VALUE OF COMPETITION IN ALLOCATION AND SEARCH PROBLEMS

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Abstract. The principal has to allocate a good to one of many firms; there are externalities that cannot be internalized through payments. The optimal allocation depends on the agents' private information, which can be verified ex-post. We design the optimal allocation rule and show that, for some $\bar{n}$, the optimal rule with $\bar{n}$ firms is superior to any rule with $n > \bar{n}$ firms. Thus, in contrast with the classic insight of Bulow and Klemperer (1996), expanding the market beyond a certain point is counterproductive and the principal should focus on learning details of the environment and designing an optimal mechanism for a small number of agents.

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> *Since the informational demands for computing optimal mechanisms are substantial, and the computations involved are complex, [...] it will often be more worthwhile for a seller to devote resources to expanding the market than to collecting the information and making the calculations required to figure out the best mechanism.*
>
> *Bulow and Klemperer (1996)*

We study a selection problem in which a principal has to choose one of several agents. The agents' value for the principal is their private information and all agents would like to be selected. The utility is not transferrable, but the principal can learn the value of the selected agent ex-post and there are penalties that can be imposed on the selected agent.

This kind of selection problem is common. Here are several examples. Multiple rebel groups are engaged in a civil war in a foreign country. The government of the United States would like to select a rebel faction which is most likely to support the US interests; the US will channel financial and political support to this group with hope of bringing it to power. The rebel groups' true allegiance is, however, uncertain. In particular, there is a risk that some groups are connected with Al-Qaeda. This information will come to light after, and if, the supported group comes to power, at which point the US can penalize the group by withdrawing all future financial aid.

A prime minister would like to choose a judge to lead a judicial inquiry into a phone hacking scandal to examine the practice and ethics of press. The government is interested in selecting a judge whose preferences and views are aligned with those of the government. If the final report is unpleasant to the government, it can penalize the judge by excluding her from lucrative appointments in the future.

Our final example is a municipal government that would like to allocate a resource, such as a subsidy or a construction permit, to a firm which would maximize the resource's social value. Each firm privately knows the social value it can generate. The government can request the interested firms to report their private information and can impose a limited financial or legal penalty if the realized social value is inconsistent with the firm's promise.

If the government could *sell* the right to be selected and all externalities could be priced, it would prefer to attract as many agents as possible and allocate the right through a competitive mechanism. A larger pool of agents would increase the maximal potential surplus for the government surplus by improving the first-order statistics of the valuation of the right to be selected among the agents. Competition among the agents would ensure efficient allocation, since higher valuations generally translate into higher bids in competitive mechanisms.

There are, however, many interesting economic environments, including the three examples above, in which the utility is not fully transferable because there exist allocative externalities, the prices are constrained for legal, agency, or budget constraint reasons, or the contracts are incomplete and the parties can (re)negotiate the distribution of surplus after the allocation decision has been made. The positive effect of expanding the market on the total surplus remains true under these circumstances.

Nevertheless, since there are externalities that cannot be priced, the government must rely on communication with the agents, and tougher competition might destroy incentives for them to reveal their private information.

We show that in such environments an optimal allocation rule is a *shortlisting procedure*: each agent submits a report about his private information and is shortlisted with a probability that depends on his reported value. Then, the choice is made from the short list at random, each agent is selected with equal probability. After the match, the principal discovers the true value of the selected agent and imposes the maximum feasible penalty if the agent has made a false report.

The central feature of the optimal rule is that the principal maximizes the probability of selecting high-value agents subject to the constraint that the low-value agents are chosen frequently enough so that they do not want to misreport their information. That is, the principal commits to choose low-social-value agents with some probability, even when better agents are available. This incentive constraint applies to each agent and induces a neutrality result: the maximal attainable payoff for the principal is independent of the size of the market as long as there are at least $\bar{n}$ agents, where $\bar{n}$ depends on the penalty size that can be imposed on the agents. The value of $\bar{n}$ could be quite small. In particular, if the penalty does not exceed half of the agent's surplus from being selected, then $\bar{n} = 2$.

These results offer a rationale for why the government might want to exclude the majority of rebel groups from negotiations about financial support or restrict access to subsidies to a select group of qualified firms. It also explains why stochastic allocation mechanisms with uniform lotteries can be optimal.

