Prize and Punishment: Optimal Contest Design with Incomplete Information[*]

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February 5, 2013

Abstract

This paper studies the optimal contest design problem when the abilities of the risk neutral contestants are independent private information. The contest designer has a fixed prize budget to elicit efforts from the contestants. We consider all possible mechanisms that allocate prizes and punishments (negative prizes) across the contestants. We find that an optimal contest mechanism does not exist. Nevertheless, the utmost total efforts (i.e. highest efforts inducible when all contestants are of the highest possible ability) can almost be achieved by mechanisms involving exploding punishments. When there is a bound on the punishment (i.e. K), an optimal contest mechanism exists and can be implemented by a modified all-pay auction with a minimum bid and a single prize consisting of the entire prize budget augmented by an entry fee of K from each participant; when no one bids, all participants share a nonnegative prize equally.

JEL Classification Numbers: D72, D82, C73

Keywords: optimal contest, mechanism design, negative prize, punishment, incomplete information, cross-type transfer, leverage

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1 Introduction

Activities and events, such as sports, promotions within organizations, school admissions, political elections and R&D races, can be viewed as contests. Contests, as a well established institution, essentially incentivize economic agents to exert costly and irreversible effort by awarding prizes to the winners.

In a contest, the contest organizer often has a fixed amount of prize budget, and uses it to induce contestants to exert the maximum amount of effort. For example, the privately funded Loebner Prize provides a cash reward to the most human-like chatterbot in the annual contest in artificial intelligence. The European Information Technology Society annually awards prizes worth 600,000 euros in total to promote novel products with high information technological contents and evidenced market potentials. These contest organizers, with predetermined budgets, aim to find certain contest rules to achieve their objective of eliciting the most efforts from the contestants with privately known abilities. This optimal contest design problem with incomplete information about the contestants is the focus of this paper.

There are many similarities between the optimal contest design with incomplete information (where the abilities of the contestants are their private information) and the optimal auction design with incomplete information (where the valuations of the bidders are their private information). The abilities of the contestants in a contest can be mathematically translated into the valuations of the bidders in an auction. The prizes in the contest can be reinterpreted as the winning probabilities in the auction. Meanwhile, efforts from the contestants in the contest can be treated as payments from the bidders in the auction. Therefore, maximizing the total efforts from the contestants by optimally allocating the prizes is equivalent to maximizing the total payments from the bidders by optimally allocating the winning probabilities. As such, we can make use of the well developed techniques on optimal auction design to analyze the optimal contest design problem.

To emphasize these similarities, in this paper we will carry out our analysis in an environment similar to the one in Myerson [25] and adopt a mechanism design approach. Suppose that there are a fixed number of risk neutral contestants. These contestants differ in their abilities, which are their own private information. We focus on the case where the contestants' abilities (i.e., types) are independently distributed.[1] In a direct mechanism, a contest designer has a fixed prize budget that he can allocate to these contestants (in the form of prizes) based on their reported types. The designer's goal is to elicit the maximum amount of total efforts from these contestants.

There is one difference, however, that sets contests apart from auctions. In contests, prizes can be negative. For example, the FCC-organized contest to set the standard for high-definition

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[1]If the contestants' types are correlated, then a full surplus extraction result can be established similarly to Crémer and McLean [10] and McAfee and Reny [19].

television was open to any firm but with a $200,000 entry fee (see Taylor [31]). Golf tournament often involves a significant amount of registration fee. As a result, the losers in these contests end up with a negative prize as they pay a fee and win nothing. A negative prize can be equivalently regarded as a punishment. Punishments can exist in contests but not in auctions. Using our parallel approach to mechanism design, negative prizes are technically equivalent to negative winning probabilities in auctions. The former is feasible, while the later is not. If prizes are not allowed to be negative, then the optimal contest design problem is mathematically equivalent to the optimal auction design problem. When negative prizes are allowed, however, the two problems are no longer the same. The contest design problem is a more relaxed problem, as it imposes no restrictions on the prizes, as opposed to its auction counterpart. In this case, Myerson's [25] technique is no longer adequate for providing a solution to this relaxed problem.

In our analysis, since both positive prizes and negative prizes are allowed, the contest designer can use them concurrently to elicit the most efforts from the contestants. This flexibility in prizes poses some challenges in the analysis, but it differentiates our paper from other papers in the contest design literature.[2]

In the optimal auction design problem, as Myerson [25] has demonstrated, the bidder with the highest virtual valuation (if it is higher than the seller's reservation value) should have a winning probability equal to 1 and all other bidders should have zero probability of winning.[3] Allowing negative prizes is equivalent to allowing negative probabilities of winning. It creates an additional venue to increase the seller's revenue (or the contestants' efforts in our case). Allocating a negative prize to a contestant with a lower (including negative) virtual valuation would increase the positive prize to a higher virtual valuation contestant by the same amount while keeping the total prize budget unchanged. This kind of leveraging on the contestants' virtual valuations relaxes the original optimization constraints and obviously increases the total level of efforts elicited. Of course, we need to ensure that the lower ability contestants still have incentive to participate and therefore need to award them with positive prizes sometimes. The optimal contest mechanism finds the optimal balance between these incentives to maximize the total efforts exerted by the contestants.

In this paper, we characterize the optimal contest mechanism in our model using the techniques of continuous linear programming. We obtain two surprising results. First, we find that an optimal contest mechanism does not exist. Second, any level of total efforts arbitrarily close to the **utmost total efforts** (i.e. highest total efforts that can be induced when all contestants are of the highest

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[2]Most papers in the contest literature do not allow the prizes to be negative. In a recent paper, Moldovanu et al [24] analyze negative prizes in a contest model, but their analysis is limited to rank-ordered tournaments. This paper will be discussed in details later.

[3]In the auction literature, a bidder's virtual valuation can be interpreted as the marginal revenue that can be elicited from the bidder, according to Bulow and Roberts [2]. Similarly, in our contest setting, a contestant's virtual valuation can be interpreted as the marginal effort that can be elicited from the contestant using one unit of prize.

possible ability) can be achieved by an incentive compatible contest mechanism that involves exploding negative contingent prizes for contestants of the lower abilities. In the contest mechanism that we will construct in the paper, there exists a threshold level of ability near the highest possible ability. If the highest ability among the contestants is above this threshold, the contestant with this ability will receive the maximum prize, which is equal to the original prize budget of the contest designer plus the extra money collected from the punishments (negative prizes) imposed on other contestants. If the highest ability of the contestants is below the threshold, then all of the contestants share the original prize budget equally. The incentive to participate for the lower ability contestants is thus maintained by the positive prizes when no contestant is above the threshold. In the equilibrium of this mechanism, those contestants with abilities below the threshold exert zero effort. Meanwhile, contestants with abilities higher than the threshold exert a large amount of efforts. The level of total expected efforts converges to the utmost total efforts and full surplus extraction can be achieved in the limit. Note that in our model, the abilities of the contestants are distributed independently. In the optimal auction design literature, the bidders' types have to be correlated in order to extract the full surplus from them. Therefore, our result illustrates that an optimal contest is very different from an optimal auction. (Almost) full surplus extraction is feasible in contests but not in auctions when players have independent private information.

In the optimal contests we discussed above, the contestants are sometimes hit with an extraordinarily large negative prize. This kind of financial punishment is not always feasible. Contestants have financial constraints, governments have regulations, and societies have laws limiting the amount of punishment one can impose. In the paper, we address this issue by analyzing a bounded punishment problem. In the analysis, we assume that any punishment (negative prize) cannot exceed a bound which is common to all contestants. Again, using the techniques of continuous linear programming, we completely characterize the optimal contest mechanism under this situation.

This optimal contest mechanism again features a threshold level of ability, and this threshold depends on the bound on the punishment. When the bound is low, contestants cannot be punished too severely, and the threshold level is exactly the same as the cut-off level in the optimal auction mechanism in Myerson [25]. When the bound is high, however, the threshold is strictly higher than the cut-off in Myerson [25]. When the abilities of all contestants are lower than the threshold, all contestants share equally a portion (in the former case) or the total (in the latter case) of the original prize budget. When the highest ability of the contestants is above the threshold, the contestant with this ability will receive the maximum prize, which is equal to the original prize budget plus the extra money collected from the punishments (negative prizes) imposed on other contestants. This optimal contest mechanism can be implemented by a modified all-pay auction with the following features: an entry fee, a minimum bid, and a grand prize equal to the original

prize budget plus the total entry fees. In this mechanism, the entry fee is equal to the bound for the punishment.

