Robust Almost Fully Revealing Equilibria in Multi-Sender Cheap Talk*

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Abstract

We show that in multi-sender cheap talk games where senders imperfectly observe the state, if the state space is large enough, then there exist equilibria arbitrarily close to full revelation of the state as the noise in the senders' observations vanishes. In the case of replacement noise, where the senders observe the true state with high probability, our equilibrium construction involves one round of communication. In the case of continuous noise, where senders observe a signal distributed according to a continuous distribution over an interval around the true state, our construction involves two rounds of communication. After the first round of communication, it becomes a common 1-belief between the senders that the state is in a small interval of the state space, even though before the communication, there is no nontrivial event that is a common p-belief between them for positive p. The results imply that when there are multiple experts from whom to solicit information, if the state space is large, then even when the state is observed imperfectly, there are communication equilibria that are strictly better for the principal than delegating the decision right to one of the experts.

1 Introduction

In sharp contrast to the predictions of cheap talk models with a single sender (Crawford and Sobel (1982), Green and Stokey (2007)), if a policymaker has the chance to consult

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multiple experts and the state space is large enough, there exist equilibria in which the policymaker always learns the true state. This observation was first made by Krishna and Morgan (2001a), while Battaglini (2002) gives necessary and sufficient conditions for the existence of such fully revealing equilibria. An important implication of these results is that with a large enough state space, under the best equilibrium for the sender, retaining the decision right and consulting multiple experts is superior to delegating the decision power to one of the informed agents. In contrast, as Dessein (2002) shows, the best outcome from communicating with one expert can be strictly worse than delegating the decision to the expert. In particular, if the expert's bias is small enough, delegation is optimal.[1]

This paper revisits the above questions by departing from the assumption that each sender observes the state exactly, and instead investigating a more realistic situation in which senders observe the state with a small amount of noise.[2] There are reasons to think that in such an environment, the qualitative conclusions from multi-sender cheap talk games would significantly change. In particular, some of the equilibrium constructions provided in the literature require the senders to exactly reveal the true state, and punish individual deviations by rendering an action that is bad for both senders in the case of nonmatching reports. The latter is possible because after an out-of-equilibrium profile of messages by the experts, nothing restricts the beliefs of the receiver. Such constructions obviously break down if the experts observe the state with noise, however small, since nonmatching reports then occur along the equilibrium path. Indeed, Battaglini (2002) shows that for a particular type of noise structure, in a 1-dimensional state space, there cannot exist any fully revealing equilibrium with two senders biased in opposite directions. However, it is not investigated how close information revelation can get to full revelation for small biases.

Our main results show that if the state space is large enough (relative to the experts' biases), then there exist equilibria arbitrarily close to full revelation of the state as the noise in the senders' observations vanishes. We consider two common types of noise. First, we investigate the case of *replacement noise*, where each sender observes the true state with high probability, but observes the realization of a random variable independent of the state with low probability. In such a context, we show that with one round of simultaneous cheap talk, as the probabilities of observing the true state approach 1, there exist equilibria arbitrarily close to full revelation under weak conditions on the noise structure. Then, we investigate the case of *continuous noise*, when the senders' signals follow a distribution with bounded density. We focus on the case where the conditional supports of the signals are small, around

[1] In the political science literature, Gilligan and Krehbiel (1987) establishes a similar point in the context of legislative decision-making.

[2] Relatedly, Krehbiel (2001) addresses the issue of empirical plausibility of equilibria in a multi-sender cheap talk context.

the true state. Constructing equilibria with fine information revelation is more involved in this environment, as it is a 0-probability event that the senders' observations coincide. Nevertheless, for a broad class of continuous noise structures, we show that two rounds of simultaneous public communication facilitate equilibria arbitrarily close to full revelation.[3]

The starting point for our basic construction is an equilibrium in the noiseless limit game where, for any message m_1 sent by sender 1 and any message m_2 sent by sender 2 along the equilibrium path, there is a set of states *with positive measure* where the prescribed message profile is (m_1, m_2). Put simply, any combination of messages that are used in equilibrium are on the equilibrium path, as in the multi-dimensional construction of Battaglini (2002), and the receiver's action following any of these message profiles can be determined by Bayes' rule. A new feature of our construction is that any pair of equilibrium messages is sent with strictly positive probability. This feature makes the equilibrium robust to introducing a small amount of noise.

The above type of construction constitutes an equilibrium whenever the set of states where (m_1', m_2) or (m_1, m_2') is prescribed (for $m_1' \neq m_1$ and $m_2' \neq m_2$) is far away, relative to the senders' biases, from the set of states where (m_1, m_2) is prescribed. We show that even when the set of states corresponding to each message pair is small, the above condition can be satisfied when the state space is sufficiently large. Thus, the action taken by the receiver can be made arbitrarily close to the true state with *ex ante* probability arbitrarily close to 1, if the state space is large enough. In such equilibria, the receiver's expected utility is arbitrarily close to her expected utility in the case of full revelation of the state.

When the state space is a large bounded interval of the real line, we propose the following construction. The state space is partitioned into n^2 intervals ("cells") with equal size, for some large n corresponding to the number of each sender's equilibrium messages. The $n \times n$ different combinations of the equilibrium messages are then assigned such that any two cells in which a sender sends the same message are far from each other. Essentially, cells are labeled with 2-digit numbers in a base-n number system in a particular way, and one sender is supposed to report the first digit of the interval from which she received a signal, while the other sender is supposed to report the second digit of the interval from which she received a signal. The construction relies on the fact that if the state space is large, then n can be taken to be such that $\frac{1}{n^2}$ times the length of the state space (the size of the cells) is small, but $\frac{1}{n}$ times the length of the state space (which is roughly the distance between cells in which a sender sends the same message) is large.

[3]Investigating robust equilibria in a multi-sender cheap talk game with continuous noise and one round of simultaneous cheap talk is outside the scope of the current paper. In a previous version of this paper, we showed that the same construction we provide for replacement noise structures remains robust for some special continuous noise structures. See Lu (2011) for some results with more general noise structures.

In the case of continous noise, the above construction breaks down, as even for very small noise, in states near a boundary between two cells, senders can be very uncertain about the cell where the other sender's signal lies. Senders getting signals right around cell boundary points may therefore deviate from prescribed play, which can cause the prescribed strategy profile to unravel far from the boundary points. To resolve this issue, we propose a construction with two rounds of cheap talk. In the second round, players play a strategy profile very similar to the basic construction above, involving combinations of messages from the senders identifying cells in a partition of the state space. The main complications are that the sizes of the partition cells vary in a specific way that depends on the noise structure, and that instead of one fixed partition, there is a continuum of partitions that can be played along the equilibrium path. The partition used in the second round is announced in the first round by sender 1. In particular, for every signal s_1 that player 1 can receive, there is exactly one equilibrium partition with a cell exactly consisting of the support of sender 2's signal s_2 conditional on s_1. We show that for large state spaces, we can take the loss from a coordination failure high enough so that sender 1 chooses to announce this partition in order to avoid coordination failure for sure.

In the latter construction, even though, initially, no small subset of the state space is common p-belief between the senders for positive p, after the first-round communication, it becomes a common 1-belief between the senders that both of their signals and the true state are in a small interval. This aspect of our construction is potentially relevant in games outside the sender-receiver framework. In particular, the infection arguments used in global games rely on the nonexistence of nontrivial events that are common p-belief among players, for p close enough to 1, as described for example in Morris *et al.* (1995).[4] We show that strategic communication can create such events, albeit in a different type of game: in global games with cheap talk, the communication stage is followed by actions from the senders of messages, while in our game they are followed by an action from a third party. As far as we know, existing work on global games with pre-play cheap talk (Baliga and Morris (2002), Baliga and Sjostrom (2004), Acharya and Ramsay (2013); see also p71 in the survey paper Morris and Shin (2003)) only consider one round of communication. Our analysis suggests the potential importance of considering multiple rounds of communication in related games.[5]

In all of our equilibrium constructions, the receiver solicits different pieces of information

[4]The concept of common p-belief for $p < 1$ was introduced by Monderer and Samet (1989).

[5]In cheap talk games without noise, it has been shown that adding rounds of communication can improve information transmission. Krishna and Morgan (2001b) make this point in two-sender games where messages are sequential, while Krishna and Morgan (2004) study a one-sender setting where the receiver can also send messages. See also *Golosov et al.* (2011), for a sender-receiver game in which there are multiple rounds of action choices, besides communication.

from the senders. Alone, each message is of very limited use, but their combination reveals the state with high precision.[6] This resembles the policymaker soliciting information along different dimensions from different experts. However, the "dimensions" in our constructions are artificial and do not correspond to natural dimensions of the state space. For this reason, we view our contribution as more normatively relevant, for situations where the policymaker can propose a mechanism, but cannot commit to action choices (so that the latter have to be sequentially rational). We note that for a fixed bounded state space, our constructions do not necessarily yield the best robust equilibrium for the receiver. However, if the state space is large enough relative to the biases, they provide a recipe to construct robust equilibria close to full revelation of information, irrespective of the fine details of the game (prior distribution of the noise, preferences of the senders), for a remarkably large class of games. In particular, the constructions allow for state-dependent biases for the senders. Moreover, we do not require either the single-crossing condition on the senders' preferences that is usually assumed in the literature, or the assumption that the *sign* of a sender's bias remains constant over the state space. Thus, our results hold for games outside the Crawford and Sobel framework.