Thus, noncompetitive allocation rules can be optimal and the value of competition is limited in that expanding the market beyond a certain point confers no additional benefit for the principal. This stands in contrast with the standard environments with private values in which the surplus can be fully internalized through payments (Bulow and Klemperer 1996). In those environments, market expansion simultaneously achieves two objectives: it increases the maximal expected surplus and strengthens competition among agents forcing them to redistribute a higher share of the surplus to the principal. In our environment, the second force is not present, which leads to the different conclusion.

An important implication of Bulow and Klemperer (1996) is that design of optimal allocation rules is a second-order issue and the principal should focus on increasing agents' participation. In our environment, however, the government's priorities are reversed and the focus should be optimal design of an allocation rule for a small number of agents.

The effect that the government cannot benefit from a larger market is general and can be obtained in many other variations of the model. The result is due to two forces: externalities that cannot be priced and the ability to provide incentives based on the realized value of the principal's payoff. The value of competition is limited because of externalities, but it is optimal to sample more than one agent because the government can provide incentives ex-post. In addition to the examples mentioned already, applications of the model include resource allocation problems, organizational decisions, choice of a public project, business ranking, and search for a political appointee.

There is a connection between the value of competition and the value of recall in a sequential search interpretation of our model. Instead of using the shortlisting procedure outlined above, the principal can sample the agents sequentially, selecting each sampled agent with probability one if she reports high value and, otherwise, selecting her with minimal probability sufficient to ensure truthtelling. This rule attains the optimal payoff but requires an infinite pool of agents. The optimal shortlisting procedure that needs only a handful of agents can be viewed as sequential search with recall. We discuss this connection in more detail in the concluding section.

Literature. *Ben-Porath, Dekel and Lipman (2013)* (henceforth, BDL) study a related model, with the key difference in modeling verification of agents' types by the principal. In BDL the principal can pay a cost and acquire information about agents' types before making and allocation decision, that is, types can be verified *ex ante*.[1] This paper complements BDL by using a different approach: after having selected an agent, the principal learns her type and can impose a penalty on that agent, that is, types are verified *ex post*.[2] As a consequence, the structures of feasible mechanisms and underlying incentives are very different, giving rise to drastically different optimal mechanisms. BDL feature an intriguing and surprisingly simple optimal rule called *favored-agent mechanism*. The principal picks a favored agent, i, and a threshold. If the highest report among agents other than i is above the threshold, then that report is checked and, if confirmed, the agent with that report is selected; in any other event the favored agent is selected. Recall that in our paper the optimal mechanism is a *shortlisting procedure*, where each agent is shortlisted with a probability that depends on her reported value, and then the choice is made from the short list at random, equally likely. One major difference between these two approaches is that BDL's optimal mechanism is deterministic and asymmetric, while ours is stochastic and symmetric; in fact, in our model deterministic allocations are generally not optimal. Another major difference concerns the value of competition, the focal issue of this paper: in BDL the principal's optimal payoff is increasing in the number of (symmetric) agents, so additional competition is always valuable, while in our model this is not the case.

Restricted competition. We believe that understanding the limits of optimality of competitive mechanisms is an important topic. A substantial literature focuses on environments where, unlike in this paper, private information is unverifiable and

[1] There is a literature on cheap talk and mechanism design in environments with evidence, e.g., Green and Laffont (1986), Bull and Watson (2007), Severinov and Deneckere (2006), Deneckere and Severinov (2008), Kartik, Ottaviani and Squintani (2007), Kartik (2009), Sher and Vohra (2011), Dziuda (2012), Ben-Porath and Lipman (2012), and Kartik and Tercieux (2012).

[2] See Mezzetti (2004) and Eraslan and Yimaz (2007) for mechanism design in environments in which the mechanisms can condition either on the agents' reports about the outcome or directly on the outcome.

monetary transfers or wasteful effort are used as design tools for truthful reporting of private types, and yet competitive allocation mechanisms need not be optimal.

