Hyperbole, Litotes and Irony: Noisy Communication with Lying Costs

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Abstract

We provide a class of tractable communication games where messages are noisy and lies are costly, in which costs are increasing in the magnitude of the lie. Games in this class admit Nash-Bayesian equilibria where both players use affine strategies. There can be between one and five affine equilibria, all of them partially informative. There is always at least one, but at most three *straight talking* equilibria, in which both players' strategies are increasing. Exactly one of the following statements holds: either (i) there is exactly one *truthful* or *exaggerating* equilibrium or (ii) there are between one and three *understated* equilibria. There can also be up to two *ironic* equilibria, where both players' strategies are decreasing. In general, the optimal level of noise is not zero and differs between the two players. The receiver may prefer to deal with a less honest sender and with a sender who is more dissimilar to him. Finally, we study the limit of the equilibria as either *(i)* the noise vanishes or *(ii)* the lying cost vanishes.

Keywords: Strategic information transmission, Noise, Lying Costs.

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1 Introduction

In this paper, we propose a new framework to think about the strategic transmission of relevant information from an informed sender to a receiver who needs to take a decision, when there is a conflict of interest. The large litterature on this type of problems originates from the seminal paper by Crawford and Sobel (1982) who modelled an extreme situation where messages are pure cheap talk: they do not directly enter the payoffs of the players. They show that games where the conflict of interest is not too large have equilibria where the sender sends different messages depending on his information and the receiver reacts differently to messages. The assumption in their model is that (i) the communication channel that is used perfectly transmits to the receiver the messages the sender decides to send and (ii) that there is no intrinsic psychological cost for the receiver in lying. We depart from this benchmark case in that we remove these two assumptions: we study communication through a noisy channel and assume that it is costly for the sender to lie.

In recent years, there has been experimental evidence that senders experience lying costs when engaged in a sender-receiver game (Gneezy, 2005; Hurkens and Kartik, 2006). Several models consider lying costs in the context of a sender receiver game (Kartik, 2009; Kartik, Ottaviani and Squintani). These papers, however, maintain the assumption that the communication channel perfectly transmits the information sent by the receiver. Introducing noise into these models enables the sender to hide in the noise. Another series of papers have considered noisy communication (Board, Blume and Kawamura, 2007; Board and Blume 2012). They do however assume that liying is costless to the receiver. One of our important contributions in this paper is that we identify a class of models with lying costs and noise that leads to simple tractable equilibria, where players use affine startegies. The sender sends a message that is an affine function of his type and the receiver chooses an action that is an affine function of the message he receives. The simplicity of the strategy space makes it possible to adress new questions that would otherwise be out of reach.

We show that in any game in the class we consider, there can be between one and five affine equilibria, all of them partially informative. There is always at least one, but at most three *straight talking* equilibria, in which both players' strategies are increasing. Exactly one of the following statements holds: either (i) there is exactly one *truthful* or *exaggerating* equilibrium or (ii) there are between one and three *understated* equilibria. There can also be up to two *ironic* equilibria, where both players' strategies are decreasing. In general, the optimal level of noise is not zero and differs between the two players. The receiver may prefer to deal with a less honest sender and with a sender who is more dissimilar to him. Finally, we study the limit of the equilibria as either *(i)* the noise vanishes or *(ii)* the lying cost vanishes.

2 The model

First, nature draws the Sender's type $\theta \in \Theta = \mathbb{R}$ and a noise level $\epsilon \in \mathbb{R}$ from a joint distribution. The Sender observes θ and sends a message $m \in M = \mathbb{R}$. The receiver then observes $y = m + \epsilon \in Y = \mathbb{R}$ and chooses an action $a \in A = \mathbb{R}$. The payoffs $U^i(a, \theta)$, for $i \in \{R, S\}$ are then realized. A pure strategy for the Sender is a function $\mu : \Theta \to M$. A pure strategy for the Receiver is a function $\alpha : Y \to A$. A Bayesian Nash-Equilibrium is a strategy profile (μ, α) such that

$$E_\epsilon \left[U^S(\alpha(\mu(\theta) + \epsilon), \theta) \mid \theta\right] \geq E_\epsilon \left[U^S(\alpha(m + \epsilon), \theta) \mid \theta\right].$$

for all $m \in M$, and all $\theta \in \Theta$ and

$$E_{\theta,\epsilon} \left[U^R(\alpha(y), \theta) \mid \mu(\theta) + \epsilon = y\right] \geq E_{\theta,\epsilon} \left[U^R(a, \theta) \mid \mu(\theta) + \epsilon = y\right].$$

for all $a \in A$, and all $y \in Y$.

We assume that the Receiver's payoff is

$$U^R(a, \theta) = -(a - [r\theta + b])^2,$$

and that the Sender's payoff is

$$U^S(a, \theta) = -(a - [s\theta + c])^2 - k(\theta - m)^2.$$

Here θ and ϵ are independent real random variables that we are assume to be normally distributed with zero zero expectation and variances $\sigma_\theta^2 > 0$ and $\sigma_\epsilon^2 > 0$. Let

$$v = \frac{\sigma_\epsilon^2}{\sigma_\theta^2} > 0.$$

The parameters b, c, r and s are real numbers. Without loss of generality we assume $s \geq 0$.[1] Following Kartik (2009), the real number $k > 0$ parametrizes the sender's cost of lying. If k is large, the cost of lying is high. In the limit $k \to 0$ the sender's message is pure cheap-talk.

3 Preliminaries

3.1 Linear equilibria

We look for equilibria in which players use "linear" strategies.

$$\mu(\theta) = \beta\theta + \mu_0$$
$$\alpha(y) = \lambda y + \alpha_0.$$

The unique best reply of the receiver to a linear strategy (β, μ_0) of the sender is linear with parameters (λ, α_0) satisfying

$$\lambda = r\frac{\beta}{\beta^2 + v} \text{ and } \alpha_0 = b - \lambda\mu_0. \tag{1}$$

These equations follow upon observing that the receiver's best response is given by $rE[\theta \mid y] + b$ and that from the linear conditional expectation property of normally distributed random variables we have $E[\theta \mid y] = \frac{\beta}{\beta^2+v}(y - \mu_0)$. The second equality has the following interpretation. The expected message sent induces the expected preferred action of the receiver.

The unique best reply of the sender to a linear strategy (λ, α_0) of the receiver is linear with parameters (β, μ_0) satisfying

$$\beta = \frac{k + s\lambda}{k + \lambda^2} \text{ and } \mu_0 = \lambda\frac{c - \alpha_0}{k + \lambda^2}. \tag{2}$$

These equations follow from substituting $a = \lambda(m + \epsilon) + \alpha_0$ into the expression for the sender's payoff, taking expectations with respect to ϵ and then maximizing with respect to m.