The closest papers to ours in the literature are Battaglini (2002), Battaglini (2004) and Eső and Fong (2008).[7] Battaglini (2002) shows the nonexistence of fully revealing equilibria robust to replacement noise in one-dimensional state spaces, but does not address the question of how close robust equilibria can get to full revelation. We show that for large state spaces, there are such equilibria arbitrarily close to full revelation.[8] Battaglini (2004) shows the existence of a fully revealing equilibrium that is robust to a specific continuous noise, if the state space is a multi-dimensional Euclidean space and the prior distribution is diffuse. This result requires restrictive assumptions, and in particular does not extend to situations where the prior distribution is proper. Eső and Fong (2008) analyze a continuous-time dynamic multi-sender game with discounting and construct a fully revealing equilibrium that is robust to replacement noise under certain assumptions, including that the receiver is more patient than the senders. Less related to our work are papers that investigate multi-sender cheap talk games in which the senders observe the state with substantial noise, such as

[6] Ivanov (2012) proposes an optimal mechanism with similar features in a communication game with one sender, in which the receiver can endogenously determine the type of information the sender can learn, and there can be multiple rounds of learning and communication.

[7] Also related to this literature is Ambrus and Takahashi (2008), showing the nonexistence of fully revealing equilibria that satisfy a robustness criterion (diagonal continuity), indirectly motivated by noisy state observations, for compact state spaces. Lai et al. (2011) and Vespa and Wilson (2012) are recent experimental contributions on multi-sender cheap talk games.

[8] To reconcile these results, note that a finer and finer interval partition of the real line segment, such as the one in our construction, does not have a well-defined limit as the sizes of the intervals in the partition go to zero.

Austen-Smith (1990a, 1990b, 1993), Wolinsky (2002) and Ottaviani and Sørensen (2006).

The rest of the paper is organized as follows. In Section 2, we introduce the model and some terminology. In Section 3, we establish our main results for one-dimensional state spaces (both bounded and unbounded) in games with replacement noise. In Section 4, we examine the case of continuous noise. Finally, in Section 5, we discuss extensions of the model. In particular, we describe how some of our results extend to multidimensional state spaces, to discrete state spaces, to models in which noise is also introduced at other points of the game, and to situations in which the receiver has commitment power.

2 The model

The model features two senders, labeled 1 and 2, and one receiver. The game starts with sender 1 observing signal s_1 and sender 2 observing signal s_2 of a random variable $\theta \in \Theta$, which we call the *state*. We refer to Θ as the *state space*, and assume that it is a closed and connected subset (not necessarily proper) of $\mathbb{R}^d$.[9] In Sections 3 and 4, we assume $d = 1$, while in Section 5, we discuss the case where $d > 1$. The prior distribution of θ is given by F, which we assume exhibits a density function f that is strictly positive and continuous on Θ.[10]

We will consider both games in which the senders observe the state perfectly (*noiseless limit games*, where $s_1 = s_2 = \theta$), and games in which senders observe the state with small noise, for two types of noise structures: *replacement noise* (Section 3), where each sender observes the true state with high probability, and *bounded continuous noise* (Section 4), where each sender observes a signal that follows a continuous distribution around the state.

After observing their signals, the senders simultaneously send public messages $m_1 \in M_1$ and $m_2 \in M_2$. We assume that M_1 and M_2 are Borel sets having the cardinality of the continuum. In the baseline game, after observing the above messages, the receiver chooses an action $y \in \mathbb{R}^d$, and the game ends. In Section 4, we consider an extended version of this game, with an additional round of cheap talk. Formally, after the senders send public messages m_1 and m_2, they send another pair of messages $m_1' \in M_1$ and $m_2' \in M_2$. After observing the sequence of message pairs (m_1, m_2), (m_1', m_2'), the receiver chooses an action and the game ends.

We assume that the receiver's utility function $v(\theta, y)$ is continuous, strictly concave in y, and that $v(\theta, \cdot)$ attains its maximum value of 0 at $y = \theta$. We also assume that sender i's

[9] Closedness is assumed for notational convenience only. None of the results depend on this assumption.

[10] Although we assume a proper prior distribution throughout, our results from Section 3 readily extend to the case where the state space is an unrestricted Euclidean space and the prior is diffuse, as in Battaglini (2004).

utility function $u_i(\theta, y)$ is continuous, that it is strictly concave in y, and that $u_i(\theta, \cdot)$ attains its maximum value of 0 at $y = \theta + b_i(\theta)$. We refer to $\theta + b_i(\theta)$ as sender i's ideal point at state θ, and to $b_i(\theta)$ as sender i's bias at state θ. Note that neither the signals or the messages directly enter the players' utility functions.

We also maintain the following two assumptions throughout the paper.

A1: For every $y \in \mathbb{R}^d$, $\int_{\mathbb{R}^d} f(\theta)v(\theta, y)d\theta$ is finite.

A2: For any $\delta \geq 0$ and $\theta \in \Theta$, there exists $K(\delta) > 0$ such that $u_i(\theta, a') < u_i(\theta, a)$ whenever $|a - \theta| \leq \delta$ and $|a' - \theta| \geq K(\delta)$, $\forall\ i = 1, 2$.

A1 requires that the expected utility of the receiver from choosing any action is well-defined under the prior. A2 posits that neither sender becomes infinitely more sensitive to the chosen action being in some directions from the true state than in other directions. In the case of symmetric loss functions around ideal points, which is assumed in most of the literature, A2 is equivalent to requiring that there is a universal bound on the magnitude of senders' biases. The assumption automatically holds in the case of state-independent biases assumed, for example, in Battaglini (2002, 2004).

3 Replacement noise

Throughout this section, we assume that the senders observe the state with replacement noise, defined as follows.

Definition: In a *game with replacement noise*, there is a random variable $\tau \in \Theta$ independent of θ and distributed according to cdf G with a continuous density function g strictly positive on Θ. Then, conditional on any $\theta \in \Theta$, $s_i = \begin{cases} \theta \text{ with probability } p \\ \tau \text{ with probability } 1-p \end{cases}$ for $i \in \{1, 2\}$, for some $p \in (0, 1)$.

The solution concept we use is weak perfect Bayesian equilibrium, defined in the context of our model as follows.

For the baseline game with one communication round, let $h(\theta, s_1, s_2)$ be the joint density function of the state and the sender's signals, and for $i \in \{1, 2\}$, let $h_i^{s_i}$ be the marginal density of (θ, s_{-i}) conditional on s_i. An *action rule* of the receiver is a function $y : M_1 \times M_2 \to \mathbb{R}^d$, and a *belief rule* of the receiver is a function $\mu : M_1 \times M_2 \to \Delta(\Theta)$. For every $i \in \{1, 2\}$, sender i's *signaling strategy* is a function $m_i : \Theta \to M_i$.[11]

[11] Since we only construct pure strategy equilibria, we do not formally introduce mixed strategies here.

Definition: Action rule $\widehat{y}$, belief rule $\widehat{\mu}$, and signaling strategies $\widehat{m}_i$ ($i \in \{1,2\}$) constitute a pure strategy weak perfect Bayesian Nash equilibrium if:

(1) $\quad \forall\, i \in \{1,2\}$ and $s_i \in \Theta$, $\widehat{m}_i(s_i)$ solves $\max_{m_i \in M_i} \int_{(\theta, s_{-i}) \in \Theta^2} u_i(\theta, y(m_i, \widehat{m}_{-i}(s_{-i})))h_i^{s_i}(\theta, s_{-i})d\theta ds_{-i}$,

(2) $\quad \forall\, (m_1, m_2) \in M_1 \times M_2$, $\widehat{y}(m_1, m_2)$ solves $\max_{y \in \mathbb{R}} \int_{\theta \in \Theta} v(\theta, y)\mu(m_1, m_2)d\theta$,

(3) $\quad \widehat{\mu}(m_1, m_2)$ is obtained from $\widehat{m}_1(\cdot)$ and $\widehat{m}_2(\cdot)$ by Bayes' rule, whenever possible.

We use this weak notion of perfect Bayesian Nash equilibrium mainly because there is no universally accepted definition of perfect Bayesian Nash equilibrium with continuous action spaces. We note that in the equilibrium below, there are no out of equilibrium message pairs, and Bayes' rule pins down the receiver's beliefs after any possible message pair. Such equilibria satisfy the requirements of any reasonable definition of perfect Bayesian equilibrium.

We henceforth refer to weak perfect Bayesian Nash equilibrium simply as equilibrium.