In Chakravarty and Kaplan (2013) and Condorelli (2012), a benevolent principal would like to allocate an object to the agent with the highest valuation, and the agents signal their private types by exerting socially wasteful effort. The central issue is the trade-off between efficient allocation and wasted resources: if truthful communication is expensive, it might be optimal to select a winner randomly or based only on the publicly available information. In an ambitious paper, Condorelli (2012) studies a general model with heterogeneuos objects and agents and characterizes optimal allocation rules where a socially wasteful cost is a part of mechanism design. Chakravarty and Kaplan (2013) restrict attention to homogeneous objects and agents, which allows them to consider interesting economic environments in which socially wasteful cost has two components: an exogenously given type and a component controlled by the principal. They characterize optimal mechanisms and, in particular, demonstrate conditions under which, surprisingly, the uniform lottery is optimal.[3]

Bar and Gordon (2011) consider a problem of project selection. For each project, the principal's and the project manager's values of the potential match are privately known to the manager. Transfers are permitted in one direction: the principal can subsidize but cannot tax projects. Even though the problem of efficient matching is isomorphic to Myerson (1981), the problem of the principal's revenue-maximizing project selection is different and yields an unexpected result. The optimal mechanism features randomization over projects whose reported value for the principal exceeds some threshold; the highest type is not guaranteed to be chosen.

There are other well-known reasons for restricting competition. If there is an entry cost for agents, the competitive allocation might be inefficient, as low-value agents have low probability to win and thus lack incentives to participate (Levin and Smith 1994, Gilbert and Klemperer 2000, Ye 2007). If the value of surplus is endogenous and is determined by the actions of the agents prior to the allocation decision, excessive competition might weaken their incentives to undertake costly actions increasing total surplus. For example, in research and development contests, it might be optimal to limit the number of participants to improve their incentives to invest in developing new technology (Taylor 1995, Fullerton and McAfee 1999, Che and Gale 2003). In financial settings, it may also be desirable to limit the number of banks to keep their incentives to screen loan applicants (Cao and Shi 2001).

In addition, competition might have negative effects within specific allocation mechanisms. In Manelli and Vincent (1995), a principal would like to procure a good from suppliers whose quality is uncertain. In their environment, a trading mechanism that selects the bidder with the lowest price might result in only low-quality goods being offered for sale, so competitive mechanisms might price out high quality suppliers. Compte and Jehiel (2002) study auctions in affiliated value environments and show

[3]See also McAfee and McMillan (1992), Hartline and Roughgarden (2008), Yoon (2011) for design environments without transfers and money burning. Money burning is also studied in Ambrus and Egorov (2012) in the context of a delegation model.

that the uncertainty about the common value component might imply that more bidders need not lead to higher welfare.

Search. The paper is also related to a growing literature on search with incomplete information. Guerrieri, Shimer and Wright (2010) analyze a competitive search model, where uninformed principals compete in mechanisms that screen privately informed agents based on their reports. In a directed-search model in Menzio (2007), agents make statements about their privately known type and engage in the bargaining game about splitting their surplus from the match. Galenianos, Pacula and Persico (2012) study a market for illicit drugs, in which dealers can dilute drugs and hence, the quality is uncertain to the consumers when they meet a dealer.

A number of papers focus on search environments in which the principal is privately informed. We refer the reader to Lauermann and Wolinsky (2011) and the references therein. Finally, in Cremer, Spiegel and Zheng (2006, 2007), an auctioneer searches for bidders who are privately informed about their valuations, and Moldovanu and Shi (forthcoming) consider a model of search by a committee where each committee member has an ability to privately evaluate the value of an attribute of an alternative.

Model. There is a principal who has to select one of many agents. The principal's payoff from a match with agent i is $x_i \in X \equiv [a, b]$, where x_i is private to agent i. The values of x_i's are i.i.d. random draws, with continuously differentiable c.d.f. F on X, whose density f is positive almost everywhere on X. Each agent i can make a statement $y_i \in X$ about his x_i.

If an agent is not selected, his payoff is 0. Otherwise, he obtains a payoff of $v(x_i) > 0$. Thus, a match with agent i generates the surplus $S(x_i) = x_i + v(x_i)$, and the distribution of the surplus between the parties is exogenously fixed.

In addition, we assume that if the agent is selected, the principal can verify x_i and impose a penalty equal to the fraction $c(x_i) \geq 0$ of the agent's share of the surplus, where c is measurable. Note that c can be nonmonotonic and greater than v. To save on notation, w.l.o.g. we normalize $v_i(x_i)$ to a unit of utility for agent i, $v_i(x_i) \equiv 1$.