If (β, λ) solves the system

$$\begin{cases} \lambda = \frac{r\beta}{\beta^2+v} \\ \beta = \frac{k+s\lambda}{k+\lambda^2} \end{cases} \tag{3}$$

[1] Treating $-\theta$ rather than θ as the type of the sender transforms a model with $s < 0$ into one with $s > 0$.

then the system

$$\begin{cases} \alpha_0 = b - \lambda\mu_0 \\ \mu_0 = \lambda\frac{c-\alpha_0}{\lambda^2+k}. \end{cases}$$

has a unique solution given by

$$\mu_0 = \frac{\lambda}{k}(c-b) \text{ and } \alpha_0 = b - \frac{\lambda^2}{k}(c-b). \tag{4}$$

Therefore the problem of finding equilibria can be reduced to the problem of finding the solutions (β, λ) to the system (3). Furthermore, taking the solution to (4) into account, we can we can write the player's expected payoffs as functions of (β, λ) and the exogenous parameters:

$$u_S(\beta, \lambda) = -\left[(\lambda\beta - s)^2 + \lambda^2 v + k(1-\beta)^2\right]\sigma_\theta^2 - \frac{k+\lambda^2}{k}(c-b)^2. \tag{5}$$

$$u_R(\beta, \lambda) = -\left[(\lambda\beta - r)^2 + \lambda^2 v\right]\sigma_\theta^2. \tag{6}$$

3.2 Existence of straight talking equilibria and non-existence of babbling equilibria

We refer to an equilibrium as *straight talking* if $\beta > 0$ holds. An equilibrium with $\beta = 0$ is *babbling* and an equilibrium with $\beta < 0$ is *ironic*.

Substituting the first equation from (3) into the second yields that β is part of and equilibrium if and only if

$$k\beta\left(\beta^2+v\right)^2 + r^2\beta^3 - k\left(\beta^2+v\right)^2 - sr\beta\left(\beta^2+v\right) = 0 \tag{7}$$

holds. Because we have assumed $k > 0$ the left side of (7) is strictly smaller than zero at $\beta = 0$ and converges to ∞ for $\beta \to \infty$. If follows that there is no babbling equilibrium and that at least one straight talking equilibrium exists.

The intuition for the non-existence of a babbling equilibrium is the following: if y had no informational content, the receiver would find it optimal to chose the response $a = c$ no matter which signal he receives. Given that lying is costly and his message does not affect the receiver's response the sender will however find it optimal to choose $m = \theta$, implying that y carries informational content.

3.3 The maximal number of straight talking and ironic equilibria

Expanding the polynomial equation (7) can be rewritten as

$$k\beta^5 - k\beta^4 + \left[2kv + r^2 - sr\right]\beta^3 - 2kv\beta^2 + \left[kv^2 - srv\right]\beta - kv^2 = 0. \tag{8}$$

Form Descartes' rule of signs it is then immediate that there is a unique straight talking equilibrium if

$$2kv + r^2 - sr \leq 0 \tag{9}$$

holds. This is because when this inequality holds, then $kv - sr \leq 0$ also holds an an implication. Similarly, there is no ironic equilibrium if

$$kv - sr \geq 0 \tag{10}$$

holds. This is because when this inequality holds, then $2kv + r^2 - sr \geq 0$ also holds as an implication.

As we will see below (10) does a good job at capturing the conditions which preclude the existence of ironic equilibria, namely high values of k and v and low values of r and s (observe, in particular, that no ironic equilibrium can exist for $s = 0$ or $r \leq 0$), but the sufficient conditions for uniqueness of a straight talking equilibrium in (9), which requires not only k and v to be small, but also $s > r > 0$, can be much improved.

We obtain the following result.

Proposition 1. *For any parameters, there can be at most three straight talking equilibria and at most two ironic talking equilibria. There are parameters for which the game has two ironic and three straight talking equilibria.*

Proof to be added.

3.4 Changing parameters

Let $\gamma = \lambda\beta$. This parameter measures how strongly the receiver's response varies with the underlying type of the sender. Multiplying both equations in the system (3) by β (which introduces an artificial root at $\beta = 0$ which does not correspond to an equilibrium) we can rewrite these equations as

$$\gamma = f(\beta) = \frac{r\beta^2}{v + \beta^2} \tag{11}$$

and

$$g(\beta, \gamma) = k\beta^2 + \gamma^2 - k\beta - s\gamma = 0. \tag{12}$$

and identify the linear equilibria of the model with the solutions (β, γ) of these equations satisfying $\beta \neq 0$.

Equation (12) is equivalent to

$$\frac{\left(\beta - \frac{1}{2}\right)^2}{\left(\sqrt{\frac{k+s^2}{4k}}\right)^2} + \frac{\left(\gamma - \frac{s}{2}\right)^2}{\left(\sqrt{\frac{k+s^2}{4}}\right)^2} = 1 \tag{13}$$

and thus describes an ellipse. Let

$$\underline{\gamma} = \frac{s}{2} - \sqrt{\frac{k+s^2}{4}},\ \overline{\gamma} = \frac{s}{2} + \sqrt{\frac{k+s^2}{4}}, \underline{\beta} = \frac{1}{2} - \sqrt{\frac{k+s^2}{4k}} \text{ and } \overline{\beta} = \frac{1}{2} + \sqrt{\frac{k+s^2}{4k}}.$$

For all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$ let $\beta_1(\gamma) \leq \beta_2(\gamma)$ be the two reals such that $(\beta_1(\gamma), \gamma)$ is the left part of the graph of the ellipse and $(\beta_2(\gamma), \gamma)$ is the right part of the graph of the ellipse. More precisely:

$$\beta_1(\gamma) = \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{s\gamma - \gamma^2}{k}} \tag{14}$$

$$\beta_2(\gamma) = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{s\gamma - \gamma^2}{k}}. \tag{15}$$

Similarly, for all $\beta \in \left[\underline{\beta}, \overline{\beta}\right]$, let $\gamma_1(\beta) \leq \gamma_2(\beta)$ be the two reals such that $(\beta, \gamma_1(\beta))$ is the lower part of the graph of the ellipse and $(\beta, \gamma_2(\beta))$ is the upper part of the graph of the ellipse. More precisely

$$\gamma_1(\beta) = \frac{s}{2} - \sqrt{\frac{s^2}{4} + k\left(\beta - \beta^2\right)} \tag{16}$$

$$\gamma_2(\beta) = \frac{s}{2} + \sqrt{\frac{s^2}{4} + k\left(\beta - \beta^2\right)}. \tag{17}$$

Observe that every equilibrium (β, γ) must satisfy $\underline{\beta} \leq \beta \leq \overline{\beta}$ and $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$.

4 Uniqueness and multiplicity of straight talking equilibria

Here we consider the case $r > 0$.[2] In addition we assume $s > 0$.[3]

We have already established that there is at least one straight talking equilibrium. Here we identify conditions under which there is a unique such equilibrium and discuss the existence of different kinds of straight talking equilibria.

Before we proceed, observe that under the assumption $r > 0$ the best response of the receiver to any strategy of the sender with $\beta > 0$ satisfies $\lambda > 0$. Consequently, we seek to identify conditions under which there cannot be more than one solution to the equations (11) and (12) satisfying $\beta > 0$ and $\gamma > 0$.

4.1 Truthful equilibrium

The conditions under which there is an equilibrium satisfying $\beta = 1$ are easy to identity: this will be an equilibrium if and only if $f(1) = s$ holds, [4] which is in turn is equivalent to $r = s(1 + v)$. Furthermore, if $f(1) = s$ holds, there can be no other straight talking equilibrium but $(\beta, \gamma) = (1, s)$ because $\gamma_2(\beta) > s > f(\beta) > 0 > \gamma_1(\beta)$ holds for all $\beta \in (0, 1)$ and $f(\beta) > s > \gamma_2(\beta) \geq \gamma_1(\beta)$ holds for all $\beta \in (1, \overline{\beta})$. Consequently, we have the following result.

