Parametric Recoverability of Preferences*

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Abstract

We recover approximate parametric preferences from consistent and inconsistent consumer choices. The procedure seeks to utilize revealed preference information contained in choices by minimizing its ranking inconsistency with the proposed parametric preferences. We prove that the goodness-of-fit of such an approximation can be decomposed into measures of inconsistency and misspecification. This provides a reasonable way to test restrictions on parametric models. An application of the method to the data set constructed by Choi et al. [2007] to study choice under risk suggests more frequent and pronounced non-expected utility preferences than previously suggested.

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1 Introduction

This paper is a contribution to the applicability of revealed preference theory to the domain of recovering stable preferences from individual choices. The need for such an application emerges from the recent availability of relatively large data sets composed of *individual* choices made directly from linear budget sets, where choices of most subjects seem to be exactly or approximately stable.[1] These rich data sets allow researchers to recover approximate *individual* stable utility functions and report the magnitude and distribution of behavioral characteristics in the subject population.

Our proposed procedure of recovering approximate preferences is based on the minimal budget adjustments required to remove inconsistencies between the ranking information induced by the suggested preferences and the revealed preference information contained in the choices. The latter summarizes *all* ranking information implied by the observed choices that has refutable predictions. That is, assuming singleton choice sets, if an alternative is chosen from a set and is available in a subset then it will be chosen from the subset; Similarly, if an alternative is not chosen from a set then it will not be chosen from any other set in which the original chosen alternative is also feasible. This procedure extends the rationale of Afriat [1972] and Varian [1990], who seek the minimal budget adjustments to remove violations of rationality (consistency with the Generalized Axiom of Revealed Preference, henceforth GARP), to the domain of parametric recoverability of preferences.

Given a data set constructed from a generic consumer choice problem, which satisfies GARP, Afriat [1967] suggests a nicely behaved piecewise linear utility function that satisfies the restrictions imposed by revealed preference. This method requires recovering twice the number of parameters as there are observations and therefore the behavioral implications of such functional forms are difficult to interpret. Varian [1982] builds on this work to construct non parametric bounds that allow partial identification of the utility function, assuming that the preferences are convex. However, in many cases, researchers assume simple functional forms with few parameters that lend themselves naturally to behavioral interpretation.[2] The drawback of this approach is that simple functional forms are often too structured to capture every nuance of individual decision making, and thus preferences recovered in

[1]Notable references are Andreoni and Miller [2002], Fisman et al. [2007], Choi et al. [2007], Ahn et al. [2011], Andreoni and Sprenger [2012].

[2]For example, the single parameter in the CRRA von Neumann-Morgenstern utility function can be interpreted as a measure of the decision maker's risk attitude.

this way are almost always misspecified. That is, the ranking implied by the recovered preferences may be inconsistent with the ranking information implied by the decision maker's choices (summarized through revealed preference). Following this line of reasoning, given a parametric utility function, we seek a measure to quantify the extent of misspecification that may also be used as a criterion for choosing the element of the functional family which minimizes this inconsistency.

A utility function rationalizes a data set, by the classical sense, if there are no inconsistencies between the order revealed by the choices and the ranking induced by the utility function. We weaken this notion by requiring the condition to hold for adjusted budget sets. A utility function $\mathbf{v}$-*rationalizes* the data, where $\mathbf{v}$ is an adjustments vector, if by reducing the budget of observation i by a proportion of v_i for all i, all inconsistencies between the proposed utility function and the revealed preference information are removed. Our proposed measure is based on an aggregation of the *minimal* budget adjustments that are required in order to remove inconsistencies between the revealed preference information implied by the data and the rankings of bundles induced by a given utility function. We show that this minimal vector can be calculated observation by observation using the money metric utility function.

If a data set satisfies GARP, the measure we propose quantifies the extent of misspecification that arises solely from considering a specific family of utility functions, rather than all possible utility functions. If the data set does not satisfy GARP, the measure can be decomposed into an inconsistency index (the Varian [1990] efficiency index) and a misspecification index, which is the difference between our measure and the Varian index. Since for a given data set the inconsistency index is constant (zero if GARP is satisfied), the measure can be used to recover parametric preferences within some parametric family by minimizing the misspecification. Similarly, it can be used to evaluate the increase in misspecification implied by restricting the set of parameters. Moreover, one can use the measure to choose among functional forms. For example, consider some parametric form of non-expected utility that includes expected utility as a special case. Given a data set of choices under risk, one can recover the values of the parameters that minimize misspecification, and evaluate the additional misspecification implied by restricting to expected utility.

To illustrate, we apply our method to recover preferences from data on choice under risk collected by Choi et al. [2007]. We recover parameters for the disappointment aversion functional form of Gul [1991][3] using both the Euclidean-distance-based NLLS and the revealed preference approach and identify several important qualitative differences in the recovered parameters. In several cases, the recovered parameters are contradictory with respect to whether subjects are elation loving or disappointment averse, and as such the behavioral conclusions of our analysis may depend critically on the chosen recovery method. Moreover, quantitative differences in the distribution of parameter values in the subject population suggest that the preferences recovered by minimizing inconsistency with revealed preference information put higher weight on first-order risk aversion and lower weight on second-order risk aversion [Segal and Spivak, 1990] than previously found using a distance-based approaches. We calculate the additional misspecification implied by restricting to expected utility, and find that for about one third of the subjects expected utility may be used as a good approximation since the restriction implies a proportional increase in misspecification of less than 10%.

Varian [1990] suggested the money metric as a "natural measure of how close the observed consumer choices come to maximizing a particular utility function" (page 133) and then recommends its usage as a criterion for recovering preferences. He argues that measuring differences in utility space has a more natural economic interpretation than measuring distances between bundles in commodity space. We augment Varian's intuition by providing theoretical foundations for the usage of the money metric as a measure of misspecification. First, we demonstrate that the money metric utilizes more preference information encoded in the observed choices to recover preferences than methods that are based on minimizing distance between observed and predicted bundles. Second, we prove that the money metric measure can be constructed observation-by-observation while maintaining most revealed preference information contained in choices. Third, we relate the budget adjustments implied by the money metric to the non-parametric measure of the inconsistency implied by revealed preference. Finally, since we show that the goodness of fit can be decomposed into an inconsistency index and a misspecification index, we introduce several novel applications including evaluating parametric restrictions and model selection.

The procedure proposed in this paper selects, for each individual independently, a deterministic utility function such that the inconsistency between its ranking information and the revealed preference information encoded in the individual's choices is minimized. A different

[3] Although in the considered setup of two-states of the world it is observationally equivalent to models of Rank Dependent Utility [Quiggin, 1982].

approach, used frequently in empirical analysis of consumer demand is the Random Utility Maximization (henceforth, RUM) model. A RUM model is a probability distribution over a set of utility functions. The theory of revealed stochastic preference is an application of RUM models to the typical field data set where a population of individuals is faced with a variety of choice problems and for every choice problem the researcher observes only the distribution of choices (the proportion of the population that chose each alternative).[4] In this context, a data set is rationalizable if it can be represented as a result of utility maximization, that is there exists an RUM model such that for every alternative, the expected frequency generated by the model coincides the observed frequency. Given a choice problem, the choice heterogeneity inherent in the typical field data set requires some stochastic element to enable rationalizability. Such heterogeneity is absent from the experimental data we examine in this paper where individuals choices are observed. Therefore, as mentioned by McFadden [2005] and Hoderlein and Stoye [2010] among others, RUM models are inadequate for recovering preferences from experimental data.

2 Non-Parametric Recoverability

Consider a decision maker (DM) who chooses bundles $x^i \in \Re^K_+$ $i = 1, \ldots, n$ out of budget menus $\left\{x : p^i x \leq 1,\, p^i \in \Re^K_{++}\right\}$. The traditional problem of recoverability is to find a utility function that rationalizes the data. This section presents the standard non-parametric approach to the problem of recoverability, following Varian [1982]. In particular, we explain why this approach constitutes partial identification only under an important restriction on preferences. This restriction of this non-parametric approach motivates the study of the parametric approach presented in the next section.

Let $D = \left\{(p^i, x^i)^n_{i=1}\right\}$ be a finite data set, where x^i is the chosen bundle at prices p^i.

Definition 1. An observed bundle $x^i \in \Re^K_+$ is

1. *directly revealed preferred* to a bundle $x \in \Re^K_+$, denoted $x^i R^0_D x$, if $p^i x^i \geq p^i x$.

2. *strictly directly revealed preferred* to a bundle $x \in \Re^K_+$, denoted $x^i P^0_D x$, if $p^i x^i > p^i x$.

[4] RUM models can be interpreted either as modeling a homogeneous population where individuals hold identical stochastic preferences (e.g. Gul and Pesendorfer [2006]) or modeling a heterogeneous population where each individual has deterministic preferences (e.g. McFadden [2005]).

3. *revealed preferred* to a bundle $x \in \Re^K_+$, denoted $x^i R_D x$, if there exists a sequence of observed bundles $\left(x^j, x^k, \ldots, x^m\right)$ such that $x^i R^0_D x^j, x^j R^0_D x^k, \ldots, x^m R^0_D x$.

4. *strictly revealed preferred* to a bundle $x \in \Re^K_+$, denoted $x^i P_D x$, if there exists a sequence of observed bundles $\left(x^j, x^k, \ldots, x^m\right)$ such that $x^i R^0_D x^j, x^j R^0_D x^k, \ldots, x^m R^0_D x$ at least one of them is strict.

The data is said to be consistent if it satisfies the General Axiom of Revealed Preference.

Definition 2. Data set D satisfies the *General Axiom of Revealed Preference (GARP)* if for every pair of observed bundles, $x^i R_D x^j$ implies not $x^j P^0_D x^i$.

The following definition relates the revealed preference information implied by observed choices to ranking induced by utility maximization.

Definition 3. A utility function $u : \Re^K_+ \rightarrow \Re$ *rationalizes* data set D, if for every observed bundle $x^i \in \Re^K_+$, $u(x^i) \geq u(x)$ for all x such that $x^i R^0_D x$. We say that D is *rationalizable* if such $u(\cdot)$ exists.

Rationalizability does not imply uniqueness. There could be different utility functions (not related by a monotonic transformation) that rationalize the same data set. Afriat's celebrated theorem provides tight conditions for the rationalizability of a data set.

Theorem. [Afriat, 1967] *The following conditions are equivalent:*

1. There exists a non-satiated utility function that rationalizes the data.

2. The data satisfies GARP.

3. There exists a non-satiated, continuous, concave, monotonic utility function that rationalizes the data.

Proof. See Afriat [1967], Diewert [1973], Varian [1982]. □

Afriat's proof of (3) is constructive: he shows that if a data set D of size n satisfies GARP then $U(x) = min_i \{U^i + \lambda^i p^i (x - x^i)\}$, where U^i and $\lambda^i > 0$ are $2n$ real numbers that satisfy a set of n inequalities: $U^i \leq U^j + \lambda^j p^j (x^i - x^j)$, rationalizes D. It is important to note that although Afriat's utility function does not rely on any parametric assumptions, it is difficult to directly learn from it about behavioral characteristics of the decision maker, which are typically summarized by few parameters (e.g. risk aversion, ambiguity aversion). Moreover, this utility function that rationalizes the data is generically non-unique. Hence, if one can find a "simpler" (parametric) utility function that rationalizes the data set - it will have equal standing in representing the ranking information implied by the data set.

If one accepts that "simple" may be superior, then one should consider paying a price in terms of misspecification. In later sections we will pursue this line of reasoning by considering the minimal misspecification implied by certain parametric specifications. Moreover, we next demonstrate that even the non-parametric method of recoverability introduced by Varian [1982] imposes some significant restrictions on the structure of preferences that may be recovered.

Assume D satisfies GARP. The following definitions follow Varian [1982].

Definition 4. $P_u(x) \equiv \{x' : u(x') > u(x)\}$ is the *strictly upper contour set* of a bundle $x \in \Re^K_+$ given a utility function $u(x)$.

Next, consider the set of prices at which an unobserved bundle - x, can be chosen such that the augmented data set would still be consistent with GARP.

