in political power affect the agreements that parties reach.

The second part of the paper uses this bargaining model to study the effect that elections have on bargaining outcomes. I show that elections might give rise to long periods of gridlock. These delays occur in spite of the fact that implementing a policy immediately is always the efficient outcome. I provide bounds on the parties' equilibrium payoffs. These bounds on payoffs become tight as the election approaches, and are easy to compute numerically in the limit as $\Delta \rightarrow 0$. I use these bounds on payoffs to analyze the dynamics of bargaining when there is an election upcoming.

A Appendix

A.1 Proof of Theorem 1

Let F^2 be the set of bounded and measurable functions on $[0, 1]$ taking values on \mathbb{R}^2 . Let $\|\cdot\|_2$ denote the sup norm on \mathbb{R}^2 . For any $f \in F^2$, let $\|f\| = \sup_{z \in [0, 1]} \|f(z)\|_2$. Fix $\Delta > 0$ and $r > 0$. Define $\psi : F^2 \rightarrow F^2$ as follows: for any $f = (f_1, f_2) \in F^2$ and for $i, j = 1, 2, i \neq j$,

$$\psi_i(f_i, f_j)(x) = \begin{cases} e^{-r\Delta} \mathbb{E}[f_i(x_{t+\Delta}) | x_t = x] & \text{if } x \notin M_i, \\ 1 - e^{-r\Delta} \mathbb{E}[f_j(x_{t+\Delta}) | x_t = x] & \text{if } x \in M_i, \end{cases}$$

Note that ψ is a contraction of modulus $e^{-r\Delta}$: for any $f, g \in F^2$, $\|\psi(f) - \psi(g)\| \leq e^{-r\Delta} \|f - g\|$.

Proof of Theorem 1. To prove Theorem 1, I start out assuming that the set of SPE of Γ_Δ is non-empty. At the end of the proof I show that Γ_Δ has a SPE. Fix a SPE of Γ_Δ and let $f_i(x)$ be party i 's payoff from this SPE when $x_0 = x$. Let $\bar{U} = (\bar{U}_1, \bar{U}_2) \in F^2$ and $\bar{u} = (\bar{u}_1, \bar{u}_2) \in F^2$ be the parties' supremum and infimum SPE payoffs, so $f_i(x) \in [\bar{u}_i(x), \bar{U}_i(x)] \forall x \in [0, 1]$.

Note that for all $x \in M_i$ party i 's SPE payoff is bounded below by $1 - e^{-r\Delta} \mathbb{E}[\bar{U}_j(x_{t+\Delta}) | x_t = x]$, since in any SPE party j always accepts an offer that gives that party a payoff equal to $e^{-r\Delta} \mathbb{E}[\bar{U}_j(x_{t+\Delta}) | x_t = x]$. On the other hand, for all $x \notin M_i$ party i 's payoffs is bounded below by $e^{-r\Delta} \mathbb{E}[\bar{u}_i(x_{t+\Delta}) | x_t = x]$, since party i can always guarantee this payoff by rejecting party j 's offer. Thus, for all $x \in [0, 1]$ it must be that $f_i(x) \geq \bar{u}_i(x) \geq \psi_i(\bar{u}_i, \bar{U}_j)(x)$.

At states $x \notin M_i$, party i 's payoff is bounded above by $e^{-r\Delta} \mathbb{E}[\bar{U}_i(x_{t+\Delta}) | x_t = x]$, since party j will never make an offer that gives party i a payoff larger than this. Consider next states $x \in M_i$, and note that $f_i(x) + f_j(x) \leq 1$. This inequality follows since the sum of the parties SPE payoffs cannot be larger than the sum of their payoffs from implementing a policy immediately, which is 1. Moreover, by the arguments in the previous paragraph, $f_j(x) \geq e^{-r\Delta} \mathbb{E}[\bar{u}_j(x_{t+\Delta}) | x_t = x]$ for all $x \in M_i$. These two inequalities imply that $f_i(x) \leq 1 - e^{-r\Delta} \mathbb{E}[\bar{u}_j(x_{t+\Delta}) | x_t = x]$ for all $x \in M_i$. Thus, $f_i(x) \leq \bar{U}_i(x) \leq \psi_i(\bar{U}_i, \bar{u}_j)(x) \forall x \in [0, 1]$.

The two paragraphs above imply that, for $i = 1, 2, i \neq j$, and for all $x \in [0, 1]$,

$$\bar{U}_i(x) - \bar{u}_i(x) \leq \psi_i(\bar{U}_i, \bar{u}_j)(x) - \psi_i(\bar{u}_i, \bar{U}_j)(x) \leq \max_{k \in \{1, 2\}} e^{-r\Delta} \mathbb{E}[\bar{U}_k(x_{t+\Delta}) - \bar{u}_k(x_{t+\Delta}) | x_t = x],$$

where the last inequality follows from the definition of ψ_i . Since the inequality above holds for $i = 1, 2$ and for all $x \in [0, 1]$, it follows from the definition of $\|\cdot\|$ that $\|\bar{U} - \bar{u}\| \leq e^{-r\Delta} \|\bar{U} - \bar{u}\|$. It follows that $\bar{U} = \bar{u}$, so SPE payoffs are unique. Let $V^\Delta = (V_1^\Delta, V_2^\Delta) \in F^2$ be the unique

SPE payoffs. Since $\psi_i(\bar{U}_i, \bar{u}_j) \geq V_i^\Delta \geq \psi_i(\bar{u}_i, \bar{U}_j)$ and since $\bar{U} = \bar{u}$, it follows that V^Δ is the unique fixed point of ψ . Note that V_1^Δ and V_2^Δ and satisfy the conditions in Theorem 1.

The arguments above show that all SPE of Γ_Δ are payoff equivalent. I now show that there exists a unique SPE. Since $V_1^\Delta(x) + V_2^\Delta(x) = 1$ for all $x \in [0, 1]$, in any SPE parties always reach an immediate agreement, and their payoffs from this agreement is $(V_1^\Delta(x_0), V_2^\Delta(x_0))$. I now use this to construct the unique SPE. Consider the following strategy profile. At every $x \in M_i$, party i makes an offer that gives parties a payoff equal to $(V_1^\Delta(x), V_2^\Delta(x))$, and party $j \neq i$ only accepts offers that give this party a payoff at least as large as $V_j^\Delta(x)$. The parties' payoffs from this strategy profile are (V_1^Δ, V_2^Δ) . Moreover, it is easy to see that no party can gain by deviating from its strategy at any $x \in [0, 1]$. Hence, this is a SPE of Γ_Δ . ■

A.2 Proof of Theorem 2

For $i = 1, 2$ and for all $x \in [0, 1]$, party i 's SPE payoffs $V_i^\Delta(x)$ satisfies equation (3) in the main text. Setting $t = 0$ in equation (3) and solving this equation forward yields

$$V_i^\Delta(x) = \mathbb{E} \left[(1 - e^{-r\Delta}) \sum_{k=0}^{\infty} e^{-rk\Delta} \mathbf{1}_{\{x_{k\Delta} \in M_i\}} \middle| x_0 = x \right] = \frac{1 - e^{-r\Delta}}{\Delta} \sum_{k=0}^{\infty} \Delta e^{-rk\Delta} P_i(\Delta k, x), \quad (\text{A.1})$$

where, for all $s \geq 0$ and all $x \in [0, 1]$, $P_i(s, x) := \mathbb{E}[\mathbf{1}_{\{x_s \in M_i\}} | x_0 = x]$ is the probability with which party i has proposal power at time s conditional on $x_0 = x$.

For all $s > 0$ and all $x, y \in [0, 1]$, let $p(x, y, s) = \text{Prob}(x_s = y | x_0 = x)$ be the transition density function of the process x_t . It is well known that $p(x, y, s)$ solves Kolmogorov's backward equation (i.e., Bhattacharya and Waymire, 2009, chapter V.6),

$$\frac{\partial}{\partial s} p(x, y, s) = \mu \frac{\partial}{\partial x} p(x, y, s) + \frac{1}{2} \sigma^2 \frac{\partial^2}{\partial x^2} p(x, y, s), \quad (\text{A.2})$$

with $\lim_{s \rightarrow 0} p(x, y, s) = \mathbf{1}_{\{y=x\}}$ and $\frac{\partial}{\partial x} p(x, y, s)|_{x=0} = \frac{\partial}{\partial x} p(x, y, s)|_{x=1} = 0$ for all $s > 0$. Note that for all $s > 0$ and for $i = 1, 2$, $P_i(s, x) = \mathbb{E}[\mathbf{1}_{\{x_s \in M_i\}} | x_0 = x] = \int_{M_i} p(x, y, s) dy$. Since $p(x, y, s)$ solves (A.2) with $\frac{\partial}{\partial x} p(x, y, s)|_{x=0} = \frac{\partial}{\partial x} p(x, y, s)|_{x=1} = 0$ and $\lim_{s \rightarrow 0} p(x, y, s) = \mathbf{1}_{\{y=x\}}$, it follows that $P_i(s, x)$ also solves (A.2), with $\lim_{s \rightarrow 0} P_i(s, x) = P_i(0, x) = \mathbf{1}_{\{x \in M_i\}}$ and $\frac{\partial}{\partial x} P_i(s, x)|_{x=0} = \frac{\partial}{\partial x} P_i(s, x)|_{x=1} = 0$ for all $s \geq 0$.²³ Note that $P_i(\cdot, x)$ is continuous on $(0, \infty)$ (being differentiable). Hence, $e^{-rs} P_i(s, x)$ is Riemann integrable, and so $\sum_{k=0}^{\infty} \Delta e^{-rk\Delta} P_i(\Delta k, x) \rightarrow \int_0^{\infty} e^{-rt} P_i(t, x) dt$ as $\Delta \rightarrow 0$. It then follows from (A.1) that $V_i^\Delta(x)$

²³Since $p(x, y, s)$ satisfies (A.2), for all $x \in [0, 1]$ and all $s > 0$, $\frac{\partial}{\partial s} P_i(s, x) = \int_{M_i} \frac{\partial}{\partial s} p(x, y, s) dy = \mu \int_{M_i} \frac{\partial}{\partial x} p(x, y, s) dy + \frac{1}{2} \sigma^2 \int_{M_i} \frac{\partial^2}{\partial x^2} p(x, y, s) dy = \mu \frac{\partial}{\partial x} P_i(s, x) + \frac{1}{2} \sigma^2 \frac{\partial^2}{\partial x^2} P_i(s, x)$.

converges pointwise to $V_i^*(x) := r \int_0^\infty e^{-rt} P(t, x) dt$ as $\Delta \rightarrow 0$.