3.1 Large bounded state space

First, we consider the case where $\Theta = [-T, T]$ for some $T \in \mathbb{R}_{++}$, and show that for every $\varepsilon, \delta > 0$, if T is large enough and the noise parameter is low enough, then there exists an equilibrium of the cheap talk game in which, at every state, the probability that the distance between the induced action and the state is smaller than δ is at least $1 - \varepsilon$.

To establish this result, we consider the following signaling profile for the senders. For any $T \geq K(\delta)$, let $n_{\delta,T}$ be the largest integer such that $\frac{T}{n_{\delta,T}} \geq K(\delta)$. Partition Θ to $n_{\delta,T}$ equal intervals, to which we will refer as *blocks*. Note that the size of each block is $\frac{2T}{n_{\delta,T}}$, which is by construction between $2K(\delta)$ and $4K(\delta)$. Next, further partition each block into $n_{\delta,T}$ equal subintervals, to which we will refer as *cells*. We will use $I_{j,k(i,j)}$ to denote the jth cell in the ith block, where $k(i,j) = \left\{\begin{smallmatrix} i+j-1 \text{ if } i+j-1 \leq n \\ i+j-1-n \text{ if } i+j-1 > n \end{smallmatrix}\right.$. Thus, block i is partitioned into the following $n_{\delta,T}$ cells: $\{(1,i), (2,i+1), ..., (n_{\delta,T} - i + 1, n_{\delta,T}), (n_{\delta,T} - i + 2, 1), ..., (n_{\delta,T}, i-1)\}$, and there is a total of $n_{\delta,T}^2$ cells. For completeness, assume that the cells in the partition are closed on the left and open on the right, with the exception of cell $(n_{\delta,T}, n_{\delta,T})$, which is closed at both ends. Define signaling profile $(m_1^{\delta,T}, m_2^{\delta,T})$ such that for every $j, k \in \{1, ..., n_T\}$, after receiving signal $s_1 \in I_{j,k}$, sender 1 sends message m_1^j, and after receiving signal $s_2 \in I_{j,k}$, sender 2 sends message m_2^k. Figure 1 below illustrates this signaling profile.

Let $y^{\delta,T}$ be an action rule that maximizes the receiver's expected payoff given $(m_1^{\delta,T}, m_2^{\delta,T})$. Note that $y(m_1^j, m_2^k)$ is uniquely defined for $j, k \in \{1, ..., n_{\delta,T}\}$ for any noise structure we consider, since the conditional beliefs of the receiver after receiving such message pairs are given by Bayes' rule, and the receiver's utility function is strictly concave. As for out-of-equilibrium messages $m_i \neq m_i^j$ for all $j \in \{1, ..., n_{\delta,T}\}$, assume that the receiver interprets each as having the same meaning as some message sent in equilibrium. No sender will then have an incentive to deviate to an out-of-equilibrium message.

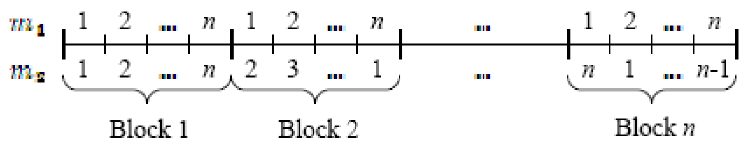

Figure 1: Signaling profile for large bounded intervals

Proposition 1: For every $\delta > 0$, there exists $T(\delta) > 0$ such that if $T > T(\delta)$, then strategy profile $(m_1^{\delta,T}, m_2^{\delta,T}, y^{\delta,T})$ constitutes an equilibrium in the noiseless limit game, and for every $\theta \in \Theta$, we have $|y^{\delta,T}(m_1^{\delta,T}(\theta), m_2^{\delta,T}(\theta)) - \theta| < \delta$.

Proof: By construction, the receiver plays a best response in the proposed profile, so we only need to check the optimality of the senders' strategies.

Note that $n_{\delta,T} \to \infty$ as $T \to \infty$. Since, by construction, $\frac{2T}{n_{\delta,T}} \leq 4K(\delta)$ for any $T \geq K(\delta)$, the above implies that the cell size, $\frac{2T}{n_{\delta,T}^2}$, goes to 0 as $T \to \infty$. Note that for every $j, k \in \{1, ..., n_{\delta,T}\}$ and $\theta \in I_{j,k}$, the assumptions on v imply that if both senders play according to the prescribed profile, then the action induced at θ lies within $I_{j,k}$.

Also by construction, if the other sender plays the prescribed strategy, all other actions that a sender could induce by sending a different message than prescribed are more than $\frac{n_{\delta,T}-2}{n_{\delta,T}}$ times the block size away. The latter is by construction at least $2K(\delta)$, so if $n_{\delta,T} > 4$, those actions are more than $K(\delta)$ away.

The above imply that there exists $T(\delta) > 0$ such that $|y^{\delta,T}(m_1^{\delta,T}(\theta), m_2^{\delta,T}(\theta)) - \theta| < \delta$ if $T > T(\delta)$, and any deviation by a sender, given strategy profile $(m_1^{\delta,T}, m_2^{\delta,T}, y^{\delta,T})$, would induce an action y by the receiver such that $|u - \theta| > K(\delta)$. By the definition of $K(\delta)$, this implies that there is no profitable deviation by either sender. ■

Intuitively, the proposed construction is an equilibrium because the cell associated with message pair (m_1^j, m_2^k) for any $j, k \in \{1, ..., n_{\delta,T}\}$ is far away from any cell in which the prescribed message pair is either (m_1, m_2^k) with $m_1 \neq m_1^j$, or (m_1^j, m_2) with $m_2 \neq m_2^k$. This holds for large T even given a small cell size, which ensures that the distance between states and induced actions is small.

Next, we show that if noise parameter $1-p$ is small enough, then profile $(m_1^{\delta,T}, m_2^{\delta,T}, y^{\delta,T})$ remains an equilibrium in a game with replacement noise.

Proposition 2: Suppose $\delta > 0$ and $T > T(\delta)$. Then for any noise distribution G, there exists $\underline{p}(G) < 1$ such that $p > \underline{p}(G)$ implies that in a game with replacement noise structure (G, p), strategy profile $(m_1^{\delta,T}, m_2^{\delta,T}, y^{\delta,T})$ constitutes an equilibrium.

Proof: Note that since both f and g are continuous and strictly positive on the compact Θ, as $p \to 1$, given signaling strategies $(\mu_1^{\delta,T}, \mu_2^{\delta,T})$, the conditional distribution of θ given message pair (m_1^j, m_2^k) in the game with replacement noise converges weakly to the conditional distribution of θ given message pair (m_1^j, m_2^k) in the noiseless limit game, for every $j, k \in \{1, ..., n_{\delta,T}\}$. Then since the expected payoff of the receiver resulting from choosing some action y after message pair (m_1^j, m_2^k) is continuous with respect to the weak topology in the conditional distribution of θ given (m_1^j, m_2^k), the theorem of the maximum implies that $y^{\delta,T}$ is continuous in p, even at $p = 1$. This implies that the expected payoff of sender i resulting from sending message m_i^l after receiving signal s_i is continuous in p, for every $i \in \{1, 2\}$, $l \in \{1, ..., n_{\delta,T}\}$ and $s_i \in \Theta$, even at $p = 1$. Moreover, in the noiseless limit game, after signals $s_1, s_2 \in I_{j,k}$, sending message m_1^j yields a strictly higher expected payoff for sender 1 than m_1^l for $l \neq j$, and sending message m_2^k yields a strictly higher expected payoff for sender 2 than m_2^l for $l \neq k$. Thus, the same holds for p close enough to 1. This establishes the claim. ■

The intuition behind Proposition 2 is that the receiver's optimal action rule given $(m_1^{\delta,T}, m_2^{\delta,T})$ is continuous in p, even at $p = 1$. Therefore, the expected payoff of a sender when sending different messages after a certain signal changes continuously in p as well. Since in the noiseless limit game, a sender strictly prefers to send the prescribed message to sending any other equilibrium message, the same holds for noisy games with p high enough.

Note that the above propositions imply that for any $\delta > 0$, if the state space is large enough, then there is an equilibrium of the noiseless limit game, where the action induced at any state is at most δ away from the state, that is robust to replacement noise in a strong sense: it can be obtained as a limit of equilibria of games with vanishing replacement noise, for any noise distribution G.

The propositions also imply the following result.

Corollary 1: Fix payoff functions $v(.,.)$ and $u_i(.,.)$, $i = 1, 2$, defined over $\mathbb{R}^2$ satisfying A1. Take any sequence of games with bounded interval state spaces $[-T_1, T_1]$,$[-T_2, T_2]$,... , state distributions $F_1, F_2, ..$, noise distributions G_1,G_2,... and payoff functions $(v^1, u_1^1, u_2^1), (v^2, u_1^2, u_2^2), ...$ such that v^j, u_1^j and u_2^j are restrictions of v, u_1 and u_2 to $[-T_j, T_j] \times \mathbb{R}$. If $T_i \to \infty$ as $i \to \infty$, then there exists a sequence of noise levels $p_1, p_2, ...$ with $p_i < 1$ for every $i \in \mathbb{Z}_{++}$ and $p_i \to 1$ as $i \to \infty$, such that there is a sequence of equilibria of the above games with equilibrium outcomes converging to full revelation in $\mathbb{R}$.