There are a few distinctive features for the prize allocation rule in the optimal contest mechanism. First, a maximal punishment is imposed on all contestants except the one with the highest ability, unless all of them have abilities lower than the cut-off. Second, the pool of contestants with abilities lower than the cut-off are treated equally, regardless of their ability ranking. They all win a positive prize when no contestant's ability is above the cut-off. These two features together ensure that the maximum incentive is generated for the high ability contestants to exert effort while the low ability contestants are still willing to participate (and provide the necessary cross-ability subsidies). Third, similar to Myerson [25], a threshold value is defined, but in a somewhat different way. The threshold in our optimal contest mechanism is always weakly higher than the threshold in Myerson [25], and strictly higher if the bound for the punishment is sufficiently high. In particular, our threshold would approach the upper limit of the support of the ability distribution when the bound for punishment becomes higher and higher. Fourth, when the bound of punishment is low and thus punishments are not allowed to be large, our threshold coincides with the one in Myerson [25]. In this case, only a portion of the original prize budget is awarded to the contestants when every contestant's ability is below the threshold. This is necessary for maintaining the incentives for those contestants with ability above the threshold. This partial award in our optimal contest resembles the optimal supply reduction by a monopoly seller in an optimal auction, even when the seller values the object at zero.

In the contest literature, many papers focus on the moral hazard issue in situations such as verifiable outputs as noisy signals of unobservable efforts. Pioneer works on perfectly discriminatory contests (i.e. all pay auctions) with incomplete contestant information but perfect effort monitoring include Moldovanu and Sela [21], and Moldovanu et al. [23], [24]. In these papers, the focus is on the adverse selection issue. Our paper belongs to this latter line of research.

The rest of this paper is organized as follows. In the rest of this section, we review the related literature. In Section 2, we present the model. In Section 3, we carry out our main analysis on the optimal contest design. We first show that there exists no optimal mechanism. We then construct mechanisms which can achieve almost the utmost total efforts. (These mechanisms require large punishments to occur with positive probabilities.) Finally, in a sequence of analytical steps, we characterize the optimal contest when punishments are bounded by K. In Section 4, we provide some concluding remarks. An appendix collects some technical proofs.

Related literature

Our paper is related to a few important strands of the literature. First, it obviously belongs to the literature on optimal contest design with incomplete information. Fullerton and McAfee [11] establish the optimality of shortlisting two most efficient firms in a single fixed-prize research tournament, where the R&D firms' research costs are the firms' private information. Moldovanu and Sela [21] are the first to study the optimal prize allocation problem within the all-pay contest framework when contestants have private information. They establish the optimality of a winner-take-all contest within a paradigm of fixed nonnegative prizes that are not contingent on the bid profile. Minor [20] reexamines the same design problem when contestants have convex costs of effort and when the contest designer has concave benefit of effort. Moldovanu and Sela [22] investigate this optimal prize allocation problem in a two-stage all-pay auction framework. Meanwhile, Moldovanu et al [23] analyze the optimal contest design in an all-pay auction framework in which contestants care about their relative status. Polishchuk and Tonis [28] and Chawla et al [3] adopt a mechanism design approach to examine optimal contest designs, focusing on nonnegative prizes. Kirkegaard [13] also uses a mechanism design approach to study the optimal favoritism in contest designs when the type distributions for the contestants are asymmetric. Our paper differentiates from these existing studies by providing a complete analysis on the optimal contest design when negative prizes are allowed. Similar to some of the existing studies, we also adopt a mechanism design approach. In the analysis, we shed light on why negative prizes are necessary in the optimal mechanism and how the optimal negative prizes can be utilized to leverage on the differences in the virtual values of the contestants. We find that we can achieve a level of total efforts arbitrarily close to the utmost total efforts by making use of negative prizes contingent on the types of the contestants in the contest mechanism we constructed in the paper.

Among the above-mentioned papers, our paper is most related to Moldovanu et al [24] who study the optimal design of prizes that can be positive or negative in a multiple-prize all-pay auction with incomplete information. There are two major differences between our paper and theirs. The first difference is that our analysis can accommodate any contest rule that allocates the budget contingent on the contestants' reported type profile, while theirs focuses on all-pay auctions. The second difference is that we model punishment differently. In their paper, a punishment can either be costly or costless to the organizer and it thus never helps the designer's budget. In our paper, both prizes and punishments are in monetary terms, and therefore the punishments collected from some contestants can be added to prize budget to relax the designer's budget constraint.

Second, our paper is related to the literature on full surplus extraction when players have private information. Myerson [25] provides a full surplus extraction example in an auction with two bidders with correlated valuations that can take only two values. Crémer and McLean [10]

formally establish the full surplus extraction result in an auction environment, where bidders have finite private types which are correlated. McAfee and Reny [19] extend Crémer and McLean's result of full surplus extraction to a continuum of types in a general mechanism design setting. Heifetz and Neeman [12] and Chen and Xiong [9] further examine the generality and robustness of the full surplus extraction result. All of the above papers require that the types of the players are correlated. When the types of the players are independently distributed, it is believed that full surplus extraction cannot be achieved. In this paper, we show that if negative prizes are allowed, almost full surplus extraction can be achieved in the contest environment when players' types are independent.[4]

Third, in our paper, we implement the "almost" utmost total efforts using a modified all-pay auction. This complements Lazear and Rosen [16], even though they consider social efficiency in their paper and we are concerned with effort level. They show that rank-ordered tournaments can achieve the first best when the effort cost functions of the contestants are public information. Riis [29] demonstrates the same if contestants learn their private types after entering the contest. We assume that the contestants' types are private information, comparing to the public information assumption in Lazear and Rosen [16]. We also assume that the type information is endowed, comparing to the ex post private information in Riis [29].

Finally, our paper is related to the literature on mechanism design with financially constrained players. Che and Gale [4] consider the revenue ranking of standard auctions when bidders are financially constrained. Benoit and Krishna [1] study the effects of budget constraints in multi-object auctions. Laffont and Robert [15] and Maskin [18] analyze the optimal auction design when bidders have a common fixed budget. Che and Gale [6] characterize the revenue maximizing pricing scheme in an environment with a single buyer, whose willingness to pay and budget are both his private information. Pai and Vohra [26] study the optimal auction with multiple bidders with two dimensional (valuation and budget) private information. In our contest model, contestants are financially constrained, and therefore the negative prizes have to be bounded in the optimal contest design.

2 The model

A risk neutral contest designer has a total prize budget of $V > 0$ to elicit efforts from $N \geq 2$ risk neutral contestants in a contest. Each contestant has an ability for the contest. The cost for

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[4]While our analysis shows the possibility of full rent extraction even when agents have independent and private types, we assume risk neutrality, unlimited liability, no collusion among the agents, and no competing principal in our model. Therefore, the critics on full rent extraction by Robert [30], Laffont and Martimort [14], Che and Kim [8], and Peters [27] remain valid.

contestant i with ability t_i to exert effort $e_i \geq 0$ is given by $c(e_i, t_i) = e_i/t_i$. This ability[5], or type, t_i is the private information of contestant i, and it follows an independent and identical distribution with cumulative distribution function $F(\cdot)$, and probability density function $f(\cdot)$ which is strictly positive on support $[a, b]$ with $a > 0$. Following convention, we assume that the distribution satisfies the following regularity condition as in Myerson [25].

Assumption 1. The virtual value function $J(t) = t - \frac{1-F(t)}{f(t)}$ is strictly increasing in t.

The payoff of a contestant is equal to the prize he receives in the contest minus his cost of effort. The contest designer uses the prize budget V to induce efforts from the contestants. At the same time, if there is money left in the budget, she values that money as well. To simplify the notation, assume that there is a linear relationship between effort and money for the contest designer. Let t_0 denote the money to effort ratio; 1 dollar is equivalent to t_0 units of effort. If the contest designer's objective is to maximize the total efforts, then $t_0 = 0$. Assume that t_0 is common knowledge.[6] Note that the cost of 1 unit of effort for the highest ability (b) contestant is $1/b$, which must be less than $1/t_0$, the value of 1 unit of effort to the contest designer; otherwise, it is obviously optimal for the designer not to spend any of the prize budget. Therefore, we assume that $t_0 < b$.

According to the revelation principle, we can focus our analysis on direct mechanisms. Let $\tilde{t}_i \in [a, b]$ be the report of contestant i regarding his own ability. Then we can define contestant i's prize and effort as functions of the profile of reports $\tilde{\mathbf{t}} = (\tilde{t}_1, \cdots, \tilde{t}_N)$ by $v_i(\tilde{\mathbf{t}})$ and $e_i(\tilde{\mathbf{t}})$, respectively. Given the profile of reports $\tilde{\mathbf{t}} = (\tilde{t}_1, \cdots, \tilde{t}_N)$, the contest designer gives a prize of $v_i(\tilde{\mathbf{t}})$ to contestant i and demands an effort of $e_i(\tilde{\mathbf{t}})$ from him. This is equivalent to its counterpart in auction designs: given the profile of type reports from the bidders, the auctioneer assigns the probability of winning to a bidder and demands a payment from him. Similarly, a direct contest mechanism can thus be denoted by $(\mathbf{v}(\tilde{\mathbf{t}}), \mathbf{e}(\tilde{\mathbf{t}}))$, where $\mathbf{v}(\tilde{\mathbf{t}}) = (v_1(\tilde{\mathbf{t}}), \cdots, v_N(\tilde{\mathbf{t}}))$ and $\mathbf{e}(\tilde{\mathbf{t}}) = (e_1(\tilde{\mathbf{t}}), \cdots, e_N(\tilde{\mathbf{t}}))$.