Lemma A1 $V_1^*(\cdot)$ and $V_2^*(\cdot)$ are continuous.

Proof. For $i = 1, 2$ and for every $\varepsilon > 0$, let $V_i^\varepsilon(x) := r \int_\varepsilon^\infty e^{-rt} P_i(t, x) dt$. Since $P_i(t, \cdot)$ is continuous for all $t > 0$ (being differentiable), $V_i^\varepsilon(\cdot)$ is continuous for all $\varepsilon > 0$. To show that $V_i^*(\cdot)$ is continuous, it suffices to show that $V_i^\varepsilon(x) \rightarrow V_i^*(x)$ uniformly as $\varepsilon \rightarrow 0$. For any $\varepsilon > 0$ and any $x \in [0, 1]$, $|V_i^*(x) - V_i^\varepsilon(x)| = r \int_0^\varepsilon e^{-rt} P_i(t, x) dt \leq r \int_0^\varepsilon e^{-rt} dt = 1 - e^{-r\varepsilon}$ (since $P_i(t, x) \in [0, 1]$). Since $\lim_{\varepsilon \rightarrow 0} 1 - e^{-r\varepsilon} = 0$, $V_i^\varepsilon(\cdot) \rightarrow V_i^*(\cdot)$ uniformly as $\varepsilon \rightarrow 0$. ■

Note that, for all $t > 0$, $P_1(t, x)$ is increasing in x , since party 1 is more likely to have proposal power at $t > 0$ if $x_0 = x$ is larger. Similarly, $P_2(t, x)$ is decreasing in x . Therefore, by (A.1), for all $\Delta > 0$ $V_i^\Delta(\cdot)$ is monotone on $[0, 1]$. Since $V_i^*(\cdot)$ is continuous, it follows from Theorem A in Buchanan and Hildebrandt (1908) that $V_i^\Delta(\cdot) \rightarrow V_i^*(\cdot)$ uniformly as $\Delta \rightarrow 0$.

Proof of Theorem 2. By the arguments above, $V_i^\Delta(x) \rightarrow V_i^*(x) = r \int_0^\infty e^{-rt} P(t, x) dt$ uniformly as $\Delta \rightarrow 0$. The rule of integration by parts implies that, for all $x \neq 1/2$,

$$V_i^*(x) = r \int_0^\infty e^{-rt} P_i(t, x) dt = -e^{-rt} P_i(t, x) \Big|_0^\infty + \int_0^\infty e^{-rt} \frac{\partial}{\partial t} P_i(t, x) dt.$$

Note that $-e^{-rt} P_i(t, x) \Big|_0^\infty = \lim_{t \rightarrow \infty} -e^{-rt} P_i(t, x) + P_i(0, x) = P_i(0, x) = \mathbf{1}_{\{x \in M_i\}}$. Hence, $\forall x \neq 1/2$, $V_i^*(x) = \mathbf{1}_{\{x \in M_i\}} + \int_0^\infty e^{-rt} \frac{\partial}{\partial t} P_i(t, x) dt$. Since $\frac{\partial}{\partial t} P_i(t, x) = \mu \frac{\partial}{\partial x} P_i(t, x) + \frac{\sigma^2}{2} \frac{\partial^2}{\partial x^2} P_i(t, x)$ for all $t > 0$ (see footnote (23)), it follows that for all $x \neq 1/2$,

$$\begin{aligned} rV_i^*(x) &= r \times \mathbf{1}_{\{x \in M_i\}} + r \int_0^\infty e^{-rt} \left(\mu \frac{\partial}{\partial x} P_i(t, x) + \frac{1}{2} \sigma^2 \frac{\partial^2}{\partial x^2} P_i(t, x) \right) dt \\ &= r \times \mathbf{1}_{\{x \in M_i\}} + \mu (V_i^*)'(x) + \frac{1}{2} \sigma^2 (V_i^*)''(x), \end{aligned} \tag{A.3}$$

where the second equality follows since $(V_i^*)'(x) = r \int_0^\infty e^{-rt} \frac{\partial}{\partial x} P_i(t, x) dt$ and since $(V_i^*)''(x) = r \int_0^\infty e^{-rt} \frac{\partial^2}{\partial x^2} P_i(t, x) dt$. Therefore, $V_i^*(\cdot)$ solves (4) for all $x \neq 1/2$. To pin down the boundary conditions, note that $(V_i^*)'(x) = r \int_0^\infty e^{-rt} \frac{\partial}{\partial x} P_i(t, x) dt$. Since $\frac{\partial}{\partial x} P_i(t, x) \Big|_{x=0} = \frac{\partial}{\partial x} P_i(t, x) \Big|_{x=1} = 0$ for all $t \geq 0$, it follows that $(V_i^*)'(0) = (V_i^*)'(1) = 0$.

Since $V_i^*(\cdot)$ is continuous (Lemma A1), it must be that $V_i^*(1/2^-) = V_i^*(1/2^+)$. Finally, integrating both sides of (A.3) yields

$$\int_0^z rV_i^*(x) dx = \int_0^z r \mathbf{1}_{\{x \in M_i\}} dx + \mu (V_i^*(z) - V_i^*(0)) + \frac{1}{2} \sigma^2 ((V_i^*)'(z) - (V_i^*)'(0)).$$

Note that the two integrals in the equation above are continuous in z . Since $V_i^*(\cdot)$ is also continuous, $(V_i^*)'(\cdot)$ must be continuous as well. Hence, $(V_i^*)'(1/2^-) = (V_i^*)'(1/2^+)$ ■

For $i = 1$, the solution to the differential equation and boundary conditions in Theorem 2 is

$$V_1^*(x) = \begin{cases} \frac{e^{\alpha/2}(\beta e^{-\alpha x} + \alpha e^{\beta x})}{(1+e^{(\alpha+\beta)/2})^{\alpha+\beta}} & x \in [0, 1/2], \\ 1 - \frac{e^{-\beta/2}(\alpha e^{\beta x} + \beta e^{(\alpha+\beta)} e^{-\alpha x})}{(1+e^{(\alpha+\beta)/2})^{\alpha+\beta}} & x \in [1/2, 1], \end{cases} \quad (\text{A.4})$$

where $\alpha = (\mu + \sqrt{\mu^2 + 2r\sigma^2})/\sigma^2$, $\beta = (-\mu + \sqrt{\mu^2 + 2r\sigma^2})/\sigma^2$. Finally, $V_2^*(x) = 1 - V_1^*(x)$.

A.3 Proof of Propositions 1 and 2

By Theorem 2, for $i = 1, 2$ and for all $x \notin M_i$, $V_i^*(\cdot)$ is a solution to the differential equation

$$rv(x) = \mu v'(x) + \frac{1}{2}\sigma^2 v''(x). \quad (\text{A.5})$$

Lemma A2 *Let U be a solution to (A.5) with parameters $(\mu, \hat{\sigma})$ such that $U'' > 0$, and let W be a solution to (A.5) with parameters $(\mu, \tilde{\sigma})$, with $\tilde{\sigma} > \hat{\sigma}$, such that $W'' > 0$. (i) If $U(y) \geq W(y)$ and $U'(y) \geq W'(y)$ for some y , then $U'(x) > W'(x)$ for all $x > y$, and so $U(x) > W(x)$ for all $x > y$. (ii) If $U(y) \geq W(y)$ and $U'(y) \leq W'(y)$ for some y , then $U'(x) < W'(x)$ for all $x < y$, and so $U(x) > W(x)$ for all $x < y$.*

Proof. I prove part (i) of the lemma. The proof of part (ii) is symmetric and omitted. To prove part (i), I first show that there exists $\eta > 0$ such that $U'(x) > W'(x)$ for all $x \in (y, y + \eta)$. Since U', W' are continuous, this is true when $U'(y) > W'(y)$. Suppose that $U'(y) = W'(y)$. Since U and W solve (A.5) (with parameters $(\mu, \hat{\sigma})$ and $(\mu, \tilde{\sigma})$, respectively), $W''(y) = 2(rW(y) - \mu W'(y))/\tilde{\sigma}^2 < 2(rU(y) - \mu U'(y))/\hat{\sigma}^2 = U''(y)$, where the inequality follows since $U(y) \geq W(y)$, $\tilde{\sigma} > \hat{\sigma}$ and $U'', W'' > 0$. Since $U'(y) = W'(y)$ and $U''(y) > W''(y)$, there exists $\eta > 0$ such that $U'(x) > W'(x) \forall x \in (y, y + \eta)$.

