

MATCHING BY LUCK OR SEARCH?

KUN LI

TOULOUSE SCHOOL OF ECONOMICS, FRANCE

MICHAEL PETERS

UNIVERSITY OF BRITISH COLUMBIA, CANADA

PAI XU

UNIVERSITY OF HONG KONG, HONG KONG

ABSTRACT. This paper provides a model of directed search in which workers have private information about type at the point where they make their applications to firms. Firms are able to observe these types once workers apply. The paper shows that for any smooth wage distribution there is a continuation equilibrium in which unemployed workers choose a reservation wage which is a strictly increasing function of their type, then apply with equal probability to all positions that offer more than that wage. We consider a case where matching occurs 'quickly', and show two main results. First, the wages at which workers are employed throughout their lives are correlated, but very imperfectly because of the fact that equilibrium involves a lot of mismatch. Second, the variance of future income of workers must be a decreasing function of the wage at which they are currently employed. In other words, high wage workers will enjoy more stable lifetime income.

These results make it possible to distinguish between the three main models of directed search empirically. The imperfect correlation - declining variance results in this paper contrast sharply with the classic directed search, where wages are uncorrelated over time, and models with assortative matching, in which wages are perfectly correlated over time.

The paper concludes with an analysis of data from the executive labor market from 1993 to 2009.

1. INTRODUCTION

This paper provides a dynamic extension of the directed search model in (Peters 2010) in which workers and firms have private information about their characteristics that drive their search behavior. The original paper was designed to illustrate the connection between unemployment duration and exit wage. However, in the context we

This is a **very** preliminary version of the paper. Though the theoretical section is more or less complete, the econometric part is still quite preliminary.

consider here, we are more interested in the matching that is supported. In particular we are interested in matching outcomes, and whether observable outcomes can be in any way understood using arguments from directed search. Directed search provides three different perspectives. In the most basic models of directed search (for example, (Peters 2000)), workers are identical but use mixed application strategies when they apply to firms, applying with highest probability at the firms who offer the highest prices. In the steady state of such a model, workers repeat this outcome in every period. Over their lifetimes, their matching outcomes will vary, but in a manner that exhibits no autocorrelation at all. A worker who is lucky enough to land a high paying job in some period will mix again after he becomes unemployed. So his future outcome will tend to see his wages fall.

At the other extreme, models that support pure assortative matching (for example (Eeckhout and Kircher 2008)) will predict that workers who land high wage jobs in one period will do so again in future periods. Theoretically, outcomes are perfectly correlated over time. The same kind of outcome could be expected from wage-ladder like models (e.g., (Delacroix and Shi 2006)) in which homogenous workers search on the job and implicitly use the current wage as a way of coordinating applications. Workers who are employed at some wage will apply to firms offering slightly higher wages until they are matched. Workers types are all the same, so there is no question about assortative matching. Yet the ladder like application behavior will mean that a worker's current outcome will be a good predictor of his future success.

In between these extremes is the model in (Peters 2010) in which workers have privately known types which matter to firms. The key difference is that workers use mixed application strategies with a simple form. Each worker adopts a 'reservation wage', which is an increasing function of the worker's type. Whenever the worker is unemployed, he or she makes applications to all the firms who offer wages above their reservation wage. Of course, the higher the worker's type, the more likely he is to be hired by the high wage firms. As a consequence, workers search outcomes are positively correlated with their type. As workers carry their types through time, search outcomes are positively correlated between periods. However, the correlation is far from perfect. Workers mixed strategies lead to mismatch. Though workers who become employed at low wage firms must have low enough types to make it sensible for them to apply to those firms, workers who find jobs at high wage firms may either have good types, or may just have gotten lucky. So good outcomes are very noisy signals of good types.

So the model predicts that the correlation between present and future outcomes fall as outcomes improve.

Moreover, since workers who have low types (and are employed at low wages) do sometimes get lucky and land high wage jobs. It doesn't work the other way around for high type workers, high wage workers aren't likely to face a large decline in income unless they are low types. As a consequence, the variance of future income should be larger for workers who have low wage outcomes than it is for workers with high wage outcomes. These correlations seem plausible, yet they aren't consistent with either assortative matching, or homogenous worker directed search. Wage ladder like on the job search suggest no difference in future income variance for low and high types. The reason is that workers who land high wage jobs by searching on the job suffer large income losses when they become unemployed.

To illustrate these things, this paper begins with a simple dynamic extension of the model in (Peters 2010). This section illustrates the properties of equilibrium that drive the main predictions. The dynamic arguments will also serve to illustrate how comparable arguments would work with the better known variants of the directed search model.

To check all these things, we turn to a dataset taken from the executive labor market. This market has a number of advantages from our perspective. First, executives' talent is a key input in the production (or, profit-generating) process. However executive talent is not captured by the number of MBA's or law degrees that an executive has. Rather, these talents seem largely interpreted as unobservables, such as connections with other executives, leadership ability, etc. Though these skills are unobservable to an outsider, firms seem to know them when they see them. Presumably reputation, reference letters, participation in successful projects are signals of managerial skill. At the same time, it is impossible to write a wage contract that conditions on these unobservables. This is the sort of environment which our theory fits.

Second, for executives in general, there are no segmented labor markets across industries, leadership skill is valuable in all industries. Thus we could effectively consider an integrated labor market for all executives.

2. FUNDAMENTALS

We begin with a description of equilibrium in a labor market in which worker types are privately known to them.

A labor market consists of measurable sets of positions and workers. We assume the measure of the set of positions is equal to the measure

of the set of workers, and normalize both to $\frac{1}{1-\gamma}$ where $0 < \gamma < 1$. The reason for this normalization will become clear below. Workers are parameterized by their *type* which is an element of some compact subset Y or $\mathbb{R}_+$. The measure of the set of workers in the population with types less than or equal to y is given by $\frac{F(y)}{1-\gamma}$, where F is some distribution function that is monotonically increasing and differentiable with support equal to some interval $Y = [\underline{y}, \bar{y}]$. Positions are characterized by some characteristic $x \in X$, where X is a compact subset of $\mathbb{R}$. The measure of position types is given $\frac{H}{1-\gamma}$ where H is a distribution function. Workers' types are private information when they apply for jobs, though we assume that workers can *show* their types to firms when they apply. Position types are assumed to be public information.

Matching and production occur over an infinite number of periods and all participants in the matching process are assumed to be infinitely lived. A match between a worker and a position results in some kind of wage payment and profit for the firm. At the end of each period, there is some exogenous probability $1 - \delta$ with which the match will be terminated. In particular, with probability 1, each match will terminate within a finite number of periods.