Proposition 2. *There exists an equilibrium with $\beta = 1$ if and only if $r = s(1 + v)$ holds. If this condition holds $(\beta, \gamma) = (1, s)$ is the unique equilibrium satisfying $\beta > 0$.*

[2] For $r = 0$ it is trivial that the unique linear equilibrium has $(\beta, \lambda) = (1, 0)$. The case $r < 0$ will be considered separately.

[3] The case $s = 0$ is covered in most of the following, so that it can be used for illustrative purposes. However, the result establishing uniqueness for low k uses the assumption $s > 0$.

[4] Assuming $b = c = 0$ the interpretation of the case $f(1) = s$ is the following: if the sender reports his type truthfully (that is, choose $\beta = 1$) the best response of the receiver is to choose $\lambda = s$ so that conditional on θ the receiver's expected action is the one the sender wants him to choose. Hence, there is no incentive for the sender to distort his message from the truth and $(\beta, \lambda) = (1, s)$ is an equilibrium. Similarly, $f(1) < s$ means that if the sender reports his type truthfully then from the sender's perspective the receiver's best response isn't sufficiently reactive to the underlying state of the world. This implies that the sender has an incentive to exaggerate by choosing $\beta > 1$.

4.2 Exaggerating equilibrium

Here we consider the possibility of a straight talking equilibrium in which the sender exaggerates his type in the sense that $\beta > 1$ holds. Because $\gamma_2(\beta) \geq s$ and $\gamma_1(\beta) \leq 0$ holds for all $\beta \in [0,1]$ and $f(\beta)$ is strictly increasing such an equilibrium exists if and only if $f(1) < s$ holds. Consequently, a necessary and sufficient condition for the existence of an exaggerating equilibrium is

$$r < s(1+v) \tag{18}$$

and if this condition holds there can be no straight talking equilibrium with $\beta \leq 1$. The proof of the following result demonstrates that, in addition, that there cannot be more than one exaggerating equilibrium. The difficult case in proving this result is the one in which $f(\overline{\beta}) < s/2$ holds, meaning that the receiver is much less reactive than the sender would like him to be.

Proposition 3. *There exists an equilibrium with $\beta > 1$ if and only if condition (18) holds. If this condition holds there is a unique equilibrium satisfying $\beta > 0$.*

Proof. Let (β^*, γ^*) be the straight talking equilibrium with the smallest $\beta > 0$. If $\beta^* = \overline{\beta}$ it is immediate that there is no other straight talking equilibrium. Suppose $1 < \beta^* < \overline{\beta}$ and $\gamma^* = \gamma_2(\beta^*)$ holds. Because f is strictly increasing and γ_2 is strictly decreasing on the interval $[\beta^*, \overline{\beta}]$ it is immediate that $f(\beta) > \gamma_2(\beta)$ holds for all $\beta \in (\beta^*, \overline{\beta}]$. Using the inequality $\gamma_2(\beta) \geq \gamma_1(\beta)$ this implies there is no straight talking equilibrium with $\beta > \beta^*$. It remains to consider the case $1 < \beta^* < \overline{\beta}$ and $\gamma^* = \gamma_1(\beta^*)$. The slope of f at $\beta \in (0, \overline{\beta})$ is

$$f'(\beta) = \frac{2rv\beta}{(\beta^2+v)^2} = 2\frac{vf(\beta)}{r\beta^2}\frac{f(\beta)}{\beta}.$$

From (11) we have $vf(\beta) < r\beta^2$, implying

$$f'(\beta) < 2f(\beta)/\beta. \tag{19}$$

The slope of γ_1 at $\beta \in (0, \overline{\beta})$ is

$$\gamma_1'(\beta) = \frac{k\left(\beta - \frac{1}{2}\right)}{\sqrt{\frac{s^2}{4} + k\left(\beta - \beta^2\right)}} = \frac{k\left(\beta - \frac{1}{2}\right)}{\frac{s}{2} - \gamma_1(\beta)}.$$

From (12) we have $k(\beta - 1) = \gamma_1(\beta)(s - \gamma_1(\beta))/\beta$. Using $\gamma_1(\beta) < s/2$ this implies

$$\gamma_1'(\beta) > \frac{\gamma_1(\beta)}{\beta} \frac{(s - \gamma_1(\beta))}{(\frac{s}{2} - \gamma_1(\beta))} > 2\frac{\gamma_1(\beta)}{\beta}. \tag{20}$$

From (19) and (20) it follows that $f(\beta) < \gamma_1(\beta)$ holds for all $\beta \in (\beta^*, \overline{\beta})$ (because at any point of intersection of f and γ_1 in the interval $[0, \overline{\beta})$ the slope of f is strictly smaller than the slope of γ_1, implying that there can be at most one such intersection). Using the inequality $\gamma_2(\beta) \geq \gamma_1(\beta)$ this implies there is no straight talking equilibrium with $\beta > \beta^*$.[5] □

4.3 Understated equilibria

Here we consider equilibria which are understated in the sense that $0 < \beta < 1$ holds. Because straight talking equilibria exist and (as we have seen above) for $r > s(1+v)$ there can be no truthful or exaggerating equilibria, this condition is necessary and sufficient for the existence of an understated equilibrium.

Proposition 4. *An equilibrium satisfying* $0 < \beta < 1$ *exists if and only if* $r > s(1+v)$ *holds.*

Because $\beta \in (0, 1)$ implies $\gamma_1(\beta) < 0$ every understated equilibrium satisfies $\gamma = \gamma_2(\beta)$.

We say that an understated equilibrium is *loud* if $\beta \in [1/2, 1)$ holds and *gentle* if $\beta \in (0, 1/2)$ holds.

Because γ_2 is strictly decreasing on the interval $[1/2, 1]$ and f is strictly increasing there can be at most one loud equilibrium and the conditions $f(0.5) \leq \gamma_2(0.5)$ and $f(1) > \gamma_2(1)$ are necessary and sufficient for existence of a loud equilibrium. As we will see below the condition $f(0.5) \leq \gamma_2(0.5)$ is not sufficient to exclude the possibility that besides the loud equilibrium there are also gentle ones. If, however, the stronger condition $f(0.5) \leq s$ holds, there can be no gentle equilibrium as $\gamma_2(\beta) > s > f(\beta)$ then holds for all $\beta \in (0, 1/2)$. We thus have the following result.

[5] This is a bit sloppy because as it stands the argument only precludes the existence of an equilibrium in $(\beta^*, \overline{\beta})$, so one should also argue that there can be no additional equilibrium at $\overline{\beta}$. But as the "slope" of γ_1 is infinite there, it seems obvious enough (but painful to write down properly) that there can be no such equilibrium.

Proposition 5. *An equilibrium satisfying* $1/2 \leq \beta < 1$ *exists if and only if*

$$(1+v)s < r \leq [1+4v]\left[\frac{s}{2} + \frac{\sqrt{s^2+k}}{2}\right]. \tag{21}$$

Furthermore, there is at most one such equilibrium and if

$$(1+v)s < r \leq (1+4v)s \tag{22}$$

holds then the unique equilibrium satisfying $1/2 < \beta < 1$ *is also the unique equilibrium satisfying* $\beta > 0$.