Suppose next that part (i) in the lemma is not true and let y_1 be the smallest point strictly above y with $U'(y_1) = W'(y_1)$. By the paragraph above, $y_1 \geq y + \eta > y$. It follows that $U'(x) > W'(x)$ for all $x \in (y, y_1)$, so $U(y_1) > W(y_1)$. Note then that $W''(y_1) = 2(rW(y_1) - \mu W'(y_1))/\tilde{\sigma}^2 < 2(rU(y_1) - \mu U'(y_1))/\hat{\sigma}^2 = U''(y_1)$, where the inequality follows since $U(y_1) > W(y_1)$, $\tilde{\sigma} > \hat{\sigma}$ and $U'', W'' > 0$. Since $U'(y_1) = W'(y_1)$ and $U''(y_1) > W''(y_1)$, it must be that $U'(y_1 - \varepsilon) < W'(y_1 - \varepsilon)$ for $\varepsilon > 0$ small, a contradiction. Thus, $U'(x) > W'(x) \forall x > y$. ■

Proof of Proposition 1. I first show that, for $\mu \leq 0$, V_1^* is increasing in σ for all $x \in [0, 1/2]$. Suppose then that $\mu \leq 0$. Using equation (A.4), it can be shown that

$$\frac{\partial V_1^*(1/2)}{\partial \sigma} = \frac{\mu}{\sigma^2(\alpha + \beta)} \frac{-2e^{(\alpha+\beta)/2}(\mu^2 + r\sigma^2)}{(1 + e^{(\alpha+\beta)/2})^2 \sigma^3 \sqrt{\mu^2 + 2r\sigma^2}} + \frac{-1 + e^{(\alpha+\beta)/2}}{1 + e^{(\alpha+\beta)/2}} \frac{-\mu r \sigma}{(\mu^2 + 2r\sigma^2)^{3/2}} \geq 0,$$

where the inequality follows since $\mu \leq 0$ and since $\alpha + \beta > 0$.²⁴ Fix $\hat{\sigma} < \tilde{\sigma}$ and let \hat{V}_1^* and \tilde{V}_1^* be party 1's limiting payoff when the volatility of x_t is $\hat{\sigma}$ and $\tilde{\sigma}$, respectively. By the derivative above, $\tilde{V}_1^*(1/2) \geq \hat{V}_1^*(1/2)$. By Theorem 2, \hat{V}_1^* and \tilde{V}_1^* solve (A.5) on $[0, 1/2]$ (but with different values of volatility), with $(\tilde{V}_1^*)'(0) = (\hat{V}_1^*)'(0) = 0$. Moreover, it can be shown using equation (A.4) that \hat{V}_1^* and \tilde{V}_1^* are strictly convex for all $x < 1/2$.

I now show that $\tilde{V}_1^*(x) > \hat{V}_1^*(x) \forall x \in [0, 1/2]$. Note first that it must be that $\tilde{V}_1^*(0) > \hat{V}_1^*(0)$: if $\hat{V}_1^*(0) \geq \tilde{V}_1^*(0)$, then Lemma A2 (i) and the fact that $(\tilde{V}_1^*)'(0) = (\hat{V}_1^*)'(0) = 0$ would imply that $\hat{V}_1^*(1/2) > \tilde{V}_1^*(1/2)$, which would contradict $\tilde{V}_1^*(1/2) \geq \hat{V}_1^*(1/2)$. Let $z > 0$ be the smallest point such that $\hat{V}_1^*(z) = \tilde{V}_1^*(z)$ (let $z = \infty$ if this point doesn't exist). Suppose by contradiction that $z < 1/2$. Since $\hat{V}_1^*(x) < \tilde{V}_1^*(x)$ for all $x \in [0, z)$, it must be that $(\hat{V}_1^*(z))' > (\tilde{V}_1^*(z))'$. Lemma A2 (i) then implies that $\hat{V}_1^*(x) > \tilde{V}_1^*(x)$ for all $x \in (z, 1/2]$, which contradicts the fact that $\tilde{V}_1^*(1/2) \geq \hat{V}_1^*(1/2)$. Hence, $\tilde{V}_1^*(x) > \hat{V}_1^*(x) \forall x \in [0, 1/2]$. The proof that V_2^* is increasing in σ for $x \in [1/2, 1]$ when $\mu \geq 0$ follows from a symmetric argument (but using part (ii) of Lemma A2 instead of part (i)). ■

Lemma A3 *Let U be a solution to (A.5) with parameters $(\hat{\mu}, \sigma)$ such that $U'' > 0$ and let W be a solution to (A.5) with parameters $(\tilde{\mu}, \sigma)$, $\tilde{\mu} > \hat{\mu}$, such that $W'' > 0$. (i) If $U', W' \geq 0$ and there exists y such that $U(y) \geq W(y)$ and $U'(y) \geq W'(y)$, then $U'(x) > W'(x)$ for all $x > y$, and so $U(x) > W(x)$ for all $x > y$. (ii) If $U', W' \leq 0$ and there exists y such that $U(y) \leq W(y)$ and $U'(y) \geq W'(y)$ for some y , then $U'(x) > W'(x)$ for all $x < y$, and so $U(x) < W(x)$ for all $x < y$.*

Proof. I prove part (i) of the lemma. The proof of part (ii) is symmetric and omitted. To prove part (i), I first show that there exists $\eta > 0$ such that $U'(x) > W'(x)$ for all $x \in (y, y + \eta)$. Since $U, W \in C^2$, this is true when $U'(y) > W'(y)$. Suppose that $U'(y) = W'(y)$, and note that in this case $W''(y) - U''(y) = \frac{2}{\sigma^2}(r(W(y) - U(y)) - (\tilde{\mu} - \hat{\mu})W'(y))$. Given the assumptions in the lemma, $W''(y) < U''(y)$ if either $U(y) > W(y)$ or $W'(y) = U'(y) > 0$. Since $W'(y) =$

²⁴Using (A.4) and the definition of α, β , $V_1^*(1/2) = \frac{1}{2} + \frac{1}{2} \frac{\mu}{\sigma^2(\alpha+\beta)} \frac{-1+e^{\frac{\alpha+\beta}{2}}}{1+e^{\frac{\alpha+\beta}{2}}}$. The derivative above can be shown using this expression and noting that $\frac{\partial(\alpha+\beta)}{\partial\sigma} = \frac{-2(\mu^2+r\sigma^2)}{\sigma^3\sqrt{\mu^2+2r\sigma^2}}$ and $\frac{\partial}{\partial\sigma} \frac{1}{\sigma^2(\alpha+\beta)} = \frac{-2r\sigma}{(\mu^2+2r\sigma^2)^{3/2}}$.

$U'(y)$ and $W''(y) < U''(y)$, in this case there exists $\eta > 0$ such that $U'(x) > W'(x) \forall x \in (y, y + \eta)$. Suppose instead that $W'(y) = U'(y) = 0$ and $W(y) = U(y)$, so that $W''(y) = U''(y)$. Note that in this case $W'''(y) = \frac{2}{\sigma^2}(rW'(y) - \tilde{\mu}W''(y)) < \frac{2}{\sigma^2}(rU'(y) - \hat{\mu}U''(y)) = U'''(y)$, where the inequality follows since $U'', W'' > 0$ and $\tilde{\mu} > \hat{\mu}$. Since $W'(y) = U'(y) = 0$, $W''(y) = U''(y)$ and $U'''(y) > W'''(y)$, in this case there also exists $\eta > 0$ such that $U'(x) > W'(x) \forall x \in (y, y + \eta)$.

Suppose next that part (i) in the lemma is not true and let y_1 be the smallest point strictly above y with $U'(y_1) = W'(y_1)$. By the paragraph above, $y_1 \geq y + \eta > y$. It follows that $U'(x) > W'(x)$ for all $x \in (y, y_1)$, so $U(y_1) > W(y_1)$. Note then that $W''(y_1) = 2(rW(y_1) - \tilde{\mu}W'(y_1))/\sigma^2 < 2(rU(y_1) - \hat{\mu}U'(y_1))/\sigma^2 = U''(y_1)$, where the inequality follows since $U(y_1) > W(y_1)$, $W', U' \geq 0$ and $\tilde{\mu} > \hat{\mu}$. Since $U'(y_1) = W'(y_1)$, this implies that $U'(y_1 - \varepsilon) < W'(y_1 - \varepsilon)$ for $\varepsilon > 0$ small, a contradiction. Thus, $U'(x) > W'(x) \forall x > y$. ■

Proof of Proposition 2. Since $V_1^*(x) + V_2^*(x) = 1$ for all x , to prove Proposition 2 it suffices to show that V_1^* is increasing in μ for all $x \in [0, 1/2]$ and that V_2^* is decreasing in μ for all $x \in [1/2, 1]$. Using equation (A.4), it can be shown that

$$\frac{\partial V_1^*(1/2)}{\partial \mu} = \frac{\mu}{\sigma^2(\alpha + \beta)} \frac{e^{(\alpha+\beta)/2}}{(1 + e^{(\alpha+\beta)/2})^2} \frac{\mu}{\sigma^2 \sqrt{\mu^2 + 2r\sigma^2}} + \frac{-1 + e^{(\alpha+\beta)/2}}{1 + e^{(\alpha+\beta)/2}} \frac{r\sigma^2}{(\mu^2 + 2r\sigma^2)^{3/2}} > 0,$$

where the inequality follows since $\alpha + \beta > 0$.²⁵ Fix $\hat{\mu} < \tilde{\mu}$ and let \hat{V}_1^* and \tilde{V}_1^* denote party 1's payoff when the drift of x_t is $\hat{\mu}$ and $\tilde{\mu}$, respectively. By the inequality above, $\hat{V}_1^*(1/2) < \tilde{V}_1^*(1/2)$. By Theorem 2, \hat{V}_1^* and \tilde{V}_1^* solve (A.5) on $[0, 1/2]$ (but with different values of drifts), with $(\tilde{V}_1^*)'(0) = (\hat{V}_1^*)'(0) = 0$. Moreover, it can be shown using equation (A.4) that \hat{V}_1^* and \tilde{V}_1^* are increasing and strictly concave for all $x < 1/2$.