Definition 5. Suppose $x \in \Re^K_+$ is an unobserved bundle, then

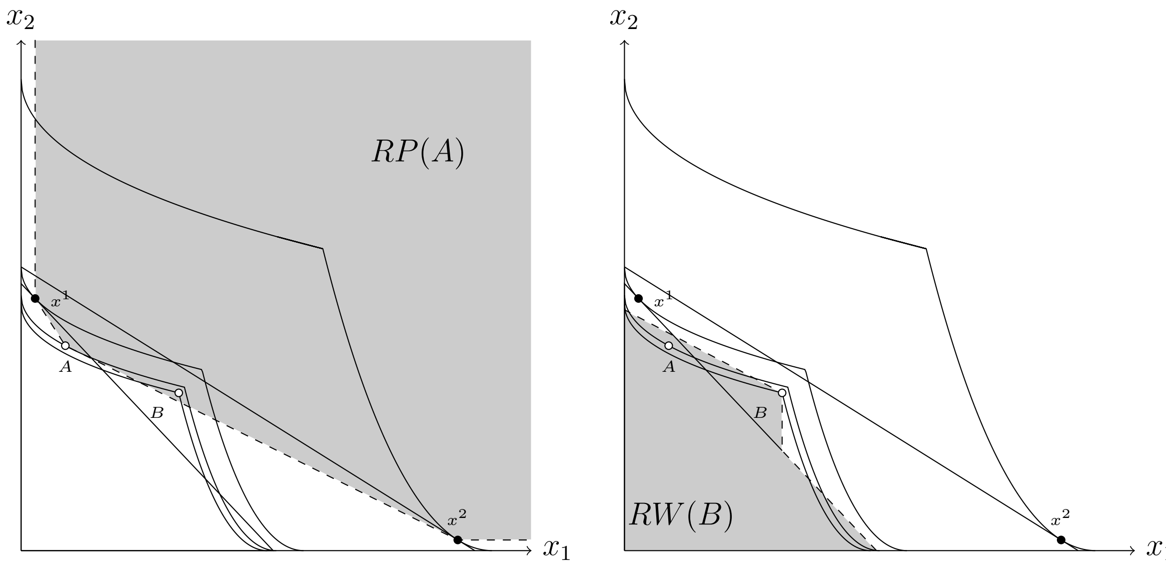
$$S(x) = \{p \,|\, \{(p, x)\} \cup D \text{ satisfies GARP and } px = 1\}$$

For every unobserved bundle x, Varian [1982] employs $S(x)$ to construct lower and upper bounds on the upper and lower contour sets through x.

Definition 6. For every unobserved bundle $x \in \Re^K_+$:

1. The *revealed worse set* is $RW(x) \equiv \left\{x' \,\middle|\, \forall p \in S(x), xP_{D\cup\{p,x\}}x' \right\}$. The *not revealed worse set,* denoted by $NRW(x)$, is the complement of $RW(x)$.
2. The *revealed preferred set* is $RP(x) \equiv \left\{x' \,\middle|\, \forall p \in S(x'), x'P_{D\cup\{p,x'\}}x \right\}$.

In Fact 5, Varian [1982] (page 953) states: "let $u(x)$ be any utility function that rationalizes the data. Then for all (unobserved bundles - HPZ) x , $RP(x) \subset P_u(x) \subset NRW(x)$". Thus, given a data set that satisfies GARP and a utility function that rationalizes these data, every indifference curve through a given unobserved bundle must be bounded between the revealed worse set and the revealed preferred set of this bundle.



(a) Violation of the Revealed Preferred Set (b) Violation of the Revealed Worse Set

Figure 2.1: Violations of Fact 5

Suppose a DM has to decide how to allocate a wealth of 1 between consumption in two mutually exclusive, exhaustive and equally probable states of the world. The allocation is attained by holding a portfolio of Arrow securities with unit prices $p = (p_1, p_2)$. Figure 2.1 presents a data set D of two observations. Portfolio $x^1 = (0.124, 2.222)$ is chosen when prices are $p^1 = (0.450, 0.425)$, and portfolio $x^2 = (3.850, 0.094)$ is chosen when prices are $p^2 = (0.250, 0.400)$. Notice that since $p^2 < p^1$,[5] every portfolio that is feasible under p^1 is also feasible when prices are p^2, therefore $x^2 R_D^0 x^1$. Now consider two unobserved portfolios $A = (0.390, 1.806)$ and $B = (1.390, 1.390)$. Portfolio A is feasible under both prices, but portfolio B is feasible only under p^2. The revealed preferred set of A and the revealed worse set of B are drawn in panels 2.1a and 2.1b, respectively. Now consider the following utility function over portfolio $x = (x_1, x_2)$:

$$u(x_1, x_2) = \sqrt{\max\{x_1, x_2\}} + \frac{1}{4}\sqrt{\min\{x_1, x_2\}} \tag{2.1}$$

which represents the preferences of an elation seeking DM [Gul, 1991] with $\beta = -0.75$ and a CRRA utility index with $\rho = 0.5$ over Arrow securities.[6] Therefore, the DM's preferences

[5] For vectors notation see Footnote 13.

[6] A reader who is not familiar with Gul [1991] model, may find the following footnote helpful: Let $p = (p_1, x_1; ...p_n, x_n)$ be a lottery such that $x_1 \leq \cdots \leq x_n$. Assuming (for simplicity) that $ce(p) \notin supp(p)$,

here are not convex and $u(\cdot)$ is not quasi-concave (let alone not concave). The indifference curves drawn in Figure 2.1 through x^1 and x^2 demonstrate that this utility function rationalizes the data (see Definition 3).

Recall that Fact 5 in Varian [1982] states that for any unobserved bundle x, if u rationalizes the data then $RP(x) \subset P_u(x) \subset NRW(x)$. However, Figure 2.1a clearly demonstrates that while $B \in RP(A)$, it is not true that $B \in P_u(A)$. Similarly, Figure 2.1b shows that while $A \in P_u(B)$ it is not true that $A \in NRW(B)$. That is, the ranking of unobserved portfolios implied by the Revealed Preferred and Revealed Worse sets is inconsistent with the ranking of portfolios induced by the utility function (2.1) that rationalizes the data. In other words, the utility function's indifference curves do not abide by Varian [1982] non-parametric bounds.

Figure 2.1 suggests the source of the above inconsistency with Varian's Fact 5: when the DM is elation seeking, her preferences are non-convex and the utility function is not concave. The failure of the nonparametric bounds can be traced back to the construction of the revealed preferred and revealed worse sets. Since by Afriat's Theorem if the data satisfies GARP there exists a concave utility function that rationalizes it, $S(x)$ (Definition 5) is non-empty for every x. However, there may exist a utility function that rationalizes the data for which there is no price vector p that will support x as an optimal choice. Therefore,

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the support of p can be partitioned into elation and disappointment sets: there exists a unique j such that for all $i < j : (x_i, 1) \prec p$ and for all $i \geq j : (x_i, 1) \succ p$. Gul's elation/disappointment decomposition is then given by $r = (x_1, r_1; \cdots; x_{j-1}, r_{j-1})$, $q = (x_j, q_j; \cdots; x_n, q_n)$ and $\alpha = \sum_{i=j}^{n} p_i$ such that $r_i = \frac{p_i}{1-\alpha}$ and $q_i = \frac{p_i}{\alpha}$. Note that $p = \alpha q + (1-\alpha) r$. Then:

$$u_{DA}(p) = \gamma(\alpha) E(v, q) + (1 - \gamma(\alpha)) E(v, r)$$

and $\exists -1 < \beta < \infty$ such that

$$\gamma(\alpha) = \frac{\alpha}{1 + (1-\alpha)\beta}$$

where $v(\cdot)$ is a utility index and $E(v, \mu)$ is the expectation of the functional v with respect to measure μ. If $\beta = 0$ disappointment aversion reduces to expected utility, if $\beta > 0$ the DM is disappointment averse ($\gamma(\alpha) < \alpha$ for all $0 < \alpha < 1$), and if $\beta < 0$ the DM is elation seeking ($\gamma(\alpha) > \alpha$ for all $0 < \alpha < 1$). Gul [1991] shows that the DM is averse to mean preserving spreads if and only if $\beta \geq 0$ and v is concave. That is, if v is concave then, by Yaari [1969], preferences are convex if and only if the DM is weakly disappointment averse.

For binary lotteries: Let $(x_1, p; x_2, 1-p)$ be a lottery. The elation component is x_2 and the disappointment component is x_1 and $\alpha = 1 - p$ (in our case $\alpha = 0.5$). Therefore:

$$u_{DA}(x_1, p; x_2, 1-p) = \gamma(1-p) v(x_2) + (1 - \gamma(1-p)) v(x_1)$$

and since $\gamma(0) = 0, \gamma(1) = 1$ and $\gamma(\cdot)$ is increasing, $\gamma(\cdot)$ can be viewed as a weighting function, and DA is a special case of Rank Dependent Utility [Quiggin, 1982].

even if x' is such that $xP_{D\cup\{p,x\}}x'$ for every $p \in S(x)$, it does not imply that the utility function that *never* chooses x will rank x above x'.[7]

Non-convex preferences are very important in domains like risk, ambiguity and other-regarding preferences. However, Varian's nonparametric recoverability approach partially identifies upper and lower bounds on the utility function subject to the restriction of convex preferences. If the data set is generated by a DM who correctly maximizes a non-convex preference relation, the ranking implied by the nonparametric bounds may be inconsistent with the underlying preferences of the DM.[8]

Unlike the non-parametric approach described above, the parametric approach to recoverability permits the observer to identify non-convex preferences within a given functional family with few parameters, at a cost of introducing misspecification. Once the misspecification is quantified, we will pursue a procedure to minimize it.

So far we have just discussed data sets that satisfy GARP and hence we know there exists a utility function that rationalizes the observed choices. What about choices that are not consistent in the sense that they do not abide by GARP? Afriat [1973, 1987] and Houtman [1995] use similar methods to those used in Afriat [1967] to recover an approximate utility function, in the sense that the existence of an underlying preference relation is maintained by allowing the DM not to exactly maximize that relation. This approach suffers from the same shortcomings of Afriat [1967] discussed above. The non-parametric approach of Varian [1982] has been extended and developed in Blundell et al. [2003, 2008] and **?**, however, to the best of our knowledge, it had not been expanded to include treatment of inconsistent data sets, and doing so will probably entail some behavioral assumptions regarding the nature of the inconsistencies. The parametric approach developed in the current paper, not only extends naturally to inconsistent data sets, but also permits an insightful decomposition of the goodness of fit into measures of inconsistency and misspecification.

[7] Definitions 5 and 6 can be trivially extended to include observed bundles, and then a similar argument can be constructed for the observed portfolio x^1 in Figure 2.1a. Note that the violation of the revealed worse set demonstrated in Figure 2.1b cannot occur for an observed bundle since there exists a price vector p that supports the bundle as an optimal choice.

[8] An alternative perspective on this problem is provided by Afriat's Theorem. He showed that if one considers *all* possible utility functions, then if any utility function rationalizes a data set, then there exists a concave utility function that rationalizes the same data. However, the convexified preferences rank unobserved bundles differently than the non-convex preferences.

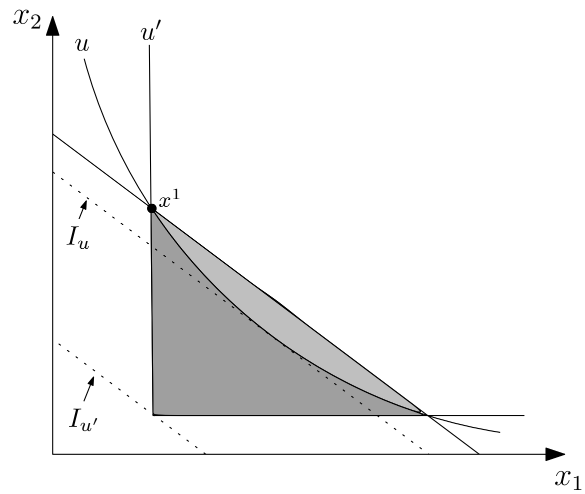


Figure 3.1: Measuring Misspecification with Budget Adjustments

3 Parametric Recoverability

This section proposes a loss-function that measures the inconsistency between the ranking information encoded in choices made within a data set and a given utility function. For a data set that satisfies GARP, this will constitute a measure of the misspecification in representing the data set by the utility function.

Consider, for example, a data set of a single observation $D = \{(p^1, x^1)\}$ and two candidate[9] utility functions u and u' as depicted in Figure 3.1. The data set includes only a single observation, hence is trivially consistent. However, both utility functions fail to rationalize the data since for both utility function there exist feasible bundles that are preferred to x^1according to the respective utility function.

Consider the unobserved bundles in the lightly shaded region of Figure 3.1. These are bundles over which x^1 is directly revealed preferred and yet are ranked higher than x^1 by the utility function u. In other words, u is misspecified since for these bundles the ranking induced by u is inconsistent with the ranking implied by choices and summarized by the revealed preference information. Yet, if we look at the union of the light and dark shaded regions, it is easy to see that all inconsistencies with the revealed preference information implied by u are also implied by u'. In this sense, we say that the utility function u dominates u' and that u' is more misspecified than u.[10]

[9]The restriction to two utility functions represents the parametric restriction in recovering preferences.

[10]Following this example of a single data point, it might be tempting to conclude that as the preferences become less convex (for the same prediction), the misspecification diminishes. However, this intuition is

Our proposed loss-function seeks the minimal adjustment to the expenditure levels such that all inconsistencies between the revealed preference information and the ranking information are removed. In Figure 3.1, I_u and $I_{u'}$ are the highest expenditure levels (keeping the prices constant) such that there is no affordable bundle that is ranked strictly higher than x^1 by the utility functions u and u' respectively. Since $I_{u'} < I_u < p^1x^1$ it is evident that although both utility functions are misspecified, the misspecification implied by u is smaller than the misspecification implied by u' relative to the data set.[11] In the following subsection we introduce theoretical foundations for this approach.

