Predictability and Power in Legislative Bargaining*

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Abstract

This paper examines the relationship between the concentration of political power in legislative bargaining and the predictability of the process governing the recognition of legislators. Our main result establishes that, for a broad class of legislative bargaining games, if the recognition procedure permits the legislators to rule out some minimum number of proposers one round in advance, then irrespective of how patient the individual legislators are, Markovian equilibria necessarily deliver all economic surplus to the first proposer. We also examine the extent to which alternative bargaining protocols can limit the concentration of power.

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1 Introduction

Motivation: Multilateral bargaining is a ubiquitous aspect of decision making within social groups. For example, in the political sphere, it governs the distribution of pork, the design of international treaties, and the formation of governments in parliamentary systems. Negotiators routinely attempt to steer these processes towards their own objectives, not only by organizing coalitions to support or block particular proposals, but also by maneuvering for control of the process through which they and others bring forth proposals. The relative power of the negotiators determines how agreements are reached and who benefits most from them.

In this paper, we investigate the ways in which the predictability of *future* bargaining power affects multilateral negotiations. Intuitively, an individual's willingness to join a coalition in support of a proposal today, rather than one aiming to block the proposal and extend negotiations, depends on how much bargaining power she expects to have if negotiations continue. If she has reason to be optimistic about her future bargaining power, she will only support proposals that are favorable to her; in contrast, if she expects to have little bargaining power in the future, she should be willing to support current proposals that provide her with modest benefits. Thus, the predictability of future bargaining power has an important bearing on negotiations.

Proposer Power: We examine these issues in the context of noncooperative multilateral bargaining models that build on Rubinstein (1982) and Baron and Ferejohn (1989). In these settings, rejection of a proposal leads to costly delay, and hence bargaining power flows from the ability to make proposals (or "set the agenda").

Most models of legislative bargaining treat proposer power as a primitive, formulated as an i.i.d. recognition process in which uncertainty as to who makes a proposal in period t is resolved immediately before the proposal is made. While this assumption can be defended as a stylized attempt to capture the somewhat inscrutable nature of the various protocols by which proposers are actually selected, it is also unrealistic. According to this assumption, the selection of each proposer is *entirely* random, and no information bearing on the identity of the period-t proposer is revealed prior to period t. In practice, important elements of the processes governing the recognition of proposers are non-random, and information concerning random elements may be revealed in advance of the round-t proposer's selection. Examples include the following:

1. Proposers may be pre-announced. For instance, an otherwise inscrutable (and hence apparently random) chair may identify the next proposer when recognizing the current one.

2. Based on the rules governing multilateral bargaining, it may be possible to rule out certain candidates for the proposer in advance. For example, the rules may require that proposers belong to pertinent committees or have seniority. Alternatively, the rules may prevent members of the same party from being recognized twice in a row or before representatives from all other

parties have had a turn, or they may specify that the proposer for period t must be selected from a slate of nominees who are announced at an earlier point in time.

3. The selection of the period-t proposer may depend upon strategic choices that are themselves predictable in equilibrium, e.g., if choice of the proposer is up to a chair who is elected in advance, and who has a known "pecking order" of favorites, or if members can compete for the right to propose through costly effort or the expenditure of political capital.

From the perspective of developing a realistic positive theory of multilateral bargaining, it is essential to understand the implications of a (somewhat) predictable recognition process. Analyzing such processes is also of normative importance: when designing institutions, rules, and constitutions that govern multilateral bargaining, institutional architects must decide whether to make these processes more or less predictable. Thus it is important to know whether predictability improves or worsens the efficiency and equity of negotiated outcomes. For instance, our analysis raises the possibility that greater procedural equity, such as rules ensuring that negotiators must take turns making proposals, can perversely produce less equitable outcomes.

Our Approach and Main Result: We study a multilateral bargaining game wherein, in each period, one of n legislators proposes a division of a fixed payoff, and needs the support of $q < n$ legislators for the proposal to pass. Initially we consider a class of recognition processes that generalizes the standard i.i.d. selection mechanism in two distinct ways. First, we allow the selection process to exhibit rich history-dependence: the identity of the current proposer may depend on the history of random shocks and past proposers. Second, information concerning the random shocks affecting the identity of the period-t proposer may be revealed in prior periods, so that negotiators become better informed about period-t bargaining power over time. These two elements give rise to a collection of rich stochastic games in which negotiations are influenced by the evolution of beliefs about future bargaining power.

We find that even a limited degree of predictability of the recognition process has stark implications for bargaining: under a relatively mild condition described in the next paragraph, the first proposer receives the *entire* prize, irrespective of negotiators' discount factors, in every subgame perfect equilibrium of the finite horizon game, and every Markov Perfect equilibrium of the infinite horizon game. Our result contrasts sharply with the characterization of stationary equilibria described in Baron and Ferejohn (1989), in which the first proposer shares surplus with a minimal winning coalition according to a formula that depends on negotiators' discount factors, with the proposer keeping roughly half the surplus in large legislatures of patient players. Thus, mild predictability of the recognition process confers considerably more bargaining power on the first proposer than existing approaches to multilateral bargaining suggest.

The predictability condition that gives rise to the aforementioned result is simple and intuitive. We say that bargaining power is *one-period predictable of degree d* if, for each history, at least d players can be ruled out as the next proposer. Our main result, Theorem 1, establishes that if the bargaining process is one-period predictable of degree q, then the first proposer captures all of the surplus. (Recall that passage of a proposal requires $q < n$ favorable votes.) The impact of predictability is continuous; if q legislators are recognized with sufficiently small probability, then the current proposer captures nearly all of the surplus.

The logic of our main result (which we also illustrate through examples in Section 2) is straightforward. When a voting rule is non-unanimous, a proposer can exclude any other negotiator from a winning coalition, and this possibility constrains the amount of surplus any particular participant anticipates obtaining from the proposer. Greater predictability of bargaining power at period $t+1$ helps the period-t proposer identify the most profitable negotiators to exclude: those who expect to be the proposer tomorrow have incentives to block the current proposal and extend negotiations, while others have no reason to reject offers unless they expect the next proposer to treat them more generously. But the proposer at $t+1$ cannot commit to such generosity. Indeed, since she will be able to predict bargaining power at $t+2$ prior to making her offer, she will, in turn, act on her own incentives to form the cheapest winning coalition by proposing to share the surplus with participants who do not expect to be recognized at $t+2$. Iterating this logic, the proposer at time t can secure the support of those who can be ruled out as period $t+1$ proposers by offering them arbitrarily small shares of the surplus, even if they expect to be recognized in subsequent rounds. Because the number of such individuals is sufficient to form a winning coalition ($d \geq q$ by assumption), the first proposer extracts all of the surplus.

Broader Implications: We do not view our findings as providing a literal description of legislative outcomes, but rather as illustrating an important principle concerning the possible effects of institutional characteristics on negotiated outcomes: the combination of predictable bargaining power and a non-unanimous voting rule can provide those with short-term control over agenda setting with extreme power. Perhaps surprisingly, neither institutional feature in isolation engenders such inequality. For instance, for the alternating-offer protocol of Rubinstein (1982), future bargaining power is perfectly predictable but unanimity is required; as a result, in both bilateral and multilateral settings, stationary equilibria yield approximately equal division as $\delta \to 1$. Similarly, in Baron and Ferejohn (1989), unanimity is not required but future bargaining power is unpredictable; as a result, the proposer shares roughly half the surplus with the winning coalition. Thus it is the combination of these two features that permits a proposer to capture the entire surplus.

As we show in Section 5, the proposer power resulting from predictability and excludability extends to general coalition structures that would arise, e.g., if individuals have different voting weights, to settings in which utility is non-transferable, and even to richer settings in which individuals can maneuver for bargaining power. Indeed, across these settings, when future bargaining power is sufficiently predictable, heterogeneity in discount factors, recognition probabilities, risk-aversion, and voting weights have little impact on the shares offered by the first proposer. Thus, the combination of even a limited degree of predictability and opportunities for profitable coalition formation confers far greater power upon the proposer than previous research would seem to imply.

Because our main result elucidates the implications of one-period predictability in the starkest case, it may raise the concern that predictability matters only if it is possible to rule out at least q negotiators as the next proposer. That is not the case. For instance, if it is possible to rule out only $q-1$ negotiators, the current proposer will still capture nearly all of the surplus, provided the group is reasonably large. We study a tractable sub-class of models falling within our framework (see Section 6) and derive results for the spectrum of possibilities between the one-period predictability of degree 0 (as in Baron and Ferejohn 1989), in which players receive no information regarding next period's proposer, and perfect one-period predictability, in which next period's proposer is revealed one period in advance. We show that increases in the degree of one-period predictability monotonically increase the first proposer's payoff until one-period predictability of degree q is reached, at which point the proposer captures the entire surplus. In large legislatures (as $n \to \infty$), only the limiting proportional degree of one-period predictability (d/n) and the limiting proportional voting rule (q/n) matter, so even if the degree of predictability trails the size of a winning coalition by a constant number of legislators, the proposer still captures nearly all of the surplus.

Our results are potentially important because they identify features of institutions (pertaining to the predictability or "scrutability" of recognition processes) that tend to concentrate power. For example, they imply that legislative processes in which agenda setters are selected from special committees or small groups of nominees can confer great power to the first proposer. Likewise, "seniority rules" that give priority to senior members can generate extreme inequality by conferring enormous power on the most senior member. Paradoxically, process-oriented reforms that promote "transparency" may exacerbate inequality because inscrutable processes tend to foster more equal sharing of social surplus.

Our analysis also potentially informs the design of institutions by identifying rules and procedures that can counter the effects of predictability. We consider two such possibilities: endowing multiple players with veto power, and allowing proposals to be amended before they are brought to a vote. When members of a set of negotiators each individually has veto power, then even if the recognition process exhibits perfect one-period predictability, each veto player has the ability to delay agreement until she is the proposer. To avoid such delays, the proposer must share some surplus with those players. Similarly, when a proposer must rely on other players to *move* her proposal in settings where they can alternatively propose *amendments*, she can secure their agreement only by sharing surplus. We explore the extent to which such provisions limit the proposer's power.

Related Literature: While legislative bargaining has been studied extensively, the implications of predictability concerning future bargaining power have not been explored systematically. Our model is related to those in which legislators are recognized with unequal (but time-invariant) probabilities, a possibility that is mentioned in the literature.[1] If the i.i.d. recognition probability for legislator

[1] Baron and Ferejohn (1989) present some simple examples with asymmetric recognition probabilities (emphasizing that a lower recognition probability does not necessarily imply a lower expected payoff) but offer no general results. Subsequent papers that study asymmetric recognition probabilities include Banks and Duggan (2000), who establish

i is sufficiently close to unity, then one can show that player i may obtain the entire prize. But such circumstances transparently establish legislator i as a (near) dictator and leave those with low recognition probabilities entirely powerless. In that setting, future bargaining power is in some sense predictable, but only because it is exactly the same as current bargaining power. Our main result differs from this observation because we demonstrate that the first proposer receives the entire prize even if (i) his probability of being selected in any future period is low, (ii) it is predictable that *someone else* (or some other group of individuals) will make the next proposal, and (iii) subsequent proposers are not at all predictable.

More broadly, the defining features of our framework are that the recognition process can be highly non-stationary and history-dependent, individuals learn about the bargaining process over time, and proposals can pass with less than unanimous consent. Merlo and Wilson (1995) consider a rich class of recognition processes, but study bargaining over a stochastic surplus when unanimity is required; unlike us, they focus on whether delays in bargaining are efficient. Simsek and Yildiz (2009) study durable bargaining power in a bilateral setting where the proposer today is very likely to be the proposer tomorrow. Our results imply that the current proposer should capture a large share of the surplus not only when bargaining power is highly durable, but also when it is fleeting, in the sense that the current and recent proposers are not expected to have bargaining power in the future.

Predictability of future bargaining power is naturally connected to the information and beliefs that individuals have about the bargaining process. One strand of work that has examined the role of these beliefs in detail departs from the common prior assumption. Yildiz (2003, 2004) and Ali (2006) study how beliefs about bargaining power can induce delays and influence the outcome of negotiations. When individuals agree to disagree, bargaining power may be predictable from each individual's perspective—for example, each negotiator may believe that others will be recognized with probability 0—but the lack of consensus may prevent the first proposer from capturing the entire surplus. Our results do not require common priors, but merely that enough people are pessimistic about their bargaining power tomorrow: if q or more players believe their recognition probabilities for the next round are less than ϵ, and if this is common knowledge, then the first proposer captures almost the entire surplus.

We follow the existing literature on dynamic multilateral bargaining in assuming that proposers and voters are sophisticated and forward-looking with respect to decisions that affect the division of economic surplus. Dynamic sophistication is at the heart of many results in the literature on legislative bargaining, such as the distinction between the consequences of employing open and closed rules emphasized by Baron and Ferejohn (1989), and is known to have important implications for proposer power. Norman (2002) has previously noted that those who are likely to be excluded from the winning coalition at $t+1$ make attractive coalition partners at time t. Kalandrakis (2004) shows

existence of stationary equilibria in the infinite horizon, Eraslan (2002), who establishes that stationary equilibria payoffs are unique, and Norman (2002), who discusses it as a perturbation that restores uniqueness in finite-horizon bargaining problems.

in a three player majoritarian bargaining game with an evolving status quo that the proposer can capture the entire surplus in each period because each non-proposer anticipates that such inequality creates cheaper coalition partners in the future. Bernheim, Rangel and Rayo (2006) study real-time agenda setting with an evolving status quo, and establish that a mildly predictable recognition process can provide the *last* proposer with dictatorial power.

Our results speak to the potential importance of commitment mechanisms when there are multiple proposers: if a future proposer could commit to an equitable distribution of surplus, others would reject any exploitative offer made by the first proposer, and so instead he too would propose a more equitable outcome.[2] Note the contrast between this implication and that of Diermeier and Fong (2011), in which a single agenda-setter's inability to commit to future proposals limits the surplus she can extract; in our case, it is the inability of future agenda-setters to commit that permits the current agenda-setter to extract surplus.