Neither workers nor firms discount payoffs. However, there is an exogenous matching cost that each party bears when they search for partners. This cost is assumed to be a proportion $(1 - \gamma)$ of their future earnings or profits. Workers care only about their total wage payments net of these matching costs. Firms care about their total profits net of matching costs. Let w be the expected payments made to a worker during a match with a firm. The expected profit earned by a firm with a position of type x who hires a worker of type y and pays him or her this wage, is given by some function $v(w, x, y)$ which is assumed decreasing in w and weakly increasing in y . To maintain an order on position types, it is assumed that for any pair (w, y) and (w', y') with $(w, y) \geq (w', y')$, if $v(w, x, y) \geq v(w', x, y')$ for some type x , then $v(w, x', y) \geq v(w', x', y')$ for any higher type $x' \geq x$. In words, this *single crossing condition* says that higher type positions generate more profits from higher type workers than lower type positions do. We assume wages are chosen from a compact interval W .

At the beginning of each period, a worker is either employed, or unemployed. If she is unemployed, the worker simply chooses where to apply with full knowledge of the expected payments she will receive from any firm who hires her. If she is hired, she expects to receive the promised payments until her employment with the firm is terminated. If she isn't hired by the firm where she applies, she applies again to

another firm in the next period and bears the cost of unemployment. The worker doesn't care directly how long it takes her to find a new job. However, she bears the cost of unemployment whenever she is forced to apply to a new position. Her application strategy is chosen to balance unemployment costs against future wage payments.

For simplicity, payoffs are assumed to be such that firms always hire the worker who applies who has the highest type - the firm doesn't have the option of refusing its best application in order to search for a worker of higher quality. The underlying presumption is that all the workers who are searching for work have the same observable qualifications like education and experience. However, we will also assume that the worker types are not verifiable. So firms can not explicitly condition their wage payments on the worker type.

3. THE MARKET

The payoffs that players receive depend on their own actions, and on the distributions of actions taken by the other players. We specify these payoffs using standard arguments from directed search. Let G be the steady state wage *offer* distribution. We'll assume throughout that G is monotonic, differentiable, and has interval support $\overline{G} = [\underline{w}, \overline{w}]$. This will differ from the overall wage distribution G^* in general because positions will be filled at rates that depend on the wage offered. Let P represent the steady state distribution of applications, where $P(w, y)$ is understood to be the measure of the set of workers who have type y or less who apply at wage w or less. From the perspective of an individual position offering wage w , we will be interested in the measure of the set of searching workers of type y or less who apply at wage w . Denote this *conditional distribution* by $p_w(y)$ and observe that the relationship between P and p_w is given by

$$(3.1) \quad P(w, y) = \int_{\underline{w}}^w p_{w'}(y) dG(w').$$

Since P must be absolutely continuous with respect to the distribution G , it should be apparent that p_w is the Radon-Nikodym derivative of $P(w, y)$ with respect to G .

Payoffs associated with each of the actions available to players depend on their types, the steady state distributions of actions of the other players G and P , and on the steady state distribution of types $\mathcal{F}$ of the searching workers. The steady state distribution of types for the searching workers must be consistent with the steady state wage offer distribution in the sense that $G(\overline{w}, \overline{y}) = \mathcal{F}(\overline{y})$. In the steady state, it is apparent that the measure of the set of wage offers is going to be

equal to $(1 - \gamma)$ times the measure of the set of firms. So we simplify from now on and assume that the steady state distribution of types has measure $F(\bar{y}) = G(\bar{w}, \bar{y})$.

To understand the payoffs of workers, observe that a worker who has current type y is always hired before workers with lower types. As a consequence, he is concerned not with the total number of applicants expected to apply at the firm where he applies (the 'queue size'), but with the measure of the set of applicants who apply whose type is at least as large as his. When he applies at wage w , this number is given by

$$\int_y^{\bar{y}} dp_w(\tilde{y}).$$

We use the familiar formula $e^{-\int_y^{\bar{y}} dp_w(\tilde{y})}$ to give the probability that the worker will be hired if he applies at wage w .

From this it is straightforward to write down the payoff to a worker of type y who is searching for a job

$$(3.2) \quad U(0, y) = \max_{w' \in \bar{W}} \left((w' + \gamma U(0, y)) e^{-\int_y^{\bar{y}} dp_{w'}(\tilde{y})} + \left(1 - e^{-\int_y^{\bar{y}} dp_{w'}(\tilde{y})} \right) \gamma U(0, y) \right).$$

For firms, an unfilled position then has value

$$(3.3) \quad V(x) = \max_w \left[\int_y^{\bar{y}} (v(w, x, y) + \gamma V(x)) e^{-\int_y^{\bar{y}} dp_w(\tilde{y})} dp_w(y) + \left(1 - \int_y^{\bar{y}} e^{-\int_y^{\bar{y}} dp_w(\tilde{y})} dp_w(y) \right) \gamma V(x) \right].$$

Each expression contains an expected profit or wage term that applies to the duration of the match, then the value to the firm or worker of finding a new match once the existing one terminates discounted to reflect the costs search. A steady state equilibrium for this model is a set of distributions $(G, P, \mathcal{F})$ having three properties: (i) at every wage w , $G(w)$ coincides with the measure of the set of position types which maximize discounted payoff by offering a wage w or less; and (ii) for each pair (w, y) , there is a set of workers of measure $P(w, y)$ whose types are less than or equal to y and who maximize expected payoff by applying at wage w or less; and given the application decisions of workers, the distribution of types for searching worker in the following

period will be $\mathcal{F}$. In other words, the distribution of best replies to the distributions G and P are G and P themselves.

4. CONTINUATION STRATEGIES

Since the workers make their application decisions conditional on the distribution of wage offers, we can begin to characterize the equilibrium by describing the equilibrium value functions conditional on distributions G and P . The continuation equilibrium we are about to describe follows (Peters 2010). The utility function $U(0, y)$ in the theorem that follows should be interpreted as the *market payoff function* since it describes the payoff to a worker of type y who follows his equilibrium strategy. The reservation wage $\omega(y)$ is the wage that would yield the worker his market payoff if he expected to be hired at that wage for sure.

In the continuation equilibrium we are about to describe, each worker applies to every wage above his reservation wage with equal probability unless he is already employed in a position that pays more than his reservation wage. That means, that for any interval of wages, the probability that the worker applies to a wage in that interval is equal to the measure of wage offers in that interval divided by the measure of wage offers above the worker's reservation wage.

