Dynamic Coalitions

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Abstract

Policy-making is a dynamic process in which policies can be changed in each period but continue in the absence of new legislation. We study a dynamic legislative bargaining game with an endogenous status quo where in each period a dollar is allocated with a proposal voted against the allocation in the previous period. We characterize for any initial status quo a class of simple Markov perfect equilibria (MPE) with dynamic coalitions, where a dynamic coalition is a decisive set of legislators whose members support the same policy, or set of policies, in at least two consecutive periods. In the basic model a dynamic coalition persists throughout the game, and coalition members share the dollar equally in every period. If uncertainty is associated with the implementation of a policy, there is a continuum of allocations supported by coalition MPE in which the originator of the coalition receives a share larger than the coalition partner receives but smaller than in sequential legislative bargaining theory. These coalition equilibria have the same allocation in every period when the coalition persists, but with positive probability the coalition dissolves due to the uncertainty. Coalition MPE also exist in which members tolerate a degree of implementation uncertainty resulting in coalition allocations that can change from one period to the next. The dynamic coalitions are minimal winning, form in the first period, and, if a coalition dissolves, a new coalition is formed in the next period. The predictions of the theory are compared to experiment results.

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1 Introduction

Most laws and government programs are continuing and remain in effect in the absence of new legislation. Rules promulgated by regulatory commissions also remain in effect until modified or rescinded. Social security, welfare, and other distributive programs are typically continuing, and distributions are governed by formulas that are changed only infrequently. Tax rates also continue in the absence of new legislation. At the state level, legislatures establish continuing policies including Medicaid eligibility and benefits as well as other state distributive programs. State regulatory commissions establish prices, rules governing lifeline and other cross-subsidization programs, and environmental and energy-efficiency policies. These programs have the property that the policy adopted or in place in the current period becomes the status quo for the next period. Policy choice thus can be viewed as a dynamic legislative bargaining game with an endogenous status quo in which a legislature has the opportunity to choose the policy in every period and agenda-setting power can change over time. Despite their dynamic nature and the opportunities for change, many policies are stable over time and are supported by a coalition that persists from period to period. This paper presents a theory of dynamic legislative bargaining with a focus on dynamic coalitions and the policies they adopt. Coalitions that persist over time are natural when the preferences of legislators are aligned, so to provide a strong test for dynamic coalitions, the policies considered are purely distributive with the preferences of legislators directly opposing.

A dynamic coalition is a decisive set of legislators whose members support the same policy, or set of policies, over time and receive a positive allocation. We characterize Markov perfect equilibria (MPE) that support dynamic coalitions beginning from any initial status quo. The game is an extension of the sequential legislative bargaining game introduced by Baron and Ferejohn (1989). In the stationary equilibrium in that static game a bargain is reached with the first proposal, and the decisive set supporting the bargain is minimal-winning. The proposer captures what otherwise would be the allocation of those legislators excluded from the decisive set and does not share the gains with other members of the decisive set. In the dynamic game with a dollar allocated in every period and an endogenous status quo, the equilibria exhibit some of the properties of the sequential game but not others. In the basic dynamic game with a sufficiently high discount factor, a dynamic coalition is formed in the first period and persists thereafter. The dynamic coalition adopts the same allocation in every period, so policy is stable. The dynamic coalition is minimal-winning, and

the originator of the coalition shares proposal power equally with the other coalition members, so all coalition members receive the same allocation.

The MPE concept has several desirable attributes. First, the equilibria are simple with strategies depending only on a state variable, and dynamic coalition MPE are particularly simple. Second, the set of dynamic coalition MPE is strictly smaller than the set of subgame perfect equilibria. Third, strategies depend only on payoff relevant information and not on other components of the history of play that may be strategy relevant. This means that player-specific, or selective, punishment strategies cannot be used. MPE is thus a stringent equilibrium concept for the establishment of dynamic coalition equilibria, since it does not allow selective punishments in response to deviations from the coalition strategies.

In dynamic coalition MPE coalition members propose the status quo when it is a coalition allocation and otherwise randomize among potential coalition partners in forming a new coalition. If a coalition dissolves, its members are collectively punished, since randomization gives all legislators, including legislators outside the coalition, the same payoff. This payoff is less than the payoff from preserving the coalition, and this difference allows the coalition to be supported as an MPE.

The coalition MPE are focal because they are simple, exhibit policy and coalition stability, and could be coordinated through straightforward communication between the originator of the coalition and potential coalition partners. The equilibrium allocation in the basic model is unique and is the same in every period.[1] It is also an equilibrium with risk averse legislators, since it involves perfect smoothing over time.

Many policies have a degree of uncertainty associated with their implementation, and this uncertainty can affect the allocation in a period and hence the status quo in the next period. The uncertainty could be due to exogenous factors or to endogenous factors associated with delegation to an administrative agency or regulatory commission or to choices made by those affected by the policy. To examine the robustness of dynamic coalitions, the basic model is extended to include implementation uncertainty that can cause allocations to differ from those in the adopted policy. A class of dynamic coalition MPE is characterized in which a coalition and its allocation persist as long as implementation uncertainty is not realized, so policy has a degree of stability. A coalition is formed in the first period, and if it dissolves as a result of realized implementation uncertainty, a new coalition is formed in the next period. The coalition implements the same policy in each period,

[1] The basic game is identical to Kalandrakis (2004).

and the originator of the coalition shares the gains from proposal power with the coalition partner but not necessarily equally. These specific-policy coalition equilibria support a set of allocations and that set is increasing in the discount factor.

A specific-policy coalition dissolves when implementation uncertainty changes the allocation, but a coalition could tolerate some changes in the allocation due to implementation uncertainty. A coalition MPE exists that supports a set of tolerated allocations where the coalition persists if the allocation remains in the set and dissolves if it is outside the set. A tolerant coalition is more valuable to its members than is the corresponding specific-policy coalition to its members. A tolerant coalition forms in the first period, and if it dissolves as a result of implementation uncertainty, a new coalition is formed in the next period. The originator of the coalition shares proposal power with the coalition partner, and because of the realized implementation uncertainty, the coalition partner could in a period have a larger allocation than the originator.

The allocations supportable by dynamic coalition equilibria are determined by stability considerations. A coalition can be threatened externally by a proposal from a non-coalition member intended to induce a coalition member to deviate from the coalition strategies. In the basic model the external threat determines a bound on the discount factor such that coalition members reject any proposal by an out legislator. A coalition also faces the internal threat that a coalition member might propose a more favorable allocation than called for in the equilibrium or vote against a coalition proposal when the status quo is favorable. This threat is present at all discount factors in the basic model and requires the originator of the coalition to share proposal power with the other coalition members.

Coalition equilibria exhibit policy stability and hence have substantive implications that differ from those in other theories. The rotating dictator equilibrium established in Kalandrakis (2004) predicts that distributive policies change depending on which legislator is recognized as the proposer, whereas the coalition equilibria identified here have stable policies. Moreover, a dynamic coalition has the same decisive set in every period, whereas the decisive sets change in a rotating dictator equilibrium.

Epple and Riordan (1987) characterize subgame perfect equilibria in a model of distributive policy with a deterministic rule for selecting the proposer, whereas we characterize a set of coalition MPE where the proposer is randomly selected. Baron (1996) considers a unidimensional policy and proves a dynamic median voter theorem. A number of recent papers have considered legislative

bargaining games with an endogenous status quo, including Baron and Herron (2003), Bernheim, Rangel and Rayo (2006), Anesi (2010), Dziuda and Loeper (2010), Bowen (2011), Diermeier and Fong (2011), Zápal (2012), Nunnari (2012), Piguillem and Riboni (2012), and Bowen, Eraslan and Chen (2012). Policies in these papers are not purely distributive with the exception of Bernheim, Rangel and Rayo, which has a finite horizon.

Kalandrakis (2004), Kalandrakis (2010), Bowen and Zahran (2012), Richter (2011), Battaglini and Palfrey (2012), and Anesi and Seidmann (2012) consider Markov perfect equilibrium in a distributive game with an endogenous status quo similar to the game considered here. Kalandrakis was the first to characterize Markov perfect equilibria in this setting, where in equilibrium a rotating dictator has all the bargaining power in a period. Bowen and Zahran (2012), Richter (2011), and Anesi and Seidmann (2012) identify equilibria that exhibit compromise where more than a minimal majority receive an allocation. Dynamic coalitions are present in Anesi and Seidmann and in Bowen and Zahran but only for particular initial status quos. Bowen and Zahran find compromise with risk-averse legislators, and Battaglini and Palfrey (2012) also find compromise in a quantal response equilibrium with risk-averse players. By allowing part of the dollar to be wasted, Richter (2011) identifies MPE in which all legislators share benefits. Duggan and Kalandrakis (2012) provide a general existence result for dynamic legislative bargaining games with an endogenous status quo and uncertainty over legislators' preferences. In the environment considered here, legislators' preferences are fixed, so existence is shown constructively.[2]

Cooperation or policy moderation in a dynamic policy-making environment has been studied in Dixit, Grossman and Gul (2000) and Lagunoff (2001). Battaglini and Coate (2007), Battaglini and Coate (2008), Acemoglu, Egorov and Sonin (2012), and Baron, Diermeier and Fong (2012) show that dynamic incentives can lead to inefficiency. In contrast to these papers we consider purely distributive policy, where there is no natural incentive to form coalitions or induce cooperation.

Battaglini and Palfrey conducted experiments that implemented the dynamic game considered here. Their findings are consistent with dynamic coalition MPE, although participants in the experiment exhibit a number of behaviors that may not correspond to an equilibrium.

The basic model is introduced in the next section, and a coalition MPE is defined and illustrated. Section 3 introduces implementation uncertainty, and coalition equilibria in which the originator

[2]Duggan (2012) proves a general existence result for MPE in noisy stochastic games that requires norm-continuity of state transition probabilities. Norm-continuity is violated with voting, so this result is not applicable to dynamic legislative bargaining games.

of the coalition receives a greater share than the coalition partner are characterized in Section 4. Section 5 considers coalitions that tolerate a degree of implementation uncertainty. Section 6 compares the dynamic coalition equilibria to the results of the Battaglini and Palfrey experiments. Conclusions are provided in the final section.

2 The Model and Coalition Equilibrium Concept

2.1 The Basic Model

The model represents a legislative process with an endogenous status quo where in each period a 3-member legislature can choose a new policy or leave the status quo policy in place. In each period a legislator is selected at random to propose a policy, which is then voted against the status quo policy from the previous period. The winner becomes the policy in place in the current period and the status quo for the next period. The process is repeated for an infinite number of periods. The legislators allocate a dollar in every period, so the policy space in each period is $X = \{x | \sum_{i=1}^{3} x_i = 1, x_i \geq 0, i = 1, 2, 3\}$. A proposal by legislator i in period t is a policy $y_i^t \in X$, and the status quo policy at the beginning of period t is denoted q^{t-1} with q^0 the initial status quo. The agenda on which legislators vote is $\{q^{t-1}, y_i^t\}$. Legislator i is assumed to have a linear utility function $U_i(x) = x_i$ in each period and maximizes the expectation of the discounted, infinite stream of utilities $\sum_{t=1}^{\infty} \delta^{t-1} U_i(x_i^t)$, where x_i^t is the ith component of the policy x^t in period t and $\delta \in [0, 1)$ is a common discount factor.

A MPE is a subgame perfect equilibrium in which strategies depend only on the payoff-relevant state variable, which at a proposal stage is the status quo q^{t-1} at the beginning of period t. A strategy for legislator i is a pair of functions (σ_i, ω_i), where $\sigma_i : X \to X$ is a proposal strategy for legislator i and $\omega_i : X \times X \to \{0, 1\}$ is legislator i's voting strategy.[3] Legislator i's proposal strategy $\sigma_i(q^{t-1}) = y_i^t$ selects a proposal y_i^t conditional on the status quo. Legislator i's voting strategy $\omega_i(q^{t-1}, y_\ell^t)$ assigns a vote conditional on the proposal and the status quo, where ℓ denotes the proposer and $\omega_i(q^{t-1}, y_\ell^t) = 1$ denotes a vote for the proposal. A simple majority is required, so the proposal is approved if and only if $\sum_{i=1}^{3} \omega_i(q^{t-1}, y_\ell^t) \geq 2$. The status quo $q^t \in X$ in period

[3] We abuse notation slightly by writing proposal strategies as pure strategies. The coalition MPE we characterize involves mixing, but the mixing among pure strategies is simple so for clarity we use pure strategy notation.

$t+1$ is then

$$q^t = \begin{cases} q^{t-1} & \text{if } \sum_{i=1}^{3} \omega_i(q^{t-1}, y_\ell^t) < 2 \\ y_\ell^t & \text{if } \sum_{i=1}^{3} \omega_i(q^{t-1}, y_\ell^t) \geq 2. \end{cases}$$

The state thus evolves as proposals are made and votes are cast.

Letting σ and ω denote the profiles of strategies, the dynamic payoff $V_i(\sigma, \omega | q^{t-1})$ for i depends on t only through the state and is defined by

$$V_i(\sigma, \omega | q^{t-1}) = E^t[U_i(q^t) + \delta V_i(\sigma, \omega | q^t)],$$

where E^t denotes expectation with respect to the selection of the proposer and any uncertainty affecting payoffs and transitions.

Voting strategies are required to be stage undominated, i.e., a legislator votes for the alternative he strictly prefers regardless of whether he is pivotal. That is,

$$\omega_i(q^{t-1}, y_\ell^t) = \begin{cases} 1 & \text{if } E^t[U_i(y_\ell^t) + \delta V_i(\sigma, \omega | y_\ell^t)] > E^t[U_i(q^{t-1}) + \delta V_i(\sigma, \omega | q^{t-1})] \\ 0 & \text{if } E^t[U_i(y_\ell^t) + \delta V_i(\sigma, \omega | y_\ell^t)] < E^t[U_i(q^{t-1}) + \delta V_i(\sigma, \omega | q^{t-1})]. \end{cases} \tag{1}$$

If the dynamic payoffs are equal, legislator i votes according to an indifference rule that is specified below as part of the equilibrium construction.

A perfect equilibrium requires that in every subgame for each t every legislator's dynamic payoff is optimal given the equilibrium strategies of the other legislators. That is, a profile (σ^*, ω^*) is a Markov perfect equilibrium in stage undominated voting strategies if and only if

$$V_i(\sigma^*, \omega^* | q^{t-1}) \geq V_i((\sigma_{-i}^*, \sigma_i), (\omega_{-i}^*, \omega_i) | q^{t-1}), \forall (\sigma_i, \omega_i), i = 1, 2, 3.$$

A dynamic coalition is defined as a decisive set of legislators each of whom receives a positive allocation and support the same policy, or set of policies, in at least two consecutive periods. A dynamic coalition MPE is an equilibrium that supports a dynamic coalition. The focus here is on equilibria that support dynamic coalitions for all initial status quos $q^0 \in X$.

Two models are considered. In the basic model in Section 2.2 payoffs and transitions are deterministic, and the unique allocation supported by a coalition MPE has equal division of the dollar within a minimal winning coalition in every period. The equilibrium is presented for three legislators and a simple majority, and an extension to n legislators and any decisiveness rule is

presented in Section 2.5. In Section 3 implementation uncertainty is introduced, and in Section 4 a class of specific-policy coalition equilibria is characterized where in every period the originator of the coalition receives a greater share than the coalition partner receives. In Section 5 coalition members tolerate moderate implementation uncertainty, and the equilibrium supports allocations in which the coalition members receive unequal shares that can vary from period to period.

2.2 The Coalition MPE

A dynamic coalition MPE, or coalition MPE for short, is introduced and illustrated in this section for the model in which payoffs and transitions are deterministic. Consider a set $Z = \{z_{12}, z_{21}, z_{13}, z_{31}, z_{23}, z_{32}\}$ of coalition allocations, where $z_{12} = z_{21} = (\frac{1}{2}, \frac{1}{2}, 0)$, $z_{13} = z_{31} = (\frac{1}{2}, 0, \frac{1}{2})$, and $z_{23} = z_{32} = (0, \frac{1}{2}, \frac{1}{2})$. Consider the coalition proposal strategy $\sigma_i^*(q^{t-1})$ for legislator i selected as the proposer in period t given by

$$\sigma_i^*(q^{t-1}) = \begin{cases} q^{t-1} & \text{if } q^{t-1} \in \{z_{ij}, z_{ik}\} \\ z_{i\ell}, \ell = j, k, \text{with probability } \frac{1}{2} & \text{if } q^{t-1} \notin \{z_{ij}, z_{ik}\}. \end{cases} \tag{2}$$

The proposer thus proposes the status quo if it is favorable and, if not, proposes a favorable policy, randomizing among the other two legislators.

These strategies identify the coalition as the legislator selected as the proposer in the first period and the other legislator to whom $\frac{1}{2}$ is offered. Thereafter, the coalition members propose the coalition allocation and choose to maintain the coalition whenever they are the proposer.

The following proposition identifies a coalition MPE that exists for all initial status quo when the future is sufficiently important.

