

The Prudent Principal*

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Abstract

This paper re-examines executive incentive compensation, using a principal-agent model in which the principal is downside risk averse, or prudent (as a number of empirical facts, regulations and scholarly works suggest it should be), instead of risk neutral (as it has been commonly assumed so far in the literature). We find that optimal incentive pay should then be ‘approximately concave’ in performance (in a precise sense), the approximation being closer the more prudent the principal is relative to the agent. This means that an executive should face higher-powered incentives while in the bad states, but be given somewhat weaker incentives when things are going well. Such a statement runs counter to current evidence that incentive compensation packages often put substantial weight on convex devices (such as stock options). We show that this disparity can be justified under certain limited liability and taxation regimes. Some empirical research directions (investigating the composition of incentive pay, notably) and public policy implications are briefly discussed.

Keywords: Executive compensation, downside risk aversion, prudence, approximately concave functions.

JEL Classification: D86, M12, M52, G38

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1 Introduction

The 2008 financial crisis has put again the spotlight on executive pay. One highlighted feature is the increasing *convexification* over the past decades - through the more and more widespread use of call options, notably - of the pay-performance relationship in incentive packages: in other words, managerial rewards have generally become very responsive to upside gains but relatively immune to poor results. This asymmetry is now being questioned by several scholars (see, e.g., Yermack 1995; Jensen and Murphy 2004; Murphy and Jensen 2011; Boyer 2011). In its January 2011 report, the National Commission in charge of investigating the causes of the financial and economic downturn maintained that:

Compensation systems - designed in an environment of cheap money, intense competition, and light regulation - too often rewarded the quick deal, the short-term gain - without considerations of long-term consequences. Often those systems encouraged the big bet - where *the payoff on the upside could be huge and the downside limited*. This was the case up and down the line - from the corporate boardroom to the mortgage broker on the street. (emphasis added)¹

One might impute this state of affairs to managerial power (Finkelstein 1992; Bebchuck and Fried 2003) or other systematic behavioral and governance failures (Gervais et al. 2011; Jensen et Murphy 2004, p. 50-81; Ruiz-Verdú 2008), although these views are increasingly being challenged (Holmström 2005, Kaplan 2012). In this paper, we reconsider the main framework to deal with executive compensation - the principal-agent model.²

The ‘principal’ in this context stands for the corporate board, which is chiefly responsible in setting top executives’ compensation.³ It has been commonly viewed so far as being risk-neutral - making gains and losses bearing the same magnitude and probability cancel. This assumption can be debated on at least three grounds. First, since Roy (1952) and Markowitz (1959), a number of economists have contended

¹As far as CEO (not traders) compensation is concerned, this assertion has now been qualified (see, e.g., Murphy 2012). Whether and when convex incentive schemes are appropriate remains, however, a fundamental issue.

²In addition to the managerial power and principal-agent approaches, other theories of incentive pay include contests and tournaments (Shen et al. 2010), career concerns (Hermalin 2005), organizational structure (Santaló and Kock 2009), pay-for-luck (Bertrand and Mullainathan 2001), firms-managers matching (Gabaix and Landier 2008), social influence (O’Reilly and Main 2010), the contracting environment (Cadman et al. 2010), and government policy (Murphy 2011). Recent accounts of these theories can be found in the indicated references. Principal-agent theory remains, however, the dominant approach to analyze executive compensation from either a positive or a normative perspective. For general surveys of CEO compensation, see Baker et al. (1988), Murphy (1999), Jensen and Murphy (2004), Edmans and Gabaix (2009), Adams et al. (2010), and Conyon and Peck (2012).

³Although the corporate board is itself an agent of shareholders, Conyon and Peck (2012, p. 464), among many others, point out that: “Clearly, shareholders do *not* set pay: they are too numerous and too diverse. In reality the board of directors sets pay.”

that investors react asymmetrically to gains and losses. Corroborating this, Harvey and Siddique (2000), Ang et al. (2006), and others report that stock returns do reflect a premium for bearing downside risk. A corporate board acting in the best interest of shareholders would likely take this into account. Second, in assessing and disclosing their firm’s prospects, corporate boards often display ‘*conservatism*’ (see, e.g., Watts 2003a and 2003b, García Lara et al. 2009, and Gao 2013), i.e. a systematic tendency to put “(...) stronger verifiability requirements for the recognition of gains than for the recognition of losses.” (Watts 2003b, p. 287). Conservatism in this sense can be seen as “(...) a potentially useful tool for directors (especially outside directors) in fulfilling their role of ratifying and monitoring key decisions.” (Ahmed and Duellman 2007, p. 415) Moreover, it “(...) helps to limit creditors’ downside risk, which in turn enables firms to obtain lower interest rates on their loans.” (Hui et al. 2012, p. 116). One might then reasonably expect board directors to consistently maintain a similar posture when dealing with compensation matters.⁴ Third, corporate law and jurisprudence endow corporate board members with fiduciary duties of loyalty and care towards their corporation (Clark 1985; Gutierrez 2003; Adams et al 2010; Lan and Herakleous 2010; Corporate Law Committee 2011). The latter charge notably confers board directors and officers a key role in preventing and managing crisis situations (Mace 1971; Williamson 2007; Adams et al 2010).⁵ Directors and officers should accordingly “(...) exercise that degree of care, skill, and diligence which an ordinary, *prudent* man would exercise in the management of his own affairs.” (Clark 1985, p. 73; emphasis added). This requirement should drive corporate boards to weigh differently the risks correlated with downside losses versus those linked to upside gains. Such behavior is inconsistent again with risk neutrality, so one might sensibly cast doubt on some of the prescriptions from current and past principal-agent analyses of incentive pay.

In what follows, we now examine executive compensation using a principal-agent model in which the principal is ‘prudent,’ in the sense introduced in economics by Kimball (1990). This indeed portrays the principal as a downside risk averse entity.⁶ A prudent decision maker dislikes mean *and* variance-

⁴As the reader will see in the upcoming sections, this paper draws further from this literature. It supports one of our key modelling assumptions (Section 2). It also suggests ways to empirically characterize more or less prudent boards (Section 6).

⁵Quoting Adams et al. (2010, p. 58): “People often question whether corporate boards matter because their day-to-day impact is difficult to observe. But when things go wrong, they can become the center of attention. Certainly this was true of the Enron, Worldcom, and Parmalat scandals. The directors of Enron and Worldcom, in particular, were held liable for the fraud that occurred: Enron directors had to pay \$168 million to investor plaintiffs, of which \$13 million was out of pocket (not covered by insurance); and Worldcom directors had to pay \$36 million, of which \$18 million was out of pocket.”

⁶An alternative way to capture loss aversion would be to assume that utility declines sharply (albeit at a decreasing rate) below some reference point, as in prospect theory (Kahneman and Tversky 1979) - which is incidentally a *descriptive*, not a normative, theory. Dittman et al (2010), for instance, use this representation of an agent’s preferences to analyze executive compensation; the principal is risk neutral in their model.

preserving transformations that skew the distribution of outcomes to the left (Menezes et al 1980; Crainich and Eeckhoudt 2008). Equivalently, she prefers additional volatility to be associated with good rather than bad outcomes (Eeckhoudt and Schlesinger 2006; Denuit et al. 2010). Formally, someone is prudent when her marginal utility function is strictly convex (it is of course constant in the risk neutral case). As a characteristic of the agent’s (not the principal’s) preferences - the agent standing here for an executive or a top manager, prudence has already been dealt with and found relevant in the literature, especially in contingent monitoring (Fagart and Sinclair-Desgagné 2007) and background risk (Ligon and Thistle 2013) situations, and to explain the composition of incentive pay (Chaigneau 2012).⁷ To our knowledge, this is the first time prudence is taken to also be an attribute of the principal.

In a benchmark model, we show that incentive compensation should then be *approximately concave* (as defined by Páles 2003) in performance, the approximation being closer the more prudent the principal is relative to the agent. This result simultaneously complements Hemmer et al (2000)’s and Chaigneau (2012)’s respective propositions that relate convexity of the agent’s remuneration to the agent’s prudence under a risk neutral principal, Hau (2011)’s converse statement that a risk averse principal may ‘concavify’ the reward function of a risk neutral agent, and Gutierrez Arnaiz and Sallas-Fumás (2008)’s justification of convex-concave bonus schemes. The principle underneath our general statement seems straightforward: whoever is relatively more prudent should bear less downside risk. Convex incentives, being very sensitive to performance in upbeat situations and rather flat in the range where results are mediocre, shelter a prudent agent against downside volatility which must then be born by the principal. Concave incentives, by contrast, reward performance improvements much more strongly under adverse circumstances and make the agent bear significant downside risk; a prudent principal thereby decreases her own exposure to downside risk by firmly pushing her agent to get away from dangerous territory.

Whether the principal is more or less prudent relative to the agent should therefore be an important practical matter in setting optimal compensation contracts. Yet, the principal’s prudence does not seem to matter much so far in most industries. According to Garvey and Milbourn (2006, p. 198), “(...) the average executive loses 25-45% less pay from bad luck than is gained from good luck.” According to Core and Guay (2010), CEO annual pay falls by about 13.7% in the lowest decile and increases by about 19.7% in the highest decile. In other words, convex executive contracts are quite common, owing notably to the widespread use of stock options (see, e.g., Hall and Murphy 2003) and performance shares (Equilar 2012 a and b).⁸ We argue below that this can be justified under some common government policies such as

⁷Empirical evidence that executives are prudent can be found in McAnally et al (2011), Garvey and Milbourn (2006), and the references therein.

⁸Notable exceptions are firms in the utility sector (Murphy 1999) and ‘socially responsible’ firms (Frye and al. 2006), where the pay-performance relation actually tends to be concave. In this context, regulation and public pressure can be

limited liability and progressive taxation.⁹ In cases where the agent/executive cannot be inflicted negative revenues or the principal/corporation can be refunded when net profits are negative (which can be seen as a rough proxy for the 2008 TARP - Trouble Asset Relief Program - rescue of financial institutions), approximately concave contracts are no longer optimal even if the principal is very prudent. A similar conclusion holds when the principal's profits are taxed (which corresponds to a British government's proposal concerning banks' profits). When executive income is subject to progressive taxation (which roughly reproduces suggestions actively debated in the U.S. and implemented in France), the upshot is even more radical: a prudent principal might squarely offer strictly *convex* rewards in order to circumvent the effect of taxation and properly encourage the agent to pursue the better states of nature.