For large enough values of k it follows from (21) that a loud equilibrium exists provided the condition $(1+v)s < r$ is satisfied. Intuition suggests that for large value of k there can be no gentle equilibrium as it is too expensive for the sender to stray far from $\beta = 1$, implying that under these circumstance a loud equilibrium not only exists but is also the unique straight talking equilibrium. The following proposition formalizes this intuition.

Proposition 6. *Suppose*

$$(1+v)s < r \leq 4\sqrt{v}\left[\frac{s}{2} + \frac{\sqrt{s^2+k}}{2}\right] \tag{23}$$

holds. Then there exists a unique straight talking equilibrium which satisfied $1/2 \leq \beta < 1$.

Proof. Define $\lambda_f : (-\infty, +\infty) \to (0, \infty)$ by $\lambda_f(\beta) = f(\beta)/\beta = r\beta/(\beta^2 + v)$. The derivative of this function is

$$\lambda_f'(\beta) = \frac{v - \beta^2}{\beta^2 + v)^2},$$

implying that $\lambda_f(\beta)$ is unimodal on $[0, \infty)$ with the unique maximum occurring at $\beta = \sqrt{v}$. Hence, we have $f(\beta) \leq \lambda_f(\sqrt{v})\beta$ for all $\beta \in [0, \overline{\beta}]$. Because γ_2 is concave and $\gamma_2(0) \geq 0$ holds, it follows that the condition $\gamma_2(1/2) \geq \lambda_f(\sqrt{v})/2$ is sufficient to imply $\gamma_2(\beta) > f(\beta)$ for all $\beta \in (0, 1/2)$. Calculating $\lambda_f(\sqrt{v})$ yields the result. □

Observe that (as must be the case) the rightmost side in (23) is smaller than the rightmost side in (21), but that for $v = 1/4$ these two expressions are identical, implying that in this special case whenever a loud equilibrium exists it is the unique straightforward equilibrium. More generally, as asserted in the following proposition, there is a unique straight talking equilibrium whenever $v \geq 1/4$ holds.

Proposition 7. *If $v \geq 1/4$ there exists a unique straight talking equilibrium.*

Proof. Define $\lambda_2 : (0, \overline{\beta}) \to (0, \infty)$ by $\lambda_2(\beta) = \gamma_2(\beta)/\beta$. Because γ_2 is strictly concave and $\gamma_2(0) \geq 0$ holds, the function λ_2 is strictly decreasing on $(0, 1/2]$ whereas for $v \geq 1/4$ the function λ_f (defined in the proof of the previous proposition) is strictly increasing on $(0, 1/2]$. It follows that the functions λ_2 and λ_f have either zero or one intersections on $(0, 1/2)$. In the first case every straight talking equilibrium satisfies $\beta \geq 1/2$ and it follows from previous results (Propositions 2, 3, and 5) that there is a unique straight talking equilibrium. In the second case there is a unique gentle equilibrium and we must have $\lambda_f(1/2) > \lambda_2(1/2)$, implying $r > [1 + 4v]\left[\frac{s}{2} + \frac{\sqrt{s^2+k}}{2}\right]$. From previous results (Propositions 2, 3, and 5) the later condition precludes the existence of a straight talking equilibrium with $\beta \geq 1/2$, implying that the unique gentle equilibrium is also the unique straight talking equilibrium. □

We know that the condition

$$r > [1 + 4v]\left[\frac{s}{2} + \frac{\sqrt{s^2 + k}}{2}\right] \tag{24}$$

is sufficient for the existence of a gentle equilibrium and ensures that all straightforward equilibria are gentle. If $v \geq 1/4$ holds, the previous result implies that (24) is necessary and sufficient for the existence of a gentle equilibrium and ensures it uniqueness. Without the additional condition $v \geq 1/4$ we are neither assured that (24) is necessary for existence of a gentle equilibrium (rather we have the weaker necessary conditions $r > \max\{(1+4v)s, 4\sqrt{v}\left[\frac{s}{2} + \frac{\sqrt{s^2+k}}{2}\right]\}$) nor do we have a uniqueness result.

At the cost of replacing (24) by a stronger condition, the following proposition extends our previous uniqueness result for gentle equilibria to the case $v < 1/4$.

Proposition 8. *If*

$$r \geq 2\left[\frac{s}{2} + \frac{\sqrt{s^2 + k}}{2}\right] \tag{25}$$

then there there exists a unique straight talking equilibrium. If $v < 1/4$ this equilibrium satisfies $\beta < 1/2$.

Proof. Condition (25) is equivalent to $f(\sqrt{v}) \geq \gamma_2(1/2)$. Suppose $v < 1/4$. Then (25) implies (24), so that all straight talking equilibria satisfy $\beta < 1/2$. Uniqueness follows by observing that both f and γ_2 are increasing on $[0, 1/2]$ so that $f(\sqrt{v}) \geq \gamma_2(1/2)$

implies $f(\beta) > \gamma_2(\beta)$ for all $\beta \in [\sqrt{v}, 1/2]$. Because λ_f is strictly increasing and λ_2 is strictly decreasing on $(0, \sqrt{v})$ it follows that f and γ have at most one intersection on $[0, 1/2]$. If $v \geq 1/4$ uniqueness follows from the preceding result. $\square$

Our results so far establish that multiple straight talking equilibria can only exist if there is at least one gentle equilibrium. We now provide an upper bound on k which ensures that if there is a gentle equilibrium it must be the unique straight talking equilibrium, thus establishing uniqueness of straight talking equilibrium for sufficiently small lying costs.

Proposition 9. *If*

$$k \leq rs \frac{v}{(v + 1/4)^2} \tag{26}$$

then there is a unique equilibrium with $\beta > 0$.

Proof. We first observe that the first and second derivatives of f are

$$f'(\beta) = 2rv \frac{\beta}{(v + \beta^2)^2}, \quad f''(\beta) = 2rv \frac{(v - 3\beta^2)}{(v + \beta^2)^3}. \tag{27}$$

Hence on $(0, \infty)$ the function f has a unique inflection point $\beta^\circ = \sqrt{v/3}$. Suppose there exist equilibria satisfying $0 < \beta < 1/2$. (Otherwise uniqueness of straight talking equilibria is immediate from the previous results.) Let β^* be the smallest such equilibrium. It must then be the case that $f'(\beta^*) \geq \gamma'(\beta^*)$ holds. If $f'(\beta^*) > \gamma_2'(\beta^*)$ and there is no other equilibrium satisfying $\beta < 1/2$, it follows that $f(1/2) > \gamma_2(1/2)$ holds, precluding the existence of any equilibrium with $\beta \geq 1/2$, so that there is a unique straight talking equilibrium. Similarly, unless there is another equilibrium satisfying $\beta < 1/2$, $f'(\beta^*) = \gamma_2'(\beta^*)$ and $f''(\beta^*) > \gamma_2''(\beta^*)$ implies $f(1/2) > \gamma_2(1/2)$ and thus uniqueness of straight talking equilibria. Hence, if there are multiple straight talking equilibria, we must either have (i) $f'(\beta^*) = \gamma_2'(\beta^*)$ and $f''(\beta^*) \leq \gamma''(\beta^*)$ or (ii) there exists a second equilibrium with $1/2 > \beta^{**} > \beta^*$ satisfying $f'(\beta^{**}) \leq \gamma_2'(\beta^{**})$. In either case we have the existence of an equilibrium satisfying $0 < \beta < 1/2$, $f'(\beta) \leq \gamma_2'(\beta)$ and $\beta > \beta^\circ$: In case (i) the conclusion $\beta > \beta^\circ$ follows because γ_2 is strictly concave, so that $f''(\beta^*) \leq g''(\beta^*)$ implies $f''(\beta^*) < 0$; in case (ii) it follows because we have $\gamma_2'(\beta^*) \leq f'(\beta^*)$ and $f'(\beta^{**}) \leq \gamma_2'(\beta^{**})$ and therefore, from the concavity of γ_2, $f'(\beta^{**}) < f'(\beta^*)$. This implies the inflection point of the f lies on $[0, \beta^{**})$. To finish the proof it thus suffices to show that condition (26) implies that there is no $\beta \in (\beta^\circ, 1/2)$ satisfying $f'(\beta) \leq \gamma_2(\beta)$. Towards this end we first observe that $\beta \in (\beta^\circ, 1/2)$ implies