Outline: Section 2 conveys the intuition for our results through some simple examples. We present our framework in Section 3 and our main results in Section 4. In Section 5, we extend our results to settings with general coalition structures, non-transferable utility, and those in which individuals can influence recognition through costly maneuvers. We develop comparative statics on imperfect predictability in Section 6. Section 7 describes the implications of veto power and open rule negotiations. Section 8 concludes. Omitted proofs are in the Appendix or the Online Appendix.

2 Examples

We begin by illustrating the stark implications of predictable recognition processes through a series of examples. Our starting point is the closed rule divide-a-dollar model of Baron and Ferejohn (1989), in which each of n players (with n odd) has an equal probability of being recognized in each period independently of the past, legislators are equally patient, and legislative approval requires a simple majority.

Example 1: One-Period-Ahead Revelation. In Baron and Ferejohn (1989), the proposer for period $t+1$ is revealed at the beginning of period $t+1$. Suppose instead that this uncertainty is resolved one period before, revealing the identity of the period-$(t+1)$ proposer at the beginning of period t.

First consider a two-period model in which proposals can be made at $t \in \{0,1\}$, and if no agreement is reached by $t=1$, the dollar is destroyed. Suppose Alice and Carol are known to be the proposers at

[2] For example, suppose the proposer at $t=1$, p^1, can commit to an offer to be made at $t=1$ before the proposer at $t=0$, p^0, makes an offer. The legislator p^1 then has a incentive to ensure that she is the most expensive member of a minimal winning coalition at $t=0$. Accordingly, she would commit to an offer providing 0 to $q-1$ players (including the first proposer), $\frac{1}{n-q}-\epsilon$ to herself, and equal shares for the remaining $n-q-1$ players. The legislator p^0 then chooses a policy in which he obtains $q-2$ players for free and proposer p^1 at a cost of $\frac{\delta}{n-q}-\delta\epsilon$. Even if commitment is costly, p^1 clearly benefits from this strategy.

$t = 0$ and $t = 1$ respectively. The unique sub-game perfect equilibrium (henceforth SPE) emerges from backward induction: if agreement is not reached at $t = 0$, Carol proposes at $t = 1$ to keep the entire dollar for herself, and a strict majority votes in favor. Thus, at $t = 0$, every player other than Carol has an expected continuation value of 0 if no agreement is reached immediately. So the unique SPE outcome involves Alice proposing to keep the entire dollar for herself at $t = 0$, and a strict majority voting in favor.

Naturally, the same conclusion applies in longer finite-horizon settings: each proposer is able to capture the entire surplus since everyone other than the next proposer expects to receive 0 if there is delay. For the infinite horizon setting, it is easy to construct an SPE in which the first proposer receives the entire prize: in every period, the selected legislator proposes to keep the prize for himself, and all legislators (except for the next proposer, who is known) vote in favor. Given the continuation equilibrium, rejecting the proposal would simply shift the prize from the current proposer to the next proposer, which is of no benefit to other legislators. Thus, such behavior is sequentially rational for every history.[3]

Because multilateral bargaining games with infinite horizons give rise to folk theorems (see Baron and Ferejohn 1989 and Osborne and Rubinstein 1990), the literature generally focuses on Markov Perfect Equilibria (henceforth MPE). We discuss MPE at length in Section 3.2; in this example, the concept implies that players can condition proposals and voting strategies only on variables that are directly payoff-relevant (rather than indirectly relevant through others' strategies)—i.e., the identities of the current and next proposers, and (when voting) the proposal currently on the table—and not on past proposals or voting decisions.

Our main result implies that the first proposer captures the entire surplus in *all* MPE of the infinite horizon model (and not merely in the particular MPE described above). The following is a sketch of the proof for this special case. Suppose each MPE involves agreement in every period (a claim we prove in Lemma 1). As before, suppose the proposers at $t \in \{0, 1\}$ are known to be Alice and Carol, and towards a contradiction that Alice has to share at least $\epsilon > 0$ to secure the support of Bob, who knows he will not be recognized at $t = 1$. The fact that Bob will not accept a lower offer from Alice implies that there must be some realization of the $t = 2$ proposer for which Carol will offer Bob at least ϵ/δ. But Carol would make such an offer only if she could not find cheaper votes at $t = 1$. It follows that there must be some party (possibly Bob) who, in that same contingency, will not be the proposer for $t = 2$, and yet demands at least ϵ/δ to vote for Carol's proposal at $t = 1$. The fact that such a player would not accept a lower offer from Carol in that contingency implies that there must be some further realization of the $t = 3$ proposer such that the $t = 2$ proposer would offer that player at least ϵ/δ^2. Since the same argument applies at $t = 2, 3, \ldots$, and $\delta < 1$, there must be a contingency under which a proposer eventually offers more than the entire surplus to another player, which is plainly a contradiction.

[3]This equilibrium does not rely on the fortuitous resolution of indifference among voters: by offering members of the winning coalition arbitrarily small shares, the current proposer can break their indifference and secure their support.

Example 2: Nominations. Next suppose that the period-t proposer is selected randomly (with equal probabilities) in period t from a set of n^* nominees, call it N_t^*. The nominees are in turn selected in period $t-1$ from the full set of legislators, and immediately announced. For concreteness, we will assume that nominees are also selected randomly (with equal probabilities), but the particular selection process is in fact irrelevant. Finally suppose that the list of nominees is not too long: $n^* \leq \frac{n-1}{2}$.

Reasoning as in the first example, we see that the following is an equilibrium: in period t, the selected legislator proposes to keep the entire prize for himself, and the proposal passes with the support of legislators belonging to $N \backslash N_{t+1}^*$. Members of that group are willing to vote for the proposal because they understand that rejecting it would simply shift the entire prize from the current proposer to some member of N_{t+1}^*. Because N_{t+1}^* has fewer than $\frac{n-1}{2}$ members, $N \backslash N_{t+1}^*$ has at least $\frac{n+1}{2}$ members, and therefore the proposal passes. The intuition for uniqueness is also essentially identical: the support of those in $N \backslash N_{t+1}^*$ is costly to secure only if the total amount at least one individual expects to be offered in the next round is even greater under some contingency, which recursively implies the existence of some eventual contingency under which a proposer offers a member of the winning coalition more than the total payoff.

This special case has immediate implications for procedural rules that favor legislators based on their seniority or membership in special committees: if proposals must come from a small subset of the entire legislature, the first proposer necessarily captures the entire surplus.

Example 3: Rotation Through Parties. Next suppose that every legislator belongs to one of P parties, where $P \geq 3$. Also assume that, despite these party affiliations, each legislator is concerned only about his own constituents; hence politics remains a zero-sum game. Let n_j (with $\sum_{j=1}^{P} n_j = n$) denote the number of legislators belonging to party j. By convention, list the parties so that $n_1 > n_2 > ... > n_P$. Also assume $n_1 < \frac{n}{2} - 1$, so that no party has a majority.

Consider a recognition rule that cycles through the parties, starting with the largest. For example, a randomly selected member of the largest party makes the first offer, followed by a randomly selected member of the second largest party, and so forth, returning to the largest party if a proposal by the smallest party is rejected. In this setting, the first proposer from the largest party receives the entire prize in any Markov-perfect equilibrium. The logic is essentially the same as for Example 2.

3 The Model

3.1 Environment

Consider a group of players, $\mathcal{N} = \{1, ..., n\}$, who are bargaining over the division of a fixed payoff (normalized to unity); i.e., the policy space is $\mathcal{X} \equiv \{x \in [0,1]^n : \sum_{i \in \mathcal{N}} x_i = 1\}$. Proposals can be made at discrete points of time in $\mathcal{T} \equiv \{t \in \mathbb{N} : t \leq \bar{t}\}$, where $\bar{t} \leq \infty$ is the deadline for bargaining. In each period, a player is recognized to propose an alternative in $\mathcal{X}$. If the group approves the proposal according to the voting rules described below, the game ends and the policy is implemented. If the group rejects the proposal, play proceeds to the next period unless $t = \bar{t}$, in which case the game ends and the policy $(0, \ldots, 0)$ is implemented.

Within period t, events unfold as follows:

1. Information concerning the selection of current and future proposers is revealed, and the proposer for time t, p^t, is determined. The proposer p^t observes this information and makes a proposal.
2. Legislators vote on the proposal.

Details concerning each of these stages follow.

Stage 1: Information and Recognition. The selection of proposer at time t may depend both on random events, as in Baron and Ferejohn (1989), and on institutional rules that constrain the possible sequences of proposers. Formally, consider a canonical probability space $(\Omega, \mathcal{F}, \mu)$ (where Ω is the state space, $\mathcal{F}$ is a σ-algebra, and μ is a probability measure) encompassing all uncertainty pertaining to the recognition process, and let $\omega \in \Omega$ denote the generic state of nature. For every $t \in \mathcal{T}$, define $h_P^t \equiv (p^\tau)_{\tau \in \mathcal{T}, \tau \leq t}$ as the history of proposers, and let H_P^t denote the set of possible proposer histories. The recognition rule is a sequence of functions $\widetilde{P}^t : H_P^{t-1} \times \Omega \to \mathcal{N}$, where $\widetilde{P}^t$ governs the selection of p^t, the proposer in round t. Of course, for a process of that type, the state of nature recursively determines the entire sequence of proposers, and hence we can rewrite the recognition rule more compactly as a sequence of functions $P^t : \Omega \to \mathcal{N}$ that only depend on ω. This formulation is extremely general, in that the proposer in any period can depend on the entire history of past proposers and random events. In Section 5.4, we enrich it further by allowing for the possibility that players can influence recognition through political maneuvers.

In stage 1 of period t, the players commonly observe a signal σ^t, where $\sigma^t(\omega)$ is an $\mathcal{F}$-measurable function. We assume that p^t is fully revealed in period t: for every t and $\omega, \omega' \in \Omega$, $P^t(\omega) \neq P^t(\omega')$ implies that $\sigma^t(\omega) \neq \sigma^t(\omega')$. For each t, we can represent the information structure induced by the stochastic process $(\sigma^\tau)_{\tau \in \mathcal{T}, \tau \leq t}$ as a partition, $\mathcal{S}^t$, of the state space Ω.[4] The partition identifies states of nature that generate exactly the same signals through period t. Formally, $\mathcal{S}^t$ satisfies two requirements: (i) it partitions Ω and therefore $\bigcup_{s^t \in \mathcal{S}^t} s^t = \Omega$; and (ii) for each $s^t \in \mathcal{S}^t$, $\{\omega, \omega'\} \subset s^t$ if and only if $\sigma^\tau(\omega) = \sigma^\tau(\omega')$ for every $\tau \leq t$.

The framework we use to describe uncertainty concerning future proposers and the revelation of pertinent information embeds all natural possibilities, with the restriction that revealed information is never forgotten; in other words, the signal structure generates a sequence of partitions $\{\mathcal{S}^t\}_{t \in \mathcal{T}}$ that are weakly finer over time. For example, the framework encompasses the extreme possibilities that the recognition order is known in advance, and that no information concerning the period-t proposer is revealed prior to period t. Between these extremes, we place no restrictions on the correlation

[4]The signal process also generates a filtration $\mathcal{F}^t \subset \mathcal{F}$; i.e., a series of sub-$\sigma$-algebras for the probability spaces governing residual uncertainty.

structure governing the selection of proposers and the generation of signals.

Stage 2: Voting. The players vote on the proposal in a fixed sequential order.[5] A proposal is implemented if and only if at least q players (including the proposer) vote in favor. A voting rule is *non-unanimous* if $q < n$.

Payoffs: Players evaluate payoffs according to conventional exponential discounting: player i's discount factor is δ_i. No player is perfectly patient and $\hat{\delta} < 1$ denotes an upper-bound on their discount factors. If proposals at every period t in $\mathcal{T}$ are rejected, each player obtains a payoff of 0. If proposal x is implemented at time t, player i's payoff is $u_i(x,t) \equiv \delta_i^t x_i$.

3.2 Solution Concept

It is well-known that when three or more players bargain, every division can be supported as the outcome of a subgame perfect equilibrium (Baron and Ferejohn 1989; Osborne and Rubinstein 1990). The literature generally avoids the implications of this "folk theorem" by restricting attention to equilibria that are stationary or Markov Perfect.[6] Because our model allows for complex history-dependent recognition processes, the state of the game evolves as proposers are recognized and information about future proposers is revealed. Accordingly, the state space is different in each period. We restrict attention to equilibria that prescribe the same continuation strategies at all structurally indistinguishable nodes (i.e., those at which the same information pertinent to the selection of subsequent proposers has been revealed). We refer to such equilibria as *Markov Perfect Equilibria* (henceforth MPE). Our focus on MPE rules out equilibrium strategies for which choices depend on past proposals and votes, inasmuch as those actions have no direct structural implications for the continuation game.[7]

We have two motivations for studying this class of equilibria. First, adopting an appropriate generalization of the solution concept that is widely used in the literature facilitates transparent comparisons with existing results and highlights the implications of predictability. Second, Markovian strategies are the simplest possible form of behavior consistent with equilibrium rationality.[8] Every equilibrium must condition choices on variables that alter structural features of the continuation

[5]Sequential rationality combined with voting in a sequential order implies that for every equilibrium, there exists an outcome-equivalent equilibrium in which each player eliminates weakly dominated strategies at the voting stage and "votes as if pivotal."

[6]The restriction to stationary or Markovian equilibria is sufficiently pervasive in political economy that we hesitate to offer an exhaustive list; several recent studies that impose this restriction with an evolving state variable are Acemoglu, Egorov and Sonin (2010), Battaglini and Coate (2007), Diermeier and Fong (2011), and Lagunoff (2009). Banks and Duggan (2000), Eraslan (2002) and Eraslan and McLennan (2013) show that stationary equilibria exist and are unique in the divide-the-dollar and coalitional bargaining settings with an i.i.d. recognition process.

[7]As is conventional, our restriction is on equilibria, and not strategies. Players have the option to consider non-Markovian deviations, and for a strategy profile to be an MPE, such deviations have to be unprofitable.

[8]In the canonical legislative bargaining environment, Baron and Kalai (1993) prove that the Markovian equilibrium is the simplest equilibrium based on an automaton notion of complexity.

game, and non-Markovian equilibria are more complex because choices *also* depend on variables that have no structural implications for the continuation game. Because non-Markovian equilibria require legislators to follow different continuation strategies in structurally identical circumstances, sustaining any such equilibrium presumably requires more coordination. Yet, because every MPE ends in immediate agreement (as we show in Lemma 1), there is no efficiency motive for selecting a more complex equilibrium. Thus, the complex coordination required for a non-Markovian equilibrium is never in the legislators' mutual interests.