I now show that $\tilde{V}_1^*(x) > \hat{V}_1^*(x) \forall x \in [0, 1/2]$. Note first that it must be that $\tilde{V}_1^*(0) > \hat{V}_1^*(0)$: if $\hat{V}_1^*(0) \geq \tilde{V}_1^*(0)$, then Lemma A3 (i) and the fact that $(\tilde{V}_1^*)'(0) = (\hat{V}_1^*)'(0) = 0$ would imply that $\hat{V}_1^*(1/2) > \tilde{V}_1^*(1/2)$, which would contradict $\tilde{V}_1^*(1/2) > \hat{V}_1^*(1/2)$. Let $z > 0$ be the smallest point such that $\hat{V}_1^*(z) = \tilde{V}_1^*(z)$, and suppose by contradiction that $z < 1/2$. Since $\tilde{V}_1^*(x) > \hat{V}_1^*(x)$ for all $x < z$, it must be that $(\tilde{V}_1^*)'(z) > (\hat{V}_1^*)'(z)$. Lemma A3 (i) then implies that $\hat{V}_1^*(x) > \tilde{V}_1^*(x)$ for all $x \in (z, 1/2]$, which contradicts the fact that $\tilde{V}_1^*(1/2) > \hat{V}_1^*(1/2)$. Hence, $\tilde{V}_1^*(x) > \hat{V}_1^*(x) \forall x \in [0, 1/2]$. The proof that V_2^* is decreasing in μ for $x \in [1/2, 1]$ follows from a symmetric argument (but using part (ii) of Lemma A3 instead of part (i)). ■

²⁵The inequality above can be shown using the fact that $V_1^*(1/2) = \frac{1}{2} + \frac{1}{2} \frac{\mu}{\sigma^2(\alpha+\beta)} \frac{-1+e^{\frac{\alpha+\beta}{2}}}{1+e^{\frac{\alpha+\beta}{2}}}$ (see footnote 24), together with $\frac{\partial(\alpha+\beta)}{\partial \mu} = \frac{\mu}{\sigma^2 \sqrt{\mu^2 + 2r\sigma^2}}$ and $\frac{\partial}{\partial \mu} \frac{\mu}{\sigma^2(\alpha+\beta)} = \frac{2r\sigma^2}{(\mu^2 + 2r\sigma^2)^{3/2}}$.

A.4 Proofs of Section 3.2

Proof of Theorem 3. Let $W_i^\Delta(x, t)$ denote party i 's SPE payoffs at time $t \in T(\Delta)$ with $x_t = x$. Note that the subgame that starts at any time $t \geq t^*$ at which parties haven't yet reached an agreement is strategically identical to the game in Section 2. Therefore, in any SPE parties will reach an agreement at time $t \geq t^*$ if they haven't done so already. Moreover, party i 's payoff from this agreement will be equal to $V_i^\Delta(x_t)$. It then follows that $W_i^\Delta(x, t^*) = V_i^\Delta(x) + K \times \mathbf{1}_{\{x \in M_i\}}$ and $W_i^\Delta(x, t) = V_i^\Delta(x)$ for all $t > t^*$.

For $i = 1, 2$ and $t \in T(\Delta), t < t^*$, let $U_i(z, x, t) = u_i(z) + e^{-r(t^*-t)}KQ_i^z(x, t)$ be the payoff that party i gets by implementing policy $z \in [0, 1]$ at time t when $x_t = x$. Note that $U_i(\cdot, x, t)$ is continuous (since $u_i(\cdot)$, $h(x, \cdot)$ and $Q_i(\cdot, t)$ are continuous). Suppose that parties have not reached an agreement by time $t^* - \Delta$. Suppose further that $x_{t^*-\Delta} = x \in M_j$ (so party j has proposal power). For $i = 1, 2$, party i 's payoff if there is no agreement at time $t^* - \Delta$ is $e^{-r\Delta}\mathbb{E}_{NA}[W_i^\Delta(x_{t^*}, t^*)|x_{t^*-\Delta} = x]$. Let $A_i^\Delta(x, t^* - \Delta) := \{z \in [0, 1] : U_i(z, x, t^* - \Delta) \geq e^{-r\Delta}\mathbb{E}_{NA}[W_i^\Delta(x_{t^*}, t^*)|x_{t^*-\Delta} = x]\}$ be the set of policies that give party i a payoff weakly larger than the payoff from delaying an agreement one round. Let $A^\Delta(x, t^* - \Delta) := A_1^\Delta(x, t^* - \Delta) \cap A_2^\Delta(x, t^* - \Delta)$. If $A^\Delta(x, t^* - \Delta) = \emptyset$, there is no policy that both parties would agree to implement. In this case there must be delay at time $t^* - \Delta$, so for $i = 1, 2$, party i 's payoff is $W_i^\Delta(x, t^* - \Delta) = e^{-r\Delta}\mathbb{E}_{NA}[W_i^\Delta(x_{t^*}, t^*)|x_{t^*-\Delta} = x]$. Otherwise, if $A^\Delta(x, t^* - \Delta) \neq \emptyset$, party j offers $z(x, t^* - \Delta) \in \arg \max_{z \in A^\Delta(x, t^* - \Delta)} U_j(z, x, t^* - \Delta)$, and its opponent accepts this offer.²⁶ In this case, for $i = 1, 2$, party i 's payoff is $U_i(z(x, t^* - \Delta), x, t^* - \Delta)$.

The first paragraph above establishes parts (i) and (ii) of Theorem 3, while the second paragraph establishes part (iii) for $t = t^* - \Delta$. Consider next time $t^* - 2\Delta$. Party i 's payoff in case of delay is $e^{-r\Delta}\mathbb{E}_{NA}[W_i^\Delta(x_{t^*-\Delta}, t^* - \Delta)|x_{t^*-2\Delta} = x]$. Let $A_i^\Delta(x, t^* - 2\Delta) := \{z \in [0, 1] : U_i(z, x, t^* - 2\Delta) \geq e^{-r\Delta}\mathbb{E}_{NA}[W_i^\Delta(x_{t^*-\Delta}, t^*)|x_{t^*-2\Delta} = x]\}$ and let $A^\Delta(x, t^* - 2\Delta) := A_1^\Delta(x, t^* - 2\Delta) \cap A_2^\Delta(x, t^* - 2\Delta)$. If $A^\Delta(x, t^* - 2\Delta) = \emptyset$, there is no policy that both parties would agree to implement. In this case there must be delay at $t^* - 2\Delta$, so for $i = 1, 2$ party i 's payoff is $W_i^\Delta(x, t^* - 2\Delta) = e^{-r\Delta}\mathbb{E}_{NA}[W_i^\Delta(x_{t^*-\Delta}, t^*)|x_{t^*-2\Delta} = x]$. If $A^\Delta(x, t^* - 2\Delta) \neq \emptyset$ the party with proposal power j offers $z(x, t^* - 2\Delta) \in \arg \max_{z \in A^\Delta(x, t^* - 2\Delta)} U_j(z, x, t^* - 2\Delta)$ and its opponent accepts this offer. In this case, for $i = 1, 2$ party i 's payoff is $U_i(z(x, t^* - 2\Delta), x, t^* - 2\Delta)$. Repeating these arguments for all $t \in T(\Delta)$ completes the proof of Theorem 3. ■

²⁶There are two things to note. First, when $A^\Delta(x, t^* - \Delta) \neq \emptyset$ the set of policies that maximize party j 's payoff is non-empty since $A^\Delta(x, t^* - \Delta)$ is compact and $U_j(\cdot, x, t^* - \Delta)$ is continuous. Second, by our restriction on SPE, when $A^\Delta(x, t^* - \Delta) \neq \emptyset$ the party with proposal power will always make an offer in $\arg \max_{z \in A^\Delta(x, t^* - \Delta)} U_j(z, x, t^* - \Delta)$ even if its indifferent between making this offer or delaying, and the responder will always accept such an offer even if its indifferent between accepting and rejecting.