Corollary 1 contrasts with Proposition 2 in Battaglini (2002), which establishes that if the senders' biases are above some threshold, then there does not exist a fully revealing equilibrium robust to replacement noise in a one-dimensional state space, no matter how large the state space.[12] To reconcile these results, it is useful to observe that although the sequence of outcomes induced by the sequence of equilibria from Corollary 1 converges to full revelation of the state in $\mathbb{R}$, such sequences of equilibrium strategy profiles do not have a well-defined limit in the noiseless limit game with state space $\mathbb{R}$. This is because the limit of a sequence of interval partitions where the sizes of the intervals converge to zero is not well-defined.

3.2 Unbounded state space

In this subsection, we analyze the case where $\Theta = \mathbb{R}$. We show that the equilibrium construction introduced in the previous subsection can be extended to this case when the prior distribution of states has thin enough tails.

The state space is still partitioned into n^2 cells, and combinations of the n equilibrium messages are allocated to different cells in the same order as before. The difference is that in the case of an unrestricted state space, only the middle $n^2 - 2$ cells can be taken to be small; the extreme cells are infinitely large. Hence, in the equilibria we construct, even with no noise, the implemented action will be far away from the state with nontrivial probability in states in the extreme cells. But if the profile is constructed such that the middle $n^2 - 2$ cells cover interval $[-T, T]$ for large enough T, then for small noise, the *ex ante* probability that the induced action is within a small neighborhood of the realized state can be made close to 1.

The extra assumption needed for this construction guarantees that for large enough block size, even in the extreme cells, the senders prefer inducing the action corresponding to the cell instead of deviating and inducing an action in a different block.

Let $y_{\theta,d,L}$ (respectively, $y_{\theta,d,R}$) be the optimal action for the receiver when her belief about the true state follows density d truncated so the true state is in $(-\infty, \theta]$ (respectively, $[\theta, \infty)$).

A3: There exist $C, Z > 0$ such that $|y_{\theta,f,L} - \theta| < Z$ for all $\theta < -C$, and $|y_{\theta,f,R} - \theta| < Z$ for all $\theta > C$.

A3 requires the tail of the prior distribution to be thin enough: it is satisfied if f converges

[12] Battaglini's result is stated in an environment where each player's loss depends only on $(a - b_i(\theta))^2$, and does so in the same way in every state. However, its proof extends to our more general setting if for every θ, each sender's bias is large enough (in the sense that if $u_i(\theta, \theta) = u_i(\theta, a)$ and $a \neq \theta$, then $|a - \theta|$ must be large), and if the extent to which the receiver's payoff's sensibility to $|a - \theta|$ varies across states is bounded.

quickly enough to 0 at $-\infty$ and ∞, relative to how fast the loss functions at moderate states diverge to infinity as the action goes to $-\infty$ or ∞. For example, if v exhibits quadratic loss invariant in θ, a sufficient condition for A3 is that $\lim_{x\to-\infty}\frac{F(x)}{f(x)}$ and $\lim_{x\to\infty}\frac{1-F(x)}{f(x)}$ exist. This is clearly true if f converges to 0 exponentially fast, so for v quadratic, A3 holds for exponential distributions or for any distribution that converges to 0 faster, such as the normal.

Proposition 3: Suppose $\frac{f(\theta)}{g(\theta)} \geq b > 0$, for every $\theta \in \Theta$. If $\Theta = \mathbb{R}$ and A3 holds, then for every $\delta, \eta > 0$, there exists $\underline{p} < 1$ such that, in a noisy game with $p > \underline{p}$, $|y - \theta| < \delta$ with *ex ante* probability at least $1 - \eta$.

Proof: Consider the following strategy profile in the noiseless limit game.

Let T be such that $F(T) - F(-T) = 1 - \frac{\eta}{2}$. As in Subsection 3.1, partition $\mathbb{R}$ into n blocks. Blocks 2 through $n-1$ are equally sized and large enough so that each is bigger than $K(\delta) + 2\delta$, and they together cover $[-T, T] \cup [-C, C]$, where C is the corresponding constant in A3.

For each $k \in \{1, ..., n\}$, we will further partition block k into n cells, labeled as in Subsection 3.1. Block 1 minus the leftmost cell and block n minus the rightmost cell are each bigger than $\max\{K(Z), K(\delta) + \delta\}$, where Z is the corresponding constant in A3. We choose n large enough so that each of the middle $n^2 - 2$ cells, which are of equal size, is smaller than δ. For the sake of completeness, let each of the middle $n^2 - 2$ cells be closed on the left and open on the right.

Label the cells as in Subsection 3.1, and consider the following strategy profile:

- when s_1 falls in cell (j, k), sender 1 sends message m_1^j;

- when s_2 falls in cell (j, k), sender 2 sends message m_2^k;

- $y(m_1^j, m_2^k)$ is an optimal response to m_1^j, m_2^k given the above strategies, for every $j, k \in \{1, ..., n\}$;

- the receiver associates any out-of-equilibrium message to a message sent by that player in equilibrium, and after any other message pair, the receiver chooses the corresponding $y(m_1^j, m_2^k)$ for some $j, k \in \{1, ..., n\}$.

This profile constitutes an equilibrium in the noiseless limit game, which has the property that $|y - \theta| < \delta$ with *ex ante* probability at least $1 - \frac{\eta}{2}$. This is because message pairs are allocated to cells in a way that at any state, any action that a sender could induce other than the prescribed one is strictly worse for the sender than the prescribed action.

Analogous arguments as the ones used in the proof of Proposition 2 establish that for large enough p, the above profile still constitutes an equilibrium. The assumption that $\frac{f(\theta)}{g(\theta)} \geq b > 0$ guarantees that for p large, senders believe with high probability that they have observed the correct state. Moreover, it is easy to see that for large enough p, conditional on the state being in the middle $n^2 - 2$ cells, the probability that $|y - \theta| < \delta$ is at least $1 - \frac{\eta}{2}$. Then, the *ex ante* probability of $|y - \theta| < \delta$ is at least $1 - \eta$, concluding the proof. ■

Proposition 3 implies that, if A3 holds, then in a game where the state space is the real line, for any $\delta > 0$, there exists an equilibrium robust to small replacement noise in which the distance between any state and the action induced in that state is less than δ with high *ex ante* probability. The thinness of the tail of the distribution then also implies that the *ex ante* expected payoff of the receiver in equilibrium can closely approximate the maximum possible payoff value 0, obtained in a truthful equilibrium.

4 Bounded continuous noise

Our construction from Section 3 requires that, regardless of their signal s_i, each sender believes that the other sender's signal s_j lies in the same cell as s_i with high probability. This occurs with replacement noise due to the high probability that both senders observe the state exactly. However, if signals follow a continuous distribution around the state and are not perfectly correlated, then the probability that they coincide is 0. As a result, when s_i is sufficiently near the boundary between two cells, the probability that s_j lies on the other side of the boundary is non-negligible. In that case, the senders may have an incentive to second-guess their signal in order to reduce the probability of a coordination failure (which we call *miscoordination*) that would result in the action being in a different block, or in order to make miscoordination less costly by changing the action that follows it. But doing so in states near the boundary can trigger a departure from the originally prescribed strategy profile in other states and lead to an unraveling of the equilibrium construction from Section 3.

This section shows that for a broad class of continuous signal distributions with bounded support, with two rounds of communication by the senders and large enough state space, there exists an equilibrium where one of the senders' signals is *perfectly* revealed. Therefore, as the noise in the senders' information goes to zero, the proposed equilibrium converges to full revelation of the state. We do not know how close equilibria with one round of cheap talk can get to full revelation in games with such noise.[13] Naturally, our result holds for situations with more than two rounds of simultaneous cheap talk, as in such games, senders can simply babble before the last two rounds of messages, and then play according to the construction proposed below.

Formally, we consider a multi-stage game. At stage 0, state θ realizes, and senders 1

[13] For an investigation along these lines, see Lu (2011).

and 2 receive noisy signals s_1 and s_2 of θ (as described in more detail below). At stage 1, the senders simultaneously send messages m_1 and m_2, which are public (*i.e.* observed by all players before the next stage). After observing these messages, in stage 2, the senders simultaneously send public messages m_1' and m_2'. Lastly, in stage 3, the receiver chooses an action.

Strategies in the game with an extra round of communication are defined as follows. The action rule of the receiver becomes $y : (M_1 \times M_2)^2 \rightarrow \mathbb{R}$, and the belief rule is now $\mu : (M_1 \times M_2)^2 \rightarrow \Delta(\Theta)$. For $i \in \{1, 2\}$, sender i's signaling strategy is a pair of functions $m_i : S_i \rightarrow \Delta(M_i)$ and $m_i' : S_i \times M_1 \times M_2 \rightarrow M_i$.[14] Furthermore, at the second round of cheap talk, sender i has beliefs $\mu_i : S_i \times M_1 \times M_2 \rightarrow \Delta(\Theta \times S_{-i})$ about the state and about sender $-i$'s signal. We use weak perfect Bayesian Nash equilibrium, defined analogously as in Section 3, as our solution concept.