Focusing on the proposal stage of period t, the *structural state* consists of the proposer's identity and all information bearing on the selection of future proposers. In our framework, s^t encapsulates that state in period t; recall in particular that it encodes the identities of all proposers through and including period t, as well as all signals pertaining to the identities of future proposers. For the voting stage of period t, the state consists of (s^t, x^t), where x^t is the period-t proposal.[9] Let $\mathcal{S}_i^t$ be the set of all $s^t \in \mathcal{S}^t$ consistent with player i being the proposer in period t, i.e., in which for every $\omega \in s^t$, $P^t(\omega) = i$. A Markov Perfect Equilibrium is an SPE in which we can write each player's equilibrium strategy as a sequence of functions $\left(\xi_P^{i,t}, \xi_V^{i,t}\right)_{t \in \mathcal{T}}$ such that $\xi_P^{i,t} : \mathcal{S}_i^t \to \Delta\mathcal{X}$ is player i's randomization over proposals when recognized in period t in structural state s^t, and $\xi_V^{i,t} : \mathcal{S}^t \times \mathcal{X} \to \Delta\{\text{yes}, \text{no}\}$ is player i's randomization over whether to vote for or against a policy $x \in \mathcal{X}$ proposed in period t and structural state s^t.

3.3 Some Examples

This framework subsumes numerous examples of recognition processes:

(i) Baron and Ferejohn (1989): Let $\Omega = \mathcal{N}^{|\mathcal{T}|}$, and $P^t(\omega) = \omega_{t+1}$. The signal at time t is $\sigma_t(\omega) = \omega_{t+1}$. The measure μ is the "uniform" measure on Ω.

(ii) *One-period-ahead revelation*: The process is as above, except the signal at time t is $\sigma_t(\omega) = \omega_{t+2}$, so that the identity of the proposer at time $t+1$ is publicly observed prior to the period t proposal.

(iii) *Fixed Order*: The role of proposer rotates through the players in a fixed order. In this case, Ω is degenerate.

(iv) *Nomination*: At most $n - q$ nominees for the period-t proposer are determined randomly in period $t' < t$, and the proposer is then chosen randomly from the nominees in period t. In this case, the signal partially reveals the identity of the period-t proposer in period t', and then fully reveals it in period t.

[9] As is conventional, we ignore the voting history at stage 2 of period t up to the time that player i votes. Although an equilibrium must specify behavior for every voting history, it is well-known that the requirement of subgame perfection selects the same outcome in the sequential voting game that would emerge if players voted simultaneously and sincerely for their preferred alternatives.

4 Bargaining with a Closed Rule

We begin by formally defining predictability and then examine its implications in closed rule negotiations.

4.1 Predictability

In each period, players forecast the distribution of future bargaining power based on all information accumulated up to the present. Suppose that in stage 1 of period t, the sequence of signals $(\sigma^0(\omega), \ldots, \sigma^t(\omega))$ indicate that the underlying state of the world ω is in s^t. Player i is recognized at time $t+1$ if and only if ω is in

$$\widehat{\Omega}_i^{t+1}(s^t) \equiv \left\{\omega \in s^t : P^{t+1}(\omega) = i\right\},$$

which has probability $r_i^{t+1}(s^t) \equiv \mu\left(\widehat{\Omega}_i^{t+1}(s^t) \mid s^t\right)$. The *losers* are those players who have 0 probability of being the proposer at $t+1$ conditional on all that is known at the proposal stage in period t:

$$L^{t+1}(s^t) \equiv \left\{i \in \mathcal{N} : r_i^{t+1}(s^t) = 0\right\}.$$

The cardinality of this set serves as a metric for the predictability of the recognition process.

Definition 1. *The recognition process exhibits* ***one-period predictability of degree*** d *if* $|L^{t+1}(s^t)| \geq d$ *for all* s^t *in* $\mathcal{S}^t$ *and* t *in* $\mathcal{T}$.

One-period predictability of degree d means that by the time the proposer is selected in period t, at least d players have been ruled out as the proposer for period $t+1$. Plainly, one-period predictability of degree d implies the same for any degree $d' < d$. We can classify the examples from Section 3.3 as follows: process (i) (Baron-Ferejohn) does not exhibit one-period predictability of degree d for any strictly positive d, processes (ii) and (iii) exhibit one-period predictability of degree $n-1$, and process (iv) exhibits one-period predictability of degree q. In general, the degree of predictability for a given process depends on the timing of decisions that influence the choice of the period-t proposer, institutional constraints on the order of proposers, and the timing of revelation for random events.

Note that one-period predictability of degree d has no implications for two-period predictability (defined in the analogous way). For example, though process (ii) exhibits one-period predictability of degree $n-1$, it does not exhibit two-period predictability of degree d for any $d > 0$. Our results for this class of models do not require an ability to predict bargaining power in any round but the next.

4.2 Main Result

In this section, we state and prove our main result.

Theorem 1. *Suppose the voting rule is non-unanimous, requiring $q < n$ votes for a proposal to pass. If the recognition process exhibits one-period predictability of degree q, the proposer selected at $t = 0$ captures the entire surplus in every MPE.*

This result illuminates the implications of combining predictability of the recognition process with a non-unanimous voting rule: the current proposer can extract all of the surplus by forming a winning coalition consisting only of those who definitely will not make a proposal in the next period. Perhaps surprisingly, the members of this minimal winning coalition may expect to have bargaining power two periods hence, but in equilibrium, the votes of such individuals are bought cheaply since they expect to obtain no surplus in the next period. Thus, one-period predictability of degree q confers complete power.

Constructing an MPE that delivers this division of surplus is straightforward. Suppose that in each period t, and in each structural state s^t, the proposer p^t offers to share nothing, each player in $L^{t+1}(s^t)$ accepts all offers, and any other players accepts an offer if and only if his share exceeds his discounted continuation value. No proposer or voter will have a strict incentive to deviate from this strategy profile.

Of course, the theorem makes the much stronger claim that all MPE generate this outcome. The proof makes use of two properties of MPE, stated formally (and proven) below as Lemmas 1 and 2: all MPE end in immediate agreement, and the proposer never offers strictly positive surplus to more than $q - 1$ other players (the smallest group needed to achieve a winning coalition). With these results in hand, the logic of the argument is similar to that outlined in Section 2.

Specifically, suppose towards a contradiction that the first (period 0) proposer, player i, does not capture the entire surplus. Let player j be a member of player i's minimal winning coalition to whom i offers (weakly) more than she does to anyone else, and let x_j^0 denote this share. Because she chooses to exclude the other $(n - q)$ players and include player j in her minimal winning coalition, each of the excluded players must be more expensive to buy out; i.e., each has an expected discounted continuation value that weakly exceeds x_j^0. Thus, at least $(n-(q-1))$ players have expected discounted continuation values no less than x_j^0. If the recognition process exhibits one-period predictability of degree q, then at least one person within that set (call her player k) has no chance of being recognized in the next period. Player k necessarily derives all of her continuation value from the payoff she expects to receive when someone else serves as the proposer in period 1. Thus, in some structural state in period 1, the period 1 proposer must offer player k a payoff of at least $x_j^0/\hat{\delta}$.

The same logic, of course, holds for the aforementioned state in period 1. So by induction, there is some period 2 state in which the proposer offers some player a payoff of at least $x_j^0/\hat{\delta}^2$. Iterating this argument, we see that, for every t, there exists a structural state in which the proposer offers another a player a share of at least $x_j^0/\hat{\delta}^t$. Because $\hat{\delta} < 1$, at least one player eventually obtains a share that exceeds the maximal feasible payoff, which is an obvious contradiction.

Necessity: The preceding logic shows that one-period predictability of degree q suffices for the first proposer to capture the entire surplus. Is this condition also necessary? We describe a setting in Section 6 where the first proposer cannot capture the entire surplus if the degree of one-period predictability is strictly less than q. Nevertheless, for that setting, greater predictability confers greater power: the first proposer's share is strictly increasing in the degree of predictability until $d = q$, at which point he captures the entire surplus.

However, as a general matter, one-period predictability of degree q is not the tightest possible condition for ensuring that the first proposer receives the entire surplus. One way to weaken this condition is to employ a "proposer-specific" notion of predictability. Suppose in particular that, in each structural state s^t, one can rule out $q-1$ players *other* than the current proposer, p^t, as the proposer for $t+1$. Then the current proposer captures the entire surplus even if the degree of one-period predictability is less than q. We return to this possibility in Section 5.2.

A Formal Proof of the Main Result: To prove Theorem 1, we first establish that every pure or mixed MPE must yield immediate agreement, and that every equilibrium proposal is directed towards the cheapest minimal winning coalition. For these purposes, it is useful to introduce some additional notation for players' continuation values in an MPE. For every t in $\mathcal{T}$ and every structural state s^t in $\mathcal{S}^t$, let $p^t(s^t)$ denote the proposer in period t. Moreover, for $t < \bar{t}$, let $V_i^{t+1}(s^t)$ denote the expected continuation value of player i at the beginning of period $t+1$ (before Stage 1) after the rejection of an offer in period t and structural state s^t; for the finite-horizon setting ($\bar{t} < \infty$), let $V_i^{\bar{t}+1}(s^{\bar{t}}) \equiv 0$. For a coalition $C \subseteq \mathcal{N} \setminus \{p^t(s^t)\}$, let $W_C^t(s^t) \equiv \sum_{i \in C} \delta_i V_i^{t+1}(s^t)$ represent the sum of discounted continuation values for the coalition. Denote the lowest cost of a coalition of size $q-1$ as

$$\underline{W}^t(s^t) \equiv \min_{\substack{C \subseteq \mathcal{N} \setminus \{p^t(s^t)\}, \\ |C| = q-1}} W_C^t(s^t),$$

the associated set of coalitions that achieve the minimum cost by

$$C^t(s^t) \equiv \left\{ C \subseteq \mathcal{N} \setminus \{p^t(s^t)\} : |C| = q-1 \text{ and } W_C^t(s^t) = \underline{W}^t(s^t) \right\},$$

and the cheapest policies required to secure the support of such coalitions as

$$\mathcal{X}^t(s^t) \equiv \left\{ x \in \mathcal{X} : \exists C \in C^t(s^t) \text{ such that } x_i = \delta_i V_i^{t+1}(s^t) \forall i \in C \text{ and } x_{p^t(s^t)} = 1 - \underline{W}^t(s^t) \right\}.$$

Plainly, every coalition C in $C^t(s^t)$ includes $q-1$ players (other than the proposer) with the weakly lowest discounted continuation value. The set of policies $\mathcal{X}^t(s^t)$ is that which offers discounted continuation values to these cheapest minimal winning coalition partners, 0 to others, and the rest to the proposer $p^t(s^t)$. Observe that the maximum offered to any player other than $p^t(s^t)$ is the same for all proposals in $\mathcal{X}^t(s^t)$: i.e., there exists $\overline{x}^t(s^t)$ such that for every offer $x \in \mathcal{X}^t(s^t)$, $\overline{x}^t(s^t) = \max_{i \neq p^t(s^t)} x_i$.

Lemma 1 (Immediate Agreement)**.** *For every t in $\mathcal{T}$ and structural state s^t in $\mathcal{S}^t$, every MPE proposal offered with strictly positive probability is accepted with probability* 1*.*

Proof. Suppose there is a structural state s^t in $\mathcal{S}^t$ such that an equilibrium proposal offered with strictly positive probability in period t, x', is rejected with strictly positive probability. Select some $x \in \mathcal{X}^t(s^t)$ and let C in $C^t(s^t)$ be an associated minimal winning coalition (excluding the proposer). Define a proposal x^ϵ for small $\epsilon \geq 0$ in which $x_i^\epsilon = x_i + \epsilon$ for every $i \in C$, $x_i^\epsilon = 0$ for every $i \notin C \cup \{p^t(s^t)\}$, and the proposer keeps the remainder. In equilibrium, the proposal x^ϵ is accepted by all members of C with probability 1 if $\epsilon > 0$. Observe that because $\sum_{j \in \mathcal{N}} V_j^{t+1}(s^t) \leq 1$, and $\hat{\delta} < 1$,

$$\sum_{i \in C} \delta_i V_i^{t+1}(s^t) + \delta_{p^t(s^t)} V_{p^t(s^t)}^{t+1}(s^t) \leq \hat{\delta} \sum_{j \in \mathcal{N}} V_j^{t+1}(s^t) < 1. \tag{1}$$

Therefore, for sufficiently small ϵ, the proposer's share of $1 - \sum_{i \in C} \delta_i V_i^{t+1}(s^t) - (q-1)\epsilon$ exceeds her discounted continuation value of $\delta_{p^t(s^t)} V_{p^t(s^t)}^{t+1}(s^t)$. Conditional on the equilibrium proposal x' being rejected, the proposer is strictly better off deviating to x^ϵ for sufficiently small $\epsilon > 0$. Conditional on the equilibrium proposal x' being accepted, the proposer's share can be no greater than that she obtains when offering x (otherwise a winning coalition would not support it). Since proposal x' is rejected with strictly positive probability, she is strictly better off offering x^ϵ for sufficiently small $\epsilon > 0$. Therefore, no equilibrium offer is rejected with strictly positive probability. □

Lemma 2 (Minimal Winning Coalition)**.** *For every t in $\mathcal{T}$ and structural state s^t in $\mathcal{S}^t$, every MPE proposal offered with positive probability provides positive payoffs only to members of the cheapest minimal winning coalition: $x \in \mathcal{X}$ is an MPE proposal in s^t only if $x \in \mathcal{X}^t(s^t)$.*

Proof. Any proposal in which the proposer shares less than $\underline{W}^t(s^t)$ with others is rejected with probability 1, and so Lemma 1 rules out such MPE proposals. If the proposer shares strictly more than $\underline{W}^t(s^t)$ with others, deviating to the proposal x^ϵ defined in the proof of Lemma 1 is strictly profitable for sufficiently small $\epsilon > 0$. □

Proof of Theorem 1. Let the structural state in Stage 1 of period 0 be s^0, and consider $\overline{x}^0(s^0)$, the highest equilibrium share that the proposer $p^0(s^0)$ offers to any player other than herself with strictly positive probability. Suppose towards a contradiction that $\overline{x}^0(s^0) > 0$. By Lemmas 1-2, we know that every MPE offer is made to a minimal winning coalition and accepted. Consider the set of players whose support cannot be secured for shares less than $\overline{x}^0(s^0)$:

$$H^0(s^0) \equiv \left\{ i \in \mathcal{N} \backslash \{p^0(s^0)\} : \delta_i V_i^1(s^0) \geq \overline{x}^0(s^0) \right\}.$$

$H^0(s^0)$ must have cardinality of at least $n - (q-1)$, because otherwise proposer $p^0(s^0)$ could form a cheaper coalition without having to offer $\overline{x}^0(s^0)$ to any player. Since the recognition process exhibits one-period predictability of degree q, $H^0(s^0) \cap L^1(s^0)$ is non-empty. Consider a generic player i in $H^0(s^0) \cap L^1(s^0)$: player i definitely will not be the proposer in the next period, and his continuation value must therefore reflect an offer he receives. So there exists some structural state $s^1 \in \mathcal{S}^1$ such that the associated proposer offers player i at least $\frac{\overline{x}^0(s^0)}{\delta_i} \geq \frac{\overline{x}^0(s^0)}{\hat{\delta}}$ with strictly positive probability. Therefore, the highest share offered by that proposer to another player, $\overline{x}^1(s^1)$, must be no less than $\frac{\overline{x}^0(s^0)}{\hat{\delta}}$.