In the following section, we will examine the existence of the optimal mechanism and offer remedies when it does not exist.

3 Optimal contest design

We will use Myerson's [25] optimal auction approach to analyze this optimal contest design problem. The model setup in the previous section hints on the connections between an optimal auction problem and an optimal contest problem. In this section, in the process of analyzing the optimal contest design, we will illustrate how the two distinguish themselves from each other.

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[5]In Moldovanu and Sela [21], a contestant's ability is defined as $c_i = \frac{1}{t_i}$, which is mathematically equivalent.

[6]As we shall see later, this t_0 is similar to the reservation value of a seller in the auction design problem.

3.1 Prize and punishment

Define the expected prize of contestant i with report $\tilde{t}_i$ as

$$V_i(\tilde{t}_i) = \int_{\mathbf{t}_{-i}} v_i(\tilde{t}_i, \mathbf{t}_{-i})\mathbf{f}_{-i}(\mathbf{t}_{-i})d\mathbf{t}_{-i}, \tag{1}$$

where $\mathbf{t}_{-i} = (t_1, ...t_{i-1}, t_{i+1}, ...t_N)$ and $\mathbf{f}_{-i}(\mathbf{t}_{-i})$ denotes the density of $\mathbf{t}_{-i}$.

Given that other contestants truthfully report their abilities, contestant i's expected payoff when reporting $\tilde{t}_i$ is

$$
\begin{aligned}
u_i(\tilde{t}_i, t_i) &= \int_{\mathbf{t}_{-i}} v_i(\tilde{t}_i, t_{-i})\mathbf{f}_{-i}(\mathbf{t}_{-i})d\mathbf{t}_{-i} - \frac{\int_{\mathbf{t}_{-i}} e_i(\tilde{t}_i, \mathbf{t}_{-i})\mathbf{f}_{-i}(\mathbf{t}_{-i})d\mathbf{t}_{-i}}{t_i} \\
&= V_i(\tilde{t}_i) - \frac{\int_{\mathbf{t}_{-i}} e_i(\tilde{t}_i, \mathbf{t}_{-i})\mathbf{f}_{-i}(\mathbf{t}_{-i})d\mathbf{t}_{-i}}{t_i}.
\end{aligned} \tag{2}
$$

The contest designer maximizes the expected total efforts from the contestants plus the effort equivalent of any money left in the prize budget. In the rest of the analysis, this designer is to maximize this total effort equivalent (or total efforts for short), which is given by the following:

$$\max_{(\mathbf{v}(\cdot), \mathbf{e}(\cdot))} R = \int_{\mathbf{t}} \left[\sum_i e_i(\mathbf{t}) + t_0(V - \sum_i v_i(\mathbf{t})) \right] \mathbf{f}(\mathbf{t})d\mathbf{t} \tag{3}$$

subject to the following feasibility constraints

$$u_i(t_i, t_i) \geq u_i(\tilde{t}_i, t_i), \forall \tilde{t}_i, t_i, \forall i, \tag{4}$$

$$u_i(t_i, t_i) \geq 0, \forall t_i, \forall i, \tag{5}$$

$$\sum_i v_i(\mathbf{t}) \leq V, \forall \mathbf{t}, \tag{6}$$

$$e_i(\mathbf{t}) \geq 0, \forall \mathbf{t}, \forall i. \tag{7}$$

The feasibility constraints consist of four parts: (4) is the incentive compatibility constraint, (5) is the participation constraint, (6) is the designer's budget constraint, and (7) is the nonnegative effort constraint.

We are now ready to compare our optimal contest design problem with the optimal auction design problem in Myerson [25]. There are three similarities. First, the prize allocations here correspond to the object winning probabilities there. The sum of the winning probabilities in an

9

auction must not exceed 1, while the sum of the prizes awarded in a contest must not exceed V. (Note that the total prize budget V can be normalized to 1.) Second, the efforts here resemble the transfer payments there. Third, the contest designer's objective function here is equivalent to the auction designer's revenue there (with t_0 being the seller's reservation value).

Despite these similarities, there are two main differences, both lie in the restrictions on the choice variables. First, in an optimal auction design problem, the object winning probabilities must be nonnegative. In our optimal contest design problem, the prizes for the contestants can be positive or negative. A positive prize is a reward, while a negative prize is a punishment. This enlarges the set of feasible mechanisms. Negative prizes provide a venue for further enhancing the contest design by leveraging the differences in the contestants' virtual values. For a given ability profile $\mathbf{t}$, the negative prizes to lower ability contestants can be used to increase the positive prizes to higher ability contestants while still balancing the prize budget. This would improve the total efforts exerted by the contestants even if the participation constraints for the lower types still need to be satisfied.

Second, the monetary transfers in the optimal auction design problem can be positive or negative. However, in our optimal contest design problem, efforts must be non-negative. This shrinks the set of feasible mechanisms and thus reduces the amount of efforts that can be induced. Putting these two conflicting effects together, it is not clear whether the optimal contest can do better than the optimal auction.

Allowing for negative prizes is seemingly a small deviation from the conventional auction design literature. But it creates significant technical challenges in the analysis. In the analysis for the optimal contest design, there is no obvious way to optimally leverage the prizes assigned to different virtual valuations of the contestants. In the rest of this section, we devote our analysis to solving this optimal contest design problem.

Define $\tilde{u}_i(\tilde{t}_i, t_i) = t_i \cdot u_i(\tilde{t}_i, t_i)$. Then

$$\tilde{u}_i(\tilde{t}_i, t_i) = t_i V_i(\tilde{t}_i) - \int_{\mathbf{t}_{-i}} e_i(\tilde{t}_i, \mathbf{t}_{-i}) \mathbf{f}_{-i}(\mathbf{t}_{-i}) d\mathbf{t}_{-i}. \tag{8}$$

Constraints (4) and (5) can be rewritten in terms of $\tilde{u}_i(\cdot, \cdot)$. From (4) and the Envelope Theorem, we have

$$\frac{d\tilde{u}_i(t_i, t_i)}{dt_i} = \frac{\partial \tilde{u}_i(\tilde{t}_i, t_i)}{\partial t_i}\Big|_{\tilde{t}_i = t_i} = V_i(t_i),$$

which leads to

$$\tilde{u}_i(t_i, t_i) - \tilde{u}_i(a, a) = \int_a^{t_i} V_i(s) ds.$$

Standard derivations such as those in Myerson [25] lead to the following lemma. The proof is

omitted here.

Lemma 1 *Mechanism* $(\mathbf{v}(\cdot), \mathbf{e}(\cdot))$ *is feasible if and only if the following conditions hold together with (6) and (7):*

$$\int_{\mathbf{t}_{-i}} e_i(t_i, \mathbf{t}_{-i})\mathbf{f}_{-i}(\mathbf{t}_{-i})d\mathbf{t}_{-i} = t_i V_i(t_i) - \int_a^{t_i} V_i(s)ds - a \cdot u_i(a, a), \forall t_i, \forall i, \tag{9}$$

$$V_i(t_i') \geq V_i(t_i), \ \forall t_i' > t_i, \forall i, \tag{10}$$

$$u_i(a, a) \geq 0, \forall i. \tag{11}$$

Condition (9) is a direct implication of the incentive compatibility constraint. It implies that the contestants' expected effort levels $\mathbf{e}(\cdot)$ can be fully pinned down by the prize structure $\mathbf{v}(\cdot)$. In other words, two mechanisms with the same prize functions would generate the same total expected efforts. Apparently, this result is parallel to the Revenue Equivalence Theorem in the auction design literature. Condition (10) requires that the expected prize must be increasing in a contestant's ability, and it is parallel to the increasing expected winning probability condition in an auction. Condition (11) implies that a contestant with the lowest ability must be willing to participate, same as in an auction.

Note that in the optimal contest, $u_i(a, a) = 0$, i.e., the lowest ability contestant must earn zero informational rent. Otherwise, the contest designer can simply decrease the informational rent for every ability and yield a higher expected total effort level.

3.2 Optimal mechanisms and utmost total efforts

We are now ready to investigate the existence of an optimal mechanism.

3.2.1 Problem (P)

Given (1) and (9), we can replace effort $\mathbf{e}(\cdot)$ by the prize function $\mathbf{v}(\cdot)$ and rewrite the contest designer's objective function as

$$\max \int_{\mathbf{t}} \sum_i \left[J(t_i) - t_0\right] v_i(\mathbf{t})\mathbf{f}(\mathbf{t})d\mathbf{t} + t_0 V. \tag{12}$$

Note that $J(t_i) = t_i - \frac{1-F(t_i)}{f(t_i)}$ is assumed to be strictly increasing in Assumption 1. Therefore, the contest designer's optimization problem can be restated as maximizing (12), subject to (6), (7), (9) and (10). We denote this maximization problem as **problem (P)** and the resulting mechanism as

the **optimal mechanism**.