The rest of the paper unfolds as follows. Section 2 presents the benchmark model - a static principal-agent model where the agent is effort and risk averse while the principal is both risk averse and prudent; we ensure throughout that the first-order approach is valid, using only some of the assumptions in Jewitt (1988). Our central result - that the optimal contract should in that case be *approximately* concave, thereby seeking a balance between the agent's and the principal's respective prudence - is established in Section 3. Sections 4 and 5 then show how limited liability and taxation can respectively produce deviations from this prescription. Section 6 next explores some avenues for empirical research. An important upshot, for instance, is that our findings about approximate concavity can be seen as making predictions on the *composition* of pay: indeed, one mean to measure approximate concavity is the relative weight an incentive package gives to convex (e.g., call options) versus concave (e.g., capped bonuses) incentives. Section 7, finally, contains concluding remarks and some policy observations. All proofs are in the Appendix.

2 The benchmark model

Consider an agent - standing for a CEO or a top executive - whose preferences can be represented by a Von Neumann-Morgenstern utility function $u(\cdot)$ defined over monetary payments. We assume this function is three-times differentiable, increasing and strictly concave, formally $u'(\cdot) > 0$ and $u''(\cdot) < 0$, so the agent is risk averse.

This agent can work for a principal - in this case, a corporate board acting for a given company - whose preferences are represented by the Von Neumann-Morgenstern utility function $v(\cdot)$ defined over net

suspected to add to fiduciary duties to ultimately make corporate boards quite prudent.

⁹Tax laws have been pointed out by many (e.g., Smith and Watts 1982; Hall and Murphy 2003; Murphy 2011) to be one explanation of the sudden wave of executive option grants which convexify incentive compensation schemes. In more recent studies, however, Kadan and Swinkels (2008) and Aboody and Kasznik (2009) report mitigated evidence of this. Concerning liability regimes, Dittman and Maug (2007)'s theoretical and empirical work suggests that bankruptcy risks tend to reduce the convexity of incentive schemes. These issues are further discussed in Sections 4 and 5 respectively.

final wealth. We suppose this function is increasing and strictly concave, i.e. $v'(\cdot) > 0$ and $v''(\cdot) < 0$, so the principal is risk averse. Moreover, let the marginal utility $v'(\cdot)$ be convex, i.e. $v'''(\cdot) > 0$, which means that the principal is downside risk averse or (equivalently) prudent.

Let us write $R_u = -\frac{u''}{u'}$ and $R_v = -\frac{v''}{v'}$ the Arrow-Pratt measures of absolute risk aversion corresponding to the agent's and the principal's utility functions u and v respectively, and $P_u = -\frac{u'''}{u''}$, $P_v = -\frac{v'''}{v''}$ the analogous measures of prudence proposed by Kimball (1990). It can be checked that prudence (i.e. a utility function with positive third derivative) is a necessary condition for absolute risk aversion to decrease with wealth. To fix intuition further about the notion of prudence, consider the following examples.

In the first one, two decision makers with respective (quadratic) utility functions $u_1(w) = 70w - w^2$ and $u_2(w) = 100w - w^2$ contemplate lotteries $X = (0, 2; 1/4, 3/4)$ and $Y = (1, 3; 3/4, 1/4)$, where the first two arguments are prizes and the last two their respective probabilities (lottery X , for instance, yields prize 0 with probability $1/4$). According to the Arrow-Pratt coefficient, the first decision maker is more risk averse since $R_{u_1} = \frac{2}{70-2w}$ and $R_{u_2} = \frac{2}{100-2w}$. The two lotteries have the same mean and variance, but they exhibit different skewness. Yet, we have that $E[u_1(X)] = E[u_1(Y)]$ and $E[u_2(X)] = E[u_2(Y)]$, so both decision makers are indifferent between X and Y . Since neither of the two decision makers is prudent ($u_1''' = u_2''' = 0$), no-one cares about third moments.

As another example, take lotteries $A = (3, (0, 2; 1/2, 1/2); 1/2, 1/2)$, $B = (1, (2, 4; 1/2, 1/2); 1/2, 1/2)$. In the first one, a prize equal to 3 obtains with probability $1/2$ or a sublottery is played with probability $1/2$ that yields prizes 0 or 2 with equal probability. In the second one, a prize equal to 1 obtains with probability $1/2$ or a sublottery is played with probability $1/2$ that yields prizes 2 or 4 with equal probability. Again, both lotteries have the same mean and variance. But a prudent - or downside risk averse - decision maker will prefer lottery B over lottery A , because in the former (the latter) the occurrence of a sublottery - or additional volatility - happens when the stakes are relatively good (bad).

Now, let the principal's profit depend stochastically on the agent's effort level a . The latter cannot be observed, however, and the agent incurs a cost of effort $c(a)$ that is increasing and convex ($c'(a) > 0$ and $c''(a) \geq 0$). The principal only gets a verifiable signal s , drawn from a compact subset $S = [\underline{s}, \bar{s}]$ of $\mathbb{R}$, which is positively correlated with the agent's effort a through the conditional probability distribution $F(s; a)$ with density $f(s; a)$ strictly positive on S . Based on observing s , she can infer a realized profit $\pi(s)$, which we suppose increasing and concave or linear in s ($\pi'(s) > 0$ and $\pi''(s) \leq 0$), and pays the agent a compensation. It is fair to assume $|w(s)| < |\pi(s)|$ for any $s \in S$. To fix intuition, one may think of s as a sales forecast; if the firm has some market power, then it is reasonable to expect profit to be concave in output, hence in sales.¹⁰ Alternatively, s might be the firm's stock market value and $\pi(s)$ a

¹⁰ We thank Justin Leroux for this example.

conservative assessment of the firm's net assets.¹¹

The principal's problem is to find a smooth reward schedule or incentive scheme $w(s)$ that maximizes profit, under the constraints that the agent will maximize his own expected utility (the incentive compatibility constraint) and must receive an expected payoff that is not inferior to some external one U_0 (the participation constraint). This can be written formally as follows:

$$\begin{aligned} & \max_{w(s), a} \int v(\pi(s) - w(s)) dF(s; a) \\ & \text{subject to} \\ & a \in \arg \max_e \int u(w(s)) dF(s; e) - c(e) \\ & \int u(w(s)) dF(s; a) - c(a) \geq U_0 \end{aligned} \tag{1}$$

Smoothness means here that all wage schedules $w(s)$ are twice continuously differentiable. In addition, let $D_1, D_2, \dots, D_n$ be a finite partition of S ; then, on each D_i , the derivatives $w'(s)$ satisfy $w'(s) < M_i$, with M_i a positive real number. At any performance signal s , therefore, the agent's marginal revenue will be bounded. Technically, this is equivalent to saying that the allowed incentive schemes are Lipschitz continuous in the specific sense that, for any $w(s)$ on each set D_i , $|w(x) - w(s)| \leq M_i |x - s|$ for all $x, s \in D_i$ with the same constant M_i . This assumption will be useful later in some proofs; it seems rather innocuous because the number n and ceilings M_i 's, although exogenous, are arbitrary, and since any continuous function can be approximated as closely as wanted by a sequence of locally Lipschitz maps (Miculescu 2000).

For tractability reasons, one usually replaces the incentive compatibility constraint by a relaxed constraint based on the first-order necessary condition on the agent's utility-maximizing effort a . This transforms the principal's initial problem into the following one:

$$\begin{aligned} & \max_{w(s), a} \int v(\pi(s) - w(s)) dF(s; a) \\ & \text{subject to} \\ & \int_{s \in S} u(w(s)) dF_a(s; a) - c'(a) \geq 0, \quad (\gamma) \\ & \int_{s \in S} u(w(s)) dF(s; a) - c(a) \geq U_0, \quad (\mu) \end{aligned} \tag{2}$$

where γ and μ are the constraints' respective Lagrange multipliers. This so-called 'first-order approach' delivers for sure a solution to problem (1) under the following conditions.

¹¹ Quoting Watts 2003b, p. 288: "Although the market values of the assets and liabilities comprising net assets change every period, all these changes are not recorded in the accounts and reflected in financial reports. Under conservatism, increases in net asset values (gains) that are not sufficiently verifiable are not recorded, while decreases of similar verifiability are recorded. The result is that net assets are understated - carried below market value."

Assumption 1: The integral $\int_{\underline{s}}^y F(s; a) ds$ is nonincreasing convex in a for each value of y in S .

Assumption 2: The integral $\int_S s dF(s; a)$ is nondecreasing concave in a .

Assumption 3: [Concave Monotone Likelihood Ratio Property]: The ratio $\frac{f_a(s; a)}{f(s; a)}$ is non decreasing and concave in s for each value of a .

These requirements correspond respectively to assumptions (2.10a), (2.10b) and (2.11) in Jewitt (1988)’s theorem 1. They are satisfied by many common distributions, such as the Poisson with mean a , the gamma with mean κa , and the chi-squared with degree of freedom parameter a . Assumption 3 “(...) suggests that variations in output at higher levels are relatively less useful in providing ‘information’ on the agent’s effort than they are at lower levels of output.” (Jewitt 1988, p. 1181) This seems again consistent with accounting conservatism (see Watts 2003a). Hemmer et al. (2000)’s analysis makes it clear, incidentally, that assuming a concave likelihood ratio does not entail that the agent’s incentive scheme will necessarily have to be concave. Note, lastly, that the fourth assumption in Jewitt’s theorem - assumption (2.12) - is not needed here. Instead of it, we have the following.

Lemma 1 *Let $h(w) = \frac{v'(\pi(w))}{u'(w)}$. If the principal is more prudent than the agent (in the sense of Definition 1 below), the function $\bar{\omega}(z) = u(h^{-1}(z))$ is non decreasing and concave in z .*

Since the multiplier γ must be positive at an optimum, by Assumption 3 and Rogerson (1985)’s lemma 5, the validity of the first-order approach ensues from Jewitt (1988)’s theorem 1.¹²

This completes the description of our benchmark model. Let us now proceed to characterize the optimal incentive scheme in this context.