4.1 Unbounded state space

For continuous noise, it is more convenient to start with the case where $\Theta = \mathbb{R}$: the construction we provide is simpler than in the case of bounded state spaces because out-of-equilibrium message pairs can be avoided, like in the case of replacement noise. We will discuss the case where Θ is a large bounded interval of $\mathbb{R}$ in the next subsection.

We impose the following assumptions regarding the distribution of signals: (i) The support of each sender's signal is an interval around the state, with a universal bound on the interval's size; (ii) The conditional distribution of sender 2's signal given any s_1 has a density function that is universally bounded, with a lower bound strictly positive; (iii) Both the lower endpoint and the higher endpoint of the support of the distribution of s_2 conditional on s_1 are increasing in s_1 at a rate bounded away from 0.

A4: There exists $\varepsilon > 0$ such that, conditional on the state θ, we have $s_i \in [\theta - \varepsilon, \theta + \varepsilon]$. Furthermore, for every s_1, the distribution of s_2 conditional on s_1 exhibits a density function such that $f(s_2|s_1) \in [\underline{\nu}, \overline{\nu}]$ everywhere on $S_2(s_1) \equiv [\underline{s_2}(s_1), \overline{s_2}(s_1)]$, for some $\underline{\nu} > 0$, and $\underline{s_2}(.)$ and $\overline{s_2}(.)$ are continuous and strictly increasing at a rate of at least $r > 0$.

An example of a class of noise that satisfies the above restrictions is the following: $s_1 = \theta + \omega_1$ and $s_2 = \theta + \omega_1 + \omega_2$, where $\omega_i \sim F_i$ independently, and F_1 and F_2 are distributions with finite interval supports and densities bounded away from 0. In particular this holds when s_2 is a mean-preserving spread of s_1 satisfying the above technical conditions.

[14] We only introduce mixed strategies for the senders' round 1 messages since that is the only place in our construction where players randomize.

Moreover, we impose the following technical assumption.

Let $a(s_1) = \arg\max_a \int v(\theta, a) f(\theta|s_1) d\theta$ be the receiver's optimal action conditional on s_1. Note that $a(s_1)$ is unique because $v(\theta, a)$ is strictly concave in a.

A5: $a(s_1)$ is strictly increasing and Lipschitz continuous in s_1, with Lipschitz constant Λ.

Lastly, we impose a weak restriction on the relation between a sender's state-conditional utility functions.

A6: For any $\varepsilon \geq 0$, there exists $L(\varepsilon) > 0$ such that for any $a, \theta \in \Theta$, and $a', \theta' \in [\theta - \varepsilon, \theta + \varepsilon]$, we have $u_i(\theta, a') - u_i(\theta, a) = u_i(\theta', a'') - u_i(\theta', a)$ for some $a'' \in [a - L(\varepsilon), a + L(\varepsilon)]$, $\forall$ $i = 1, 2$.

A6 bounds the extent to which senders care more about the outcome in one state of the world than the outcome in another state. Like A2, A6 is automatically satisfied if preferences depend only on $a - \theta$.

Our result for this subsection is stated in Proposition 4.

Proposition 4: Suppose A1, A2, A4, A5 and A6 are satisfied and $\Theta = \mathbb{R}$. Then in a game with two rounds of simultaneous public messages, there exists an equilibrium where s_1 is exactly revealed.

The proof of Proposition 4 can be found in the Appendix. Note that unlike in Proposition 3, the receiver learns the state with high precision (if ε is small) even for extreme values of θ. This is because Proposition 4 uses an infinite partition, as described below, while Proposition 3 uses a finite partition. Using an infinite partition with replacement noise would create the following problem: if a state θ is unlikely to occur (for example, θ is in a tail of the distribution), then a sender observing $s_i = \theta$ would believe that her signal is noise. This problem does not arise here because noise is bounded.

We now provide an outline of the equilibrium construction. In stage 1, sender 1 sends a message that selects a partition among a continuum of possible partitions, while sender 2 babbles. Each of these partitions has infinitely many cells, each designated by a message pair (m_1', m_2'), where $m_1' \in \{1, 2, ..., n\}$ and $m_2' \in \mathbb{Z}$. As in our construction from Section 3, a cell designated by message pair (m_1', m_2') is located far from any cell of the form (x, m_2') for $x \in \{1, 2, ..., n\} \backslash \{m_1'\}$ or of the form (m_1', x) for $x \in \mathbb{Z} \backslash \{m_2'\}$. The continuum of possible partitions is such that for any s_1, exactly one partition has a cell that contains $S_2(s_1)$. For this partition, sender 1 knows for sure which cell s_2 lies in. In stage 2, the senders play a continuation strategy profile analogous to the one from Section 3. The combination of stage 1 and stage 2 messages exactly reveals s_1 to the receiver, and does not reveal any additional information. Hence the receiver plays the optimal action conditional on sender 1's signal being s_1.

The partitions are constructed as follows. Each partition has a countable number of *sets*, indexed by integers k. Within each set k, there are n blocks with n cells each, where sender 1's message for each cell is the same as in Section 3's construction, while sender 2's message for each cell is kn plus the message from Section 3's construction. As before, given a partition, as long as the blocks are sufficiently large and the cell where signals are located is known, no deviation is profitable.

We build the collection of partitions for the first stage as follows. First, note that A4 ensures that for any $s_1, s_1' \in S_1$, $S_2(s_1) \nsubseteq S_2(s_1')$. This implies that one can build a set $\mathcal{P}$ of partitions such that, for every s_1, there is a unique partition within $\mathcal{P}$ where sender 1 puts probability 1 on s_2 lying in the same cell as s_1. Furthermore, for every s_1, there exist $s_1' < s_1$ and $s_1'' > s_1$ such that $\underline{s_2}(s_1) = \overline{s_2}(s_1')$ and $\overline{s_2}(s_1) = \underline{s_2}(s_1'')$. Therefore, it is possible to construct $\mathcal{P}$ such that, in every partition in $\mathcal{P}$, every cell can occur on the equilibrium path. Assumptions A4 (through $\underline{\nu}$ and r) and A5, together with the cost of miscoordination in our construction, ensure that sender 1 chooses the partition where the probability of miscoordination is 0. To see this, note that since both $\underline{s_2}(s_1)$ and $\overline{s_2}(s_1)$ are increasing at a rate bounded away from 0, and the density of $S_2(s_1)$ is bounded away from 0 on its support, any small deviation from the prescribed partition increases the probability of miscoordination by a rate bounded away from 0. Finally, the receiver's action is $a(s_1)$: she receives no information about s_2 other than the fact that it lies in $S_2(s_1)$.

We refer the reader to the proof of Proposition 4 in the Appendix for a complete description of the strategy profile and the demonstration that it is indeed an equilibrium.

4.2 Large bounded state space

As before, we define the *underlying* game (distribution of θ and s_i, preferences) over all of $\mathbb{R}$. In this subsection, we show that Proposition 4 continues to hold when the state space is truncated to $[-T, T]$, for sufficiently large T.

Proposition 5: Suppose A1, A2, A4, A5 and A6 are satisfied in the underlying game and in truncated games. Then there exists T^* such that whenever Θ is truncated to $[-T, T]$, where $T > T^*$, then in a game with two rounds of simultaneous public messages, there exists an equilibrium where s_1 is exactly revealed.

The proof of Proposition 5, which can be found in the Appendix, uses the same idea as the proof of Proposition 4, except now each possible partition has n^2 cells, like in our basic construction from Section 3. As in Section 3, T needs to be large relative to the senders' biases such that the partition's blocks can be made large enough to discourage deviations.

There are two main difficulties for this extension: i) since the size of the cells is determined by the noise structure, to make the number of cells in each partition square, we need to modify the construction from Section 4.1; ii) the cells at the ends of the partitions may now be out of equilibrium. We discuss these issues below.

To make the number of cells in each partition square, we divide the set of partitions $\mathcal{P}$ into two subsets, L and R. Each partition in L is constructed starting with a cell at the left end of $S_2 \subseteq [-T-\varepsilon, T+\varepsilon]$. This first cell is called a *small extreme* cell if its right endpoint is less than $\overline{s_2}(\min_{s_1 \in S_1} s_1)$, which implies that there is no s_1 for which this cell contains $S_2(s_1)$; otherwise, it is a *regular* cell. Each subsequent cell, except for the rightmost one, is $(\underline{s_2}(z), \overline{s_2}(z)]$ for some $z \in S_1$, where $\underline{s_2}(z)$ is the right boundary of the previous cell; these are all *regular* cells. Once regular cells and, if applicable, the small extreme cell together cover $\frac{3}{4}$ of S_2 and number n^2-1 for some integer n, we cease creating new cells, and the last cell, called the *large extreme* cell, covers the rest of S_2. By construction, there are n^2 cells in each partition. Partitions in R are constructed in a similar fashion, but starting at the right end of S_2. Notice that for every element of L, there is a corresponding element of R that is identical in the area where regular cells of L-partitions and regular cells of R-partitions overlap, and vice versa. A typical partition is illustrated in Figure 2. The top of the picture displays the partition in L prescribed for signal s_1, while the bottom of the picture displays the corresponding partition in R. Note that in both partitions, the cell that s_1 belongs to is exactly $S_2(s_1)$.