Our primary interpretation of c is the *maximal* penalty that can be imposed on the agent, conditional on his report y_i and type x_i; that is, we assume that x_i of the selected agent can be verified ex-post. The main message of the paper is robust to extensions that allow for stochastic verification of x_i, assume that the value of x_i is verified with some noise, or make verification costly for the principal or the agent. The value $c(x_i)$ can also capture the share of the lost surplus from the match if the principal can burn the surplus or break up the match with some probability or delay after learning the payoff. An alternative interpretation consistent with the results is that c represents the intrinsic disutility borne by the agent for being dishonest in case he is selected.

The principal has full commitment power and can choose any stochastic allocation rule that determines a probability of selecting each agent conditional on the report profile and the penalty conditional on the report profile and the type of the selected agent after it is verified ex-post. The allocation rule is common knowledge among the agents. The solution concept is perfect Bayesian equilibrium.

Interpretations. *Resource allocation.* The principal is a government that has a subsidy or a construction permit to allocate to one of several firms. There are no payments between the firms and the government, for example, because of legal or political constraints. The firms have private information about the social impact of their use of the resource. With some probability, the government can verify this information ex-post, in which case it can impose a penalty on the firm if it lied in its application.

Organizational decisions. A related application, which is inspired by an application in Ben-Porath, Dekel and Lipman (2013), is a dean who has a job slot to allocate to one of the departments. Each department has private information that determines the value to the dean of giving the post to the department. There are no monetary transfers, but the dean will eventually learn the value of the hired faculty and can penalize the department in the long run.

Contest. The principal is a ranking agency that selects the best business in an industry. Her objective is to select the best business based on customer opinions and referee reports. The businesses can manipulate the ranking algorithm by establishing cozy connections with referees and encouraging multiple votes from their patrons. The winner gets the spotlight; the relevant disutility cost for the winner is the probability of being caught manipulating the ranking and excluded in the future.

Public project. A regulator would like to reorganize a failing bank. Its objective is to maximize the social value of the restructured bank. There are multiple stakeholders who can make proposals about how to reorganize the bank. The quality of their proposals is their private information. Once a proposal is adopted, its quality is revealed; there is a legal penalty for deliberate distortion of information if there are negative consequences for public and/or other stakeholders.

Search. The principal is a politician that would like to appoint a loyal and competent bureaucrat to an agency position. The payoff to the bureaucrat is the experience, visibility, and connections he or she acquires in the position. The competence and loyalty of the bureaucrat are eventually revealed and the principal can choose to fire the bureaucrat. The proofs are not affected if we assume that the penalty also affects the principal. In particular, the agent can be fired and the penalty can be the unrealized share of the match surplus. Furthermore, the characterization of the optimal rule is not affected if we allow the principal to restart the search after firing the agent to recover the unrealized share of the surplus.

Upper bound. Let n be the number of agents. An allocation rule (p, ξ) associates with every profile of statements $y = (y_1, \ldots, y_n)$ a probability distribution $p(y)$ over $\{1, 2, \ldots, n\}$, where for each agent i, $p_i(y)$ stands for the probability of choosing i, and a family of functions $\xi_i(x_i, y) \in \{0, 1\}$, $i = 1, \ldots, n$, which determine whether agent i is penalized if he is selected given his type and the report profile.

Let x^* be the unique solution of

$$\int_a^{x^*}(x^*-x)\max\{1-c(x),0\}f(x)\mathrm{d}x=\int_{x^*}^b(x-x^*)f(x)\mathrm{d}x. \tag{1}$$

Proposition 1. *In any allocation rule, the principal's payoff is at most x^*.*

By the revelation principle, we can focus on incentive-compatible rules in which truthtelling is an equilibrium. Since x_i of the selected agent is verifiable, it is optimal to penalize the selected agent whenever he lies, $y_i \neq x_i$, and not penalize him otherwise.

If the penalty exceeds the surplus, $c(x) \geq 1$ for all x, then this is sufficient to deter lying. Then, the principal payoff is uncapped and $x^* = b$.

Otherwise, the penalty might be insufficient to ensure truthful reporting. Since there are no transfers, the incentives have to be provided through allocative distortion: each agent must be chosen with a high enough probability so that the benefit of making a false report is less than the penalty. At the extreme, if there is no penalty, $c(x) \equiv 0$, then the principal cannot do better than to select an agent at random, with $x^* = \int_a^b xf(x)\mathrm{d}x$.

The proof shows that this incentive constraint pins down the minimal probability with which low-value agents are selected and caps the maximum ex-ante payoff available to the principal at x^*.