3 Approximately concave incentive schemes

This section will now establish that a principal who is sufficiently prudent compared with the agent (in a sense to be made precise very soon) should set an incentive compensation package that is *approximately* concave in outcome. The implications of such a contract are discussed below. To begin with, note that the Kuhn-Tucker necessary and sufficient conditions require that a solution to program (2) meet the equation:

$$\frac{v'(\pi(s) - w(s))}{u'(w(s))} = \mu + \gamma \frac{f_a(s; a)}{f(s; a)}, \quad \forall s \quad (3)$$

¹²Rogerson (1985) actually shows that the Monotone Likelihood Ratio Property implies the constraint corresponding to γ in problem (2) is binding. Were γ then equal to 0, some tiny perturbation of the principal’s objective would suffice to make it positive.

The multiplier γ being positive, Assumption 3 entails that the right-hand-side of (3) is increasing and concave in the signal s . This allows to state the following.

Lemma 2 *The optimal reward schedule $w^*(s)$ is increasing in the performance signal s .*

The proof consists in taking the first derivative of the left-hand-side of expression (3), knowing it must be positive. Similarly taking the second derivative, which must in turn be negative, will yield the central result of this section. Beforehand, we need to define two key items.

The first one concerns the principal's and the agent's relative prudence. Observe that $P_v R_v = \frac{v'''(\cdot)}{v'(\cdot)} = d_v$, a coefficient introduced by Modica and Scarsini (2005) to measure someone's degree of local downside risk aversion (or local prudence). A higher coefficient d_v means the principal would be ready to pay more to insure against a risk with greater negative skewness. As shown by Crainich and Eeckhoudt (2008), d_v increases if the utility function v becomes more concave while the marginal utility v' is more convex.¹³

Definition 1 *If $k \cdot d_u < d_v$ for some real number $k \geq 1$, the principal is said to be more prudent than the agent by a factor k .*

We next borrow from the litterature on approximately concave functions (Hyers and Ulam 1952; Páles 2003).

Definition 2 *If I is a subinterval of the real line $\mathbb{R}$ and δ, ρ are nonnegative numbers, a function $g : I \longrightarrow \mathbb{R}$ is called (δ, ρ) -concave on I if $tg(x) + (1-t)g(y) \leq g(tx + (1-t)y) + \delta t(1-t) |x - y| + \rho$ for all $x, y \in I$ and $t \in [0, 1]$.*

Clearly, the function g is concave when $\delta = \rho = 0$. The literature uses the term ρ -concave when $\delta = 0 < \rho$. The following characterization, which combines Páles (2003)'s theorems 3 and 4, will help visualize better the case where $\delta > 0 = \rho$, which is the relevant one in this paper.

Lemma 3 *Let I be a subinterval of the real line $\mathbb{R}$ and δ a nonnegative number. A function $g : I \longrightarrow \mathbb{R}$ is $(\delta, 0)$ -concave at $x \in I$ if and only if there exists a non-increasing function $q : I \longrightarrow \mathbb{R}$ such that $g(y) \leq g(x) + q(x)(y - x) + \frac{\delta}{2} |y - x|$ for all $y \in I$.*

The function q in Lemma 3 bears a close resemblance to a subgradient, and the literature indeed says that g is $(\delta, 0)$ -subdifferentiable when such a function exists. If the lemma holds on each subinterval of the entire domain of the function g , then g 's graph might look like the one shown in Figure 1.

¹³A somewhat different measure is the 'index of downside risk aversion' $S_v = d_v - \frac{3}{2}R_v^2$ due to Keenan and Snow (2005). This index does not have the properties d_v has, but it recalls the Arrow-Pratt measure of risk aversion in the sense that its value increases under monotonic downside risk averse transformations of the utility function v .

Insert Figure 1 about here.

Our main result is now at hand.

Theorem 1 *Suppose that the principal is more prudent than the agent by a factor k . Then the optimal wage schedule $w^*(s)$ is $(\delta(k), 0)$ -concave at any $s \in S$, where the number $\delta(k)$ decreases with k and tends to 0 as k grows.*

The proof shows, actually, that convergence to concavity is not asymptotic: when $\left(\frac{\pi'(s)-w'(s)}{w'(s)}\right)^2 \geq \frac{1}{k}$, i.e. the CEO's earnings do not grow too fast with respect to the firm's net profit, then we must have $\delta(k) = 0$ so $w^*(s)$ concave at s . Meanwhile, moreover, the $(\delta(k), 0)$ -subgradient of the wage schedule $w^*(s)$ will be the derivative $\pi'(s)$ of the profit function, so the CEO's incentives will remain well-aligned on the firm's interest.

The theorem's conclusion holds vacuously - hence the optimal incentive scheme is concave - when the agent is not prudent (for $u''' \leq 0$, hence $d_u \leq 0$, in this case). If the agent is prudent (i.e. $u''' > 0$), the theorem says that he may still have to bear more downside risk when the principal exhibits enough local prudence. In this case, incentive compensation will be approximately concave, so generally more responsive to performance under unfavorable than under positive circumstances. By offering such a contract, the prudent principal motivates the agent to keep away from, not only the bad, but indeed the very bad outcomes.¹⁴

While the theorem recommends to set approximately concave contracts under certain conditions, non-concave or even convex compensation modes seem to be quite common. Hall and Murphy (2003, p. 49), for instance, report that: "In 1992, firms in the Standard & Poor's 500 granted their employees options worth a total of \$11 billion at the time of grant; by 2000, option grants in S&P 500 firms increased to \$119 billion." This phenomenon per se does not invalidate our result, since we adopt here a normative standpoint. The current model may simply not capture key elements of the corporate landscape that would make non-concave incentive schemes optimal. In the following sections, we successively examine two sets of reasons which, when added to the benchmark model, might indeed justify why pay-performance concavity should not be that frequent.

¹⁴We don't mean to say here that pay-performance concavity renders the agent less eager to take risks. As Ross (2004, p. 209-211) pointed out, the overall effect of an incentive scheme $w(s)$ compared to an alternative $z(s)$ on the agent's behavior towards risk depends on whether the utility function $u(w(s))$ displays more or less risk aversion than the utility function $u(z(s))$. Suppose, for instance, that the latter scheme takes the form of a call option (a convex contract) $z(s) = \max\{s - r, 0\}$ with r the exercise price, while the former is the put option (a concave contract) $w(s) = \min\{b - r + s, b\}$ with b a fixed fee and r the exercise price. An agent whose risk aversion decreases with wealth (prudence is a necessary condition for this) will then be *less* locally risk averse at the exercise price r under contract $w(\cdot)$ than under contract $z(\cdot)$.

4 Limited liability and non-concavity

As a first departure from our benchmark model, let's allow either the agent or the principal to bear limited losses. In the first subsection, the agent will always earn nonnegative revenue. In the second subsection, the principal will be rescued whenever net profits are falling below zero.

4.1 The judgment-proof agent

Suppose the agent's revenue is bounded from below, so he cannot bear very high penalties when performance is bad. Management remuneration is frequently subject to this type of constraint.¹⁵ An agent with limited wealth, for instance, can file for bankruptcy if he cannot afford paying some penalty. Golden parachutes and other devices (like retirement benefits) have also been introduced to compensate top managers in case employment is terminated. Executives who own large amounts of their company's stock can now often hedge their holdings to contain losses if the value of the stock plunges (Bisin et al. 2008; Acharya and Bisin 2009; Gao 2010). Recently, some CEOs have also been offered insurance to offset the costs of investigations or liability for certain criminal mischiefs such as foreign corruption and bribery (Boyer and Tennyson 2011).¹⁶ And in certain contexts, institutions that prevent an agent from breaching his contract under bad circumstances might simply not exist.

Without loss of generality, let us then normalize the agent's minimum revenue to zero. The principal's optimization problem now becomes:

$$\begin{aligned} & \max_{w(s), a} \int v(\pi(s) - w(s)) dF(s; a) \\ & \text{subject to} \\ & \int_{s \in S} u(w(s)) dF_a(s; a) - c'(a) \geq 0, \quad (\gamma_1) \\ & \int_{s \in S} u(w(s)) dF(s; a) - c(a) \geq U_0, \quad (\mu_1) \\ & w(s) \geq 0, \forall s \quad (\lambda(s)) \end{aligned} \tag{4}$$

where $\lambda(s)$ is the Lagrange multiplier associated with the nonnegative wage constraint in state s .

Let a_1 denote the agent's new choice of effort (to be soon compared with a^* , the agent's optimal effort in the benchmark model). The Kuhn-Tucker conditions for a solution to this problem (4) lead this time

¹⁵Hence, since Holmstrom (1979) and especially Sappington (1983)'s seminal works, analyzing the impact of an agent's limited liability remains a rather well-covered topic in the principal-agent literature. For a recent account of this literature, see Poblete and Spulber (2012). In most articles, however, both the principal and the agent are assumed to be risk neutral.

¹⁶Marsh & McLennan created such a policy, in order to allow people and businesses to cover the cost of investigations under the U.S. Foreign Corrupt Practices Act and the U.K.'s Bribery Act.

to the equation:

$$\frac{v'(\pi(s) - w(s))}{u'(w(s))} = \mu_1 + \gamma_1 \frac{f_a(s; a)}{f(s; a)} + \frac{\lambda(s)}{f(s; a)u'(w(s))}, \quad \forall s \quad (5)$$

with $\lambda(s)w(s) = 0$ at all s .

Let s_1 be the signal value for which a zero wage is optimal. Since the reward function is increasing on the positive subspace, this signal is unique. Take the two subsets

$$\begin{aligned} \overline{S}_1 &= \{s \in S; s > s_1\} \\ \underline{S}_1 &= \{s \in S; s \leq s_1\}, \end{aligned}$$

and let $w_1(s)$ be the optimal reward function. Some extra computations based on (5) lead to the following statement.