3.1 v-Rationalizability and the Money Metric Index

The following definition is a generalization of Definition 1. Similar concepts have been introduced into the literature on consistency (Afriat, 1972, 1987, Varian, 1990, 1993) in order to measure how close is a DM to satisfying GARP[12]. As will be evident from the rest of the current subsection, we employ these relations in order to eliminate the part of the revealed preference information contained in the choices, which is inconsistent with the ranking implied by a specific utility function under consideration. The minimal eliminated part will serve as a measure of the distance between the utility function and the data set.[13]

Definition 7. Let D be a finite data set. Let $\mathbf{v} \in [0,1]^n$. An observed bundle $x^i \in \Re^K_+$ is

1. $\mathbf{v}-$*directly revealed preferred* to a bundle $x \in \Re^K_+$, denoted $x^i R^0_{D,\mathbf{v}} x$, if $v^i p^i x^i \geq p^i x$.

2. $\mathbf{v}-$*strictly directly revealed preferred* to a bundle $x \in \Re^K_+$, denoted $x^i P^0_{D,\mathbf{v}} x$, if $v^i p^i x^i > p^i x$.

3. $\mathbf{v}-$*revealed preferred* to a bundle $x \in \Re^K_+$, denoted $x^i R_{D,\mathbf{v}} x$, if there exists a sequence of observed bundles $(x^j, x^k, \ldots, x^m)$ such that $x^i R^0_{D,\mathbf{v}} x^j, x^j R^0_{D,\mathbf{v}} x^k, \ldots, x^m R^0_{D,\mathbf{v}} x$.

misleading since in larger data sets the variability in prices may be high enough so that less convex preferences will result in more misspecification than the more convex one.

[11] Obviously, this measure is not unique. For example, an alternative measure could use the area contained in the intersection of the upper counter set that goes through x^1 and the budget line. We differ the discussion of this alternative measure to section 5.2.

[12] A different but related concept of inconsistency is presented in Echenique et al. [2011]. The main difference is that in most of the literature a single adjustment is enough to "break" a cycle, while in their work, all the relevant budget lines must be adjusted.

[13] Throughout the paper we use bold fonts (as $\mathbf{v}$ or $\mathbf{1}$) to denote vectors of scalars in $\Re^n$. For $\mathbf{v}, \mathbf{v}' \in \Re^n$ $\mathbf{v} = \mathbf{v}'$ if $\forall i : v_i = v'_i$, $\mathbf{v} \geqq \mathbf{v}'$ if $\forall i : v_i \geq v'_i$, $\mathbf{v} \geq \mathbf{v}'$ if $\mathbf{v} \geqq \mathbf{v}'$and $\mathbf{v} \neq \mathbf{v}'$ and $\mathbf{v} > \mathbf{v}'$ if $\forall i : v_i > v'_i$. We continue to use regular fonts to denote vectors of prices and goods.

4. $\mathbf{v}-$*strictly revealed preferred* to a bundle $x \in \Re^K_+$, denoted $x^i P_{D,\mathbf{v}} x$, if there exists a sequence of observed bundles $\left(x^j, x^k, \ldots, x^m\right)$ such that $x^i R^0_{D,\mathbf{v}} x^j, x^j R^0_{D,\mathbf{v}} x^k, \ldots, x^m R^0_{D,\mathbf{v}} x$ at least one of them is strict.

Afriat [1967] showed that D satisfies GARP if and only if there exists a non-satiated utility function that rationalizes the data. However, if we consider a specific utility function, it will generically not rationalize the data (even if choices are consistent).
Next we define the following generalization of rationalizability:

Definition 8. Let $\mathbf{v} \in [0,1]^n$. A utility function $u(x)$ $\mathbf{v}$-*rationalizes* D, if for every observed bundle $x^i \in \Re^K_+$, $u(x^i) \geq u(x)$ for all x such that $x^i R^0_{D,\mathbf{v}} x$.

That is, the intersection between $P_u\left(x^i\right)$, the set of bundles strictly preferred to x^i according to $u\left(\cdot\right)$, and the set of bundles to which x^i is $\mathbf{v}-$directly revealed preferred when the budget constraint is adjusted by v^i, is empty. Notice that $\mathbf{1}-$rationalizability reduces to Definition 3, and every utility function $\mathbf{0}$-rationalizes D.

To illustrate, consider Figure 3.1, where x^1 is chosen but is not optimal according to utility function u. For every v^1 such that $0 \leq v^1 p^1 x^1 \leq I_u$ there is no x that satisfies $v^1 p^1 x^1 \geq p^1 x$ and is strictly preferred to x^1 according to u. In this case we say that u $v-$rationalizes x^1. Of course this condition can be trivially satisfied by setting $v^1 = 0$; thus we define the minimum adjustment (supremum v) as the basis for our measure of misspecification. In Figure 3.1 the minimal adjustment required to $v-$rationalize x^1 by utility function u is given by $\frac{I_u}{p^1 x^1}$. Naturally, we would expect utility functions that represent the decision maker's preferences more closely, i.e. less misspecified, to require smaller budget adjustments in order to $v-$rationalize the observed choices. This is evident in Figure 3.1 where $I_{u'} < I_u$ and captures the intuition that u is less misspecified than u'.

When the data set contains more than a single point, we must aggregate the adjustments required for each observation using an aggregator function.

Definition 9. $f : [0,1]^n \rightarrow [0, M]$, where M is finite, is an *Aggregator Function* if $f(\mathbf{1}) = 0$, $f(\mathbf{0}) = M$ and $f(\cdot)$ is continuous and weakly decreasing.[14]

The minimal adjustment to the budget set for every observation is given by the Money Metric Utility Function [Samuelson, 1974]:

[14] For every $\mathbf{v}, \mathbf{v}' \in [0,1]^n$: $\begin{array}{lcl} \mathbf{v} \geq \mathbf{v}' & \Longrightarrow & f(\mathbf{v}) \leq f(\mathbf{v}') \\ \mathbf{v} > \mathbf{v}' & \Longrightarrow & f(\mathbf{v}) < f(\mathbf{v}') \end{array}$.

Definition 10. The *normalized money metric vector for a utility function* $u(\cdot)$*,* $\mathbf{v}^{\star}(D,u)$*, is such that* $v^{\star i}(D,u) = \frac{m(x^i,p^i,u)}{p^i x^i}$ *where* $m(x^i,p^i,u) = min_{\left\{y \in \Re_+^K : u(y) \geq u(x^i)\right\}} p^i y$*. The Money Metric Index for a utility function* $u(\cdot)$ *is* $f\left(\mathbf{v}^{\star}\left(D,u\right)\right)$.

Thus, the money metric vector and the money metric utility function upon which it is based, measure, for a given utility function, the minimal expenditure required to achieve at least the same level of utility as the observed choices.[15]

Proposition 1. *Let* $D = \left\{(p^i, x^i)_{i=1}^n\right\}$ *and let* $u\left(\cdot\right)$ *be a continuous and locally non-satiated utility function.*

1. $u(\cdot)$ $\mathbf{v}^{\star}(D,u)$*-rationalizes* D*.*

2. $\mathbf{v}^{\star}\left(D,u\right) = \mathbf{1}$ if and only if $u\left(\cdot\right)$ rationalizes D.

3. Let $\mathbf{v} \in [0,1]^n$. $u\left(\cdot\right)$ $\mathbf{v}$*-rationalizes* D if and only if $\mathbf{v} \leqq \mathbf{v}^{\star}(D,u)$.

Proof. See Appendix A. □

Proposition 1 establishes that $f\left(\mathbf{v}^{\star}\left(D,u\right)\right)$ may be viewed as a measure of the distance between the data set and a given utility function. Part 3 shows that $\mathbf{v}^{\star}\left(D,u\right)$ measures the minimal adjustments to the budget sets required to $\mathbf{v}-$rationalize D by u, that is - to remove inconsistencies between the revealed preference information contained in D and the ranking information induced by u.

Part 3 also implies that each coordinate of $\mathbf{v}^{\star}\left(D,u\right)$ is calculated independently of the other observations in the data set. This is a crucial feature of this procedure which deserves some discussion. Based on the intuition contained in the nonparametric recoverability literature and Fact 5 in Varian [1982] in particular[16], one may be led to believe that a utility function can be consistent with the directly revealed preference information but fail to rationalize the data based on the indirect revealed preference information. However, since R_D is the transitive closure of R_D^0, it follows that a utility function is consistent with the directly revealed preference information if and only if it is consistent with all the indirectly revealed

[15] We include (D,u) in the definition to emphasize that the optimal budget set adjustments depend on both the observed choices and on the specific utility function.

[16] Recall that the focus of Varian [1982] is primarily on constructing tight non-parametric bounds on utility for unobserved bundles.

preference information. In other words, if the utility function is inconsistent with some indirect revealed preference information, it must be inconsistent with some directly revealed preference information as well.

Figure 3.2 demonstrates this point, and how the data is $\mathbf{v}^{\star}-$rationalized. The data set includes two observations, where x^1 is directly revealed preferred to x^2. Although the utility function $u(\cdot)$ ranks x^1 above x^2 it fails to rationalize the data since $u(y) > u(x^1)$ although x^1 is strictly indirectly revealed preferred to y (which is feasible when x^2 is chosen). First, note that if this is the case, it must be that $u(y) > u(x^2)$. That is, $u(\cdot)$ does not rationalize the direct revealed preference information. Second, as is evident from Figure 3.2a, D will be $\mathbf{v}^{\star}-$rationalized by adjusting *only* observation 2's budget set to remove inconsistencies between the utility ranking and the *direct* revealed preference information.[17] More generally, the $\mathbf{v}^{\star}-$ adjustments can be calculated observation-by-observation: for each observation the minimal adjustment is independent of the required adjustments for other observations.[18] Moreover, since $R_{D,\mathbf{v}^{\star}(D,u)}$ is just the transitive closure of $R^0_{D,\mathbf{v}^{\star}(D,u)}$, Figure 3.2b[19] demonstrates that $\mathbf{v}^{\star}(D,u)$ retains most of the indirect revealed preference information that is consistent with the ranking encoded in the utility function under consideration.

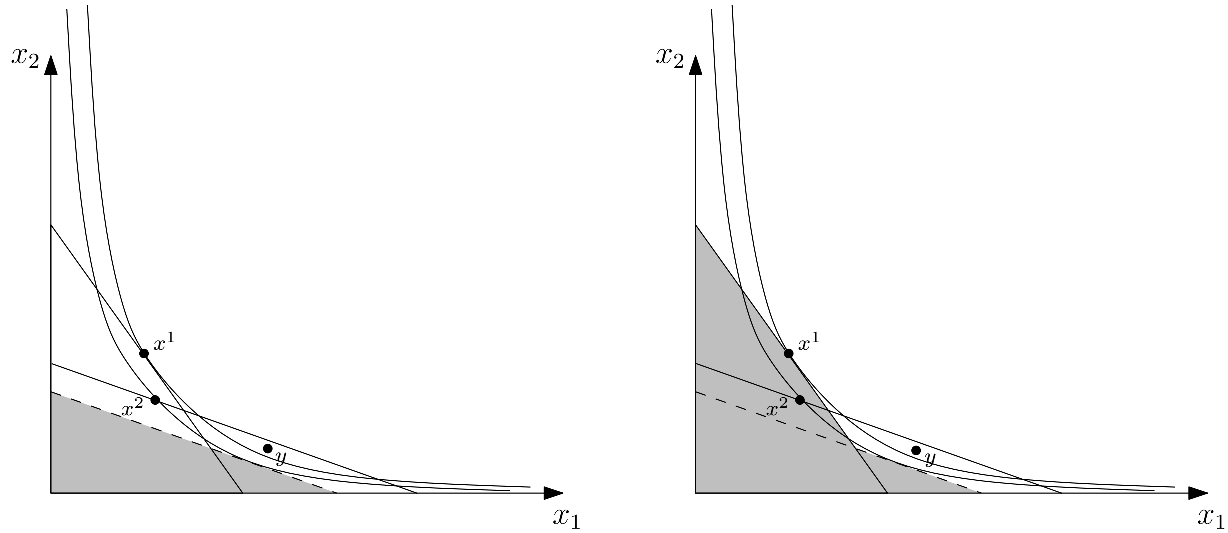
Part 2 of Proposition 1 is merely a restatement of the familiar definition of rationalizability using the money metric as a criterion. It shows that a non-satiated and continuous utility function $u(\cdot)$ rationalizes the observed choices if and only if it is the case that for all observations there exist no affordable bundles that achieve strictly higher level of utility than the observed choices themselves. In this case we would say that the utility function is correctly specified.

Recall that given an aggregator function $f(\cdot)$, $f(\mathbf{v}^{\star}(D,u))$ measures the distance between a data set D and a specific preference relation represented by the utility function u. Let $\mathcal{U}^c$ be the set of all continuous and locally non-satiated utility functions. Given a set of utility functions $\mathcal{U} \subseteq \mathcal{U}^c$, the Money Metric Index measures the distance between $\mathcal{U}$ and the data set D:

[17] There are several different ways we could interpret the nature of this adjustment in behavioral terms, we postpone this discussion to Section 5.3.