$$f'(\beta) \geq f'\left(\frac{1}{2}\right) = \frac{rv}{\left(v+\frac{1}{4}\right)^2}.$$

Second, since γ_2' is strictly decreasing, we have that $f'(\beta) \leq \gamma_2'(\beta)$ implies

$$f'(\beta) < \gamma_2'(0) = \frac{k}{s}.$$

Therefore, if both of these conditions holds we have be that

$$\frac{v}{\left(v+\frac{1}{4}\right)^2} < \frac{k}{rs},$$

contradicting (26). $\square$

4.4 Summary of sufficient conditions for the uniqueness of straight talking equilibria

Uniqueness of straight talking equilibrium is assured if the communication schedule is sufficiently noisy, that is $v \geq 1/4$. Depending on the other parameters this case is consistent with the equilibrium value of β taking any value in $(0, \overline{\beta}]$.

For small noise, that is $v < 1/4$, the following are sufficient for uniqueness:

- r is sufficiently small relative to s, that is $r \leq (1+4v)s$. In this case the equilibrium value of β satisfies $\beta > 1/2$. Observe, in particular, that if $r \leq s$ uniqueness holds independently of the other parameter values.

- r is sufficiently large relative to s, that is $r > s+\sqrt{s^2+k}$. Under the assumption $v < 1/4$ in this case the equilibrium value of β satisfies $\beta < 1/2$.

- Large lying costs k, that is $r \leq 4\sqrt{v}\left[\frac{s}{2}+\frac{\sqrt{s^2+k}}{2}\right]$. In this case the equilibrium value of β satisfies $\beta \geq 1/2$.

- Small lying cost k, that is $k \leq rs\frac{v}{(v+1/4)^2}$.

4.5 Existence of multiple straight talking equilibria

From the above sufficient conditions for uniqueness, multiple straight talking equilibria can exist only if v is small and the other parameters are in some intermediate range. So far we we have noted that multiplicity of straight talking equilibria requires that there exists at least one gentle equilibrium and that in this case all other equilibria must be understated, too, and that there is at most one loud equilibrium among those. We have not, however, provided explicit conditions on the underlying parameters which are sufficient for the existence of multiple straight talking equilibria. Here we fill this gap.

The basic idea is very simple: Assuming $r < \frac{s}{2} + \frac{\sqrt{s^2+k}}{2}$ ensures that there exists a straightforward equilibrium satisfying $\beta > 1/2$. On the other hand, as the proof of the following proposition demonstrates, for sufficiently low v a gentle equilibrium exists whenever $s < r$ holds.

Proposition 10. *Let*

$$s < r < \frac{s}{2} + \frac{\sqrt{s^2+k}}{2} \tag{28}$$

and suppose

$$\sqrt{v} \leq \min\{\frac{s}{k}\hat{\rho}, \frac{1}{2\hat{x}}\} \tag{29}$$

where $\hat{\rho} = \max_{x \geq 0} \left[r\frac{x}{1+x^2} - s\frac{1}{x} \right] > 0$ *and* $\hat{x} > 0$ *is a solution to this maximization problem. Then there exist multiple straightforward equilibria.*

Proof. We first demonstrate that $\hat{\rho}$ and $\hat{x}$ are well defined and satisfy $\hat{\rho} > 0$ and $\hat{x} > 0$. Towards this end observe that

$$\rho(x) = r\frac{x}{1+x^2} - s\frac{1}{x} = \frac{1}{x(1+x^2)} \left[(r-s)x^2 - s \right],$$

so that the assumption $r > s$ in (28) implies that $\rho(x)$ is strictly positive for sufficiently large x. As $\rho(0) < 0$ and $\lim_{x\to\infty} \rho(x) = 0$ it follows that $\hat{\rho}$ and $\hat{x}$ are well-defined and satisfy $\hat{\rho} > 0$ and $\hat{x} > 0$. Now set $\hat{\beta} = \hat{x}\sqrt{v}$. From (29) we have $\hat{\beta} \leq 1/2$. Because γ_2 is strictly concave, $\gamma_2(0) = s$, and $\gamma_2'(0) = \frac{k}{s}$ we have $\gamma_2(\hat{\beta}) < s + \frac{k}{s}\hat{x}\sqrt{v}$. We also have $f(\hat{\beta}) = r\hat{x}^2/(1+\hat{x}^2)$, so that (29) implies $f(\hat{\beta}) > g(\hat{\beta})$. It follows that there exists a gentle straight talking equilibrium. As the second inequality in (28) implies that there exists an equilibrium with $\beta > 1/2$, it follows that there are multiple straight talking equilibria. □

5 Ironic equilibria

Ironic equilibria can only exists if $r > 0$ and $s > 0$ holds, so we impose these parameter restrictions throughout this section.

A simple necessary condition for the existence of ironic equilibria is that the inequality

$$-\frac{r}{2\sqrt{v}}\beta \geq \gamma_1(\beta) \tag{30}$$

holds for some $\beta \in (\underline{\beta}, 0)$. The expression on the left side of this inequality is obtained by observing that - by the same argument as in the one used in the proof of Proposition 6 for the case $\beta > 0$ - the inequality $f(\beta) \leq \frac{r}{2\sqrt{v}}\beta$ holds for all $\beta < 0$.

Because $\gamma_1(0) = 0$, $\gamma_1'(0) = -\frac{k}{s}$ and γ_1 is strictly convex inequality (30) holds for some $\beta < 0$ if and only if $rs \geq 2k\sqrt{v}$, so that the condition $rs < 2k\sqrt{v}$ precludes the existence of an ironic equilibrium. Suppose on the other hand that the inequality $rs \geq 2k\sqrt{v}$ holds and that, in addition, $-\sqrt{v} \geq \underline{\beta}$ holds. Then we have

$$f(-\sqrt{v}) = \frac{r}{2} \geq \frac{k}{s}\sqrt{v} > \gamma_1(-\sqrt{v}),$$

implying the existence of (at least) two ironic equilibria. We have thus shown

Proposition 11. *If $rs < 2k\sqrt{v}$ then no ironic equilibrium exists. If $rs \geq 2k\sqrt{v}$ and*

$$\sqrt{v} \leq \frac{1}{2}\sqrt{1 + \frac{s^2}{k}} - \frac{1}{2} \tag{31}$$

then at least two ironic equilibria exist.