Proposition 1 *Suppose that the principal is more prudent than the agent by a factor k . If the agent is protected by limited liability, then:*

a) The optimal wage schedule is such that (i) $w_1(s) = 0$ for any signal s lower than some threshold s_1 , and (ii) $w_1(s)$ is $(\delta_1(k), 0)$ -concave in $s > s_1$, where the number $\delta_1(k)$ decreases with k and tends to 0 as k grows.

b) The incentive scheme $w_1(s)$ is less approximately concave on the subset $\overline{S}_1$ than the one $w^(s)$ obtained in Theorem 1. In other words, $\delta_1(k) < \delta(k)$ for any given k .*

Unsurprisingly, greater convexity is introduced in the incentive scheme through the floor payment $w(s) = 0$ if $s \leq s_1$ imposed by the agent's limited liability. In the range where performance signals induce strictly positive rewards, that is for any s in $\overline{S}_1$, the optimal wage schedule remains qualitatively similar to the one established in the theorem. However, the pay-performance relationship in this range is now closer to a concave one. In practice, this means, for instance, that the CEO's compensation package should give relatively more weight to capped bonuses than stock options. The agent being fully protected on the downside, the principal can indeed save on offering potentially expensive rewards on the upside.

4.2 The sheltered principal

Consider now a situation where it is the principal's losses which are limited. Many countries actually possess, implicitly or explicitly, rescue programs aimed at supporting their so-called 'strategic' or 'too big to fail' enterprises when they are on the verge of collapse. In 2008, for example, at the heart of the financial crisis, the United States government - under the Troubled Assets Relief Program (TARP) - purchased hundreds of billions of dollars in assets and equity from distressed financial institutions. In 2004, the engineering and manufacturing company Alstom, which had experienced a string of business disasters, received 2.5 billion euros in rescue money from the French government (as part of a plan previously

approved by the European Commission). Such state interventions usually raise concerns that they will fuel moral hazard from the sheltered firms. As we shall see, they might at first have an effect on the incentives of senior executives and CEOs.

Suppose there is a state of nature s_2 in which $\pi(s_2) = 0$; profits being increasing in s by assumption, we have that $\pi(s) < 0$ when $s < s_2$ and $\pi(s) \geq 0$ for $s \geq s_2$. We postulate that the principal will be rescued after she has compensated the agent.¹⁷ Recall that $|w(s)| < |\pi(s)|$ for any $s \in S$; net profits $\pi(s) - w(s)$ remain therefore positive for $s > s_2$, equal to zero at $s = s_2$ and negative when $s < s_2$.

Consider now the two subsets:

$$\begin{aligned}\overline{S_2} &= \{s \in S; \pi(s) - w(s) \geq 0\} \\ \underline{S_2} &= \{s \in S; \pi(s) - w(s) < 0\},\end{aligned}$$

A sheltered principal must then solve the following problem:

$$\max_{w(s), a} \int_{s \in \overline{S_2}} v(\pi(s) - w(s)) dF(s; a) + v(0) \cdot F(s_2; a) \quad (6)$$

subject to

$$\begin{aligned}\int_{s \in S} u(w(s)) dF_a(s; a) - c'(a) &\geq 0, \quad (\gamma_2) \\ \int_{s \in S} u(w(s)) dF(s; a) - c(a) &\geq U_0, \quad (\mu_2)\end{aligned}$$

The Kuhn-Tucker conditions applied to this problem give rise to two distinct expressions. That is:

$$\frac{v'(\pi(s) - w(s))}{u'(w(s))} = \mu_2 + \gamma_2 \frac{f_a(s; a)}{f(s; a)}, \quad \forall s \in \overline{S_2} \quad (7)$$

and

$$u'(w(s)) \cdot f(s; a) \left(\mu_2 + \gamma_2 \frac{f_a(s; a)}{f(s; a)} \right) \geq 0 \text{ for } \widehat{s} < s \leq s_2 \text{ and } \leq 0 \text{ for } s \leq \widehat{s} \quad (8)$$

Condition (7) is the same as in the benchmark case while condition (8) entails a constant floor wage. The optimal incentive scheme is thus similar, on the upside, to the one prescribed in Proposition 1.

Proposition 2 *Suppose that the principal is more prudent than the agent by a factor k . If net profits are prevented to fall below zero, then:*

a) *The optimal wage schedule is such that (i) $w_2(s) = w_2(s_2) = 0$ for all adverse signals $s \in \underline{S_2}$, and (ii) $w_2(s)$ is $(\delta_2(k), 0)$ -concave on $\overline{S_2} = S \setminus \underline{S_2}$, where the number $\delta_2(k)$ decreases with k and tends to 0 as k grows.*

b) *The incentive scheme $w_2(s)$ is less approximately concave on the subset $\overline{S_2}$ than the one $w^*(s)$ in Theorem 1. In other words, $\delta_2(k) < \delta(k)$ for any given k .*

¹⁷As an illustration, recall that, during the 2008 financial crisis, some banks disclosed their financial losses after having set aside provisions to pay their traders.

Limiting the principal's losses induces therefore additional convexity in the agent's wage schedule, as the CEO also benefits from the firm's rescue when profits $\pi(s)$ turn negative. This allows again the principal to save on costly upside payments, so the pay-performance relationship should approach concavity more closely on the upside.

These prescriptions may not coincide with the prevailing practices of some industries (investment banking, notably). We shall now examine whether taxation can better support the evidence.

5 Taxation and convexity

The second departure from our benchmark model is to introduce personal and corporate taxes. Tax laws are often pointed out as a key rationale for the popularity of stock options and other components of executive compensation (see, e.g., Smith and Watts 1982; Murphy 1999; Hall and Murphy 2003; Aboody and Kasznik 2009; Dittman and Maug 2007; Murphy 2011) and for discrepancies in executive pay across countries (Thomas 2008). Taxation - progressive taxation notably - is also periodically mentioned as a means to moderate what many people regard as excessive rewards to some corporate members (see Rose and Wolfram 2002 for an empirical study). In 2009, for example, U.S. President Barack Obama and U.K. Prime Minister Gordon Brown respectively gazed at taxing traders or banks to precisely meet this concern. In what follows, we investigate the ramifications such proposals could have for the agent's incentive scheme.¹⁸

5.1 Income taxation

Assume that the agent has to pay a tax when her income is positive, the tax rate τ being positive and nondecreasing in the agent's revenue. Such progressive taxation exists in several countries. In the United States, for example, the effective tax rate for revenues around \$266,000 is 20.1%, while it is 20.9% for incomes around \$610,000.¹⁹ In the Netherlands, the first 200,000 euros of taxable income are subject to a tax rate of 20%, and the rate on further income is 25,5%. Meanwhile, France's president François

¹⁸Taxation can influence the level and composition of CEO compensation in many more ways than the ones explored in this section (by applying different tax rates to different sources of income, notably). A thorough study of all possible channels is of course beyond the scope of this article. For a recent survey of the empirical literature, we refer the interested reader to Aboody and Kasznik (2009)'s section 4.

¹⁹The US Congressional Budget Office calculates effective tax rates by dividing taxes paid with comprehensive household income. The latter "equals pretax cash income plus income from other sources. Pretax cash income is the sum of wages, salaries, self-employment income, rents, taxable and nontaxable interest, dividends, realized capital gains, cash transfer payments, and retirement benefits (...). Other sources of income include all in-kind benefits (Medicare, Medicaid, employer-paid health insurance premiums, food stamps, school lunches and breakfasts, housing assistance, and energy assistance)."

Hollande has made it a binding electoral promise to impose a special tax rate of 75% on annual incomes larger than one million euros.

Since the agent's payoff grows with the signal s , let us then write the tax rate as a function $\tau(s)$ of s with $\tau'(s) \geq 0$ at all s where $w(s) > 0$. To keep matters simple, we suppose that $\tau''(s) = 0$. Using the notation $\overline{S}_3 = \{s \in S; w(s) > 0\}$ and $\underline{S}_3 = \{s \in S; w(s) \leq 0\}$, the optimal incentive scheme must now solve the following problem:

$$\max_{w(s), a} \int v(\pi(s) - w(s)) dF(s; a) \quad (9)$$

subject to

$$\int_{s \in \overline{S}_3} u((1 - \tau(s))w(s)) dF_a(s; a) + \int_{s \in \underline{S}_3} u(w(s)) dF_a(s; a) - c'(a) \geq 0, \quad (\gamma_3)$$

$$\int_{s \in \overline{S}_3} u((1 - \tau(s))w(s)) dF(s; a) + \int_{s \in \underline{S}_3} u(w(s)) dF(s; a) - c(a) \geq U_0, \quad (\mu_3)$$

The first-order conditions are then given by:

$$\frac{v'(\pi(s) - w(s))}{(1 - \tau(s))u'((1 - \tau(s))w(s))} = \mu_3 + \gamma_3 \frac{f_a(s; a)}{f(s; a)}, \quad \forall s \in \overline{S}_3 \quad (10)$$

and

$$\frac{v'(\pi(s) - w(s))}{u'(w(s))} = \mu_3 + \gamma_3 \frac{f_a(s; a)}{f(s; a)}, \quad \forall s \in \underline{S}_3 \quad (11)$$

Taking the first and second derivatives of the left-hand side of the latter expressions leads to a perhaps surprising (albeit intuitive) conclusion.

Proposition 3 *Assume that the principal is more prudent than the agent by a factor k , that a non-decreasing linear tax rate $\tau(s)$ applies to the agent's positive income, and that the agent's net income $(1 - \tau(s))w(s)$ is nondecreasing in s . Then:*

(i) *When the agent is risk neutral and the tax rate is constant, the optimal wage schedule is concave at any state s in S .*

(ii) *When the agent is risk-averse, the principal being more prudent than the agent by a factor k no longer suffices to make the optimal wage schedule approximately-concave on S .*

This proposition compares a situation with (i) a constant tax rate to another (ii) where it is progressive. Each fiscal policy has of course a specific impact on the agent's behavior and affects therefore the optimal compensation scheme set by the principal. In the former case ($\tau'(s) = 0$), the principal's prudence prevails and the pay-performance relationship remains concave when the agent is risk neutral. For any remuneration package, however, an increasing tax function ($\tau'(s) > 0$) weakens more and more the agent's incentives as his efforts yield better results. This might induce even a principal that is a lot more prudent than the agent to find approximately concave incentive pay inappropriate and offer instead a reward

function that becomes steeper as s goes up. Progressive taxation might thus bring about convex reward schemes, despite the fact that the principal is prudent, and despite the often-explicit intent of such fiscal policy to curb executive revenues.²⁰

5.2 Profit taxation

Suppose now that the principal's positive net profit is subject to a constant tax rate θ . An optimal incentive scheme must then solve:

$$\max_{w(s), a} \int_{s \in \overline{S_4}} v((1-\theta)(\pi(s) - w(s))) dF(s; a) + \int_{s \in \underline{S_4}} v(\pi(s) - w(s)) dF(s; a) \quad (12)$$

subject to

$$\int_{s \in S} u(w(s)) dF_a(s; a) - c'(a) \geq 0, \quad (\gamma_4)$$

$$\int_{s \in S} u(w(s)) dF(s; a) - c(a) \geq U_0, \quad (\mu_4)$$

where $\overline{S_4} = \{s \in S; \pi(s) - w(s) \geq 0\}$ and $\underline{S_4} = \{s \in S; \pi(s) - w(s) < 0\}$.