Proposition 1. *There exists a $\delta^o < 1$ such that for all $\delta > \delta^o$ and any $q^0 \in X$: (i) The continuation values $v_\ell(z_{ij})$, where $\ell \in \{i, j\}$ is the proposer, are $v_i(z_{ij}) = v_j(z_{ij}) = \frac{1}{2(1-\delta)}$ and $v_k(z_{ij}) = 0, k \neq i, j$. The continuation value for $q^{t-1} \notin Z$ is $\hat{v} = \frac{1}{3(1-\delta)}$ for $i = 1, 2, 3$. (ii) The strategies $\{(\sigma_i^*(q^{t-1}), \omega_i^*(q^{t-1}, \sigma_i(q^{t-1}))), i = 1, 2, 3, \}$ in (1) and (2) with legislators voting for the status quo when indifferent between it and a proposal constitute a MPE.*

A sketch of the main steps of the argument is presented here, and in Section 4 Proposition 1 is formally proven. In the equilibrium the coalition persists because a deviation results in a collective punishment in which all legislators receive $\hat{v} = \frac{1}{3(1-\delta)}$, whereas the coalition members receive $v_i(z_{ij}) = v_j(z_{ij}) = \frac{1}{2(1-\delta)}$ on the equilibrium path. The identity of the coalition is determined by

which legislator is selected as the first proposer. Suppose that the proposer in the first period is legislator 1 and proposes z_{12}, so the coalition is $\{1, 2\}$. Thereafter, the coalition members propose z_{12} and vote for $q^{t-1} = z_{12}$.[4] The continuation value $v_\ell(z_{12})$ on the equilibrium path is then

$$v_\ell(z_{12}) = \frac{1}{2} + \delta v_\ell(z_{12}),$$

which yields

$$v_\ell(z_{12}) = \frac{1}{2(1-\delta)}$$

establishing the first part of (i). Since all legislators are in symmetric positions, the dynamic payoff from randomization must be $\hat{v} = \frac{1}{3(1-\delta)}$, establishing the second part of (i).[5] To identify the bound δ^o on the discount factor, consider an initial status quo $q^0 = (1, 0, 0)$. Legislator 1 can propose $(1, 0, 0)$ and obtain 1 in the first period, and the status quo for period 2 is $q^1 = q^0 = (1, 0, 0)$. Given $q^1 \notin Z$, whichever legislator is recognized in period 2 randomizes. Legislator 1 prefers to propose z_{12} in the first period rather than $(1, 0, 0)$ if and only if

$$\frac{1}{2} + \delta v_1(z_{12}) > 1 + \delta \hat{v},$$

which is satisfied if and only if $\delta > \delta^o = \frac{3}{4}$. Legislator 1 cannot offer less than $\frac{1}{2}$ to legislator 2, since then $q^1 \notin \{z_{12}, z_{13}\}$ and legislators would randomize in the next period, in which case the continuation value would be $\hat{v}$.

Next, suppose that $q^1 = z_{12}$ and legislator 3 is selected as the proposer in period 2. If legislator 3 proposes $y_3 \notin Z$, coalition members strictly prefer q^1 to that proposal for $\delta > \delta^o$. Suppose 3 proposes z_{23}, which gives $\frac{1}{2}$ to coalition member 2, and if 2 votes for the proposal, legislators 2 and 3 would constitute a coalition thereafter. Legislator 2 is indifferent between voting for the status quo and the proposal, and, according to the indifference rule, votes against the proposal and for the present coalition $\{1, 2\}$. This shows (ii) in Proposition 1.

The coalition equilibrium in Proposition 1 exists because coalition members each receive $\frac{1}{2}$ on the equilibrium path, and although a coalition member selected as proposer can obtain (almost) 1 by offering the out legislator a positive allocation, doing so results in a status quo outside Z. In the

[4] On the equilibrium path the coalition members could make any proposal and then vote for the status quo against the proposal, so there is a continuum of equilibria that support a dynamic coalition and the equal division allocation.

[5] This is proven formally in Lemma 1 below.

following period the proposer randomizes in selecting a coalition partner resulting in a continuation value $\hat{v} = \frac{1}{3(1-\delta)}$ for all legislators. A coalition member who so deviates thus is punished by all legislators by the difference $\frac{\delta}{2(1-\delta)} - \frac{\delta}{3(1-\delta)} = \frac{\delta}{6(1-\delta)}$. For $\delta > \delta^o = \frac{3}{4}$ the current period gain of $1 - \frac{1}{2} = \frac{1}{2}$ is less than the punishment. This punishment is not selective, since both coalition members are punished, and it is implemented with Markov strategies.

The coalition equilibrium in Proposition 1 has equal allocations for the coalition members, and the following proposition establishes that no other allocation can be supported as a coalition equilibrium.

Proposition 2. *Only allocations in Z can be supported by an MPE with coalition proposal strategies when legislators vote for the status quo when indifferent.*

Proof. Consider coalition proposal strategies in (2) supporting coalition allocations for a set $\hat{Z}$ with typical element $\hat{z}_{ij} = (\frac{1}{2} + \beta, \frac{1}{2} - \beta, 0)$ where the proposer proposes to take a share $\frac{1}{2} + \beta$, $\beta > 0$, and offer $\frac{1}{2} - \beta$ to another legislator selected at random. The two legislators vote for the status quo if it gives them $\frac{1}{2} + \beta$ and $\frac{1}{2} - \beta$, respectively. Suppose the initial proposer is legislator 1 and 2 is the coalition partner. When 2 is selected as the proposer, she has a strict incentive to propose $\frac{1}{2} + \beta$ for herself and offer $\frac{1}{2} - \beta$ to legislator 3. Both 2 and 3 vote for the proposal, so only if $\beta = 0$ is there a coalition MPE. ■

The dynamic coalition equilibrium established in Proposition 1 has a minimal winning coalition that forms in the first period and continues thereafter. The policy selected by the originator of the coalition is unique and stable, which are also the predictions of sequential legislative bargaining theory. In the dynamic model, however, the only allocation that can be supported as a coalition equilibrium is equal division, so the proposer and the coalition partner share proposal power equally.

2.3 The Indifference Rule

The indifference rule used in Proposition 1 specifies that a legislator votes for the status quo when indifferent. This rule supports a coalition MPE in which coalition members receive equal shares of the dollar in every period. The role of the indifference rule is to preclude a coalition member from deviating to an apparently equivalent proposal by a non-coalition legislator. Legislator 3 could propose $\left(0, \frac{1}{2}, \frac{1}{2}\right)$, and if the continuation value to legislator 2 were $\frac{1}{2(1-\delta)}$ as it would be in a coalition of legislators 2 and 3, legislator 2 would be indifferent between the status quo and the

proposal. If legislators were to vote for a proposal when indifferent, however, a dynamic coalition cannot be supported as a MPE using coalition proposal strategies, since legislator 3 can make a proposal that induces legislator 2 to defect from the coalition $\{1, 2\}$. Then, the continuation value for a coalition member would be less than $\frac{1}{2(1-\delta)}$. The indifference rule of voting against the proposal when indifferent thus is needed to support a coalition equilibrium in the dynamic legislative bargaining game. As shown in Sections 4 and 5, this indifference rule is not needed to support coalition equilibria when there is implementation uncertainty that affects current period payoffs and transitions.

The indifference rule typically used in game theoretic models is for a player to vote for a proposal when indifferent between it and another alternative, in this case the status quo.[6] This indifference rule is used to avoid open set issues where if a legislator would not vote for the proposal when indifferent, the proposer would simply increase its offer by a small amount and the player would then vote for it. As Battaglini and Palfrey (p. 750) observe, "the [Kalandrakis] equilibrium must have voters voting in favor of the proposal when they are indifferent. (Otherwise, the proposer would have an incentive to sweeten the offer [to a player receiving 0] by an infinitessimal amount)."

In the equilibria presented here, a coalition member votes for the alternative with the higher dynamic payoff (current-period allocation plus the discounted continuation value) but if indifferent between the proposal and the status quo legislators vote for the status quo rather than the proposal. A legislator not in the coalition cannot sweeten the offer because no more than the entire dollar can be offered, and with the discount factor sufficiently high the entire dollar is not enough to elicit a vote for the proposal. The out member thus cannot overturn the coalition MPE for any initial status quo. As Proposition 2 indicates, the indifference rule is not innocuous in a dynamic game.

Indifference rules thus can support at least two MPE. If legislators vote for the proposal when indifferent, a rotating dictatorship is a MPE. If the legislators vote no when indifferent, a coalition equilibrium exists with equal division and a rotating dictator is not a MPE. Which equilibrium is a better predictor of how people would play this dynamic game is an empirical matter, and the experiments by Battaglini and Palfrey provide evidence about that play. They conclude that the experiment results do not support the rotating dictator equilibrium and do support a Markov Logistic Quantal Response Equilibrium (MLQRE) (Mckelvey and Palfrey (1995) McKelvey and

[6]The MPE established by Kalandrakis, Bowen and Zahran, Duggan and Kalandrakis, and others specify that if a legislator is indifferent between the status quo and the proposal, he votes for the proposal.

Palfrey (1998)) when players are sufficiently risk-averse. The payments to the participants in the experiment were small, and whether players were actually risk averse is not clear. In Section 6 we show that the Battaglini and Palfrey experiment show evidence of dynamic coalition behavior. The principal difference between the experiment results and the predictions of the coalition equilibrium is the frequency with which a universal allocation results, since the universal allocation is never a coalition Markov perfect equilibrium even for highly risk-averse players. The universal allocation can, however, be supported by a subgame perfect equilibrium, and that equilibrium is a dynamic coalition equilibrium.[7]

2.4 Communication

Several experiments implementing the static sequential legislative bargaining game introduced by Baron and Ferejohn find behavior that only weakly supports the theory.[8] Agranov and Tergiman (2012) show that allowing participants in a legislative bargaining experiment to communicate results in behavior that strongly supports the theory. The cheap talk communication allowed in the experiment takes place after a proposer has been selected, and any player can send a message. Participants in the experiment used the communication opportunity to learn about the reservation values of the other players and to induce the proposer to include them in the majority.

Simple communication could allow legislators to coordinate on a coalition equilibrium. Suppose that the status quo is $q^{t-1} = (1, 0, 0)$ and legislator 1 in the experiment is selected as the proposer. Legislator 1 could propose $(1, 0, 0)$ as in a rotating dictatorship and receive 1 in the current period with $q^t = (1, 0, 0)$ the next status quo. Legislator 1 could alternatively propose $(\frac{1}{2}, \frac{1}{2}, 0)$ accompanied by the following message to legislator 2. "I am offering you $\frac{1}{2}$ in the hope that you will also propose $(\frac{1}{2}, \frac{1}{2}, 0)$ when you are selected and along with me vote for this allocation in every period and against any other proposal. I cannot commit to vote for this allocation nor to propose it in the future, but you should understand that it is in my, and your, interest to do so, and it is in my interest to have you believe that I will do so. I will have an incentive to vote for it, since I am sufficiently patient that I cannot be tempted away from following through on my intentions even if I were offered 1 by legislator 3. I also understand that legislator 3 could propose $(0, \frac{1}{2}, \frac{1}{2})$, but you can do no better

[7] The MLQRE predicts that a universal allocation in which all players receive at least one-quarter of the dollar can occur with high probability when players are risk averse.

[8] See Frechette, Kagel and Lehrer (2003), Frechette, Kagel and Morelli (2005*a*), and Frechette, Kagel and Morelli (2005*b*).

than to vote for $(\frac{1}{2}, \frac{1}{2}, 0)$ when it is the status quo, so I believe you will stick with me. I signaled my confidence in you by sacrificing in the current period to initiate this coalition." The simplicity of the coalition MPE should facilitate communication, making this an appealing equilibrium.

2.5 Extensions

The coalition MPE in Proposition 1 can be extended in a straightforward manner to an n-member legislature and to legislators with preferences exhibiting risk aversion. These extensions are described briefly here along with a direction in which the equilibrium cannot be extended.

For a legislature with n members and $m \in [\frac{n+1}{2}, n)$ votes to approve a proposal, the legislator selected in the first period offers $\frac{1}{m}$ to herself and any $m-1$ other legislators selected randomly. A member of the coalition votes for the coalition allocation over any other allocation if

$$\frac{1}{m} + \delta \frac{1}{m(1-\delta)} > 1 + \delta \frac{1}{n(1-\delta)},$$

where $\frac{1}{n(1-\delta)}$ is the continuation value corresponding to randomization. If a coalition member is indifferent between the status quo and a proposal, he votes for the status quo when it is the coalition allocation. A coalition MPE then exists for all $\delta > \delta^o = \frac{n(m-1)}{m(n-1)}$.

The coalition equilibrium strategies for a three-member legislature yield a payoff of $\frac{1}{2}$ in every period for the coalition members, so there is perfect risk smoothing over time. The equilibrium strategies then constitute a MPE for risk averse legislators with δ sufficiently high. Letting $U_\ell(x) = u(x), \ell = i, j, k$, be a strictly concave utility function, the lower bound δ_u^o on the discount factor is $\delta_u^o = \frac{3u(1)-3u(\frac{1}{2})}{3u(1)-2u(\frac{1}{2})-u(0)}$. Since risk smoothing over time benefits the coalition members, the discount factor δ_u^o is strictly less than δ^o for risk averse legislators.

The coalition equilibria in Proposition 1 have the same allocation in every period, but with risk-neutral legislators allocations that are unbalanced within a period but balanced over time as each member of the coalition is selected as the proposer would provide equivalent payoffs. This conjecture, however, cannot be a coalition equilibrium when legislators vote against the proposal when indifferent. To show this, consider a coalition of legislators 1 and 2 with legislator 1 when selected proposing $z_{12}(z) = (1-z, z, 0)$ and legislator 2 when selected proposing $z_{21}(z) = (z, 1-z, 0)$, where $z \in [0, \frac{1}{2})$. Legislator 3 proposes any $y \in X$, and legislators 1 and 2 vote against that proposal for δ sufficiently high. The continuation values for the conjectured equilibrium for coalition member

1 when $q^{t-1} = z_{12}(z)$ are given by

$$v_1(z_{12}(z)) = \frac{1}{3}\left(z + \delta v_1(z_{21}(z))\right) + \frac{2}{3}\left(1 - z + \delta v_1(z_{12}(z))\right)$$

$$v_1(z_{21}(z)) = \frac{2}{3}\left(z + \delta v_1(z_{21}(z)) + \frac{1}{3}\left(1 - z + \delta v_1(z_{12}(z))\right),\right.$$

which yield

$$v_1(z_{12}(z)) = \frac{1 + (1-z)(1-\delta)}{(3-\delta)(1-\delta)}$$

and

$$v_1(z_{21}(z)) = \frac{1 + z(1-\delta)}{(3-\delta)(1-\delta)}.$$

Similarly, $v_2(z_{12}(z)) = v_1(z_{21}(z))$ and $v_2(z_{21}(z)) = v_1(z_{12}(z))$.

Consider the votes by legislators 1 and 2. Suppose $q^{t-1} = z_{12}(z)$, and legislator 2 is the proposer and proposes $z_{21}(z)$. Legislator 1 votes for $z_{21}(z)$ if and only if

$$z + \delta v_1(z_{21}(z)) > 1 - z + \delta v_1(z_{12}(z)),$$

which simplifies to

$$-\delta(1 - 2z) > 1 - 2z,$$

which is satisfied only if $z > \frac{1}{2}$. Since $z \in [0, \frac{1}{2})$, legislator 1 votes against legislator 2's proposal when the status quo favors 1. Similarly, legislator 2 also votes against 1's proposal when the status quo favors 2. Consequently, there is no unbalanced, alternating-allocation coalition equilibrium.

3 Implementation Uncertainty

The unique coalition equilibrium allocation in the deterministic dynamic game has equal division and the supporting equilibrium requires an indifference rule in which coalition members vote against a proposal and for the status quo when indifferent. If there is publicly-observable uncertainty resulting from the implementation of a policy, however, there are coalition equilibria that support a set of unequal allocations and legislators can vote arbitrarily when indifferent between a proposal and the status quo. Uncertainty can affect not only the payoff in the current period but also the status quo for the following period.

Uncertainty resulting from implementation represents the observation that policy does not always work as intended. The implementation of legislation is typically delegated to administrative agencies or regulatory commissions that develop the details of the application of the legislation. A degree of uncertainty can be associated with that delegation, and legislators take that uncertainty into account in choosing a policy. The uncertainty could also be associated with the response to the enacted policy by those affected by it, and the realization of that uncertainty can affect the status quo and the strategies of legislators in the future. As an extreme example, with the support of the American Association of Retired Persons Congress overwhelmingly enacted the Medicare Catastrophic Coverage Act of 1988 which provided generous benefits for catastrophic care under Medicare and financed the benefits through increases in Medicare premiums. Before the change could be fully implemented, Medicare recipients began protesting the forthcoming premium increases, and, facing a revolt, Congress quickly repealed the Act it had passed in the previous session.

Uncertainty is also an integral part of dynamic legislative bargaining theory. In their MLQRE Battaglini and Palfrey assume that players use behavioral strategies that place positive probability on every available action (on a grid). That probability is proportional to the continuation value, and as that proportion increases the limit points correspond to MPE. This uncertainty affects strategies and hence payoffs but does not affect state transitions other than through the strategies. Duggan and Kalandrakis (2012) show the existence of stationary MPE in pure strategies for a class of dynamic games that accommodate uncertainty in the current period payoffs and in the transitions from one state to another. Uncertainty that affects the transitions is necessary for existence with an infinite policy space, and preference uncertainty is needed to show uniqueness of optimal proposals. The uncertainty considered here affects the implemented policy and hence both the payoffs in the current period and the status quo for the next period. Since the uncertainty affects the policy that is implemented, the status quo can move away from the coalition allocations in which case the legislators as proposers all randomize in forming new coalitions.