The same logic applies in period 1 and in the structural state s^1. So by induction, there exists a sequence of states $\{s^t\}_{t \in \mathcal{T}}$ such that for each t, $s^t \in \mathcal{S}^t$, and $\overline{x}^t(s^t) \geq \frac{\overline{x}^0(s^0)}{\hat{\delta}^t}$. If $\overline{t} = \infty$, $\hat{\delta} < 1$ implies that $\overline{x}^t(s^t)$ eventually exceeds 1; if $\overline{t} < \infty$, the same argument implies that the proposer in the final round offers a strictly positive share to another player. In both cases, we have reached a contradiction. □

5 Extensions

In this section, we consider several extensions of our framework, pointing out that (i) the proposer captures nearly all of the surplus if he can "almost" rule out q other players; (ii) our analysis can be generalized to encompass more general coalitional structures; (iii) the proposer captures the entire surplus even if payoffs are not transferrable and more equitable distributions generate greater aggregate surplus; and (iv) our findings generalize to settings in which the recognition of proposers depends not only on procedural rules and random events, but also on political maneuvering.

5.1 Robust Predictability

A feature of our main result is that it requires a degree of certainty (concerning the identities of individuals who will not make the next proposal). In this section, we show that this strict requirement is qualitatively inessential: if q players are recognized with probability of at most $\epsilon > 0$ tomorrow, the first proposer's does not offer more than $\frac{\hat{\delta}\epsilon}{1-\hat{\delta}(1-\epsilon)}$ to any player. Formally, for every $\epsilon \in [0, 1)$, we define the set of *almost losers*:

$$L_\epsilon^{t+1}(s^t) \equiv \left\{ i \in \mathcal{N} : r_i^{t+1}(s^t) \leq \epsilon \right\}.$$

In other words, based on all available information available at the period t proposal stage, the probability that any player in $L_\epsilon^{t+1}(s^t)$ will be recognized as the period $t+1$ proposer is at most ϵ. The cardinality of this set determines the degree of ϵ-predictability.

Definition 2. *The recognition process exhibits* ***one-period*** ϵ***-predictability of degree*** **d** *if* $|L_\epsilon^{t+1}(s^t)| \geq d$ *for all* s^t *in* $\mathcal{S}^t$ *and* t *in* $\mathcal{T}$.

Theorem 2. *Suppose the voting rule is non-unanimous, requiring $q < n$ votes for a proposal to pass. If the recognition process exhibits one-period ϵ-predictability of degree q, then in every MPE, the proposer selected at $t = 0$ does not offer more than $\frac{\hat{\delta}\epsilon}{1-\hat{\delta}(1-\epsilon)}$ to any other player.*

Thus, if there are sufficiently many players who are unlikely to be recognized, the first proposer captures almost the entire surplus in every MPE. The logic of the argument closely resembles that given for Theorem 1: we show that if the proposer at $t = 0$ offers any player a share that exceeds $\frac{\hat{\delta}\epsilon}{1-\hat{\delta}(1-\epsilon)}$, then there must exist some future state in which the proposer offers at least one player more than the entire surplus. In contrast to the proof of Theorem 1, the maximum amount offered to an almost loser does not increase geometrically; however, when initialized at a level exceeding $\frac{\hat{\delta}\epsilon}{1-\hat{\delta}(1-\epsilon)}$, it grows according to an "expansive" mapping that upon repeated iteration escapes the feasible set.

5.2 General Coalition Structures

Our main result generalizes to settings with more complex coalitional structures, as studied in the context of multilateral bargaining by Banks and Duggan (2000). We will say that a coalition of players is *decisive* if approval of an offer by all members of the coalition results in its implementation. Let $\mathcal{D} \subset 2^{\mathcal{N}}$ denote the set of *decisive coalitions*. As is conventional, we assume that $\mathcal{D}$ satisfies *monotonicity*: if D is decisive and $D \subseteq D'$, then D' is decisive. This general structure provides tremendous flexibility for modeling coalitional power; e.g., it encompasses settings in which players have unequal voting weights, as well as those with individual or coalitional veto power.[10]

We generalize our notion of one-period predictability as follows:

Definition 3. *The recognition process exhibits* ***one-period decisive predictability*** *if for all s^t in $\mathcal{S}^t$ and t in $\mathcal{T}$, there exists a decisive coalition D in $\mathcal{D}$ such that:*

(a) *D includes the proposer at time t, $p^t(s^t)$, and*

(b) *every other player in D definitely will not be recognized at $t+1$, i.e., $D \backslash \{p^t(s^t)\} \subseteq L^{t+1}(s^t)$.*

For the special class of anonymous aggregation rules—i.e. D is in $\mathcal{D}$ if and only if $|D| \geq q$—one-period decisive predictability actually *weakens* one-period predictability of degree q, insofar as it requires only that $q-1$ players *other than the current proposer* definitely will not be the next proposer. The vital implication of predictability is that the current proposer can form a decisive coalition with players who definitely will not be recognized in the next period; for Theorem 1, we invoked predictability of degree q (rather than $q-1$) so as to avoid stating a condition that depends on the identity of the proposer. The following theorem generalizes our earlier result:

Theorem 3. *If the recognition process exhibits one-period decisive predictability, the proposer selected at $t = 0$ captures the entire surplus in every MPE.*

While the logic is similar to that of Theorem 1, the argument has to address the constraints a proposer faces in choosing a decisive coalition. In particular, it is no longer the case that all those excluded from an equilibrium coalition necessarily have lower discounted continuation value than those who are included. Instead, the argument proceeds by analyzing the cost of each coalition. We establish that coalitions formed with players who definitely will not be the next proposer must have zero cost; otherwise, there is some state in which a proposer offers more than is feasible.

[10] In the related literature on coalitional bargaining (Chatterjee, Dutta, Ray and Sengupta 1993; Ray 2007), each coalition is assigned an aggregate value that it can achieve irrespective of whether others oppose it. The framework that we adopt here implicitly assigns coalitions in $\mathcal{D}$ a value of unity and all other coalitions a value of zero.

5.3 Beyond Pure Distribution: Non-Transferable Utility

The implications of predictable recognition processes extend to environments with non-transferable utility. Suppose the policy space is $\mathcal{X}$, and player i's stage payoff from policy x is $u_i(x_i)$ where, for each i, $u_i(\cdot)$ is strictly increasing, continuous, and concave, with $u_i(0) = 0$. Player i's discount factor is δ_i, and perpetual disagreement yields a payoff of 0. The following analog of Theorem 1 holds in this setting:

Theorem 4. *If the recognition process exhibits one-period predictability of degree q, the proposer selected at $t = 0$, player i, obtains a payoff of $u_i(1)$ in every MPE.*

Two interesting implications follow from Theorem 4. First, heterogeneity in risk-aversion, as captured by the concavity of $u_i(\cdot)$, may be less important in negotiations when bargaining power is predictable and unanimity is not required.[11] Second, even in settings where equality promotes aggregate efficiency (e.g., because all utility functions are identical and strictly concave), a high degree of inequality still prevails. Thus, predictability of the recognition process exacerbates both inequity and inefficiency.

5.4 Political Maneuvers

Our results extend seamlessly to environments in which players can maneuver for bargaining power or otherwise influence the selection of future proposers. Suppose that in each period t, prior to the arrival of information and the selection of a proposer, each player i (potentially including a Chair, denoted $i = 0$, in addition to the negotiators) chooses a (potentially) costly maneuver m_i^t from some set M_i, and that the entire history of maneuvers up to that point (in addition to past random shocks and proposers) influences recognition in period t.

In this setting, it is useful to distinguish between two forms of predictability. The first is *unconditional predictability*, defined as follows: at the end of period t, it is possible to rule out a fixed set of q players as the next proposer irrespective of period $t+1$ maneuvers. Theorem 1 applies to this setting with only slight modification. A weaker notion is that of *conditional predictability*, defined as follows: at the end of period t, it is possible to rule out some set of q players as the next proposer for each profile of period $t+1$ maneuvers (where the set may depend on the maneuver profile). Under that condition, our main result follows for all pure strategy MPE: because players can accurately predict future maneuvers in any such equilibrium, the period-t proposer can still form a winning coalition with players who will definitely not be the next proposer. Since the logic of these arguments mirror that of Theorem 1, we relegate formalizations to the Online Appendix.

As an application, suppose a Chair is endowed with the power to choose the proposer at the outset of each period from a set of eligible candidates (which may be history-dependent). Because the recognition process satisfies conditional predictability of degree $n - 1 \geq q$, the first proposer

[11] Our analysis thus complements previous work on the implications of risk-aversion in bilateral bargaining problems wherein recognition is perfectly predictable (Roth 1985; Binmore, Rubinstein and Wolinsky 1986).

captures the entire surplus in every pure strategy MPE. While randomization on the part of the Chair could overturn this conclusion, a deterministic choice is more intuitively compelling when the Chair has favorites among the negotiators; i.e., for every policy x and pair i and j such that $x_i \neq x_j$, the Chair has strict preferences between x, and the policy $x_{i\leftrightarrow j}$ that exchanges the allocations for i and j. Thus, a Chair who fails to project inscrutability may have to choose between highly unequal distributions of surplus even if she favors equality.[12]

6 Imperfect Predictability

This section examines a tractable subclass of the environments subsumed by our framework, with the object of illuminating the spectrum of possibilities between one-period predictability of degree zero, as in Baron and Ferejohn (1989), and of degree q, as in Theorem 1. We show that increases in the degree of predictability increase the expected share captured by the first proposer.

As in Baron and Ferejohn (1989), suppose that proposer recognition is governed by an i.i.d. process, and that in every period, each player has an equal chance $\left(\frac{1}{n}\right)$ of being recognized. Suppose that players receive a potentially informative signal about the period-t proposer in period $t-1$. We represent the signal by the posterior beliefs it induces: for a fixed vector of probabilities λ (such that $\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n$) governing the selection of the period-t proposer, the signal reveals which player i is assigned to which probability λ_j.[13] For simplicity, we suppose that all players are equally patient, with a common discount factor of δ. The Baron and Ferejohn (1989) framework corresponds to the special case where $\lambda_1 = \ldots = \lambda_n = \frac{1}{n}$, which exhibits one-period predictability of degree 0. Example 1 in Section 2 ("One-Period-Ahead Revelation") corresponds to $0 = \lambda_1 = \ldots = \lambda_{n-1} < \lambda_n = 1$, which exhibits one-period predictability of degree $n-1$.

We characterize the MPE for this model as follows. Let w_i denote the continuation value for a player who will be selected as the next proposer with probability λ_i. In equilibrium, proposers will assemble the cheapest possible minimal winning coalitions by including the players who are least likely to be recognized tomorrow. Thus, we obtain the following recursive formula for the continuation value:

$$w_i = \underbrace{\lambda_i \left(\frac{n-q+1}{n} \left(1 - \delta \sum_{j=1}^{q-1} w_j \right) + \frac{1}{n} \sum_{k=1}^{q-1} (1 - \delta \sum_{j=1, j\neq k}^{q} w_j) \right)}_{\text{Expected Proposer Surplus}} + \underbrace{(1-\lambda_i)\delta \left(\sum_{j=1}^{q-1} \frac{w_j}{n} + \frac{(q-1)w_q}{n(n-1)} \right)}_{\text{Included in MWC}}.$$

The first term in this expression represents the player's continuation value conditional upon being recognized as the proposer. It encompasses two distinct possibilities: either she is among the $n-q+1$ players most likely to be recognized, in which case she purchases the cheapest $q-1$ votes (i.e., those of

[12] Even if the negotiators are not entirely sure of the Chair's preferences, similar conclusions would follow provided they can confidently rule out a sufficient number of possibilities.

[13] It is straightforward to formulate a signal space with $n!$ possible realizations that induces this family of posterior beliefs.

the players least likely to be recognized), or she is among the $q-1$ players least likely to be recognized, in which case she purchases the $q-1$ cheapest votes other than her own. The second term represents the player's continuation value conditional upon another player being recognized as the proposer. It reflects the same considerations: the player assigned the recognition probability λ_q is included in the minimal winning coalition if and only if the current proposer is assigned a weakly lower probability. This recursive formulation generates n linear equations with n unknowns, and consequently has a unique solution.

We use this approach to derive the closed-form solution of the three player game.

Example 1. *Suppose there are three players who make decisions based on simple majority rule. As $\delta \to 1$, the expected proposer surplus converges to*

$$PS(\lambda_1, \lambda_2) \equiv \frac{2}{3}(1 - w_1) + \frac{1}{3}(1 - w_2) = \frac{2\lambda_1 + \lambda_2 + 3}{6\lambda_1 + 3\lambda_2 + 3}.$$

When $\lambda_1 = \lambda_2 = \frac{1}{3}$, the preceding term is $\frac{2}{3}$, which coincides with the solution in Baron and Ferejohn (1989). Greater predictability monotonically increases the proposer's share: if $\lambda_1 \geq \lambda_1'$ and $\lambda_2 \geq \lambda_2'$, then $PS(\lambda_1, \lambda_2) < PS(\lambda_1', \lambda_2')$. Thus, an increases in predictability results in the proposer capturing a larger share of the surplus.