The first-order conditions in this case are given by:

$$\frac{(1-\theta)v'((1-\theta)(\pi(s) - w(s)))}{u'(w(s))} = \mu_4 + \gamma_4 \frac{f_a(s; a)}{f(s; a)}, \forall s \in \overline{S_4} \quad (13)$$

$$\frac{v'(\pi(s) - w(s))}{u'(w(s))} = \mu_4 + \gamma_4 \frac{f_a(s; a)}{f(s; a)}, \forall s \in \underline{S_4} \quad (14)$$

The principal's and the agent's relative prudence, measured by the factor k , will now have to be considered with respect to a given taxation policy. Proceeding as before yields our last result.

Proposition 4 *Assume that the principal is more prudent than the agent by a factor k , and that a constant tax rate θ applies to the principal's net profit when it is positive. Then:*

- (i) *the optimal wage schedule is $(\delta_4(k), 0)$ -concave when net profits are negative, where the number $\delta_4(k)$ gets smaller with k and tends to 0 as k grows;*
- (ii) *the optimal wage schedule is $(\delta_4(k, \theta), 0)$ -concave when net profits are positive, where the number $\delta_4(k, \theta)$ gets smaller with k and tends to 0 as k grows;*
- (iii) *$\delta_4(k, \theta)$ increases with θ , so the higher the tax rate θ on corporate profits the cruder the pay-performance concavity.*

Statements (i) and (ii) coincide with the Theorem's conclusion. Part (iii) raises again the possibility of convexified incentive schemes. This end result has already received some empirical support (Babenko

²⁰ The same distortive power of taxation has already been emphasized by Eeckhoudt et al. (2009) in a different setting (with perfect information). Their result is that a progressive (increasing and concave) taxation scheme can actually make an otherwise risk neutral manager behave as if he were risk averse and prudent.

and Tserlukevich 2009). The rationale we offer here runs as follows. Recall that a prudent or downside risk averse principal worries more about the variability of net profits ($\pi(s) - w(s)$) in bad states than in good states. Taxation on positive net profits makes the good states no longer as good as before. The principal then becomes more sensitive to net profit variations in the now not-so-good states. Temptation to have the agent bear more of these variations grows and might finally prevail under very high taxes. When corporate profits are taxed, in sum, a prudent board might nevertheless prefer to grant the CEO more and more revenue as the firm's performance gets better and better, especially if this motivates him further without changing the expected tax bill by much.

With these final remarks, Section 5 is now complete. Although the above propositions were set to be normative statements, they can also be seen as making a number of testable empirical predictions. To make this clearer, the next section will briefly look at how certain key notions such as approximately concave incentive wages and prudent boards might be used by empiricists.

6 Empirical ramifications

This paper's main theorem and four propositions relate a corporate board's relative prudence to how well the resulting incentive scheme will approximate a concave function.

A natural way to assess the approximate concavity of a CEO's pay with respect to performance would be to check the relative weight the actual incentive package gives to capped bonuses and similar concave devices, compared to other means like call options that tend to convexify remuneration.

To estimate a board's prudence, one might draw from the empirical literature on accounting conservatism. As we argued in the introduction, conservatism is akin to prudence.²¹ Factors that drive conservative practices could then be seen as proxies for a board's degree of prudence. For instance, Garcia Lara et al. (2009) find that strong governance firms - characterized by low level of antitakeover protection, more outside directors, more frequent board meetings (which may incidentally signal downside risk aversion), etc. - show significantly higher levels of accounting conservatism. Ahmed and Duellman (2007) report that the percentage of inside directors is negatively related to conservatism, while the percentage of outside directors' shareholdings is positively related to conservatism. According to Ramalingegowda and Yu (2012), ownership by monitoring institutions is positively associated with conservatism, and the relationship holds more strongly for firms facing better growth prospects and higher information asymmetry. Hui et al. (2012) add on that a firm displays greater conservatism in financial reporting when its suppliers or customers have more bargaining power and important long-term stakes with it. There is also

²¹In their empirical study of the relationship between institutional ownership and conservatism, Ramalingegowda and Yu (2012, p. 100) even assert that managerial incentives should be somewhat concave under accounting conservatism: "To the extent that managerial compensation is tied to earnings, conservatism penalizes managers for their failures (economic losses) in a timely manner but defers rewards for their successes (economic gains) until benefits are realized (...)."

evidence of more conservative accounting when the firm’s external auditor has greater bargaining power. Some recent works suggest, finally, that female CFOs tend to adopt more conservative financial reporting policies (Francis et al. 2009).

What matters, however, is not only whether the board is prudent but also (and more importantly) how much more prudent it is than the CEO. This can in turn be captured by gauging the influence the CEO has on the board (higher CEO turnover, for instance, will indicate less influence, but whether the same person holds both the CEO and chairman of the board titles would signal the opposite). The greater the CEO’s power, the lower the factor k in the above theorem and propositions, so the more incentive pay could depart from real concavity.²²

As the discussions following our results suggested, finally, in running empirical tests one might have to control for regulation, institutions, and the firm’s industry. Drawing again from the literature on accounting conservatism, it has been found, for instance, that “(...) litigation under the Securities Acts encourages conservatism because litigation is much more likely when earnings and net assets are overstated, not understated.” (Watts 2003a, p. 216) Also, “(...) common law countries’ use of published financial accounting statement numbers in contracts causes those countries’ earning numbers to be more conservatives than those of code law countries.” (Watts 2003b, p. 293) Further public policy implications are discussed in the next final section.

7 Concluding remarks and some policy observations

Assuming the principal is not risk neutral is somewhat unusual in the managerial accounting and corporate governance literatures. Yet, prudence for example - now understood and formally defined in economics as aversion to downside risk (Menezes et al. 1980; Eeckhoudt and Schlesinger 2006; Crainich and Eeckhoudt 2008) - corresponds to well-documented behavioral characteristics of investors (Harvey and Siddique 2000; Ang et al. 2006) and the fact that financial reporting is often subject to conservatism (Watts 2003ab). It also seems to capture fairly well the requirement that corporate boards exercise a *fiduciary duty of care* to protect their corporation’s interest (Clark 1985; Blair and Stout 1999; Gutierrez 2003; Lan and Herakleous 2010). Taking stock of these observations and analyses, this paper examined what should happen to an agent/CEO’s incentive compensation under a *prudent* principal/corporate board. We found that the CEO’s contract should then be approximately concave (in the sense of Páles 2003) in the performance signal. As the board exhibits greater prudence relative to its top manager, moreover, the pay-performance relationship should lean more towards real concavity.

²²The analogy with accounting conservatism comes short here, as Ahmed and Duellman (2007, p. 412) find that “CEO/chair separation is unrelated to accounting conservatism in all specifications.”

When the principal is prudent, the optimal contract trades off *downside risk* and incentives. An approximately concave incentive scheme clearly shifts some downside risk upon the agent, as remuneration follows the pattern shown in Figure 1 and is on the whole more sensitive to improved performance at the lower levels than across the highpoints. By doing so, a prudent principal increases the agent's motivation to keep the firm away from the worst.

Interestingly, several government policies that were adopted in the aftermath of the 2008 financial debacle can be viewed as surrogates for having prudent corporate boards. The 2010 *Dodd-Frank Wall Street Reform and Consumer Protection Act* in the United States, for instance, contains three key measures to 'concavify' the incentive compensation of CEOs and top executives.²³ First, empirical work so far suggests that providing shareholders with a regular vote on pay, as the 'Say-on-Pay' clause does, is likely to increase the sensitivity of CEO remuneration to poor accounting performance (see Ferri and Maber 2013). Second, mandatory 'hedging policies' - which ask companies to disclose whether or not employees or directors are allowed to use financial instruments to offset a fall in the market value of equity securities granted as compensation - will certainly deter some means to avoid downside risk which would otherwise tend to 'convexify' the pay-performance relationship (as shown in subsection 4.1 above). Third, asking firms to both disclose total payments made to the CEO and the median of overall compensation awarded to all employees may well put an implicit cap on the growth rate of CEO rewards. To see if this truly happens is left to future empirical work, of course, and the above Section 6 makes suggestions on how this could be done. Whether such regulations are more effective and efficient than fostering fiduciary duties and good governance in the first place also remains to be seen.²⁴

To be sure, however, not all public policies that preceded or followed financial crises have been supportive of exposing CEOs to more downside risk. As mentioned in the previous subsection, the 1993 *Omnibus Budget Reconciliation Act* in the U.S., for instance, which made CEO gains above \$1 million non-deductible if not performance-based, is well-known to have encouraged companies to 'convexify' incentive pay by granting more stock options (Murphy 2011). As Proposition 2 shows, firm rescue programs such as TARP tend similarly to support sheltering CEOs from downside risk. Proposition 3's prescription, finally, suggests that beefing up the tax rate on high incomes, as put forward by the French government, may well lead even prudent corporate boards to go for convex incentive schemes.²⁵

²³ For a summary and some comments on these measures, see Murphy (2012, p. 47-51) and Equilar (2010).

²⁴ The matter of regulatory efficiency seems particularly relevant nowadays, considering the recent sharp declines in initial public offerings (IPOs) and the number of public companies. Reporting on this phenomenon in its May 19th 2012 edition, *The Economist* (p. 30) contends that: "Public companies have always had to put up with more regulation than private ones because they encourage ordinary people to risk their capital. But the regulatory burden has become heavier, especially after the 2007-08 financial crisis."