A useful benchmark effort level is the **utmost total efforts,** which is the highest amount of total efforts inducible given budget V. This level of effort is achieved when all contestants are of the highest ability b, and it is equal to bV. We can see this from the following arguments. First, the total efforts induced cannot be higher than bV from the participation constraints of the contestants. Second, the effort level bV can be obtained by asking each of the N contestants to exert effort $e = bV/N$ and awarding each of them a prize of V/N. In the rest of the analysis, we will refer to this utmost total efforts from time to time.

We will approach problem (P) using the following method. We first examine a relaxed problem and establish an upper bound for the expected total effort level in this relaxed problem. Second, we show that there exists no optimal mechanism in this relaxed problem. Third, we construct a feasible mechanism in this relaxed problem and show that the expected total effort level in this mechanism can approach arbitrarily close to the utmost total efforts. Finally, we demonstrate that the same conclusion holds for the original problem (P).

3.2.2   Problem (P-Relax)

We start by considering the following optimization problem, denoted as (P-Relax):

$$\max \int_{\mathbf{t}} \sum_i \left[J(t_i) - t_0 \right] v_i(\mathbf{t}) \mathbf{f}(\mathbf{t}) d\mathbf{t} + t_0 V \tag{13}$$

subject to

$$\sum_i v_i(\mathbf{t}) \leq V, \ \forall \mathbf{t}, \tag{14}$$

$$V_i(t_i) = \int_{\mathbf{t}_{-i}} v_i(t_i, \mathbf{t}_{-i}) \mathbf{f}_{-i}(\mathbf{t}_{-i}) d\mathbf{t}_{-i} \geq 0, \ \forall t_i, \forall i. \tag{15}$$

This is a relaxed problem of problem (P): the objective function is the same but the feasibility constraints are less restrictive than the original ones. To see this, constraint (14) follows directly from (6). We next argue that constraint (15) is implied by the feasibility constraints in (P). Note that from the monotonicity condition (10), it is sufficient to show that $V_i(a) \geq 0, \forall i$. From (9), evaluating at $v_i = a$, we obtain $aV_i(a) = \int_{\mathbf{t}_{-i}} e_i(t_i, \mathbf{t}_{-i}) \mathbf{f}_{-i}(\mathbf{t}_{-i}) d\mathbf{t}_{-i}$, which is non-negative from the non-negative effort constraint (7).

The relaxed problem (P-Relax) is a continuous linear programming problem. We construct the Lagrangian by applying multiplier $\lambda(\mathbf{t})$ to constraint (14) and $\mu_i(t_i)$ to constraint (15) before

integrating them and adding them to the objective function:

$$L = \int_{\mathbf{t}} \sum_i [J(t_i) - t_0] v_i(\mathbf{t}) \mathbf{f}(\mathbf{t}) d\mathbf{t} + t_0 V + \int_{\mathbf{t}} \lambda(\mathbf{t}) \left[V - \sum_i v_i(\mathbf{t}) \right] \mathbf{f}(\mathbf{t}) d\mathbf{t}$$

$$+ \sum_i \int_{t_i} \mu_i(t_i) \left(\int_{\mathbf{t}_{-i}} v_i(t_i, \mathbf{t}_{-i}) \mathbf{f}_{-i}(\mathbf{t}_{-i}) d\mathbf{t}_{-i} \right) f(t_i) dt_i. \tag{16}$$

Suppose that an optimal solution exists. Then it must satisfy the following Kuhn-Tucker conditions:

$$[J(t_i) - t_0] - \lambda(\mathbf{t}) + \mu_i(t_i) = 0, \forall \mathbf{t}, \forall i, \tag{17}$$

$$\lambda(\mathbf{t}) \geq 0, \ V - \sum_i v_i(\mathbf{t}) \geq 0, \ \text{and} \ \lambda(\mathbf{t}) \left[V - \sum_i v_i(\mathbf{t}) \right] = 0, \forall \mathbf{t}, \tag{18}$$

$$\mu_i(t_i) \geq 0, \ \int_{\mathbf{t}_{-i}} v_i(t_i, \mathbf{t}_{-i}) \mathbf{f}_{-i}(\mathbf{t}_{-i}) d\mathbf{t}_{-i} \geq 0, \ \text{and} \ \mu_i(t_i) \int_{\mathbf{t}_{-i}} v_i(t_i, \mathbf{t}_{-i}) \mathbf{f}_{-i}(\mathbf{t}_{-i}) d\mathbf{t}_{-i} = 0, \forall t_i, \forall i. \tag{19}$$

Lemma 2 *An optimal solution does not exist for problem (P-Relax).*

Proof: From (17), $\lambda(\mathbf{t})$ should not depend on any $\mathbf{t}_{-i}, \forall i$. Thus, $\lambda(\mathbf{t})$ must be a constant and does not depend on $\mathbf{t}$. Since $\lambda(\mathbf{t}) = \lambda$ is a constant, and since $\mu_i(t_i) \geq 0$, from (17) we must have $J(t_i) - t_0 \leq \lambda$ for every t_i. This means that $\lambda \geq b - t_0 > 0$, as $J(b) = b$. Since $[J(t_i) - t_0]$ is increasing, we have $[J(t_i) - t_0] < b - t_0$ for all $t_i < b$. Then $\mu_i(t_i) = \lambda - [J(t_i) - t_0] > 0$ for all $t_i < b, \forall i$. Thus, (15) must be binding for all $t_i < b$ at the optimal solution. This implies that $V_i(t_i) = 0, \ \forall t_i < b, \forall i$.

Since $\lambda > 0$, (14) must be binding for any $\mathbf{t}$, which implies that $\int_{\mathbf{t}} \sum_i v_i(\mathbf{t}) \mathbf{f}(\mathbf{t}) d\mathbf{t} = V$. We thus have $\sum_i \int_a^b V_i(t_i) f(t_i) dt_i = V$. This means that at least one $V_i(b)$ should be infinity. However, $V_i(b) = \infty$ is impossible. Thus an optimal solution does not exist for the relaxed problem (P-Relax). $\square$

Although the above lemma shows that an optimal contest mechanism does not exist for the relaxed problem (P-Relax), we can nevertheless establish that the utmost total efforts bV is an upper bound for the total efforts in all feasible mechanisms in problem (P-Relax). In problem (P-Relax), the abilities of the contestants are usually less than b, and the total efforts elicited in any feasible mechanism must be less than bV. We have the following lemma.

Lemma 3 *The expected total efforts elicited in problem (P-Relax) are strictly less than bV.*

Proof: See Appendix. $\square$

Next, we establish a sequence of feasible mechanisms $(\mathbf{v}(\cdot;K),\mathbf{e}(\cdot;K))$, with $K \geq 0$ in the relaxed problem (P-Relax). When K goes to infinity, these mechanisms achieve levels of total efforts arbitrarily close to the upper bound bV.

Define

$$\hat{t}(K) = F^{-1}((\frac{NK}{V+NK})^{\frac{1}{N-1}}),\tag{20}$$

where $K \geq 0$ is an arbitrary non-negative real number. Define set $S = \{j : t_j > \hat{t}\}$, and let $t^{(1)}$ denote the first order statistics of $\mathbf{t}$.

Define the following prize allocation function:

$$v_i(\mathbf{t};K) = \begin{cases} \frac{V}{N}, & \text{if } S = \emptyset; \\ -K, & \text{if } S \neq \emptyset \ \text{and } t_i < t^{(1)}; \\ V + (N-1)K, & \text{if } S \neq \emptyset \ \text{and } t_i = t^{(1)}. \end{cases}\tag{21}$$

In this function, ties are broken randomly and fairly.

One can verify that for this prize allocation function,

$$V_i(t_i;K) = \begin{cases} 0, & \text{if } t_i \leq \hat{t}(K); \\ (V+NK)F^{N-1}(t_i) - K > 0, & \text{if } t_i > \hat{t}(K). \end{cases}\tag{22}$$

and it is increasing in t_i.

Define effort functions from (9) with $u_i(a,a) = 0$ as follows:

$$e_i(\mathbf{t};K) = t_i V_i(t_i) - \int_a^{t_i} V_i(s)ds \begin{cases} = 0, & \text{if } t_i \leq \hat{t}(K); \\ > 0, & \text{if } t_i > \hat{t}(K). \end{cases}\tag{23}$$

We have $t_i V_i(t_i) - \int_a^{t_i} V_i(s)ds > 0$ for $t_i > \hat{t}(K)$, as $V_i(s)$ is strictly increasing given $t_i > \hat{t}(K)$ and $a > 0$.

It is straight-forward to verify that all constraints of (P-Relax) are satisfied and therefore, the above mechanism $(\mathbf{v}(\cdot;K),\mathbf{e}(\cdot;K))$ is feasible for every $K \geq 0$. The calculations are standard and are omitted here. We have the following lemma.

Lemma 4 *When K goes to ∞, the expected total efforts elicited by mechanism $(\mathbf{v}(\cdot;K),\mathbf{e}(\cdot;K))$ go to bV and the expected payoffs of the contestants go to zero.*

Proof: See Appendix. $\square$

This lemma shows that even when abilities of the contestants are their own private information and their abilities are normally less than the upper limit b, the contest designer can still elicit

14

almost the utmost total efforts bV using the above mechanisms. In each of these mechanisms, there is a cut-off ability. If none of the contestants have an ability higher than this cut-off, then every contestant gets an equal share of the prize budget V. But if at least one of the contestants has an ability higher than the cut-off, then every contestant will be punished by a negative prize $-K$, except the highest ability contestant who gets the prize budget V plus the extra money generated from the negative prizes of other contestants.