Theorem 4.1. *For any differentiable wage offer distribution G , there is a continuation equilibrium characterized by a monotonically increasing reservation wage strategy $\omega(y)$ in which each worker applies with equal probability at every wage at or above $\max[\underline{w}, \omega(y)]$. The reservation wage is characterized by the solution to the differential equation*

$$(4.1) \quad \omega'(y) = \frac{\omega(y) F'(y)}{G(\bar{w}) - G(\omega(y))}$$

through the point $(\bar{y}, \bar{w})$. The market payoff is given by $U(0, y) = \frac{\omega(y)}{1-\gamma}$ when $\omega(y) > \underline{w}$, and by

$$U(0, y) = \frac{\underline{w} e^{-\int_y^{\omega^{-1}(\underline{w})} \frac{1}{G(\bar{w}) - G(\omega(y'))} dF(y')}}{1 - \delta}$$

otherwise. Finally for every wage w in the support of G , the queue size faced by a worker who applies for a position offering wage w is

$$(4.2) \quad \int_y^{\bar{y}} dp_w(\tilde{y}) = \int_y^{\omega^{-1}(w)} \frac{1}{G(\bar{w}) - G(\omega(y'))} dF(y')$$

Proof. Fix a continuous non-decreasing rule $\omega : Y \rightarrow W$. Notice that ω is not required in this definition to have range contained in $\bar{G}$, so the

proper interpretation is that $\omega(y)$ is the wage that yeilds the worker his market payoff if he is hired for sure at that wage. If all searching workers apply to all wages at or above their reservation wage, then $P(w, y) = \int_{\underline{y}}^{\min[\omega^{-1}(w), y]} \frac{G(w) - G(\omega(y'))}{G(\bar{w}) - G(\omega(y'))} dF(y')$. The 'queue size' $p_w(y)$ has to satisfy (3.1), so

$$p_w(y) = \int_{\underline{y}}^{\min[\omega^{-1}(w), y]} \frac{1}{G(\bar{w}) - G(\omega(y'))} dF(y').$$

To see this observe that for any w ,

$$\begin{aligned} \int_{\underline{w}}^w p_{\tilde{w}}(y) dG(w) &= \int_{\underline{w}}^w \int_{\underline{y}}^{\min[\omega^{-1}(\tilde{w}), y]} \frac{1}{G(\bar{w}) - G(\omega(y'))} dF(y') dG(\tilde{w}) \\ &= \int_{\underline{y}}^{\min[\omega^{-1}(w), y]} \int_{\omega(y')}^w \frac{dG(\tilde{w})}{G(\bar{w}) - G(\omega(y'))} dF(y') = \\ &\quad \int_{\underline{y}}^{\min[\omega^{-1}(w), y]} \frac{G(w) - G(\omega(y'))}{G(\bar{w}) - G(\omega(y'))} dF(y'). \end{aligned}$$

This implies that

$$(4.3) \quad \int_y^{\bar{y}} dp_w(\tilde{y}) = \int_y^{\omega^{-1}(w)} \frac{1}{G(\bar{w}) - G(\omega(y'))} dF(y').$$

So hiring probabilities will be given by (4.2) provided that workers all use the application strategy described. Given this matching probability we can now describe the condition that $\omega(y)$ has to satisfy in order for them to be willing to follow this strategy. In order for a searching worker of type $\omega(y) > \underline{w}$ to be indifferent between all wages above his reservation wage, it should be that for each $w' > \omega(y)$

$$\begin{aligned} (w' + \gamma U(0, y)) e^{-\int_y^{\bar{y}} dp_{w'}(\tilde{y})} + \left(1 - e^{-\int_y^{\bar{y}} dp_{w'}(\tilde{y})}\right) \gamma U(0, y) \\ = \omega(y) + \gamma U(0, y), \end{aligned}$$

or

$$w' e^{-\int_y^{\bar{y}} dp_{w'}(\tilde{y})} = \omega(y).$$

Taking logs yields

$$\int_y^{\bar{y}} dp_{w'}(\tilde{y}) = \log(w') - \log(\omega(y)).$$

By the fundamental theorem of calculus this implies

$$(4.4) \quad \int_{\omega(y)}^{w'} \frac{1}{\tilde{w}} d\tilde{w} = \int_y^{\bar{y}} dp_{w'}(\tilde{y}).$$

Substituting (4.3), then gives the identity

$$\int_{\omega(y)}^w \frac{1}{\tilde{w}} d\tilde{w} = \int_y^{\omega^{-1}(w)} \frac{1}{G(\bar{w}) - G(\omega(y'))} dF(y')$$

is satisfied for all y . Differentiating both sides with respect to w gives the differential equation

$$(4.5) \quad \omega'(y) = \frac{\omega(y) F'(y)}{G(\bar{w}) - G(\omega(y))}.$$

The reservation wage function ω will support the continuation equilibrium if it has a solution with $\omega(\bar{y}) = \bar{w}$. This is not immediate since the right hand side does not have a continuous derivative around the point $(\bar{y}, \bar{w})$.

However it does have a solution through the point $(\bar{y}, \bar{w} - \epsilon)$ for any $\epsilon > 0$. Denote the solution for $\epsilon > 0$ as $\omega^\epsilon(y)$. Observe that each ω^ϵ is strictly increasing and that ω^ϵ and $\omega^{\epsilon'}$ cannot cross, therefore the sequence $\{\omega^\epsilon\}_{\epsilon \rightarrow 0}$ is an increasing sequence of increasing functions. As the sequence $\omega^\epsilon(y)$ is a bounded increasing sequence of real numbers, ω^ϵ converges pointwise, therefore uniformly (Dini's Theorem) to some function ω . If (4.5) fails at some point y , then by uniform convergence, it must fail for small ϵ . So ω is a solution to (4.5).