The proposer's ability to capture rents also depends on the relative power of the other players. Holding fixed λ_3, the first proposer's share increases along with the size of the disparity between λ_1 and λ_2: using $\Delta \equiv \lambda_2 - \lambda_1$, the first proposer's share can be re-written as

$$PS = \frac{9 - 3\lambda_3 - \Delta}{15 - 9\lambda_3 - 3\Delta},$$

which is increasing in both λ_3 and Δ. Intuitively, greater inequality in predicted bargaining power decreases the cost of buying the vote of the weakest coalition partner.

We can use this approach to investigate the implications of one-period predictability of degree d for $d < q$, in the special case where d players learn one period in advance that they definitely will not the next proposer, while the remaining $n-d$ players learn that they are equally likely to be the next proposer; formally, $\lambda_1 = \ldots = \lambda_d = 0$ and $\lambda_{d+1} = \ldots = \lambda_n = \frac{1}{n-d}$.

In this setting, we can write continuation values as $\underline{w}$ for players $1, \ldots, d$ and $\overline{w}$ for players $d+1, \ldots, n$. To solve for these values, we compute $\underline{w}$ recursively and then make use of the fact that all continuation values must sum to 1 (since there is no delay). Relegating the algebra to the Online Appendix, we find that

$$\overline{w} = \frac{(n-1)(n-\delta d)}{n((n-1)(n-d) - \delta d(n-q))}, \text{ and}$$
$$\underline{w} = \frac{\delta(n(q-(d+1)) + d)}{n((n-1)(n-d) - \delta d(n-q))}.$$

The above terms are strictly positive for non-unanimous rules ($q < n$) when the degree of predictability d is strictly less than q. The first proposer's expected share is

$$1 - \delta + \delta(n - q)\overline{w} + \delta/n.$$

Using this solution, we can determine the effect of d, the number of players who definitely will not make the next offer, on the proposer's payoff for the case of $d \le q$:

Theorem 5. *Suppose the voting rule is non-unanimous, and the recognition process exhibits one-period predictability of degree $d \le q$. For every $\delta > 0$, the share obtained by the first proposer is strictly increasing in d.*

Thus, the conceptual message of Theorem 1 generalizes beyond the case of $d \ge q$: when $d \le q$, greater one-period predictability (measured according to the degree d) implies greater proposer power.

Our second result characterizes the proposer's (approximate) share in large legislatures. Consider a sequence of games $(G_n)_{n=3,4,...}$ such that game G_n has n players, requires q_n votes for approval of a proposal, and exhibits one-period predictability of degree d_n. We say that the sequence is *convergent* if there exists α_v and α_p such that $q_n/n \to \alpha_v$ and $d_n/n \to \alpha_p$. Our next result identifies the proposer's limiting share in a convergent sequence of games.

Theorem 6. *Consider a convergent sequence of games $(G_n)_{n=3}^{\infty}$ in which α_v is the limiting proportional voting rule and α_p is the limiting proportional degree of one-period predictability. For every $\epsilon > 0$, there exists $\overline{n}$ such that if $n > \overline{n}$, the share of the surplus captured by the first proposer is within ϵ of $1 - \frac{\delta(\alpha_v - \alpha_p)}{1 - \alpha_p(1 + \delta(1 - \alpha_v))}$ if $\alpha_p \le \alpha_v$, and 1 otherwise.*

This expression shows how one-period predictability of a less-than-decisive degree influences the first proposer's share in the limit. For $\alpha_p = 0$, the proposer's share corresponds to that found by Baron and Ferejohn (1989). Increases in the limiting degree of one-period predictability (as measured by α_p) improve the outcome for the first proposer, consistent with the conceptual message of Theorem 5. Moreover, for $\alpha_p < \alpha_v$, the proposer's share is a *convex* function of α_p.[14]

Two additional implications of Theorem 6 merit emphasis. First, even if the votes required for passage (q_n) exceed the degree of one-period predictability (d_n), the proposer's share will converge to unity as the legislature becomes arbitrarily large provided the difference between q_n and d_n remains bounded (because then $\frac{q_n}{n} - \frac{d_n}{n}$ converges to zero). Second, as the voting rule converges to unanimity, the first proposer's limiting share converges to $1 - \delta$ irrespective of the degree of one-period predictability. Thus, we see once again that the source of the proposer's power is the *combination* of a predictable recognition process and an ability to exclude some players from a minimal winning coalition.

[14] The second derivative of the proposer's share with respect to α_p is $\frac{2\delta(1-\alpha_v)(1-\delta\alpha_v)(1+\delta(1-\alpha_v))}{(1-\alpha_p(1+\delta(1-\alpha_v)))^3} > 0$.

7 Institutions That Counter Predictability

In this section, we discuss how offering players opportunities to veto or amend proposals can counter the tendency for multilateral negotiations to generate inequitable outcomes when future bargaining power is highly predictable.

7.1 Vetoes

Veto rights can play a critical role in moderating a proposer's power, and when the recognition process exhibits predictability, the difference in outcomes for those with and without veto power is more stark. Because a player with veto power can delay agreement for multiple periods, she can hold out for a strictly positive share of the total surplus even if she definitely will not be the next proposer.

Suppose that passage of a proposal requires the support of players $1, \ldots, k$ and at least $q - k$ of the remaining $n - k$ players, where necessarily $k \leq q < n$. This is a special case of the coalitional framework studied in Section 5.2, but instead of focusing on one-period decisive predictability, we examine the implications of one-period predictability of degree q (which is weaker).

Theorem 7. *If the recognition process exhibits one-period predictability of degree q, then in every MPE, the proposer selected at $t = 0$ shares the surplus only with veto players.*

A player without veto power obtains a strictly positive payoff *only* if she is the first proposer. The contrast between the shares obtained by veto and non-veto players is more stark than for completely unpredictable recognition processes; in the latter settings, some non-veto players may receive positive shares even when another player is the proposer.

The amount that veto players extract in equilibrium depends on the recognition process. To illustrate their power, suppose that all players are equally patient and consider the "One-Period-Ahead Revelation" recognition process described in Section 2: in each period $t \geq 1$, each player has a $1/n$ chance of being the proposer, but uncertainty is fully resolved in the preceding period (which implies one-period predictability of degree $n - 1$). In the MPE of this model, the proposer today must share surplus with all veto players (because they can block agreement today), and must share a larger fraction of the surplus with any veto player who is identified as tomorrow's proposer (because he has an even greater incentive to block agreement today); in contrast, the proposer today does not need to share surplus with any non-veto player who is identified as tomorrow's proposer, since he can exclude that player from the minimal winning coalition. With these considerations in mind, we derive the recursive equations and solutions for the continuation values of a veto player identified as the proposer at $t+1$ (w_P), and a veto player not identified as the proposer at $t+1$ (w_V):

$$w_P = \frac{k-1}{n}(1-\delta(k-2)w_V - \delta w_P) + \frac{n-k+1}{n}(1-\delta(k-1)w_V) = \frac{n(1-\delta)+\delta}{n(1-\delta)+\delta k},$$
$$w_V = \delta\left(\frac{w_P + (n-1)w_V}{n}\right) = \frac{\delta}{n(1-\delta)+\delta k}.$$

The first equation, which describes the continuation value of a veto player who is identified as tomorrow's proposer, encompasses two distinct possibilities, defined according to whether he learns that another veto player is the proposer at $t+2$ (which occurs with probability $(k-1)/n$). The second equation, which describes the continuation value of a veto player who is definitely not the proposer at $t+1$, encompasses the possibility that he is identified as the proposer at $t+2$ (which occurs with probability $1/n$), and that he is not.

Thus, the proposer at $t=0$ shares 0 with each non-veto player, δw_V with each veto player other than the proposer for $t=1$, and δw_P with the veto player who will be the proposer at $t=1$ (if one exists). As $\delta \to 1$, all veto players split the surplus equally as both w_P and w_V converge to $\frac{1}{k}$.[15]

7.2 Amendments

So far we have analyzed a "closed rule" setting in which there is no opportunity to amend a proposal before voting on it. In contrast, many legislatures employ "open rule" procedures that allow for amendments and require a motion to bring any (possibly amended) proposal to a vote. As we explain in this section, such procedures weaken the first proposer's ability to capitalize on a predictable recognition process, but only to a limited degree.

We model open-rule bargaining by generalizing the framework of Baron and Ferejohn (1989). For simplicity, we take the number of legislators to be odd and assume they employ simple majority rule. At the beginning of period 0, the first proposer p^0 names a policy x in $\mathcal{X}$. A slate of k distinct amenders $A^0(p^0) = (a_1^0, \ldots, a_k^0)$ is then drawn at random (with equal probabilities) from $\mathcal{N}\backslash\{p^0\}$. First a_1^0 chooses whether to offer an amendment or move the proposal. To offer an amendment, a_1^0 names an alternative policy x' in $\mathcal{X}\backslash\{x\}$. The legislature then votes between x and x'. Round 0 ends and round 1 begins, with the winning policy (either x or x') serving as the proposal on the table. A new list of amenders ($A^1(p^0)$ or $A^1(a_i^0)$) is chosen, and the process starts over. If instead a_1^0 moves the proposal, a_2^0 is recognized, and must likewise either offer an amendment or join the pending motion. As long as amenders join the motion, the process moves sequentially through $A^0(p^0)$. If every amender joins the motion, then the policy x is put to a vote. Should a strict majority vote in favor, the policy is implemented; otherwise, round 0 ends and round 1 begins with the random selection of a new proposer p^1, as well as amenders $A^1(p^1)$. Players discount payoffs across (but

[15] As $\delta \to 1$, predicted shares are the same regardless of whether the recognition process is predictable: veto players in Baron and Ferejohn (1989) also collectively appropriate the entire surplus whenever players are perfectly patient. However, in contrast to our setting, non-veto players obtain some share of the surplus from the proposer whenever $\delta < 1$.

not within) rounds at a common rate ($\delta < 1$), and consequently incur the costs of delay whenever a proposal is amended or rejected.

Baron and Ferejohn (1989) study a special case of this open-rule procedure in which the slate of amenders consists of a single individual ($k = 1$), and the amender in period t also serves as the proposer in period t if there is no proposal on the table. We consider, as they do, a symmetric recognition process in which each player has the same probability of becoming a proposer, and conditional on the choice of proposer, each list of amenders is drawn from the remaining players with uniform probabilities. Because our objective is to determine whether an amendment process counters the effects of a predictable recognition process, we depart from Baron and Ferejohn (1989) by assuming that the bargaining process satisfies *perfect one-period predictability*, defined as follows: in each period t, players know the identities of the proposer in period $t+1$ (who becomes active only if proposal in period t is moved and then rejected) and the set of amenders in period $t+1$ for each possible contingency. The following result characterizes the extent to which an open rule moderates the tendencies identified in Theorem 1.

Theorem 8. *Suppose the recognition process for open-rule bargaining exhibits perfect one-period predictability. Then there exists a pure strategy MPE that reaches the following agreement without delay: the first proposer offers $\frac{\delta}{1+\delta k}$ to each amender and 0 to every other player, keeping $\frac{1}{1+\delta k}$ for herself.*

Thus, when bargaining power is predictable, an open rule promotes greater equity than a closed rule.[16] The open-rule process ensures that a proposer must share surplus with those who can offer amendments. The special case of a single amender ($k = 1$), studied by Baron and Ferejohn (1989), deserves emphasis, in that the proposer offers $\frac{\delta}{1+\delta}$ to the amender and keeps $\frac{1}{1+\delta}$ for himself. Interestingly, this outcome coincides exactly with the result of two-player bargaining in Rubinstein (1982).

8 Concluding Remarks

In practice, bargaining power flows from a variety of sources. Often future bargaining power is predictable, at least to some extent. The central observation motivating our analysis is that such predictability can dramatically influence the outcomes of multilateral negotiations when passage of a proposal does not require unanimous consent. Predictability of future bargaining power becomes a critical source of current power, one that can dominate the effects of heterogeneity in patience, risk-aversion, or voting weights. Predictability need not be perfect to influence negotiations. On the contrary, a modest degree of predictability ensures that the first proposer receives the entire surplus,

[16]Through an argument that involves comparing the best and worst possible equilibrium payoffs for a proposer, one can show that the MPE outcome is unique among all those in which the proposer shares surplus only with amenders. We conjecture that this is the unique MPE outcome more generally, but have not yet proven that every MPE outcome offers 0 to those not in $p^0 \cup A^0(p^0)$.

and below that threshold greater predictability implies a larger share for the proposer. Thus, our theory yields implications that are both testable and useful for understanding why certain groups divide resources less equally than others. Our results also offer insight into institutional design; for example, they explains how veto rights and amendment processes can limit the power of the first proposer and thereby promote more equitable outcomes.

As in many models of coalition formation and multilateral bargaining, all information is public. Our results nevertheless extend to settings in which there is some private information but sufficient common knowledge. For example, suppose it is common knowledge that each of at least q players privately learns that he will not be the next proposer. Then there exists an MPE in which each proposer offers to share nothing, and each of the aforementioned players vote in favor. More generally, when individuals privately learn about the bargaining process, each has a motive to signal information through his bargaining posture; however, each player has an incentive not to appear too greedy or powerful lest he be excluded from the minimal winning coalition. The signaling motives that may emerge when individuals are privately informed, and that complicate the process of competition in coalition formation, are intricate and important subjects for further study.

