The Most Reasonable Solution for an Asymmetric Three-firm Oligopoly

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Abstract: The most reasonable solution for an oligopoly is a stable partition of the firms (i.e., a premerger or postmerger equilibrium) that is free of subsequent mergers or breakups. This paper characterizes three possible solutions for an asymmetric three-firm linear Cournot oligopoly: 1) monopoly (if its merging cost is low and cost differentials are large); 2) a profitable two-firm merger (if monopoly is unprofitable, its smaller member is not too small, and its larger member's share of the merger's gain is large); and 3) original Cournot equilibrium (if no merger is profitable, possibly caused by high merging costs).

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"It would be an error in interpretation to imagine that the players in a non-cooperative game can somehow select whichever strategic equilibrium [SE] they prefer. That would imply a context in which bargaining and joint action are permitted, making it rather unlikely that the players would feel constrained to restrict their strategy selections to SEs" (Shapley 1987, *Lecture Notes on Game Theory*, page 1.29).

"If there are at least three players and if the concept is that of a cooperative game then it seems to me that there isn't any theory yet that seems acceptable as providing a solution concept for the game" (Nash 1998, Presentation at *Cowles Foundation Seminar*).

"There is no single universally accepted solution. There may be many solutions that appear to be reasonable if judged from a specific context point of view" (Shubik 2010, Video presentation at *Inaugural Chinese Game Theory Conference*).

1. Introduction

Consider the four possible mergers in an asymmetric linear Cournot oligopoly with three firms: 12, 13, 23 and 123. Which one, if any, will be formed and be free of subsequent takeovers or spin-offs? Although an answer to the question has been wanting, it has not, as implied in the above Nash and Shubik quotations, been fully answered, because the problem is much more complicated than it appears at first glance.

This paper attempts to provide a complete answer by studying the most reasonable solution and its characterizations. Briefly, there are three classes of solutions determined by the parameters: *1*) monopoly will be formed if its merging cost is sufficiently low and cost differentials are sufficiently large (equivalently, if the largest or most efficient firm is sufficiently large, or if the two small firms are sufficiently small); *2*) each of the three two-firm mergers will be formed if the merger is profitable, the monopoly merger is unprofitable due to high merging costs, its smaller member is not too small, and its larger member's share of the merger's gain is sufficiently large; and *3*) the original Cournot equilibrium will be *the* *solution* (i.e., no merger will be formed) if none of the four mergers is profitable.[1]

These results form a complete characterization for the problem as it covers the set of all three-firm linear oligopolies with seven parameters. In addition, the results also represent two other major advances in the literature. First, it advances the previous works in both cooperative approach (e.g., Lekeas 2013, Lardon 2012, Yong 2004, and Rajan 1989) and non-cooperative approach (e.g., Ray and Vohra [2013,1999], Xue and Zhang 2012, and Bloch 1996) from symmetry to asymmetry by showing how cost differentials affect a stable partition.[2] Second, it advances merger studies by showing for the first time how merging cost or transaction cost (i.e., the cost of forming a coalition)[3] affect merger stability.

The rest of the paper is organized as follows. Section 2 describes the model and merger contracts, section 3 defines reasonable solutions for an oligopoly with n firms, section 4 characterizes the most reasonable solution with three firms, section 5 concludes, and the appendix provides proofs.

2. Description of the model and merger contracts

For notational simplicity, I describe the model and concepts with n firms. A linear Cournot oligopoly for a homogeneous good is given by an inverse demand $p(\Sigma x_j) = a - \Sigma x_j$ and

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1 If the conditions for solutions fail to hold, there will be no solution and one is left with an endless cycle of merger-breakups (such as the recent AOL-TimeWarner or Arby-Wendy merger-breakups). Such cycles had been summarized in the opening of *The Romance of the Three Kingdoms*, as famously cited in *On China* (Kissinger 2011, p. 6): "The empire, long divided, must unite; long united, must divide." This seems to be a new and unstudied driving force behind business cycles.

2 See Lekeas (2013) for recent survey on cooperative approach, Ray and Vohra (2013) and Xue and Zhang (2012) for recent surveys on both approaches.

3 A major part of merging costs are the fees paid to accounting firms to access the value of targeted firms, which runs from 3% to 9% of the targeted firm's value.

n cost functions: $C_i(x_i)=c_i x_i, 0\le x_i\le z_i,\ i=1,\dots,n$, or by a $(2n+1)$-vector $(a,c,z)\in\mathbf{R}^{2n+1}_{++}$, with $a>0$ as the intercept of inverse demand, and $c=(c_1,\dots,c_n)$ and $z=(z_1,\dots,z_n)>>0$ as vectors of marginal costs and capacities. This model is equivalent to a normal form game given by

$$\Gamma=\{N, Z_i, \pi_i\}, \tag{1}$$

where $N=\{1, 2, \dots, n\}$; for each firm $i\in N$, $Z_i=[0, z_i]$ is its production set bounded by its capacity $z_i>0$, and $\pi_i(x)=p(\Sigma x_j)x_i-C_i(x_i)=(a-\Sigma x_j-c_i)x_i$ is its profit function.