The following two sections identify coalition equilibria with unequal allocations among coalition members when there is uncertainty associated with the implementation of proposals. The uncertainty reflects factors that intervene between the passage of a policy and the consequences once the policy is implemented. This uncertainty affects the allocation when a coalition continues and when it dissolves. The uncertainty could be greater when a policy different from the status quo is

implemented than when the status quo policy continues in place. This uncertainty allows coalition MPE with unequal allocations to coalition members. In Section 4 this is established for coalitions that implement the same, unbalanced allocation in every period and in Section 5 for coalitions that tolerate allocations that can vary from one period to the next. When there is no implementation uncertainty, in both cases the unique equilibrium policy has a balanced allocation in which coalition members receive equal shares of the dollar in every period, as characterized in Proposition 1.

Consider a set $Z(z)$ of coalition allocations that is analogous to the set Z in the equal allocation case. Let $z_{ij}(z)$ denote the allocation where i receives $1-z$, j receives z, and k receives 0. Let $Z(z) = \{z_{12}(z), z_{13}(z), z_{21}(z), z_{23}(z), z_{31}(z), z_{32}(z)\}$ denote the set of proposals of this form, where $z \in [0, \frac{1}{2}]$. The model of implementation uncertainty is chosen to facilitate comparative statics analysis of the set of allocations supported as coalition equilibria and to simplify the analysis of the incentive constraints. Implementation uncertainty is assumed to be present with a positive probability, η or γ, in Assumption 1 below, and with the complementary probability there is no uncertainty and hence the allocation equals the policy adopted by the legislature. When implementation uncertainty is realized, its magnitude is represented by a continuous, mean zero, random shock. The former specification allows comparative statics analysis in terms of a single parameter, and the latter specification means that the probability is zero that the shocked allocation equals the policy adopted by the legislature. A legislator cannot receive more that 1 or less than 0, so the shocked allocation may be truncated with a reallocation of the truncated amount to other legislators.

The following assumptions specify the representation of the implementation uncertainty.

Assumption 1. *If a proposal $y_i^t = q^{t-1}$ is adopted, with probability $1-\eta$ the policy implemented equals the proposal, and with probability $1 > \eta \geq 0$ the policy is distorted by a uniformly distributed shock $\tilde{\theta}^t$ with mean zero and support $[-\underline{\theta}, \underline{\theta}]$. (i) For $y_i^t \in Z(z)$, if the realization θ^t is such that $z - \theta^t \geq 0$, the legislators in the coalition receive $1 - z + \theta^t$ and $z - \theta^t$, respectively. If $z - \theta^t < 0$ for legislator ℓ, ℓ receives 0 and the other coalition member ℓ' receives 1. (ii) For a proposal $y_i^t = q^{t-1} \notin Z(z)$, if a legislator ℓ receives 1 in y_i^t and $1 + \theta^t \geq 1$, ℓ receives 1. If $1 + \theta^t < 1$, ℓ receives $1 + \theta^t$ and one other legislator selected at random receives $-\theta^t$. If only two legislators ℓ and ℓ' receive positive allocations in $y_i^t = (1 - x_\ell, x_\ell, 0)$, where $0 < x_\ell \leq \frac{1}{2}$, they receive $1 - x_\ell + \theta^t$ and $x_\ell - \theta^t$, respectively, if $x_\ell - \theta^t \geq 0$. If $x_\ell - \theta^t \leq 0$, ℓ receives 1 and ℓ' receives 0. If all three legislators receive positive allocations in y_i^t, the allocations with the shock are $x_\ell + \alpha_\ell \theta^t, \ell = i, j, k$, where $|\alpha_\ell| \leq 1, \ell = i, j, k$, and $\alpha_i + \alpha_j + \alpha_k = 0$. If $x_{\ell'} + \alpha_{\ell'}\theta^t \leq 0$ for some ℓ', ℓ' receives 0 and*

$-\alpha_{\ell'}\theta^t$ *is allocated randomly among the other legislators.*

If a proposal $y_i^t \neq q^{t-1}$ *is adopted, with probability* $1-\gamma$ *the policy implemented equals the proposal, and with probability* $1 > \gamma \geq 0$ *the policy is distorted from the proposal by a uniformly distributed shock* $\tilde{\varepsilon}^t$ *with support* $[-\underline{\varepsilon}, \underline{\varepsilon}]$. *(iii) For* $y_i^t \in Z(z)$ *the allocations are as in (i) with the realization* ε^t *replacing* θ^t. *(iv) For* $y_i^t \notin Z(z)$ *the allocations are as in (ii) with the realization* ε^t *replacing* θ^t.

Assumption 2. *Implementation of a new policy* $y_i^t \neq q^{t-1}$ *has a higher probability of a shock than implementing the status quo policy, i.e.,* $\gamma \geq \eta$, *and a stochastically larger shock, i.e.,* $\underline{\varepsilon} \geq \underline{\theta}$.

4 Specific-Policy Coalition MPE with Implementation Uncertainty

This section shows by construction the existence of a coalition MPE that supports a specific policy with unequal allocations of the form $(1-z, z, 0), z < \frac{1}{2}$, where the coalition persists with probability $1-\eta$ and dissolves with probability η when implementation uncertainty is realized. The following bound on the shocks to the policy facilitates the exposition by simplifying the expressions for the continuation values.

Assumption 3. $\underline{\varepsilon} \leq \frac{1}{3}$.[9]

The equilibrium strategies for a specific-policy coalition MPE are identified in the following proposition.

Proposition 3. *Under Assumptions 1-3, the following strategies constitute a specific-policy coalition MPE for some parameter values and some* $z \in (0, \frac{1}{2}]$:

$$\sigma_i^*(q^{t-1}) = \begin{cases} q^{t-1} & \text{if } q^{t-1} \in \{z_{ij}(z), z_{ji}(z), z_{ik}(z), z_{ki}(z)\} \\ z_{i\ell}(z), \ell \in \{j,k\}, \text{with probability } \frac{1}{2} & \text{if } q^{t-1} \notin \{z_{ij}(z), z_{ji}(z), z_{ik}(z), z_{ki}(z)\}. \end{cases} \tag{3}$$

If $\gamma = \eta$, *legislators vote no when indifferent. If* $\gamma > \eta$, *legislators vote no or yes when indifferent.*

The strategies in Proposition 3 identify a class of specific-policy coalition equilibria that are indexed by the share z of the dollar going to the the coalition partner, where the originator of the

[9] The requirement that $\underline{\varepsilon} \leq \frac{1}{3}$ assures that the payoffs to coalition members are in $[0,1]$ with probability 1. This is shown in Corollary 4 in the Appendix.

coalition receives the larger share $1-z$. For $z=\frac{1}{2}$ and $\gamma=\eta=0$ (no uncertainty), the strategies in Proposition 3 are the same as strategies in Proposition 1.

The equilibrium strategies in Proposition 3 support minimal winning dynamic coalitions that form in the first period and continue to the next period with probability $1-\eta$ and with probability η dissolve as a result of the implementation uncertainty. That is, when implementation uncertainty is present, with probably 1 the shock moves the policy outside the set $Z(z)$, so that the coalition dissolves. A new coalition then forms in the next period.

The originator of a specific-policy coalition has proposal power but may not share it equally with the coalition partner. In an experiment, a specific-policy coalition equilibrium could be coordinated by communication among the legislators, as in the experiment by Agranov and Tergiman.

The strategies in Proposition 3 are shown in four steps to constitute a coalition MPE. In Lemma 1 the continuation values corresponding to the equilibrium strategies are derived. In Lemma 2 bounds on z and δ are established such that the strategies are impervious to one-step deviations. Lemma 3 establishes restrictions on the implementation uncertainty such that the bound on δ identified in Lemma 2 is less than one. In Lemma 4 existence is shown of a non-empty set of $z \leq \frac{1}{2}$ that satisfy the bounds identified in Lemma 2, which completes the proof.

Let $v_\ell(y^t), \ell=1,2,3$, denote the continuation value corresponding to the equilibrium strategies in Proposition 3 when the proposal y^t is implemented and no shock is realized. Let $v_\ell(y^{\varepsilon^t})$ denote the continuation value when y^t is implemented with a shock ε^t, and $v_\ell(y^{\theta^t})$ denote the continuation value when y^t is implemented when a shock θ^t is realized.

Lemma 1. *For $q^{t-1} \notin Z(z)$ the strategies in Proposition 3 yield continuation values*

$$\hat{v} = \frac{1}{3(1-\delta)}, i=1,2,3.$$

For $q^{t-1} = z_{ij}(z) \in Z(z)$ the strategies in Proposition 3 yield continuation values $v_\ell(z_{ij}^{\theta^t}) = v_\ell(z_{ij}^{\varepsilon_t}) = \hat{v}, \ell=1,2,3$, conditional on receiving a shock, and continuation values $v_\ell(z_{ij}(z)), \ell=1,2,3$, condi-

tional on not receiving a shock, where

$$v_i(z_{ij}(z)) = \frac{3(1-\delta)(1-z)+\eta\delta}{3(1-\delta)(1-\delta(1-\eta))} \tag{4}$$

$$v_j(z_{ij}(z)) = \frac{3(1-\delta)z+\eta\delta}{3(1-\delta)(1-\delta(1-\eta))} \tag{5}$$

$$v_k(z_{ij}(z)) = \frac{\eta\delta}{3(1-\delta)(1-\delta(1-\eta))}. \tag{6}$$

The proof is presented in the Appendix.

The incentive for the coalition members to preserve the coalition is the same as that identified in the sketch of the proof of Proposition 1. The originating coalition member receives a payoff of $1-z \geq \frac{1}{2}$ every period in which the coalition persists, whereas outside the coalition he receives this payoff only with probability $\frac{1}{3}$. The continuation value $v_i(z_{ij}(z))$ is decreasing in η, but the difference $v_i(z_{ij}(z)) - \hat{v}$ is positive, which provides a collective punishment for deviating from the coalition strategies. In addition, the dynamic payoff from switching coalitions is lower because the implementation uncertainty is greater when a policy changes.

The continuation value $v_i(z_{ij})$ for the originating coalition member i is greater than the continuation value $v_j(z_{ij})$ of the coalition partner. Although the coalition partner j has a lower continuation value, the continuation value for remaining in the coalition is greater than when the coalition dissolves, provided that $z > \frac{1}{3}$. It is the difference between $v_j(z_{ij}(z))$ and $\hat{v}$ that creates the incentive to accept the lower payoff .

The values $v_\ell(z_{ij}(z))$ of the game on the equilibrium path have the expected comparative statics properties. A higher probability η of implementation uncertainty reduces the probability of staying on the equilibrium path, since it reduces the continuation value to the coalition members and increases the continuation value to the out legislator. The continuation values are increasing in δ and independent of γ.

When z is not too small and the discount factor is high enough, the coalition partner has an incentive both to accept the coalition proposal and to maintain the status quo once the coalition has formed. Lemma 2 identifies the bounds on the coalition equilibrium allocations.

Lemma 2. *For* $\delta > \delta^o \equiv \frac{3-\frac{3}{2}\eta\underline{\theta}}{4-\gamma-3\eta-\frac{3}{2}(1-\eta)\eta\underline{\theta}}$, *there is no incentive for a legislator to deviate from the*

strategies given in Proposition 3 if and only if z satisfies[10]

$$\max\{z^*, z^o\} \leq z \leq \min\left\{\hat{z}_1, \frac{1}{2}\right\}, \tag{7}$$

where

$$\begin{aligned} z^* &= \frac{3 - 2\delta(\gamma - \eta)}{3(2 - \delta(\gamma - \eta))} \\ z^o &= \frac{3 - \delta(2 + \gamma - 3\eta) - \frac{3}{4}\eta\underline{\theta}(1 - \delta(1 - \eta))}{3(1 - \delta(\gamma - \eta))} \\ \hat{z}_1 &= \frac{2\delta(1 - \gamma) + \frac{3}{4}\eta\underline{\theta}(1 - \delta(1 - \eta))}{3(1 - \delta(\gamma - \eta))}. \end{aligned} \tag{8}$$

The proof of Lemma 2 is presented in the Appendix.

The lower bound z^* ensures that the coalition partner j has no incentive to propose (or accept) $z_{ji}(z)$ or $z_{jk}(z)$ if the status quo is $z_{ij}(z)$. Legislator j has a higher allocation $1 - z$ in $z_{ji}(z)$ and $z_{jk}(z)$, but changing the policy results in increased implementation uncertainty and a lower expected dynamic payoff. The bound z^* indicates the importance of the implementation uncertainty for sustaining unbalanced coalition allocations, since $z^* = \frac{1}{2}$ for $\gamma = \eta = 0$, in which case the only coalition allocation that can be supported has equal sharing of proposal power.

The bound z^o ensures that the coalition partner accepts the coalition proposal for status quos not in $Z(z)$. Remaining at a status quo outside $Z(z)$ has a continuation value of $\hat{v} = \frac{1}{3(1-\delta)}$, and for $z \geq z^o$, the coalition partner prefers to vote for the proposal and obtain $z + \delta v_j(z_{ij}(z))$ with probability $1 - \gamma$. The upper bound $\hat{z}_1$ is the coalition originator's analogue of z^o, and the intuition is similar.

It remains to determine if $\delta^o < 1$. Consider a status quo allocation $(0, 1, 0)$ for which legislator 1 makes the proposal $(\frac{1}{2}, \frac{1}{2}, 0)$. The status quo is very attractive for legislator 2, so with sufficient uncertainty about the new allocation relative to the status quo (γ high relative to η), legislator 2 could reject this proposal regardless of the discount factor. The following lemma establishes conditions on γ and η such that $\delta^o < 1$.

Lemma 3. *For* $(\gamma, \eta) \in R(\underline{\theta}) \equiv \left\{(\gamma, \eta) | \gamma \in [0, 1], \eta \in [0, \gamma],\ \textit{and}\ 1 - \gamma - 3\eta\left(1 - \frac{\eta\underline{\theta}}{2}\right) > 0\right\}$, $\delta^o < 1$.

Proof. Since $\delta^o = \frac{3 - \frac{3}{2}\eta\underline{\theta}}{4 - \gamma - 3\eta - \frac{3}{2}(1-\eta)\eta\underline{\theta}}$, it is straightforward to show that $\delta^o < 1 \Leftrightarrow (\gamma, \eta) \in R(\underline{\theta})$. ■

[10] Assumption 3 simplifies the expressions for the continuation values and the incentive constraints by keeping the shocked allocation in $[0, 1]$. Lemma 2 indicates that the assumption can be weakened to $\underline{\varepsilon} \leq \min\{z^*, z^o\}$ for $\delta > \delta^o$.

When $\gamma = \eta$, $z^* = \frac{1}{2}$ and for η small enough $(\eta, \eta) \in R(\underline{\theta})$, so $\delta^o < 1$ and $z^o \leq \frac{1}{2}$. So when $\gamma = \eta$ the unique allocation that a coalition equilibrium supports is $z = \frac{1}{2}$ by Lemma 2.[11] Hence the equal allocation coalition equilibrium can be supported when the probability of implementation is the same if the policy remains the same or changes.

The following lemma establishes that for $\delta \in (\delta^o, 1)$ there exists a z that satisfies (7).

Lemma 4. *For $\delta \in (\delta^o, 1)$, there exists a $z \leq \frac{1}{2}$ satisfying* (7)*. That is,*

$$\begin{aligned} \max\{z^*, z^o\} &\leq \frac{1}{2}, \text{ and} \\ \hat{z}_1 &> \frac{1}{2}. \end{aligned}$$

The proof is provided in the Appendix. Lemma 4 establishes that if legislators are sufficiently patient, there is a non-empty set of allocations that can be supported by the strategies given in Proposition 3. That is, the left side of (7) is no greater than one-half and the right side is at least one-half. This completes the proof of Proposition 3.

To summarize, Lemma 2 shows that for $\delta > \delta^o$ there is no incentive to deviate from the strategies in Proposition 3 for $z \leq \frac{1}{2}$ satisfying (7). Lemma 3 states that $\delta^o < 1$ for $(\gamma, \eta) \in R(\underline{\theta})$. Lemma 4 shows that for $\delta \in (\delta^o, 1)$ a non-empty set of $z \leq \frac{1}{2}$ satisfying (7) exists. Since $z \leq \frac{1}{2}$, the right-hand side of (7) is non-binding and the upper bound on z is $\frac{1}{2}$. Hence the allocations that can be supported as a specific-policy coalition MPE are all $z \in [z^+, \frac{1}{2}]$, where $z^+ = \max\{z^*, z^o\}$.

Both z^* and z^o are strictly decreasing in δ, which establishes the following corollary.

Corollary 1. *The set of allocations supported by specific-policy MPE is strictly decreasing in δ for $\delta \in (\delta^o, 1)$.*

The least upper bound and the greatest lower bound on the set of z for which the strategies constitute an equilibrium are characterized in Lemma 5 in terms of a cut-point on δ and in Lemma 6 in terms of a cut-point on γ. The following lemma establishes that $z^+ = z^*$ for sufficiently high discount factors and γ not too large. That is, z^* is the greater lower bound on z for δ sufficiently high.

[11] A more complete statement is presented in Corollary 2.