In particular, fixing all other parameter values ironic equilibria exist if

- v is small
- k is small
- s is large

Provided that v and k are sufficiently small relative to s to ensure the inequality in (31), ironic equilibria will also exist whenever r is large enough.

6 The case $s = 0$

The special case $s = 0$ in which the sender does not care about the state θ is of interest because here the effect of lying costs on the equilibrium analysis are particularly easy to understand. We first observe that in this case there are no ironic equilibria. All equilibria have to be straight talking (because $\underline{\beta} = 0$) and understated (because $\overline{\beta} = 1$ and $\gamma_1(\overline{\beta}) = \gamma_2(\overline{\beta}) = 0$.

The uniqueness results from the previous section apply to the case $s = 0$ (with the "small k"-bound stated in Proposition 9 and the bound $r \leq [1 + v]$ never being satisfied for any values of the remaining parameters). In particular, equilibrium is unique if $v \geq 1/4$ holds, with the equilibrium being loud if $r \leq [1 + 4v]\sqrt{k}$ and gentle otherwise. If $v < 1/4$ there is a unique equilibrium if $r \leq 2\sqrt{v}\sqrt{k}$ (in which case the equilibrium is loud) or $r \geq \sqrt{k}$ (in which case the equilibrium is gentle). Observe that the latter of these conditions implies that as in the case $s > 0$ equilibrium is unique for sufficiently small lying costs.

Even in the case $s = 0$ multiple equilibria may occur. To see this, suppose the inequality $r \leq \frac{1}{2}\sqrt{k}$ holds, ensuring that for all $v > 0$ a loud equilibrium exists. Provided that $v < 1/4$ holds, it follows that there must be multiple equilibria if the inequality

$$f(\sqrt{v}) \geq \gamma_2(\sqrt{v}) \Leftrightarrow r \geq 2\sqrt{\sqrt{v} - v}\sqrt{k} \tag{32}$$

is satisfied. For values of v satisfying the inequalities $\sqrt{v} - v < 1/16$ and $v < 1/4$ the interval $(2\sqrt{\sqrt{v} - v}\sqrt{k}, \frac{1}{2}\sqrt{k})$ is non-empty and for all r in this interval multiple equilibria exist. Hence, provided that $v < \underline{v} \approx 0,0044$ holds, we have multiple equilibria.

7 The case $r < 0$

We first observe that in this case there are no ironic equilibria. From our general existence result a straight talking equilibrium exists. Because $f(\beta) < 0$ holds for $\beta > 0$ all straight talking equilibria must feature $\gamma < 0$ and must thus be understated and satisfy $\gamma = \gamma_1(\beta)$.

Following the same logic as in the case $r > 0$ some results are clear:

- Loud equilibria exist if and only if $f(1/2) > \gamma_1(1/2)$ which translates to

$$r \geq [4v+1]\left[\frac{s}{2} - \frac{\sqrt{s^2+k}}{2}\right].$$

Furthermore, there is at most one loud equilibrium.

- The condition

$$r \geq 4\sqrt{v}\left[\frac{s}{2} - \frac{\sqrt{s^2+k}}{2}\right]$$

ensures that there is no gentle equilibrium. Hence, if this condition holds there is a unique equilibrium which is loud.

- If $v \geq 1/4$ there is a unique equilibrium.

8 Comparing equilibria

8.1 Stability

We say that an equilibrium (β^*, λ^*) is stable if there exists a neigborhood of β^* such that for any initial condition β_0 in the neighborhood, the composed best response dynamic

$$\begin{cases} \lambda_n = \frac{r\beta_n}{\beta_n^2+v} \\ \beta_{n+1} = \frac{k+s\lambda_n}{k+\lambda_n^2} \end{cases}$$

converges to β^*.

In the generic situation in which we have an odd number of straight talking equilibria, the smallest and the loudest straight talking equilibrium may or may not be stable. If there are ironic equilibria the larger of these (in absolute value) may or may not be stable. The smaller one is necessarily unstable. If there is more than one straight talking equilibrium the first and (in case this is possible) the third of these will be stable. Moreover, when there are there are three straight talking equilibria, we know that all of them are understated. The loudest equilibrium is necessarily stable and the middle one unstable. We summarize this result in the following proposition.

Proposition 12. *If there are three traight talking equilibria, the loudest one is necessarily stable. Moreover, in that case, the iteration of the composed best response dynamic from $\beta^* = 1$ converges to the loudest straight talking equilibrium. The medium* *one is necessarily unstable. If there are two ironic equilibria, the gentle one is necessarily unstable.*

Proof. The presence of three straight talking equilibria implies that all three are understated. In particular, the loudest one (β^*, λ^*) is such that the best response of the sender is negative at λ^*. This and the presence of three straight talking equilibria further implies that the best response of the receiver is also negative ta β^*. Finally, the best-response of the sender cross the receiver's from above, which implies that (β^*, λ^*) is stable. □

If one considers "truth-telling" as the natural starting point of a dynamic, then one would tend to focus on the largest straight talking equilibrium. If ones considers babbling as the natural starting point of a dynamics, then one would tend to focus on the smallest straight talking equilibrium. It is not immediately apparent how one would want to tell a story about the emergence of ironic equilibria.

8.2 Informational content

A standard question in models such as ours is "How much information is transmitted in equilibrium?" In a model with normally distributed random variables it is natural to measure "how much information" by considering the ratio of the precisions of the receiver's posterior (after having observed the message) and prior (before having observed the message) forecast of θ. The prior precision is $1/\sigma_\theta^2$. The posterior precision after having observed the signal $\beta\theta + \epsilon$ is $1/\sigma_\theta^2 + \beta^2/\sigma_\epsilon^2$ (this uses standard formulas for the precision of normally distributed random variables). Hence, the ratio of the precisions is simply

$$1 + \beta^2/v$$

(to be evaluated at the equilibrium value of β).[6]

The following result follows directly from the definitions.

[6] As we have seen before, the value $\beta = \sqrt{v}$ plays a special role in our analysis as this is the value of β at which the receiver's best response λ takes it maximal value. We may observe that at this value of β the measure of informational efficiency is equal to 2 – which happens to be its equilibrium value in the Kyle model. Observe too, that as far as the cost resulting from the presence of noise is concerned the value $\beta = \sqrt{v}$ is the worst possible one as it leads to the maximal possible value of $\lambda^2 v$.

Proposition 13. *The informational content of an equilibrium is proportional to* β^2. *The loudest straight talking equilibrium is also the most informative one. Any straight talking equilibrium is more informative than any ironic equilibrium.*

8.3 Equilibrium utilities

Multiplying both sides of the equation for the receiver's best response by λ and using the definition $\gamma = \lambda\beta$ we obtain that for any profile (β, λ) on th receiver's best response, the relation

$$\lambda^2 v = (r - \gamma)\gamma \tag{33}$$

holds. Substituting this into the formulas for player's expected utilities given in (5) and (6) we obtain.