Proof of Proposition 3. Note that $W_1^\Delta(x, t) + W_2^\Delta(x, t) \leq 1 + Ke^{-r(t^*-t)}$ for all $t < t^*$ and all $x \in [0, 1]$; that is, the sum of the parties' payoffs is bounded above by the total payoff they would get if they implemented a policy today, which is equal to $u_1(z) + u_2(z) = 1$, plus the sum of the parties' discounted payoff coming from the fact that one party will win the election, which is equal to $Ke^{-r(t^*-t)}$. Therefore, there exists $s > 0$ such that $e^{-r\Delta}\mathbb{E}_{NA}[W_1^\Delta(x_{t+\Delta}, t + \Delta) + W_2^\Delta(x_{t+\Delta}, t + \Delta)|x_t = x] \leq e^{-r\Delta}(1 + Ke^{-r(t^*-t)}) < 1$ for all t with $t^* - t > s$ and all $x \in [0, 1]$; i.e., s solves $1 + Ke^{-rs} = e^{r\Delta}$. Note that s is strictly increasing in K . Note further that, for all t such that $t^* - t > s$ and for all $x \in [0, 1]$, there exists a policy $z(x, t) \in [0, 1]$ such that $u_i(z(x, t)) \geq e^{-r\Delta}\mathbb{E}_{NA}[W_i^\Delta(x_{t+\Delta}, t + \Delta)|x_t = x]$ for $i = 1, 2$. Since $U_i(z(x, t), x, t) \geq u_i(z(x, t))$, it follows that $z(x, t) \in A_i^\Delta(x, t)$ for $i = 1, 2$. Therefore, $A^\Delta(x, t) = A_1^\Delta(x, t) \cap A_2^\Delta(x, t) \neq \emptyset$, so parties reach an agreement at t . ■

A.5 Proofs of Section 3.3

Proof of Lemma 1. I first show that $W_i^\Delta(x, t) \geq \underline{W}_i^\Delta(x, t)$ for all $t < t^*$ and all $x \in [0, 1]$. To see this, note that party i can always unilaterally generate delay at each time $t < t^*$, either by rejecting offers when $x_t \notin M_i$ and by choosing to pass on its right to make offers when $x_t \in M_i$. At times $t < t^*$, the payoff that party i gets by unilaterally delaying an agreement until time t^* is equal to $\mathbb{E}_{NA}[e^{-r(t^*-t)}V_i^\Delta(x_{t^*})|x_t = x] + e^{-r(t^*-t)}KQ_i(x, t) = \underline{W}_i^\Delta(x, t)$. Therefore, it must be that $W_i^\Delta(x, t) \geq \underline{W}_i^\Delta(x, t)$ for all (x, t) with $t < t^*$.

Next, I show that $W_i^\Delta(x, t) \leq \overline{W}_i^\Delta(x, t)$ for all $t < t^*$ and all $x \in [0, 1]$. To see this, note first that $W_1^\Delta(x, t) + W_2^\Delta(x, t) \leq 1 + Ke^{-r(t^*-t)}$ for all $t < t^*$ and all $x \in [0, 1]$: at any time $t < t^*$, the sum of the parties' payoffs cannot be larger than what they would jointly get by implementing a policy at t . From this inequality it follows that for all $t < t^*$ and all $x \in [0, 1]$

$$\begin{aligned} W_i^\Delta(x, t) &\leq 1 + Ke^{-r(t^*-t)} - W_j^\Delta(x, t) \\ &\leq 1 + Ke^{-r(t^*-t)} - \mathbb{E}_{NA}[e^{-r(t^*-t)}V_j^\Delta(x_{t^*})|x_t = x] - e^{-r(t^*-t)}KQ_j(x, t) \\ &= 1 - e^{-r(t^*-t)} + \mathbb{E}_{NA}[e^{-r(t^*-t)}V_i^\Delta(x_{t^*})|x_t = x] + e^{-r(t^*-t)}KQ_i(x, t), \end{aligned}$$

where the second inequality follows since $W_j^\Delta(x, t) \geq \underline{W}_j^\Delta(x, t)$ and the equality follows since $V_1^\Delta(x) + V_2^\Delta(x) = 1$ for all x and since $Q_1(x, t) + Q_2(x, t) = 1$ for all x and all $t < t^*$. Hence, $W_i^\Delta(x, t) \leq \overline{W}_i^\Delta(x, t)$ for all $t < t^*$ and for all $x \in [0, 1]$. ■

Lemma A4 For $i = 1, 2$ and for all $x, t \in [0, 1] \times [0, t^*)$, $\underline{W}_i^*(x, t)$ solves

$$r\underline{W}_i^*(x, t) = \frac{\partial}{\partial t}\underline{W}_i^*(x, t) + \mu \frac{\partial}{\partial x}\underline{W}_i^*(x, t) + \frac{1}{2}\sigma^2 \frac{\partial^2}{\partial x^2}\underline{W}_i^*(x, t),$$

with $\lim_{t \rightarrow t^*} \underline{W}_i^*(x, t) = V_i^*(x) + K\mathbf{1}_{\{x \in M_i\}}$ and $\frac{\partial}{\partial x}\underline{W}_i^*(x, t)|_{x=0} = \frac{\partial}{\partial x}\underline{W}_i^*(x, t)|_{x=1} = 0$.

Proof. For $i = 1, 2$ and $t < t^*$ let $w_i(x, t) := \mathbb{E}_{NA}[V_i^*(x_{t^*})|x_t = x]$, so that $\underline{W}_i^*(x, t) = e^{-r(t^*-t)}(w_i(x, t) + KQ_i(x, t))$. Note first that $Q_i(x, t) = P_i(t^* - t, x)$, where $P_i(s, x) = \Pr(x_s \in M_i|x_0 = x) = \int_{M_i} p(x, y, s)dy$ (recall from the proof of Theorem 2 that $p(x, y, s) = \Pr(x_s = y|x_0 = x)$). Since $P_i(s, x)$ solves (A.2) with $P_i(0, x) = \mathbf{1}_{\{x \in M_i\}}$ and $\frac{\partial}{\partial x}P_i(s, x)|_{x=0} = \frac{\partial}{\partial x}P_i(s, x)|_{x=1} = 0$ (see footnote 23), it follows that $Q_i(x, t) \in C^{2,1}$ for all $(x, t) \in [0, 1] \times [0, t^*)$, with $Q_i(x, t^*) = \mathbf{1}_{\{x \in M_i\}}$ and $\frac{\partial}{\partial x}Q_i(x, t)|_{x=0} = \frac{\partial}{\partial x}Q_i(x, t)|_{x=1} = 0$.

Note next that $w_i(x, t) = \mathbb{E}_{NA}[V_i^*(x_{t^*})|x_t = x] = \int_0^1 V_i^*(y)p(x, y, t^* - t)dy$. Hence, for all $(x, t) \in [0, 1] \times [0, t^*)$, $\frac{\partial}{\partial t}w_i(x, t) = \int_0^1 V_i^*(y)\frac{\partial}{\partial t}p(x, y, t^* - t)dy$, $\frac{\partial}{\partial x}w_i(x, t) = \int_0^1 V_i^*(y)\frac{\partial}{\partial x}p(x, y, t^* - t)dy$ and $\frac{\partial^2}{\partial x^2}w_i(x, t) = \int_0^1 V_i^*(y)\frac{\partial^2}{\partial x^2}p(x, y, t^* - t)dy$; i.e., $w_i(x, t) \in C^{2,1}$ for all $(x, t) \in [0, 1] \times [0, t^*)$. Since $\lim_{s \rightarrow 0} p(x, y, s) = \mathbf{1}_{\{y=x\}}$ and $\frac{\partial}{\partial x}p(x, y, s)|_{x=0} = \frac{\partial}{\partial x}p(x, y, s)|_{x=1} = 0$, it follows that $\lim_{t \rightarrow t^*} w_i(x, t) = V_i^*(x)$ and $\frac{\partial}{\partial x}w_i(x, t)|_{x=0} = \frac{\partial}{\partial x}w_i(x, t)|_{x=1} = 0$.

The analysis above implies that $\underline{W}_i^*(x, t) = e^{-r(t^*-t)}(w_i(x, t) + KQ_i(x, t)) \in C^{2,1}$ for all $(x, t) \in [0, 1] \times [0, t^*)$, with $\frac{\partial}{\partial x}\underline{W}_i^*(x, t)|_{x=0} = \frac{\partial}{\partial x}\underline{W}_i^*(x, t)|_{x=1} = 0$ and $\lim_{t \rightarrow t^*} \underline{W}_i^*(x, t) = V_i^*(x) + K\mathbf{1}_{\{x \in M_i\}}$. Note that by the law of iterated expectations, the process $Y_t = e^{-rt}\underline{W}_i^*(x, t) = \mathbb{E}_{NA}[e^{-rt^*}(V_i^*(x_{t^*}) + K \times \mathbf{1}_{\{x_{t^*} \in M_i\}})|x_t = x]$ is a martingale for all $t < t^*$. By Ito's lemma, for all $(x, t) \in [0, 1] \times [0, t^*)$,

$$dY_t = e^{-rt} \left[-r\underline{W}_i^*(x, t) + \frac{\partial}{\partial t}\underline{W}_i^*(x, t) + \mu \frac{\partial}{\partial x}\underline{W}_i^*(x, t) + \frac{1}{2}\sigma^2 \frac{\partial^2}{\partial x^2}\underline{W}_i^*(x, t) \right] dt + \sigma \frac{\partial}{\partial x}\underline{W}_i^*(x, t)dB_t.$$

Since Y_t is a martingale, the term inside the square brackets must be zero. This implies that $\underline{W}_i^*(x, t)$ solves the equation in the statement of the lemma. ■

Lemma A5 Fix a time $t \in T(\Delta)$, $t < t^*$ and an $x \in [0, 1]$. If there exists an offer $\hat{z} \in [0, 1]$ and a party $j \in \{1, 2\}$ such that $U_j(\hat{z}, x, t) = \mathbb{E}_{NA}[e^{-r\Delta}W_j^\Delta(x_{t+\Delta}, t + \Delta)|x_t = x]$, then parties reach an agreement at time t if $x_t = x$.