Lemma 5. *For $\gamma \leq \frac{2}{3}$, $z^+ = z^*$ for $\delta \geq \delta^+$ and $z^+ = z^o$ for $\delta < \delta^+$, where $\delta^+ > \delta^o$ and*

$$\delta^+ \equiv \frac{(4(1-\eta) + \frac{3}{4}\eta\underline{\theta}(2+\gamma-3\eta))}{2(\gamma-\eta)(2-\gamma-\eta-\frac{3}{4}\eta\underline{\theta}(1-\eta))} - \frac{\sqrt{(4(1-\eta)+\frac{3}{4}\eta\underline{\theta}(2+\gamma-3\eta))^2 - 4(3-\frac{3}{2}\eta\underline{\theta})(\gamma-\eta)(2-\gamma-\eta-\frac{3}{4}\eta\underline{\theta}(1-\eta))}}{2(\gamma-\eta)(2-\gamma-\eta-\frac{3}{4}\eta\underline{\theta}(1-\eta))}. \tag{9}$$

Proof. The difference $z^* - z^o$ is increasing in δ for $\gamma \leq \frac{2}{3}$. To show this, differentiation yields

$$\begin{aligned} \frac{\partial(z^* - z^o)}{\partial\delta} &= -\frac{\gamma-\eta}{3[2-\delta(\gamma-\eta)]^2} + \frac{(2-\frac{3}{4}\eta\underline{\theta})(1-\delta(1-\eta))}{3[1-\delta(\gamma-\eta)]^2} \\ &> -\frac{\gamma-\eta}{3[2-\delta(\gamma-\eta)]^2} + \frac{(2-\frac{3}{4}\eta\underline{\theta})(1-\delta(1-\eta))}{3[2-\delta(\gamma-\eta)]^2}. \end{aligned} \tag{10}$$

If $\gamma = \eta$, the first line of (10) is positive. If $\gamma = \eta$, the second line is positive if $2 - 3\gamma + \eta[1 - \frac{3}{4}\underline{\theta}(1-\gamma)] > 0$, which is the case for $\gamma \leq \frac{2}{3}$. The greater lower bound is then z^* if and only if $\delta \geq \delta^+$, where δ^+ in (9) is obtained by equating z^o and z^* in (8). ■

The following proposition characterizes z^+ in terms of the probability γ of implementation uncertainty with $z^+ = z^*$ for low γ and $z^+ = z^o$ for higher γ.

Lemma 6. *$z^+ = z^*$ for $\gamma \leq \gamma^e$ and $z^+ = z^o$ for $\gamma > \gamma^e$, where*

$$\gamma^e \equiv 1 + \frac{1}{8\delta}\left[3\eta\underline{\theta}(1-\delta(1-\eta))\right] - \frac{1}{8\delta}\left[(8-3\eta\underline{\theta})[1-\delta(1-\eta)]\left[16+(8-3\eta\underline{\theta})[1-\delta(1-\eta)]\right]^{\frac{1}{2}}\right]. \tag{11}$$

Proof. The bound z^* is decreasing in γ, and z^o is increasing in γ, so the difference $z^* - z^o$ is decreasing in γ. The greater lower bound is then z^* if and only if $\gamma \geq \gamma^e$, where γ^e in (11) is obtained by equating z^o and z^* in (8). ■

Lemmas 5 and 6 establish that the lower bound on the set of specific-policy coalition equilibrium is z^* when $\delta \geq \delta^+$ and $\gamma \leq \frac{2}{3}$ or when $\gamma \leq \gamma^e$, and the lower bound is z^o otherwise. When the discount factor is high ($\delta \geq \delta^+$) or the probability of implementation uncertainty when the policy changes is low ($\gamma \leq \gamma^e$), the binding dynamic incentive constraint (that establishes the lower bound z^*) is for the coalition partner to stay on the equilibrium path; i.e., to accept the allocation z and not propose a policy in $Z(z)$ that would yield $1 - z$. When $\delta \in (\delta^o, \delta^+)$ or $\gamma > \gamma^e$, the binding incentive

constraint is for the potential coalition partner to accept the coalition originator's offer for any status quo. The former represents an internal threat to the coalition, whereas the latter represents an external threat. The binding incentive constraints thus are associated with the coalition member who receives the lower allocation.

In a specific-policy coalition equilibrium with $z < \frac{1}{2}$ the originator of the coalition does not share proposal power equally with the coalition partner. Both z^* and z^o are greater than $\frac{1}{3}$, however, so the coalition partner receives more in each period than in sequential legislative bargaining theory. The dynamic incentive arising from internal and external threats from the coalition partner force the proposer to take less than in sequential bargaining theory.

The following corollary shows that when the probability of implementation uncertainty is the same when the policy changes as when it remains the same, the set of allocations supported by a coalition equilibrium is a singleton with equal division among the coalition members.

Corollary 2. *For $\gamma = \eta$, the set of z that can be supported as a coalition MPE is the singleton $\{\frac{1}{2}\}$ for $\eta < \frac{1}{3\underline{\theta}}[4 - 2(4 - \frac{3}{2}\underline{\theta})^{\frac{1}{2}}]$.*

Proof. Substituting $\gamma = \eta$ into z^* given in (8) gives $z^* = \frac{1}{2}$, so the only allocation that can be supported by a specific-policy coalition MPE has equal division between the coalition members. The condition $\eta < \frac{1}{3\underline{\theta}}[4 - 2(4 - \frac{3}{2}\underline{\theta})^{\frac{1}{2}}]$ implies $(\eta, \eta) \in R(\underline{\theta})$, so $\delta^o < 1$ by Lemma 3 and hence $z^o < \frac{1}{2}$. ■

Corollary 2 establishes that when $\gamma = \eta$, the lower bound $z^* = \frac{1}{2}$, so the only allocation supported by a coalition equilibrium is an equal division of the dollar among the coalition members. The equal division equilibrium in Proposition 1 is thus robust to implementation uncertainty provided that the probability that uncertainty is realized is the same when the policy is changed as when it remains the same.

Proposition 1 then follows from Corollary 2.

Proposition 1(ii) (restated). *If $\gamma = \eta = 0$, the strategies in Proposition 3 are the same as the strategies in Proposition 1(ii) for $z = \frac{1}{2}$ and are a coalition MPE for $1 > \delta > \delta^o = \frac{3}{4}$.*

Proof. Corollary 2 implies that only $z = \frac{1}{2}$ is supported with $\gamma = \eta$. For $\gamma = \eta = 0$, $(0, 0) \in R(\underline{\theta})$ and $\delta^o = \frac{3}{4}$. Then $z^+ = \frac{1}{2}$, and by Lemma 5 the strategies in Proposition 3 are a coalition MPE for $1 > \delta > \delta^o$. ■

For $\gamma = \eta = 0$, the unique coalition MPE allocation has $z = \frac{1}{2}$. By Lemma 1 when $\gamma = \eta = 0$, the continuation values for $q^{t-1} = z_{ij}(\frac{1}{2})$ are $v_i(z_{ij}(\frac{1}{2})) = v_i(z_{ij}(\frac{1}{2})) = \frac{1}{2(1-\delta)}$, and $v_k(z_{ij}(\frac{1}{2})) = 0$, and for $q^{t-1} \notin Z$, $\hat{v} = \frac{1}{3(1-\delta)}$. Hence, Lemma 1 provides the formal proof of part (i) of Proposition 1.

5 Tolerant Coalitions

The specific-policy coalition in Proposition 3 dissolves if implementation uncertainty is realized and moves the implemented policy away from the coalition allocation and hence off the equilibrium path. A coalition could, however, tolerate some changes in policy due to implementation uncertainty. This section identifies tolerant coalition MPE in which coalition members tolerate a degree of change in the coalition allocation, i.e., the coalition persists if the allocation remains in a tolerated set of allocations and dissolves if it is outside the set.

The value of a tolerant coalition is greater than the value of a corresponding coalition that dissolves with probability one when there is implementation uncertainty, since if the realized uncertainty leaves the allocation in the tolerated set, the coalition persists. This is formalized in Proposition 5 below. Tolerant coalitions are also more durable than coalitions that dissolve whenever the implemented policy differs from the policy adopted. As shown in Proposition 7, the set of allocations for which there is a tolerant coalition equilibrium can be strictly smaller, however, than the set of allocations characterized in Proposition 2 for specific-policy coalitions that do not tolerate any deviation from the coalition allocation. This results because toleration of a set of allocations strengthens the incentive of the coalition partner to deviate from the coalition strategies.

A tolerant coalition equilibrium has allocations in $Z(z)$ for $z \in \zeta^m \equiv [z^m, 1-z^m]$, $z^m \leq \frac{1}{2}$, so the coalition continues when the realization θ^t of the implementation uncertainty satisfies $z - \theta^t \in \zeta^m$. The following assumption assures that the implementation uncertainty is sufficiently great that the coalition can dissolve for all $z \in \zeta$ in any period from either a very high or a very low realization of $\tilde{\theta}^t$. This simplifies the expressions for the continuation values and facilitates the comparison between tolerant coalitions and specific-policy coalitions.

Assumption 4. $1 - 2z^m \leq \underline{\theta} \leq \underline{\varepsilon} \leq z^m$.

The following proposition identifies tolerant coalition MPE strategies.

Proposition 4. *Under Assumptions 1, 2, and 4 the following strategies constitute a tolerant coalition MPE for some $z^m \in [0, \frac{1}{2}]$, where the set of z^m is characterized in Lemma 9:*

$$y_i^t = \begin{cases} q^{t-1} & \text{if } q^{t-1} \in \{z_{ij}(z), z_{ik}(z), z_{ji}(z), z_{ki}(z), z \in \zeta^m\} \\ z_{i\ell}(z^m), \ell = j, k, \text{with probability } \frac{1}{2} & \text{otherwize} \end{cases}$$

If $\gamma = \eta$, legislators vote no if indifferent. If $\gamma > \eta$, legislators vote no or yes if indifferent.

Tolerant coalition equilibria exist and are simple with coalition members in period $t+1$ proposing the status quo when the payoff $z - \theta^t$ to a coalition member in period t is in ζ^m. If the realized implementation uncertainty θ^t is such that $z - \theta^t$ is not in ζ^m, the coalition dissolves and the legislator i selected in the next period proposes a policy $z_{i\ell}(z^m)$ in which i receives $1 - z^m$. Tolerant coalitions thus form immediately with the composition of the coalition determined by the selection of a proposer and the random selection of the coalition partner. As with a specific-policy coalition, the originator of a tolerant coalition receives a larger share than the coalition partner. After the first period of a coalition, however, the coalition partner could have the larger share as a result of the realization of the implementation uncertainty.

Proposition 4 is proven through a series of lemmas with Lemma 8 identifying the proposal made when a coalition forms and Lemma 9 identifying the set of z^m such that the strategies in Proposition 4 are an equilibrium. Lemma 7 establishes the continuation values corresponding to the conjectured equilibrium strategies in Proposition 4, and its proof is presented in the Appendix.

Lemma 7. *(i) For all $q^{t-1} \neq z_{ij}(z)$ for any $z \in \zeta^m$, the strategies in Proposition 4 yield a continuation value $\bar{v}_\ell(q^{t-1})$ for all legislators given by*

$$\bar{v}_\ell(q^{t-1}) = \hat{v} = \frac{1}{3(1-\delta)}. \tag{12}$$

(ii) For $q^{t-1} = z_{ij}(z), z \in \zeta^m$, the strategies in Proposition 4 yield continuation values $\bar{v}_\ell(z_{ij}(z))$ given by

$$\bar{v}_i(z_{ij}(z)) = \frac{3(1-\delta)(1-z)+\eta\delta+3(1-\delta)\eta\delta\nu(z^m)}{3(1-\delta)(1-\delta(1-\eta))} \tag{13}$$

$$\bar{v}_j(z_{ij}(z)) = \frac{3(1-\delta)z+\eta\delta+3(1-\delta)\eta\delta\nu(z^m)}{3(1-\delta)(1-\delta(1-\eta))} \tag{14}$$

$$\bar{v}_k(z_{ij}(z)) = \frac{\eta\delta-6(1-\delta)\eta\delta\nu(z^m)}{3(1-\delta)(1-\delta(1-\eta))}, \tag{15}$$

where

$$\nu(z^m) = \frac{1-2z^m}{6[2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2z^m)]}. \tag{16}$$

(iii) $\nu(z^m) \geq (>) 0$ *if* $1-2z^m \geq (>) 0$.

The continuation value for a specific-policy coalition corresponding to z^+ can be compared to the continuation value for a tolerant coalition with $z^m = z^+$.

Proposition 5. *Consider a* $z^m = z^+ < \frac{1}{2}$ *such that both a specific-policy coalition equilibrium and a tolerant coalition equilibrium exist. If* $\eta > (=) 0$*, the continuation value in* (4) *or* (5) *for a specific-policy coalition member* i *or* j *is strictly less than (equal to) the continuation value in* (13) *or* (14) *for tolerant coalition members* i *and* j*.*

Proof. The difference between the continuation values in (13) and (4) for the coalition originator i is

$$\bar{v}_i(z_{ij}(z^m)) - v_i(z_{ij}(z^m)) = \frac{\delta\eta\nu(z^m)}{1-\delta(1-\eta)}, \tag{17}$$

which is positive for $\eta > 0$ and $z^m < \frac{1}{2}$. If $\eta = 0$, the continuation values are the same. The same argument establishes the result for the coalition partner j. ■

With $\eta = 0$ in a specific-policy coalition equilibrium, once formed the coalition continues with probability one, as does a tolerant coalition. The continuation values thus are the same in the two equilibria. For $\eta > 0$ the shocked allocation has a positive probability of remaining in ζ^m in which case the coalition continues. A tolerant coalition thus is more valuable to its members than is the corresponding specific-policy coalition.

When $q^{t-1} \notin z_{ij}(z)$ for any $z \in \zeta^m$, the proposer i can choose any $z' \in \zeta^m$ and have a coalition form. The proposer benefits from a low z', but a low z' means that the probability that the coalition dissolves is higher than if z' were lower. The following lemma shows that the first

incentive dominates.

Lemma 8. *For $q^{t-1} \neq z_{ij}(z), z \in \zeta^m$, the optimal proposal by the originator i of a tolerant coalition is $y_i^t = z_{i\ell}(z^m), \ell = j, k$.*

Proof. Legislator i proposes $z_{ij}(z) \in Z(z), z \in \zeta^m$, which yields an expected utility $EU_i(z)$ given by

$$EU_i(z) = (1-\gamma)\left(1 - z + \delta\bar{v}_i(z_{ij}(z))\right) + \gamma\left(1 - z + E^t\tilde{\varepsilon}^t + \delta\bar{v}_i(z_{ij}^{\varepsilon^t})\right), \tag{18}$$

where $\bar{v}_i(z_{ij}(z))$ is given in (13) and $\bar{v}_i(z_{ij}^{\varepsilon^t})$ is given in (55) in the Appendix. From Lemma 11 in the Appendix, $\bar{v}_i(z_{ij}^{\varepsilon^t})$ does not depend on z, so differentiating (18) yields

$$\frac{dEU_i(z)}{dz} = -1 - \frac{\delta(1-\gamma)}{1-\delta(1-\eta)} < 0.$$

Consequently, i prefers the lowest $z \in \zeta^m$, so $z = z^m$ is optimal. ■

The bound z^m of the set ζ^m is obtained from the tolerant coalition incentive constraints as in the proof of Lemma 2 in the Appendix. The incentive constraints identify a set of bounds $z^m \in [\hat{z}^m, \frac{1}{2}]$ on the tolerated allocations, where $\hat{z}^m$ is the analogue to z^+ in the specific-policy equilibrium. The analogue z^{**} of z^* in (8) is given by[12]

$$z^{**} = z^* - \frac{\delta\left(\delta\eta(\gamma-\eta) - (1-\delta(1-\eta))\left(\gamma\frac{\theta}{\underline{\varepsilon}} - \eta\right)\right)\nu(z^{**})}{2-\delta(\gamma-\eta)}, \tag{19}$$

and the analogue z^{oo} of z^o is

$$z^{oo} = z^o - \frac{\delta\left((1-\gamma)\eta\delta + \gamma(1-\delta(1-\eta))\frac{\theta}{\underline{\varepsilon}}\right)\nu(z^{oo})}{1-\delta(\gamma-\eta)}. \tag{20}$$

The analogue $z^{\ell\ell}$ of z^ℓ defined in the proof of Lemma 2 in the Appendix is

$$z^{\ell\ell} = z^\ell - \eta\delta\nu(z^{\ell\ell}), \tag{21}$$

which is derived from the constraint corresponding to the coalition partner's incentive to deviate to a policy not in $Z(z), z \in \zeta^m$. Note that z^{oo} and $z^{\ell\ell}$ are less than their counterparts with a

[12]The bound z^{**} is derived from the incentive of a player who receives the allocation $z = z^m$, since that provides the strongest incentive to deviate from the coalition proposal strategy.

specific-policy coalition allocation, so $z^{oo} < \frac{1}{2}$ and $z^{\ell\ell} < \frac{1}{2}$ for $\delta > \delta^o$. It is straightforward to show that $z^{**} < \frac{1}{2}$ for all δ.

The set of tolerant coalition equilibria is characterized in the following lemma.

Lemma 9. *For $\delta > \delta^o$ a tolerant coalition equilibrium exists with $\zeta^m = [z^m, 1 - z^m]$, where z^m satisfies*

$$z^m \in \left\{ z | \max\{z^{**}, z^{oo}, z^{\ell\ell}\} \leq z \leq \frac{1}{2} \right\}. \tag{22}$$

The most tolerant equilibrium has $\hat{\zeta}^m = [\hat{z}^m, 1 - \hat{z}^m]$, where $\hat{z}^m$ is the minimum z^m of the class of tolerant equilibria. That is,

$$\hat{z}^m \equiv \min_z \left\{ z | \max\{z^{**}, z^{oo}, z^{\ell\ell}\} \leq z \leq \frac{1}{2} \right\}. \tag{23}$$

The strategies in Proposition 4 then are immune to deviations for $z \in \zeta^m, z^m \geq \hat{z}^m$.