Lemma 1. *Let* (β, λ) *be an equilibrium with* $\beta\lambda = \gamma$. *Then the corresponding equilibrium utilities are given by*

$$u_S(\beta, \lambda) = -\left[(\gamma - s)^2 + (r - \gamma)\gamma) + k(1 - \beta)^2\right]\sigma_\theta^2 - \frac{kv + (r - \gamma)r}{kv}(c - b)^2.$$
$$u_R(\beta, \lambda) = -r\,(r - \gamma)\,\sigma_\theta^2.$$

Using these eexpressions, one can immediately rank the equilibria, from the point of view of the receiver's welfare. This is because in any equilibrium, $\gamma \leq r$ holds and u_R is increasing in γ. Therefore the receiver prefers the equilibrium which has the highest value of γ, which is also the most informative one. This gives the following result.

Proposition 14. *If these equilibria exist, the receiver ranks equilibria as follows. The gentle ironic is the worst (if there is any), followed by the loud ironic (if there is any), followed by the gentle straight talking (if there is any), followed by the middle straight talking (if there is any), followed by the loudest equilbrium, which is the receiver's preferred equilibrium.*

In the simple case where $b = c$, the sender's utility over pairs (β, γ) on the receiver's best response curve is given by

$$(2s - r)\,\gamma - k(1 - \beta)^2.$$

If $2s > r$, this implies that the sender's ideal point $(\beta^\circ, \gamma^\circ)$ on the receiver's best response curve is such that $\beta^\circ > 1$. In this case, the sender's utility is single-peaked in β for positive values of β. Moreover, between two points (β, γ) and $(-\beta', \gamma')$, with $0 < \beta' \leq \beta \leq 1$, on the receiver's best response curve, the sender prefers the first. In other words, if an ironic pair is preferred to a straight talking one, the former must be louder. But when $2s > r$, we know that there is a unique straight talking equilibrium and either no or two ironic equilibria. In this case, it is clear that the unique straight talking equilibrium is preferred to the two ironic equilibria, if they exist. How the sender ranks the two ironic equilibria is not immediately clear.

If $r > 2s$, this imples that the sender's ideal point is $(1, 0)$, while if $r < s$, his ideal point is $(1, +\infty)$. In both cases, it is not immediately clear how the sender ranks equilibria. In the second case, if the loudest equilibrium is understated (or if it does not exaggerate too much), this will be the sender's preferred equilibrium. If it exagerates a lot, then the middle straight talking equilibrium is preferred. In any case, one of these two equilibria is preferred to all the others. The most gentle straight talking equilibrium comes next. The ranking between the two ironic equilibria is again unclear.

9 Comparative statics

9.1 Comparative statics with respect to k

The effect of increasing k is easy to understand by thinking in terms of the $(\beta.\gamma)$ diagram. We keep the points $(0, 0), (s, 0), (1, 0), (1, s)$ fixed and "stretch" the ellipse by pulling at the points $(1/2, \overline{\gamma})$ and $(1/2, \underline{\gamma})$ is the vertical direction. Consequently, the effect will be the following:

- $r > 0$, Straight talking, exaggerated equilibria: β and γ fall as k increases.
- $r > 0$, Straight talking, understated equilibria: If there is a unique straight talking equilibrium β and γ increase as k increases. If $v \geq 1/4$ the equilibrium will thus simply trace along the f-curve. If $v < 1/4$ more interesting situations may arise: For sufficiently low k we have a unique gentle equilibrium which then may either morph continuously into a loud equilibrium or at some critical value

of k "suddenly" a second larger gentle equilibrium appears which then "splits" into two equilibria with the larger of these equilibria then moving smoothly into the loud domain and the smaller one of this pair (for which β and γ are decreasing with k) merging with the smallest gentle equilibrium, leaving the loud equilibrium as the unique one for sufficiently high k.

- $r > 0$, ironic equilibria: Starting from a situation in which two of these exists, the louder one (bigger absolute value of β) moves closer to the origin as k increases and the more gentle one will move in the opposite direction until they both merge and disappear.

- $r < 0$: if there is a unique one β and the absolute value of γ increase as k increases. Multiplicity story is akin to the one for the $r > 0$ case.

An interesting implication of these results and the ones obtained in the previous section is the fact that when $0 < r < s(1 + v)$ the receiver's equilibrium utility is actually decreasing in k. One might have thought that it is always to the receiver's advantage if the sender's incentive to mislead him is reduced.

Proposition 15. *Suppose that $0 < r < s(1+v)$ holds. Then the informational content and the receiver's expected utility are decreasing in k at the (unique) exaggerating equilibrium.*

The intuition of this result is simple. As moral norms against lying becomes weaker, the equilibrium involves more exaggaration. In equilibrium, information is encoded in a more exaggerated language, which implies a better transmission of information, as the relative importance of the noise decreases, due to how "loud" the sender speaks.

9.1.1 Limit as $k \to \infty$

For large enough k we have a unique equilibrium (β_k, γ_k) satisfying $\beta_k \to 1$ and $\gamma_k \to r/(1 + v)$.

In terms of the welfare analysis it is not immediately obvious whether the term $k(1 - \beta)^2$ converges to zero. However, using (12) we know that

$$k(1 - \beta)^2 = (\gamma - s)\gamma \left[\frac{1}{\beta} - 1\right] \tag{34}$$

holds in every equilibrium. As $\beta \to 1$ and γ has a finite limit, it follows that $k(1-\beta)^2$ converges to zero as k converges to infinity.

$$u_S = -\frac{(r-s)^2 + s^2 v}{1+v}\sigma_\theta^2 + (c-b)^2 \text{ and } u_R = -\frac{r^2 v}{1+v}\sigma_\theta^2$$

(One should check whether the above is what one gets "in the limit", that is, by simply presuming that the sender is an automaton who has to tell the truth. I think it should. That would help in clarifying that the remaining costs result from the fact that (a) the seller gets his way in expectation with (b) the noisiness of the communication channel imposing an additional cost hurting both players.)

9.1.2 Limit as $k \to 0$

For $r < 0$ there will be a unique equilibrium for k small enough and this converges to babbling, that is the limit is $(0,0)$.

For $r > 0$ and sufficiently small k there are exactly three equilibria, two ironic ones and a straight talking equilibrium. The gentler (unstable) ironic equilibrium converges to $(\beta, \lambda) = (0,0)$, which is the babbling equilibrium. (As none of the other equilibria converges to babbling this establishes a sense in which the babbling equilibrium – that always exists when $k = 0$ – is not stable.)

For the loud ironic and the straight talking equilibria, there are two cases to consider: $s \geq r$ and $s < r$.