I assume that each merger generates a weak synergy as in part (i) of A0 below:

A0 (Assumption 0): (i) *For each merger $S\subseteq N$, its capacity and cost function are:*

$$z_S=\Sigma_{j\in S}z_j,\ C_S(q)=c_S q,\ q\le z_S, \text{ where } c_S=\mathrm{Min}\{c_j|\ j\in S\}; \text{ and} \tag{2}$$

(ii) *at any equilibrium, the optimal supply by each $S\subseteq N$ is an interior solution.*

Under A0, a merger removes its inefficient members and raises its efficient member's capacity to z_S. Let π_i denote firm i's premerger profit, $\bar{x}$ and π_m the monopoly supply and monopoly profits. Define a monopoly merger contract as a pair $(\bar{x};\lambda)$ of its supply and a split of its profits. For this merger contract to be successful, it must meet two preconditions. The first one is the well studied profitability precondition (or incentive to merge), which requires that no firm be worse off and total profits be higher (i.e., $\lambda_i\ge\pi_i$, all i, and $\Sigma\lambda_i=\pi_m>\Sigma\pi_i$).

The second one is the relatively new non-empty core precondition[4] (Zhao 2009), which requires that no coalition receive less than its worst (or guaranteed) profits (see (5) below) or that no other merger be more profitable than their share of the monopoly profits. In

[4] These two preconditions are independent of each other. As an example, let $n=3$, $(a;c;z)=(6; 0.5, 0.5, 0.5; 2,2,2)$, and merging costs be $MC_N=2$, $MC_S=0$, $S\neq N$. By $\pi_i=1.89$, $\pi_m=7.56$, $\Sigma\pi_i=5.67>v(N)=(\pi_m\text{-}MC_N)=5.56>4.59=MNBP$ (see (17) for definition of $MNBP$), the monopoly is unprofitable and has a non-empty core.

other words, it requires that the profit split λ be in the monopoly's core, or equivalently the core of the oligopoly (1), or precisely the core of the following coalitional game

$$\Gamma_C = \{N, v\}, \text{ with } v(S) \text{ given in (5),} \tag{3}$$

where the core (= α-core = β-core)[5] is given by

$$\text{Core}(\Gamma_C) = \{\lambda \in \boldsymbol{R}^n_+ \mid \Sigma\lambda_i = v(N), \text{ and } \Sigma_{i\in S}\lambda_i \geq v(S), \text{ all } S \neq N\}. \tag{4}$$

In (3)-(4), $v(N) = \pi_m$, and $v(S)$ is the guaranteed (or worst) profit for each $S \neq N$ given by

$$v(S) = \underset{x_S}{\text{Max}}\,\underset{y_{-S}}{\text{Min}}\,\Sigma_{i\in S}\,\pi_i(x_S, y_{-S}) = \underset{y_{-S}}{\text{Min}}\,\underset{x_S}{\text{Max}}\,\Sigma_{i\in S}\,\pi_i(x_S, y_{-S}) = \underset{x_S}{\text{Max}}\,\Sigma_{i\in S}\,\pi_i(x_S, z_{-S}), \tag{5}$$

where the Min is taken over $Z_{-S} = \Pi_{j\notin S}Z_j$, the Max over $X_S = \{x_S \in \boldsymbol{R}^S_+ \mid \Sigma_{j\in S}x_j \leq z_S\}$, and $(x_S, y_{-S}) = w$ is a vector with $w_i = x_i$ if $i \in S$, $= y_i$ if $i \notin S$.

It is useful to note that the first precondition requires rationality only for singletons and grand coalition, while the second the rationality for all coalitions, and also that individual rationality in the first precondition is stronger than that in the second precondition.

Now we are ready to define the contracts for non-monopoly mergers. Let Π be the set of all partitions or market structures of N (i.e., all sets of simultaneous mergers)[6]. Given $\Delta \in \Pi$, let $\tilde{x}(\Delta) = \{\tilde{x}_S(\Delta) \mid S \in \Delta\}$ and $\tilde{\pi}(\Delta) = \{\tilde{\pi}_S(\Delta) \mid S \in \Delta\}$ be the supplies and profits at its post-merger equilibrium. Define the merger contracts for Δ as a list of pairs $\{\tilde{x}_S(\Delta);\ \lambda_S(\Delta)\}$ of

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5 There is no need to make the α- and β-distinction here and we can simply use the term core, because α-core = β-core always holds in an oligopoly (Zhao 1999). In more general situations, one would have

$$v_\alpha(S) = \underset{x_S}{\text{Max}}\,\underset{y_{-S}}{\text{Min}}\,\Sigma_{i\in S}\,\pi_i(x_S, y_{-S}) < v_\beta(S) = \underset{y_{-S}}{\text{Min}}\,\underset{x_S}{\text{Max}}\,\Sigma_{i\in S}\,\pi_i(x_S, y_{-S}),$$

which implies β-core $\subset \alpha$-core (Aumann 1959, Scarf 1971).

6 The traditional definition of market structure is "*the number and size distribution of firms*" or the finest partition $\Delta_0 = \{(1), \ldots, (n)\}$ (Bain 1959), which is now upgraded to "*the number and size distribution of mergers*" or a general partition $\Delta = \{S_1, \ldots, S_k\}$ of the firms (i.e., $\cup S_j = N$, $S_i \cap S_j = \varnothing$, all $i \neq j$).

post-merger supply and split of postmerger profits[7] (i.e., $\lambda_S \geq 0$, $\Sigma_{j\in S}\lambda_j = \tilde{\pi}_S(\Delta)$) for each $S \in \Delta$.