The remaining bits of the theorem then follow by using (3.2) along with the reservation wage. $\square$

Then assuming workers use the reservation wage strategy in the continuation, the payoff functions for firms can then be given for any wage in the support of the wage offer distribution as

$$(4.6) \quad \max_w \left[\int_{\underline{y}}^{\omega^{-1}(w)} v(w, x, y') e^{-\int_{y'}^{\omega^{-1}(w)} \frac{dF(\bar{y})}{G(\bar{w}) - G(\omega(\bar{y}))}} \frac{dF(y')}{G(\bar{w}) - G(\omega(y'))} + \gamma V(x) \right].$$

This expression has a very convenient interpretation. Once a firm chooses a wage, it will receive applications from workers whose types are such that their reservation wage in the continuation equilibrium does not exceed the wage the firm offers. A slightly simpler formulation is to think of the firm as choosing the highest worker type it wants to try to attract, then offering the reservation wage of that worker type to all workers. The profit function then has the slightly simpler form

$$V(x) =$$

$$(4.7) \quad \max_{y^*} \left[\int_{\underline{y}}^{y^*} v(\omega(y^*), x, y') e^{-\int_{y'}^{y^*} \frac{dF(\bar{y})}{G(\bar{w}) - G(\omega(\bar{y}))}} \frac{dF(y')}{G(\bar{w}) - G(\omega(y'))} \right. \\ \left. + \gamma V(x) \right].$$

In the latter formulation of profits, the firm is maximizing its profit when the iso-profit line (in (y, w) space) associated with the argument in the maximization above is tangent to the reservation price rule.

Of course, this only defines payoffs in the support of the wage offer distribution. We should define payoffs outside this support. To keep things simple, we just define payoffs outside the support to ensure that a tangency with $\omega(y)$ is sufficient for profit maximization.¹

EARNINGS HISTORIES

The main implications of this theory for the matching data stems from the fact that the type dependent application strategy imposes restrictions on what happens to workers as they move between jobs. The implications are quite straightforward. Workers in the model bear an unemployment cost that is proportional to their earnings whenever they look for a job. However, the matching process itself doesn't take them any time. They simply apply until they find a job. Since their application strategy has them making applications to every firm whose wage is above their reservation wage, the future wage of a worker of type y is a random variable whose distribution is just $\frac{G(w) - G(\omega(y))}{G(\bar{w}) - G(\omega(y))}$. As $\omega(y)$ is increasing, workers with higher types are drawing their future wage from the same distribution but conditional on an event that is a strict subset of the conditioning event for workers of lower types. As a consequence, the variance of future income is smaller the larger is a worker's type.

Of course, the workers' types are not observed directly, but they can be indirectly observed by looking at the wage at which a worker was last employed. Again, using the reservation wage strategy and the fact that the matching probability for a worker of type y at wage w is $\frac{\omega(y)}{w}$, the probability distribution of types employed at wage w is given by

$$\int_{\underline{y}}^y \frac{\omega(y')}{w} \frac{dF(y')}{G(\bar{w}) - G(\omega(y'))}$$

¹This is equivalent to imposing the usual market utility assumption in directed search, though the payoff when an offer is made above the support of the distribution of G requires some subtle considerations. See (Peters 2010) for details.

divided by the measure of the set of types who are employed at wage w (the formulae above with $y = \omega^{-1}(w)$). Since $\omega(y)$ is strictly increasing, an increase in w supports a new distribution the first order stochastically dominates the initial distribution. Putting this together with the fact that the variance of future income is falling with worker type, illustrates that the variance of future income is a declining function of the wage at which a worker is currently employed. This forms the basis of the empirical test in the data that follows below.

5. EMPIRICAL APPLICATION - THE EXECUTIVE MARKET

//**** very preliminary incomplete *****/

5.1. Data. Our dataset is collected from Compustat and Execucomp. The sample comprises the observed characteristics of both executives and firms for the period from 1993 to 2009. Especially, we observe executives' age, tenure, gender and turnover on his/her career path. On the firm side, we observe the total assets (AT), sales, employment size (EMP), income before extraordinary (OIBDP), standard industry classification code (SIC) and total current compensation, which is the sum of bonus and salary.² All monetary terms are converted to the dollar value of year 1992.

One unusual characteristic of the executive data is that there is no apparent unemployment. The executives searching for new positions do so 'on the job', so there is no clear definition of who is unemployed and who isn't. The way we handle this here is to assume that any executive whose pay is below the average pay of her peers is searching for new employment. Otherwise, executives aren't searching.

We use the following algorithm to find relevant peer executives in the labor market. We first order each executive by their current compensation within a firm, then give a ranking to each executive within firm by assigning them to one ordered quintile of the wage distribution within the firm. Then, for an executive j in quintile k in firm i at year t , we suppose that all the executives in the k^{th} quintile of the wage distribution at firm i' are her 'peers', provided

- (1) Firms i and i' belong to the same industry in the Fama-French category (defined in accordance with SIC) at year t ;
- (2) Total assets of firm i' are within 20% of the total assets of Firm i in year t ; and

²There are other information on accounting status in the sample period, such as net income (NI), earnings before interests and taxes (EBIT), etc. We decide not to use these variables, as they do not seem appealing to explain the wage variation in our application.

- (3) Return on assets (ROA) of firm i' is within 20% of the return on assets of firm i in year t .³

Following this procedure, for executive j we could find a set of peer executives in this executive labor market, denoted as Γ_{jt} . Note that it is possible that for some j we have $\Gamma_{jt} = \emptyset$. Denote executive j 's wage at year t as w_{jt} and the mean value of her peers wages as $\bar{w}_{jt}$. For those executives with $\Gamma_{jt} \neq \emptyset$, we define their labor market status d_{jt} as follows

$$d_{jt} = \begin{cases} 0 & w_{jt} \geq \bar{w}_{jt} \\ 1 & otherwise \end{cases}$$

i.e., executive j 's earning is relatively larger than his peers and we consider him as an overpaid executive ($d_{jt} = 0$), who would not actively search for jobs. However, if his earning is lower than his peers, we consider him as an underpaid executive ($d_{jt} = 1$) who is actively searching for a job.

Our definition of job market participants is a natural extension of (?). They find that employee's job search intentions hinge on the current job satisfaction, which is highly correlated with the differences between how much the worker and her peers make. Our algorithm identifying peers is consistent with recent literature discussing relative performance evaluation on CEO compensation. (?) examines the peer choice in CEO compensation, and finds that peers are more likely to be in the same industry and be closer in revenues and market capitalization to the firm. Their findings support the labor market view of CEO compensation that firms will select peers that represent the CEO's outside employment opportunities. We extend this method to all the executives in our sample, by matching them with "hypothetical" peer firms and executives, to evaluate their labor market status.

One of the natural interpretations associated with this characterization is that any searching worker has a natural reservation utility, which is just the wage he gets from his current firm. For this reason, we use the executives current pay as a measure of his type in the model above.

One implication of this identifying assumption is that executives should move from low paid jobs to high paid jobs. Not all of them do. There are 62 cases in the sample of executives who move to lower paid jobs. We exclude these observations from our analysis.

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(a) The ROA is calculated as $ROA_t = \frac{OIBDP_t}{AT_{t-1}}$.