[18] An additional implication of this property is that given m data sets D_i of n_i observations, and utility function $u(\cdot)$, if u $\mathbf{v}^{\star}(D_i,u)-$ rationalizes D_i for every i, then u $\mathbf{v}^{\star}(\bigcup_{i=1}^m D_i,u)-$ rationalizes $\bigcup_{i=1}^m D_i$ where $\mathbf{v}^{\star}(\bigcup_{i=1}^m D_i,u) = \left(\mathbf{v}^{\star}(D_1,u)^T,\ldots,\mathbf{v}^{\star}(D_m,u)^T\right)^T$. Moreover, if $f(\cdot)$ is additive separable (as are all the aggregators mentioned in this paper) then $f(\mathbf{v}^{\star}(\bigcup_{i=1}^m D_i,u)) = \sum_{i=1}^m f(\mathbf{v}^{\star}(D_i,u))$.

[19] The shaded area represents those bundles that are directly and indirectly $\mathbf{v}^{\star}(D,u)-$ dominated by x^1.



(a) (b)

Figure 3.2: Removing Violations of Revealed Preference

Definition 11. For a data set D and an aggregator function $f(\cdot)$, let $\mathcal{U} \subseteq \mathcal{U}^c$. The *Money Metric Index of* $\mathcal{U}$ is

$$I_M(D, f, \mathcal{U}) = \inf_{u \in \mathcal{U}} f\left(\mathbf{v}^\star\left(D, u\right)\right)$$

The following observation follows directly from the definition of $I_M(D, f, \mathcal{U})$.

Fact 1. *For every* $\mathcal{U}' \subseteq \mathcal{U} : I_M(D, f, \mathcal{U}) \leq I_M(D, f, \mathcal{U}')$.

In particular, it implies that for every $\mathcal{U} \subseteq \mathcal{U}^c$: $I_M(D, f, \mathcal{U}^c) \leq I_M(D, f, \mathcal{U})$. That is, the value of the Money Metric Index calculated for *all* continuous and locally non-satiated utility functions is a lower bound on $I_M(D, f, \mathcal{U})$ for every subset of utility functions.

3.2 Decomposing the Money Metric Index

Thus far we have been primarily concerned with GARP-consistent data sets that can be rationalized by some utility function. Given such data sets we argued that $I_M(D, f, \mathcal{U})$ is a natural measure of the misspecification induced by the choice to recover the utility function of the DM using the parametric family $\mathcal{U}$. By Afriat's Theorem, data sets that do not satisfy GARP cannot be rationalized by any utility function. Were we to restrict our analysis to only consistent data sets, the scope of our analysis would be somewhat limited.[20]

[20] Andreoni and Miller [2002], one of the first experimental papers that utilizes revealed preference approach with moderate price variation, finds that a great majority of the subjects satisfy GARP. However, in

The method we propose to construct $\mathbf{v}^{\star}(D,u)$ does not depend on the consistency of the data set D. Therefore, even if a decision maker does not satisfy GARP, we can recover preferences (within the parametric family $\mathcal{U}$) that *approximate* the consistent revealed preference information encoded in the choices. The difficulty with this arises from the fact that $I_M(D,f,\mathcal{U})$ includes both the inconsistency with respect to GARP and the misspecification implied by the chosen parametric family. In this section we study how we can decompose our measure into these two components.

Our strategy in developing the decomposition is to employ Varian [1990] efficiency index as a measure of inconsistency, which is independent of the parametric family under consideration. Then, we prove that the money metric index calculated for *all* locally non-satiated and continuous utility function - $I_M(D,f,\mathcal{U}^c)$ coincides with Varian's efficiency index. It follows that $I_M(D,f,\mathcal{U}) - I_M(D,f,\mathcal{U}^c)$ is a measure of misspecification.

Consider the following generalization of GARP [Varian, 1990]:

Definition 12. Let $\mathbf{v} \in [0,1]^n$. D satisfies the *General Axiom of Revealed Preference Given* $\boldsymbol{v}$ *(GARP$_{\mathbf{v}}$)* if for every pair of observed bundles, $x^i R_{D,\mathbf{v}} x^j$ implies not $x^j P^0_{D,\mathbf{v}} x^i$.

The vector $\mathbf{v}$ is used to generate the adjusted relation $R_{D,\mathbf{v}}$ that is acyclic although R_D may contain cycles. It is important to note that unlike the definition of $\mathbf{v}-$rationalizability, such adjustments are independent of any ranking implied by a utility function. Moreover, usually there are many vectors such that D satisfies $GARP_{\mathbf{v}}$. Following are two useful and trivial properties of $GARP_{\mathbf{v}}$:

Fact 2. *Every D satisfies $GARP_{\mathbf{0}}$.*[21]

Fact 3. *If $\mathbf{v},\mathbf{v}' \in [0,1]^n$ and $\mathbf{v} \geq \mathbf{v}'$ and D satisfies $GARP_{\mathbf{v}}$ then D satisfies $GARP_{\mathbf{v}'}$.*

Varian [1990] proposed an inefficiency index that measures the minimal adjustments of the budget sets which remove cycles implied by choices.[22] While Varian suggests to aggregate the adjustments using the sum of squares, we define this index with respect to an arbitrary

several recent experimental studies that employ a graphical interface with considerable price variation (Choi et al. [2007], Ahn et al. [2011], Choi et al. [2011]), about 75 percent of subjects did not satisfy GARP. Most of them can be shown to be very nearly consistent with GARP according to various measures of consistency as Afriat [1972] Critical Cost Efficiency Index, Varian [1990] Efficiency Index, and the Houtman and Maks [1985] Index.

[21] Recall that $P^0_{D,\mathbf{0}}$ is the empty relation.

[22] Afriat [1972, 1973] Critical Cost Efficiency Index employs a uniform adjustment for all budgets.

aggregator function as in Definition 9. Thus, for a given aggregator function, this index is a measure of the decision maker's inconsistency.

Definition 13. Let $f : [0,1]^n \to [0,M]$ be an aggregator function. *Varian's Efficiency Index* is[23],

$$I_V(D,f) = \inf_{\mathbf{v}\in[0,1]^n : D \text{ satisfies } GARP_{\mathbf{v}}} f(\mathbf{v})$$

Fact 4. *$I_V(D,f)$ always exists.*[24]

Note that the Varian Index is independent of any preference ranking, and as defined is just a measure of the inconsistency incorporated in the data set. On the other hand, recall that for a family of utility functions $\mathcal{U}$, the Money Metric Index measures the distance between $\mathcal{U}$ and the data set. The following Theorem establishes that the Varian Index can be viewed as the distance between a data set and the set of *all* continuous and locally non-satiated utility functions.

Theorem 1. *For every finite data set $D = \left\{(p^i, x^i)_{i=1}^n\right\}$ and aggregator function $f : [0,1]^n \to [0,M]$:*

$$I_V(D,f) = I_M(D,f,\mathcal{U}^c)$$

where $\mathcal{U}^c$ is the set of continuous and locally non-satiated utility functions.

Proof. See Appendix B. □

The proof proceeds to show that $I_V(D,f) \le I_M(D,f,\mathcal{U}^c)$ since if $I_V(D,f) > I_M(D,f,\mathcal{U}^c)$ there exists a utility function $u(\cdot)$ such that $I_M(D,f,\mathcal{U}^c) \le f(\mathbf{v}^\star(D,u)) < I_V(D,f)$ and D satisfies $GARP_{\mathbf{v}^\star(D,u)}$ in contradiction to the definition of $I_V(D,f)$. On the other hand, we show that if D satisfies $GARP_{\mathbf{v}}$ then $I_M(D,f,\mathcal{U}^c) \le f(\mathbf{v})$. Moreover, we show that there exists a vector of adjustments $\mathbf{v}$ such that $f(\mathbf{v}) = I_V(D,f)$ and for every $0 \le \lambda < 1$ D satisfies $GARP_{\lambda\mathbf{v}}$, and therefore we conclude that $I_M(D,f,\mathcal{U}^c) \le I_V(D,f)$.

[23] The following example, due to Alcantud et al. [2010], clarifies the use of the infimum. A DM makes four choices ($n=4$) of bundles of three goods ($K=3$) with varying prices. The resulting choices are $D = \left\{\left((1,1,\frac{1}{2}),(8,1,8)\right);\left((1,1,\frac{3}{2}),(5,5,6)\right);\left((1,\frac{1}{2},1),(5,6,5)\right);\left((1,2,2),(8,8,1)\right)\right\}$. It is easy to see that $(8,8,1)$ is feasible when the prices are $(1,\frac{1}{2},1)$ and therefore $(5,6,5)R^0_D(8,8,1)$ and in the same way $(8,8,1)R^0_D(8,1,8)$ and $(8,1,8)R^0_D(5,5,6)$, that is $(5,6,5)R_D(5,5,6)$. However, $(5,5,6)P^0_D(5,6,5)$. Therefore, these choices do not satisfy $GARP$, or $GARP_{\mathbf{1}}$. However, consider the series $\mathbf{v}_l = (1-\frac{1}{l},1,1,1)$ where $l \in \mathbb{N}_{>0}$. It is easy to verify that for every $l \in \mathbb{N}_{>0}$, D satisfies $GARP_{\mathbf{v}_l}$.

[24] $f(\cdot)$ is bounded and by Fact 2, the set $\{\mathbf{v} \in [0,1]^n : D \textit{ satisfies } GARP_{\mathbf{v}}\}$ is non-empty.

Theorem 1 enables to decompose the Money Metric Index into a familiar measure of inconsistency (The Varian Index) and a natural measure of misspecification that quantifies the cost of restricting preferences to a subset of utility functions (possibly through a parametric form). By monotonicity of I_M (Fact 1), for every $\mathcal{U} \subseteq \mathcal{U}^c$:

$$I_V(D, f) = I_M(D, f, \mathcal{U}^c) \leq I_M(D, f, \mathcal{U})$$

Therefore, we can write $I_M(D, f, \mathcal{U})$ as the sum of $I_V(D, f)$ and $I_M(D, f, \mathcal{U}) - I_V(D, f)$. The former is a measure of the cost associated with inconsistent choices that is *independent* of any parametric restriction and depends only on the DM, while the latter measures the cost of restricting the preferences to a specific parametric form by the researcher who tries to recover the DM's preferences. This decomposition has the advantage that the two measures are comparable (same units) and are constructed to maintain revealed preference information encoded in the choices. As such, $I_M(D, f, \mathcal{U}) - I_V(D, f)$ serves as a natural measure of misspecification that is rooted in economic theory. Two reasons lead us to believe that such a decomposition is essential for any method of recovering preferences of a DM who is inconsistent, although we are not aware of its existence elsewhere in the literature. First, since for a given data set, the inconsistency index is constant (zero if GARP is satisfied) we can be certain that $I_M(D, f, \mathcal{U})$ can be used to recover parametric preferences within some parametric family $\mathcal{U}$ by minimizing the misspecification. Second, only when the decomposition exists, one can truly evaluate the cost of restricting preferences to some parametric family compared to the cost incurred by the inconsistency in the choices.

Figure 3.3 demonstrates the decomposition graphically. Consider a data set of size 2: $D = \{(p^1, x^1), (p^2, x^2)\}$ where $p^i x^i = 1$. The choice is inconsistent since $x^i R x^j$ and $x^j P^0 x^i$ for $i, j \in \{1, 2\}$ $i \neq j$. It is easy to see that for any anonymous aggregator, the Varian Index will be $I_V(D, f) = f(1, v^2)$. Hence, the dashed line (together with the original budget line from which x^1 was chosen) represents graphically the minimal adjustments required for D to satisfy $GARP_{\mathbf{v}}$. Now consider, for example, the singleton set of utility functions that includes the monotonic and continuous function u. We would like to find $\mathbf{v}^\star(D, u)$. Since for this utility function $P_u(x^1) \cap RW(x^1) = \phi$, then $v^{\star 1}(D, u) = 1$. $v^{\star 2}(D, u)$ is the minimal expenditure required to achieve utility level of $u(x^2)$ under prices p^2, which is represented graphically by the dotted line. $I_M(D, f, \{u\}) = f(1, v^{\star 2}(D, u))$ and since $v^{\star 2}(D, u)$ is smaller than v^2, it implies that $I_M(D, f, \{u\})$ is weakly greater than $I_V(D, f)$.