The two earlier preconditions can be similarly defined. In particular, the non-empty core precondition requires that the split of each merger's postmerger profits be in the merger's core, or that each λ_S be in the core of the following normal form game

$$\Gamma_S(\tilde{x}_{-S}) = \{S, Z_i, \pi_i(x_S, \tilde{x}_{-S})\}, \tag{6}$$

where $\pi_i(x_S, \tilde{x}_{-S}) = (a - \Sigma_{j\in S}x_j - \Sigma_{j\notin S}\tilde{x}_j - c_i)x_i, i \in S$, are parameterized by outsiders' fixed supply $\tilde{x}_{-S}$.

It is useful to make three observations on the above two merger preconditions. First, both are only necessary conditions. So failing either or both will result in a merger failure, and meeting both will not guarantee a merger success. Second, the core for each merger $S \in \Delta$ is defined only for a fixed outsiders' supply $\tilde{x}_{-S}$. Otherwise, the problem of dividing its joint profits would not exist. Third, the non-empty core precondition has failed for nearly two decades to be accepted by the previous industrial organization literature. This resulted from, the author believes, the following four misconceptions about cooperative game theory in general and the above core in particular.

Misconception no. 1: *Strategic form games are noncooperative games, hence they can't be used to study cooperation*. In most textbooks, normal form games have been called strategic form games, which inherently suggest that a normal form game is strategic or noncooperative (so Nash equilibrium is the only solution!). Whether a normal form game is cooperative or noncooperative depends on whether joint actions are available: it is noncooperative if no joint actions are available, cooperative if joint actions are available to

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[7] $\tilde{x}(\Delta)$ is a quasi-hybrid solution (i.e., for each $S \in \Delta$, $0 \leq \Sigma_{j\in S}\tilde{x}_j \leq z_S$, $\tilde{\pi}_S(\Delta) = \Sigma_{j\in S}\pi_j(\tilde{x}_S, \tilde{x}_{-S}) \geq \pi_S(y_S; \tilde{x}_{-S})$ for all $0 \leq \Sigma_{j\in S}y_j \leq z_S$), and $(\tilde{x}(\Delta), \lambda(\Delta))$ under the two preconditions is a hybrid equilibrium (Zhao 1991, 1992). This includes monopoly merger and Cournot equilibrium (for $\Delta_m = \{N\}$ and $\Delta_0 = \{(1), \ldots, (n)\}$) as two special cases.

the grand coalition, and hybrid if joint actions are available to each coalition in a given partition (also called alliance structure or coalitional structure).[8]

Misconception no. 2: *Cooperative games study cooperative behavior.* Cooperative game theory studies coalitional rationality or strategic cooperation, it doesn't study "being cooperative in the layman's language", which differs from its meaning in game theory. Players in cooperative games take joint actions (such as enforcing binding agreements for coalitions) to choose or negotiate outcomes that are desirable for all coalitions, including each singleton or individual player. In sharp contrast, players in noncooperative games are not allowed to take any form of joint actions (such as communications between players).

Misconception no. 3: *The noncooperative approach is central to cooperative game theory. Hence a theory of cooperative games is unacceptable unless it has a noncooperative foundation*. This is falsely derived from two facts: unselfish or cooperative behavior has been observed in noncooperative situations (e.g., prisoner's dilemma games), and confrontational or noncooperative behavior has been observed in cooperative situations or negotiations. A seemingly logical conclusion of these facts is that cooperative games need a noncooperative foundation, but the inferred conclusion is false and invalid because the observed behavior in

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8 Another inherent problem is that it makes no distinction between choice and strategy. The strategies in strategic form games are just feasible choices, or sequences of actions/moves from beginning to end. Since a military strategy should involve a study of one's goals and enemies' moves, we may define a strategy in game theory as a rationalized selection of one's choices. In this sense, players in a prisoner's dilemma game have infinitely many strategies (e.g., undominated strategy, maxmin strategy, best-response strategy, and mixed strategies), although they have only two feasible choices.

Note that McDonald's claim (1950), "an exact description of the nature of strategy has been wanting," might still be and remain true, as our definition is open to debate and none of the masterpieces on strategies contain a formal definition. For example, the closest reference to (or meaning of) strategy is: "The highest form of warfare is to attack [the enemy's] strategy itself" in *Art of War* (Sun-Tzu 500 B.C.; see Kissinger 2011, p. 28, for comments), "policy" in *Principles of War* (Clausewitz 1812; see McDonald 1950, p.12, for a summary), and "the general principles governing his choices" in von Neumann and Morgenstern (1944, p.49).

both facts are not what studied in game theory, and because one falsely equates, for example, the Israel-Palestine conflicts with conflicts among Israeli ministers in a cabinet meeting. So applying noncooperative approach (which allows no negotiation nor joint action) to cooperative problems (such as alliances or mergers, which allow negotiations and joint actions) is the same as adjusting the world to fit our theories.