To identify the firms who offer vacancies at any point, we assume that any firm who hires a new executive or promotes an executive from within as if they had a vacancy in the previous year.

We incorporate all the observations that satisfy the above selection process as the working sample of the executive labor market. It is worth noting that we indeed have two samples to work with for our analysis. The first is the search sample, which includes all the observations for workers searching on the market. There are in total of 66,147 observations in this sample. The other sample is the move sample, which has only those executives who actually moved (internal or external) and consists of 12,629 observations. All the variables used in this paper are defined in Table A. The basic summary statistics of these variables in the sample are reported in Table B in appendix. The market tightness is defined as the ratio of the number of searching workers and the number of hiring firms, which corresponds to τ in the search model. Table C in appendix contains the information on the market tightness over the sample period. There is variation on this measure, largely between 6 and 8. The information on executives involving moves, internal or external, are summarized by years in Tables C and by industries in Table D, respectively.

5.2. Empirical evidence on the model.

5.2.1. *Direct evidence: model predictions.* First, we examine whether there is empirical evidence from the working sample to support the model predictions and structures.

Duration of search We counted the number of consecutive years of underpay that an executive took to get either internal promotion or external move, and we define this as the duration of search.⁴ We regress the exit wages on the covariates, including duration of search. The result is reported in Column (1) of Table 1. There we identify a statistically significant and negative impact of duration on the exit wage. In general, a one percentage increase in search duration is associated with 3.6 percent decrease in the exit wage. We further did the same regression with some more observed worker characteristics (such as age, tenure), which consequently comprises a much smaller sample. Not only the negative relation of exit wage with duration of search still

⁴It should be noted that the duration of search here is identical to the unemployment duration predicted by the theoretical model. The subtle difference arises from the different outside options for the worker. In theoretical model, a worker is actively searching for the job as being unemployed. Whereas an executive in application is indeed doing on-the-job search.

holds, but such elasticity increases to -4.7%. (See Column (2) of Table 1.)

Probability of finding a job In the working sample, we did a probit regression of the probability of being hired on the observed covariates, including the reservation wages. The result is reported in Columns (3) and (4) in Table 1. The coefficients for reservation wage are positive and significant. In particular, the results reported in Column (3) suggest an average marginal effect of 4% for the reservation wage on the probabilities of finding jobs. Moreover, with a much restricted sample, the results in Column (4) show an average marginal effect of 12%.

Whether higher types have higher matching probability in our data depends on how we define type within our sample. Using the prior wage as a measure of the worker's type is natural enough. However, given that definition of type, and its empirical relationship with matching probability, the model predicts that the function (??) should be monotonically increasing. The function (??) depends only on the wage distribution from the sample. This prediction then provides a simple test of the model. The function (??) as derived from the wage distribution data in our sample is given by the blue dotted in Figure 1. If we discount the data noise, this function is evidently increasing as required.

5.2.2. Indirect evidence: The mixed application strategy at equilibrium.

The application strategy in the model supports the imperfect matching as equilibrium outcome. Such imperfect matching is implied by the unique mixed-strategy equilibrium, in which the workers apply with probability to all firms who set wages above their reservation wage. For this reason the usual frictions associated with directed search will occur. High type employees may remain unmatched searching simply because they were unlucky, and applied at locations where other higher quality workers also applied. Low type workers, on the other hand, use quite different applications strategies and apply to high wage firms where they have little chance of finding a job.

Approach 1: shrinking variances. In the equilibrium of Peters' model, a worker with better type will use a higher cutoff as her reservation wage, therefore smaller range of jobs will be applied. This implies that *ex-post*, we expect to see smaller variance of exit wages for workers of better types. To explore the likelihood of such a pattern in our data, we did the following exercise. We first evenly divide the workers into five groups by their percentiles of reservation wage. For example,

Group 1 represents the workers whose reservation wages are below 20-th percentile. Then for each group, we compute the mean, median, 90-th and 10-th percentile of their exit wages (on the next job). We plot all these measures in Figure 2. There is a clear pattern that the variances of exit wages for each group shrinks as the worker groups get to better types. The bars in Figures 2 explicitly indicate such a pattern of shrinking variances over these groups. This is in great support to a mixed application strategy.

For the following approaches, we choose the pure-strategy equilibrium in a model close to ours, by (?) to compare with our mixed-strategy equilibrium.⁵

Approach 2: correlation of rankings. The pure strategy equilibrium says that a worker would simply apply for the job exactly fits her type on the firm type distribution. If it were the case for our data, we should expect that the relative ranking of the wage at new job should be perfectly and positively correlated with the worker’s relative ranking in his types. To this end, we investigate the relations between the CDF of worker’s exit wage and the CDF of his reservation wage. Of course, we implicitly assume here the reservation wage represents the worker types. First of all, the correlation parameter between the two is 0.82. Then we regress the CDF of exit wage on the CDF of reservation wages, including all other observed covariates. The results are reported in Columns (1) and (2) in Table 2. Based on the regression parameters, we conduct one-sided tests with null hypotheses of parameters at question (CDF of reservation wage) being one. The null are rejected at all reasonable significance levels.

Following the same logic, we repeat the same exercise but using CDF of worker’s previous exit wage as the proxy for worker’s relative types, instead of reservation wage. The correlation parameters now is 0.67. The regression results are reported in the last two columns in Table 2. Again, all the null hypotheses of parameters at question being one are rejected at all reasonable significance levels.

Approach 3: distributional tests. Different application strategies (pure vs. mixed) may result in subtle difference on equilibrium wage distributions in a particular situation. Suppose in the two samples, the worker types are distributed differently but the firm types follow same distributions, the pure strategy equilibrium should never result in the same wage distributions. Instead, the mixed strategy equilibrium would rather be able to possibly produce identical equilibrium wage

⁵The choice of (?) is due to the fact that there is heterogeneity on agents in their model.

distributions across the two samples. The same argument runs through, when the firm types are distributed differently but the worker type is same.

We implement a series of KS tests on our data, (a test on the identity of two distributions). We first use the workers reservation wage as the proxy for worker type and the past year average wages by the firms as a proxy for firm types. In a mixed-strategy equilibrium, though imperfect matching, the better type of firms are still more likely to hire a better worker, in a probability sense. Given these proxies for the types, we compare the worker type distribution, firm type distribution and resulting (equilibrium) wage distribution over subsamples. We construct the subsamples over two dimensions, one on industry and the other on time (years). The results listed on Table 3, indicate that there are cases in which one of the type distributions is not identical, while the equilibrium wage distribution is yet same. For example, Industry 1 and 4 have same worker type distribution, different firm type distribution, but identical equilibrium wage distribution. Another example worth mentioning involve the sample years of 2000 and 2005. Across these two years, the worker and firm distributions are identical. However, the resulting equilibrium wage distribution are different. This case surely goes against the pure-strategy equilibrium argument.