Lemmas 3 and 4 establish that for problem (P-Relax), there exists no optimal contest mechanism, but the contest designer can obtain an expected total effort level arbitrarily close to the utmost total efforts bV. The following proposition shows that the same result is valid for the original problem (P) as well.

Proposition 1 *There exists no optimal contest mechanism in problem (P). However, the contest designer can achieve an expected total effort level that is arbitrarily close to the utmost total efforts bV and the contestants' surplus can be extracted arbitrarily close to the full extent.*

Proof: Similar to Lemma 3, we can establish that the expected total effort level in the original problem (P) is also bounded by bV. We can also verify that the above constructed mechanisms are also feasible in the original problem. Note that these mechanisms generate expected total effort levels arbitrarily close to bV. Therefore, if an optimal contest mechanism exists for problem (P), then the expected total efforts elicited must be exactly bV. However, since problem (P-Relax) is a relaxed problem of problem (P), that optimal contest mechanism must also be feasible for problem (P-Relax) and generate expected total efforts bV. From Lemma 3, we know that it is not possible. Therefore, there exists no optimal contest mechanism in problem (P). Lemma 4 implies the second half of this proposition. $\square$

The results in this proposition are seemingly contradicting to the full surplus extraction results in the optimal auction literature. There, the valuations of the bidders must be interdependent. Full surplus cannot be extracted if bidders' valuations are independent. Our results demonstrate that almost full surplus extraction is feasible with independent valuations in contests. More surprisingly, the utmost total efforts that correspond to the effort level when ALL contestants have the upper bound ability b can almost be reached. In auctions, the maximal achievable expected total revenue is the expectation of the maximum virtual valuation. Even in the case where bidders' valuations are common knowledge, the seller's expected total revenue is just the maximum valuation. From these differences, we can see that an optimal contest problem is very different from an optimal auction problem.

3.3 Optimal contest design with bounded punishment

The previous subsection establishes a surprising finding that the utmost total efforts bV can be achieved asymptotically when we use larger and larger punishments (negative prizes). It is not difficult to imagine that large punishments are not practical. Contestants do not have an infinite amount of wealth to pay for the punishments. Furthermore, large punishments may not be lawful. In this subsection, we investigate the optimal contest design problem when there is a bound on the punishments.

3.3.1 Problem (P-K)

Suppose that the bound on any punishment for a contestant is $K \geq 0$. We impose this additional restriction on the original Problem (P),

$$v_i(\mathbf{t}) \geq -K, \forall \mathbf{t}, \forall i. \tag{24}$$

We call this bounded punishment optimization problem (P-K).

We adopt a multi-step procedure to solve optimization problem (P-K). Here is our road map for solving this problem. First, we consider a relaxed problem of problem (P-K), denoted by problem (P-K-Relax), and establish some necessary conditions for the optimization. Second, we add these necessary conditions to the constraints of problem (P-K-Relax) and obtain an equivalent problem of the relaxed problem, denoted by problem (P-K-Relax-Equivalent). Note that the optimal solutions of problems (P-K-Relax) and (P-K-Relax-Equivalent) are the same. Third, we further relax problem (P-K-Relax-Equivalent) and examine problem (P-K-Relax-Equivalent-Relax). Fourth, we fully characterize the solution to problem (P-K-Relax-Equivalent-Relax). Finally, we construct a feasible mechanism of the original problem (P-K) that achieves the maximal effort level of problem (P-K-Relax-Equivalent-Relax).

In problem (P-K), define $\hat{t}_i = \sup\{t_i | V_i(t_i) = 0\}$. Note that $V_i(t_i)$ is nonnegative and increasing. Without loss of generality, assume that $V_i(t_i)$ is left-continuous at $\hat{t}_i$. Therefore, $V_i(t_i) = 0$ for $t_i \leq \hat{t}_i$ and $V_i(t_i) > 0$ for $t_i > \hat{t}_i$.

3.3.2 Problem (P-K-Relax)

We start our analysis by considering the following relaxed optimization problem (P-K-Relax) of problem (P-K):

$$\max_{\{v_i(\mathbf{t}), \hat{t}_i, \forall i\}} \int_{\mathbf{t}} \sum_i \left[J(t_i) - t_0 \right] v_i(\mathbf{t}) \mathbf{f}(\mathbf{t}) d\mathbf{t} + t_0 V \tag{25}$$

16

subject to

$$\int_{\mathbf{t}} \sum_i v_i(\mathbf{t})\mathbf{f}(\mathbf{t})d\mathbf{t} \quad \leq \quad V, \tag{26}$$

$$V_i(t_i) \quad = \quad \int_{\mathbf{t}_{-i}} v_i(t_i, \mathbf{t}_{-i})\mathbf{f}_{-i}(\mathbf{t}_{-i})d\mathbf{t}_{-i} = 0, \ \forall t_i \leq \hat{t}_i, \forall i, \tag{27}$$

$$V_i(t_i) \quad = \quad \int_{\mathbf{t}_{-i}} v_i(t_i, \mathbf{t}_{-i})\mathbf{f}_{-i}(\mathbf{t}_{-i})d\mathbf{t}_{-i} > 0, \ \forall t_i > \hat{t}_i, \forall i, \tag{28}$$

$$v_i(\mathbf{t}) \quad \geq \quad -K, \ \forall \mathbf{t}, \forall i, \tag{29}$$

$$a \quad \leq \quad \hat{t}_i \leq b, \forall i. \tag{30}$$

This is a relaxed problem of problem (P-K). This is because the objective functions are the same in both problems, and the feasibility constraints are less restrictive than those in problem (P-K). To see this, constraint (26) follows from (6) by integrating over $\mathbf{t}$. Constraints (27) and (28) directly follow the definition of $\hat{t}_i$. Constraint (29) is the same as (24) in (P-K). Constraint (30) allows for all possible threshold values of $\hat{t}_i$.

We next characterize a key property for the optimal solutions $\{v_i^{\circledast}(\mathbf{t}), \hat{t}_i^{\circledast}, \forall i\}$ of problem (P-K-Relax).

Consider problem (P-K-Relax) for a fixed $\hat{t}_i = \hat{t}_i^{\circledast}$. We construct the Lagrangian by introducing multipliers λ for constraint (26), $\mu_i(t_i)$ for constraints (27) and (28), and $\xi_i(\mathbf{t})$ for constraint (29):

$$\begin{aligned}
L \quad = \quad & \int_{\mathbf{t}} \sum_i \left[J(t_i) - t_0\right] v_i(\mathbf{t})\mathbf{f}(\mathbf{t})d\mathbf{t} + t_0 V + \lambda \int_{\mathbf{t}} \left[V - \sum_i v_i(\mathbf{t})\right]\mathbf{f}(\mathbf{t})d\mathbf{t} \\
& + \sum_i \int_{t_i} \mu_i(t_i) \left(\int_{\mathbf{t}_{-i}} v_i(t_i, \mathbf{t}_{-i})\mathbf{f}_{-i}(\mathbf{t}_{-i})d\mathbf{t}_{-i}\right) f(t_i)dt_i \\
& + \sum_i \int_{\mathbf{t}} \xi_i(\mathbf{t}) \left[v_i(\mathbf{t}) + K\right]\mathbf{f}(\mathbf{t})d\mathbf{t}.
\end{aligned}$$

The Kuhn-Tucker conditions for the optimization are:

$$\varsigma_i(\mathbf{t}) = \left[J(t_i) - t_0\right] - \lambda + \mu_i(t_i) + \xi_i(\mathbf{t}) = 0, \forall \mathbf{t}, \forall i,$$

$$\lambda \geq 0, \ V - \int_{\mathbf{t}} \sum_i v_i(\mathbf{t})\mathbf{f}(\mathbf{t})d\mathbf{t} \geq 0, \ \text{and} \ \lambda \left[V - \int_{\mathbf{t}} \sum_i v_i(\mathbf{t})\mathbf{f}(\mathbf{t})d\mathbf{t}\right] = 0, \forall \mathbf{t},$$

17

$$\mu_i(t_i) \geq 0, \ \int_{\mathbf{t}_{-i}} v_i(\mathbf{t})\mathbf{f}_{-i}(\mathbf{t}_{-i})d\mathbf{t}_{-i} \geq 0, \ \text{and} \ \mu_i(t_i) \int_{\mathbf{t}_{-i}} v_i(\mathbf{t})\mathbf{f}_{-i}(\mathbf{t}_{-i})d\mathbf{t}_{-i} = 0, \forall t_i, \forall i,$$

$$\xi_i(\mathbf{t}) \geq 0, \ v_i(\mathbf{t}) + K \geq 0, \ \text{and} \ \xi_i(\mathbf{t})\left[v_i(\mathbf{t}) + K\right] = 0, \forall \mathbf{t}, \forall i.$$

These Kuhn-Tucker conditions lead to the following important necessary conditions for the optimal solutions $\{v_i^{\circledast}(\mathbf{t}), \hat{t}_i^{\circledast}, \forall i\}$ for problem (P-K-Relax).