- $r \leq s$. In this case, the sequence of straight talking equilibria (β_k, r) converges to $(+\infty, r)$ and the sequence of load ironic equilibria converges to $(-\infty, r)$.In either case, the informational content of equilibrium goes to infinity and the equilibrium utility of the receiver converges to 0 – which is the receiver's ideal outcome. Understanding what happens to the sender's payoff is a bit more challenging. Suppose, first, that $b = c$ holds, so we can ignore the last term in the sender's payoff. Using (34) and $\beta \to \infty$, $\gamma \to r$, we find that the term $k(1-\beta)^2$ converges to $(s-r)r$ – hence, unless $r = s$ the expected lying costs to be borne by the sender do not converge to zero. The sender's utility converges to $(s-r)r$. [7] Now, let's assume $b \neq c$. The question then is what happens

[7] Observe that keeping s fixed this term is maximized for $r = s/2$. This is consistent with the highest possible amount of exaggeration occurs when $f(\overline{\beta}) = s/2$.

to the term $(r-\gamma)r/kv$ as k converges to zero. This is not obvious as $r-\gamma$ and k both go to zero, so we might want to take a closer look at $(r-\gamma)/k$. In fact, rather than doing that let us return to the expression λ^2/k as the one describing the expected cost of "lying about the intercept." This term can be rewritten [8] as

$$\frac{\lambda^2}{k} = \frac{\beta-1}{\beta}\frac{\gamma}{s-\gamma}. \tag{35}$$

As β goes to infinity, the first fraction goes to 1, demonstrating that in the case $r < a$ we get a strictly positive limit given by $r/(s-r)$. (I find it somewhat puzzling that this expression is strictly increasing in r.) In the special case $s = r$ we get that the cost converges to infinity. (So if there is only conflict about the intercept the costs go off to infinity. If, however, there is an additional conflict about slope that this effect gets tempered - provided $r < s$ holds.)

- $r > s$. In this case, the two equilibrium values of β converges to the positive and negative solution of

  $$\frac{s}{r} = \frac{\beta^{*2}}{\beta^{*2}+v},$$

  i.e.

  $$\beta^* = \pm\sqrt{\frac{v}{1-\frac{r}{s}}}$$

  and $\gamma^* = s$.

  Observe the limit of the straight talking equilibrium can be gentle, loud, truthful, or exaggerated depending on how the value of the ratio r/s, e.g. $r = s(1+v)$ implies $\beta^* = 1$ etc. (the case distinction is exactly in line with what we have seen before).

  Observe: In the case $b = c$ there are no lying costs in the limit and in expectation the sender gets his most preferred action. Nevertheless, the sender does not obtain his bliss utility as λ converges to a finite limit, implying that the noise cost-term $\lambda^2 v$ does not vanish in the limit, but converges to $(r-s)s > 0$. If $b \neq c$ it is clear from the calculations that we did above and the fact that β

[8] Here is a somewhat roundabout way of doing this. Multiply the sender's best response condition by λ rather than β to obtain $\lambda^2(s-\gamma) = k(\gamma-\lambda)$ and then eliminate the λ on the right side by using $\lambda = \gamma/\beta$.

cannot converge to 1 that the expected cost of lying about the intercept go to infinity.

9.2 Comparative statics with respect to v

Increasing v flattens the function f.

Consider straight talking equilibria for $r > 0$ and suppose equilibrium is unique. The equilibrium value of β will then be increasing in v until we hit the point at which $f(\overline{\beta}) = s/2$. Thereafter the equilibrium value of β is decreasing and converges to 1 as $v \to \infty$. If v is sufficiently small (and r sufficiently large) that $\beta < 1/2$ holds for small enough v, then the equilibrium value of γ will first be increasing in v and then – once $\beta = 1/2$ has been hit – decreasing in v.

Considering the ironic equilibria for $r > 0$,it is clear that these will cease to exist for v sufficiently large. As long as they exist, the gentle (unstable) one will move away from babbling when v increases, whereas the comparative statics of the louder ironic equilibrium (I am again taking it for granted that there are at most two ironic equilibria) are determined by whether equilibrium sits on γ_2 or γ_1. If it sits on γ_2, then the absolute value of β is increasing in v until it reaches $\beta = \underline{\beta}$ is reached; thereafter we move on γ_1 with β decreasing until we bump into the unstable equilibrium and both disappear. Throughout γ is decreasing in v for the stable equilibrium.

Consider the case $r < 0$ under the additional assumption that we have uniqueness. Then β is increasing in v and γ is decreasing in v for $\beta < 1/2$ and increasing thereafter.

9.2.1 Limiting behavior of all equilibria when v is small

Let $r > 0$.

Keeping all other parameters fixed, in the limit where v goes to 0 :

- If
$$\frac{r}{s} \leq 1,$$
there is a unique positive equilibrium, that converges to $(\beta_2(r), r)$, i.e. $\beta = \beta_2(r)$ and $\lambda = \frac{r}{\beta_2(r)}$. There are also two negative equilibria. One that converges to $(0,0)$, i.e. $\beta = 0$ and $\lambda = -\frac{k}{s}$, and another one that converges to $(\beta_1(r), r)$, i.e. $\beta = \beta_1(r)$ and $\lambda = \frac{r}{\beta_1(r)}$.

- If
$$1 < \frac{r}{s} < \frac{1}{2} + \frac{\sqrt{\frac{k}{s^2}+1}}{2},$$
there are three positive equilibria. Two of them converge respectively to $(\beta_1(r), r)$ and to $(\beta_2(r), r)$. The third one converges to $(0, s)$, i.e. $\beta = 0$ and $\lambda = +\infty$. There are also two negative equilibria. One that converges to $(0, 0)$, i.e. $\beta = 0$ and $\lambda = -\frac{k}{s}$, and another one that converges to $(0, s)$, i.e. $\beta = 0$ and $\lambda = -\infty$.

- If
$$\frac{1}{2} + \frac{\sqrt{\frac{k}{s^2}+1}}{2} < \frac{r}{s},$$
there is again a unique positive equilibrium that converges to $(0, s)$, i.e. $\beta = 0$ and $\lambda = +\infty$. There are also two negative equilibria. One that converges to $(0, 0)$, i.e. $\beta = 0$ and $\lambda = -\frac{k}{s}$, and another one that converges to $(0, s)$, i.e. $\beta = 0$ and $\lambda = -\infty$.

9.2.2 Optimal noise level

Noise has a direct negative on the welfare of both players. It has also an indirect strategic effect. For positive (negative) equilibria that lie on some increasing (decreasing) portion of the ellipse, a noise increase has a positive effect on the receiver. The opposite is true for equilibria that lie on some decreasing (increasing) portion of the ellipse.

The welfare effect on the sender is more complex, because the lying cost also plays a role.

Assume $r > 0$ and focus on straight talking equilibria.

If $r \leq s$ (so that the unique straight talking equilibrium is always exaggerating) it is clear that the receiver prefers noise to be as small as possible and in the limit for $v \to 0$ obtains his bliss point with $\gamma = r$.

If $r > s$ the question of the optimal noise level is more interesting (and we have to grapple with the problem that there might be multiple equilibria).

In particular, when $s < r < s/2 + \sqrt{s^2 + k}/2$ we have already seen that there are two equilibria such that the receiver obtains his bliss point in the limit as $v \to 0$. There is, however, also a third equilibrium in which γ converges to s. So there is a risk that the receiver may end up in the "wrong" equilibrium.

If r is greater than the upper bound just given, pushing v to zero is not what the receiver wants to do. Rather he wants to choose v such that equilibrium occurs at $\beta = 1/2$ – which is the best the receiver can hope for. It is not immediately obvious to me, though, that this equilibrium must then be unique. If it is not, the same question as in the previous case arise. This is summarized in the following result.

Proposition 16. *Suppose that $r > \overline{\gamma}$. The informational content and the expected utility of the receiver are both increasing in v in $\left(0, \frac{r-\overline{\gamma}}{4\overline{\gamma}}\right]$ and decreasing in v on $\left[\frac{r-\overline{\gamma}}{4\overline{\gamma}}, +\infty\right)$.*

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