Throughout our analysis, we have assumed that recognition probabilities depend only on the history of random events and proposers (plus the history of political maneuvers in Section 5), but not on past proposals and voting decisions. If the latter choices directly affected the subsequent recognition process, they would become part of the (structural) state variable; Markov strategies could then condition subsequent choices upon them, and a folk theorem would be difficult to avoid. A tractable variation of the bargaining protocol that avoids these issues involves the notion of "rejector-friendliness" (Chatterjee, Dutta, Ray and Sengupta 1993; Ray 2007): the first rejector of a proposal (according to the sequential voting order) in period t is recognized with probability μ in the next period, whereas all others are recognized with probability $\frac{1-\mu}{n-1}$. The probability μ is meant to capture "rejector-power," which is maximized at $\mu = 1$ and minimized at $\mu = 0$. Ray (2007) exhibits an interesting example in which the first proposer can capture the entire surplus when $\mu = 0$ and unanimous consent is required. Although our setting is quite different—in our model, bargaining power is non-stationary and independent of prior voting decisions, rather than the other way around—there appears to be an intriguing connection, which we hope to explore in future work.[17]

An important assumption made throughout our analysis is that each individual is indifferent between all outcomes for which she receives the same share, irrespective of how the residual share is distributed among other negotiators. If externalities are present, one can reformulate policies as points in utility space and proceed as in Section 5.3. However, in some instances natural restrictions on the policy space will defeat the logic of Theorem 4. To illustrate, suppose the parties are negotiating over the level of a public good, and no side-payments are possible. In that case, the policy space is one-dimensional, and it is natural to assume that each player has single-peaked preferences, so that a Condorcet winner exists. With majority rule and standard (unpredictable) recognition processes, the

[17] We thank Debraj Ray for drawing our attention to rejector-friendliness and its connection to our work.

negotiated outcome cannot stray far from the Condorcet winner as players become patient (Jackson and Moselle 2002; Cho and Duggan 2009). We conjecture that a similar result holds even with predictable recognition processes. As a general matter, when the negotiators have preferences that are more congruent than is the case for the settings studied herein, they may naturally coordinate so as to block the first proposer from exploiting power that would otherwise flow from an ability to predict future bargaining strengths.

A Appendix

Proof of Theorem 2 on p. 16. We first describe the function that we use as a lower bound on the amount a proposer must share with at least one other party. Consider the function $f : \mathfrak{R} \to \mathfrak{R}$ defined by $f(y) \equiv \frac{y - \hat{\delta}\epsilon}{\hat{\delta}(1-\epsilon)}$. Observe that f has a unique fixed point, namely $\hat{y} = \frac{\hat{\delta}\epsilon}{1-\hat{\delta}(1-\epsilon)}$. The function f is both strictly increasing and *expansive*: for each $y > \hat{y}$, an induction argument establishes that

$$f^k(y) - \hat{y} = \left(\frac{1}{\hat{\delta}(1-\epsilon)}\right)^k (y - \hat{y}).$$

Since $\hat{\delta}(1-\epsilon) < 1$, it follows that for each $y > \hat{y}$, there exists a finite $\overline{k}$ such that for every $k > \overline{k}$, $f^k(y) > 1$. We use this observation to prove this result.

Let the structural state in Stage 1 of period 0 be s^0 and consider $\overline{x}^0(s^0)$, the highest equilibrium share that the proposer $p^0(s^0)$ offers to any player other than herself. Suppose towards a contradiction that $\overline{x}^0(s^0) > \frac{\hat{\delta}\epsilon}{1-\hat{\delta}(1-\epsilon)}$. Exactly as by the argument of Theorem 1, if the recognition process exhibits one-period ϵ-predictability of degree q, there must exist some player i in $H^0(s^0) \bigcap L^1_\epsilon(s^0)$. Player i's continuation value $V^1_i(s^0)$ emerges from the ϵ probability that he is recognized and the rents he then captures, and the amounts offered to him if someone else is recognized. Since player i's payoff from being the proposer is bounded above by 1, there must exist some structural state $s^1 \in \mathcal{S}^1$ in which player i is not the proposer and his expected share $\tilde{x}^1_i(s^1)$ satisfies

$$V^1_i(s^0) \leq \epsilon + (1-\epsilon)\tilde{x}^1_i(s^1).$$

Because the greatest share offered to any non-proposer, $\overline{x}^1(s^1)$ must exceed $\tilde{x}^1_i(s^1)$, and player i's discounted continuation value in state s^0 weakly exceeds $\overline{x}^0(s^0)$, it follows that

$$\frac{\overline{x}^0(s^0)}{\hat{\delta}} \leq V^1_i(s^0) \leq \epsilon + (1-\epsilon)\overline{x}^1(s^1)$$

or re-arranging that $\overline{x}^1(s^1) \geq f\left(\overline{x}^0(s^0)\right)$. Since f is strictly increasing and expansive, we are guaranteed that $f\left(\overline{x}^0(s^0)\right) > \overline{x}^0(s^0)$, which is greater than $\frac{\hat{\delta}\epsilon}{1-\hat{\delta}(1-\epsilon)}$. Therefore, the same argument applies in state s^1. Accordingly, there exists a sequence of states $\{s^t\}_{t\in\mathcal{T}}$ such that for each t, we have $\overline{x}^t(s^t) \geq f^t\left(\overline{x}^0(s^0)\right)$, and $\overline{x}^0(s^0) > \hat{y}$. Our earlier observation implies that if $\overline{t} = \infty$, a proposer eventually offers a share exceeding 1 to another player in some state, or if $\bar{t} < \infty$, a proposer in the final round offers a strictly positive share to another player. In both cases, we have reached a contradiction. $\square$

Proof of Theorem 3 on p. 17. Lemma 1 extends seamlessly to this environment, so every MPE proposal is accepted with probability 1. To extend Lemma 2 to this setting, recall that $W_C^t(s^t)$ is the sum of discounted continuation values for a coalition C. Let

$$\tilde{W}^t(s^t) \equiv \min_{\substack{C \subseteq \mathcal{N} \setminus \{p^t(s^t)\}, \\ C \cup \{p^t(s^t)\} \in \mathcal{D}}} W_C^t(s^t),$$

be the cost of the cheapest decisive coalitions for proposer $p^t(s^t)$. We denote the cheapest decisive coalition partners as

$$\tilde{C}^t(s^t) \equiv \left\{ C \subseteq \mathcal{N} \setminus \{p^t(s^t)\} : C \cup \{p^t(s^t)\} \in \mathcal{D} \text{ and } W_C^t(s^t) = \tilde{W}^t(s^t) \right\},$$

and the proposals that involve creating such coalitions as

$$\tilde{\mathcal{X}}^t(s^t) \equiv \left\{ x \in \mathcal{X} : \exists C \in \tilde{C}^t(s^t) \text{ such that } x_i = \delta_i V_i^{t+1}(s^t) \forall i \in C \text{ and } x_{p^t(s^t)} = 1 - \tilde{W}^t(s^t) \right\}.$$

Lemma 2 generalizes insofar as every MPE proposal offered with positive probability in state s^t must be in $\tilde{\mathcal{X}}^t(s^t)$. Recall that $\xi_P^{i,t}(s^t)$ is the equilibrium mixed action selected by proposer $p^t(s^t)$ at state s^t: for a proposal x in $\tilde{\mathcal{X}}^t(s^t)$, let $\xi_P^{i,t}(s^t)(x)$ denote the equilibrium probability with which proposer $p^t(s^t)$ makes that proposal at state s^t. We prove an additional lemma for this setting bounding the continuation value at time t for the coalition of losers.

Lemma 3. *Consider a time period $t < \bar{t}$ and a structural state s^t. The following relates costs of coalitions across periods:*

$$W_{L^{t+1}(s^t)}^t(s^t) \leq \hat{\delta} \int_{\mathcal{S}^{t+1}} \tilde{W}^{t+1}(s^{t+1}) d\mu\left(s^{t+1} \mid s^t\right).$$

Proof. Observe that by definition of $W_C^t(s^t)$, and using $\delta_i \leq \hat{\delta}$,

$$W_{L^{t+1}(s^t)}^t(s^t) = \sum_{i \in L^{t+1}(s^t)} \delta_i V_i^{t+1}(s^t) \leq \hat{\delta} \sum_{i \in L^{t+1}(s^t)} V_i^{t+1}(s^t). \tag{2}$$

Consider any player i in $L^{t+1}(s^t)$: such a player is recognized with probability 0 in period $t+1$. In other words, given s^t, for each feasible continuation structural state in period $t+1$, $s^{t+1} \subset s^t$, player i is distinct from the proposer $p^{t+1}(s^{t+1})$. Therefore, player i can only expect to obtain strictly positive payoffs in period $t+1$ in structural states s^{t+1} in which the proposer $p^{t+1}(s^{t+1})$ makes an offer that offers a strictly positive share to player i. In such a scenario, he is offered his discounted continuation value, namely $\delta_i V_i^{t+2}(s^{t+1})$. Therefore, for every player i in $L^{t+1}(s^t)$,

$$V_i^{t+1}(s^t) = \int_{\mathcal{S}^{t+1}} \delta_i V_i^{t+2}(s^{t+1}) \sum_{x \in \tilde{\mathcal{X}}^{t+1}(s^{t+1})} \mathbf{1}_{x_i>0} \xi_P^{i,t+1}(s^{t+1})(x) d\mu \left(s^{t+1} \mid s^t\right). \tag{3}$$

We substitute (3) into (2):

$$\begin{aligned}
W^t_{L^{t+1}(s^t)}(s^t) &\leq \hat{\delta} \sum_{i \in L^{t+1}(s^t)} \int_{\mathcal{S}^{t+1}} \delta_i V_i^{t+2}(s^{t+1}) \sum_{x \in \tilde{\mathcal{X}}^{t+1}(s^{t+1})} \mathbf{1}_{x_i>0} \xi_P^{i,t+1}(s^{t+1})(x) d\mu \left(s^{t+1} \mid s^t\right) \\
&= \hat{\delta} \int_{\mathcal{S}^{t+1}} \sum_{i \in L^{t+1}(s^t)} \delta_i V_i^{t+2}(s^{t+1}) \sum_{x \in \tilde{\mathcal{X}}^{t+1}(s^{t+1})} \mathbf{1}_{x_i>0} \xi_P^{i,t+1}(s^{t+1})(x) d\mu \left(s^{t+1} \mid s^t\right) \\
&\leq \hat{\delta} \int_{\mathcal{S}^{t+1}} \sum_{i \in \mathcal{N} \setminus p^{t+1}(s^{t+1})} \delta_i V_i^{t+2}(s^{t+1}) \sum_{x \in \tilde{\mathcal{X}}^{t+1}(s^{t+1})} \mathbf{1}_{x_i>0} \xi_P^{i,t+1}(s^{t+1})(x) d\mu \left(s^{t+1} \mid s^t\right), \\
&= \hat{\delta} \int_{\mathcal{S}^{t+1}} \sum_{x \in \tilde{\mathcal{X}}^{t+1}(s^{t+1})} \xi_P^{i,t+1}(s^{t+1})(x) \sum_{i \in \mathcal{N} \setminus p^{t+1}(s^{t+1})} \mathbf{1}_{x_i>0} \delta_i V_i^{t+2}(s^{t+1}) d\mu \left(s^{t+1} \mid s^t\right), \\
&= \hat{\delta} \int_{\mathcal{S}^{t+1}} \sum_{x \in \tilde{\mathcal{X}}^{t+1}(s^{t+1})} \xi_P^{i,t+1}(s^{t+1})(x) \tilde{W}^{t+1}(s^{t+1}) d\mu \left(s^{t+1} \mid s^t\right), \\
&= \hat{\delta} \int_{\mathcal{S}^{t+1}} \tilde{W}^{t+1}(s^{t+1}) d\mu \left(s^{t+1} \mid s^t\right).
\end{aligned}$$

in which the first line is the substitution, the second line interchanges the sum and integral, the third line uses the fact that for each $s^{t+1} \subset s^t$, $L^{t+1}(s^t)$ is a subset of $\mathcal{N} \setminus p^{t+1}(s^{t+1})$, the fourth line re-arranges terms by interchanging summation, the fifth line uses the fact that by definition, for each x in $\tilde{\mathcal{X}}^{t+1}(s^{t+1})$, $\sum_{i \in \mathcal{N} \setminus p^{t+1}(s^{t+1})} \mathbf{1}_{x_i>0} \delta_i V_i^{t+2}(s^{t+1}) = \tilde{W}^{t+1}(s^{t+1})$, and the sixth line uses the generalized Lemma 2 to note that $\sum_{x \in \tilde{\mathcal{X}}^{t+1}(s^{t+1})} \xi_P^{i,t+1}(s^{t+1})(x) = 1$. □

We now prove the theorem by contradiction. Suppose the state in Stage 1 of period 0 is s^0, and that a policy proposed with positive probability in which the proposer $p^0(s^0)$ offers a strictly positive amount, x, to another player, in which case $\tilde{W}^0(s^0) \geq x$. Since the recognition process exhibits one-period decisive predictability, there exists a set of coalition partners C that excludes $p^0(s^0)$ such that $C \bigcup \{p^0(s^0)\}$ is in $\mathcal{D}$, and C is a subset of $L^1(s^0)$. By definition of $\tilde{W}$ that $\tilde{W}^0(s^0) \leq W_C^0(s^0)$ and by monotonicity that $W_C^0(s^0) \leq W^0_{L^1(s^0)}$. Therefore, $W^0_{L^1(s^0)}$ must be no less than x. Lemma 3 implies that there must exist a structural state s^1 such that $\tilde{W}^1(s^1) \geq x/\hat{\delta}$. Since $\tilde{W}^1(s^1)$ is defined to be the cost of the cheapest decisive coalition partners for proposer $p^1(s^1)$, the same argument as above implies that $W^1_{L^2(s^1)}(s^1)$ must also be no less than $x/\hat{\delta}$. Therefore, by induction, there exists a sequence of states $\{s^t\}_{t \in \mathcal{T}}$ such that for each t, $s^t \in \mathcal{S}^t$, and $\tilde{W}^t(s^t) \geq \frac{x}{\hat{\delta}^t}$. If $\bar{t} = \infty$, $\hat{\delta} < 1$ implies that $\tilde{W}^t(s^t)$ eventually exceeds 1; if $\bar{t} < \infty$, the same argument implies that the proposer at $\bar{t}$ does not appropriate the entire surplus in some state $s^{\bar{t}}$. In both cases, we have reached a contradiction. □

Proof of Theorem 4 on p. 18. In this setting, we re-define the cost of a coalition, $W_C^t(s^t)$: for a state s^t and coalition $C \subseteq \mathcal{N} \backslash \{p^t(s^t)\}$, let

$$W_C^t(s^t) \equiv \sum_{i \in C} u_i^{-1}\left(\delta_i V_i^{t+1}\left(s^t\right)\right)$$

Given that $V_i^{t+1}(s^t) \in [0, u_i(1)]$, $\delta_i \in (0,1)$, $u_i(0) = 0$, and u_i is strictly increasing and continuous, we know that $W_C^t(s^t)$ is well-defined. Having re-defined $W_C^t(s^t)$, we define $\underline{W}^t(s^t)$ and $C^t(s^t)$ exactly as before. We now write

$$\mathcal{X}^t(s^t) \equiv \left\{ \begin{array}{r} x \in \mathcal{X} : \exists C \in C^t(s^t) \text{ such that } \forall i \in C,\ u_i(x_i) = \delta_i V_i^{t+1}(s^t) \\ \text{and } x_{p^t(s^t)} = 1 - \underline{W}^t(s^t) \end{array} \right\}.$$

In an equilibrium, let $a(s^t)$ denote the (undiscounted) average of policies that are selected in the continuation after rejection of the proposal in state s^t. Because u_i is concave for each i and $\delta_i < 1$, we necessarily have $u_i(a_i(s^t)) > \delta_i V_i^{t+1}(s^t)$ for all i. Consequently, for any coalition C, we have $W_C^t(s^t) < \sum_{i \in C} a_i(s^t) \leq 1$. It follows that $1 - \underline{W}^t(s^t) > 0$, and hence that $\mathcal{X}^t(s^t)$ is non-empty.