²⁵ While the French government's intention clearly seems to tackle what are deemed to be illegitimate pay levels in times

Exploring the tradeoff between downside risk and incentives is only beginning, of course. Approximately concave incentive schemes have their supporters (e.g., Benmelech et al. 2010) and their detractors. Siding with the latter, Murphy and Jensen (2011) point out, for instance, that managers subject to concave bonuses are encouraged to smooth performance across periods, thereby hiding superior results at one time in order to use them later when facing harsher circumstances. The process of setting incentive contracts for CEOs is indeed a complex one. It involves negotiations and third-party inputs (from consultants, other employees, etc.) which are open to manipulations, power struggles and conflicts of interest. Shareholders, directors and other stakeholders can also be quite heterogeneous, so it might not be clear whose risk preferences should be taken into account after all. The choice of an incentive scheme is furthermore subject to other criteria, such as attracting and retaining talented people. This paper thus represents only one additional step towards building a complete, integrated and operational normative framework for the design of executive incentive contracts.

APPENDIX

Proof of Lemma 1.

We want to show that $\omega(z) = u(h^{-1}(z))$ concave. Recall that $(h^{-1})'(z) = \frac{1}{h'(h^{-1}(z))}$. By the Chain Rule:

$$\begin{aligned} \frac{du(h^{-1}(z))}{dz} &= u'(h^{-1}) \cdot \frac{1}{\frac{-v''u' - v'u''}{(u')^2}} \\ &= -\frac{(u')^3}{v''u' + v'u''} > 0 \end{aligned}$$

so

$$\begin{aligned} &\frac{d^2u(h^{-1}(z))}{dz^2} \\ &= -\frac{3(u')^2u''(v''u' + v'u'') - (u')^3 \cdot (-v'''u' + v''u'' - v'u''' + v'u''')}{(v''u' + v'u'')^2} \\ &= (-3u''(v''u' + v'u'') + u' \cdot (-v'''u' + v''u'' - v'u''' + v'u''')) \cdot \frac{(u')^2}{(v''u' + v'u'')^2} \\ &= (-3R_u(R_v + R_u) + (d_u - d_v)) \cdot \frac{(u')^2}{v' \cdot (u')^2 \cdot (v''u' + v'u'')^2} . \end{aligned}$$

The latter term is certainly negative if $d_v > d_u$, i.e. as long as the principal is more prudent than the agent in the sense of Definition 1. ♦

of crisis, the upshot will be to restore the incentive structure that might have contributed to the pre-2008 financial bubble.

Proof of Lemma 2.

Risk aversion of at least one player is sufficient to obtain that $w^{*'}(s) \geq 0$. Indeed we have, with $v(\pi(s) - w(s))$ denoted as $v(\cdot)$ and $u(w(s))$ denoted as $u(\cdot)$:

$$\begin{aligned} & \frac{\partial}{\partial s} \left(\frac{v'(\pi(s) - w(s))}{u'(w(s))} \right) \\ = & \frac{(\pi'(s) - w'(s)) \cdot v''(\cdot) u'(\cdot) - v'(\cdot) \cdot u''(\cdot) \cdot w'(s)}{(u'(\cdot))^2} \\ = & \frac{-w'(s) \cdot (v''(\cdot) \cdot u'(\cdot) + v'(\cdot) \cdot u''(\cdot)) + \pi'(s) \cdot v''(\cdot) \cdot u'(\cdot)}{u'(\cdot)^2} \end{aligned}$$

The latter must be positive, by Assumption 1, in order to satisfy equation (3). A necessary condition for this is $w'(s) \geq 0$. ♦

Proof of the theorem.

Let us now compute the second derivative of the left-hand side term in equation (3). Assumption 1 entails it must be negative.

$$\begin{aligned} & \frac{\partial^2}{\partial s^2} \left(\frac{v'(\pi(s) - w(s))}{u'(w(s))} \right) \\ = & \frac{1}{(u'(\cdot))^4} \cdot (\{-w'' \cdot (v''u' + v'u'') - w' \cdot [(\pi' - w')v'''u' + w'v''u'' + (\pi' - w')v''u'' + w'v'u'''] \\ & + \pi''v''u' + \pi'[(\pi' - w')v'''u' + v''w'u'']\} \cdot u'^2 + 2u''u' \cdot w' \cdot [w'(v''u' + v'u'') - \pi'v''u']) \\ = & \frac{1}{u'^3} \cdot (\{-w'' \cdot (v''u' + v'u'') + (\pi' - w')^2v'''u' + (\pi' - w')w'v''u'' - w'^2v'u''' + \pi''v''u'\} \cdot u' \\ & - (\pi' - w')v''u''w'u'(u' + 2) + 2u''^2w'^2v') \\ = & \frac{1}{u'^3} \cdot (\{-w'' \cdot (v''u' + v'u'') + (\pi' - w')^2v'''u' - w'^2v'u''' + \pi''v''u'\} \cdot u' \\ & - 2w'u'' \cdot ((\pi' - w')v''u' - v'u''w')) \\ = & \frac{1}{u'^2} \cdot [-w'' \cdot (v''u' + v'u'') + (\pi' - w')^2v'''u' - w'^2v'u''' + \pi''v''u'] \\ & + 2w'R_u \cdot \frac{((\pi' - w')v''u' - v'u''w')}{u'^2} \end{aligned}$$

The last term here is in fact $\frac{\partial}{\partial s} \left(\frac{v'(\pi(s) - w(s))}{u'(w(s))} \right)$, and it is positive by Lemma 2. Then:

$$\begin{aligned}
& \frac{\partial^2}{\partial s^2} \left(\frac{v'(\pi(s) - w(s))}{u'(w(s))} \right) \\
= & 2w'R_u \cdot \frac{\partial}{\partial s} \left(\frac{v'(\pi(s) - w(s))}{u'(w(s))} \right) \\
& + \frac{1}{u'^2} \cdot [-w'' \cdot (v''u' + v'u'') + (\pi' - w')^2 v'''u' - w'^2 v'u''' + \pi''v''u'] \\
= & 2w'R_u \cdot \frac{\partial}{\partial s} \left(\frac{v'(\pi(s) - w(s))}{u'(w(s))} \right) \\
& + \frac{v'}{u'} \cdot [w'' \cdot (R_v + R_u) + (\pi' - w')^2 P_v R_v - w'^2 P_u R_u - \pi'' R_v] \\
= & 2w'R_u \cdot \frac{\partial}{\partial s} \left(\frac{v'(\pi(s) - w(s))}{u'(w(s))} \right) \\
& + \frac{v'}{u'} \cdot [w'' \cdot (R_v + R_u) - \pi'' R_v + (\pi' - w')^2 P_v R_v - w'^2 P_u R_u]
\end{aligned}$$

The sign of this last expression depends on the sign of $(\pi' - w')^2 P_v R_v - w'^2 P_u R_u$. Two cases are possible. Either

$$\left(\frac{\pi'(s) - w'(s)}{w'(s)} \right)^2 \geq \frac{1}{k} ,$$

and it is then necessary that $w''(s) < 0$ so w is concave at s , or

$$\left(\frac{\pi'(s) - w'(s)}{w'(s)} \right)^2 < \frac{1}{k} . \quad (15)$$

Recall that the derivatives $w'(s)$ are Lipschitz continuous by assumption. Thus there exist positive real numbers M_i associated with each D_i , $i = 1..n$, of a finite partition $D_1, D_2, \dots, D_n$ of S such that $w'(s) < M_i$ on each D_i . In the situation (15), let $M = \max_i M_i$, and take $\delta(k) > 0$ so that $\left(\frac{\delta}{4M} \right)^2 = \frac{1}{k}$. Inequality (15) is equivalent to

$$(\pi'(s) - w'(s))^2 < \left(\frac{\delta(k)}{4M} \right)^2 (w'(s))^2 .$$

For all $x \in S$, $x \neq s$, then:

$$\begin{aligned}
|(\pi'(s) - w'(s))(x - s)| & < \left(\frac{\delta(k)}{4M} \right) w'(s) |x - s| \\
& < \frac{\delta(k)}{4} |x - s| ,
\end{aligned}$$

since we have $\pi'(s) \geq w'(s)$ by assumption and, in particular, $\max_{s \in S} \pi'(s) \geq w'(s)$. So

$$w'(s)(x - s) < \pi'(s)(x - s) + \frac{\delta(k)}{4} |x - s| .$$

Now, since w is differentiable at s , we have (by definition) that

$$w(x) = w(s) + w'(s)(x - s) + r(x)$$

with the residual $r(x)$ such that $\lim_{x \rightarrow s} \frac{r(x)}{x - s} = 0$. The last inequality entails that

$$\begin{aligned} w(x) &< w(s) + \pi'(s)(x - s) + \left(\frac{r(x)}{|x - s|} + \frac{\delta(k)}{4} \right) |x - s| \\ &\leq w(s) + \pi'(s)(x - s) + \frac{\delta(k)}{2} |x - s| \end{aligned}$$

if x is sufficiently close to s . Since $\pi'(s)$ is decreasing in s by assumption, the latter and Lemma 3 mean that $w(s)$ is $(\delta(k), 0)$ -concave on a subinterval of S that contains s . Since this is to be true at any point s , keeping the same number $\delta(k)$, then $w^*(s)$ is $(\delta(k), 0)$ -concave on S . ♦

Proof of Proposition 1.

For any state s in $\overline{S_1}$, we have $\lambda(s) = 0$ and, following the proof of the theorem, the optimal reward function is increasing and $(\delta_1(k) - 0)$ -concave with $\delta_1(k)$ defined below.

Besides, the limited liability constraint is binding for any signal s in $\underline{S_1}$. This proves a).

To show b), let $M' = \max_{D_i \cap \overline{S_1} \neq \emptyset} M_i$ on the neighborhoods D_i having a nonempty intersection with $\overline{S_1}$. Clearly, we have $M' = \max_{D_i \cap \overline{S_1} \neq \emptyset} M_i \leq M = \max_{all D_i} M_i$. From the proof of the theorem, we have $(\frac{\delta}{4M})^2 = \frac{1}{k} = (\frac{\delta_1}{4M'})^2$ implies $\delta_1(k) \leq \delta(k)$ for any given k . ♦

Proof of Proposition 2.