The proof that the upper bound in (22) and (23) is $\frac{1}{2}$ is given in the Appendix. The proof of Lemma 9 parallels that for Proposition 2. Moreover, an argument analogous to that for Lemma 5 establishes that there is a non-empty set of parameter values such that tolerant coalition MPE exist.

A tolerant coalition equilibrium corresponding to each z^m satisfying (22) exists, so there is a continuum of tolerant coalition equilibria. The most tolerant equilibrium corresponds to $\hat{z}^m$ in (23). The characterization of $\hat{z}^m$ is complex, so the following section considers the case in which $\gamma > \eta = 0$. A closed-form characterization of $\hat{z}^m$ is provided and the underlying intuition is developed.

For $\gamma = \eta$ and $\underline{\varepsilon} = \underline{\theta}$ the unique tolerant coalition allocation has equal shares for the coalition members, so the coalition is no more tolerant than the specific-policy coalition in Proposition 3. This is formalized in the following corollary.

Corollary 3. *For $\gamma = \eta$ and $\underline{\varepsilon} = \underline{\theta}$, $z^{**} = z^* = \frac{1}{2}$, so a tolerant coalition is no more tolerant than the specific-policy coalition.*

When a tolerant coalition forms with $z = z^m$, the probability that it dissolves due to implementation uncertainty is $\frac{1}{2}\eta$, since the distribution of $\tilde{\theta}^t$ is symmetric about 0. If the coalition persists beyond the first period, the probability that it dissolves in the next period is smaller, since $z \geq z^m$. Also, when a tolerant coalition forms, the originator of the coalition receives a strictly greater

allocation than the coalition partner, but following a tolerated realization of the implementation uncertainty, the allocation to the other coalition member and the corresponding continuation value can be the larger. When the coalition approves a policy equal to the status quo and implementation uncertainty is tolerated, the coalition supports the new status quo policy in the next period.

5.1 Implementation Uncertainty Only When Policy Changes ($\eta = 0$)

To provide a further characterization of tolerant coalition equilibria, consider the case in which there is implementation uncertainty ($\gamma > 0$) associated with a change in policy but no implementation uncertainty ($\eta = 0$) when the status quo policy is continued. This will allow a complete characterization of the set of allocations supported by a tolerant coalition equilibrium as well as the comparative statics properties on the bound on that set. The most tolerant coalition for high discount factors is determined by the constraint arising from the incentive of the coalition member with the smaller allocation in the status quo to propose a policy different from the status quo. The bound on the most tolerant coalition is decreasing in the discount factor and in the probability γ that implementation uncertainty is realized when the policy is changed. The most tolerant coalition, however, results in allocations that are a strict subset for the allocation supported by specific-policy equilibria. A comparison between the set of specific-policy coalition equilibria and the set of tolerant coalition equilibria is also given.

As in Lemma 2, the most tolerant coalition has $\hat{z}^m$ in (23) equal to z^{**}, z^{oo}, or $z^{\ell\ell}$. Solving (19) for z^{**} yields

$$z^{**} = \frac{3 - \delta\gamma(2 - \frac{1}{4\underline{\varepsilon}})}{3(2 - \delta\gamma(1 - \frac{1}{6\underline{\varepsilon}}))}. \tag{24}$$

Solving (20) for z^{oo} yields

$$z^{oo} = \frac{3 - 2\delta - \delta\gamma(1 + \frac{1}{4\underline{\varepsilon}})}{3(1 - \delta\gamma(1 + \frac{1}{6\underline{\varepsilon}}))}. \tag{25}$$

Similarly, $z^{\ell\ell} = z^{\ell}$ from (21).

The following lemma identifies the discount factors such that the equilibrium allocation of the most tolerant coalition is identified by z^{**}.

Lemma 10. *There exists a $\delta^{\zeta} < 1$ such that for all $\delta > \delta^{\zeta}$ the set of allocations supported by a tolerant coalition MPE is $Z(z), z \in \zeta^{**} = [z^{**}, 1 - z^{**}]$.*

Proof. The difference between z^{**} and z^{oo} is

$$z^{**} - z^{oo} = \frac{3 - 2\delta\gamma + \frac{\delta\gamma}{4\underline{\varepsilon}}}{3(2 - \delta\gamma(1 - \frac{1}{6\underline{\varepsilon}}))} - \frac{3 - \delta(2+\gamma) - \frac{\delta\gamma}{4\underline{\varepsilon}}}{3(1 - \delta\gamma(1 + \frac{\delta\gamma}{6\underline{\varepsilon}}))}. \tag{26}$$

Evaluating (26) at $\delta = 0$ yields $(z^{**} - z^{oo})_{|\delta=0} = -\frac{1}{2}$. Taking the limit as $\delta \to 1$ yields

$$\sup_{\delta\to 1}(z^{**} - z^{oo}) = \frac{(1-\gamma)^2 + \frac{\gamma}{12\underline{\varepsilon}}}{9(2 - \gamma(1 - \frac{1}{6\underline{\varepsilon}}))(1 - \gamma(1 + \frac{1}{6\underline{\varepsilon}}))} > 0.$$

By the mean value theorem there exists one or more solutions to $z^{**} - z^{oo} = 0$ in $(0, 1)$. Let the largest of these be denoted by δ^*.

Similarly, $z^{**} - z^{\ell}$ is positive as $\delta \to 1$ and negative for $\delta = 0$. Let $z^{\ell} \in (0, 1)$ denote the largest δ such that $z^{**} - z^{\ell} = 0$.

The difference $z^{**} - z^*$ is positive, since

$$\begin{aligned} z^{**} - z^* &= \frac{3 - 2\delta\gamma + \frac{\delta\gamma}{4\underline{\varepsilon}}}{3(2 - \delta\gamma) + \frac{\delta\gamma}{2\underline{\varepsilon}}} - \frac{3 - 2\delta\gamma}{3(2 - \delta\gamma)} \\ &= \frac{\delta^2\gamma^2}{36\underline{\varepsilon}(2 - \delta\gamma)(3(2 - \delta\gamma) + \frac{\delta\gamma}{2\underline{\varepsilon}})} > 0. \end{aligned} \tag{27}$$

To determine the relation between δ^o and δ^*, evaluate the difference $z^{**} - z^{oo}$ at $\delta = \delta^o = \frac{3}{4-\gamma}$, which yields

$$(z^{**} - z^{oo})_{|\delta=\delta^o} = -\frac{\gamma}{2(8 - 5\gamma + \frac{\gamma}{2\underline{\varepsilon}})} < 0. \tag{28}$$

This implies that $\delta^* > \delta^o$.[13] Since $\delta^* > \delta^o$, $z^* > z^{\ell}$ from the proof of Lemma 2, so $z^{**} > z^{\ell}$.

Then, for $\delta > \delta^{\zeta} \equiv \max\{\delta^*, \delta^{\ell}\}$, z^{**} is the greatest lower bound in (23). ■

For some $\delta < \delta^{\zeta}$ there are equilibria for some $z^m \in [z^{oo}, z^{**})$. For $\delta \geq \delta^o = \frac{3}{4-\gamma}$ the lower bound for allocations for specific-policy equilibria is $z^o \leq \frac{1}{2}$, and the same is true for the bound z^{oo} for tolerant coalition equilibria. That is,

$$(z^{oo})_{|\delta=\delta^o} = \frac{1}{2}, \tag{29}$$

[13] The lower bound $\underline{\theta}$ of the support of $\tilde{\theta}^t$ ensuring that a coalition can dissolve for all $z \in \zeta^m$ is $1 - 2z^{**}$ for $\delta > \delta^*$, and evaluating the lower bound yields

$$1 - 2z^{**} = \frac{\delta\gamma}{3(2 - \delta\gamma(1 - \frac{1}{6\underline{\varepsilon}}))},$$

which is less than $\frac{\gamma}{3(2-\gamma)}$. For example, if $\gamma = \frac{1}{2}$, the lower bound $1 - 2z^{**}$ is less than $\frac{1}{9}$.

and z^{oo} is strictly decreasing in δ. Consequently, for δ in at least some subset of (δ^o, δ^ζ) the lower bound in (23) is z^{oo}.

The allocation proposed by the originator i of the coalition is thus $z_{ij}(z^{**}) = (1 - z^{**}, z^{**}, 0)$ for $\delta > \delta^\zeta$. The following proposition characterizes the allocation.

Proposition 6. *For $\delta > \delta^\zeta$ and $\gamma > 0$, the allocation z^{**} to the coalition partner is (i) strictly less than $\frac{1}{2}$, (ii) strictly greater than $\frac{1}{3}$, (iii) strictly decreasing in γ, and (iv) strictly decreasing in δ.*

Proof. To prove (iii) and (iv), differentiate z^{**} to obtain

$$\frac{dz^{**}}{d\delta} = \frac{\delta}{\gamma}\frac{dz^{**}}{d\gamma} = -\frac{\gamma}{3(2\delta\gamma(1 - \frac{1}{6\underline{\varepsilon}}))^2} < 0. \tag{30}$$

Properties (i) and (ii) are also straightforward to show. ■

The originator of a tolerant coalition thus receives a strictly larger share of the dollar in the first period of the coalition than does the coalition partner, but that share is less than in sequential legislative bargaining theory. The proposer in a tolerant dynamic coalition thus shares proposal power with the coalition partner, but the implementation uncertainty allows the originator to take the larger share but limits that share. The allocation to the originator is greater the more important is the future and hence the more valuable is the coalition to the coalition partner. As with specific-policy coalition equilibria, the set of allocations supported by the most tolerant coalition with z^{**} is increasing in the discount factor. Similarly, the coalition is more valuable the greater is the probability γ that policy uncertainty materializes when the coalition dissolves because of a change in the policy.

Specific-policy equilibria, however, can support a larger divergence between the allocation to the coalition originator and the partner than in the most tolerant coalition equilibrium.

Proposition 7. *For $\gamma > \eta = 0$, $q^0 \in X$, $(\gamma, 0) \in R(\underline{\theta})$ and $\delta > \max\{\delta^\zeta, \delta^+\}$, the set $[z^*, \frac{1}{2}]$ of coalition partner allocations supported by the class of specific-policy MPE strictly contains the set $[z^{**}, \frac{1}{2}]$ of coalition partner allocations supported by a tolerant coalition MPE.*

Proof. The difference $z^{**} - z^*$ is positive from (27), and the result follows from Lemmas 5 and 10. ■

For $\gamma > \eta = 0$ and $(\gamma, 0) \in R(\underline{\theta})$ the set of allocations supported by the most tolerant coalition equilibrium is strictly contained in the corresponding set for specific-policy equilibria, so tolerant

coalition equilibria can have less disparity in the allocations than can specific-policy equilibria. The proposal power of the originator of the coalition is shared more evenly in the most tolerant coalition equilibrium than in the most disparate specific-policy equilibrium.

The intuition underlying Proposition 7 is as follows. When $\eta = 0$ the value of a specific-policy coalition corresponding to z^m equals the value of a tolerant coalition corresponding to z^m, since once on the equilibrium path the allocation does not change provided no coalition member deviates from the equilibrium strategies.[14] A deviation from the tolerant coalition equilibrium strategies, however, is not as costly to the coalition members as is a deviation from the specific-policy equilibrium strategies because the former deviation could result in an allocation in the set ζ^m as a result of the realization of ε^t, whereas with a specific-policy coalition the shocked allocation equals z^m with probability 0.[15] The incentive constraint is thus tighter for a tolerant coalition equilibrium than for a specific-policy coalition equilibrium, so $z^{**} > z^*$.

The continuation value in (13) for the originator of the coalition is greater than the continuation value in (14) for the coalition partner, since $z^m < \frac{1}{2}$, so proposal power is shared although not equally. When a coalition persists, however, the allocation to the originator i of the coalition can be less than the allocation to the other coalition partner j due to implementation uncertainty. The continuation value for j is then given in (13), and the continuation value for i is then given in (14).

When $\eta > 0$, the probability that a tolerant coalition persists is higher than the probability a specific-policy coalition persists since the former policy can remain in the set η^m. The higher probability means that the continuation value for a tolerant coalition is higher. This effect is in the opposite direction of the effect characterized in Proposition 7, and the bound z^{**} can be lower than z^* for a specific-policy coalition. A tolerant coalition equilibrium thus can have greater inequality in the allocation among the coalition members than in a specific-policy coalition.

6 The Battaglini-Palfrey Experiments

6.1 The Experiments

Battaglini and Palfrey conducted two types of experiments using the same deterministic model considered here. In the first, referred to as the no-Condorcet winner (NCW) experiment, the

[14] Note that $\overline{v}_\ell(z_{ij}(z^m)) = v_\ell(z_{ij}(z^m))$ for $\ell = i, j, k$

[15] That is, $\bar{v}_\ell(z^{\varepsilon^t}) > v_\ell(z^{\varepsilon^t})$ for coalition members $\ell = i, j$.

policy space consisted of four allocations, and the second, referred to as the continuous experiment, was an approximation to the dynamic game considered here.[16] The experiments provide evidence about how people would actually play the game. As Battaglini and Palfrey note, "status quo outcomes have great inertia," and the theory presented here provides one explanation for the inertia–the status quo is an equilibrium allocation. In the experiment players have the opportunity to adopt coalition equilibrium strategies, but the game itself is complex and players may approach strategy formulation myopically, which should make it more difficult to support coalition equilibria. Battaglini and Palfrey find, for example, that players voted myopically for the alternative with the higher current period payoff. Players may also use non-Markov strategies or non-equilibrium strategies that can result in allocations not supported as MPE. Despite these qualifications players frequently used coalition strategies.

6.2 NCW Discrete Allocation Space Experiment

The NCW experiment consisted of two sessions separated by 2 months, and the subjects were undergraduate students at Princeton University. The first session consisted of 10 matches in which 9 participants were randomly assigned to 3-person committees, and the second session consisted of 10 matches with 12 participants randomly assigned to three-person committees for a total of 70 committees. Each match continued to the next round (period) with probability 0.75, and matches lasted between 1 and 10 rounds for a total of 291 rounds. Each committee played the same stage game in every round with the status quo equal to the policy in place at the end of the previous round. The initial status quo was chosen randomly from among the alternatives. In every round each player chose a provisional proposal, and one proposal was selected randomly from those provisional proposals. In each round each committee had 60 units of experiment currency to allocate by majority rule. There was no communication among the participants.

In the NCW experiment the alternatives were the allocations $\{(30, 30, 0), (0, 30, 30), (30, 0, 30), (20, 20, 20)\}$. The NCW experiment is of interest because it provides evidence about the formation and persistence of coalitions in a simplified game. It also provides direct evidence about whether players behave in a manner consistent with the indifference rule; i.e., whether they voted for the status quo when indifferent between it and a proposal. A dynamic coalition for the NCW experiment is a set

[16] Battaglini and Palfrey also conducted a discrete allocation experiment with one of the alternatives a Condorcet winner. That experiment is not considered here.

of players such that a policy persists and gives them the same payoff in at least two consecutive periods.

A MPE exists for the set of discrete allocation for all $\delta \in [0, 1)$, where the player selected proposes keeping 30 and allocates 30 to another player selected at random. Players use stage undominated voting strategies and vote for the proposal if indifferent. Battaglini and Palfrey show that voting myopically for the alternative giving the strictly higher current round payoff and randomizing when the allocations are the same is also a MPE. With either far-sighted or myopic voting strategies the allocations can change when a player is recognized, and each player has a continuation value of $\frac{20}{1-\delta}$. On the equilibrium path of play it is not possible to distinguish between myopic and farsighted voting.

A dynamic coalition MPE also exists for all $\delta \in [0, 1)$ with the player selected proposing q^{t-1} if the allocation gives the proposer 30 and, if the allocation gives the proposer 0 or 20, proposing keeping 30 and allocating 30 randomly to one of the other players. The coalition MPE voting strategy is to vote for the alternative with the strictly greater dynamic payoff and to vote for the status quo when indifferent.[17] This is also a coalition MPE with players voting myopically but for the status quo when indifferent. Allocations do not change after the first round, and farsighted and myopic voting yield the same behavior.

The equilibria in the NCW game thus depend on the indifference rule. In the experiment there is nothing that tells a player how to vote when indifferent between two alternatives in which she receives 30, so the players could be thought of as randomizing between the two alternatives. An allocation then would persist from one period to the next with probability one-third (since the universal allocation policy never results in equilibrium). With an indifference rule of voting for the status quo when indifferent, the coalition equilibrium prediction is that an allocation should persist from one period to the next with probability one.

The two indifference rules and the corresponding theories can be assessed using the frequency with which policies persist from one period to the next. The MPE identified by Battaglini and Palfrey should have coalitions persisting with probability one-third, whereas the coalition MPE should have coalitions present with probability one. Any statistical test would reject the coalition MPE, since coalitions are not present in all rounds. Coalitions are present in 74.2% of the rounds,

[17] If the allocation space is augmented to include the three dictatorial allocations in which the legislator selected proposes to keeps 60, this is also a dynamic coalition MPE for $\delta \geq \frac{3}{4}$.

and if the rounds with a universal initial status quo are excluded, 78.5% of the rounds have coalitions with policies that persist from one round to the next. Table 4 in Battaglini and Palfrey gives the frequency with which an allocation persisted from one round to the next. The frequencies for coalitions {12}, {13}, and {23} were 0.77, 0.79, and 0.75, respectively, so coalitions were present in approximately 77% of the rounds. The frequency with which coalitions persist is closer to the coalition MPE prediction than to the Battaglini and Palfrey MPE prediction, providing a degree of support for the coalition MPE prediction.