Lemma 5 *(i)* $\hat{t}_i^{\circledast} \geq \tilde{t}_0 = F^{-1}((\frac{K}{V+NK})^{\frac{1}{N-1}})$;

(ii) For $t_i > \hat{t}_i^{\circledast}$, we must have $0 < V_i^{\circledast}(t_i) \leq (V + NK)F^{N-1}(t_i) - K$.

Proof: We claim $v_i^{\circledast}(t_i, \mathbf{t}_{-i}) = -K$ for $t_i > \hat{t}_i^{\circledast}$ if there exists some contestant $j \neq i$ such that $t_j > t_i$. Suppose not, then $v_i^{\circledast}(t_i, \mathbf{t}_{-i}) > -K$, which means $\xi_i(\mathbf{t}) = 0$. In addition, we have $\mu_i(t_i) = 0$ from the fact $V_i^{\circledast}(t_i) > 0$. Thus $[J(t_i) - t_0] - \lambda = 0$. Note $[J(t_j) - t_0] - \lambda + \mu_j(t_j) + \xi_j(t_i, t_j, \mathbf{t}_{-ij}) = 0$. Thus $J(t_j) - t_0 = \lambda - \mu_j(t_j) - \xi_j(t_i, t_j, \mathbf{t}_{-ij}) \leq \lambda = [J(t_i) - t_0]$, which contradicts the assumption that $J(\cdot)$ is a strictly increasing function.

When t_i is the highest among all contestants, contestant i can at most collect $V + (N-1)K$; when $t_i > \hat{t}_i^{\circledast}$ is not the highest, $v_i^{\circledast}(t_i, \mathbf{t}_{-i}) = -K$. For contestant i, when $t_i > \hat{t}_i^{\circledast}$, we must have

$$0 < V_i^{\circledast}(t_i) \leq [V + (N-1)K]F^{N-1}(t_i) - K \cdot (1 - F^{N-1}(t_i)) = (V + NK)F^{N-1}(t_i) - K. \quad (31)$$

(31) implies that $\hat{t}_i^{\circledast} \geq \tilde{t}_0 = F^{-1}((\frac{K}{V+NK})^{\frac{1}{N-1}})$. □

3.3.3 Problem (P-K-Relax-Equivalent)

Lemma 5 provides a set of necessary conditions for the optimal solution of (P-K-Relax). If we add these necessary conditions to the constraints in (P-K-Relax), we obtain a revised optimization problem (P-K-Relax-Equivalent). The solutions to these two problems are the same. This is because the optimal solution of (P-K-Relax) must satisfy all of the constraints (the original feasibility constraints and the additional necessary conditions) in problem (P-K-Relax-Equivalent). Thus the solution to problem (P-K-Relax-Equivalent) cannot be worse than problem (P-K-Relax). Meanwhile, problem (P-K-Relax-Equivalent) is more restrictive and therefore its solution cannot be better than problem (P-K-Relax).

The equivalent problem (P-K-Relax-Equivalent) can be rewritten as follows:

$$\max_{\{v_i(\mathbf{t}), \hat{t}_i, \forall i\}} \sum_{i=1}^{N} \int_a^b \left[J(t_i) - t_0\right] V_i(t_i)f(t_i)dt_i + t_0 V \quad (32)$$

18

subject to

$$\sum_{i=1}^{N} \int_{a}^{b} V_i(t_i)f(t_i)dt_i \leq V, \tag{33}$$

$$0 < V_i(t_i) \leq (V + NK)F^{N-1}(t_i) - K, \text{ if } t_i > \hat{t}_i , \; \forall i, \tag{34}$$

$$V_i(t_i) = 0, \text{ if } t_i \leq \hat{t}_i, \forall i, \tag{35}$$

$$v_i(\mathbf{t}) \geq -K, \; \forall \mathbf{t}, \forall i, \tag{36}$$

$$\tilde{t}_0 \leq \hat{t}_i \leq b, \forall i. \tag{37}$$

3.3.4 Problem (P-K-Relax-Equivalent-Relax)

Problem (P-K-Relax-Equivalent) can be relaxed to problem (P-K-Relax-Equivalent-Relax) by dropping constraint (36):

$$\max_{\{V_i(\cdot), \, \hat{t}_i, \forall i\}} \sum_{i=1}^{N} \int_{a}^{b} [J(t_i) - t_0] \, V_i(t_i)f(t_i)dt_i + t_0 V \tag{38}$$

subject to

$$\sum_{i=1}^{N} \int_{a}^{b} V_i(t_i)f(t_i)dt_i \leq V, \tag{39}$$

$$0 < V_i(t_i) \leq (V + NK)F^{N-1}(t_i) - K, \text{ if } t_i > \hat{t}_i , \; \forall i, \tag{40}$$

$$V_i(t_i) = 0, \text{ if } t_i \leq \hat{t}_i, \forall i, \tag{41}$$

$$\tilde{t}_0 \leq \hat{t}_i \leq b, \forall i. \tag{42}$$

Note that in problem (P-K-Relax-Equivalent-Relax), the choice variables are merely $\{V_i(\cdot), \hat{t}_i, \forall i\}$.

We define $t^* = \max \{\hat{t}, t^M\}$, where $\hat{t} = F^{-1}((\frac{NK}{V+NK})^{\frac{1}{N-1}})$, and $t^M = J^{-1}(t_0)$ if $J(a) \leq t_0$, $t^M = a$ if $J(a) > t_0$. Note that $t^* \geq \hat{t} \geq \tilde{t}_0$.

We are now ready to present the following lemma, which is the key to the analysis of the optimal contest with bounded punishment.

Lemma 6 $\quad \bar{V}_i(t_i) = \begin{cases} (V + NK)F^{N-1}(t_i) - K, & \text{if } t_i > t^*, \\ 0, & \text{if } t_i \leq t^*. \end{cases} \quad \forall i, \quad$ *is an optimal solution to problem (P-K-Relax-Equivalent-Relax).*

Proof: See Appendix. $\square$

Note that (P-K-Relax-Equivalent-Relax) is a relaxed problem of (P-K). Suppose that we find a $\bar{v}_i(\mathbf{t})$ such that it generates $\bar{V}_i(t_i)$ and satisfies the constraints in the original problem (P-K). Then $\{v_i^*(\mathbf{t}), i = 1, 2, ..., N\}$ and the effort functions $\{e_i^*(\mathbf{t}), i = 1, 2, ..., N\}$ that support $\bar{v}_i(\mathbf{t})$ and satisfy $u_i(a, a) = 0, \forall i$ would constitute an optimal solution to problem (P-K). The supporting effort functions $\{e_i^*(\mathbf{t}), i = 1, 2, ..., N\}$ can be constructed based on (9) with $u_i(a, a) = 0$.

We now construct a mechanism $(\mathbf{v}^*(\cdot), \mathbf{e}^*(\cdot))$ that meets the above requirements.

Define $\Lambda(K) = \begin{cases} \frac{K}{F^{N-1}(t^*)} - K, & \text{if } K > 0; \\ 0, & \text{if } K = 0. \end{cases}$. When $K = 0$, we have $\Lambda(K) = 0 \leq \frac{V}{N}$. When $K > 0$, as $t^* \geq \hat{t}$, we have $0 < \Lambda(K) \leq \frac{V}{N}$, and $\Lambda(K) = \frac{V}{N}$ if and only if $t^* = \hat{t}$.

Let $S^* = \{j : t_j > t^*\}$, and $t^{(1)}$ denote the first order statistics of $\mathbf{t}$.

Proposition 2 *The following is an optimal mechanism for problem (P-K). The prize allocation function $\mathbf{v}^*(\cdot)$ is given by*

$$v_i^*(\mathbf{t}) = \begin{cases} \Lambda(K), & \text{if } S^* = \emptyset; \\ -K, & \text{if } S^* \neq \emptyset \text{ and } t_i < t^{(1)}; \\ V + (N-1)K, & \text{if } S^* \neq \emptyset \text{ and } t_i = t^{(1)}. \end{cases} \tag{43}$$

The effort function is given by

$$e_i^*(\mathbf{t}) = \begin{cases} 0, & \text{if } t_i \leq t^*; \\ t_i V_i^*(t_i) - \int_{t^*}^{t_i} V_i^*(s) ds, & \text{if } t_i > t^*. \end{cases} \tag{44}$$

Proof: It is easy to verify that the prize allocation function generates

$$V_i^*(t_i) = \begin{cases} 0, & \text{if } t_i \leq t^*; \\ [(V + NK)F^{N-1}(t_i) - K] > 0, & \text{if } t_i > t^*. \end{cases} \tag{45}$$

which is the optimal $\bar{V}_i(t_i)$ of Lemma 6. Note that $V_i^*(t_i)$ is discontinuous at $t_i = t^*$.