Misconception no. 4: *The core in (4) is unacceptable because outsiders produce at full capacity* z_{-S} (see $v(S) = Max\{\Sigma_{i\in S}\pi_i(x_S, z_{-S}) | x_S\}$ in (5)), *which is not credible*. No outsiders actually produce, nor they are required to produce, at full capacity. The core just requires that monopoly profits be split in such a way that no S receives less than $v(S)$. The only drawback of using $v(S)$ in (5) is that it makes the core possibly large. However, the largeness of the core strengthens, rather than weakens, the requirement that $\lambda\in$Core(Γ_C) must hold.

3. Reasonable solutions for a Cournot oligopoly

The reasonable solutions for our oligopoly (1) are the set of its stable hybrid solutions or precisely the set of stable partitions and payoffs of its *partition function game* (Thrall and Lucas 1963) given by

$$\Gamma_{PF} = \{N, \phi\}, \tag{7}$$

where for each partition or market structure $\Delta\in\Pi$, $\phi(\Delta) = \tilde{\pi}(\Delta) = \{\phi(T,\Delta) = \tilde{\pi}_T(\Delta) | T\in\Delta\}$ is its unique postmerger profit vector.

Given Δ and its merger contract $(\tilde{x}(\Delta), \lambda(\Delta)) = \{(\tilde{x}_T, \lambda_T) | T\in\Delta\}$, consider the deviation by a different merger $S\notin\Delta$. This implicitly assumes that firms in S could breakup old and reach new binding agreements at the beginning of next virtual period.[9] Let

$$\Pi(S) = \{\mathcal{B} \in \Pi \mid \mathcal{B} = \{S, T_1, \ldots, T_m\}\} \tag{8}$$

be the set of partitions that include S as a member. S has incentives to move to a new partition $\mathcal{B} \in \Pi(S)$ if $\phi(S, \mathcal{B}) = \tilde{\pi}_S(\mathcal{B}) > \Sigma_{j\in S}\lambda_j(\Delta)$. Hence, "whether $S \notin \Delta$ will deviate from Δ" or "whether a new merger $S \notin \Delta$ will be formed" depends on two factors: *i*) its members' current profits on the contract $\lambda(\Delta)$, and *ii*) its joint profits at the new partition $\mathcal{B} \in \Pi(S)$.

In reality, merger contracts, although binding, usually have clauses allowing members to break up the deal under penalty and specifying how the remaining members will react to a breakup. Two popular reactions to breakups (Hart and Kurz 1983) are: *i*) remaining members remain loyal to each other and stay together as a smaller coalition, and *ii*) remaining members breakup into singletons. These are formally given as below:

$$\begin{aligned} \mathcal{B}_\delta(S,\Delta) &= \{S, T_1^\delta, \ldots, T_{m(\delta)}^\delta\} \in \Pi(S) \text{ (for loyal belief), and} \\ \mathcal{B}_\gamma(S,\Delta) &= \{S, T_1^\gamma, \ldots, T_{m(\gamma)}^\gamma\} \in \Pi(S) \text{ (for breakup belief),} \end{aligned} \tag{9}$$

where for $j = 1, \ldots, m(\delta)$, $T_j^\delta = T/S = \{i \mid i \in T,\ i \notin S\}$ for some $T \in \Delta$; and for $i = 1, \ldots, m(\gamma)$, T_i^γ $= T$ for each $T \in \Delta$ with $S \cap T = \varnothing$, $= \{j\}$ for each $j \in T \backslash S$ and each $T \in \Delta$ with $S \cap T \neq \varnothing$.

As an example, for $\Delta = \{1, (2,3,4,5)\}$ and $S = (1,2)$, one has: $\mathcal{B}_\gamma(S,\Delta) = \{(1,2), 3, 4, 5\}$, $\mathcal{B}_\delta(S,\Delta) = \{(1,2), (3, 4, 5)\}$. A third popular reaction is the cautious partition given by

$$\mathcal{B}_{\alpha*}(S,\Delta) \equiv \mathcal{B}_{\alpha*}(S) = \{S, T_1^{\alpha*}, \ldots, T_{m(a*)}^{\alpha*}\} \in \Pi(S) \text{ (for cautious belief),} \tag{10}$$

which solves $\text{Min}\{\phi(S,\mathcal{B}) \mid \mathcal{B} \in \Pi(S)\}$. This partition is often called the worst partition, which is independent of the current partition Δ. Definition 1 below defines stable partitions.

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[9] This is similar to the virtual dynamics of tâtonnement process for reaching a competitive equilibrium or the process of reaching a Nash equilibrium in static normal form games.

Definition 1: *A partition Δ is stable under loyal* (*breakup, cautious*) *belief or δ-stable (γ-, α^*-stable) if it has a contract* ($\tilde{x}(\Delta), \lambda(\Delta)$) *such that for all $S \notin \Delta$, $\Sigma_{j\in S}\lambda_j(\Delta) \geq \phi(S,\mathcal{B}_\delta(S,\Delta))$ ($\geq \phi(S,\mathcal{B}_\gamma(S,\Delta))$, $\geq \phi(S,\mathcal{B}_{\alpha^*}(S))$), where ϕ, $\mathcal{B}_\delta$, $\mathcal{B}_\gamma$ and $\mathcal{B}_{\alpha^*}$ are given in (7)-(10).*