Our last empirical examination before moving into structural approach concerns the explanatory power of unobservable heterogeneities on the wage variations. As previously argued, the firm's past wage can be used as a proxy for firm type (in a probability sense). We then incorporated this measure into the wage regression. The result is reported in Table 4. Starting with Column (2) in the table, we use the average of firm's past wages in the regression and change in R-square occurs rather small. As another check on the possibility of perfect matching, we add the CDF of firm type into the regression, which stands for the measures of worker types. If the equilibrium is of perfect matching, then the order of firm types should be identical with the worker types (i.e., the best firm is matched with the best worker, and so on.). However, the R-square is not further changed in the regression. This reinforces the previous empirical evidence on the support of our model. Moreover, these results also suggest the necessity of a structural analysis.

The above empirical results indicate that the Peters' model is consistent with the data. We in the next section, consider a structural approach through which we are to recover the firm and worker types from the data. Eventually, we will use the recovered unobserved heterogeneities to investigate whether (and by how much) the wage variation can be explained by the imposed equilibrium structures of the search

model. Effectively, that will enable us to conclude whether the matches of firms and workers can be explained by an economic mechanism.

6. IDENTIFICATION AND ESTIMATION

6.1. The econometric model. We assume our observed labor market data are generated through the following process.

- (1) $\{\{X\}_{M \in \mathbb{N}}, \{Y\}_{N \in \mathbb{N}}\}$ is a bivariate stochastic process, which is independently and identically distributed. The marginal distributions are H and F , respectively.
- (2) $\{W\}$ is a univariate process generated by the probability law $p_w(y)$, which is in accordance with the model equilibrium by a Peters' model.

The model primitives are H and F , while econometricians observe only the equilibrium wages in general.

Nonparametric identification. The model is generally unidentified. To see this more clearly, we can substitute the equation ?? into ??. From there, we can see that $F(y)$ and $h(y)$ (i.e., x) are not jointly identified. To get some intuition, we revisit the equilibrium condition that $G(\omega(y)) = H(h(y))$. It is an impossible task for one to decompose the observed equilibrium wage into x or y without knowing the other. To put simply, we do not have enough information to recover both distributions F and H from the only observed distribution G .

To overcome the identification problem, we will have to impose some restrictions on the model. First, we assume that F is uniform on $[\underline{y}, \bar{y}]$. Then, equation ?? implies:

$$\frac{\tau}{1 - G(\omega(y))} \cdot \frac{1}{\bar{y} - \underline{y}} = \frac{\omega'(y)}{\omega(y)} = (\log \omega(y))'$$

Denote $z(y) = \log \omega(y)$. We then have

$$\frac{\tau}{1 - G(\exp z(y))} \cdot \frac{1}{\bar{y} - \underline{y}} = z'(y)$$

Since G is a cumulative distribution function, $0 \leq G \leq 1$, the inverse function exists. This further implies,

$$\frac{dy}{dz} = \frac{1 - G(\exp z(y))}{\tau} \cdot (\bar{y} - \underline{y}).$$

Integrating both sides from z to $\bar{z}$.

$$\begin{aligned}\int_z^{\bar{z}} \frac{dy}{dz} dz &= \frac{(\bar{y} - \underline{y})}{\tau} \cdot \int_z^{\bar{z}} [1 - G(\exp z(y))] dz \\ y(\bar{z}) - y(z) &= \frac{(\bar{y} - \underline{y})}{\tau} \cdot [\bar{z} - z - \int_z^{\bar{z}} G(\exp u) du]\end{aligned}$$

where the second line integrates out the elements, and with a slight risk of abuse the notations, $y(z)$ denotes the worker type y whose logarithm of reservation wage is z .

Imposing initial condition of $y(\bar{z}) = \bar{y}$ and rearranging terms, we derive a closed-form expression for y ,

$$(6.1) \quad y = \bar{y} - \frac{\bar{y} - \underline{y}}{\tau} \cdot (\bar{z} - z) + \frac{\bar{y} - \underline{y}}{\tau} \int_z^{\bar{z}} G(\exp u) du$$

We assume that the support of F is a unitary interval, i.e., $|\bar{y} - \underline{y}| = 1$. Furthermore, the constant term in equation 6.1 is normalized to zero, which implies that $\bar{y} = \bar{z}/\tau$ and $\underline{y} = \bar{y} - 1$. Therefore, we derive a computable formula for y ,

$$(6.2) \quad y = \frac{1}{\tau} z + \frac{1}{\tau} \int_z^{\bar{z}} G(\exp u) du$$

Equation 6.2 suggests that y can be recovered, if z , or alternatively ω , can be observed. Fortunately, we come up a plausible way to define the reservation wages in our application.

A few remarks are in order. First, in our identification strategy, the specification of uniform distributions on F is innocent. To recover two generally unidentified distributions, a uniform distribution is assumed for one, then the shape of the other will be completely determined by the model equilibrium. Moreover, even though we impose shape and norm restrictions to help identify y , the boundaries of F are pinned down by the data and equilibrium restrictions.

Second, note that two further restrictions were imposed when proceeding from equation 6.1 to equation 6.2. The length restriction is rather for simplification and convenience. We should have chosen any other arbitrary number, and the entire previous argument carries over. We indeed experimented our estimation with different choices of such length choices. Our major empirical findings remain same. The other restriction, which however is technically necessary, is the normalization of constant term in equation 6.1. Without it, the estimation of equation 6.2 becomes impossible. However, it should be reminded that our major task of this empirical exercise is to see how recovered variation y can contribute to increase the explanatory power of residual wage

variations. To this end, the normalization of constant term plays a negligible role for the recovered variation in y .

Lastly, it is worth mentioning that y can be nonparametrically estimated according to equation 6.2. We however will use a parametric framework estimating y to circumvent data noise issues and choices of smoothing parameters. In particular, we will assume the wage distribution G follows a lognormal distribution. Figure A in appendix shows that lognormal specification can serve as a decent approximation of wage distributions at work.