As we noted earlier, the supporting effort functions $\{e_i^*(\mathbf{t}), i = 1, 2, ..., N\}$ can be constructed using (9) with $u_i(a, a) = 0$. We thus have

$$e_i^*(\mathbf{t}) = \begin{cases} 0, & \text{if } t_i \leq t^*; \\ t_i V_i^*(t_i) - \int_{t^*}^{t_i} V_i^*(s) ds, & \text{if } t_i > t^*. \end{cases}$$

Note that $e_i^*(\mathbf{t})$, the effort contestant i exerts, is independent of the types of other contestants. We can therefore denote it as $e^*(t_i)$ as the effort function is also independent of contestants' identities. One can further verify that $e^*(t_i)$ is strictly positive and increases with t_i for $t_i > t^*$. Note

20

also that because $V_i^*(t_i)$ is discontinuous at $t_i = t^*$, $e^*(t_i)$ is also discontinuous at $t_i = t^*$.

We can easily verify that all constraints in problem (P-K) are satisfied in mechanism $(\mathbf{v}^*(\cdot), \mathbf{e}^*(\cdot))$. We thus have established that $(\mathbf{v}^*(\cdot), \mathbf{e}^*(\cdot))$ is indeed an optimal solution for problem (P-K). $\square$

The following corollary provides a condition for the prize budget V to be spent completely in the optimal mechanism.

Corollary 1 *The prize budget of the contest designer is always completely spent for every* $\mathbf{t}$ *when and only when* $K > 0$ *and* $t^* = \hat{t}$.

When $K = 0$, or $K > 0$ but $t^* > \hat{t}$, the budget constraint is not binding when every t_i is lower than t^*. Note that this result holds even when the contest designer does not derive any benefit from any unspent prize budget, i.e. when $t_0 = 0$.

The optimal mechanism we characterized in the above proposition can be implemented by a simple all-pay auction with entry fees and minimum bids. Define $e^*(t_+^*) = \lim_{t_i \to t_+^*} e^*(t_i)$.

Proposition 3 *The optimal mechanism* $(\mathbf{v}^*(\cdot), \mathbf{e}^*(\cdot))$ *can be implemented by a modified all-pay auction with entry fee* K *and minimum bid* $e^*(t_+^*)$. *The highest bidder wins* V *plus all entry fees collected from all of the participants. When no one bids, all participants equally share prize* $N\Lambda(K) \leq V$; *all entry fees are still collected. All non-participants get zero prize.*

Proof: It is straight-forward to verify that it is optimal for all types of contestants to pay K and participate. For abilities above t^*, the optimal bid (in terms of effort) is given by $e_i^*(\mathbf{t})$ that is defined by (44). Contestants with abilities below t^* participate but do not bid. Note that $e_i^*(\mathbf{t})$ depends only on t_i and strictly increases with $t_i(> t^*)$. $\square$

Obviously, an optimal mechanism for a smaller K is also feasible when K becomes larger. We thus can conclude that the expected total efforts elicited from problem (P-K) must be increasing in K. When $K = 0$, negative prize is not allowed in the contest. In this case, the constraints in the contest design problem (P-K) are more restrictive than those in the optimal auction design, as negative efforts are not allowed in contests but negative payments are allowed in auctions. However, the contest designer can do equally well in this case simply because the Myerson optimal auctions need not involve negative monetary payments. These results are formalized in the following corollary.

Corollary 2 *(i) The expected total efforts elicited in the optimal contest mechanism of problem (P-K) increase in* K, *the bound on punishments.*

(ii) When $K = 0$ *(i.e., when no punishment is allowed), the optimal contest mechanism resembles the Myerson optimal auction. In particular, when* $J(a) \geq t_0$, *the optimal contest mechanism*

can be implemented by a standard all-pay auction with a single prize V for the winner. (This is the optimal mechanism in Moldovanu and Sela [21].)

4 Concluding remarks

In this paper, we examine the issue of optimal contest design with private information and completely characterize the optimal or almost optimal contests in various situations. In the model, contestants differ in their abilities, which are their own private information. We adopt a mechanism design approach to accommodate all possible prize allocation rules. We focus on the effect of negative prizes (punishments): our analysis allows for both positive and negative prizes given a fixed prize budget. We find that the utmost total efforts can be achieved in the limit when the size of punishments becomes larger and larger. In the limit, all surpluses from the contestants are extracted. In addition, we fully characterize the optimal contest when there is a bound on the punishments. It is noteworthy that the (almost) full extraction result is obtained in an environment of independent private information, in contrast to similar results in the auction literature that require interdependent private information.

Compared to an optimal auction mechanism, one distinct feature of the optimal contest mechanism is worth highlighting. The prizes assigned to the contestants' different virtual valuations are being leveraged in the optimal contest mechanism to achieve the efficiency of the contest. The adoption of negative prizes allows the contest designer to expand the original prize budget for the more able contestants. The participation constraint for each type of contestants can be accommodated by using appropriately designed cross-type transfers. The incentives for the more able contestants to exert effort are enhanced by the cross-type transfers through positive and negative prizes. Such transfers may dampen the incentive for the less able contestants to exert effort. But since it is more efficient for the more able contestants to exert effort, the overall effects of cross-type transfers unambiguously increase the expected total efforts in the mechanism.

Our current study provides a few possibilities for future research. First, in the analysis, we consider symmetric contestants only. It is not trivial to generalize the analysis to accommodate asymmetric contestants. Second, in the optimal contest with bounded punishments, we assume a common and fixed bound for the negative prizes. In many situations, this bound is heterogenous among the contestants. The bound can even be the private information of the contestants. Future analysis might provide new insights on how optimal leveraging on different virtual valuations should be applied according to these heterogeneities and private information. Third, similar to the optimal auctions with risk averse players (c.f. Maskin and Reily [17]), one can investigate the optimal contest design problem with risk averse contestants (or equivalently, with convex effort cost functions). Fourth, our analysis currently focuses on an environment with pure adverse selection. Extending

the analysis to a setting of mixed adverse selection and moral hazard problem would be a natural direction. Finally, alternative objective functions for the contest designer can be examined. For example, the contest designer may value only the highest effort among the contestants. All these are interesting topics for future studies.

5   Appendix

Proof of Lemma 3: We will establish that bV is an upper bound of the expected total efforts in problem (P-Relax). First note that

$$\int_{\mathbf{t}} \sum_i \left[J(t_i) - t_0 \right] v_i(\mathbf{t}) \mathbf{f}(\mathbf{t}) d\mathbf{t} = \sum_i \int_{t_i} \left[J(t_i) - t_0 \right] V_i(t_i) f(t_i) dt_i.$$

Since $V_i(t_i) \geq 0$, $J(\cdot)$ is increasing and $t_0 < b = J(b)$, we have

$$\sum_i \int_{t_i} \left[J(t_i) - t_0 \right] V_i(t_i) f(t_i) dt_i + t_0 V$$

$$\leq \quad [J(b) - t_0] \sum_i \int_{t_i} V_i(t_i) f(t_i) dt_i + t_0 V$$

$$= \quad (b - t_0) \int_{\mathbf{t}} [\sum_i v_i(\mathbf{t})] f(\mathbf{t}) d\mathbf{t} + t_0 V$$

$$\leq \quad (b - t_0) \int_{\mathbf{t}} V f(\mathbf{t}) d\mathbf{t} + t_0 V$$

$$= \quad bV.$$

This upper bound bV cannot be reached by any mechanism as Lemma 2 showed that an optimal solution does not exist for problem (P-Relax). $\square$

Proof of Lemma 4: The expected total efforts induced by mechanism $(\mathbf{v}(\cdot; K), \mathbf{e}(\cdot; K))$ are given by

$$
\begin{aligned}
R &= N \int_{\hat{t}(K)}^{b} \left[J(t_i) - t_0 \right] V_i(t_i; K) dF(t_i) + t_0 V \\
&= N \int_{\hat{t}(K)}^{b} \left[J(t_i) - t_0 \right] \left[(V + NK) F^{N-1}(t_i) - K \right] dF(t_i) + t_0 V \\
&= N \int_{\hat{t}(K)}^{b} \left[J(t_i) - t_0 \right] V F^{N-1}(t_i) dF(t_i) + NK \int_{\hat{t}}^{b} \left[J(t_i) - t_0 \right] [N F^{N-1}(t_i) - 1] dF(t_i) + t_0 V.
\end{aligned}
$$

When $K \to +\infty$, $\hat{t}(K)$ goes to b. Therefore, the first part in the last expression goes to zero. The third part is a constant. To show that the expected total efforts converge to bV when $K \to +\infty$, it suffices to show that the second part converges to $(b - t_0)V$ when $K \to +\infty$. For the second part,

24

note that $F^{N-1}(\hat{t}(K)) = \frac{NK}{V+NK}$. That leads to

$$\frac{d\hat{t}(K)}{dK} = \frac{NV}{(V+NK)^2(N-1)F^{N-2}(\hat{t}(K))f(\hat{t}(K))}.$$

Therefore,

$$\lim_{K\to+\infty} NK \int_{\hat{t}(K)}^{b} [J(t_i) - t_0] [NF^{N-1}(t_i) - 1]dF(t_i)$$

$$= \lim_{K\to+\infty} N\frac{\int_{\hat{t}(K)}^{b} [J(t_i) - t_0] [NF^{N-1}(t_i) - 1]dF(t_i)}{\frac{1}{K}}$$

$$= \lim_{K\to+\infty} N\frac{-[J(\hat{t}(K)) - t_0][NF^{N-1}(\hat{t}(K)) - 1]f(\hat{t}(K))\frac{d\hat{t}}{dK}}{-\frac{1}{K^2}} \quad \text{(by L'Hospital's rule)}$$

$$= \lim_{K\to+\infty} N\frac{[J(\hat{t}(K)) - t_0][NF^{N-1}(\hat{t}(K)) - 1]\frac{NV}{(V+NK)^2(N-1)F^{N-2}(\hat{t}(K))}}{\frac{1}{K^2}}$$

$$= \lim_{K\to+\infty} N\frac{[J(b) - t_0][NF^{N-1}(b) - 1]\frac{NV}{(V+NK)^2(N-1)F^{N-2}(b)}}{\frac{1}{K^2}}$$

$$= \lim_{K\to+\infty} N(b - t_0)(N - 1)\frac{NVK^2}{(V+NK)^2(N-1)}$$

$$= (b - t_0)V.$$

Hence, the expected total efforts R converge to bV.