Proof. Suppose such an offer \hat{z} exists, and note that $\hat{z} \in A_j^\Delta(x, t)$. Since $W_1^\Delta(x_{t+\Delta}, t + \Delta) + W_2^\Delta(x_{t+\Delta}, t + \Delta) \leq 1 + Ke^{-r(t^*-t-\Delta)}$ for all $x_{t+\Delta} \in [0, 1]$, it follows that,

$$\mathbb{E}_{NA} [e^{-r\Delta}W_i^\Delta(x_{t+\Delta}, t + \Delta)|x_t] \leq e^{-r\Delta} + Ke^{-r(t^*-t)} - \mathbb{E}_{NA} [e^{-r\Delta}W_j^\Delta(x_{t+\Delta}, t + \Delta)|x_t]. \quad (\text{A.6})$$

Since $U_i(z, x, t) + U_j(z, x, t) = 1 + e^{-r(t^*-t)}K$ for all $z \in [0, 1]$, party i 's payoff from implementing policy \hat{z} at time $t < t^*$ with $x_t = x$ is

$$U_i(\hat{z}, x, t) = 1 + Ke^{-r(t^*-t)} - U_j(\hat{z}, x, t) = 1 + Ke^{-r(t^*-t)} - \mathbb{E}_{NA} [e^{-r\Delta} W_j^\Delta(x_{t+\Delta}, t + \Delta) | x_t].$$

This equation together with (A.6) implies that $U_i(\hat{z}, x, t) > \mathbb{E}_{NA}[e^{-r\Delta} W_i^\Delta(x_{t+\Delta}, t + \Delta) | x_t = x]$, so that $\hat{z} \in A_i^\Delta(x, t)$. Hence, $A^\Delta(x, t) = A_1^\Delta(x, t) \cap A_2^\Delta(x, t) \neq \emptyset$, so parties reach an agreement at time t if $x_t = x$. ■

Proof of Proposition 4. Let (x, t) be a state satisfying the conditions in part (i) of the proposition, and suppose by contradiction that parties reach an agreement at time t when $x_t = x$. Since $U_i(z, x, t) < \underline{W}_i^\Delta(x, t)$ for all $z \in [0, 1]$, this implies that party i 's SPE payoff at state (x, t) is strictly lower than $\underline{W}_i^\Delta(x, t)$, a contradiction to the fact that that $\underline{W}_i^\Delta(x, t)$ is a lower bound to party i 's payoff at state (x, t) . Thus, there must be delay at state (x, t) .

Next, let (x, t) be a state satisfying the conditions in Proposition 4 (ii), and note that

$$\mathbb{E}_{NA}[e^{-r\Delta} \underline{W}_i^\Delta(x_{t+\Delta}, t + \Delta) | x_t] \leq \mathbb{E}_{NA}[e^{-r\Delta} W_i^\Delta(x_{t+\Delta}, t + \Delta) | x_t] \leq \mathbb{E}_{NA}[e^{-r\Delta} \overline{W}_i^\Delta(x_{t+\Delta}, t + \Delta) | x_t].$$

By the law of iterated expectations, $\mathbb{E}_{NA}[e^{-r\Delta} \underline{W}_i^\Delta(x_{t+\Delta}, t + \Delta) | x_t] = \mathbb{E}_{NA}[e^{-r(t^*-t)}(V_i^\Delta(x_{t^*}) + K\mathbf{1}_{\{x_{t^*} \in M_i\}}) | x_t = x] = \underline{W}_i^\Delta(x_t, t)$ and $\mathbb{E}_{NA}[e^{-r\Delta} \overline{W}_i^\Delta(x_{t+\Delta}, t + \Delta) | x_t = x] = e^{-r\Delta} - e^{-r(t^*-t)} + \mathbb{E}_{NA}[e^{-r\Delta} \underline{W}_i^\Delta(x_{t+\Delta}, t + \Delta) | x_t] = e^{-r\Delta} - e^{-r(t^*-t)} + \underline{W}_i^\Delta(x_t, t) < \overline{W}_i^\Delta(x_t, t)$. Since $U_i(\cdot, x, t)$ is continuous and since $U_i(z', x, t) \leq \underline{W}_i^\Delta(x, t)$ and $U_i(z'', x, t) \geq \overline{W}_i^\Delta(x, t)$ for $z', z'' \in [0, 1]$, there exists $\hat{z} \in [0, 1]$ such that $U_i(\hat{z}, x, t) = \mathbb{E}_{NA}[e^{-r\Delta} W_i^\Delta(x_{t+\Delta}, t + \Delta) | x_t = x]$. By Lemma A5, there is agreement at state (x, t) . ■

A.6 Proofs of Section 3.4

Proof of Proposition 5. Note first that either $V_1^\Delta(1) > 1/2$ or $V_2^\Delta(0) > 1/2$ (or both).²⁷ Note further that $\mathbb{E}_{NA}[e^{-r(t^*-t)} V_i^\Delta(x_{t^*}) | x_t = x] \rightarrow V_i^\Delta(x)$ as $t \rightarrow t^*$. Therefore, there exist $i \in \{1, 2\}$ and $(x, t) \in [0, 1] \times T(\Delta)$, $t < t^*$ with $x \in M_i$ such that $\mathbb{E}_{NA}[e^{-r(t^*-t)} V_i^\Delta(x_{t^*}) | x_t = x] > 1/2$. Fix such a state (x, t) , and assume that $i = 1$; i.e., assume that $x \in M_1$ and $\mathbb{E}_{NA}[e^{-r(t^*-t)} V_1^\Delta(x_{t^*}) | x_t = x] > 1/2$. I now show that, if K is large enough, parties will delay at state (x, t) . The proof that there is delay when $x \in M_2$ and $\mathbb{E}_{NA}[e^{-r(t^*-t)} V_2^\Delta(x_{t^*}) | x_t = x] > 1/2$ is symmetric and omitted.

²⁷Proof: If $V_2^\Delta(0) > 1/2$, then the statement is true. Otherwise, if $1/2 \geq V_2^\Delta(0) = 1 - V_1^\Delta(0)$, then it must be that $V_1^\Delta(1) > V_1^\Delta(0) \geq 1/2$ (since V_1^Δ is increasing).

Party 1's payoff from implementing policy z at time t when $x_t = x$ is $U_1(z, x, t) = z + Ke^{-r(t^*-t)}Q_1^z(x, t)$. Let $z' \in [0, 1]$ be such that $z' = \mathbb{E}_{NA}[e^{-r(t^*-t)}V_1^\Delta(x_{t^*})|x_t = x] > 1/2$. Since $h(\cdot, \cdot)$ satisfies (5) and since $x \in M_1$, it follows that $Q_1^z(x, t) \leq Q_1(x, t)$ for all $z \in [0, 1]$, with strict inequality if $z \neq 1/2$. Therefore, for all $z \in [0, z']$, $U_1(z, x, t) = z + Ke^{-r(t^*-t)}Q_1^z(x, t) < \mathbb{E}_{NA}[e^{-r(t^*-t)}V_1^\Delta(x_{t^*})|x_t = x] + Ke^{-r(t^*-t)}Q_1(x, t) = \underline{W}_1^\Delta(x, t)$. Moreover, for all $z \in [z', 1]$,

$$\underline{W}_1^\Delta(x, t) - U_1(z, x, t) = \mathbb{E}_{NA}[e^{-r(t^*-t)}V_1^\Delta(x_{t^*})|x_t = x] - z + Ke^{-r(t^*-t)}[Q_1(x, t) - Q_1^z(x, t)].$$

Since the term in squared brackets is strictly negative for all $z \in [z', 1]$, for all such z there exists $K(z) > 0$ such that $\underline{W}_1^\Delta(x, t) > U_1(z, x, t)$ if $K > K(z)$. Moreover, it is clear that $K(z)$ can be chosen to be bounded for all $z \in [z', 1]$. Letting $\bar{K} = \sup_{z \in [z', 1]} K(z)$, it follows that $\underline{W}_1^\Delta(x, t) > U_1(z, x, t)$ for all $z \in [0, 1]$ whenever $K > \bar{K}$, so by Proposition 4 (i) parties delay an agreement at time t if $x_t = x$. ■

Proof of Proposition 6. Fix a state (x, t) with $x \in M_i \cap (0, 1)$ and $t < t^*$. Note that, when $h(\cdot, \cdot)$ is given by (6), $z_j \in [0, 1]$ is the policy that maximizes $U_j(\cdot, x, t)$ for all $x \in M_i$ (where z_j is party j 's ideal policy). Moreover, note that for all such states (x, t) ,

$$\underline{W}_j^\Delta(x, t) - U_j(z_j, x, t) = \mathbb{E}_{NA}[e^{-r(t^*-t)}V_j^\Delta(x_{t^*})|x_t = x] - 1 + Ke^{-r(t^*-t)}[Q_j(x, t) - Q_j^{z_j}(x, t)].$$

Since the term in squared brackets is negative, there exists $\bar{K} > 0$ such that $\underline{W}_j^\Delta(x, t) > U_j(z_j, x, t) = \max_{z \in [0, 1]} U_j(z, x, t)$ whenever $K > \bar{K}$. It then follows from Proposition 4 (i) that there must be delay at state (x, t) if $K > \bar{K}$. ■

Proof of Proposition 7. I first show that parties reach an immediate agreement when $h(\cdot, \cdot)$ satisfies the conditions in the statement of Proposition 7. To prove this, I start by showing that for all (x, t) with $t \in T(\Delta), t < t^*$ and for $i = 1, 2$, $\min_{z \in [0, 1]} U_i(z, x, t) < \underline{W}_i^\Delta(x, t)$ and $\max_{z \in [0, 1]} U_i(z, x, t) > \bar{W}_i^\Delta(x, t)$. To see this, recall that $z_1 = 1$ and $z_2 = 0$ are, respectively, the ideal policies of parties 1 and 2. Note that in this setting $z_j = \arg \min_z U_i(z, x, t)$ and $z_i = \arg \max_z U_i(z, x, t)$. Note further that, for all $x \in [0, 1]$ and all $t < t^*$, $U_i(z_j, x, t) = e^{-r(t^*-t)}KQ_i(x + h(z_j, x), t) \leq e^{-r(t^*-t)}KQ_i(x, t) \leq \underline{W}_i^\Delta(x, t)$ and $U_i(z_i, x, t) = 1 + e^{-r(t^*-t)}KQ_i(x + h(z_i, x), t) \geq 1 + e^{-r(t^*-t)}KQ_i(x, t) \geq \bar{W}_i^\Delta(x, t)$. Therefore, by Proposition 4 (ii) parties reach an agreement at all states (x, t) with $t \in T(\Delta), t < t^*$.