To recover firm heterogeneities, we further assume the value function has the following form: $v(h, y, \omega) = h \cdot y - \omega$, where h is abbreviation for $h(y)$, i.e., the firm type who offers y 's reservation wage. This functional form effectively assumes the complementarity between the two unobserved heterogeneities in productivity.⁶ Substituting it into equation ?? and rearranging terms, we get

$$h \cdot y - \omega(y) = - \int_{\underline{y}}^y \left[1 - \frac{h \cdot y' - \omega(y')}{\omega(y)} \right] \omega'(y') dy'$$

This leads to a closed-form solution for h :

$$(6.3) \quad h = \frac{\omega^2(y)}{\underline{y}\omega(y) + \int_{\underline{y}}^y \omega(y') dy'}$$

In implementation of computing h , we use the polynomial functions of order five to approximate the reservation wage function ω . $\underline{y}$ is the minimum of recovered y . Thus, the distribution H is identified through h .

6.2. Estimation results. Equations 6.2 and 6.3 provide structural estimates for the unobserved heterogeneities in our model. F is uniformly distributed between estimated bounds 0.5 and 1.5. The kernel density estimate on H with recovered $h(y)$ is plotted in Figure 3. It entails a sharp hike-up at the lower end, but a rather long tail towards the higher end. This in turn may suggest higher values of the firm heterogeneities be more scarce resources in market.

We next include these estimates (x and y) into wage regressions as what has been used in Table 4. The results are reported in Table 5. We first include recovered y or x into the regressions. See Columns (2) and (3) in Table 5. R^2 goes up by around three percentage points only when adding y to the wage regression, whereas adding x alone increases R^2 by about 16 percentage points. This finding appears consistent with results in the literature. (?) conclude in his work that “the variation in

⁶Such an assumption is not uncommon in the literature, see for example, (?).

CEO pay is found to be mostly due to variation in firm characteristics, whereas implied differences in managerial ability are small and make relatively little difference to shareholder value.” Moreover, (?) shows small dispersion in CEO talent and large firm size go together justify many of the observed CEO pay patterns.

Including both x and y increases the explanatory power of wage regression from 60.8% to 77.9%. The same pattern can be observed, when considering more observed covariates and thus working with a smaller sample. (Columns (5)-(8) in Table 5.) We then did the same exercise with the same sample used in Table 4. With corresponding x and y , the wage variation explanatory power can increase from 61.9% to 80.1%. This result is listed in the last few columns in Table 4. Such subtle difference on the explanatory power caused by the recovered heterogeneities indicates the usefulness of our structural analysis.

Next, we study the impacts of the unobserved heterogeneities in different industries. We repeat the same exercises over several categories of industries. The increments of R^2 in different categories are listed in Table 6.⁷ It is found that for the industries with “Finance, insurance and real estate”, “retail trade” and “services”, the unobserved heterogeneities can help to increase the wage variation by about 20 percentage points, while the “manufacturing” industry however only entails an increase 14 percentage points. This result is not surprising. Comparing with other industries like public utilities, the business practice in trade, finance and service may reflect more of enterprise cultural and strategic components, rely more on the executive’s vision, ability and social networks, and the matches of the merits from both sides. These account for the unobserved heterogeneities and matches under study.

For a better understanding of recovered heterogeneities, we take a closer look at their variations and relations to other model elements or observables. Figure 4 first plots the relationship between worker type (y) and her reservation wage ($\omega(y)$). It is monotonically increasing. It is of a (almost) linear pattern in a large part of the range, but with a kink point around 1.2 ensuing a convex segment towards the top end of the graph, which implies much increasing marginal payoff of unobserved characteristic for top workers. This echoes the established fact of “superstar effect” in the literature. The works on superstars attempt to offer explanations of why “relatively small numbers of people earn

⁷For this exercise, we keep using Fama-French categories of industries. However, there are a few of them endowed with much less data observation and therefore variation. We therefore decided to focus on the listed five categories instead, for the matter of fair comparison.

enormous amounts of money and seem to dominate the fields in which they engage.” Rosen (1981) argues that in superstar markets, “small differences in talent at the top of the distribution will translate into large differences in revenue.” Our empirical findings enrich the story of superstars. In our model where workers are uniformly distributed, the enormous increase on the superstars’ pay is therefore more seemingly attributed to the long tail of firm heterogeneities.

Figure 5 indicates the relations between x and y as matching pairs in equilibrium. We presented the variation in the way as we did it on Figure 2. That is, we sort the workers by their types y into five quintiles. Then within each group, we look at their mean, median, 10-th and 90-th percentile of matched firm types x . The bar plots in Figure 5 also show a shrinking pattern of variance of x . Figure 5 clearly shows the consistency with Figure 2, both of which support the mixed application strategy.

Next concern arises with respect to the specified value function form v . One may wonder whether the productivity generated by matched pairs $x \cdot y$ make sense in application, rather than a simplifying assumption in identification strategy. To address this concern, we first report the correlation matrix between $x \cdot y$ and other observed characteristics of firms in Table 7. The first column indicates that the generated value from matching pairs is statistically significant in correlations with other firm characteristics, all with intuitive and reasonable signs. It is worth noting that the generated value is correlated with firm’s income only at a value of 0.11 (from Table 7). We further regress the firm’s income on other firm characteristics in Table 8. It is shown that when adding the generated value (of output by matchings) to the regression, the R^2 does not have any increase, though the coefficient itself is statistically significant.

The last concern in line is on the robustness of our major empirical finding, that is the increase of explanatory power by unobserved heterogeneities. One may suspect that both the increase and its magnitude found in Table 5 may occur from luck. We then conduct a robustness check by using Racine and White (2010). We first randomly split the entire sample into two subsamples and use the first subsample as a training sample to estimate the wage equations. In next step, we use the estimated parameters from training sample to compute predicting values in the other subsample (calling it predicted sample). We then regress the observed variation in the predicted sample on the predicting values and obtain its R^2 . We compare the R^2 in both training sample and predicted sample. We repeated the exercises for 10,000 times. The basic distributional measures on these R^2 in these experiments show

that the increase and its magnitudes of R^2 by including recovered x and y are robust.

All in all, our recovered variations on the heterogeneities from both sides of market agents, have economically meanings. This sheds light on the necessity of doing our structural analysis on the unobserved heterogeneities.

7. CONCLUSION

(?) extends a typical directed search model to consider unobserved heterogeneities in both workers and firms. It is designed to explain the residual part of wage variation left over after the impact of all observed characteristics has been removed. This new model is in strict contrast to the usual directed search stories in two ways: first, workers don't apply with higher probability to higher wage jobs; second, it explains why high wages are often associated with short unemployment spells.