A stronger test for the presence of a coalition is for the players favored by the status quo to vote against any proposal that differs from the status quo. Battaglini and Palfrey report that almost all (96%) of the votes cast by participants were for the alternative offering the larger payoff in the round. They conclude that voting was myopic and self-interested. To examine the voting and the alternative selection rules in more detail, the votes have been examined for those players who were indifferent between a proposal and the status quo when the two allocations differed. In those rounds one player voted for the status quo in 23 of 35 rounds, a second player in 27 of 53 rounds, and the third player in 19 of 30 rounds. In the aggregate, the players voted for the status quo with probability 0.585. For those rounds in which a coalition was present, the corresponding numbers are 20 of 27, 24 of 45, and 16 of 23, respectively, so in the aggregate the players in a coalition voted for the status quo with probability 0.632.[18] Then, under the indifference rule of voting for the proposal when indifferent, only 36.8% of the participants voted for the proposal. These data also provide a degree of support for the indifference rule of voting for the status quo when indifferent between it and a proposal.

Table 1 presents data on the duration of coalitions. The 70 committees experienced 78 coalitions. Forty-seven coalitions lasted only 1 or two rounds, and of those 18 were due to the match ending. Note that a coalition cannot be present in the first round if the initial status quo is (20,20,20), so the probability that a coalition could form in the first round is bounded above by three-quarters. The longest lasting coalitions were one of 9 rounds and 3 of 8 rounds. For 2 of the coalitions with 8 rounds the game ended with the 8th round. Twenty-two coalitions lasted throughout the match. The longest-lived coalition was formed in the second round and continued for 9 rounds until the match ended.

[18]For rounds in which a player who was indifferent between the proposal and the status quo and the coalition was preserved, that player was always pivotal since indifference can only be between two minimal winning majorities. In all such cases the player voted for the status quo.

A coalition with an allocation equal to the status quo q^{t-1} can result for one of two reasons. First, the proposal could equal q^{t-1}, in which case the vote does not matter. Second, the proposal could be different from the status quo, but a majority votes for the status quo. The experiment provides evidence on the reason for the persistence of a coalition. For the rounds in which the status quo benefits the first player, 68.5% of that player's proposals equaled the status quo, and for the other two players the corresponding figures were 74.6% and 70.6%, respectively, for an average of 71.2%. The frequency with which some coalition member was selected as the proposer was approximately two-thirds, so in approximately 47% of the rounds the proposal should equal the status quo. In the experiment the observed frequency was 45.7%. In 28.5% of the rounds, the proposal was different from the status quo and was rejected, so 52.5% of the proposals that differed from the status quo were rejected. (Note that 45.7+28.5 = 74.2.) Consequently, in about half the rounds in which there was a coalition, the coalition persisted because the status quo was proposed, and in about half the rounds it persisted because a proposal different from the status quo was rejected by the coalition members.

Three-player coalitions were present in 5 of 70 committees, and all but one resulted from an initial status quo of (20, 20, 20). The universal allocation is not a MPE, but playing it in every period yields the same average payoff as does rotating or random minimal winning majorities. Moreover, the universal allocation is also supported as a subgame perfect equilibrium of the game in the discrete experiment for $\delta \geq \frac{1}{3}$. The universal allocation is more prevalent in the first experiment session than in the second, and the explanation may be that in the first session the initial status quo was the universal allocation for one of the three committees in each of the first five matches, whereas in the second session the universal allocation was not an initial status quo for any of the four committees for the first 4 matches.[19] The universal allocation resulted in only 21 of the 291 rounds or 7.2% of the rounds. Two committees sustained the universal coalition for 4 rounds and another for 6 rounds. For the two committees with coalitions that lasted 4 rounds the universal allocation was the initial status quo.

[19]The universal allocation was the initial status quo for 8 committees in each experiment.

6.3 Continuous Allocation Space Experiment

The continuous allocation space experiment was implemented with a discrete grid with 60 available units allocable in increments of one and a discount factor of $\delta = \frac{5}{6}$.[20] With this richer set of allocations participants would likely be more uncertain about how the status quo is likely to evolve. The experiment consisted of 10 matches in which 12 participants were randomly assigned to three-person committees.[21]

With this more complex allocation space, participants played the universal allocation of $(20, 20, 20)$ with frequency 0.37. The universal allocation is never a MPE, even with risk averse players, nor is it a subgame perfect equilibrium when only collective punishments are available.[22] The universal allocation, however, can be supported as a subgame perfect equilibrium for sufficiently high δ. The participants in the experiment could have been playing a universal allocation subgame perfect equilibrium, but they could also be playing a natural focal allocation in which each player receives the same payoff. This focal allocation yields the same dynamic payoff as the expected payoff of a rotating dictator equilibrium or randomized proposals with myopic voting, but the expected payoff is strictly less than in a coalition equilibrium.

Majoritarian allocations were played more frequently than the universal allocation, and Table 2 presents the transition probabilities for majoritarian, universal, and dictator allocations. A universal allocation has each player receiving at least 15, a dictator allocation has one player receiving at least 50, and for the other allocations the majoritarian allocation M_{ij} has i and j receiving at least as much as k.[23] The definition of a majoritarian set corresponds to a tolerant coalition MPE, although the third member can receive a positive allocation in M_{ij}. The probability of transitioning to the same majoritarian allocation allocation set, which is then interpreted as a coalition, was 0.522 for M_{12} and 0.582 for M_{13}. The transition probability for M_{23} was only 0.313, presumably due to randomness, and the probability of transitioning to the universal allocation was 0.281.[24] Battaglini and Palfrey use somewhat larger sets M_{ij} and report (Table 3) transition

[20] Battaglini and Palfrey also conducted an experiment with $\delta = \frac{3}{4}$, but $\delta^o = \frac{3}{4}$, so there is no coalition MPE corresponding to the experiment.

[21] A programming error resulted in the loss of the initial status quo in 4 of the 10 matches, so the data discussed here include only the 6 matches for which the initial status quo is known.

[22] Battaglini and Palfrey show that allocations in which all players receive between 20 and 40 occurs with high probability in a logistic quantal response equilibrium when players are highly risk averse. Richter (2011) shows that the universal allocation can be supported by a MPE in a model in which part of the dollar can be wasted off the equilibrium path.

[23] These definitions are slightly different from those used by Battaglini and Palfrey.

[24] The transition probabilities for M_{ij} are much lower for the $\delta = \frac{3}{4}$ experiment than for the $\delta = \frac{5}{6}$ experiment, as

probabilities of 0.55, 0.67, and 0.39 for M_{12}, M_{13}, and M_{23}, respectively. Both of these sets of transition probabilities are consistent with the presence of dynamic coalitions. With the exception of M_{23}, the probabilities are substantially higher than with random transitions. In the experiment none of the initial status quos was in the set Z, so first-period transitions should be random. Table 3 thus reports the transition probabilities for rounds after the first. The transition probabilities for majoritarian coalitions are slightly higher than in Table 2 for M_{12} and M_{13}, and slightly lower for M_{23}. In Tables 2 and 3 the transition probabilities for the universal coalition are 0.848 and 0.875, respectively.

The continuous allocation experiment provides a degree of support for coalition behavior, although the actual allocations differ from those in coalition equilibria, particularly with respect to the frequency of the universal allocation. As noted above, the logistic quantal response equilibrium in which players' best response functions are subject to random shocks predicts that the universal allocation occurs with high probability when players are sufficiently risk averse.

As in the experiment by Agranov and Tergiman, communication among experiment participants could increase the frequency with which dynamic coalitions are formed and extend their duration. As in that experiment the participant recognized as the initial proposer could initiate communication before making a proposal, and that communication could explain the benefits of a coalition and invite the other participants to join in a coalition, understanding that the coalition partner would be selected at random. The other participants would then compete to be in the coalition, but the proposer would not bargain them down because the resulting coalition would not be durable. The proposer thus could simply announce the continuation value for a coalition partner with a speech as in Section 2.4. An experiment could be extended by introducing implementation uncertainty to study whether participants tolerated a degree of uncertainty.

7 Conclusions

Public policymaking is a dynamic process in which the opportunity to set the agenda gives legislators temporary power that can be used to change policy to their advantage. Distributive policy in particular could be prone to opportunistic behavior, and shifting agenda-setters could lead to policy instability. Yet most policies exhibit a measure of stability that is unexplained by dynamic

would be expected from the theory developed here.

legislative bargaining theory. This paper shows that dynamic coalitions can be expected to form beginning from any status quo and once formed to support stable policies over time.

The originator of a dynamic coalition has agenda-setting or proposal power as in (static) sequential bargaining theory, but in contrast to that theory the originator of a dynamic coalition must share proposal power with the other members of the coalition. Sharing is required to satisfy dynamic incentive constraints resulting from the opportunity of coalition members to propose alternative policies and to vote against the coalition policy when the status quo is favorable. In the basic, deterministic model the dynamic incentives require the originator to share proposal power equally with the coalition partner. The dynamic coalition is minimal winning, and the coalition persists from one period to the next with coalition members receiving equal allocations in every period. Despite changing agenda-setters, the coalition persists and its members vote for the coalition policy over all policy alternatives when the future is sufficiently important. The equilibrium allocation in the deterministic model is unique, since any other allocation generates an incentive to break the coalition for some initial status quo.

Uncertainty can be associated with the implementation of policies, and that uncertainty can be greater when the policy changes than when it remains unchanged. Dynamic coalition equilibria are present with implementation uncertainty. The equal sharing allocation continues to be supported by a coalition equilibrium, but other specific-policy coalition equilibria exist where in every period the originator of the coalition receives more than the coalition partner. The originator shares proposal power relative to the outcome in sequential legislative bargaining theory, but the coalition partner accepts a smaller allocation than the originator receives rather than break the coalition and face increased uncertainty when the policy changes. The set of allocations supported by specific-policy coalition equilibria is increasing in the discount factor.

Coalitions in specific-policy equilibria dissolve when implementation uncertainty arises, but a coalition could tolerate a degree of implementation uncertainty and continue to the next period without dissolving. A coalition continues when the implemented policy remains in a set of tolerated policies, but if the implemented policy differs too much from the adopted policy, the coalition dissolves. A tolerant coalition thus can persist over time, and policies have a degree of stability, provided that the realized implementation uncertainty is not too large. A tolerant coalition equilibrium supports a set of allocations, and those allocations can be a subset of the allocations that are supported by some specific-policy equilibrium.

Dynamic coalition equilibria have particularly simple strategies with legislators proposing either the status quo or a new coalition with the coalition partner selected randomly. These equilibria all have the property that coalitions are minimal winning, form in the first period, and dissolve only when implementation uncertainty causes the implemented policy to fall outside the set of supported coalition allocations. These equilibria could be expected to arise in real play, since the strategies of legislators could be coordinated through straightforward communication between the originator and potential coalition partners.

Dynamic coalition equilibria be extended in a number of directions. As in Section 2.5 the specific-policy and tolerant coalition equilibria can be extended to a legislature with n members and a majority requirement of m. In the theory presented here, legislators have been assumed to be risk neutral, and if legislators were risk averse, the equal sharing equilibrium in the deterministic model remains an equilibrium since it involves perfect smoothing over time. With implementation uncertainty coalition equilibria should also exist with risk averse legislators who would prefer the smaller uncertainty associated with the status quo policy. In the pure distribution game considered here, the preferences of legislators are directly opposing yet coalitions can form and be stable, and in an extension to a policy space in which preferences are partially aligned, dynamic coalitions should also be present although their properties could differ. In particular, the extent to which proposal power is shared would depend on the preference alignment, and coalition might not be minimal winning. A natural extension is to quasi-linear preferences in which in every period the legislature allocates resources between a public good and a distributive good.

Appendix

Specific-policy equilibrium allocations

Proof of Lemma 1

Proof. The continuation value $\hat{v}$ when $q^{t-1} \notin Z(z)$ for legislator 1 is given by

$$\begin{aligned}
\hat{v} \;=\; & (1-\gamma)\left[\frac{1}{3}\left[\frac{1}{2}(1-z+\delta v_1(z_{12}(z)))\right]+\left[\frac{1}{2}(1-z+\delta v_1(z_{13}(z)))\right]\right] \\
& +\frac{1}{3}\left[\frac{1}{2}\left(z+\delta v_1(z_{21}(z))\right)+\frac{1}{2}\delta v_1(z_{23}(z))\right] \\
& +\frac{1}{3}\left[\frac{1}{2}\left(z+\delta v_1(z_{31}(z))\right)+\frac{1}{2}\delta v_1(z_{32}(z))\right] \\
& +\gamma\left[\frac{1}{3}\left[\frac{1}{2}(1-z+E^t\tilde{\varepsilon}^t+\delta v_1(z_{12}^{\varepsilon^t}))+\frac{1}{2}(1-z+E^t\tilde{\varepsilon}^t+\delta v_1(z_{13}^{\varepsilon^t}))\right]\right] \\
& +\frac{1}{3}\left[\frac{1}{2}(z-E^t\tilde{\varepsilon}^t+\delta v_1(z_{21}^{\varepsilon^t}))+\frac{1}{2}\delta v_1(z_{23}^{\varepsilon^t})\right] \\
& +\frac{1}{3}\left[\frac{1}{2}(z-E^t\tilde{\varepsilon}^t+\delta v_1(z_{31}^{\varepsilon^t}))+\frac{1}{2}\delta v_1(z_{32}^{\varepsilon^t})\right], \qquad (31)
\end{aligned}$$

and the continuation values for the other legislators are analogous. Given $q^{t-1} \notin Z(z)$, the allocation $z_{ij}^{\varepsilon^t}$ resulting from a proposal $y^t \neq q^{t-1}$ is not in $Z(z)$ with probability one, so the continuation values $v_\ell(z_{ij}^{\varepsilon^t}) = \hat{v}, \ell, i, j = 1,2,3,\ i \neq j$. For $q^{t-1} = z_{ij}(z)$ the allocation $z_{ij}^{\theta^t} \notin Z(z)$ with probability one, so the continuation value $v_\ell(z_{ij}^{\theta^t}) = \hat{v}, \ell, i, j = 1,2,3,\ i \neq j$. For $q^{t-1} \in Z(z)$ the dynamic payoffs $v_i(z_{12}(z)), i = 1,2,3$, are given by

$$v_1(z_{12}(z)) = (1-\eta)[1-z+\delta v_1(z_{12}(z))]+\eta[1-z+E^t\tilde{\theta}^t+\delta v_1(z_{12}^{\theta^t})], \qquad (32)$$

$$v_2(z_{12}(z)) = (1-\eta)[z+\delta v_2(z_{12}(z))]+\eta[z+E^t\tilde{\theta}^t+\delta v_2(z_{12}^{\theta^t})], \qquad (33)$$

$$v_3(z_{12}(z)) = (1-\eta)\delta v_3(z_{12}(z))+\eta\delta v_3(z_{12}^{\theta^t}). \qquad (34)$$

Continuation values when the allocation is any element of $Z(z)$ are defined analogously. Substituting $\hat{v}$, $v_i(z_{ij}(z))$, $v_j(z_{ij}(z))$ and $v_k(z_{ij}(z))$ from Lemma 1, into (31), (32), (33) and (34) verifies the equilibrium conditions. ■

Proof of Lemma 2

Proof. The proof proceeds by checking incentives to deviate from the strategies in Proposition 2.

1. Consider $q^{t-1} = z_{ij}(z)$.

   (a) Consider voting strategies.

      i. If i or j is the proposer, on the equilibrium path the agenda is $A = \{z_{ij}(z), z_{ij}(z)\}$, so $q^t = z_{ij}(z)$ regardless of the votes.

      ii. If k is the proposer, the agenda is $A = \{z_{ij}(z), z_{ki}(z)\}$ or $A = \{z_{ij}(z), z_{kj}(z)\}$. As shown below, legislators i and j reject both $z_{ki}(z)$ and $z_{kj}(z)$ since these involve a change in the status quo and do not provide higher continuation payoffs.