Proof of Proposition 1. By the revelation principle, it is sufficient to consider allocation rules in which it is a perfect Bayesian equilibrium for all agents to make truthful statements. Furthermore, without loss of generality, we focus on the rules in which reports are punished if and only if they are dishonest. So, henceforth, we set $\xi_i(x_i, y) = 0$ if $y_i = x_i$ and 1 otherwise and drop ξ in the description of the allocation rules.

Fix allocation rule p. Denote by $\bar{F}$ the joint c.d.f. of all n agents and by $\bar{F}_{-i}$ the joint c.d.f. of all agents except i. Denote by $g_i(y_i)$ the expected probability that agent i is chosen in the truthful equilibrium after reporting y_i,

$$g_i(y_i)=\int_{x_{-i}\in X^{n-1}}p_i(y_i,x_{-i})\mathrm{d}\bar{F}_{-i}(x_{-i}). \tag{2}$$

The payoff of agent i whose type is x_i and who reports y_i is equal to

$$V_i(x_i,y_i)=g_i(y_i)(1-\mathbf{1}_{y_i\neq x_i}c(x_i))$$

where $\mathbf{1}$ is the indicator function that equals one if the selected agent is punished, $y_i \neq x_i$, and zero otherwise. Hence, each i's incentive constraint is $V_i(x_i, x_i) \geq V_i(x_i, y_i)$ for all $x_i, y_i \in X$, or equivalently,

$$g_i(x)\geq(1-c(x))\bar{g}_i \quad \text{for all } x\in X, \tag{3}$$

where $\bar{g}_i = \sup g_i(x)$. The principal's problem is to

$$\max_p \sum_{i=1}^n \int_{x_i \in X} x_i g_i(x_i) f(x_i) \mathrm{d}x_i$$
$$\text{s.t. (2) and (3) for all } i = 1, \dots, n.$$

By symmetry, we can use the same g_i for all agents, $g_i = g$. The relevant incentive constraint for agent i is, therefore,

$$g(x) \geq (1 - c(x))\bar{g} \quad \text{for all } x \in X. \tag{3$'$}$$

The expected payoff for the principal is equal to

$$W(g) = n \int_{x \in X} x g(x) f(x) \mathrm{d}x.$$

Let g^* be a maximizer of the relaxed principal's problem in which we maximize $W(g)$ over g subject to (3$'$) and the interim feasibility constraint[4]

$$n \int_{x \in X} g(x) f(x) \mathrm{d}x = 1. \tag{4}$$

We optimize $W(g)$ by assigning the highest feasible weight (a constant $\bar{g}$) to high values and the lowest feasible, incentive compatible weight (equal to $\bar{g}\max\{1-c(x), 0\}$) to low values. So, there exists threshold $x^* \in X$ such that almost everywhere on X

$$g^*(x) = \begin{cases} \max\{1 - c(x), 0\}\bar{g}^*, & \text{if } x < x^*, \\ \bar{g}^*, & \text{if } x \geq x^*. \end{cases}$$

Otherwise, we can redistribute the probability mass from low values of x for which there is slack in (3$'$) to higher values of x for which $g^*(x) < \bar{g}^*$ and increase the principal's payoff value.

Denote by h_x^* the transformed density function which coincides with the original density f for values above x^* and is minimized s.t. the incentive constraint for values below x^*:

$$h_{x^*}(x) = \begin{cases} \max\{1 - c(x), 0\}, & x < x^*, \\ 1, & x \geq x^*. \end{cases}$$

Let

$$H(x^*) = \int_X h_{x^*}(x) f(x) dx \quad \text{and} \quad V(x^*) = \frac{1}{H(x^*)} \int_X x h_{x^*}(x) f(x) dx. \tag{5}$$

Note that $\frac{h_{x^*}(x) f(x)}{H(x^*)}$ is a probability density, and $V(x^*)$ is the expectation w.r.t. that density.

Since $g^*(x) = \bar{g}^* h_{x^*}(x)$, we have $W(g^*) = n\bar{g}^* H(x^*) V(x^*)$, and feasibility constraint (4) reduces to

$$n\bar{g}^* H(x^*) = 1. \tag{6}$$

[4]We relax the constraint that there should exist p consistent with g.