No Delay: We first extend Lemma 1. Suppose there is a structural state s^t in $\mathcal{S}^t$ such that an equilibrium proposal offered with strictly positive probability, x', is rejected with strictly positive probability. Select some $x \in \mathcal{X}^t(s^t)$ and let $C \in C^t(s^t)$ be the associated minimal winning coalition (excluding the proposer). Define a proposal x^ϵ for small $\epsilon \geq 0$ in which $x_i^\epsilon = x_i + \epsilon$ for every $i \in C$, $x_i^\epsilon = 0$ for every $i \notin C \cup \{p^t(s^t)\}$, and keeps $1 - \underline{x}(s^t) - (q-1)\epsilon$ for himself (which is feasible in light of the the fact that $1 - \underline{x}(s^t) > 0$). In the equilibrium, the proposal x^ϵ must be accepted by all members of C with probability 1 if $\epsilon > 0$. Observe that because $\sum_{i \in \mathcal{N}} u_i^{-1}\left(\delta_i V_i\left(s^t\right)\right) < \sum_{i \in \mathcal{N}} a_i(s^t) \leq 1$ and $\hat{\delta} < 1$,

$$\begin{aligned} \underline{x}(s^t) + u^{-1}\left(\delta_{p^t(s^t)} V_{p^t(s^t)}^{t+1}\left(s^t\right)\right) &= \sum_{i \in C} u_i^{-1}\left(\delta_i V_i^{t+1}\left(s^t\right)\right) + u^{-1}\left(\delta_{p^t(s^t)} V_{p^t(s^t)}^{t+1}\left(s^t\right)\right) \\ &\leq \sum_{i \in \mathcal{N}} u_i^{-1}\left(\delta_i V_i^{t+1}\left(s^t\right)\right) \\ &< 1. \end{aligned}$$

Therefore, for sufficiently small $\epsilon > 0$, we have

$$x_{p^t(s^t)}^\epsilon = 1 - \underline{x}(s^t) - (q-1)\epsilon > u^{-1}\left(\delta_{p^t(s^t)} V_{p^t(s^t)}^{t+1}\left(s^t\right)\right).$$

Thus, conditional on x' being rejected, the proposer is discretely better off deviating to $x_\epsilon(s^t)$ for sufficiently small $\epsilon > 0$. Conditional on x' being accepted, the proposer's share can be no greater than she obtains when offering x (because that offer delivers the share $1 - \underline{x}(s^t)$). Since proposal x' is rejected with strictly positive probability, she is strictly better off offering x^ϵ for sufficiently small $\epsilon > 0$. Therefore, no equilibrium offer x' can be rejected with strictly positive probability. □

Minimal Winning Coalition: Lemma 2 extends readily to this setting: if the proposer $p^t(s^t)$ chooses a policy outside $\mathcal{X}^t(s^t)$, then she can profitably deviate to such a policy (plus tiny additional payments to members of the minimal winning coalition) to obtain immediate agreement at a strictly lower cost.

Establishing the Contradiction: Suppose that in an MPE, the first proposer $p^0(s^0)$ chooses a policy with $x_{p^0(s^0)} < 1$ with strictly positive probability. Reasoning exactly as in the proof of Theorem 1 (and defining $\overline{x}^0(s^0)$ analogously), there must exist a player i in $L^1(s^0)$ such that $\delta_i V^1(s^0) \geq u_i(\overline{x}^0(s^0))$ for $\overline{x}^0(s^0) > 0$. Therefore, there must exist some state s^1 such that player i is offered at least $u_i^{-1}(u_i(\overline{x}^0(s^0))/\delta_i)$ by $p^1(s^1)$, which implies that $\overline{x}^1(s^1) \geq u_i^{-1}(u_i(\overline{x}^0(s^0))/\delta_i)$. Induction then implies that there exists a sequence of states $(s^t)_{t\in\mathcal{T}}$ such that for each $t \geq 1$, there exists j such that $\overline{x}^t(s^t) \geq u_j^{-1}(u_j(\overline{x}^{t-1}(s^{t-1}))/\delta_j)$. If $\overline{t} < \infty$, then this fact implies that there must exist some state in $\mathcal{S}^{\overline{t}}$ such that the proposer in the final round chooses a policy that offers a strictly positive share to another player, which is a contradiction. If $\overline{t} = \infty$, observe that:

1. For each player i, $u_i^{-1}(u_i(x_i)/\delta_i) > x_i$ for every $x_i > 0$, which implies that the sequence $(\overline{x}^t(s^t))_{t=0}^{\infty}$ is strictly increasing.
2. Since there are finitely many players, there exists a player i^* and an infinite subsequence of periods $\{t_k\}_{k=0}^{\infty}$ such that $u_{i^*}(\overline{x}^{t_k-1}(s^{t_k-1})) \leq \delta_{i^*} u_{i^*}(\overline{x}^{t_k}(s^{t_k}))$.

Plainly $\overline{x}^t(s^t)$ is bounded above by 1 and since it is increasing, it must converge to a limit $x^* \leq 1$. The two observations imply that $u_{i^*}(\overline{x}^{t_0}(s^{t_0})) \leq \delta_{i^*}^k u_{i^*}(x^*)$ for every $k = 1, 2, \ldots$, which implies that $\overline{x}^{t_0}(s^{t_0}) = 0$. Since $\overline{x}^0(s^0) \leq \overline{x}^{t_0}(s^{t_0})$, we have contradicted our initial assumption that $\overline{x}^0(s^0) > 0$. □

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B Online Appendix

B.1 Omitted Proofs

Proof of Theorem 5 on p. 21. We begin by describing the system of equations used to solve for $\underline{w}$ and $\overline{w}$. It follows by recursive calculation that

$$\underline{w} = \left(\frac{d}{n}\right)\delta\underline{w} + \left(\frac{n-d}{n}\right)\left(\frac{d(q-d)}{(n-1)(n-d)} + \frac{(n-d-1)(q-1-d)}{(n-1)(n-d-1)}\right)\delta\overline{w},$$

where $\frac{d}{n}$ is the probability the individual will be a member of $L^{t+1}(s^t)$ in the next period; $\left(\frac{n-d}{n}\right)\left(\frac{d}{n-1}\right)\left(\frac{q-d}{n-d}\right)$ is the probability the individual will not be a member of $L^{t+1}(s^t)$ in the next period, the next period's proposer will be a member of $L^{t+1}(s^t)$, and next period's proposer will include the individual in the winning coalition; and $\left(\frac{n-d}{n}\right)\left(\frac{n-d-1}{n-1}\right)\left(\frac{q-1-d}{n-d-1}\right)$ is the probability the individual will not be a member of $L^{t+1}(s^t)$ in the next period, the next period's proposer will not be a member of $L^{t+1}(s^t)$, and the next period's proposer will include the individual in the willing coalition.

Combining this equation with

$$d\underline{w} + (n-d)\overline{w} = 1 \tag{4}$$

yields the solutions in the text. Finally, the first proposer's expected share can be represented as

$$\begin{aligned}
&\left(\frac{n-d}{n}\right)(1 - d\delta\underline{w} - (q-d-1)\delta\overline{w}) + \left(\frac{d}{n}\right)(1-(d-1)\delta\underline{w} - (q-d)\delta\overline{w}) \\
&= \left(\frac{n-d}{n}\right)(1 - \delta(1-(n-d)\overline{w}) - (q-d-1)\delta\overline{w}) + \frac{d}{n}(1-\delta(1-(n-d)\overline{w}) + \delta\underline{w} - (q-d)\delta\overline{w}) \\
&= \left(\frac{n-d}{n}\right)(1-\delta+\delta(n-q)\overline{w} + \delta\overline{w}) + \frac{d}{n}(1-\delta+\delta(n-q)\overline{w} + \delta\underline{w}) \\
&= 1-\delta+\delta(n-q)\overline{w} + \frac{\delta}{n},
\end{aligned}$$

where the first line follows from the fact that the first proposer is not a member of $L^1(s^0)$ with probability $(n-d)/n$, and is a member with probability $\frac{d}{n}$; the second line uses (4); the third line simplifies the expression; and the fourth line uses (4) again. The derivative of the proposer's share with respect to d is

$$\frac{\delta(n-1)(n-q)(n-\delta q-(1-\delta))}{((n-1)n-d(\delta(n-q)+(n-1)))^2},$$

which being strictly positive implies that the first proposer's share is strictly increasing in d for $d < q$. □

Proof of Theorem 6 on p. 21. If $\alpha_p > \alpha_v$, then it follows that for sufficiently large n, $d_n > q_n$ in which case Theorem 1 implies that the first proposer captures the entire surplus. Suppose that

$\alpha_p \le \alpha_v$. By our earlier result, the first proposer's share is

$$
\begin{aligned}
&1-\delta+\frac{\delta}{n}+\delta(n-q_n)\overline{w}_n \\
=&1-\delta+\frac{\delta}{n}+\frac{\delta(n-q_n)(n-1)(n-\delta d_n)}{n(n(n-1)-d_n(\delta(n-q_n)+n-1))} \\
=&1-\delta+\frac{\delta}{n}+\frac{\frac{\delta(n-q_n)}{n}\frac{(n-1)}{n}\frac{(n-\delta d_n)}{n}}{\frac{(n-1)}{n}-\frac{\delta d_n(n-q_n)}{n^2}-\frac{d_n}{n}+\frac{d_n}{n^2}}
\end{aligned}
$$

Taking limits as $n\to\infty$, $q_n/n\to\alpha_v$, and $d_n/n\to\alpha_p$, we obtain

$$
1-\delta+\frac{\delta(1-\alpha_v)(1-\delta\alpha_p)}{1-\delta\alpha_p(1-\alpha_v)-\alpha_p}=1-\frac{\delta(\alpha_v-\alpha_p)}{1-\delta\alpha_p(1-\alpha_v)-\alpha_p}.
$$

□

Proof of Theorem 7 on p. 22. The coalitional structure of vetoes and votes is a special case of the framework studied in Section 5.2: every $D\in\mathcal{D}$ must contain players $1,\ldots,k$ and at least q of the remaining $n-k$ players. We invoke the extensions of Lemma 1 and Lemma 2 described in the proof of Theorem 3: for every state s^t, every MPE proposal generated in state s^t is accepted with probability 1, and is in $\tilde{\mathcal{X}}^t(s^t)$.

Observe that if $k=q$, Theorem 7 follows from Lemma 2: any proposal in which a proposer offers a strictly positive amount to a non-veto player is not in $\tilde{\mathcal{X}}^t(s^t)$.

Now suppose that $k<q<n$: it must be that there are at least two non-veto players. Observe that for every state s^t, there exists $\tilde{x}^t(s^t)$ such that for every offer $x\in\tilde{\mathcal{X}}^t(s^t)$, $\tilde{x}^t(s^t)=\max_{i\notin(\{p^t(s^t)\}\cup\{1,\ldots,k\})}x_i$. Our claim is that for every $s^0\in\mathcal{S}^0$, $\tilde{x}^0(s^0)=0$. Suppose towards a contradiction that $\tilde{x}^0(s^0)>0$. Consider the set of non-veto players whose support cannot be secured for shares less than $\tilde{x}^0(s^0)$:

$$
\tilde{H}^0(s^0)\equiv\left\{i\in\{k+1,\ldots,n\}\backslash\{p^0(s^0)\}:\delta_iV_i^1(s^0)\ge\tilde{x}^0(s^0)\right\}.
$$

$\tilde{H}^0(s^0)$ must have cardinality at least $n-(q-1)$ because otherwise proposer $p^0(s^0)$ would be able to form a coalition of veto and non-veto players without having to offer $\tilde{x}^0(s^0)$ to any player. Therefore, $\tilde{H}^0(s^0)\bigcap L^1(s^0)$ is no-empty. Therefore, there must exist some state s^1 such that player i offered at least $\tilde{x}^0(s^0)/\delta_i$, which implies that $\tilde{x}^1(s^1)\ge\tilde{x}^0(s^0)/\hat{\delta}$. By induction (as before), there must then exist a state in which a proposer shares more than the entire surplus (if $\overline{t}=\infty$) or offers a strictly positive share in $\overline{t}$ (if $\overline{t}<\infty$), both of which are contradictions. □

Proof of Theorem 8 on p. 24. Define a policy x proposed by player p to be *movable* in period t if $x_j\ge\frac{\delta}{1+\delta k}$ for each j in $A^t(p)$. We write $M^t(p)$ for the set of movable policies by player p in period t. Consider a strategy profile in which:

1. In every period t for which there is no proposal on the table, the proposer p^t offers $\frac{\delta}{1+\delta k}$ to each amender and 0 to all others.