To put it differently, mergers in Δ will be formed in the δ-fashion (γ-, α^*-fashion) if no other merger S could make more profits by moving to $\mathcal{B}_\delta(S,\Delta)$ ($\mathcal{B}_\gamma(S,\Delta)$, $\mathcal{B}_{\alpha^*}(S)$). Because all profitable deviations (i.e., breakups and mergers) are ruled out, such stable partitions or stable market structures are reasonable solutions for (1). For each market structure Δ, let

$$X_\delta(\Delta),\ X_\gamma(\Delta),\ \text{and}\ X_{\alpha^*}(\Delta) \tag{11}$$

denote the sets of its δ-, γ- and α^*-stable profit vectors. If Δ is the coarsest partition $\Delta_m=\{N\}$, the above sets become the δ-, γ- and α^*-core of the monopoly merger or (1) given by

$$\delta\text{-}Core=X_\delta(\Delta_m),\ \gamma\text{-}Core=X_\gamma(\Delta_m),\ \text{and}\ \alpha^*\text{-}Core=X_{\alpha^*}(\Delta_m), \tag{12}$$

which are all refinements of the core in (4). Proposition 1 below summarizes the relationship among the above sets of stable profit vectors.

Proposition 1: *Given (1), let its cores be given in (4) and (12), and for each Δ, let $X_\delta(\Delta)$, $X_\gamma(\Delta)$, $X_{\alpha^*}(\Delta)$ be given in (11). Then, under A0, the following two claims hold:*

(i) $\delta\text{-}Core \subset \gamma\text{-}Core = \alpha^*\text{-}Core \subset Core(\Gamma)$(= β-core = α-core), and

(ii) $X_\delta(\Delta) \subset X_\gamma(\Delta) \subset X_{\alpha^*}(\Delta)$.

By the proposition, δ-stability is stronger than γ-stability, which is stronger than the α^*-stability. By part (i) of the proposition, the α^*-core is a significant refinement of the α-core in linear oligopolies[10], because the believed actions by outsiders are now credible (i.e., there

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[10] See Yong (2004) for e-core which assumes an efficient partition of $N\backslash S$ for outsiders. Let $v_e(S)$, $v_{\alpha^*}(S)$ and $v_\delta(S)$ be the values of S (from Δ_m) under efficient-, cautious- and loyal-beliefs. By $v_e(S) \geq v_{\alpha^*}(S)$ and $v_e(S) \gtreqless v_\delta(S)$, one has: $\{\delta\text{-}Core \cup e\text{-}Core\} \subset \alpha^*\text{-}Core \subset \alpha\text{-}Core$. See also Lardon (2012) for γ-core with capacity constraints, and

exist strategic interactions between each S and outsiders in $N \backslash S$).

4. The most reasonable solution with three asymmetric firms

The difference between γ- and α^*-stability and disagreements over a final selection of stable solutions all disappear with three firms, so δ-stability yields the most reasonable solution for a three-firm oligopoly given by $(a; c; z) = (a; c_1, c_2, c_3; z_1, z_2, z_3) \in \mathbf{R}^7_{++}$.

Our task now is to evaluate the δ-stability for each of the five partitions of $N=\{1,2,3\}$ over $\mathbf{R}^7_{++}$. We simplify the task to a two-dimensional problem with $c_1 \leq c_2 \leq c_3$ under $A0$ by introducing two intermediate variables (keep in mind that firm 1 is the most efficient):

$$\varepsilon_2 = (c_2-c_1)/(a-c_1) \geq 0, \text{ and } \varepsilon_3 = (c_3-c_1)/(a-c_1) \geq 0, \tag{13}$$

which represent firm 1's relative cost advantages over firms 2 and 3. The larger the ε_i, the smaller a firm i. The usual assumptions imply $\varepsilon_2 \leq \varepsilon_3 \leq 0.5$, so we only need to study one half of the half-unit square (above 45^0 line) in the ε_2-ε_3 space. By $A0$ or no binding capacities, firm sizes are uniquely determined by efficiency or cost differentials.

The case of $\varepsilon_2=\varepsilon_3=0$ gives a symmetric model, a small ε_3 represents small cost differentials (i.e., the smallest firm is not too small and the largest firm is not too large), a large ε_2 represents large cost differentials (i.e., the two small firms are sufficiently small). Even with such simplification, the task would still have been impossible without the help of mathematical software (see (A27) for a taste of its complexity). Indeed, our polynomial equations are solved by Scientific WorkPlace, and the predictions have also been separately confirmed by numerical examples using Excel. Needless to say, the complexity is caused by

Lekeas (2013) for j-core which assumes that $N \backslash S$ is partitioned into j coalitions.

asymmetry, which has the offsetting advantage over symmetric models in that predictions under symmetry often collapse in the presence of asymmetry (see, for example, Stamatopoulos and Tauman 2009).

The monopoly's stability and optimality are characterized by comparing ε_3 against the following two functions of ε_2, respectively:

$$\omega_1 = \omega_1(\varepsilon_2) = \{2-\sqrt{1+8\varepsilon_2-20\varepsilon_2^{\,2}}\}/4, \text{ and}$$
$$\omega_2 = \omega_2(\varepsilon_2) = (7+31\varepsilon_2)/69. \tag{14}$$

Here, the optimality of a partition is in the sense of second best, which has the maximal welfare (= total profits + consumer surplus) among the five partitions.