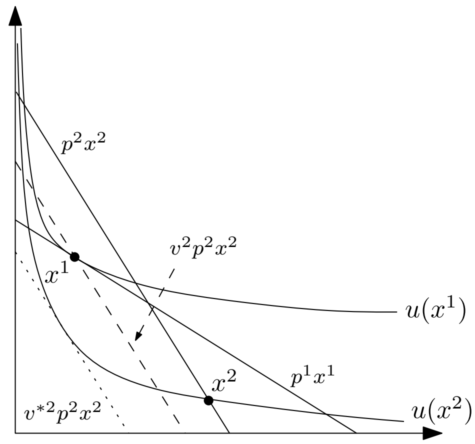


Figure 3.3: Decomposition

The difference between the original budget line from which x^2 was chosen and the dashed line - $v^2p^2x^2$, represents graphically the inconsistency implied by D, while the difference between the dashed line and the dotted line - $v^{\star 2}p^2x^2$, represents the misspecification implied by u. Their sum is the goodness of fit measured by the money metric index.

It is crucial to note that since, for a given data set, the inconsistency index is constant, the goodness of fit measure can be used to recover parametric preferences within some parametric family. The same idea can be applied to hypotheses testing and model selection. Consider two parametric families $\mathcal{U}$ and $\mathcal{U}'$. A researcher will calculate $I_M(D, f, \mathcal{U}')$ and $I_M(D, f, \mathcal{U})$. As argued before, both incorporate the same inconsistency measure - $I_V(D, f)$, hence the data set D may be as better approximated by $\mathcal{U}$ or $\mathcal{U}'$ depending on the relative magnitude of the money metric index. Moreover, an important implication of Fact 1 is that if we impose an additional parametric restriction on preferences (hence reduce the set of possible utility functions we consider), the misspecification will necessarily (weakly) increase. That is, if $\mathcal{U}'$ is a subset of $\mathcal{U}$ that is generated by some parametric restriction, then $\frac{I_M(D,f,\mathcal{U}')-I_M(D,f,\mathcal{U})}{I_M(D,f,\mathcal{U})-I_V(D,f)}$ is a measure of the relative marginal misspecification implied by the restriction of $\mathcal{U}$ to $\mathcal{U}'$. We will tend to accept the restriction if this ratio is low. This methodology resembles statistical hypothesis testing, although the current study does not incorporate any error structure. Inclusion of such structure may provide an interesting avenue for future research, but is not pursued here.[25]

[25]See related discussions in Afriat [1972] and in Varian [1985].

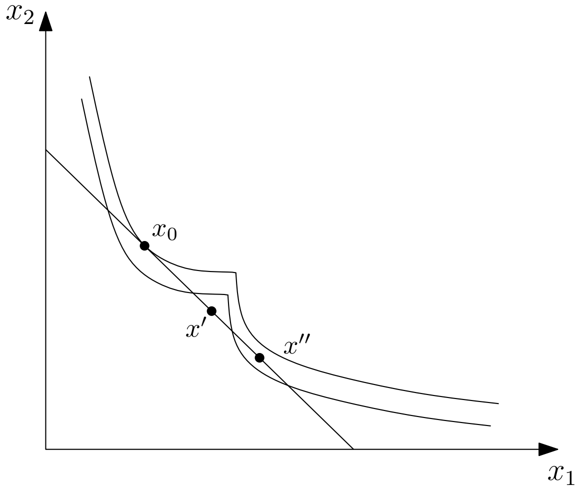


Figure 4.1: Non-convex preferences and a distance-based loss-function

4 Application to Choice under Risk

The goal of this section is to demonstrate the empirical differences between the recovery method proposed in the current paper and a recovery method that utilizes a loss-function that is based on the distance between observed and predicted choices in the commodity space. Specifically, we compare to NLLS, which belongs to this class.

Recovery that employs such loss function fails to account for all the ranking information encoded in the choices, since it compares only the distance between predictions and choices and does not incorporate all other bundles that were feasible but were not chosen. Moreover, if the "true" (unobserved) preferences are not convex, the ranking information induced by a utility function that generates a closer prediction may be more inconsistent with the "true" ranking of bundles. In other words, the intuition that a closer prediction represents less misspecification relies crucially on the assumption of convex preferences, which is not part of revealed preference theory. Figure 4.1 demonstrates this argument. Consider a choice of x_0 generated by the non-convex preferences depicted in the figure. These preferences would imply that had the DM faced the menu $\{x', x''\}$, she would choose x''. Since x' is closer to x_0 than x'', every recovery method that is based on a distance between observed and predicted choices, would assign a lower loss to preferences with predicted choice at x' than to preferences with predicted choice at x''. This would imply that x' is preferred to x'' - contrary to the "true" preferences that generated the data.

We apply the parametric recoverability method developed in this study and NLLS to a data set of portfolio choice problems collected by Choi et al. [2007]. In their experiment, subjects were asked to choose the optimal portfolio using a combination of Arrow securities from convex budget sets with varying prices. We focus our analysis only on the treatment where the two states are equally probable. For each subject, the authors collect 50 observations and proceed to test these choices for rationality (i.e. GARP) as well as estimate a parametric utility function in order to determine the magnitude and distribution of risk attitudes in the population. Choi et al. [2007] estimate a Disappointment Aversion functional form introduced by Gul [1991] (for equally probable states):[26]

$$u(x^i) = \gamma w\left(\max\left\{x_1^i, x_2^i\right\}\right) + (1-\gamma)w\left(\min\left\{x_1^i, x_2^i\right\}\right) \tag{4.1}$$

where

$$\gamma = \frac{1}{2+\beta} \qquad \beta > -1$$

and

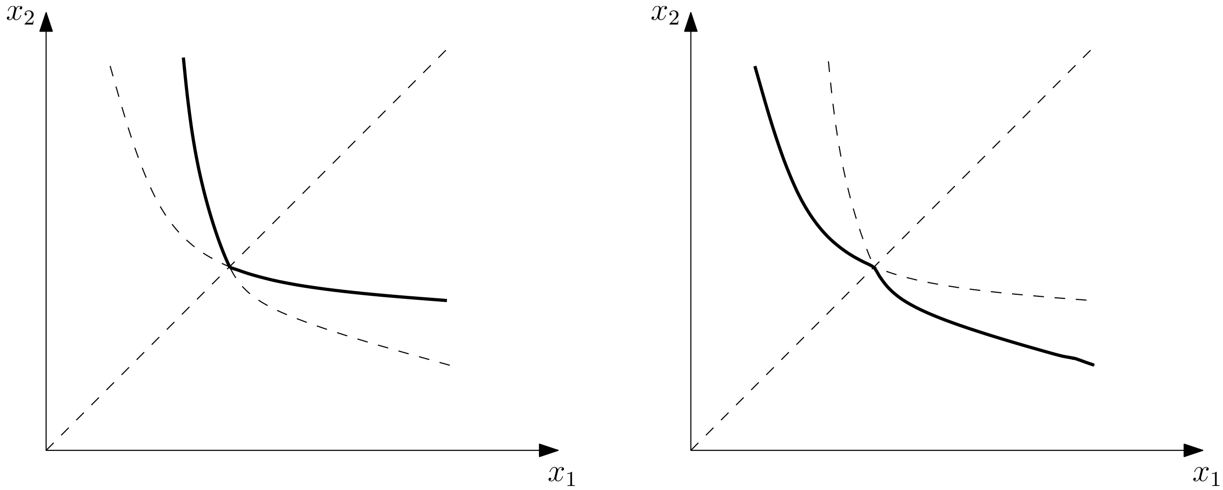
$$w(z) = \frac{z^{1-\rho}}{1-\rho}$$

The parameter γ is the weight placed on the better outcome. For $\beta > 0$, the better outcome is under-weighted relative to the objective probability (of 0.5) and the decision maker is *disappointment averse.* For $\beta < 0$, the better outcome is over-weighted relative to the objective probability (of 0.5) and the decision maker is *elation seeking.*[27] In the knife-edge case, when $\beta = 0$, (4.1) reduces to expected utility. β has important economic implication: if $\beta > (=)0$ the decision maker exhibits *first-order (second order) risk aversion* [Segal and Spivak, 1990]. That is, the risk premium for small fair gambles is proportional to the standard deviation (variance) of the gamble.[28] First order risk aversion can account for important empirical regularities that expected utility (with its implied second-order risk aversion) cannot, such as in portfolio choice problems [Segal and Spivak, 1990], calibration of risk aversion in the small and large and disentangling intertemporal substitution from risk aversion (see Epstein, 1992 for a survey). Figure 4.2 illustrates characteristic indiffer-

[26] Note that for two states of the world, one cannot distinguish between Rank Dependent Preferences and Disappointment Aversion preferences. See also Footnote 6.

[27] Note that we allow for elation seeking while the original work of Choi et al. [2007] assumes that $\beta \geq 0$.

[28] $-1 < \beta < 0$ implies local risk-seeking behavior.



(a) Disappointment Aversion, $\beta > 0$

(b) Elation Loving, $-1 < \beta < 0$

Figure 4.2: Non-Expected Utility, Gul [1991]

ence curves for disappointment aversion and elation seeking (locally non-convex) subjects, respectively. Additionally, $w(x)$ is a standard utility for wealth function and is represented here by the CRRA functional form.

We recover preferences (β and ρ) using two different methods: the standard statistical Non-Linear Least Squares (NLLS) based on the Euclidean distance and the Money Metric method developed here. To calculate the Varian Index, $I_V(D, f)$, and the Money Metric Index, $I_M(D, f, \mathcal{U})$, we use the quadratic aggregator:

$$f(\mathbf{v}) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(1 - v^i)^2}$$

In both methods we use numerical optimization algorithm that allow us to overcome the difficulties associated with using analytical method (as those based on the first order conditions) when preferences are not convex or the slope of the indifference curve at the axis approaches infinity. In addition, this enables switching the designated family of parametric utility function easily. For technical and computational details see Appendix **??**.

The recovered parameters for all subjects who satisfy GARP (12 out of 47), using both the statistical method and money metric method, are reported in Table 1.[29] In addition, the

[29]Note that the recovered parameters for the statistical method may differ from those reported in Choi et al. [2007] for several reasons: we allow for elation loving ($-1 < \beta < 0$); we permit boundary observations

	Statistical		Money Metric		
Subject	β	ρ	β	ρ	$I_M(D, f, \mathcal{U})$
304	0.575	176.227	0.613	26.802	0.00164
317	-0.058	1.556	0.182	0.979	0.00766
316	0.642	0.635	1.145	0.520	0.01944
326	0.673	0.339	0.698	0.146	0.02137
216	0.010	0.355	0.212	0.295	0.02328
314	0.365	0.0005	0.367	0.027	0.02817
219	0.136	0.580	0.958	0.171	0.03022
214	-0.994	15.208	0.093	0.943	0.03144
306	-0.315	6.245	0.521	1.566	0.03513
303	0.0014	1	0.893	0.247	0.04373
205	-0.991	2.722	-0.639	1.074	0.11104
320	-1	17.943	-0.508	0.968	0.13224
Mean (GARP)	-0.0797	18.5673	0.37781	2.8115	0.04045
Median (GARP)	0.0057	1.2795	0.4442	0.7314	0.02919
Mean (all)	0.2695	5.9934	0.3835	1.0570	0.06178
Median (all)	0.1710	0.6349	0.3674	0.3562	0.05012

Table 1: Recovering Preferences - Results

goodness of fit, expressed as the Money Metric Index is reported in the last column. This number represents the "inefficiency" implied by using the functional form (note that these subjects are consistent). The lower rows report the average and median value of parameters for the consistent and for all 47 subjects.

Notice that in all cases the recovered parameters differ between the two methods. While these differences cannot be tested for statistical significance, as our framework does not include any stochastic component, we can interpret them in terms of economic significance. In other words, numerical differences in the recovered parameters are suggestive of important qualitative differences in behavior. Consider, for example, subject 317: the statistical method results in $\beta < 0$ implying that the decision maker is elation loving, where as the money metric method suggests the opposite, i.e. the decision maker is disappointment averse. There are significant behavioral differences between these types of preferences. In particular, elation loving subjects will display local risk seeking behavior along the certainty line, but disappointment averse subjects will display first-order risk aversion in the same region. In light of

$(x^i = 0)$; we use numerical optimization with 5 to 10 initial values (instead of a single initial value) ; we use Euclidean norm (instead of the geometric mean). For robustness we made sure that we are able to replicate Choi et al. [2007] reported results.

the fact that our recovery method incorporates more revealed preference information than the statistical method, it presents more welfare information that pertains to comparisons between unobserved portfolios.

The last column provides a metric for quantifying the extent of misspecification and thus can be used by a researcher to determine if a certain parametric form is an adequate representation of the decision maker's preferences. For example, we can interpret this number as the average percentage of income wasted if the decision maker were to make choices according to the recovered utility function rather than their true preferences. For Subject 317 this implies an average waste of about 0.8% or approximately \$0.80 out of \$100 of income. On the other hand, for Subject 320 this implies an average inefficiency of more than \$13 out of \$100. These numbers have immediate economic interpretation as they quantify the percentage of the portfolio that should be paid to an investment manager who has disappointment aversion preferences, instead of making the investment decisions independently. What is the critical value for this variable remains to the researcher's discretion. In the current sample 10 out of the 12 consistent subjects have a misspecification of less than 10% of their income.[30]

4.1 Recovering Preferences for Inconsistent Decision Makers

In Section 3.2 we proved the decomposition of the Money Metric Index into the Varian Index - which serves as a measure of consistency, and a remainder - which is a measure of misspecification. As such, we can use this approach to recover approximate preferences for those subjects who fail GARP and for whom there exists no utility function that can strictly rationalize their choices. The approximate preference is the closest, within the parametric family considered, to rationalizing the choices according to the money metric and the quadratic additive aggregator function used.