This paper uses Peters' framework to structurally decompose the residual variation in wages left over after controlling for the observed characteristics using data from an executive labor market. We first investigate how unobserved heterogeneity on both sides of the market can be identified from the observed equilibrium wage distribution. We then apply the methodology to study the matching behavior. We find that both the randomized application strategy and the negative relationship between wages and unemployment spell are supported by the data. We further include the estimated unobserved heterogeneity to the wage regression, which increases the explanatory power of wage equation by up to 22 percentage points.

This paper has shown how directed search models can be used to analyse data from executive markets. We developed structural estimates for the unobserved heterogeneities of both sides of market agents in association with the defined market equilibrium. Our work should be largely viewed as a first attempt in applying a directed search model to the market data. In this spirit, we explicitly or implicitly had to impose restrictions or assumptions here and there to make an empirical use of the model. To name a few -

- (1) Dynamics, timing and details of contracting in terms of rewarding scheme, and many other interesting features of the executive markets were assumed away in our analysis;
- (2) To resolve the problem that the model generically is unidentified non-parametrically, we arbitrarily specified one of type distributions is uniform so that the model can stay in a reasonably tractable format;

- (3) To fully utilize our recovered estimates on heterogeneities, we used the linear regression of wages as benchmark and consider only R^2 as scientific measures for any judgemental claims.

Clearly, as all other structural analysis, our empirical results should be taken with a grain of salt: it follows only when one takes a firm belief on the structural model at outset.

Admittedly, there are a number of directions this work can be challenged and therefore extended. We however believe that the unobserved heterogeneities of market agents and their matchings play a genuinely important role in allocating the resources and driving the market equilibrium. Peters model offers an interesting mechanism, so to speak, an aspect towards a better understanding of the matching behavior in labor markets. While much more doubts to solve, we hope to have shown that directed search models have empirical values and thus can offer much to help understanding market performance.

8. APPENDIX

Table 1: Exit Wage and Unemployment Duration

VARIABLES	(1) lnwage	(2) lnwage	(3) move	(4) move
log(duration)	-0.03*** (0.012)	-0.04* (0.021)		
plnwage			0.15*** (0.018)	0.44*** (0.041)
size	0.23*** (0.006)	0.24*** (0.010)	0.04*** (0.011)	0.03 (0.019)
roa	0.84*** (0.080)	0.82*** (0.152)	-0.16 (0.159)	-0.52* (0.268)
female	-0.11*** (0.019)	-0.08* (0.046)	0.02 (0.030)	-0.02 (0.073)
sp500	0.03** (0.016)	0.04* (0.027)	0.10*** (0.032)	0.04 (0.051)
prod	0.00 (0.000)	0.00 (0.000)	-0.00 (0.000)	-0.00 (0.000)
ceo	0.79*** (0.008)	0.67*** (0.017)	0.17*** (0.019)	0.06* (0.035)
cfo	-0.01 (0.019)	-0.04 (0.034)	0.05 (0.030)	0.08 (0.060)
age		0.00* (0.001)		0.00 (0.002)
tenure		0.00** (0.001)		-0.01*** (0.002)
Constant	4.42*** (0.051)	4.09*** (0.101)	-2.14*** (0.123)	-3.58*** (0.256)
Adjusted R-squared	0.607	0.643		
Regression	OLS	OLS	Probit	Probit
Observations	12,285	3,437	63,459	13,278

Robust standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1

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T3.pdf

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Figure 1: Direct Model Support

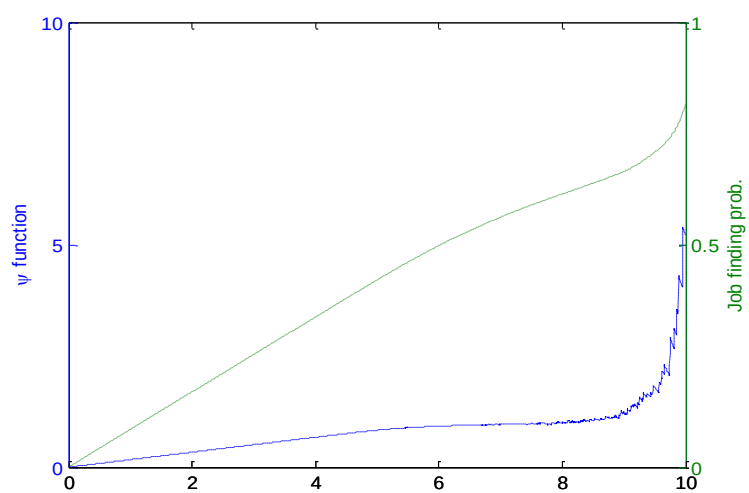
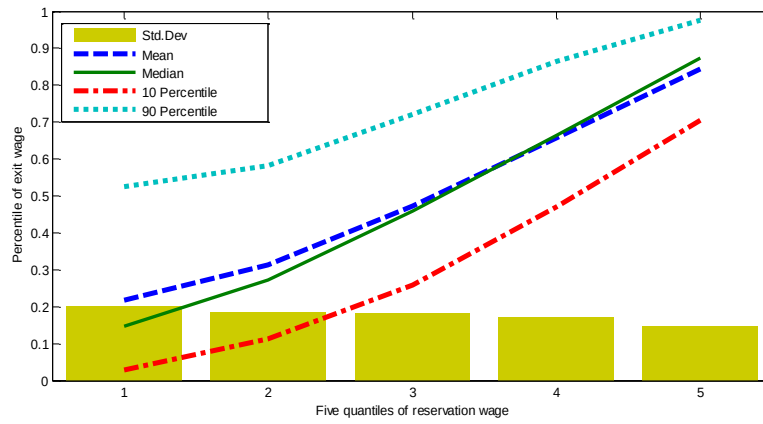


Figure 2. Mixed Application Strategy



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Table C. Labour Market Summary Statistics

year	# workers	# jobs	tightness	Internal	External	Total
1993	4841	806	6.01	521	6	527
1994	5261	754	6.98	750	56	806
1995	5612	774	7.25	695	59	754
1996	6009	929	6.47	689	85	774
1997	6107	941	6.49	809	120	929
1998	6505	907	7.17	839	102	941
1999	6360	893	7.12	815	92	907
2000	6083	980	6.21	797	96	893
2001	5811	927	6.27	886	94	980
2002	5603	808	6.93	876	51	927
2003	5865	783	7.49	755	53	808
2004	5572	715	7.79	726	57	783
2005	5018	881	5.70	669	46	715
2006	6028	1000	6.03	827	54	881
2007	6017	823	7.31	905	95	1,000
2008	5385	625	8.62	732	91	823
2009	4831	531	9.10	575	50	625
Total				1,207	12,866	14,073

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