2. When voting on a proposal in period t that has been moved by each amender in A^t, each player votes to accept the proposal unconditionally unless he is either the proposer p^{t+1} or an amender in $A^{t+1}(p^{t+1})$. The proposer in period $t+1$ votes to accept the proposal if and only if he obtains at least $\frac{\delta}{1+\delta k}$, and the amender votes to accept if and only if he obtains at least $\frac{\delta^2}{1+\delta k}$. Define a proposal to be *passable* if it satisfies these conditions.

3. In period t, if the proposal on the table is movable, then each amender moves the proposal. If it is neither movable nor passable, then assuming previous amenders have moved the proposal, each a_i^t offers an amendment to keep $\frac{1}{1+\delta k}$ for himself and share $\frac{\delta}{1+\delta k}$ with each amender in the set $A^{t+1}(a_i^t)$. In the case where the proposal is passable but not movable, let i' denote the last amender for whom the amount offered is strictly less than $\frac{\delta}{1+\delta k}$. For all $i \leq i'$, a_i^t offers the same amendment just described. For all $i > i'$ (if any), a_i^t moves the proposal.

4. When voting in period t between a proposal x proposed by player p and an amendment x' by player p', each player i votes for x if and only if
   - $x \in M^{t+1}(p)$ and $x' \in M^{t+1}(p')$, and $x_i > x'_i$,
   - or $x \in M^{t+1}(p)$ and $x' \notin M^{t+1}(p')$,
   - or $x \notin M^{t+1}(p)$, $x' \notin M^{t+1}(p')$, and i is in $A^{t+1}(p)$.

First, as a preliminary observation, we note that all movable proposals are passable. If $k \geq \frac{n-1}{2}$, then $p^t \cup A^t(p^t)$ has cardinality of at least $\frac{n+1}{2}$, so the current proposer and amenders can pass a proposal with no other support. According to the strategies, all members of that group will vote in favor of a movable proposal, so it is passable. If $k < \frac{n-1}{2}$, the set of players not in $p^{t+1} \cup A^{t+1}(p^{t+1})$ has cardinality of at least $\frac{n+1}{2}$, and can pass a proposal with no other support. According to the strategies, all members of that group will vote in favor of a movable proposal, so it is passable.

We prove that, for this strategy profile, no player has a profitable deviation for any history by considering each of the three roles separately: proposer, amender, and voter.

- **Proposer:** Suppose there is no offer on the table, so the proposer p^t must make an offer: any proposal that offers less than $\frac{\delta}{1+\delta k}$ to a player j in A^t is amended by that player and defeated. Since no proposal accepted in equilibrium in the continuation game offers a higher discounted expected payoff to the proposer p^t than $\frac{1}{1+\delta}$, he has no incentive to deviate to any proposal that offers less to amender j than $\frac{\delta}{1+\delta k}$. Of the proposals that are accepted in equilibrium, the equilibrium proposal maximizes the proposer's payoff.

- **Amender:** Suppose first that the current proposal on the table in period t is movable. The proposal is also passable, so moving it leads to its implementation (given continuation strategies), yielding a payoff of at least $\frac{\delta}{1+\delta k}$ for the amender. Amending the proposal cannot generate a strictly higher payoff for the amender given prescribed behavior in the continuation game.

Next suppose the current proposal is neither movable nor passable. Moving the proposal results in implementation of some other policy one period hence, with an expected discounted payoff no greater than $\frac{\delta}{1+\delta k}$ (given continuation strategies). By proposing the amendment prescribed by the equilibrium strategies, the amender can achieve a discounted payoff of $\frac{\delta}{1+\delta k}$, which is (weakly) greater.

Finally suppose the current proposal is passable but not movable. Amender $a^t_{i'}$ (where i' is defined in part 3 of the description of the equilibrium strategies) plainly has a strict incentive to amend the proposal by offering to keep $\frac{1}{1+\delta k}$ for himself and share $\frac{\delta}{1+\delta k}$ with each amender in the set $A^{t+1}(a^t_{i'})$ (given that this proposal will be implemented one period hence, and that no proposal more favorable to i' would be implemented). Anticipating this successful amendment, each amender i playing prior to i' has a strict incentive (by induction) to offer an analogous amendment. For $i > i'$, a^t_i can obtain an immediate payoff not less than $\frac{\delta}{1+\delta k}$ by moving the proposal (because subsequent amenders will move it and it is passable), and cannot obtain a greater discounted payoff by offering an amendment.

- **Voting Decisions:** By construction, players cast votes in favor of the alternative that yields their highest continuation payoff.

□

B.2 Formal Results for Political Maneuvers

We formalize the conclusions discussed in Section 5.4, permitting legislators $i = 1, \ldots, n$ and the Chair, $i = 0$, to choose (potentially) costly maneuvers m_i in each period from some set M_i that has persistent effects on recognition. We describe in order the timing of maneuvers, the recognition rule, and the payoff relevant state. We then describe the appropriate predictability conditions and our formal results.

Timing: At the beginning of period t, players engage in political maneuvers. Each player i simultaneously chooses an action variable m^t_i from the feasible set of maneuvers, M_i, a non-empty and compact subset of a Euclidean space. We write $M \equiv M_0 \times \ldots \times M_n$. The selected vector of maneuvers in period t is $m^t = (m^t_0, ..., m^t_n)$, which is observed by all players. We let $h^t_m = (m^0, \ldots, m^t)$ denote the full history of maneuvers up to and including that of period t.

After the maneuvers are selected, players proceed to the **Information and Recognition** stage described in Section 3. We let H^t_m denote the set of possible histories of maneuvers up to and including those of time t, and $H_m = \bigcup_{t\in\mathcal{T}} H^t_m$ denote the set of all possible histories of maneuvers. The period t recognition rule is represented by a deterministic function $\widetilde{P}^t : H^t_m \times H^{t-1}_P \times \Omega \to \mathcal{N}$ in which H_P^{t-1} is the set of possible proposer histories, and Ω is the state space. Because the state of nature and the history of maneuvers recursively determines the entire sequence of proposers, we can write the recognition rule more compactly as $P^t : H_m^t \times \Omega \to \mathcal{N}$. After the revelation of information and recognition, the proposer p^t proposes a policy in $\mathcal{X}$ and other votes in a fixed sequential order. The proposal is implemented if and only at least q players (including the proposer) vote in favor.

Payoffs: We augment each legislator's payoff in Section 3 with that from maneuvers; substantively we assume that no legislator has any interest in prolonging negotiations because he intrinsically enjoys the process of political maneuvering. Formally, for a history $h_m^t \in \mathcal{H}_m$, let $v_i : \mathcal{H}_m \to \mathfrak{R}$ represent player i's costs from that history incurred at time t. If offer x is accepted at time t, legislator i's payoff is

$$u_i\left(x, t, h_m^t\right) = \delta^t x_i - \sum_{\tau=0}^{t} \delta_i^\tau v_i\left(h_m^\tau\right).$$

We assume that for all t, and all $h_m^t \in \mathcal{H}_m$, $v_i\left(h_m^t\right) \geq 0$. Thus, maneuvering is (potentially) costly, and prolonging negotiations cannot be motivated by the desire for further maneuvering. For many applications, it suffices to consider a special case of v_i in which the only dimension of the history of maneuvers, h_m^t, that is costly at time t is the current individual maneuver, m_i^t. However, our results also accommodate settings in which the cost of maneuvering is affected by the maneuvers of others and one's own past maneuvers.

For the Chair's preferences, we write

$$u_0\left(x, t, h_m^{(t)}\right) = \delta_0^t W(x) - \sum_{\tau=0}^{t} \delta_0^\tau v_0\left(h_m^\tau\right),$$

in which $W(x)$ represents her payoffs from a policy x. We make no restrictions on v_0.

Markov Perfect Equilibria: We augment our description of structural states and equilibria to account for the possibilities for maneuvering. In the maneuvering stage of period t, let $\tilde{s}_M^t \equiv \left(h_m^{t-1}, s^{t-1}\right)$ denote all past maneuvers and all that is known after period $t-1$ about future recognition. We write $\tilde{s}_P^t \equiv \left(h_m^t, s^t\right)$ as the state at the proposal stage, in which both the maneuvers and information revealed at period t are included. Let S_M^t denote the set of possible states for the maneuvering stage of period t. We let $S_{P,i}^t$ denote the collection of all states for the proposal stage consistent with player i being the proposer. An MPE is an SPE in which each player's equilibrium strategy can be written as a sequence of function $\left(\xi_M^{i,t}, \xi_P^{i,t}, \xi_V^{i,t}\right)_{t\in\mathcal{T}}$ such that $\xi_M^{i,t} : S_M^t \to \Delta M_i$ is player i's randomization over maneuvers in period t in structural state $\tilde{s}_M^t$, $\xi_P^{i,t} : S_{P,i}^t \to \Delta X$ is player i's randomization over proposals when recognized in period t in structural state $\tilde{s}_P^t$, and $\xi_V^{i,t} : S_P^t \times \mathcal{X} \to \Delta\{\text{yes}, \text{no}\}$ is player i's randomization whether to vote in favor of a policy $x \in \mathcal{X}$ proposed in period t in structural state s_P^t.

Predictability: Using the above notation, we can extend our notions of predictability to account for political maneuvers. If the profile of maneuvers at $t+1$ is m^{t+1}, then and the sequence of signals identify that the member of the partition $\mathcal{S}^t$ that ω is in is s^t, then player i is recognized at $t+1$ if and only if ω is in

$$\Omega_i^M(h_m^t, m^{t+1}, s^t) \equiv \left\{\omega \in s^t : P^{t+1}\left((h_m^t, m^{t+1}), \omega\right) = i\right\},$$

which has probability $r_i^M(h_m^t, s^t, m^{t+1}) \equiv \mu(\Omega_i^M(h_m^t, m^{t+1}, s^t)|s^t)$. A player is a *loser conditional* on m^{t+1} in structural state $s_P^t = (h_m^t, s^t)$ if in period $t+1$, he is definitely not the proposer if the period-$t+1$ profile of maneuvers is m^{t+1}:

$$L_C^{t+1}(s_P^t, m^{t+1}) \equiv \left\{i \in \mathcal{N} : r_i^M(s_P^t, m^{t+1}) = 0\right\}.$$

The player is an *unconditional loser* if he is not the proposer regardless of m^{t+1}:

$$L_U^{t+1}(s_P^t) \equiv \bigcap_{m^{t+1} \in M} L_C^{t+1}(s_P^t, m^{t+1}).$$

We offer two distinct notions of predictability.

Definition 4. *The recognition process exhibits one-period* ***unconditional predictability*** *of degree* **d** *if* $|L_U^{t+1}(s_P^t)| \geq d$ *for all* s_P^t *in* S_P^t *and* t *in* $\mathcal{T}$*.*

Definition 5. *The recognition process exhibits one-period* ***conditional predictability*** *of degree* **d** *if* $|L_C^{t+1}(s_P^t, m^{t+1})| \geq d$ *for all* s_P^t *in* S_P^t*,* $m^{t+1} \in M$*, and* t *in* $\mathcal{T}$*.*

With conditional predictability, the players are able to rule out d legislators in period t when they can predict the maneuvers in period $t+1$. Unconditional predictability is stronger (and implies conditional predictability) as the players need not predict the maneuvers played in period $t+1$ to rule out d legislators from being proposer. The following describes the implications of each condition.

Theorem 9. *If the recognition process exhibits one-period unconditional (respectively conditional) predictability of degree* q*, the proposer selected at* $t = 0$ *captures the entire surplus in every (respectively every pure strategy) MPE.*

Proof. For every state s_P^t in S_P^t, let $V_i^{t+1}(s_P^t)$ denote the expected continuation value of player i before Stage 1 of the next period, after the rejection of an offer in state s_P^t, and excluding maneuvering costs that have already been incurred (at period t or before). Lemmas 1 and 2 extend to this setting immediately, so in every MPE proposal is accepted with probability 1.

Case 1: *Unconditional Predictability of Degree* q: Constructing $\overline{x}^0(s_P^0)$ and $H^0(s_P^0)$ as in the proof of Theorem 1, it follows that $H^0(s_P^0) \cap L_U^1(s_P^0)$ is non-empty. Consider a generic player i in $H^0(s^0) \cap L^1(s^0)$. For a generic player i in $H^0(s_P^0) \cap L_U^1(s_P^0)$, his continuation value is a combination of offers that he receives in states in S_P^1 and maneuvering costs that he incurs in period 1. Since maneuvering can be only costly, it must be that there exists some structural state s_P^1 in S_P^1 such that the associated proposer offers player i at least $\frac{\overline{x}^0(s_P^0)}{\delta_i} \geq \frac{\overline{x}^0(s_P^0)}{\hat{\delta}}$, which implies that $\overline{x}^1(s_P^1) \geq \frac{\overline{x}^0(s_P^0)}{\hat{\delta}}$. Induction (as before) implies that there exists a state in which a proposer shares more than the entire surplus (if $\overline{t} = \infty$) or offers a strictly positive share in the final period (if $\overline{t} < \infty$), both of which are contradictions.

Case 2: *Conditional Predictability of Degree q*: Construct $\overline{x}^0(s_P^0)$ and $H^0(s_P^0)$ as in the proof of Theorem 1. The state for maneuvers in period 1, $s_M^1 = (h_m^0, s_P^0)$, which is identical to s_P^0. Since the MPE is in pure strategies, there is a profile of maneuvers m^1 that is chosen in s_M^1 that is perfectly predictable in state s_P^0. Since the recognition process exhibits predictability of degree q, it follows that $|L_C^1(s^0, m^1)| \geq q$. Since $H^0(s_P^0)$ must have cardinality of at least $n-(q-1)$, $H^0(s_P^0) \bigcap L^C(s^0, m^1)$ is non-empty. It follows exactly as in the argument above that there exists some structural state s_P^1 in S_P^1 such that $\overline{x}^1(s_P^1) \geq \frac{\overline{x}^0(s_P^0)}{\hat{\delta}}$. Induction, as before, implies a contradiction. $\square$

Finally, we note that the example that we discuss in which the Chair selects proposers is that in which $M_0 = \mathcal{N}$, $\theta^t(\omega) = \theta$ for all t and ω, and $P^t(h_m^t, \theta^t) = m_0^t$. This is a recognition process that satisfies conditional predictability of degree $n-1$, and so in every pure strategy MPE, the first proposer captures the entire surplus.