Computing the Varian Index is a hard computational problem (see discussion in Section 5.1). We implemented an algorithm that over-estimates the real Varian Index (the details of the implementation are in Appendix **??**). The implication of this overestimation is that in most of the results that follow, the decomposition of the Money Metric Index overestimates the irrationality component and underestimates the misspecification component. An un-

[30]Inspection of the choices made by subjects 205 and 320 reveals a consistent pattern in their decision making that is easily represented by a utility function. That said, this utility function is not a member of the parametric family being estimated and this is the source of the misspecification. It should be noted, however, that even if we did not know the true preferences of the decision maker, the criterion would still provide evidence of the type of misspecification described above.

Subject	I_V	β	ρ	I_M
205	0	-0.639	1.074	0.111
207	0.00982	0.205	0.180	0.020

Table 2: Rationalizing the Inconsistent

Misspecification % of income	≤ 0	1	2	3	4	5	6	7	8	9	10	11+
Frequency	5	6	6	12	5	4	2	2	1	0	0	4

Table 3: Misspecification Distribution

avoidable consequence of this computational bias is that in some cases the misspecification component will be negative.

To illustrate, consider Table 2 that compares the recovered parameters using the Money Metric method for two subjects taken from Choi et al. [2007]. Subject 205's choices are consistent with GARP while subject 207's are inconsistent. In spite of the fact that 205 is consistent, the parametric preferences considered cannot encode the ranking implied by her choices, as it requires 11% wasted income on average. On the other hand, the revealed preference information implied by 207's choices is well captured by the parametric family, since it implies inefficiency of only 2%, in spite of the fact that her choices are not strictly consistent ($I_V = 0.00962 > 0$, 13 $GARP$ violations). In other words, the misspecification implied by the parametric family is much smaller for 207 than for 205. As such, the Money Metric recoverability can be applied uniformly to all data sets, and the misspecification and inconsistency can be evaluated on a common scale ex-post.

Using the decomposition of the Money Metric Index into the Varian Index (measure of consistency) and a residual which measures misspecification, we can calculate the misspecification for each subject (recall that these are underestimations).

Table 3presents the distribution of misspecification in the Choi et al. [2007] sample. Due to the underestimation of the misspecification and the lack of information about the properties of this bias, all that can be learned in certainty is that at least 25% of the sample is represented poorly by the disappointment aversion model with CRRA, as the misspecification implied by the use of the functional form is higher than 5%.

4.2 Behavioral Implications

We find that for *all* consistent subjects and 31 out of the 47 subjects overall, the money metric method recovers β value higher than the statistical method, and for 36 out of the 47 subjects (11 out of the 12 consistent subjects) the money metric method recovers lower value of ρ than the estimated statistical method value. That implies much higher degree of first-order risk aversion, and a much lower curvature of the indifference curve away from the certainty line using the money metric recovery. Naturally, this is a limited sample, but if this result would generalize (the graphical interface developed by Choi et al. 2007 enables the collection of large data sets relatively quickly and cheaply) it could have far reaching implications on the quantitative estimates used in the macroeconomics and finance literature, employing for example the Epstein and Zin [1989] framework.

4.3 Evaluating a Restriction to Expected Utility

Using the decomposition result we can evaluate the additional misspecification implied by restricting preferences to be expected utility with constant relative risk aversion. This requires recovering ρ under the restriction that $\beta = 0$. Obviously, the restriction implies increase in the misspecification. As proposed in Section 3.2, we use the ratio $\frac{I_M(D,f,EU)-I_M(D,f,DA)}{I_M(D,f,DA)-I_V(D,f)}$ where DA stands for disappointment averse with $CRRA$ utility function, EU stands for expected utility with $CRRA$ and f is standardized sum of squares aggregator. We use a critical value of 10%. That is, if the restriction to expected utility implies a proportional increase in the misspecification of more than 10% then, we tend to reject the expected utility specification. Note that since $I_V(D,f)$ is an overestimation of the Varian Index, the calculated ratio is also an overestimation of the real ratio, meaning that the test is actually more strict than it seems.

Five subjects have Varian Index of more than 10%, so their choices are too inconsistent to consider any reasonable recoverability. Four other subjects (two of them consistent) have a Money metric Index of more than 10%, implying that the disappointment aversion specification does not capture their behavior. Out of the reminder 38 subjects, the choices made by 15 subjects are well approximated by the expected utility model with CRRA, as the restriction $\beta = 0$ implies an increase of less than 10% relative to the misspecification of the disappointment aversion model (that is, the absolute additional misspecification is less than 1%). The reminder 23 subjects are not well approximated by the expected utility model with CRRA as the increase in the misspecification implied by restricting to expected utility is higher than 10%.

5 Short Discussions

5.1 The Computation of the Varian Index

Afriat [1972, 1973, 1987] and Varian [1990] discuss non-uniform adjustments of the budget lines so that the inconsistencies in the data are removed. Varian [1990] argues that given an aggregator function an optimal vector of adjustments can be found. Moreover, the value of this vector can be interpreted as the inconsistency level of a given data set. The problem of finding this exact value is equivalent to the *minimum cost feedback arc set problem*.[31] **?** shows that the minimum cost feedback arc set problem is NP-Hard and therefore finding the exact Varian's index is also NP-Hard as suggested in Varian [1990].

Three algorithms to compute a polynomial approximation were suggested in the economics literature. The first algorithm (Tsur [1989] and Algorithm 1 in Alcantud et al. [2010]) suggests to report the vector $\mathbf{v}$ such that v_j is the minimal adjustment required to exclude all x_i such that $x_i R x_j$ from the budget set of observation j. The second algorithm (Algorithm 2 in Alcantud et al. [2010]) is such that v_j is the minimal adjustment required to exclude one x_i such that $x_i R x_j$ from the budget set of observation j. If the data satisfies $GARP_{\mathbf{v}}$, $\mathbf{v}$ is reported, otherwise another point is removed for each observation j and so on until $GARP_{\mathbf{v}}$ is satisfied. The third algorithm (Varian [1993] and Algorithm 3 in Alcantud et al. [2010]) suggests to calculate the minimal adjustment to one of the budget sets, such that one violation of $GARP$ is removed. This minimal value should be substituted into $\mathbf{v}$ and $GARP_{\mathbf{v}}$ should be checked. If the data satisfies $GARP_{\mathbf{v}}$, $\mathbf{v}$ is reported, otherwise another point is removed and the procedure is repeated until the data satisfies $GARP_{\mathbf{v}}$.

Alcantud et al. [2010] show that Algorithms 2 and 3 are better approximations than Algorithm 1 and that they do not dominate each other. Moreover, Alcantud et al. [2010] show that D satisfies $GARP_{\mathbf{v}}$ for the $\mathbf{v}$ found by Algorithms 2 and 3. This implies that these approximations are overestimations of the actual Varian's index. We do not know of any measure for the quality of this approximation. Also, note that none of these algorithms

[31] Given a directed and weighted graph, find the "cheapest" subset of arcs such that its removal turns the graph into an acyclic graph

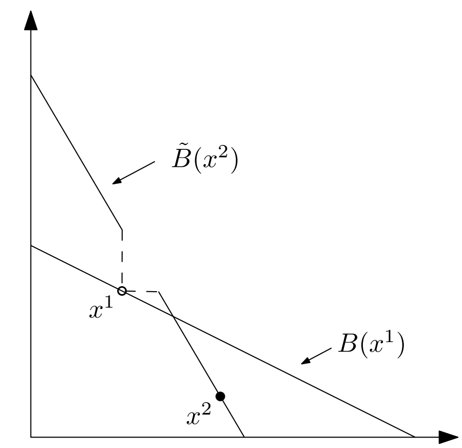


Figure 5.1: Modified Budget Sets

uses the chosen aggregator function as part of its iterative mechanism. We believe that incorporating the computer science literature on the "minimum cost feedback arc set problem" and using the chosen aggregator may improve considerably the quality of approximation.

5.2 Area-based Parametric Recoverability

Figure 3.1 suggests an obvious alternative to the money metric as a foundation for measuring misspecification: a measure that is based on the area of overlap between the upper contour set corresponding to a specific utility function, and the set of alternatives that are revealed worse than the observed choice. This measure is related to the Minimal Swaps Index, which is a measure of inconsistency proposed recently by Apesteguia and Ballester [2012] for the case of finite number of alternatives. To generalize their method to infinite alternatives set as studied in the current paper, one needs to calculate an index that is based on the area above for the entire set of acyclic relations. When the set of utility functions is restricted to a single parametric family the Minimal Swaps Index could then be decomposed into separate measures of inconsistency and misspecification. While there was an obvious way to achieve this goal with respect to the Money Metric Index and the corresponding Varian Efficiency Index, it is not entirely clear how to measure inconsistency directly using areas.

We can define a measure of inconsistency based on the area of overlap between the revealed preferred set and the budget set corresponding to an observed choice. However, since we are dealing with inconsistent data set, we differ from Varian [1982] and define the revealed preferred set as *only* those bundles which are either revealed preferred or monotonically dominate a bundle that is revealed preferred to a given bundle. Hence, as illustrated in Figure 5.1, violations of consistency are removed by modifying budget sets so as to eliminate the area of overlap between the budget set and those bundles which are revealed preferred. Hence, we can use this measure to decompose the Area Index into separate measures of inconsistency and misspecification just as we did with the Money Metric Index.

Nevertheless, the Area Index is not ideal. First, there does not exist an elegant theoretical analog to Afriat's Theorem with respect to the modified budget sets in Figure 5.1 as there does for the specific type of budget set adjustments utilized in calculating the Money Metric Index. Second, it is not clear that computing the inconsistency index suggested above would be any easier than computing the Varian Efficiency Index, a problem which is NP-hard. Third, it is a simple exercise to show that choices with modified budget sets as in Figure 5.1 can be easily rationalized by non-convex preferences and, in fact, any recoverability procedure based on the Area Index would be biased towards these types of non-convexities. Put another way, with the Area metric as a criterion, any convex preferences which rationalize the modified data set can be improved upon with similar non-convex preferences. Lastly, the simple Area Index may lack intuitive interpretation that the money metric index enjoys. All these are surmountable difficulties, that we think are worthwhile pursuing in future work. Ultimately, since the Money Metric Index does not appear to suffer from the same issues we currently believe it dominates the Area Metric both as a measure of misspecification and as a method for recovering preferences.

5.3 From Inefficiency to Consideration Sets

In the consistency literature, Afriat [1972] and Varian [1990, 1993] view the extent of the adjustment of the budget line as the amount of income wasted by a decision maker relative to a fully consistent one (hence the term "efficiency index"). A related interpretation, mentioned by Houtman [1995], holds that the DM overestimates prices and hence does not consider all feasible alternatives. An alternative interpretation (due to Manzini and Mariotti, 2007, 2012, Apesteguia and Ballester, 2012, Masatlioglu et al., 2012, Cherepanov et al., 2012), views the adjusted budget set as a consideration set which includes only the alternatives from the original budget menu that the DM compares to the chosen alternative. By construction, those bundles not included in the attention set are irrelevant for revealed preference consideration. Another line of interpretation for inconsistent choice data, is measurement error [Varian, 1985, Tsur, 1989]. These errors could be the result of various circumstances as (literally) trembling hand, indivisibility, omitted variables etc.

All above interpretations take literally the existence of an underlying "welfare" preferences that generate the data [Bernheim and Rangel, 2009]. Since there exist other plausible data generating processes that result in approximate (and even exact) consistent choices [Rubinstein and Salant, 2012], we do not find a clear reason to favor one interpretation over the other, and would rather remain agnostic about the nature of the adjustments required to measure inconsistency.

More importantly, this paper studies the problem of recoverability of preferences and not consistency. That is, we take the data set as the primitive and the utility function as an approximation. As such, the adjustments serve us as a measurement tool ("ruler") for quantifying the extent of misspecification. We view the current work as contributing to the measurement of misspecification rather than to the literature that explains how inconsistency arises.

A Proof of Proposition 1

Notation. Let $x \in \Re^K$ and $\delta > 0$. $B_\delta(x) = \left\{ y \in \Re^K : \|y - x\| < \delta \right\}$.

Definition. A utility function $u : \Re^K \to \Re$ is

1. *locally non-satiated* if $\forall x \in \Re^K$ and $\forall \epsilon > 0$ $\exists y \in B_\delta(x)$ such that $u(y) > u(x)$.
2. *continuous* if $\forall x \in \Re^K$ and $\forall \epsilon > 0$ there exists $\delta > 0$ such that for $y \in B_\delta(x)$ implies $u(y) \in B_\epsilon(u(x))$.