The Lagrangian function for this problem is given by

$$\begin{aligned} \mathcal{L} &= \int_{s \in \overline{S_2}} v(\pi(s) - w(s)) dF(s; a) + v(0) \cdot F(s_2; a) \\ &\quad + \gamma_2 \left(\int_{s \in S} u(w(s)) dF_a(s; a) - c'(a) \right) + \mu_2 \left(\int_{s \in S} u(w(s)) dF(s; a) - c(a) - U_0 \right) \end{aligned}$$

Two expressions must be considered when verifying the first-order conditions for maximizing $\mathcal{L}$. If $s \in \overline{S_2}$, we have

$$\frac{v'(\pi(s) - w(s))}{u'(w(s))} = \mu_2 + \gamma_2 \frac{f_a(s; a)}{f(s; a)}, \quad \forall s \in \overline{S_2} \quad (16)$$

and, in particular,

$$\begin{aligned} \lim_{s \downarrow s_2} \frac{v'(\pi(s) - w(s))}{u'(w(s))} &= \lim_{s \downarrow s_2} \left(\mu_2 + \gamma_2 \frac{f_a(s; a)}{f(s; a)} \right) \\ \frac{v'(\pi(s_2) - w(s_2))}{u'(w(s_2))} &= \mu_2 + \gamma_2 \frac{f_a(s_2; a)}{f(s_2; a)} \end{aligned} \quad (17)$$

For $s \in \underline{S}_2$, we have

$$\frac{\partial \mathcal{L}}{\partial w(s)} = u'(w(s)) \cdot f(s; a) \left(\mu_2 + \gamma_2 \frac{f_a(s; a)}{f(s; a)} \right), \quad \forall s \in \underline{S}_2 \quad (18)$$

To Expression (16), one can apply the reasoning used in the proof of the theorem. Thus, the optimal reward function is $(\delta_2(k), 0)$ -concave on $\overline{S}_2$. This is point (ii) in a).

For point i), notice that, from (17), we have that $\mu_2 + \gamma_2 \frac{f_a(s; a)}{f(s; a)} > 0$. This expression being continuous and non decreasing in s (Assumption 3), there exists a state $\hat{s} < s_2$ such that $\mu_2 + \gamma_2 \frac{f_a(s; a)}{f(s; a)} > 0 \forall s \in]\hat{s}, s_2[$ and $\mu_2 + \gamma_2 \frac{f_a(s; a)}{f(s; a)} \leq 0 \forall s \leq \hat{s}$. With $\frac{\partial^2 \mathcal{L}}{\partial w(s)^2} = u''(w(s)) \cdot f(s; a) \left(\mu_2 + \gamma_2 \frac{f_a(s; a)}{f(s; a)} \right)$, this implies that

$$\frac{\partial \mathcal{L}}{\partial w(s)} > 0 \quad \text{and} \quad \frac{\partial^2 \mathcal{L}}{\partial w(s)^2} < 0, \quad \forall s \in]\hat{s}, s_2[\quad (19)$$

$$\frac{\partial \mathcal{L}}{\partial w(s)} \leq 0 \quad \text{and} \quad \frac{\partial^2 \mathcal{L}}{\partial w(s)^2} > 0, \quad \forall s \leq \hat{s} \quad (20)$$

The Lagrangian function $\mathcal{L}$ being concave in $w(s)$ on $] \hat{s}, s_2[$ and convex otherwise, the optimal revenue is the maximum possible one for any $s \leq s_2$. More precisely, $w(s)$ being continuous and increasing on $\overline{S}_2$, the reward schedule satisfies $w_2(s) = w_2(s_2) = \pi(s_2) = 0$ for any $s \leq s_2$. This is point (i).in a).

Finally, we can apply the reasoning of the proof of Point b) of Proposition 1 in order to show that $\delta_2(k) < \delta(k)$ for any given k . ♦

Proof of Proposition 3.

Consider condition (10). Let $A = \frac{\partial}{\partial s}((1 - \tau(s))w(s))$, which is assumed to be positive. We have that

$$\begin{aligned} & \frac{\partial}{\partial s} \left(\frac{v'(\pi(s) - w(s))}{(1 - \tau(s))u'((1 - \tau(s))w(s))} \right) \\ &= \frac{(\pi' - w')v''(1 - \tau)u' - v'(u''A(1 - \tau) - \tau'u')}{(1 - \tau)^2 u'^2} \\ &= \frac{(\pi' - w')v'' + v'(R_u A + \tau'/(1 - \tau))}{(1 - \tau)u'} \end{aligned}$$

Now, write $B = \frac{\partial}{\partial s}(\frac{\tau'(s)}{(1 - \tau(s))}) = \frac{\tau''(1 - \tau) + 2\tau'}{(1 - \tau)^2}$. With $\tau'' = 0$ by assumption, the latter simplifies to $B = \frac{2\tau'}{(1 - \tau)^2}$. Then:

$$\begin{aligned}
& \frac{\partial^2}{\partial s^2} \left(\frac{v'(\pi(s) - w(s))}{(1 - \tau(s))u'((1 - \tau(s))w(s))} \right) \\
&= \frac{1}{(1 - \tau)u'} \cdot [(\pi'' - w'').v'' + (\pi' - w')^2 v'''] \\
&\quad + v''(\pi' - w')(R_u A + \tau'/(1 - \tau)) + v'(R'_u A + R_u A' + B)] \\
&\quad - \frac{1}{(1 - \tau)^2 u'^2} \cdot [((1 - \tau)A u'' - \tau' u')((\pi' - w')v'' + v'(R_u A + \tau'/(1 - \tau)))] \\
&= \frac{1}{(1 - \tau)u'} \cdot [(\pi'' - w'').v'' + (\pi' - w')^2 v'''] \\
&\quad + v''(\pi' - w')(R_u A + \tau'/(1 - \tau)) + v'(R'_u A + R_u A' + B) \\
&\quad + (AR_u + \tau'/(1 - \tau))((\pi' - w')v'' + v'(R_u A + \tau'/(1 - \tau)))]
\end{aligned}$$

Note that $A' = (1 - \tau)w'' - 2\tau'w'$. The latter expression becomes

$$\begin{aligned}
& \frac{\partial^2}{\partial s^2} \left(\frac{v'(\pi(s) - w(s))}{(1 - \tau(s))u'((1 - \tau(s))w(s))} \right) \\
&= \frac{1}{(1 - \tau)u'} \cdot [(\pi''v'' + w''(v'R_u(1 - \tau) - v'')) + (\pi' - w')^2 v'''] \\
&\quad + v''(\pi' - w')(R_u A + \tau'/(1 - \tau)) + v'(R'_u A - 2R_u \tau'w' + B) \\
&\quad + (AR_u + \tau'/(1 - \tau))((\pi' - w')v'' + v'(R_u A + \tau'/(1 - \tau)))] \\
&= \frac{1}{(1 - \tau)u'} \cdot [(\pi''v'' + w''v'(R_u(1 - \tau) + R_v)) \tag{21}
\end{aligned}$$

$$+ (\pi' - w')^2 v''' + v''(\pi' - w')(R_u A + \tau'/(1 - \tau)) + v'(R'_u A - 2R_u \tau'w' + B) \tag{22}$$

$$+ (AR_u + \tau'/(1 - \tau))((\pi' - w')v'' + v'(R_u A + \tau'/(1 - \tau)))] \tag{23}$$

The first term in line (21), namely $\pi''v''$, is positive by assumption. The sign of the second term depends on the sign of w'' . Lines (22) and (23) now remain to be signed. The sum of these lines can be written:

$$\begin{aligned}
& (\pi' - w')^2 v''' + v''(\pi' - w')(R_u A + \tau'/(1 - \tau)) + v'(R'_u A - 2R_u \tau'w' + B) \\
& + (AR_u + \tau'/(1 - \tau))((\pi' - w')v'' + v'(R_u A + \tau'/(1 - \tau))) \\
&= v' \cdot [(\pi' - w')^2 .d_v + R'_u A - R_v(\pi' - w')(R_u A + \tau'/(1 - \tau)) - 2R_u \tau'w' + B \\
& + (AR_u + \tau'/(1 - \tau)) \cdot (R_u A + \tau'/(1 - \tau) - (\pi' - w').R_v)]
\end{aligned}$$

With $R'_u = \frac{\partial R_u}{\partial s} = AR_u(R_u - P_u) = AR_u^2 - A.d_u$, and $B = \frac{2\tau'}{(1 - \tau)^2}$, it becomes

$$\begin{aligned}
& v' \cdot [(\pi' - w')^2 \cdot d_v - (1 - \tau)^2 \cdot w'^2 \cdot d_u \\
& + A^2 R_u^2 - R_v(\pi' - w')(R_u A + \tau'/(1 - \tau)) - 2R_u \tau' w' + B \\
& + (AR_u + \tau'/(1 - \tau)) \cdot (R_u A + \tau'/(1 - \tau) - (\pi' - w') \cdot R_v)] \\
= & v' \cdot [(\pi' - w')^2 \cdot d_v - (1 - \tau)^2 \cdot w'^2 \cdot d_u \\
& + A^2 R_u^2 + (R_u A + \tau'/(1 - \tau))^2 \\
& - 2 \cdot (R_u \tau' w' + R_v(\pi' - w')(R_u A + \tau'/(1 - \tau)) - \tau'/(1 - \tau)^2)]
\end{aligned} \tag{24}$$

If the Agent is risk neutral ($R_u = 0$), and the tax rate is constant ($\tau'(s) = \tau \forall s$), the latter reduces to

$$v' \cdot [(\pi' - w')^2 \cdot d_v - (1 - \tau)^2 \cdot w'^2 \cdot d_u] \tag{25}$$

In this case, the sign of $\frac{\partial^2}{\partial s^2} \left(\frac{v'(\pi(s) - w(s))}{(1 - \tau(s))u'((1 - \tau(s))w(s))} \right)$ depends on the sign of $(\pi' - w')^2 \cdot d_v - (1 - \tau)^2 \cdot w'^2 \cdot d_u$. By assumption, the net revenue of the agent is non decreasing, which implies that $0 < \tau < 1$. Thus we have:

$$\left(\frac{\pi'(s) - w'(s)}{(1 - \tau) \cdot w'(s)} \right)^2 > \left(\frac{\pi'(s) - w'(s)}{w'(s)} \right)^2$$

If the principal is more prudent than the agent by a factor k , we have $d_u/d_v < 1/k$. Then if $\left(\frac{\pi'(s) - w'(s)}{w'(s)} \right)^2 \geq \frac{1}{k}$ we have always $\left(\frac{\pi'(s) - w'(s)}{(1 - \tau) \cdot w'(s)} \right)^2 > \frac{1}{k}$. Thus, the function $w_3(s)$ is $(\delta_3(k), 0)$ -concave. Besides, if the agent is risk-neutral, k tends towards infinity and, from the theorem, $\delta_3(k)$ tends towards zero. The incentive wage schedule $w_3(s)$ is then $(0, 0)$ -concave, so concave. This demonstrates (i).