Substituting (6) into the payoff, we get

$$W(g^*) = \max_{x^* \in X} V(x^*).$$

The first-order condition reduces to (1). By the standard argument, the solution is unique.[5] A straightforward calculation establishes that $W(g^*) = x^*$.[6] The value of x^* is the upper bound on the principal's payoff, because we have not verified that there exists p consistent with g^*. ∎

Implementation. How many agents are required to attain the payoff of x^*? The following rule achieves this payoff with infinitely many agents. Let $n = \infty$ and consider the allocation rule p_N^* that samples agents sequentially, until some agent is selected. In every period a new agent is drawn and selected with probability $h_{x^*}(y)$, given report y.[7]

Proposition 2. *The rule p_N^* attains the payoff of x^*.*

Proof. Let $H(x^*)$ and $V(x^*)$ be as in (5). The principal's payoff $W(p_N^*)$ is stationary:

$$W(p_N^*) \;=\; \int_X \Big(h_{x^*}(x)x + (1 - h_{x^*}(x))W(p_N^*)\Big) f(x)\mathrm{d}x,$$

hence $W(p_N^*) = \frac{1}{H(x^*)} \int_X x h_{x^*}(x) f(x)\mathrm{d}x = V(x^*) = x^*$. ∎

Could the same payoff be achieved with a smaller number of agents? Denote by $\bar{n}$ the smallest positive integer such that[8]

$$c(x) \le 1 - \frac{1}{\bar{n}} \quad \text{for all } x \le x^* \tag{7}$$

and consider the following *shortlisting procedure*. Let each agent $i = 1, \ldots, n$ be short-listed with some probability $q(y_i)$ given report y_i. The rule chooses an agent from the short list with equal probability. If the short list is empty, then the choice is made at random, uniformly among all n agents.

Proposition 3. *If $n \ge \bar{n}$, then there exists a shortlisting procedure that attains the payoff of x^*.*

Proof. Define the probability of shortlisting an agent who reports x as

$$q(x) = \frac{A h_{x^*}(x) - B}{A - B},$$

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[5] Since f is a.e. positive, the left-hand side of (1) is strictly increasing in x^*, moreover, for $x^* = a$ we have $-(\int_a^b x f(x)\mathrm{d}x - a) < 0$ and for $x^* = b$ we have $b - \int_a^b x \max\{1 - c(x), 0\} f(x)\mathrm{d}x > 0$.

[6] $W(g^*) = x^* + \frac{1}{H(x^*)}\Big(-\int_a^{x^*} (x^* - x)\max\{1 - c(x), 0\} f(x)\mathrm{d}x + \int_{x^*}^b (x - x^*) f(x)\mathrm{d}x \Big) = x^*$ since the second term is zero by (1).

[7] Note that this rule is asymmetric; the proof of the upper bound x^* applies to all rules, including asymmetric ones.

[8] Note that $\bar{n}$ exists if and only if $\sup_{x \in X} c(x) < 1$.

where A and B are the expected probabilities to be chosen conditional on being short-listed and conditional on not being short-listed, respectively. Let $Q = \int_X q(x)f(x)\mathrm{d}x$ be the ex-ante probability to be short-listed. Then,

$$A = \sum_{k=1}^{n} \frac{1}{k}\binom{n-1}{k-1} Q^{k-1}(1-Q)^{n-k} \quad \text{and} \quad B = \frac{1}{n}(1-Q)^{n-1}.$$

The value of Q is implicitly defined by[9] $Q = \frac{AH(x^*)-B}{A-B}$.

The rule is feasible. It is straightforward that $q(x) \le 1$ for all x and that $q(x) \ge 0$ for $x \ge x^*$. To verify $q(x) \ge 0$ for $x < x^*$, observe that $nB \le A$, and hence, by (7)

$$Ah_{x^*}(x) - B \ge \frac{A}{n} - B \ge 0.$$

The rule is incentive compatible. An agent's payoff conditional on his truthful report x is equal to the probability to be chosen,

$$q(x)A + (1-q(x))B = Ah_{x^*}(x),$$

whereas his payoff from reporting $y \ge x^*$, $y \ne x$, is equal to $A(1-c(x)) \le Ah_{x^*}(x)$.