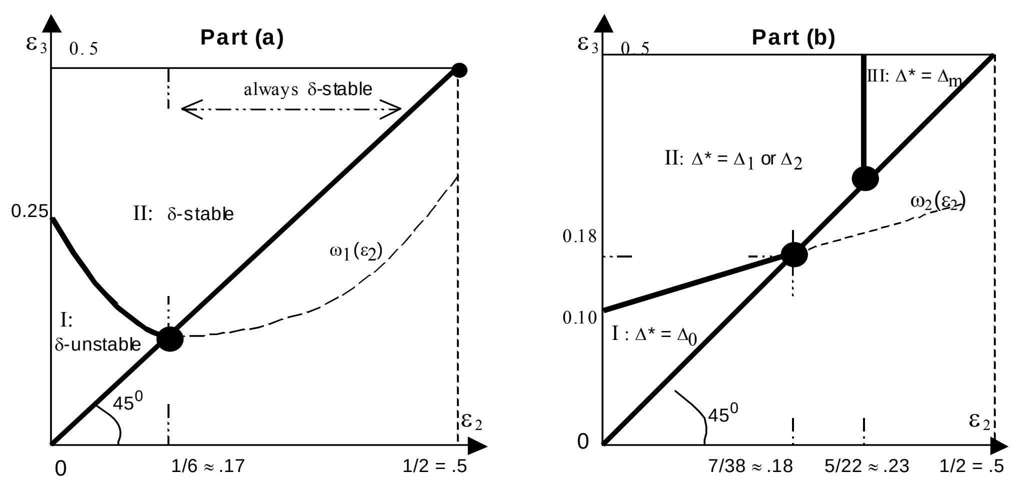

Figure 1. **(a)** The δ-stability of Δ_m = {123}; **(b)** the optimal partitions for n = 3. In both parts, the feasible region is in the area above the 45^0 line.

Proposition 2: *Under **A0**, the following two claims hold:* (*i*) *Δ_m = {123} is always α^*-stable;* (*ii*) *Δ_m is δ-stable $\Leftrightarrow$ $\varepsilon_3 \geq \omega_1(\varepsilon_2)$, and $\varepsilon_3 \geq \omega_1(\varepsilon_2)$ always holds if $\varepsilon_2 \in [1/6, 1/2]$.*

By Proposition 2, monopoly will be formed in δ–fashion if firms 2 and 3 are sufficiently small (e.g., $\varepsilon_3 \geq \varepsilon_2 \geq 1/6$), and it will be formed in α^*–fashion but not δ–fashion if firms 2 and 3 are sufficiently large (i.e., $\varepsilon_3<\omega_1(\varepsilon_2)$), which are illustrated by Regions I and II in Figure 1a. In particular, Δ_m will not be formed in δ–fashion in symmetric markets (i.e., $\varepsilon_2=\varepsilon_3=0$).[11]

Proposition 3: *Under* ***A0****, the optimal partition Δ* and maximal welfare W* are*:

$$\Delta^*=\begin{cases}\Delta_m=\{123\} & \text{if } 5/22 \le \varepsilon_2 \le 1/2\\ \Delta_1=\{1;23\} \text{ or } \Delta_2=\{13;2\} & \text{if } 7/38<\varepsilon_2<5/22 \text{ or if } \varepsilon_2\le 7/38 \ \&\ \varepsilon_3\ge\omega_2\\ \Delta_0=\{1;2;3\} & \text{if } \varepsilon_2\le 7/38;\ \varepsilon_3<\omega_2;\end{cases} \quad (15)$$

$$W^*=\begin{cases}3(a-c_1)^2/8 & \text{if } 5/22 \le \varepsilon_2 \le 1/2\\ (a-c_1)^2(8-8\varepsilon_2+11\varepsilon_2^2)/18 & \text{if } 7/38<\varepsilon_2<5/22 \text{ or if } \varepsilon_2\le 7/38 \ \&\ \varepsilon_3\ge\omega_2\\ (a-c_1)^2[15-10(\varepsilon_2+\varepsilon_3)-18\varepsilon_2\varepsilon_3+23(\varepsilon_2^2+\varepsilon_3^2)]/32 & \text{if } \varepsilon_2\le 7/38;\ \varepsilon_3<\omega_2.\end{cases} \quad (16)$$

Propositions 2 and 3 lead directly to the following corollary:

Corollary 1: *If $\varepsilon_2\ge 5/22$, $\Delta_m=\{123\}$ is both δ-stable and socially optimal.*

Hence, monopoly is both δ–stable and optimal if cost savings are sufficiently large (i.e., $\varepsilon_2\ge 5/22>1/6$). In such cases, no anti-trust regulation is needed as monopoly is the best market structure. This is illustrated in Figure 1b and in Example 1 below.