Lemma. *If $u(\cdot)$ is a locally non-satiated utility function that rationalizes $D = \left\{ (p^i, x^i)_{i=1}^n \right\}$, then $x^i P^0 x$ implies $u(x^i) > u(x)$.*

Proof. Suppose $x^i P_D^0 x$ ($p^i x^i > p^i x$). Then by the definition of the revealed preference relations (Definition 1), $x^i R_D^0 x$. Since $u(\cdot)$ rationalizes D, $x^i R_D^0 x$ implies $u(x^i) \geq u(x)$. Suppose that $u(x^i) = u(x)$. Since $p^i x^i > p^i x$ $\exists \epsilon > 0$ such that $\forall y \in B_\epsilon(x) : p^i x^i > p^i y$. By local non-satiation $\exists y' \in B_\epsilon(x)$ such that $u(y') > u(x) = u(x^i)$. Thus, y' is a bundle such that $p^i x^i > p^i y'$ and $u(y') > u(x^i)$, in contradiction to $u(\cdot)$ rationalizes D. Therefore, $u(x^i) > u(x)$. □

For what follows, let $D = \left\{ (p^i, x^i)_{i=1}^n \right\}$ and let $u(\cdot)$ be a continuous and locally non-satiated utility function.

Part 1: $u(\cdot)$ $\mathbf{v}^\star(D, u)$***-rationalizes*** D

Proof. Suppose that for some observation $(p^i, x^i) \in D$ there exists a bundle x such that $x^i R^0_{D,\mathbf{v}^\star(D,u)} x$ and $u(x^i) < u(x)$. By the definition of the revealed preference relations induced by adjusted data sets (Definition 7.1), $v^{\star i}(D,u) p^i x^i \geq p^i x$. By the normalized money metric definition (Definition 10), $m(x^i, p^i, u) \geq p^i x$. Since $m(x^i, p^i, u)$ is the minimal expenditure required to achieve a utility level of at least $u(x^i)$, the case where the inequality is strict contradicts Definition 10. If $m(x^i, p^i, u) = p^i x$ and $u(x^i) < u(x)$, by continuity of $u(\cdot)$ there exists $\gamma > 0$ such that $u(x^i) < u((1-\gamma)x)$. However, since $p^i(1-\gamma)x < p^i x = m(x^i, p^i, u)$, we reach a contradiction to Definition 10. □

Part 2: $\mathbf{v}^{\star}(D,u)=\mathbf{1}$ if and only if $u(\cdot)$ rationalizes D.

Proof. First, let us show that if $u(\cdot)$ rationalizes D then $\mathbf{v}^{\star}(D,u)=\mathbf{1}$. Suppose that for observation $(p^i,x^i)\in D$, $v^{\star i}(D,u)<1$, that is: $m(x^i,p^i,u)<p^ix^i$. By Definition 10, there exists a bundle x such that $p^ix<p^ix^i$ and $u(x)\geq u(x^i)$. However, since by Definition 1.2, $x^iP_D^0x$, and since $u(\cdot)$ is a locally non-satiated utility function that rationalizes D, the above proven lemma implies, in contradiction, that $u(x^i)>u(x)$. Thus, for every observation $(p^i,x^i)\in D$, $m(x^i,p^i,u)\geq p^ix^i$. But by Definition 10 $m(x^i,p^i,u)\leq p^ix^i$ for all i. Combining the two weak inequalities leads to $m(x^i,p^i,u)=p^ix^i$ for every observation, that is: $v^{\star i}(D,u)=1$ for all i. Thus, $\mathbf{v}^{\star}(D,u)=\mathbf{1}$.

Next, let us show that if $\mathbf{v}^{\star}(D,u)=\mathbf{1}$ then $u(\cdot)$ rationalizes D. By Definition 10, $\mathbf{v}^{\star}(D,u)=\mathbf{1}$ implies $m(x^i,p^i,u)=p^ix^i$ for every $(p^i,x^i)\in D$. Suppose that $u(\cdot)$ does not rationalize the data. That is, for some observation (p^i,x^i), there exists a bundle x such that $u(x)>u(x^i)$ and $x^iR_D^0x$. By continuity of $u(\cdot)$ there exist $\gamma>0$ such that $u((1-\gamma)x)>u(x^i)$. However, since $p^i(1-\gamma)x<p^ix^i=m(x^i,p^i,u)$ we reach a contradiction to Definition 10. $\square$

Part 3: Let $\mathbf{v}\in\Re^{\mathbf{n}}$, $\mathbf{0}\leqq\mathbf{v}\leqq\mathbf{1}$. $u(\cdot)$ $\mathbf{v}$-*rationalizes* D if and only if $\mathbf{v}\leqq\mathbf{v}^{\star}(D,u)$.

Proof. First, let us show that if $u(\cdot)$ $\mathbf{v}$-rationalizes D then $\mathbf{v}\leqq\mathbf{v}^{\star}(D,u)$. Suppose that $\mathbf{v}$ is such that $u(\cdot)$ $\mathbf{v}$-rationalizes D and for observation i, $v^i>v^{\star i}(D,u)$. By Definition 8, $u(x^i)\geq u(x)$ for all x such that $x^iR^0_{D,\mathbf{v}}x$ or equivalently $v^ip^ix^i\geq p^ix$. By Definition 10 and since $v^i>v^{\star i}(D,u)$ we get that $v^ip^ix^i>m(x^i,p^i,u)=p^ix^{i\star}$ where $x^{i\star}\in argmin_{\{y\in\Re^K_+:u(y)\geq u(x^i)\}}p^iy$. It follows that $\exists\epsilon>0$ such that $\forall y\in B_\epsilon(x^{i\star}):v^ip^ix^i>p^iy$. By local non-satiation $\exists y'\in B_\epsilon(x^{i\star})$ such that $u(y')>u(x^{i\star})\geq u(x^i)$. Thus, y' is a bundle such that $v^ip^ix^i>p^iy'$ and $u(y')>u(x^i)$ contradicting that $u(\cdot)$ $\mathbf{v}$-rationalizes D.

Next, let us show that if $\mathbf{v}\leqq\mathbf{v}^{\star}(D,u)$ then $u(\cdot)$ $\mathbf{v}$-rationalizes D. By Part 1: $u(\cdot)$ $\mathbf{v}^{\star}(D,u)$-rationalizes D. That is, for every observation $(p^i,x^i)\in D$, $v^{\star i}(D,u)p^ix^i\geq p^ix$ implies $u(x^i)\geq u(x)$. Since $\mathbf{v}\leqq\mathbf{v}^{\star}(D,u)$, for every observation $(p^i,x^i)\in D$, $v^{\star i}(D,u)p^ix^i\geq v^ip^ix^i$. Therefore, for every observation $(p^i,x^i)\in D$, $v^ip^ix^i\geq p^ix$ implies $u(x^i)\geq u(x)$. Hence, $u(\cdot)$ $\mathbf{v}$-rationalizes D. $\square$

B Proof of Theorem 1

Notation. We use the following notations throughout the proof:

- Let $\mathbf{v} \in [0,1]^n$ and $\delta > 0$. $\bar{B}_\delta(\mathbf{v}) = \{\mathbf{v}' \in [0,1]^n : \|\mathbf{v}' - \mathbf{v}\| < \delta\}$.
- $E_v = \{\mathbf{v} \in [0,1]^n : f(\mathbf{v}) = I_V(D,f)\}$
- $\forall \epsilon < M - I_V(D,f): \quad E_{v+\epsilon} = \{\mathbf{v} \in [0,1]^n : f(\mathbf{v}) = I_V(D,f) + \epsilon\}$.
- $E_G = \left\{\mathbf{v} \in [0,1]^n : \forall r > 0,\ \exists \mathbf{v}' \in \bar{B}_r(\mathbf{v}),\ D \text{ satisfies } GARP_{\mathbf{v}'}\right\}$.
- $\hat{E} = E_v \cap E_G$.

Lemma 1. *E_v is non-empty, bounded and closed.*

Proof. First, by Fact 4, $I_V(D,f)$ always exists. Second, by Definition 9 $f(\cdot)$ is continuous and bounded. By the Intermediate Value Theorem, for every value of $I_V(D,f)$ there exists a vector $\mathbf{v}$ such that $f(\mathbf{v}) = I_V(D,f)$, concluding that E_v is non-empty. Third, $E_v \subseteq [0,1]^n$ and therefore it is bounded. Finally, since $f(\cdot)$ is continuous it induces a continuous ordering on $[0,1]^n$. Therefore, for every $I_V(D,f)$, the upper contour set and the lower contour set are closed and their intersection, E_v, is closed as well. □

Lemma 2. *$\hat{E}$ is non-empty.*

Proof. Assume $I_V(D,f) < M$. Suppose that $\hat{E}$ is empty, that is $\mathbf{v} \in E_v \Rightarrow \mathbf{v} \notin E_G$ (due to Lemma 1, this condition is not vacuous). Thus, $\forall \mathbf{v} \in E_v,\ \exists r > 0,\ \forall \mathbf{v}' \in \bar{B}_r(\mathbf{v}),\ D$ violates $GARP_{\mathbf{v}'}$. Let $r(\mathbf{v}) = \sup_{\{r \in (0,\sqrt{n}] : \forall \mathbf{v}' \in \bar{B}_r(\mathbf{v}),\, D \text{ violates } GARP_{\mathbf{v}'}\}} r$. $r(\mathbf{v})$ is uniform continuous on E_v since $\forall \mathbf{v}, \mathbf{v}' \in E_v$ if $\| \mathbf{v} - \mathbf{v}' \| < \epsilon$ then by the triangle inequality $|r(\mathbf{v}) - r(\mathbf{v}')| < \epsilon$.[32] Let $\bar{r} = \min_{\mathbf{v} \in E_v} r(\mathbf{v})$. $\bar{r}$ exists since $r(\mathbf{v})$ is continuous on E_v and E_v is bounded and closed (by Lemma 1). In addition, $\bar{r} > 0$ since $\forall \mathbf{v} \in E_v:\ r(\mathbf{v}) > 0$. Then, $\forall \mathbf{v} \in E_v,\ \forall r < \bar{r},\ \forall \mathbf{v}' \in \bar{B}_r(\mathbf{v}),\ D$ violates $GARP_{\mathbf{v}'}$. Thus, we established that for every $I_V(D,f) < M$ if $\hat{E}$ is empty there exists a hypercylinder H of radius $\bar{r} > 0$ around E_v such that if $\mathbf{v}'$ is an interior point in H then D violates $GARP_{\mathbf{v}'}$.

[32] The distance between $\mathbf{v}$ and $\mathbf{v}'$ is at most ϵ, the distance between $\mathbf{v}$ and some $\mathbf{w}$ such that D satisfies $GARP_{\mathbf{w}}$ is $r(\mathbf{v})$ and by the triangle inequality the distance between $\mathbf{v}'$ and $\mathbf{w}$, which serves as a bound on $r(\mathbf{v}')$, is between $r(\mathbf{v}) - \epsilon$ and $r(\mathbf{v}) + \epsilon$.

The next step is to show that there exists $0 < \epsilon < M - I_V(D)$ such that $E_{v+\epsilon}$ is contained in H (by Lemma 1 $E_{v+\epsilon}$ is non-empty). Suppose that for every $0 < \epsilon < M - I_V(D,f)$ there exists $\mathbf{v}'_\epsilon \in E_{v+\epsilon}$ such that $\mathbf{v}'_\epsilon \notin H$. Then, $\mathbf{v}'_\epsilon$, where $\epsilon \to 0$, is an infinite bounded sequence in $[0,1]^n$ and therefore it has a convergent subsequence. Denote the limit of this subsequence by $\hat{\mathbf{v}}$. Since $\hat{\mathbf{v}}$ is not an interior point of H it must be that $f(\hat{\mathbf{v}}) \neq I_V(D,f)$. However, by construction, $\lim_{\epsilon \to 0} f(\mathbf{v}'_\epsilon) = I_V(D,f)$, suggesting that $f(\cdot)$ is not continuous. Thus, there exists $\bar{\epsilon}$ such that $E_{v+\bar{\epsilon}} \subset H$. Moreover, since $f(\cdot)$ is continuous $\forall \epsilon \in [0,\bar{\epsilon}):\ E_{v+\epsilon} \subset H$.

That is, there exists $\bar{\epsilon} > 0$ such that for every $\mathbf{v}' \in [0,1]^{\mathrm{n}}$ that satisfies $I_V(D,f) \leq f(\mathbf{v}') < I_V(D,f) + \bar{\epsilon} < M$, D violates $GARP_{\mathbf{v}'}$. Since $I_V(D,f)$ is an infimum there is no $\mathbf{v} \in [0,1]^{\mathrm{n}}$ such that $f(\mathbf{v}) < I_V(D,f)$ and D satisfies $GARP_{\mathbf{v}}$. Thus, there exists $I_V(D,f) < m < M$ such that for every $\mathbf{v} \in [0,1]^{\mathrm{n}} : f(\mathbf{v}) < m$ and D violates $GARP_{\mathbf{v}}$. That contradicts the maximality of $I_V(D,f)$ as an infimum. Therefore, we have shown that if $I_V(D,f) < M$ then $\hat{E}$ is non-empty.