Next, I show that the parties' SPE payoffs satisfy equation (7). As a first step to establish this, I show that for all $x \in M_i$ and all $t \in T(\Delta), t < t^*$ there exists an offer $z \in [0, 1]$ such that $U_j(z, x, t) = \mathbb{E}_{NA}[e^{-r\Delta}W_j^\Delta(x_{t+\Delta}, t + \Delta)|x_t = x]$. To see this, note that

by the paragraph above there exists z', z'' such that $U_j(z', x, t) \leq \underline{W}_j^\Delta(x, t)$ and $U_j(z'', x, t) \geq \overline{W}_j^\Delta(x, t)$. Moreover, note also that $\underline{W}_j^\Delta(x, t) = \mathbb{E}_{NA}[e^{-r\Delta}\underline{W}_j^\Delta(x_{t+\Delta}, t + \Delta)|x_t = x]$ and that $\overline{W}_j^\Delta(x, t) > \mathbb{E}_{NA}[e^{-r\Delta}\overline{W}_j^\Delta(x_{t+\Delta}, t + \Delta)|x_t = x]$ (the proof of Proposition 4 shows that these hold). Then, by continuity of $U_j(\cdot, x, t)$ and by the fact that $W_j^\Delta(x, t + \Delta) \in [\underline{W}_j^\Delta(x, t + \Delta), \overline{W}_j^\Delta(x, t + \Delta)]$ for all x (Lemma 1), there must exist an offer $z \in [0, 1]$ such that $U_j(z, x, t) = \mathbb{E}_{NA}[e^{-r(t^*-t)}W_j^\Delta(x_{t+\Delta}, t + \Delta)|x_t = x]$. Note that this offer maximizes party i 's payoff among the offers that party j finds acceptable at state (x, t) with $x \in M_i$, and hence is the offer that party i makes in equilibrium.

I now use the observation in the previous paragraph to show that the parties' payoffs satisfy equation (7). The proof is by induction. Consider time $t = t^* - \Delta$. By the previous paragraph, for all $x \in M_i$ party i makes an offer z such that $U_j(z, x, t) = \mathbb{E}_{NA}[e^{-r\Delta}W_j^\Delta(x_{t^*}, t^*)|x_{t^*-\Delta} = x] = \mathbb{E}_{NA}[e^{-r\Delta}V_j^\Delta(x_{t^*})|x_{t^*-\Delta} = x] + e^{-r\Delta}KQ_j(x, t^* - \Delta)$, and party j accepts such an offer. Since $\mathbb{E}_{NA}[e^{-r\Delta}V_j^\Delta(x_{t^*})|x_{t^*-\Delta} = x] = V_j^\Delta(x)$ for all $x \in M_i$,²⁸ it follows that party j 's payoff at any state $(x, t^* - \Delta)$ with $x \in M_i$ is equal to $V_j^\Delta(x) + e^{-r\Delta}KQ_j(x, t^* - \Delta)$. Since parties reach an agreement at $t^* - \Delta$, the sum of their payoffs is $1 + Ke^{-r\Delta}$. Hence, party i 's payoff at any state $(t^* - \Delta, x)$ with $x \in M_i$ is equal to $1 + Ke^{-r\Delta} - V_j^\Delta(x) - e^{-r\Delta}KQ_j(x, t^* - \Delta) = V_i(x) + e^{-r\Delta}KQ_i(x, t^* - \Delta)$, where the equality follows since $V_i^\Delta(x) + V_j^\Delta(x) = 1$ for all x and since $Q_i(x, t) + Q_j(x, t) = 1$ for all (x, t) with $t < t^*$. Therefore, the parties' payoffs satisfy equation (7) at $t = t^* - \Delta$.

Suppose next that the parties' payoffs satisfy (7) for $t = t^* - \Delta, t^* - 2\Delta, \dots, t^* - n\Delta$. Let $s = t^* - n\Delta$. At states $(x, s - \Delta)$ with $x \in M_i$, party i makes an offer z such that $U_j(z, x, s - \Delta) = \mathbb{E}_{NA}[e^{-r\Delta}W_j^\Delta(x_s, s)|x_{s-\Delta} = x] = \mathbb{E}_{NA}[e^{-r\Delta}V_j^\Delta(x_s)|x_{s-\Delta} = x] + e^{-r(t^*-(s-\Delta))}KQ_j(x, s - \Delta)$ (where the last equality follows from the induction hypothesis), and party j accepts such an offer. Since $\mathbb{E}_{NA}[e^{-r\Delta}V_j^\Delta(x_s)|x_{s-\Delta} = x] = V_j^\Delta(x)$ for all $x \in M_i$, it follows that party j 's payoff at any state $(x, s - \Delta)$ with $x \in M_i$ is equal to $V_j^\Delta(x) + e^{-r(t^*-(s-\Delta))}KQ_j(x, s - \Delta)$. Since parties reach an agreement at $s - \Delta$, the sum of their payoffs is $1 + Ke^{-r(t^*-(s-\Delta))}$. Therefore, party i 's payoff at any state $(x, s - \Delta)$ with $x \in M_i$ is equal to $1 + Ke^{-r(t^*-(s-\Delta))} - V_j^\Delta(x) - e^{-r(t^*-(s-\Delta))}KQ_j(x, s - \Delta) = V_i^\Delta(x) + e^{-r(t^*-(s-\Delta))}KQ_i(x, s - \Delta)$, so the parties' payoffs also satisfy equation (7) at $s - \Delta = t^* - (n + 1)\Delta$. ■

²⁸This follows from Theorem 1 and from the fact that the expectation operator $\mathbb{E}_{NA}[f(x_s)|x_t = x]$ is equal to the expectation operator $\mathbb{E}[f(x_s)|x_t = x]$ from Section 2; see footnote 13.

A.7 Extensions

A.7.1 General bargaining protocols

Consider the game without elections as in Section 2, but with the bargaining protocol described in the extension “General bargaining protocols.” By arguments similar to those in Theorem 1, this game has a unique SPE. Moreover, parties always reach an immediate agreement in the unique SPE. Let \hat{V}_i^Δ denote party i 's SPE payoff when the time period is Δ . It can be shown that this payoff satisfies

$$\begin{aligned}\hat{V}_i^\Delta(x) &= p_i(x) \left(1 - e^{-r\Delta} \mathbb{E} \left[\hat{V}_j^\Delta(x_{t+\Delta}) \middle| x_t = x \right] \right) + (1 - p_i(x)) e^{-r\Delta} \mathbb{E} \left[\hat{V}_i^\Delta(x_{t+\Delta}) \middle| x_t = x \right] \\ &= p_i(x)(1 - e^{-r\Delta}) + e^{-r\Delta} \mathbb{E} \left[\hat{V}_i^\Delta(x_{t+\Delta}) \middle| x_t = x \right],\end{aligned}\tag{A.7}$$

where the second equality follows since $\hat{V}_1^\Delta(y) + \hat{V}_2^\Delta(y) = 1$ for all $y \in [0, 1]$ (since parties always reach an immediate agreement). When party i makes offers, its payoff is equal to 1 minus the expected continuation payoff of party $j \neq i$. When party i is responder, its payoff is equal to its expected continuation payoff. Note that equation (A.7) has the same interpretation as equation (3) in the main text: in this setting party i extracts an expected rent equal to $p_i(x)(1 - e^{-r\Delta})$ when $x_t = x$, since it makes offers with probability $p_i(x)$.

Setting $t = 0$ in (A.7) and solving this equation forward yields

$$\hat{V}_i^\Delta(x) = \mathbb{E} \left[\frac{1 - e^{-r\Delta}}{\Delta} \sum_{k=0}^{\infty} \Delta e^{-rk\Delta} p_i(x_{k\Delta}) \middle| x_0 = x \right].$$

In this setting, it can be shown that $\hat{V}_i^*(x) := \lim_{\Delta \rightarrow 0} \hat{V}_i^\Delta(x) = \mathbb{E}[r \int_0^\infty e^{-rt} p_i(x_t) dt | x_0 = x]$. Finally, by Corollary 2.4 in chapter 5 of Harrison (1985), $\hat{V}_i^*(x)$ satisfies $r\hat{V}_i^*(x) = rp_i(x) + \mu(\hat{V}_i^*)'(x) + \frac{1}{2}\sigma^2(\hat{V}_i^*)''(x)$ for all $x \in [0, 1]$, with $(\hat{V}_i^*)'(0) = (\hat{V}_i^*)'(1) = 0$.