The principal's expected payoff is equal to

$$W = \frac{\int_X x\,(q(x)A + (1-q(x))B)\,f(x)\mathrm{d}x}{\int_X (q(x)A + (1-q(x))B)\,f(x)\mathrm{d}x} = \frac{1}{H(x^*)}\int_X xh_{x^*}(x)f(x) = V(x^*) = x^*.$$

■

Value of competition. Thus, the value of competition is limited and expanding the market beyond $\bar{n}$ agents confers no benefit to the principal. We obtain the following corollary.

Corollary 1. *The optimal rule with $\bar{n}$ agents is superior to any rule with $n > \bar{n}$ agents.*

If expanding the market is costly, this result strengthens: the optimal rule with $\bar{n}$ agents is *strictly* superior to any rule with $n > \bar{n}$ agents.

Thus, in contrast with the classic insight in Bulow and Klemperer (1996), competition has limited value, and the principal should focus on learning details of the environment and designing an optimal mechanism for a small number of agents.

Moreover, under fairly broad conditions, $\bar{n}$ is very small. For example, suppose that the penalty is bounded by half of the utility of matching:

$$c(x) \le \tfrac{1}{2} \text{ for all } x \in X.$$

Then, it is optimal for the principal to look at most *two* agents. On the other hand, $\bar{n} \to \infty$ as $c(x)$ converges to one for all x.

Connection with search. Time is discrete, $t = 1, 2, \ldots, \infty$, and one agent arrives in each period. The rest of the model is identical. If there are no waiting costs and no recall, the optimal rule for the principal is to choose the agent with probability one

[9]Using $H(x^*) \ge 1/n$ by (7), it can be verified that equation $Q = \frac{AH(x^*)-B}{A-B}$ has a unique solution.

if he reports $x \geq x^*$ and probability $h(x)$ otherwise. This rule implements the payoff of x^*. For positive waiting costs, the optimal rule without recall has the same cutoff structure but a lower value of cutoff. For example, if the principal does not discount the future but there is a positive cost γ of sampling a new agent, the optimal cutoff is the unique solution of[10]

$$\int_a^{x^*} (x^* - x) \max\{1 - c(x), 0\} f(x) \mathrm{d}x + \gamma = \int_{x^*}^{b} (x - x^*) f(x) \mathrm{d}x.$$

If recall is allowed, then the principal need to sample $\bar{n}$ agents at most. In particular, if $c(x) \leq 1/2$, there is no need sample more than two agents. Hence, the result that competition has limited value can be viewed from the perspective of the value of recall in the search interpretation of our model.[11]

References

Ambrus, Attila and Georgy Egorov, "Delegation and Nonmonetary Incentives," *mimeo*, July 2012.

Bar, Talia and Sidartha Gordon, "Optimal Project Selection Mechanisms," *mimeo*, 2011.

Ben-Porath, Elchanan and Barton L. Lipman, "Implementation with partial provability," *Journal of Economic Theory*, 2012, *147* (5), 1689–1724.

_______, Eddie Dekel, and Barton L. Lipman, "Optimal Allocation with Costly Verification," *mimeo*, March 2013.

Bull, Jesse and Joel Watson, "Hard evidence and mechanism design," *Games and Economic Behavior*, January 2007, *58* (1), 75–93.

Bulow, Jeremy and Paul Klemperer, "Auctions versus Negotiations," *American Economic Review*, March 1996, *86* (1), 180–94.

Cao, Melanie and Shouyong Shi, "Screening, Bidding, and the Loan Market Tightness," *European Finance Review*, 2001, *5*, 21–61.

Chakravarty, Surajeet and Todd R. Kaplan, "Optimal allocation without transfer payments," *Games and Economic Behavior*, 2013, *77* (1), 1–20.

Che, Yeon-Koo and Ian Gale, "Optimal Design of Research Contests," *American Economic Review*, June 2003, *93* (3), 646–671.

Compte, Olivier and Philippe Jehiel, "On the Value of Competition in Procurement Auctions," *Econometrica*, January 2002, *70* (1), 343–355.

Condorelli, Daniele, "What money can't buy: Efficient mechanism design with costly signals," *Games and Economic Behavior*, 2012, *75* (2), 613–624.

Cremer, Jacques, Yossi Spiegel, and Charles Z. Zheng, "Optimal search auctions with correlated bidder types," *Economics Letters*, October 2006, *93* (1), 94–100.

_______, _______, and Charles Zhoucheng Zheng, "Optimal search auctions," *Journal of Economic Theory*, May 2007, *134* (1), 226–248.