Finally, suppose $I_V(D,f) = M$. By Definition 9, $\mathbf{0} \in E_v$. By Fact 2, $\mathbf{0} \in E_G$. Thus, also if $I_V(D,f) = M$ then $\hat{E}$ is non-empty. $\square$

Lemma 3. *Let $\mathbf{v} \in [0,1]^n$. If $\tilde{\mathbf{v}} \in \bar{B}_\delta(\mathbf{v})$ and D satisfies $GARP_{\tilde{\mathbf{v}}}$, there exists $\hat{\mathbf{v}} \in \bar{B}_\delta(\mathbf{v})$ where $\hat{\mathbf{v}} \leq \mathbf{v}$ such that D satisfies $GARP_{\hat{\mathbf{v}}}$.*

Proof. If $\tilde{\mathbf{v}} \leq \mathbf{v}$ then the lemma is trivial. If $\mathbf{v} \leq \tilde{\mathbf{v}}$ then by Fact 3 D satisfies $GARP_{\mathbf{v}}$. By the same fact, D satisfies $GARP_{\hat{\mathbf{v}}}$ for every $\hat{\mathbf{v}} \in \bar{B}_\delta(\mathbf{v})$ where $\hat{\mathbf{v}} \leq \mathbf{v}$. Otherwise, define $\hat{\mathbf{v}}$ such that $\forall i \in \{1,\ldots,n\}:\ \hat{v}^i = \min\{v^i, \tilde{v}^i\}$. By construction, $\hat{\mathbf{v}} \leq \mathbf{v}$ and $\hat{\mathbf{v}} \leq \tilde{\mathbf{v}}$. Since $\forall i \in \{1,\ldots,n\}:\ |\hat{v}^i - v^i| \leq |\tilde{v}^i - v^i|$ then $\hat{\mathbf{v}} \in B_\delta(\mathbf{v})$. In addition, since $\mathbf{v}, \tilde{\mathbf{v}} \in [0,1]^n$ then $\hat{\mathbf{v}} \in [0,1]^n$. Therefore, $\hat{\mathbf{v}} \in \bar{B}_\delta(\mathbf{v})$. Finally, since $\hat{\mathbf{v}} \leq \tilde{\mathbf{v}}$ and D satisfies $GARP_{\tilde{\mathbf{v}}}$ by Fact 3 D satisfies $GARP_{\hat{\mathbf{v}}}$. Thus, we constructed $\hat{\mathbf{v}} \in \bar{B}_\delta(\mathbf{v})$ where $\hat{\mathbf{v}} \leq \mathbf{v}$ and such that D satisfies $GARP_{\hat{\mathbf{v}}}$. $\square$

Lemma 4. *Let $\mathbf{v}^\star \in \hat{E}$. D satisfies $GARP_{\lambda\mathbf{v}^\star}$ for all $\lambda \in [0,1)$.*

Proof. By Lemma 2 $\mathbf{v}^\star$ exists. If $\mathbf{v}^\star = \mathbf{0}$ the Lemma is trivial by Fact 2. Suppose $\mathbf{v}^\star \geq \mathbf{0}$. Denote $v^\star_{min} = \min_{\{v^\star_i > 0\}} v^\star_i$ and let $\delta \in (0, v^\star_{min})$. Let $\tilde{B}_\delta(\mathbf{v}^\star) = \{\mathbf{v} : \mathbf{v} \leq \mathbf{v}^\star\} \cap \bar{B}_\delta(\mathbf{v}^\star)$. Let $\tilde{\mathbf{v}} \in \tilde{B}_\delta(\mathbf{v}^\star)$ such that D satisfies $GARP_{\tilde{\mathbf{v}}}$. By Lemma 3 such $\tilde{\mathbf{v}}$ exists and by construction $\tilde{\mathbf{v}} \neq \mathbf{v}^\star$. Note that choice of δ implies that for every $i \in \{1,\ldots,n\}$, $v^\star_i > 0 \implies \tilde{v}_i > 0$. Define $\lambda = \{\lambda^i\}_{i=1}^n$ such that if $v^\star_i = 0$ then $\lambda^i = 0$ and otherwise $\lambda^i = \frac{\tilde{v}_i}{v^\star_i} > 0$. Then, $\lambda \in [0,1]^n \backslash \{\mathbf{0}\}$. Denote $\bar{\lambda} = \min_{\{\lambda^i > 0\}} \lambda^i$. Then, $0 < \bar{\lambda} < 1$. For every $i \in \{1,\ldots,n\}$ define

$\hat{v}_i = \bar{\lambda} v_i^\star$. First, note that $\forall i \in \{1,\dots,n\}: \ \hat{v}_i \leq \tilde{v}_i$ (if $v_i^\star = 0$ then $\hat{v}_i \leq \tilde{v}_i = v_i^\star = 0$, otherwise, $\hat{v}_i = \bar{\lambda} v_i^\star \leq \frac{\tilde{v}_i}{v_i^\star} v_i^\star = \tilde{v}_i$) and by Fact 3 since D satisfies $GARP_{\tilde{\mathbf{v}}}$ then D satisfies $GARP_{\hat{\mathbf{v}}}$. Second, $\hat{\mathbf{v}} = \bar{\lambda} \mathbf{v}^\star$. Finally, note that $\forall i \in \{1,\dots,n\}: \ v_i^\star - \delta \leq \tilde{v}_i \leq v_i^\star$. Therefore, $\forall i \in \{1,\dots,n\}: \ 1 - \frac{\delta}{v_i^\star} \leq \lambda^i \leq 1$ and $1 - \frac{\delta}{v_{min}^\star} \leq \bar{\lambda} < 1$. Thus, for every $\epsilon > 0$ there exists $\bar{\lambda} > 1 - \epsilon$ such that $\hat{\mathbf{v}} = \bar{\lambda} \mathbf{v}^\star$ and D satisfies $GARP_{\lambda \mathbf{v}^\star}$. By Fact 3 for every $0 \leq \lambda \leq \bar{\lambda}$ D satisfies $GARP_{\lambda \mathbf{v}^\star}$. Hence, D satisfies $GARP_{\lambda \mathbf{v}^\star}$ for all $\lambda \in [0,1)$. ∎

Definition. Let $\mathbf{v} \in [0,1]^n$. D satisfies *$\mathbf{v}$-Cyclical Consistency* if

$$\begin{aligned} & v^r p^r x^r \geq p^r x^s, v^s p^s x^s \geq p^s x^t, \dots, v^q p^q x^q \geq p^q x^r \\ \implies \ & v^r p^r x^r = p^r x^s, v^s p^s x^s = p^s x^t, \dots, v^q p^q x^q = p^q x^r \end{aligned}$$

Lemma 5. *(Following Fact 1 in Varian [1982]) Let $\mathbf{v} \in [0,1]^n$. D satisfies $\mathbf{v}$-Cyclical Consistency if and only if it satisfies $GARP_{\mathbf{v}}$.*

Proof. Suppose D violates $\mathbf{v}$-Cyclical Consistency. Then, there exists a sequence of observations such that $v^r p^r x^r \geq p^r x^s, v^s p^s x^s \geq p^s x^t, \dots, v^q p^q x^q \geq p^q x^r$ and $v^s p^s x^s > p^s x^t$. By Definition 7 $x^r R^0_{D,\mathbf{v}} x^s, x^s R^0_{D,\mathbf{v}} x^t, \dots, x^q R^0_{D,\mathbf{v}} x^r$ and therefore $x^t R_{D,\mathbf{v}} x^s$. However, by the same definition $x^s P^0_{D,\mathbf{v}} x^t$. Thus, D violates $GARP_{\mathbf{v}}$. On the other hand, suppose D violates $GARP_{\mathbf{v}}$. There exists a pair of observations (p^t, x^t) and (p^s, x^s) such that $x^t R_{D,\mathbf{v}} x^s$ and $x^s P^0_{D,\mathbf{v}} x^t$. Again, by Definition 7 there exists a subset of observations such that $x^t R^0_{D,\mathbf{v}} x^u, x^u R^0_{D,\mathbf{v}} x^v, \dots, x^q R^0_{D,\mathbf{v}} x^s$ and since $x^s P^0_{D,\mathbf{v}} x^t$ implies $x^s R^0_{D,\mathbf{v}} x^t$ there is a subset of observations such that $v^t p^t x^t \geq p^t x^u, v^u p^u x^u \geq p^u x^v, \dots, v^s p^s x^s \geq p^s x^t$. In addition, since $x^s P^0_{D,\mathbf{v}} x^t$ we have $v^s p^s x^s > p^s x^t$. However, this combination violates $\mathbf{v}$-Cyclical Consistency. ∎

Lemma 6. $I_V(D,f) \leq I_M(D,f,\mathcal{U}^c)$

Proof. If $I_V(D,f) = 0$ the lemma follows from definitions 9 and 11. Otherwise, suppose that $I_V(D,f) > I_M(D,f,\mathcal{U}^c)$. Since $I_M(D,f,\mathcal{U}^c) = \inf_{u \in \mathcal{U}^c} f(\mathbf{v}^\star(D,u))$ there exists $u \in \mathcal{U}^c$ such that $f(\mathbf{v}^\star(D,u)) < I_V(D,f)$. By Proposition 1.1 $u(\cdot)$ $\mathbf{v}^\star(D,u)$-rationalizes D. By Theorem 6.3.I in Afriat [1987] (p. 179)[33] $u(\cdot)$ $\mathbf{v}^\star(D,u)$-rationalizes D if and only if D satisfies $\mathbf{v}^\star(D,u)$-

[33] Afriat [1987] does not provide a proof for this theorem. Afriat [1973] provides a proof for the uniform case (same adjustments for all observations) which can be generalized to this theorem. Houtman [1995] studies general cost functions that include the uniform case, the non-uniform case that we use and many other cases. He provides a proof for a general form of Theorem 6.3.I in Afriat [1987] that applies here as well. Note that while Houtman [1995] elaborates on the uniform case, all his statements on this case apply also to the non-uniform linear case that is considered here.

Cyclical Consistency, which is equivalent, by Lemma 5, to D satisfies $GARP_{\mathbf{v}^\star(D,u)}$. However, since D satisfies $GARP_{\mathbf{v}^\star(D,u)}$ and $f(\mathbf{v}^\star(D,u)) < I_V(D,f)$, $I_V(D,f)$ cannot be the infimum of $f(\cdot)$ on the set of all $\mathbf{v} \in [0,1]^n$ such that D satisfies $GARP_{\mathbf{v}}$. □

Lemma 7. *Let $\mathbf{v} \in [0,1]^n$ be such that D satisfies $GARP_{\mathbf{v}}$. Then $I_M(D,f,\mathcal{U}^c) \leq f(\mathbf{v})$.*

Proof. By Lemma 5, D satisfies $GARP_{\mathbf{v}}$ if and only if D satisfies $\mathbf{v}$-Cyclical Consistency. By Theorem 6.3.I in Afriat [1987] (p. 179) D satisfies $\mathbf{v}$-Cyclical Consistency if and only if there exists a non-satiated continuous utility function $u \in \mathcal{U}^c$ that $\mathbf{v}$-rationalizes D. By Proposition 1.3, $\mathbf{v} \leq \mathbf{v}^\star(D,u)$. Since $f(\cdot)$ is weakly decreasing $f(\mathbf{v}^\star(D,u)) \leq f(\mathbf{v})$. Therefore, by Definition 11, $I_M(D,f,\mathcal{U}^c) \leq f(\mathbf{v})$. □

Theorem. *For every finite data set $D = \left\{(p^i,x^i)_{i=1}^n\right\}$ and aggregator function $f : [0,1]^n \to [0,M]$:*

$$I_V(D,f) = I_M(D,f,\mathcal{U}^c)$$

where $\mathcal{U}^c$ is the set of contionuous and locally non-satiated utility functions.

Proof. Let $\mathbf{v}^\star \in \hat{E}$. By Lemma 2 such point exists. For every $\lambda \in [0,1]$ denote $F_\lambda = f(\lambda \mathbf{v}^\star)$. Consider the sequence of intervals $[I_V(D,f), F_\lambda)$. By Lemma 4, D satisfies $GARP_{\lambda\mathbf{v}^\star}$ for all $\lambda \in [0,1)$. Therefore, by Lemma 7, $\forall\lambda \in [0,1) : \; I_M(D,f,\mathcal{U}^c) \leq F_\lambda$. In addition, by Lemma 6, $I_V(D,f) \leq I_M(D,f,\mathcal{U}^c)$. Hence, $\forall\lambda \in [0,1) : \; I_M(D,f,\mathcal{U}^c) \in [I_V(D,f), F_\lambda)$. Since $\lim_{\lambda\to 1} F_\lambda = I_V(D,f)$ we get $I_V(D,f) = I_M(D,f,\mathcal{U}^c)$. □

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