Example 1: *For $(a; c; z) = (6; 0.5, 1, 1.2; 2, 2, 2)$, one has: $\varepsilon_2 = 0.09$, $\varepsilon_3 = 0.127$, $\omega_1 = 0.19$, $\omega_2 = 0.14$. By $\varepsilon_3<\omega_1$ and Proposition 2, Δ_m is δ-unstable. By $\varepsilon_2<7/38$ and $\varepsilon_3<\omega_2$, and by (15), Δ_0 is optimal. Let costs be increased to $c=(0.5, 1.9, 2)$ and (a, z) be unchanged, then $\varepsilon_2 = 0.26 > 5/22$. By Corollary 1, monopoly now is both δ-stable and optimal.*

Our results are proved using the minimum no-blocking payoff (*MNBP*) method for core existence (Zhao 2001): $Core(\Gamma)\neq\varnothing \Leftrightarrow v(N)\ge MNBP$, where *MNBP* is given by

[11] Rajan (1989) reported such symmetric case with $n=3$ and $n=4$.

$$MNBP(\Gamma) = \begin{cases} \text{Min } \Sigma x_i \\ \text{subject to } x \in \boldsymbol{R}^n_+; \Sigma_{i\in S} x_i \geq v(S) \text{ for all } S \neq N. \end{cases} \tag{17}$$

Computing the above MNBP allows us to analyze the effects of merging costs (or costs of coalition formation) on the stability of each partition. For each $S \subseteq N$, let $MC_S \geq 0$ denote its merging costs. Since an analysis of $MC_S > 0$ for $S \neq N$ requires a separate study, I only study the effects of monopoly merging costs under the following assumption:

A1 (Assumption 1): *$MC_N \geq 0$, and $MC_S = 0$ for all $S \neq N$.*

Corollary 2: *Let $v(N)$, $MNBP_\delta$ and $MNBP_{\alpha^*}$ be given in (A5)-(A13) in appendix. Under **A0-A1**, $\Delta_m = \{123\}$ is α^*-(δ-)stable $\Leftrightarrow MC_N \leq [v(N)\text{-}MNBP_{\alpha^*}]$ ($\leq [v(N)\text{-}MNBP_\delta]$).*

Hence, the difference between monopoly's profits and its MNBP defines an upper bound for its merging costs above which monopoly merger will not be formed, see Zhao (2009) on the estimation of such merging costs. The MNBP method also allows us to study monopoly's external stability or analyze whether a monopoly will remain stable in face of outside perturbations.

Corollary 3: *Under **A0-A1**, an α^*-(δ-)stable monopoly remains as a stable monopoly for small perturbations in market parameters $\Leftrightarrow$ $MC_N < [v(N)\text{-}MNBP_{\alpha^*}]$ ($< [v(N)\text{-}MNBP_\delta]$).*[12]

In other words, a monopoly merger will unravel in face of small perturbations if it is unstable or if it is stable with $MC_N = [v(N)\text{-}MNBP_{\alpha^*}]$ ($= [v(N)\text{-}MNBP_\delta]$). It is straightforward to extend Corollaries 2 and 3 on merging costs and sensitivity to non-monopoly partitions, we therefore will skip such extensions in the rest of this paper.

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12 Precisely, a stable Δ_m in $(a, c, z) = t \in \boldsymbol{R}^7_{++}$ remains stable against small perturbations if there exists $\varepsilon > 0$ such that Δ_m is stable for all $t' \in B_\varepsilon(t)$, where for $t \in \boldsymbol{R}^7$, $B_\varepsilon(t) = \{y \in \boldsymbol{R}^7 \mid \|t - y\| < \varepsilon\}$ and $\|t\|^2 = \Sigma t_i^2$.

We now study the stability of $\Delta_1 = \{1; 23\}$, $\Delta_2 = \{13; 2\}$, and $\Delta_3 = \{12; 3\}$. Because the α*-, γ- and δ-stabilities for each of these three partitions are identical, there is no need to make such distinction here. The outsiders' or the single firms' postmerger profits at each Δ_i ($i = 1, 2, 3$) are equal to

$$\tilde{\pi}_1(\Delta_1) = (a-c_1)^2(1+\varepsilon_2)^2/9, \quad \tilde{\pi}_2(\Delta_2) = (a-c_1)^2(1-2\varepsilon_2)^2/9, \text{ and}$$
$$\tilde{\pi}_3(\Delta_3) = (a-c_1)^2(1-2\varepsilon_3)^2/9. \tag{18}$$

Denote the merger's gain for each $S = \{12\}$, $\{13\}$ and $\{23\}$ by

$$d_{23} = \tilde{\pi}_{23}(\Delta_1) - (\pi_2+\pi_3),\ d_{13} = \tilde{\pi}_{13}(\Delta_2) - (\pi_1+\pi_3),\ d_{12} = \tilde{\pi}_{12}(\Delta_3) - (\pi_1+\pi_2); \tag{19}$$

and denote the efficient member's share of the above gains by $t \in [0, 1]$. Then, the three dimensional postmerger profit vector $\lambda(t)$ for each Δ_i can be given as

$$\text{for } \Delta_1,\ \lambda_1 = \tilde{\pi}_1(\Delta_1),\ \lambda_2 = \pi_2 + t\, d_{23}, \text{ and } \lambda_3 = \pi_3 + (1-t)\, d_{23};$$
$$\text{for } \Delta_2,\ \lambda_1 = \pi_1 + t\, d_{13},\ \lambda_2 = \tilde{\pi}_2(\Delta_2), \text{ and } \lambda_3 = \pi_3 + (1-t)\, d_{13}; \text{ and} \tag{20}$$
$$\text{for } \Delta_3,\ \lambda_1 = \pi_1 + t\, d_{12},\ \lambda_2 = \pi_2 + (1-t) d_{12}, \text{ and } \lambda_3 = \tilde{\pi}_3(\Delta_3).$$