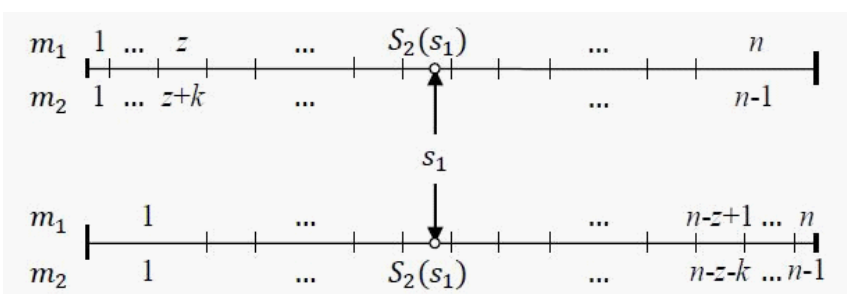

Figure 2: The L and R partitions prescribed for a typical signal of sender 1

In the proposed equilibrium, sender 1 chooses a partition where one of the cells is exactly $[\underline{s_2}(s_1), \overline{s_2}(s_1)]$. By construction, there is at least one such partition in $\mathcal{P}$, and at most one such partition in each of L and R. When a partition from L and a partition from R both satisfy the criterion, which occurs when s_1 is a regular cell of both partitions, sender 1 randomizes 50/50 between them; this is consistent with equilibrium since both partitions lead to the same action, $a(s_1)$.

The second difficulty relates to extreme cells being out of equilibrium: on the equilibrium path, the senders' round 2 messages (m_1', m_2') never point to an extreme cell. This is especially problematic when (m_1', m_2') corresponds to the large extreme cell due to its size. We specify the receiver's beliefs after such message pairs such that no profitable deviation is created.[15] Specifically, if $P \in L$, then the large extreme cell is $(n, n-1)$. In our equilibrium, the receiver's belief after seeing $P \in L$ and $(n, n-1)$ is the same as if (m_1', m_2') had instead been $(n, n-2)$, which corresponds to a cell a block away. As a result:

- When $m_1' = n$, the rightmost action that sender 2 can induce is in $(n, n-2)$ (by sending $m_2' = n-1$ or $n-2$). So when s_2 is in cell $(n, n-1)$, sending $m_2' = n-1$ is still optimal for sender 2.

- When $m_2' = n-1$, the rightmost action that sender 1 can induce is in $(n, n-2)$ (by sending $m_1' = n$). This is because, apart from $(n, n-1)$, which cannot be induced, the closest cell to $(n, n-2)$ where $m_2' = n-1$ is $(1, n-1)$, which is almost a block to the left of $(n, n-2)$.[16] Therefore, when s_1 is in cell $(n, n-1)$, sending $m_1' = n$ is still optimal for sender 1.

The case where $P \in R$ is analogous.

The proof of Proposition 5 in the Appendix contains a complete description of the strategy profile and establishes that it is an equilibrium.

5 Extensions and discussion

5.1 Multidimensional state spaces

Our basic construction can be readily extended to multidimensional state spaces for replacement noise if there are no restrictions on the state space (the state space is the whole Euclidean space). In particular, for any $\delta > 0$, instead of partitioning a high-probability portion of the state space into $n^2 - 2$ intervals of size δ, we partition in into $(n^2 - 2)^d$ d-dimensional hypercubes with edges of size δ. Now take n^d messages for each sender, and index them by $\{1, 2, ..., n\}^d$. Hence, a typical message for sender i is labeled as $m^i_{j_1,...,j_d}$ $(j_1, ..., j_d \in \{1, 2, ..., n\})$. The lth component j_l of each sender's message is determined by the l-coordinate of the cell where that sender's signal is located, in the same way as in

[15] Moreover, in round 2, sender 2 is aware that sender 1 has deviated in round 1 if s_2 is in an extreme cell of the announced partition. At such histories, in our equilibrium, sender 2's beliefs are such that m_1' is consistent with the cell where s_2 is located.

[16] By construction, $m_2' = n-1$ is skipped at the boundary between the last two blocks.

Subsection 3.2. Proceeding like this for all d dimensions results in a strategy profile of the senders such that each pair of possible messages is identified with a unique cell in the above partition. Moreover, the profile is constructed such that at every state, sending a different message than the one corresponding to the cell containing the state results in a message pair identified with a cell far away from the original state, whether the sender deviates in one or more dimensions from the prescribed message. Proving that this profile constitutes an equilibrium for small enough replacement noise is analogous to the proof of Proposition 3.

For bounded continuous noise, the construction for Proposition 4 can be extended similarly if the supports $S_2(.)$ are all orthotopes (hyperrectangles) with the same dimensions (for example, $s_i = \theta + \omega_i$, where $\omega_i \overset{i.i.d.}{\sim} F_i$, and F_1 and F_2 are distributions with finite orthotope supports and densities bounded away from 0).

For large bounded state spaces that are d-dimensional orthotopes, the above constructions can be extended in a straightforward manner. For different types of bounded state spaces in $\mathbb{R}^d$, our basic construction cannot be applied directly. However, the same qualitative insight still holds. Suppose the state space can be partitioned into n^2 cells with diameter at most δ, and that there is a bijection from $M \equiv \{m_1^1, ..., m_1^n\} \times \{m_2^1, ..., m_2^n\}$ to the cells in the partition such that for any $(m_1, m_2) \in M$ and for any $(m_1', m_2') \in M$ with either (i) $m_1 = m_1'$ and $m_2 \neq m_2'$, or (ii) $m_1 \neq m_1'$ and $m_2 = m_2'$, it holds that the distance between the partition cells associated with (m_1, m_2) and with (m_1', m_2') are at least $K(\delta)$ away from each other. Then there is an equilibrium of the noiseless limit game that is robust to a small amount of replacement noise.

5.2 Introducing noise at different stages of the game

We have been investigating equilibria robust to perturbations of a multi-sender cheap talk game in the observations of the senders. This is the type of perturbation most discussed in the literature. However, similar perturbations can be introduced at various other stages of the communication game: in the communication phase (the actual message received by the receiver is not always exactly the intended message by a sender) and in the action choice phase (the policy chosen by the receiver is not exactly the same as the intended policy choice).[17]

The equilibria we propose in this paper are robust with respect to the above perturbations as well. To see this for the case of small perturbations in the receiver's action choice, note

[17] For an analysis of noisy communication in one-sender cheap talk, see Blume et *al.* (2007). See also Chen et *al.* (2008) for a one-sender cheap talk game in which both the sender and the receiver are certain behavioral types with small probability, a model resembling one in which there is a small replacement noise in both the communication and the action choice stages.

that in the equilibria we construct, senders strictly prefer sending the prescribed message to any other equilibrium message (while out-of-equilibrium messages lead to the same intended actions as equilibrium messages). Hence both for replacement noise and in stage 2 of the continuous noise construction, if the noise in the action choice is small enough, senders still strictly prefer sending the prescribed message to sending any other message along the path of play. To ensure that there is no deviation in stage 1 of the continuous noise construction, one would need regularity conditions on the change in the distribution of the realized action when the intended action changes.

For noise in the communication phase, it is more standard to introduce replacement noise, as in Blume *et al.* (2007), since there typically is no natural metric defined on the message space (messages obtain their meanings endogenously, through the senders' strategies). A small modification of our equilibrium construction makes the profile robust with respect to such noise. In particular, take any equilibrium we constructed in Sections 3 and 4, partition each sender's message space into subsets, one for each equilibrium message, and associate each subset with a distinct equilibrium message. Then replace senders' strategies from the previous equilibrium with strategies where, after any signal, the sender selects an action randomly (according to a uniform distribution) from the subset of messages associated with the original equilibrium message. The resulting profile remains an equilibrium and induces exactly the same outcome. Moreover, this equilibrium is robust to a small amount of replacement noise, subject to regularity conditions guaranteeing that, when the above strategy profile is played, any message is much more likely to be intended than to be the result of noise.[18]

5.3 Commitment power

If the receiver can credibly commit to an action scheme as a function of messages received, then there exist constructions simpler than the ones we proposed that are robust to small amount of noise and achieve exact truthful revelation of the state. Here, we only discuss the case of replacement noise and one-dimensional state spaces. Mylovanov and Zapechelnyuk (2009) show that a necessary and sufficient condition for the existence of a fully revealing equilibrium in a noiseless two-sender cheap talk game with commitment power and bounded interval state space $[-T, T]$ is the existence of a lottery with support $\{-T, T\}$ with the property that at every $\theta \in [-T, T]$, both senders prefer action θ to the above lottery. The sufficiency of this condition is easy to see: the receiver can commit to an action scheme that

[18]For a bounded state space, this is the case if the distribution of the replacement noise has a bounded density function, and the probability of replacement noise is small enough.

triggers the above lottery in case of differing messages from the senders.