We now turn to the contestants' expected payoffs. Recall that the contestants' expected total payoffs are at most the difference between V and the expected total effort costs.[7] Since expected total efforts converge to bV, we must have the total effort costs converge to $bV/b = V$ since only those types within a small neighborhood of b would exert positive effort. It follows that the contestants' expected total payoffs must converge to zero. $\quad\square$

Proof of Lemma 6: It is straight-forward to verify that $\bar{V}_i(t_i)$ satisfies all the conditions in the maximization problem (P-K-Relax-Equivalent-Relax). We next consider two cases to show the optimality of $\bar{V}_i(t_i)$. Case 1: $t^* = \hat{t}$, i.e., $J(\hat{t}) \geq t_0$. Case 2: $t^* = t^M$, i.e., $J(\hat{t}) \leq t_0$.

First, we consider Case 1 where $J(\hat{t}) \geq t_0$, i.e. $t^* = \hat{t} \geq t^M$. We shall show that for any

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[7]When the budget constraint is binding, the contestants' expected total payoffs are equal to the difference between V and the expected total effort costs.

functions $V_i(t_i)$ satisfying (39) to (42), we have

$$\sum_{i=1}^{N} \int_a^b \left[J(t_i) - t_0\right] V_i(t_i) f(t_i) dt_i \le \sum_{i=1}^{N} \int_a^b \left[J(t_i) - t_0\right] \bar{V}_i(t_i) f(t_i) dt_i = \sum_{i=1}^{N} \int_{t^*}^b \left[J(t_i) - t_0\right] \bar{V}_i(t_i) f(t_i) dt_i.$$
(46)

This is equivalent to

$$\sum_{i=1}^{N} \int_a^{t^*} \left[J(t_i) - t_0\right] V_i(t_i) f(t_i) dt_i \le \sum_{i=1}^{N} \int_{t^*}^b \left[J(t_i) - t_0\right] \left(\bar{V}_i(t_i) - V_i(t_i)\right) f(t_i) dt_i.$$
(47)

Note that $V_i(t_i) \ge 0$. In addition, when $t_i > t^* = \hat{t}$, $\bar{V}_i(t_i) = (V + NK)F^{N-1}(t_i) - K$. So $\bar{V}_i(t_i) - V_i(t_i) \ge 0$ for $t_i > t^* = \hat{t}$.

Since $J(\cdot)$ is strictly increasing and $J(t^*) \ge t_0$, we have

$$\sum_{i=1}^{N} \int_a^{t^*} \left[J(t_i) - t_0\right] V_i(t_i) f(t_i) dt_i \le \left[J(t^*) - t_0\right] \sum_{i=1}^{N} \int_a^{t^*} V_i(t_i) f(t_i) dt_i,$$

and

$$\left[J(t^*) - t_0\right] \sum_{i=1}^{N} \int_{t^*}^b \left(\bar{V}_i(t_i) - V_i(t_i)\right) f(t_i) dt_i \le \sum_{i=1}^{N} \int_{t^*}^b \left[J(t_i) - t_0\right] \left(\bar{V}_i(t_i) - V_i(t_i)\right) f(t_i) dt_i.$$

Thus, in order for (47) to hold, we only need to show that

$$\sum_{i=1}^{N} \int_a^{t^*} V_i(t_i) f(t_i) dt_i \le \sum_{i=1}^{N} \int_{t^*}^b \left(\bar{V}_i(t_i) - V_i(t_i)\right) f(t_i) dt_i,$$

which is equivalent to

$$\sum_{i=1}^{N} \int_a^b V_i(t_i) f(t_i) dt_i \le \sum_{i=1}^{N} \int_{t^*}^b \bar{V}_i(t_i) f(t_i) dt_i.$$
(48)

According to constraint (39), the LHS of (48) must be bounded by V. For the RHS of (48), we

have

$$\sum_{i=1}^{N} \int_{t^*}^{b} \bar{V}_i(t_i) f(t_i) dt_i$$

$$= \quad N \int_{t^*}^{b} [(V + NK) F^{N-1}(t_i) - K] f(t_i) dt_i$$

$$= \quad (V + NK) F^{N}(t_i)|_{t_i=t^*}^{b} - NK(1 - F(t^*))$$

$$= \quad (V + NK)(1 - F^{N}(t^*)) - NK + NKF(t^*)$$

$$= \quad V - (V + NK) F^{N-1}(t^*) F(t^*) + NKF(t^*)$$

$$= \quad V - (V + NK) \cdot \frac{NK}{V + NK} \cdot F(t^*) + NKF(t^*)$$

$$= \quad V.$$

Hence (48) holds.

Second, we consider Case 2: $J(\hat{t}) \le t_0$, i.e. $\hat{t} \le t^* = t^M$. We shall show that for any $V_i(t_i)$ that satisfies (39) to (42), we have

$$\sum_{i=1}^{N} \int_{a}^{b} [J(t_i) - t_0] V_i(t_i) f(t_i) dt_i \le \sum_{i=1}^{N} \int_{a}^{b} [J(t_i) - t_0] \bar{V}_i(t_i) f(t_i) dt_i,$$

which is equivalent to

$$\sum_{i=1}^{N} \int_{\hat{t}_i}^{b} [J(t_i) - t_0] V_i(t_i) f(t_i) dt_i \le \sum_{i=1}^{N} \int_{t^M}^{b} [J(t_i) - t_0] \bar{V}_i(t_i) f(t_i) dt_i.$$

Consider contestant i. Suppose that $\hat{t}_i \ge t^M$. Note that when $t_i > t^M$, we have $V_i(t_i) \le (V + NK) F^{N-1}(t_i) - K = \bar{V}_i(t_i)$ and $[J(t_i) - t_0] > 0$. Therefore,

$$\int_{\hat{t}_i}^{b} [J(t_i) - t_0] V_i(t_i) f(t_i) dt_i = \int_{t^M}^{b} [J(t_i) - t_0] V_i(t_i) f(t_i) dt_i \le \int_{t^M}^{b} [J(t_i) - t_0] \bar{V}_i(t_i) f(t_i) dt_i.$$

Now suppose that $\hat{t}_i < t^M$. Note that $[J(t_i) - t_0] < 0$ when $t_i < t^M$. We have

$$\int_{\hat{t}_i}^b [J(t_i) - t_0] \, V_i(t_i) f(t_i) dt_i$$

$$= \int_{\hat{t}_i}^{t^M} [J(t_i) - t_0] \, V_i(t_i) f(t_i) dt_i + \int_{t^M}^b [J(t_i) - t_0] \, V_i(t_i) f(t_i) dt_i$$

$$\leq \int_{t^M}^b [J(t_i) - t_0] \, V_i(t_i) f(t_i) dt_i$$

$$\leq \int_{t^M}^b [J(t_i) - t_0] \, \bar{V}_i(t_i) f(t_i) dt_i.$$

The last inequality holds because when $t_i > t^M$, $V_i(t_i) \leq (V + NK) F^{N-1}(t_i) - K = \bar{V}_i(t_i)$ and $[J(t_i) - t_0] > 0$.

To conclude, either when $\hat{t}_i \geq t^M$ or when $\hat{t}_i < t^M$, we always have

$$\int_{\hat{t}_i}^b [J(t_i) - t_0] \, V_i(t_i) f(t_i) dt_i \leq \int_{t^M}^b [J(t_i) - t_0] \, \bar{V}_i(t_i) f(t_i) dt_i.$$

Thus,

$$\sum_{i=1}^N \int_a^b [J(t_i) - t_0] \, V_i(t_i) f(t_i) dt_i \leq \sum_{i=1}^N \int_a^b [J(t_i) - t_0] \, \bar{V}_i(t_i) f(t_i) dt_i.$$

This completes the proof. $\square$

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