Now, consider a risk-averse, or risk-neutral, agent ($R_u \geq 0$) and a linear, non-decreasing and non-constant tax function: it satisfies $\tau' > 0$ and $\tau'' = 0$. Now, expression (24) holds. It is immediate to see that its sign does no longer only depend on the sign of $(\pi' - w')^2 \cdot d_v - (1 - \tau)^2 \cdot w'^2 \cdot d_u$. Thus despite the fact that $(\pi' - w')^2 \cdot d_v - (1 - \tau)^2 \cdot w'^2 \cdot d_u > 0$, it is no longer sufficient that the principal be more prudent than the agent as defined in Definition 1 to obtain the $(\delta_3(k), 0)$ -concavity of $w_3(s)$. This is Point ii). ♦

Proof of Proposition 4.

First write $C(s) = \frac{\partial}{\partial s} [(1 - \theta)(\pi(s) - w(s))] = (1 - \theta)(\pi'(s) - w'(s))$ We have for any $s \in \overline{S_4}$

$$\begin{aligned}
& \frac{\partial}{\partial s} \left(\frac{v'((1 - \theta)(\pi(s) - w(s)))}{u'(w(s))} \right) \\
= & \frac{(1 - \theta)(\pi' - w')v''u' - v'u''w'}{u'^2} \\
= & \frac{-w'(1 - \theta)(v''u' + v'u'') + (1 - \theta)\pi'v''u'}{u'^2} \\
= & \frac{v'(R_u w' - C R_v)}{u'}
\end{aligned} \tag{26}$$

>From (26), $w'(s) > 0$ is a necessary condition for $\frac{\partial}{\partial s} \left(\frac{v'((1 - \theta)(\pi(s) - w(s)))}{u'(w(s))} \right) > 0$.

Computation of the second derivative now gives

$$\begin{aligned}
& \frac{\partial^2}{\partial s^2} \left(\frac{v'((1-\theta)(\pi(s)-w(s)))}{u'(w(s))} \right) \\
&= \frac{C'v'' + C^2v''' + (R'_u v' + R_u C v'') w' + R_u v' w''}{u'} \\
&\quad - \frac{(Cv'' + R_u v' w') u'' w'}{(u')^2} \\
&= \frac{C'v'' + C^2v''' + (R'_u v' + R_u C v'') w' + R_u v' w'' + (Cv'' + R_u v' w') R_u w'}{u'} \\
&= \frac{v'(C^2 R_v P_v - C' R_v) + (R'_u - R_u C R_v) v' w' + R_u v' w'' + (R_u w' - C R_v) R_u w' v'}{u'} \\
&= \frac{v'}{u'} \cdot (C^2 R_v P_v - C' R_v + (R'_u - R_u C R_v) w' + R_u w'' + (R_u w' - C R_v) R_u w') \\
&= \frac{v'}{u'} \cdot [R_v(C^2 P_v - C') + (R'_u - R_u C R_v) w' + R_u(w'' + (R_u w' - C R_v) w')] \tag{27}
\end{aligned}$$

where $\frac{dR_u}{ds} = R'_u = -w'(d_u - R_u^2)$.

We must analyze the sign of the three components in the brackets in Equation (27). With $C = (1-\theta)(\pi' - w')$, we have:

$$\begin{aligned}
& R_v(C^2 P_v - C') + (R'_u - R_u C R_v) w' + R_u(w'' + (R_u w' - C R_v) w') \\
&= C^2 d_v - C' R_v - w'^2 d_u + w'^2 R_u^2 - R_u C R_v w' + R_u w' \cdot (R_u w' - C R_v) + R_u \cdot w'' \\
&= ((1-\theta)^2(\pi' - w')^2 \cdot d_v - w'^2 d_u) + 2w' \cdot R_u \cdot (R_u w' - C R_v) - (1-\theta)\pi'' R_v + ((1-\theta)R_v + R_u) \cdot w'' \tag{28}
\end{aligned}$$

The second term in (28) is equal to $\frac{2R_u w'(s) u'(s)}{v'(s)} \cdot \frac{\partial}{\partial s} \left(\frac{v'((1-\theta)(\pi(s)-w(s)))}{u'(w(s))} \right)$ and is positive. The third term is positive by assumption. Then the sign of $w''(s)$ for any $s \in \overline{S_4}$ depends on the sign of the first term, namely $(1-\theta)^2(\pi' - w')^2 \cdot d_v - w'^2 d_u$.

As for the proof of the theorem, two cases are possible. Either

$$\left(\frac{(1-\theta) \cdot (\pi'(s) - w'(s))}{w'(s)} \right)^2 \geq \frac{1}{k} ,$$

and, since $\frac{1}{k} > \frac{d_u}{d_v}$, the expression $(1-\theta)^2(\pi' - w')^2 \cdot d_v - w'^2 d_u$ is positive. It is then necessary that $w''(s) < 0$ so w_4 is concave at $s \in \overline{S_4}$, or

$$\left(\frac{(1-\theta) \cdot (\pi'(s) - w'(s))}{w'(s)} \right)^2 < \frac{1}{k} .$$

It can also be written

$$\left(\frac{\pi'(s) - w'(s)}{w'(s)} \right)^2 < \frac{1}{k \cdot (1-\theta)^2} .$$

In the latter situation, let $M'' = \max_{D_i \cap \overline{S_4} \neq \emptyset} M_i$, and take $\delta_4(k, \theta) > 0$ so that

$$\left(\frac{\delta_4(k, \theta)}{4M''} \right)^2 = \frac{1}{k \cdot (1 - \theta)^2} . \quad (29)$$

The latter inequality is equivalent to

$$(\pi'(s) - w'(s))^2 < \left(\frac{\delta_4(k, \theta)}{4M''} \right)^2 (w'(s))^2 .$$

For all $x \in \overline{S_4}$, $x \neq s$, then:

$$\begin{aligned} |(\pi'(s) - w'(s))(x - s)| &< \left(\frac{\delta_4(k, \theta)}{4M''} \right) w'(s) |x - s| \\ &< \frac{\delta_4(k, \theta)}{4} |x - s| , \end{aligned}$$

so

$$w'(s)(x - s) < \pi'(s)(x - s) + \frac{\delta_4(k, \theta)}{4} |x - s| .$$

Now, since w is differentiable at s , we have (by definition) that

$$w(x) = w(s) + w'(s)(x - s) + r(x)$$

with the residual $r(x)$ such that $\lim_{x \rightarrow s} \frac{r(x)}{x - s} = 0$. The last inequality entails that

$$\begin{aligned} w(x) &\leq w(s) + \pi'(s)(x - s) + \left(\frac{r(x)}{|x - s|} + \frac{\delta_4(k, \theta)}{4} \right) |x - s| \\ &\leq w(s) + \pi'(s)(x - s) + \frac{\delta_4(k, \theta)}{2} |x - s| \end{aligned}$$

if x is sufficiently close to s . Since $\pi'(s)$ is decreasing in s by assumption, the latter and Lemma 3 mean that $w(s)$ is $(\delta_4(k, \theta), 0)$ -concave in $s \in \overline{S_4}$. Concerning the states $s \in \underline{S_4}$ no taxation applies because net profits are negative, we have $\delta_4(k, 0) = \delta_4(k)$ (see (29) above). Thus $w(s)$ is $(\delta_4(k), 0)$ -concave at $s \in \underline{S_4}$. This proves Point (i).

Point (ii) is immediate from (29): $\delta_4(k, \theta)$ is increasing in θ . ♦

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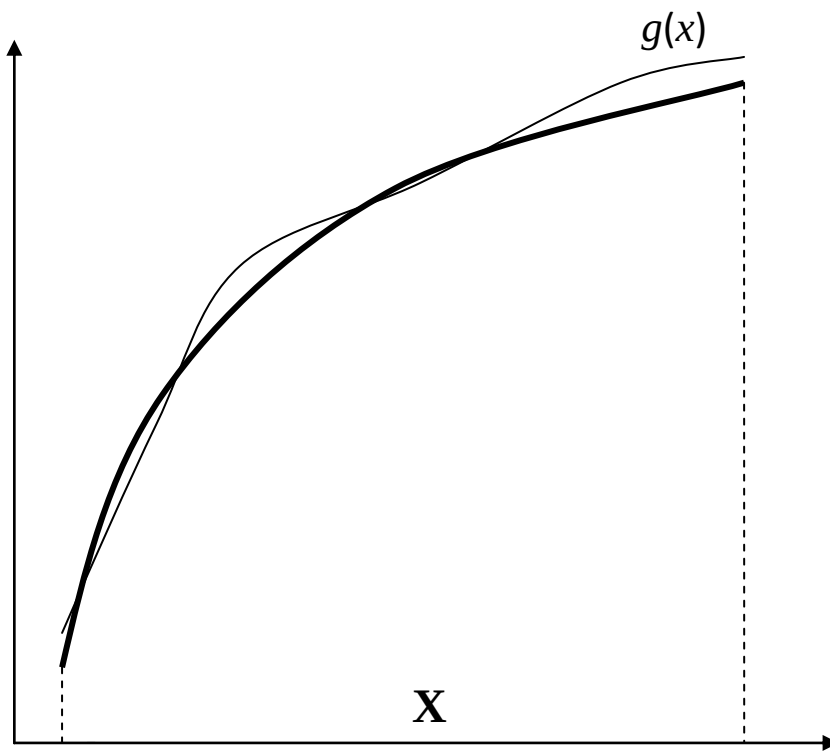


Figure 1. The function $g: X \rightarrow \mathbb{R}$ is $(\delta, 0)$ -concave on X .