We observe that given the above action scheme, truthtelling by the senders remains an equilibrium for small enough replacement noise. This is because if the other sender follows a truthtelling strategy, then after receiving signal θ, sending any other message than θ induces the threat lottery with probability 1, while sending message θ induces θ with high probability. The latter outcome is, by construction, preferred by the sender if the state is likely to be θ. The above implies that in case of commitment power, there exists a fully revealing equilibrium robust to replacement noise, even if the state space is relatively small. For example, if senders have symmetric and convex loss functions and are biased in opposite directions, then there exists an equilibrium construction like the one above whenever biases are less than T in absolute value.

5.4 Discrete state spaces

The constructions in 3.2 and 3.3 extend in a straightforward manner to large discrete state spaces. Consider first the case when the state space is a coarse finite grid of a large bounded interval: $\Theta = \{\theta \in [-T, T] | \theta = -T + k \cdot \varepsilon\}$, where $T \in \mathbb{R}_+$ is large and $\varepsilon \in \mathbb{R}_+$ is small. Define $n_{2\varepsilon,T}$ as in Subsection 3.2, and partition $[-T, T]$ to $n^2_{2\varepsilon,T}$ equal-sized subintervals. By construction, each partition contains at least one state from Θ. Then it is easy to see that the strategy profile presented in 3.2 gives an equilibrium robust to a small amount of noise, in which the supremum of the absolute distance between any possible state and the action induced in that state is at most 2ε. This construction readily extends to other finite one-dimensional state spaces and implies almost full revelation of the state whenever the distance between the two extreme states is large enough, and the maximum distance between two neighboring states is small enough.

6 Appendix

6.1 Proof of Proposition 4

1. Constructing partitions for stage 1 messages by sender 1.

Note that by assumption A4a, $\overline{s_2}(s_1) - \underline{s_2}(s_1) \geq \frac{1}{\nu}$ for all s_1.

Let X be such that $\underline{s_2}(X) = \overline{s_2}(0)$.

Define partition $P(x)$, which consists of infinitely many cells labeled as described in the main text, as follows for all $x \in (0, X]$. The starting cell, designated by $(1,1)$, is $(\underline{s_2}(x), \overline{s_2}(x)]$. Both to the right and to the left, each other cell is $(\underline{s_2}(z), \overline{s_2}(z)]$, where $\underline{s_2}(z)$ is the boundary of the previous cell. Note that, in each case, z is well-defined because $\underline{s_2}(.)$ and $\overline{s_2}(.)$ are continuous and strictly increasing, and that this construction will cover all of $\mathbb{R}$ since the size of cells is bounded below by $\frac{1}{\nu}$. Note that since $\underline{s_2}(s_1)$ and $\overline{s_2}(s_1)$ are strictly increasing, there exists a unique $x \in (0, X]$ such that sender 1 puts probability 1 on s_1 and s_2 being in the same cell. Denote this quantity $x(s_1)$.

Let n (the number of cells per block) be large enough so that the following hold for all s, feasible s_2 given $s_1 = s$, and actions a located $n-3$ cells away from $a(s)$:

(i) $\min_{\theta \in [s-\varepsilon, s+\varepsilon]} \{u_1(\theta, a(s)) - u_1(\theta, a)\} > \frac{\Lambda}{\underline{\nu} r} \max_{\theta \in [s-\varepsilon, s+\varepsilon], a' \in [a(s-4\varepsilon), a(s+4\varepsilon)]} |\frac{\partial}{\partial \alpha} u_1(\theta, a')| \equiv \frac{\Lambda}{\underline{\nu} r} M(s)$, where $\frac{\partial}{\partial \alpha}$ denotes the partial with respect to the receiver's action; and

(ii) $E[u_2(\theta, a(s))|s_1 = s, s_2] - E[u_2(\theta, a)|s_1 = s, s_2] > 0$.

Condition (ii) simply ensures that in stage 2, after sender 2 has learned $s_1 = s$, sender 2 has no incentive to deviate since inducing a cell almost a block away (as happens when she unilaterally deviates given a partition) is not profitable. Condition (i) does the same for sender 1 and - as shown in the last part of the proof - provides incentives for sender 1 to announce a partition where no miscoordination is possible. Without the min and max functions in condition (i), the existence of n would be guaranteed by A2 and the concavity of u_1 with respect to the action. A6 links sender 1's preferences at any $\theta \in [s - \varepsilon, s + \varepsilon]$ to her preferences at the state where $|\frac{\partial}{\partial \alpha} u_1(\theta, a')|$ is maximized, which allows us to take the min and max functions in condition (i).

2. The strategy profile

- Stage 1: Sender 1 announces partition $P(x(s_1))$ from the set $\{P(x)\}_{x \in (0,X]}$. For the remainder of this proof, we will let P be the announced partition. Sender 2 babbles.

- Stage 2: We distinguish two cases:

a) On the equilibrium path for sender 1, and always for sender 2: The senders send m_i' corresponding to the cell of P where s_2 lies, which, by construction, is known to sender

1. Beliefs μ_i are determined according to Bayes' rule. Note that (m_1', m_2') reveals s_1 when combined with sender 1's announcement in stage 1.

b) Off the equilibrium path for sender 1 (following deviation in stage 1): μ_1 is unchanged from stage 1, and sender 1 plays a best response.

- Stage 3: the receiver's beliefs are determined by Bayes' rule (since every combination of messages from stages 1 and 2 occurs on the equilibrium path), and she chooses $a(s_1)$, where s_1 is inferred from P, m_1' and m_2'.

We now verify the optimality of this strategy profile for each player.

3a. Optimality for the receiver

The receiver has learned s_1 exactly, but no information on s_2 other than the fact that it lies in $[\underline{s_2}(s_1), \overline{s_2}(s_1)]$. It is therefore optimal for the receiver to choose $a(s_1)$.

3b. Optimality for sender 2

Sender 2 believes that, with probability 1, sender 1's message will correspond to the cell where s_2 lies. For the same reason as in Section 3's basic construction, sender 2 has no profitable deviation.

3c. Optimality for sender 1

For the same reasons as for sender 2, there is no profitable deviation if sender 1 followed the equilibrium prescription in stage 1.

It remains to be shown that if sender 1 is supposed to announce $P(x)$, then announcing $P(y)$ instead is not a profitable deviation.

Suppose sender 1's signal is s_1 and that she is supposed to announce $P(x)$. Let $c(s_1)$ be the cell of $P(x)$ that would be communicated in stage 2 on the equilibrium path. Then announcing $P(y)$ instead would make sender 1 uncertain about sender 2's message in stage 2. Specifically, the revealed s_1 will be either:

(a) slightly different, if coordination is successful (*i.e.* one of the two cells of $P(y)$ that overlaps with $c(s_1)$ is communicated in stage 2); or

(b) almost one block away or further, in the event of miscoordination.

The probability of miscoordination is at least $\underline{\nu} r\Delta$, so the expected loss from miscoordination (point b) is at least $\underline{\nu} r\Delta\frac{\Lambda}{\underline{\nu} r}M(s_1) = \Lambda\Delta M(s_1)$ by the definition of n in step 1. We will show below that the gain from point (a) cannot exceed this amount.

We assume that in stage 2, the cell communicated by sender 1 is the rightmost of the two cells of $P(y)$ that overlaps with $c(s_1)$. This is done without loss of generality as the left case is symmetric, and any other announcement causes miscoordination for sure, which cannot be profitable.

Also without loss of generality, let $P(y)$ be the partition that should be announced if sender 1 had signal $s_1 + \Delta$, where $\underline{s_2}(s_1 + \Delta) \in (\underline{s_2}(s_1), \overline{s_2}(s_1))$. Note that $\Delta < 4\varepsilon$, which allows us to use the bound $M(s_1)$. The expected gain from point (a) is thus:

$$\begin{aligned}\int [u_1(\theta, a(s_1 + \Delta)) - u_1(\theta, a(s_1))] f(\theta | s_1, s_2 &\in [\underline{s_2}(s_1 + \Delta), \overline{s_2}(s_1)]) d\theta \\ &\leq \Lambda \Delta M(s_1)\end{aligned}$$

It is therefore not profitable for sender 1 to deviate. ■

6.2 Proof of Proposition 5

1. Constructing partitions for stage 1 messages by sender 1.

Let $S_2 = [-\underline{B}, \overline{B}] \subseteq [-T - \varepsilon, T + \varepsilon]$, and let $X = \max_{\{s_1 : \underline{s_2}(s_1) = -T\}} \overline{s_2}(s_1)$.