   (b) Consider i's incentives to propose a deviation.

      i. Proposing $z_{ji}(z)$ or $z_{ki}(z)$ changes the status quo if approved, and since $z \leq \frac{1}{2}$ and $\eta \leq \gamma$, $z_{ij}(z)$ is preferred by i. Formally,

$$\begin{aligned}&(1-\eta)[1 - z + \delta v_i(z_{ij}(z))] + \eta[1 - z + E^t\tilde{\theta}^t + \delta v_i(z_{ij}^{\theta^t})]\\ &\geq (1-\gamma)[z + \delta v_i(z_{ji}(z))] + \gamma[z - E^t\tilde{\varepsilon}^t + \delta v_i(z_{ji}^{\varepsilon^t})].\end{aligned}$$

      ii. Proposing z_{ik} results in a change in the status quo if approved, so i prefers to propose $z_{ij}(z)$, since by stochastic dominance

$$\begin{aligned}&(1-\eta)[1 - z + \delta v_i(z_{ij}(z))] + \eta[1 - z + E^t\tilde{\theta}^t + \delta v_i(z_{ij}^{\theta^t})]\\ &\geq (1-\gamma)[1 - z + \delta v_i(z_{ik}(z))] + \gamma[1 - z + E^t\tilde{\varepsilon}^t + \delta v_i(z_{ik}^{\varepsilon^t})].\end{aligned}$$

      iii. Proposing z_{jk} or z_{kj} changes the status quo if approved, and i prefers to propose $z_{ij}(z)$, since $v_i(z_{ij}(z)) > v_i(z_{jk}(z)) = v_k(z_{ij}(z))$ from (13) and (15). That is,

$$\begin{aligned}&(1-\eta)[1 - z + \delta v_i(z_{ij}(z))] + \eta[1 - z + E^t\tilde{\theta}^t + \delta v_i(z_{ij}^{\theta^t})]\\ &\geq (1-\gamma)\delta v_i(z_{jk}(z)) + \gamma\delta v_i(z_{jk}(z^{\varepsilon^t})).\end{aligned}$$

      iv. The best proposal deviation for i outside the set $Z(z)$ gives 1 to i with i and k

voting for the proposal. Proposer i prefers not to deviate if and only if

$$(1-\eta)[1-z+\delta v_i(z_{ij}(z))]+\eta[1-z+E^t\tilde{\theta}^t+\delta v_i(z_{ij}^{\theta^t})]\geq 1-\gamma\frac{\underline{\varepsilon}}{4}+\delta\frac{1}{3(1-\delta)},$$

where $1-\gamma\frac{\underline{\varepsilon}}{4}=1-\gamma+\gamma\left[\int_{-\underline{\varepsilon}}^{0}(1+\varepsilon^t)\frac{d\varepsilon^t}{2\underline{\varepsilon}}+\int_0^{\underline{\varepsilon}}\frac{d\varepsilon^t}{2\underline{\varepsilon}}\right]$ is i's expected truncated payoff in period t. Then legislator i does not deviate if

$$z\leq z^u\equiv\frac{\underline{\varepsilon}\gamma(1-\delta(1-\eta))}{4}+\frac{2\delta(1-\eta)}{3}.$$

(c) Consider j's incentives to propose a deviation.

i. If j proposes z_{ki} or z_{ik}, these proposals give legislator j the same payoff since they do not favor j. Legislator j will not deviate to these allocations since

$$\begin{aligned}&(1-\eta)[z+\delta v_j(z_{ij}(z))]+\eta[z+E^t\tilde{\theta}^t+\delta v_j(z_{ij}^{\theta^t})]\\&\geq(1-\gamma)[\delta v_j(z_{ki}(z))]+\gamma[E^t\tilde{\varepsilon}^t+\delta v_j(z_{ki}^{\varepsilon^t})].\end{aligned}$$

ii. If j proposes z_{kj} and it is approved, the status quo changes, which is worse than the payoff under the equilibrium strategies, since $\gamma\geq\eta$.

iii. If j proposes $z_{ji}(z)$ or $z_{jk}(z)$ and it is approved, j receives $1-z$ in expectation in the current period, and with probability $1-\gamma$ the continuation value is $v_j(z_{ji}(z))=v_i(z_{ij}(z))$ and with probability γ the continuation value is $\hat{v}=\frac{1}{3(1-\delta)}$ by Lemma 1. Legislator j has no incentive to deviate if and only if

$$\begin{aligned}&(1-\eta)[z+\delta v_j(z_{ij}(z))]+\eta[z-E^t\tilde{\theta}^t+\delta v_j(z_{ij}^{\theta^t})]\\&\geq(1-\gamma)[1-z+\delta v_j(z_{jk}(z))]+\gamma[1-z+E^t\tilde{\varepsilon}^t+\delta v_j(z_{jk}^{\varepsilon^t})]\\\Leftrightarrow\quad&z\geq\frac{3-2\delta(\gamma-\eta)}{3(2-\delta(\gamma-\eta))}=z^*.\end{aligned}\tag{35}$$

Note that $z^*=\frac{1}{2}$ for $\gamma=\eta$ and $z^*<\frac{1}{2}$ for all $\delta\in(0,1)$ if $\gamma>\eta$.

iv. If j proposes $y_j^t\notin Z(z)$, the best proposal gives 1 to j with j and k voting for the proposal, and the expected truncated payoff is $1-\gamma\frac{\underline{\varepsilon}}{4}+\delta\frac{1}{3(1-\delta)}$. Legislator j prefers

the conjectured equilibrium strategies if and only if

$$\begin{aligned}
&(1-\eta)[z+\delta v_j(z_{ij}(z))]+\eta[z-E^t\tilde{\theta}^t+\delta v_j(z_{ij}^{\theta^t})] \\
&\geq 1-\gamma\frac{\underline{\varepsilon}}{4}+\delta\frac{1}{3(1-\delta)} \\
&\Leftrightarrow z\geq z^\ell\equiv\frac{3-2\delta(1-\eta)}{3}-\frac{\underline{\varepsilon}\gamma(1-\delta(1-\eta))}{4}. \qquad (36)
\end{aligned}$$

Note that $z^\ell+z^u=1$.

(d) Consider k's incentive to propose a deviation.

i. If $z\in[\max\{z^*,z^o\},\frac{1}{2}]$, i and j prefer $z_{ij}(z)$ to any other allocation. Hence, any proposal by k different from $z_{ij}(z)$ will be rejected. Legislator k's payoff is the same if he proposes $z_{ij}(z)$ and it is accepted, or proposes another allocation that is rejected, hence legislator k has no incentive to deviate from the equilibrium strategies.

2. Consider $q^{t-1}\notin Z(z)$

(a) Consider j's incentive to vote for the equilibrium proposal $z_{ij}(z)$. The best status quo for j, gives 1 to j. Legislator j votes for $z_{ij}(z)$ rather than the status quo if and only if

$$1-\eta\frac{\underline{\theta}}{4}+\delta\frac{1}{3(1-\delta)}\leq(1-\gamma)[z+\delta v_j(z_{ij}(z))]+\gamma[z-E^t\tilde{\varepsilon}^t+\delta v_j(z_{ij}^{\varepsilon^t})],\ \text{for } \gamma>0$$

where $1-\eta\frac{\underline{\theta}}{4}=1-\eta+\eta\left[\int_{-\underline{\theta}}^{0}(1+\theta^t)\frac{d\theta^t}{2\underline{\theta}}+\int_0^{\underline{\theta}}\frac{d\theta^t}{2\underline{\theta}}\right]$ is the expected truncated payoff in period t, and

$$1+\delta\frac{1}{3(1-\delta)}<z+\delta v_j(z_{ij}(z)),\ \text{for } \gamma=\eta=0. \qquad (37)$$

Legislator j accepts the proposal if

$$z\geq\frac{3-\delta(2+\gamma-3\eta)-\frac{3}{4}\eta\underline{\theta}(1-\delta(1-\eta))}{3(1-\delta(\gamma-\eta))}=z^o,\ \text{for } \gamma>0\ .$$

For $\gamma=\eta=0$, (37) is satisfied for $z>z^o=1-\frac{2}{3}\delta$. From (35) at $\gamma=\eta=0$, $z^*=\frac{1}{2}$. Hence if $\delta>\frac{3}{4}$, $z^o<z^*=\frac{1}{2}$. More generally, for $\delta>\delta^o=\frac{3-\frac{3}{2}\eta\underline{\theta}}{4-\gamma-3\eta-\frac{3}{2}(1-\eta)\eta\underline{\theta}}$, $z^o<\frac{1}{2}$ for any value of γ and η. Hence for $\delta>\delta^o$, $\max\{z^o,z^*\}=z^*$ when $\gamma=\eta=0$ and hence (37) is satisfied if (35) is satisfied. The restriction $\delta>\delta^o$ is necessary for $z^o<\frac{1}{2}$ so δ^o

is the lower bound on δ for a coalition MPE to exist. The lower bound δ^o also assures that there is no incentive to deviate if (7) is satisfied.

(b) Consider i's incentive to make a proposal other than $z_{ij}(z)$.

i. By $v_i(z_{ij}(z)), v_j(z_{ij}(z)), v_k(z_{ij}(z))$ given in Lemma 1 $z_{ij}(z)$ gives i the highest payoff among proposals in $Z(z)$, so there is no incentive to make any other proposal in $Z(z)$.

ii. The best proposal $y_i^t \notin Z(z)$ gives 1 to i.

A. Assume that the status quo also gives 1 to i. Legislator i prefers $z_{ij}(z)$ if and only if

$$\begin{aligned} 1 - \eta\frac{\underline{\theta}}{4} + \delta\frac{1}{3(1-\delta)} &\leq (1-\gamma)[1 - z + \delta v_i(z_{ij}(z))] + \gamma[1 - z + E^t\tilde{\varepsilon}^t + \delta v_i(z_{ij}^{\varepsilon^t})] \\ \Leftrightarrow z &\leq \hat{z}_1 \equiv \frac{2\delta(1-\gamma) + \frac{3}{4}\eta\underline{\theta}(1-\delta(1-\eta))}{3(1-\delta(\gamma-\eta))}. \end{aligned}$$

Note that $z^o + \hat{z}_1 = 1$, so $\hat{z}_1 > \frac{1}{2}$ for $\delta > \delta^o$. It is also straightforward to show that $z^u \geq \hat{z}_1$ for $\eta\underline{\theta} \leq \gamma\underline{\varepsilon}$, so z^u is not binding. Since $\eta\underline{\theta} \leq \gamma\underline{\varepsilon}$, $z^u \geq \hat{z}_1$ and hence $z^o \geq z^\ell$, so z^ℓ is not binding.

B. Consider the case in which i does not receive 1 in q^{t-1}. Legislator i prefers a proposal $z_{ij}(z)$ to a proposal that gives 1 to i if and only if

$$\begin{aligned} 1 - \gamma\frac{\underline{\varepsilon}}{4} + \delta\frac{1}{3(1-\delta)} &\leq (1-\gamma)[1 - z + \delta v_i(z_{ij}(z))] + \gamma[1 - z + E^t\tilde{\varepsilon}^t + \delta v_i(z_{ij}^{\varepsilon^t})], \\ \Leftrightarrow z &\leq \hat{z}_2 \equiv \frac{2\delta(1-\gamma) + \frac{3}{4}\gamma\underline{\varepsilon}(1-\delta(1-\eta))}{3(1-\delta(\gamma-\eta))}. \end{aligned}$$

Note that $\hat{z}_1 \leq \hat{z}_2$, since $\eta\underline{\theta} \leq \gamma\underline{\varepsilon}$.

■

Given the restriction in (7) on z, assumption 3 is sufficient to guarantee that allocations in the specific-policy equilibrium remain in $[0, 1]$.

Corollary 4. *$\underline{\varepsilon} \leq \frac{1}{3}$ and $\underline{\theta} \leq \frac{1}{3}$ are sufficient for the coalition allocation to be in $[0, 1]$.*

Proof. It is straightforward to show that z^* is decreasing in γ and increasing in η. Evaluating z^* at $\gamma = 1$ and $\eta = 0$ yields $z^* = \frac{3-2\delta}{3(2-\delta)}$, which implies that $z^* > \frac{1}{3}$, for all $\gamma \in [0, 1)$ and $\eta \in [0, 1)$.

Consequently, if $(1-z, z, 0)$ is a coalition MPE proposal, $z - \underline{\varepsilon} > 0$, and hence $z - \underline{\theta} > 0$, and $1 - z + \underline{\varepsilon} < 1$, and hence $1 - z + \underline{\theta} < 1$. ■

Proof of Lemma 4

Proof. The lower bound $z^* \leq \frac{1}{2}$ for all $\gamma \geq \eta$, $\delta > 0$. The lower bound z^o is strictly less than $\frac{1}{2}$ for $\delta \in (\delta^o, 1)$. Since $(\gamma, \eta) \in R(\underline{\theta})$, there exists a $\delta \in (\delta^o, 1)$ such that $z^o < \frac{1}{2}$. Note that $\hat{z}_1 + z^o = 1$, so $\hat{z}_1 > \frac{1}{2}$ for $\delta \in (\delta^o, 1)$. ■

Tolerant equilibrium allocations

Proof of Lemma 7

Proof. For $z \in \zeta^m$ the dynamic payoffs are given by

$$\bar{v}_i(z_{ij}(z)) = (1-\eta)\left[1 - z + \delta\bar{v}_i(z_{ij}(z))\right] + \eta[1 - z + E^t\tilde{\theta}^t + \delta\bar{v}_i(z_{ij}^{\theta^t})] \tag{38}$$

$$\bar{v}_j(z_{ij}(z)) = (1-\eta)\left[z + \delta\bar{v}_j(z_{ij}(z))\right] + \eta[z - E^t\tilde{\theta}^t + \delta\bar{v}_j(z_{ij}^{\theta^t})] \tag{39}$$

$$\bar{v}_k(z_{ij}(z)) = (1-\eta)\delta\bar{v}_k(z_{ij}(z)) + \eta\delta\bar{v}_k(z_{ij}^{\theta^t}), \tag{40}$$

where

$$\begin{aligned}\bar{v}_\ell(z_{ij}^{\theta^t}) = & \int_{-\underline{\theta}}^{z+z^m-1} \bar{v}_\ell(z_{ij}(z-\theta^t))\frac{1}{2\underline{\theta}}d\theta^t + \int_{z+z^m-1}^{z-z^m} \bar{v}_\ell(z_{ij}(z-\theta^t))\frac{1}{2\underline{\theta}}d\theta^t \\ & + \int_{z-z^m}^{\underline{\theta}} \bar{v}_\ell(z_{ij}(z-\theta^t))\frac{1}{2\underline{\theta}}d\theta^t.\end{aligned} \tag{41}$$

In the first and third integrals in (41), $z - \theta^t \notin \zeta^m$, so $\bar{v}_\ell(z_{ij}(z-\theta^t)) = \bar{v}_\ell(z_{ij}(z'))$ for some $z' \notin \zeta^m$, where $\bar{v}_\ell(z_{ij}(z'))$ is not a function of z or θ according to the equilibrium strategies.

In the second integral in (41) $z - \theta^t \in \zeta^m$. Conjecture that for $z \in \zeta^m$, $\bar{v}_i(z_{ij}(z))$ is linear in $1-z$, $\bar{v}_j(z_{ij}(z))$ is linear in z, and these are given by $\bar{v}_i(z_{ij}(z)) = a_i + b_i(1-z)$ and $\bar{v}_j(z_{ij}(z)) = a_j + b_j z$. Then $\bar{v}_i(z_{ij}(z-\theta^t)) = a_i + b_i(1 - z + \theta^t)$, and $\bar{v}_j(z_{ij}(z-\theta^t)) = a_j + b_j(z - \theta^t)$. Conjecture that $\bar{v}_k(z_{ij}(z))$ is constant in z. Then for $\ell = i, j$

$$\bar{v}_\ell(z_{ij}^{\theta^t}) = \frac{\bar{v}_\ell(z_{ij}(z'))(2z^m - 1 + 2\underline{\theta})}{2\underline{\theta}} + \frac{(1-2z^m)(2a_\ell + b_\ell)}{4\underline{\theta}} \tag{42}$$

$$\bar{v}_k(z_{ij}^{\theta^t}) = \frac{\bar{v}_\ell(z_{ij}(z'))(2z^m - 1 + 2\underline{\theta})}{2\underline{\theta}} + \frac{(1-2z^m)a_k}{2\underline{\theta}}. \tag{43}$$

Substituting into (38)-(40) and matching coefficients gives

$$\begin{aligned}
a_i = a_j &= \frac{\delta\eta(1-2z^m)}{2[1-\delta(1-\eta)][2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2z^m)]} + a_k \\
a_k &= \frac{\delta\eta v_k(z_{ij}(z'))(2z^m-1+2\underline{\theta})}{2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2z^m)} \\
b_i = b_j &= \frac{1}{1-\delta(1-\eta)}.
\end{aligned}$$

Substituting the coefficients and simplifying (42)-(43) gives

$$\bar{v}_\ell(z_{ij}^{\theta^t}) = \frac{(1-2z^m)[1-2\bar{v}_\ell(z_{ij}(z'))(1-\delta)]}{2[2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2z^m)]} + \bar{v}_\ell(z_{ij}(z')) \tag{44}$$

$$\bar{v}_k(z_{ij}^{\theta^t}) = \frac{(1-2z^m)\bar{v}_k(z_{ij}(z'))(1-\delta)}{2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2z^m)} + \bar{v}_k(z_{ij}(z')). \tag{45}$$

Simplifying (38)-(40) gives

$$\bar{v}_i(z_{ij}(z)) = \frac{1-z}{1-\delta(1-\eta)} + \beta_i \tag{46}$$

$$\bar{v}_j(z_{ij}(z)) = \frac{z}{1-\delta(1-\eta)} + \beta_j \tag{47}$$

$$\bar{v}_k(z_{ij}(z)) = \beta_k, \tag{48}$$

where for $\ell = i, j$,

$$\begin{aligned}
\beta_\ell &= \frac{\eta\delta\bar{v}_\ell(z_{ij}(z'))}{1-\delta(1-\eta)} + \frac{\eta\delta(1-2z^m)[1-2(1-\delta)\bar{v}_\ell(z_{ij}(z'))]}{2[1-\delta(1-\eta)][2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2z^m)]} \text{ and} \\
\beta_k &= \frac{\eta\delta\bar{v}_k(z_{ij}(z'))}{1-\delta(1-\eta)} + \frac{\eta\delta(1-2z^m)(1-\delta)\bar{v}_j(z_{ij}(z'))}{[1-\delta(1-\eta)][2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2z^m)]}.
\end{aligned}$$