Deneckere, Raymond and Sergei Severinov, "Mechanism design with partial state verifiability," *Games and Economic Behavior*, November 2008, *64* (2), 487–513.

Dziuda, Wioletta, "Communication with Detectable Deceit," *mimeo*, 2012.

Eraslan, Hülya K. K. and Bilge Yimaz, "Deliberation and Security Design in Bankruptcy," *mimeo*, 2007.

[10] Compare it with the optimal cutoff in the sequential search model with observable types given by $\gamma = \int_{x^*}^{b} (x - x^*) f(x) \mathrm{d}x$.

[11] For a survey of search literature, see Rogerson, Shimer and Wright (2005).

Fullerton, Richard L. and R. Preston McAfee, "Auctioning Entry into Tournaments," *Journal of Political Economy*, 1999, *107*, 573–605.

Galenianos, Manolis, Rosalie Liccardo Pacula, and Nicola Persico, "A Search-Theoretic Model of the Retail Market for Illicit Drugs," *Review of Economic Studies*, 2012, *79* (3), 1239–1269.

Gilbert, Richard J. and Paul Klemperer, "An Equilibrium Theory of Rationing," *RAND Journal of Economics*, 2000, *31*, 1–21.

Green, Jerry R and Jean-Jacques Laffont, "Partially Verifiable Information and Mechanism Design," *Review of Economic Studies*, July 1986, *53* (3), 447–56.

Guerrieri, Veronica, Robert Shimer, and Randall Wright, "Adverse Selection in Competitive Search Equilibrium," *Econometrica*, November 2010, *78* (6), 1823–1862.

Hartline, Jason D. and Tim Roughgarden, "Optimal Mechanism Design and Money Burning," *Proceedings of the 40th Annual ACM Symposium on Theory of Computing*, 2008, pp. 75–84.

Kartik, Navin, "Strategic Communication with Lying Costs," *Review of Economic Studies*, October 2009, *76* (4), 1359–1395.

——— **and Olivier Tercieux**, "Implementation with evidence," *Theoretical Economics*, May 2012, *7* (2).

———**, Marco Ottaviani, and Francesco Squintani**, "Credulity, lies, and costly talk," *Journal of Economic Theory*, May 2007, *134* (1), 93–116.

Lauermann, Stephan and Asher Wolinsky, "Search with Adverse Selection," *mimeo*, 2011.

Levin, Dan and James L. Smith, "Equilibrium in Auctions with Entry," *American Economic Review*, 1994, *84*, 585–599.

Manelli, Alejandro M and Daniel R Vincent, "Optimal Procurement Mechanisms," *Econometrica*, May 1995, *63* (3), 591–620.

McAfee, R Preston and John McMillan, "Bidding Rings," *American Economic Review*, June 1992, *82* (3), 579–99.

Menzio, Guido, "A Theory of Partially Directed Search," *Journal of Political Economy*, October 2007, *115* (5), 748–769.

Mezzetti, Claudio, "Mechanism Design with Interdependent Valuations: Efficiency," *Econometrica*, 09 2004, *72* (5), 1617–1626.

Moldovanu, Benny and Xianwen Shi, "Specialization and Partisanship in Committee Search," *Theoretical Economics*, forthcoming.

Myerson, Roger B., "Optimal Auction Design," *Mathematics of Operations Research*, 1981, *6*, 58–73.

Rogerson, Richard, Robert Shimer, and Randall Wright, "Search-Theoretic Models of the Labor Market: A Survey," *Journal of Economic Literature*, December 2005, *43* (4), 959–988.

Severinov, Sergei and Raymond Deneckere, "Screening when some agents are nonstrategic: does a monopoly need to exclude?," *RAND Journal of Economics*, December 2006, *37* (4), 816–840.

Sher, Itai and Rakesh Vohra, "Price Discrimination through Communication," *mimeo*, June 2011.

Taylor, Curtis R., "Digging for Golden Carrots: An Analysis of Research Tournaments," *American Economic Review*, 1995, *85*, 872–890.

Ye, Lixin, "Indicative Bidding and A Theory of Two-Stage Auctions," *Games and Economic Behavior*, 2007, *58*, 181–207.

Yoon, Kiho, "Optimal mechanism design when both allocative inefficiency and expenditure inefficiency matter," *Journal of Mathematical Economics*, 2011, *47* (6), 670–676.