Define partition $L(x)$, which consists of n^2 cells labeled as in Section 3, as follows for all $x \in (-\underline{B}, X]$. The leftmost cell $(1, 1)$ is $[-\underline{B}, x]$. Each subsequent cell, except for the rightmost one $(n, n-1)$, is $(\underline{s_2}(z), \overline{s_2}(z)]$, where $\underline{s_2}(z)$ is the boundary of the previous cell; note that this is well-defined because $\underline{s_2}(.)$ and $\overline{s_2}(.)$ are continuous and strictly increasing. Let n_L be large enough so that if $n = n_L$, the left boundary of the rightmost cell $(n, n-1)$ is greater than $\frac{T}{2}$ for all $L(x)$. This is possible since, by assumption, the size of cells is bounded below by $\frac{1}{\nu}$ and above by 2ε.[19] We will refer to the rightmost cell as the *large extreme* cell, and in partitions where $x < \overline{s_2}(\min_{s_1 \in S_1} s_1)$, we refer to the leftmost cell as the *small extreme* cell. Moreover, call all others cells *regular*. Note that since $\underline{s_2}(s_1)$ and $\overline{s_2}(s_1)$ are strictly increasing, there exists a unique $x \in (-\underline{B}, X]$ such that sender 1 puts probability 1 on s_1 and s_2 being in the same regular cell.

Let n_L be large enough in an analogous way to n in the proof of Proposition 4.

Similarly, let $Y = \min_{\{s_1 : \underline{s_2}(s_1) = T\}} \underline{s_2}(s_1)$, and define the partition $R'(y)$ for all $y \in [Y, T)$, starting from the right, so that its rightmost $n^2 - 1$ cells cover at least $[-\frac{T}{2}, \overline{B}]$. Define n_R analogously, and let $n = \max\{n_L, n_R\}$.

Note that, for any x, y, the regular cells from $L(.)$ and $R'(.)$ will overlap over at least $[-\frac{T}{2}, \frac{T}{2}]$. Because $\underline{s_2}(s_1)$ and $\overline{s_2}(s_1)$ are strictly increasing over the relevant range, for every

[19] To see that for T large enough, it is possible for the size of the first $n^2 - 1$ cells to lie between $\underline{B} + \frac{T}{2}$ and $\underline{B} + \overline{B}$, we need to show that as n increases, the size of the first $n^2 - 1$ cells will not "skip over" this range. A partition's $n^2 - 2$ middle cells will cover at least $(n^2 - 2)\eta$. If we increase n by 1, we are adding $2n + 1$ cells, which cover at most $(2n + 1)\varepsilon$. Note that $\frac{(n^2-2)\eta + (2n+1)\varepsilon}{(n^2-2)\eta} \overset{n \to \infty}{\longrightarrow} 1$, while $\frac{\underline{B} + \overline{B}}{\underline{B} + \frac{T}{2}} \geq \frac{(T+\varepsilon)+(T-\varepsilon)}{(T+\varepsilon)+\frac{T}{2}}$, which increases in T and converges to $\frac{4}{3} > 1$.

$y \in (0, Y]$, there exists $\varphi(y) \in (0, X]$ such that in the range of the overlap, the cells of $L(\varphi(y))$ have the same boundaries as the cells from $R'(y)$. Define $R(x) = R'(\varphi^{-1}(x))$, so that in the area of the overlap, the cells of $L(x)$ and $R(x)$ have the same boundaries.

As a result, for every s_1, there exists a unique x such that sender 1 puts probability 1 on s_1 and s_2 being in the same regular cell for at least one of $L(x)$ and $R(x)$. Denote this quantity $x(s_1)$.

2. The strategy profile

- Stage 1: Sender 1 announces a partition from the set $\{L(x), R(x)\}_{x \in (0,X]}$. This partition is such that sender 1 knows which regular cell s_2 lies in - the partition must be either $L(x(s_1))$ or $R(x(s_1))$. If both of these partitions work, sender 1 randomizes 50/50 between them. For the remainder of this proof, we will let P be the announced partition. Sender 2 babbles.

- Stage 2: We distinguish three cases:

a) On the equilibrium path (for sender 1, if no deviation in stage 1; for sender 2, if s_2 lies in a regular cell of P): The senders send m_i' corresponding to the cell of P where s_2 lies, which, by construction, is known to sender 1. Beliefs μ_i are determined according to Bayes' rule. Note that (m_1', m_2') must correspond to a regular cell of P and reveals s_1 when combined with sender 1's announcement in stage 1.

b) Off the equilibrium path for sender 1 (following deviation in stage 1): μ_1 is unchanged from stage 1, and sender 1 plays a best response.

c) Off the equilibrium path for sender 2 (if s_2 lies in an extreme cell of P): Sender 2 sends m_2' corresponding to the cell of P where s_2 lies. Her belief μ_2 remains unchanged with respect to θ, while with respect to s_1, it is such that $m_1'(s_1, m_1, m_2) = 1$ (if s_2 lies in the leftmost cell of P) or $m_1'(s_1, m_1, m_2) = n$ (if s_2 lies in the rightmost cell of P). Note that this is always possible because, on the equilibrium path, $m_1 = P'$ and $m_1' = k$ can occur for all $P' \in \{L(x), R(x)\}_{x \in (0,X]}$ and $k \in \{1, ..., n\}$.

- Stage 3:

a) If (m_1', m_2') corresponds to a regular cell of P, the receiver chooses $a(s_1)$, where s_1 is inferred from P, m_1' and m_2'. Note that this is always the case on the equilibrium path.

b) If, instead, (m_1', m_2') points to the small extreme cell of P, the receiver believes that the state is the endpoint ($-T$ or T) within that extreme cell, and chooses that action.

c) If, finally, (m_1', m_2') points to the large extreme cell of P, the receiver believes that the state is $a(s_1)$ and chooses that action, for s_1 determined as follows:

- if $P = L(x)$ for some x, then s_1 is inferred as if sender 2 had sent $m_2' = n - 2$, so that the cell is the regular cell $(n, n-2)$ instead of the large extreme cell $(n, n-1)$;

- if $P = R(x)$ for some x, then s_1 is inferred as if sender 2 had sent $m_2' = 2$, so that the cell is the regular cell $(1, 2)$ instead of the large extreme cell $(1, 1)$.

We now verify the optimality of this strategy profile for each player. For sender 2, we will assume that $P = L(x)$ for some x. A symmetric argument applies if $P = R(x)$ instead.

3a. Optimality for the receiver

On the equilibrium path, the receiver has learned s_1 exactly, but no information on s_2 other than the fact that it lies in $[\underline{s_2}(s_1), \overline{s_2}(s_1)]$. It is therefore optimal for the receiver to choose $a(s_1)$.

Off the equilibrium path, sequential rationality for the receiver's action choice directly follows from the specified beliefs.

3b. Optimality for sender 2 on the equilibrium path

If s_2 lies in a regular cell of P, then sender 2 believes that m_1' will refer to the cell where s_2 lies. For the same reason as in Section 3's basic construction, sender 2 has no profitable deviation to another regular cell or to the small extreme cell. Deviating to the large extreme cell $(n, n-1)$ is also not profitable: message pair $(n, n-1)$ now leads to the same action as for message pair $(n, n-2)$. Obviously, sender 2 has no incentive to deviate is she is supposed to send $n-2$. When sender 1 is supposed to send n and sender 2 is supposed to send any message other than $n-1$ and $n-2$, then s_2 is at least almost a block away from cell $(n, n-2)$, so once again, there is no incentive to deviate.

3c. Optimality for sender 2 off the equilibrium path

Given μ_2, sender 2 believes, as on the equilibrium path, that coordination will be successful if m_2' corresponds to the cell of P where s_2 lies. If this cell is the small extreme cell, this is optimal as deviating would lead to an action almost a block away. If s_2 is in the large extreme cell $(n, n-1)$, then sender 2 expects $m_1' = n$. The best that sender 2 can send is to send $n-1$ or $n-2$, which results in an action about a block to the left of cell $(n, n-1)$. If sender 2 sends anything else, she would expect the action to be even further left, which is undesirable.

3d. Optimality for sender 1

For the same reasons as for sender 2, there is no profitable deviation to a regular cell or the small extreme cell in stage 2 if sender 1 followed the equilibrium prescription in stage 1. Deviating to the large extreme cell $(n, n-1)$ is also not profitable: this is only possible when $m_2' = n-1$, which implies that s_2 is located in cell $(1, n-1)$ or further to the left. Therefore, inducing an action in cell $(n, n-2)$, which is almost a block to the right of $(1, n-1)$, is suboptimal.

Moreover, it cannot be profitable for sender 1 to announce $L(x)$ instead of $R(x)$ or vice versa - it would either lead to the same action, or to a far away action (if sender 1 also deviates in stage 2, or if the announcement causes s_2 to be in the large extreme cell).

By the same argument as in the proof of Proposition 4, if sender 1 is supposed to announce $L(x)$, then announcing $L(y)$ instead is not a profitable deviation (and similarly with $R(x)$ and $R(y)$).

Combining the two arguments above implies that announcing $R(y)$ instead of $L(x)$ is also not a profitable deviation. ■

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