A.7.2 General stochastic processes

This appendix shows how the models in the paper can be generalized to settings in which political power evolves as

$$dx_t = \mu(x_t)dt + \sigma(x_t)dB_t, x_0 \in [0, 1],\tag{A.8}$$

with reflecting boundaries at 0 and 1 (where $\mu(\cdot)$ and $\sigma(\cdot)$ are both C^2 and $\sigma(x) > 0$ for all $x \in [0, 1]$). This generalization permits modeling situations in which the drift and volatility

of political power varies over time. One such example is a setting in which x_t has a tendency to revert to its long-run mean: when $\mu(x) = -\gamma(x - 1/2)$ with $\gamma > 0$ and $\sigma(x) = \sigma > 0$, x_t has a tendency to revert to $1/2$.

Consider first the game without elections and assume that x_t evolves as (A.8). Suppose further that the bargaining protocol is the same as in Sections 2 and 3: at each time $t \in T(\Delta)$, party i has proposal power if $x_t \in M_i$. The same arguments as in the proof of Theorem 1 can be used to show that, for any $\Delta > 0$, this game has a unique SPE. Moreover, parties always reach an immediate agreement in the unique SPE. Let $\tilde{V}_i^\Delta(x)$ denote party i 's SPE payoff. Following the same steps as in the derivation of equation (A.1), these payoffs satisfy

$$\tilde{V}_i^\Delta(x) = \mathbb{E} \left[(1 - e^{-r\Delta}) \sum_{k=0}^{\infty} e^{-rk\Delta} \mathbf{1}_{\{x_{k\Delta} \in M_i\}} \middle| x_0 = x \right] = \frac{1 - e^{-r\Delta}}{\Delta} \sum_{k=0}^{\infty} \Delta e^{-rk\Delta} \tilde{P}_i(k\Delta, x),$$

where for all $s \geq 0$ and all $x \in [0, 1]$, $\tilde{P}_i(s, x) = \mathbb{E}[\mathbf{1}_{\{x_s \in M_i\}} | x_0 = x]$ is the probability with which party i makes offers at time $s \geq 0$ conditional on $x_0 = x$ when x_t evolves as (A.8). When x_t evolves as (A.8), $\tilde{P}_i(s, x)$ solves the following version of Kolmogorov's backward equation,

$$\frac{\partial}{\partial s} \tilde{P}_i(s, x) = \mu(x) \frac{\partial}{\partial x} \tilde{P}_i(s, x) + \frac{1}{2} \sigma(x)^2 \frac{\partial^2}{\partial x^2} \tilde{P}_i(s, x),$$

with $\lim_{s \rightarrow 0} \tilde{P}_i(s, x) = \mathbf{1}_{\{x \in M_i\}}$ and $\frac{\partial}{\partial x} \tilde{P}_i(s, x)|_{x=0} = \frac{\partial}{\partial x} \tilde{P}_i(s, x)|_{x=1} = 0$ for all $s > 0$. Since $e^{-rs} \tilde{P}_i(s, x)$ is Riemann integrable, $\tilde{V}_i^*(x) := \lim_{\Delta \rightarrow 0} \tilde{V}_i^\Delta(x) = \mathbb{E}[r \int_0^\infty e^{-rt} \tilde{P}_i(t, x) | x_0 = x]$. Moreover, it can be shown that this convergence is also uniform.

From here, one can use the same arguments as in the proof of Theorem 2 to establish that, for all $x \neq 1/2$, $\tilde{V}_i^*(x)$ solves

$$r\tilde{V}_i^*(x) = \begin{cases} \mu(x)(\tilde{V}_i^*)'(x) + \frac{1}{2}\sigma(x)^2(\tilde{V}_i^*)''(x) & \text{if } x \notin M_i, \\ r + \mu(x)(\tilde{V}_i^*)'(x) + \frac{1}{2}\sigma(x)^2(\tilde{V}_i^*)''(x) & \text{if } x \in M_i, \end{cases} \quad (\text{A.9})$$

with boundary conditions $(\tilde{V}_i^*)'(0) = (\tilde{V}_i^*)'(1) = 0$, $\tilde{V}_i^*(1/2^-) = \tilde{V}_i^*(1/2^+)$ and $(\tilde{V}_i^*)'(1/2^-) = (\tilde{V}_i^*)'(1/2^+)$. Indeed, the same arguments as in Lemma A1 can be used to show that \tilde{V}_1^* and \tilde{V}_2^* are continuous. Moreover, the same arguments as in the proof of Theorem 2 can be used to show that $\tilde{V}_i^*(x)$ solves (A.9) with boundary conditions $(\tilde{V}_i^*)'(0) = (\tilde{V}_i^*)'(1) = 0$, $\tilde{V}_i^*(1/2^-) = \tilde{V}_i^*(1/2^+)$ and $(\tilde{V}_i^*)'(1/2^-) = (\tilde{V}_i^*)'(1/2^+)$. Equation (A.9) has the same interpretation as equation (4) in the main text: the left-hand side is party i 's limiting payoff measured in flow terms, while the right-hand side shows the sources of this flow payoff.

Consider next the model with elections. The arguments in the proof of Theorem 3 can

be used to establish existence and uniqueness of equilibrium payoffs. Moreover, by the same arguments in Section 3.3, for all (x, t) with $t < t^*$ party i 's payoffs are bounded below by $\mathbb{E}[e^{-r(t^*-t)}\tilde{V}_i^*(x_{t^*})|x_t = x] + Ke^{-r(t^*-t)}Q_i(x, t)$ and are bounded above by $1 - e^{-r(t^*-t)} + \mathbb{E}[e^{-r(t^*-t)}\tilde{V}_i^*(x_{t^*})|x_t = x] + Ke^{-r(t^*-t)}Q_i(x, t)$. These bounds on payoffs also become tight as the election approaches, and can be used to study how the likelihood of gridlock depends on the time left until the election and on the parties' level of political power.

A.7.3 Elections, proposal power and multiple issues

I now study the game described under the title ‘‘Elections, proposal power and multiple issues’’ in Section 4. Consider a history of this game under which parties reach an agreement on the first issue before t^* . Suppose that $x_{t^*} \in M_k$, so that party $k \in \{1, 2\}$ wins the election. In this case, the subgame that starts immediately after the election is essentially identical to a game without elections and with bargaining protocol $p_i^k(\cdot)$; the only difference is that parties' now bargain over a surplus of size γ . Therefore, parties will immediately reach an agreement over the second issue at time t^* , and party i 's payoff from this agreement will be $\gamma\hat{V}_i^\Delta(x, k)$, where $\hat{V}_i^\Delta(x, k)$ is party i 's SPE payoffs in the game without elections with time period Δ and with bargaining protocol $p_i^k(\cdot)$ (see the extension on ‘‘General bargaining protocols’’). Let $\hat{W}_i^\Delta(x, t^*, A)$ be party i 's payoff at time t^* if parties had reached an agreement over the first issue before the election. By the arguments above,

$$\hat{W}_i^\Delta(x, t^*, A) = (K + \gamma\hat{V}_i^\Delta(x, i)) \times \mathbf{1}_{\{x \in M_i\}} + \gamma\hat{V}_i^\Delta(x, j) \times \mathbf{1}_{\{x \notin M_i\}}.$$

Party i 's payoff at t^* is equal to $K + \gamma\hat{V}_i^\Delta(x, i)$ if $x \in M_i$ (i.e., it wins the election), and is equal to $\gamma\hat{V}_i^\Delta(x, j)$ otherwise.

Consider next a history under which parties have not reached an agreement over the first issue by time t^* . Let $\hat{W}_i^\Delta(x, t^*, N)$ be party i 's payoff at time t^* under such history, and note that

$$\hat{W}_i^\Delta(x, t^*, N) = (K + (\gamma + 1)\hat{V}_i^\Delta(x, i)) \times \mathbf{1}_{\{x \in M_i\}} + (\gamma + 1)\hat{V}_i^\Delta(x, j) \times \mathbf{1}_{\{x \notin M_i\}}.$$

In this case, party i 's payoff at time t^* is $K + (\gamma + 1)\hat{V}_i^\Delta(x, i)$ if it wins the election, since in this case parties will bargain over the two issues simultaneously after the election (so they effectively bargain over a surplus of size $\gamma + 1$). Otherwise, if party i loses the election, its payoff at t^* is $(\gamma + 1)\hat{V}_i^\Delta(x, j)$.

Consider next times $t < t^*$. As in Section 3, suppose that political power jumps by $h(x, z)$

if parties reach an agreement to implement policy z when political power is x . Note that party i 's equilibrium payoff at time $t < t^*$ is bounded below by the payoff party i would get by delaying an agreement until after the election; i.e., by $\mathbb{E}_{NA}[e^{-r(t^*-t)}\hat{W}_i^\Delta(x_{t^*}, t^*, N)|x_t = x]$. Moreover, by arguments similar to those in Lemma 1, party i 's payoff at time $t < t^*$ is bounded above by $1 - e^{-r(t^*-t)} + \mathbb{E}_{NA}[e^{-r(t^*-t)}\hat{W}_i^\Delta(x_{t^*}, t^*, N)|x_t = x]$. These bounds on payoffs also become tight as the election approaches, and are easy to compute numerically in the limit as $\Delta \rightarrow 0$. Finally, these bounds on payoffs can be used to derive an analog result to Proposition 4 in this setting, and to study how the likelihood of gridlock depends on the time left until the election and on the level of political power.

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