For $q^{t-1} \neq z_{ij}(z)$ for all $z \in \zeta^m$, the continuation value $\bar{v}_\ell(q^{t-1}) \equiv \bar{v}_\ell(q^{t-1}|q^{t-1} \neq z_{ij}(z))$ is,

using the equilibrium strategies,

$$\begin{aligned}
\bar{v}_\ell(q^{t-1}) &= (1-\gamma)\left[\frac{1}{3}\left[\frac{1}{2}(1-z+\delta\bar{v}_\ell(z_{ij}(z)))+\frac{1}{2}(1-z+\delta\bar{v}_\ell(z_{ik}(z)))\right]\right.\\
&+\frac{1}{3}\left[\frac{1}{2}\left(z+\delta\bar{v}_\ell(z_{ji}(z))\right)+\frac{1}{2}\delta\bar{v}_\ell(z_{jk}(z))\right]\\
&\left.+\frac{1}{3}\left[\frac{1}{2}\left(z+\delta\bar{v}_\ell(z_{ki}(z))\right)+\frac{1}{2}\delta\bar{v}_\ell(z_{kj}(z))\right]\right]\\
&+\gamma\left[\frac{1}{3}\left[\frac{1}{2}\left(1-z+E^t\tilde{\varepsilon}^t+\delta\bar{v}_l(z_{ij}^{\varepsilon^t})\right)+\frac{1}{2}\left(1-z+E^t\tilde{\varepsilon}^t+\delta\bar{v}_\ell(z_{ik}^{\varepsilon^t})\right)\right]\right.\\
&+\frac{1}{3}\left[\frac{1}{2}\left(z-E^t\tilde{\varepsilon}^t+\delta\bar{v}_\ell(z_{ji}^{\varepsilon^t})\right)+\frac{1}{2}\delta\bar{v}_\ell(z_{jk}^{\varepsilon^t})\right]\\
&\left.+\frac{1}{3}\left[\frac{1}{2}(z-E^t\tilde{\varepsilon}^t+\delta\bar{v}_\ell(z_{ki}^{\varepsilon^t}))+\frac{1}{2}\delta\bar{v}_\ell(z_{kj}^{\varepsilon^t})\right]\right], \qquad (49)
\end{aligned}$$

where $\bar{v}_\ell(z_{ij}(z)), \ell = i, j, k$, are given by (46)-(48) and

$$\begin{aligned}
\bar{v}_\ell(z_{ij}^{\varepsilon^t}) &= \int_{-\underline{\varepsilon}}^{z+z^m-1}\bar{v}_\ell(z_{ij}(z-\varepsilon^t))\frac{1}{2\underline{\varepsilon}}d\varepsilon^t+\int_{z+z^m-1}^{z-z^m}\bar{v}_\ell(z_{ij}(z-\varepsilon^t))\frac{1}{2\underline{\varepsilon}}d\varepsilon^t\\
&+\int_{z-z^m}^{\underline{\varepsilon}}\bar{v}_\ell(z_{ij}(z-\varepsilon^t))\frac{1}{2\underline{\varepsilon}}d\varepsilon^t. \qquad (50)
\end{aligned}$$

In the first and third integrals in (50), $z-\varepsilon^t \notin \zeta^m$, so $\bar{v}_\ell(z_{ij}(z-\varepsilon^t)) = \bar{v}_\ell(q^{t-1})$. In the second integral in (50) $z-\varepsilon^t \in \zeta^m$, so $\bar{v}_\ell(z_{ij}(z-\varepsilon^t))$ is given by (46)-(48). Then substituting from (46)-(48) and simplifying gives

$$\bar{v}_i(z_{ij}^{\varepsilon^t}) = \left(\frac{1-2z^m}{2\underline{\varepsilon}}\right)\frac{\underline{\theta}[1-2(1-\delta)\bar{v}_i(q^{t-1})]}{2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2z^m)}+\bar{v}_i(q^{t-1}) \qquad (51)$$

$$\bar{v}_j(z_{ij}^{\varepsilon^t}) = \left(\frac{1-2z^m}{2\underline{\varepsilon}}\right)\frac{\underline{\theta}[1-2(1-\delta)\bar{v}_j(q^{t-1})]}{2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2z^m)}+\bar{v}_j(q^{t-1}) \qquad (52)$$

$$\bar{v}_k(z_{ij}^{\varepsilon^t}) = -\left(\frac{1-2z^m}{3\underline{\varepsilon}}\right)\frac{\underline{\theta}(1-\delta)\bar{v}_k(q^{t-1})}{2\underline{\theta}(1-\delta(1-\eta))-\delta\eta(1-2z^m)}+\bar{v}_k(q^{t-1}). \qquad (53)$$

By symmetry $\bar{v}_i(q^{t-1}) = \bar{v}_j(q^{t-1}) = \bar{v}_k(q^{t-1}) = \bar{v}_\ell(q^{t-1})$. Substituting (51)-(53) into (49) and solving gives

$$\bar{v}_\ell(q^{t-1}) = \hat{v} = \frac{1}{3(1-\delta)}, \ell = i, j, k. \qquad (54)$$

This proves part (i) of the lemma.

To prove part (ii), by part (i) $\hat{v} = \frac{1}{3(1-\delta)}$ is the continuation payoff for any allocation such that $z \notin \zeta^m$, hence $\bar{v}_\ell(z_{ij}(z')) = \hat{v} = \frac{1}{3(1-\delta)}$, for $z' \notin \zeta^m$. Substituting $\bar{v}_\ell(z_{ij}(z')) = \frac{1}{3(1-\delta)}$ into (46)-(48)

yields (13)-(15).

To prove part (iii), first note that the numerator of (16) is nonnegative, since $\theta \geq 1 - 2z^m$. Using this inequality in the denominator of (16) yields

$$2\underline{\theta}(1 - \delta(1 - \eta)) - \delta\eta(1 - 2z^m) \geq \underline{\theta}[2 - \delta(2 - \eta)] > 0,$$

so $\nu(z^m) \geq 0$. If $z^m < \frac{1}{2}$, the numerator is strictly positive. ■

The following lemma is used in Lemma 8 to identify the proposal made by the originator of the coalition.

Lemma 11. *If $q^{t-1} \neq z_{ij}(z)$ for all $z \in \zeta^m$, $\bar{v}_\ell(z_{ij}^{\varepsilon^t})$, the continuation value conditional on a shock ε^t such that $z - \varepsilon^t < z^m$ occurring when proposal $z_{ij}(z)$ is made, is given by*

$$\bar{v}_i(z_{ij}^{\varepsilon^t}) = \bar{v}_j(z_{ij}^{\varepsilon^t}) = \frac{1}{3(1-\delta)} + \frac{\underline{\theta}}{\underline{\varepsilon}}\nu(z^m) \tag{55}$$

$$\bar{v}_k(z_{ij}^{\varepsilon^t}) = \frac{1}{3(1-\delta)} - 2\frac{\underline{\theta}}{\underline{\varepsilon}}\nu(z^m). \tag{56}$$

Proof. This follows from substituting $\bar{v}_\ell(q^{t-1}) = \frac{1}{3(1-\delta)}$ from (12) into (51)–(53). ■

The continuation values $\bar{v}_i(z_{ij}^{\varepsilon^t}) = \bar{v}_j(z_{ij}^{\varepsilon^t})$ are greater than $\frac{1}{3(1-\delta)}$ because a proposal $z_{ij}(z)$ could result in tolerant coalition allocation whereas it equals the specific-policy allocation with probability 0.

The following lemma identifies the continuation values when a coalition dissolves.

Lemma 12. *If $q^{t-1} = z_{ij}(z)$ for $z \in \zeta^m$, the continuation value $\bar{v}_\ell(z_{ij}^{\theta^t})$ conditional on a shock θ^t such that $z - \theta^t < z^m$ occurring is given by*

$$\bar{v}_i(z_{ij}^{\theta^t}) = \bar{v}_j(z_{ij}^{\theta^t}) = \frac{1}{3(1-\delta)} + \nu(z^m) \tag{57}$$

$$\bar{v}_k(z_{ij}^{\theta^t}) = \frac{1}{3(1-\delta)} - 2\nu(z^m). \tag{58}$$

Proof. This follows from substituting $\bar{v}_\ell(z_{ij}(z')) = \frac{1}{3(1-\delta)}$ from the proof of Lemma 7 into (44) and (45) in the Appendix. ■

Note that $\bar{v}_\ell(z_{ij}^{\theta^t}) \geq \bar{v}_\ell(z_{ij}^{\varepsilon^t}), \ell = i, j$, and $\bar{v}_k(z_{ij}^{\theta^t}) \geq \bar{v}_k(z_{ij}^{\varepsilon^t}), \ell = i, j$, since $\underline{\varepsilon} \geq \underline{\theta}$. If $\underline{\theta} = \underline{\varepsilon}$, $\bar{v}_\ell(z_{ij}^{\theta^t}) = \bar{v}_\ell(z_{ij}^{\varepsilon^t}), \ell = i, j, k$.

Upper Bounds on z^m and $\hat{z}^m$

The upper bounds are all greater than their single allocation counterparts because the coalition is preserved with higher probability, and hence the payoffs from entering the equilibrium path and staying on it are higher. Moreover, the payoff of the coalition partner is greater because he has the opportunity to obtain more than z^m. The upper bounds are

$$z^{uu} \equiv z^u + \eta\delta\nu(z^m). \tag{59}$$

$$\hat{\hat{z}}_1 \equiv \hat{z}_1 + \frac{\delta\left((1-\gamma)\eta\delta + \gamma(1-\delta(1-\eta))\right)\nu(z^m)}{1-\delta(\gamma-\eta)}. \tag{60}$$

$$\hat{\hat{z}}_2 \equiv \hat{z}_2 + \frac{\delta\left((1-\gamma)\eta\delta + \gamma(1-\delta(1-\eta))\right)\nu(z^m)}{1-\delta(\gamma-\eta)}. \tag{61}$$

Note that $z^{oo} + \hat{\hat{z}}_1 = z^o + \hat{z}_1 = 1$, and $z^{uu} + z^{\ell\ell} = z^u + z^\ell = 1$. For $\delta > \delta^o$, $z^{\ell\ell} < \frac{1}{2}$, so $z^{uu} > \frac{1}{2}$, and $z^{oo} < \frac{1}{2}$, so $\hat{\hat{z}}_1 > \frac{1}{2}$. Since $\hat{z}_1 \leq \hat{z}_2$, $\hat{\hat{z}}_1 \leq \hat{\hat{z}}_2$, so $\hat{\hat{z}}_2$ is not binding. Consequently, the least upper bound on z^m is $\frac{1}{2}$.

Battaglini-Palfrey experiment results

Table 1: Duration of Coalitions

		Frequency									
Match	**# Rounds**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	10	3	2	0	1	0	1	0	0	0	0
2	4	1	1	1	0						
3	10	1	0	0	0	1	0	0	0	1	0
4	5	1	0	0	2	1					
5	2	1	2								
6	1	2									
7	8	0	0	0	0	0	0	1	2		
8	1	2									
9	3	0	1	2							
10	5	0	0	0	2	1					
11	7	0	2	0	2	0	1	1			
12	2	2	1								
13	1	0									
14	5	2	0	0	2	1					
15	3	2	3	0							
16	5	1	1	0	2	1					
17	1	3									
18	2	3	1								
19	7	3	3	2	0	0	0	0			
20	3	2	1	2							
Total		29	18	7	11	5	2	2	3	1	0

NB: 70 committees

Table 2: Transition Probabilities – Continuous Experiment (All Rounds)

	Outcome q^t				
Status quo q^{t-1}	M12	M13	M23	U	D
M12 (46)	0.522	0.130	0.217	0.087	0.043
M13 (55)	0.145	0.582	0.073	0.091	0.109
M23 (32)	0.063	0.250	0.313	0.281	0.094
U (92)	0.087	0.054	0.011	0.848	0.000
D (23)	0.217	0.130	0.087	0.043	0.522

Table 3: Transition Probabilities – Continuous Experiment (Rounds after First)

	Outcome q^t				
Status quo q^{t-1}	M12	M13	M23	U	D
M12 (42)	0.524	0.119	0.214	0.095	0.048
M13 (47)	0.128	0.596	0.085	0.064	0.128
M23 (24)	0.083	0.333	0.292	0.208	0.083
U (88)	0.080	0.034	0.011	0.875	0.000
D (23)	0.217	0.130	0.087	0.043	0.522

References

Acemoglu, D., Egorov, G. and Sonin, K. (2012), 'Dynamics and Stability of Constitutions, Coalitions and Clubs', *American Economic Review* .

Agranov, M. and Tergiman, C. (2012), 'The Effect of Negotiations on Bargaining in Legislatures', *Working paper* .

Anesi, V. (2010), 'Noncooperative Foundations of Stable Sets in Voting Games', *Games and Economic Behavior* **70**, 488–493.

Anesi, V. and Seidmann, D. J. (2012), 'Bargaining in Standing Committees', *Working paper* .

Baron, D. P. (1996), 'A Dynamic Theory of Collective Goods Programs', *American Political Science Review* **90**(2), 316–330.

Baron, D. P., Diermeier, D. and Fong, P. (2012), 'Policy Dynamics and Inefficiency in a Parliamentary Democracy', *Economic Theory* **49**, 703–738.

Baron, D. P. and Herron, M. (2003), 'A Dynamic Model of Multidimensional Collective Choice', in *Computational Models of Political Economy* (K. Kollman, J. H. Miller, and St. E. Page, eds), 13-47.

Battaglini, M. and Coate, S. (2007), 'Inefficiency in Legislative Policy-Making: A Dynamic Analysis', *American Economic Review* **97**(1), 118–149.

Battaglini, M. and Coate, S. (2008), 'A Dynamic Theory of Public Spending, Taxation and Debt', *American Economic Review* **98**(1), 201–236.

Battaglini, M. and Palfrey, T. (2012), 'The Dynamics of distributive politics', *Economic Theory* **49**, 739–777.

Bernheim, D., Rangel, A. and Rayo, L. (2006), 'The Power of the Last Word in Legislative Policy Making', *Econometrica* **74**(5), 1161–1190.

Bowen, T. R. (2011), 'Legislated Protection and the WTO', *Working paper* .

Bowen, T. R., Eraslan, H. and Chen, Y. (2012), 'Mandatory Versus Discretionary Spending: The Status Quo Effect', *Working paper* .

Bowen, T. R. and Zahran, Z. (2012), 'On Dynamic Compromise', *Games and Economic Behavior* **76**, 391–419.

Diermeier, D. and Fong, P. (2011), 'Legislative Bargaining with Reconsideration', *Quarerly Journal of Economics* **126**(2), 895–946.

Dixit, A., Grossman, G. M. and Gul, F. (2000), 'The Dynamics of Political Compromise', *Journal of Political Economy* **108**(3), 531–568.

Duggan, J. (2012), 'Noisy Stochastic Games', *Econometrica* **80**(5), 2017–2045.

Duggan, J. and Kalandrakis, T. (2012), 'Dynamic Legislative Policy Making', *Journal of Economic Theory* **147**, 1653–1688.

Dziuda, W. and Loeper, A. (2010), 'Dynamic Collective Choice with Endogenous Status Quo', *Working Paper* .

Epple, D. and Riordan, M. H. (1987), 'Cooperation and Punishment under Repeated Majority Voting', *Public Choice* **55**(1/2), 41–73.

Frechette, G. R., Kagel, J. H. and Lehrer, S. F. (2003), 'Bargaining in legislatures: An experimental investigation of open versus closed amendment rules', *American Political Science Review* **97**, 221–232.

Frechette, G. R., Kagel, J. H. and Morelli, M. (2005*a*), 'Behavioral identification in coalitional bargaining: An experimental analysis of demand bargaining and alternating offers', *Econometrica* **73**, 1893–1938.

Frechette, G. R., Kagel, J. H. and Morelli, M. (2005*b*), 'Nominal bargaining power, selection protocol and discounting in legislative bargaining', *Journal of Public Economics* **89**, 1497–1517.

Kalandrakis, T. (2004), 'A Three-Player Dynamic Majoritarian Bargaining Game', *Journal of Economic Theory* **116**(2), 294–322.

Kalandrakis, T. (2010), 'Minimum winning coalitions and endogenous status quo', *International Journal of Game Theory* **39**, 617–643.

Lagunoff, R. (2001), 'A Theory of Constitutional Standards and Civil Liberties', *The Review of Economic Studies* **68**(1), 109–132.

Mckelvey, R. D. and Palfrey, T. R. (1995), 'Quantal Response Equilibria for Normal Form Games', *Games and Economic Behavior* **10**, 6–38.

McKelvey, R. D. and Palfrey, T. R. (1998), 'Quantal response equilibria for extensive form games', *Experimental Economics* **1**, 9–41.

Nunnari, S. (2012), 'Dynamic Legislative Bargaining with Veto Power', *Working Paper* .

Piguillem, F. and Riboni, A. (2012), 'Dynamic Bargaining over Redistribution in Legislatures', *Working paper* .

Richter, M. (2011), 'Perfectly Absorbing Dynamic Compromise', *Working paper* .

Zápal, J. (2012), 'Explicit and Implicit Status-quo Determination in Dynamic Bargaining: Theory and Application to FOMC Directive', *Working paper* .