The stability of each Δ_i requires two preconditions: its merger S is profitable (i.e., $d_S > 0$) and monopoly is unprofitable due to high merging costs (i.e., $(\pi_m - MC_N) < \Sigma\lambda_j$). Under these two preconditions, the stability of each Δ_i is determined by the magnitude of cost differential ε_j and by the size of the share t, which are captured by a critical level $\mu_i(\varepsilon_j, t)$ given in appendix: $\mu_1(\varepsilon_2, t)$ in (A27), $\mu_2(\varepsilon_2, t)$ in (A29), and $\mu_3(\varepsilon_3, t) = \mu_2(\varepsilon_3, t)$.

Proposition 4: *Given $(a, c, z) \in \mathbf{R}^7_{++}$, suppose $\Sigma\lambda_j > (\pi_m - MC_N)$ and $d_S > 0$ for each S = 12, 13, 23. Under **A0** and **A1**, the following three claims hold:*

(i) $\Delta_1 = \{1; 23\}$ with $\lambda(t)$ is stable $\Leftrightarrow$ $\varepsilon_3 \leq \mu_1(\varepsilon_2, t)$; $\varepsilon_3 \leq \mu_1(\varepsilon_2, t)$ holds if $0 \leq \varepsilon_2 \leq 1/11$; and $\varepsilon_3 > \mu_1(\varepsilon_2, t)$ holds if $113/316 < \varepsilon_2 \leq$ ½.

(ii) $\Delta_2 = \{13;\ 2\}$ *with* $\lambda(t)$ *is stable* $\Leftrightarrow \varepsilon_3 \le \mu_2(\varepsilon_2, t)$. *Let* $e_2(t) = (2t-9)/[14(2t-3)]$. *Then,* $\varepsilon_3 \le \mu_2(\varepsilon_2, t)$ *holds if* $0 \le \varepsilon_2 < 1/11$; $\varepsilon_3 > \mu_2(\varepsilon_2, t)$ *holds if* $e_2(t) < \varepsilon_2 \le ½$.

(iii) $\Delta_3 = \{12;\ 3\}$ *with* $\lambda(t)$ *is stable* $\Leftrightarrow \varepsilon_2 \le \mu_3(\varepsilon_3, t)$; *and it holds if* $0 \le \varepsilon_3 \le 3/14$.

To see these results intuitively, consider $\Delta_1 = \{1;23\}$ in part (*i*). Because Δ_0 and Δ_m are ruled out by preconditions and $\Delta_2 = \{13,2\}$ has the same profit vector of Δ_1, one only needs to evaluate the deviation to $\Delta_3 = \{12,3\}$, when $S = \{12\}$ forms after the breakup of $T = \{23\}$. In this light, part (*i*) is transparent: since a larger share t by firm 2 or a smaller ε_3 makes the formation of $S = \{12\}$ less profitable, Δ_1 with $\lambda(t)$ will be stable with a smaller ε_3 or a larger t (i.e., $\varepsilon_3 \le \mu_1(\varepsilon_2, t)$, note $\mu_1(\varepsilon_2, t)$ is increasing in t).

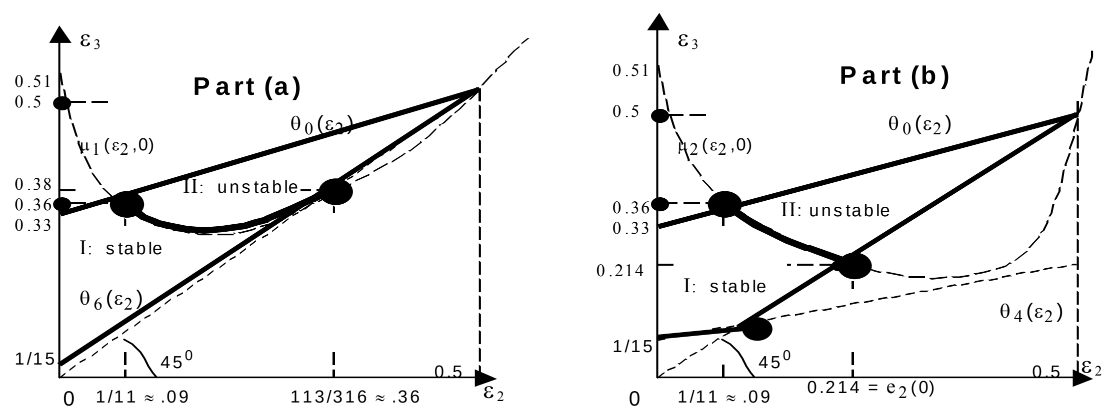

Figure 2. **(a)** Stability of Δ_1: feasible region is $\max\{\varepsilon_2, \theta_6\} \le \varepsilon_3 \le \theta_0$; **(b)** stability of Δ_2: feasible region is $\max\{\varepsilon_2, \theta_4\} \le \varepsilon_3 \le \theta_0$. In both cases, t is set at 0.

Inverting t in $\varepsilon_3 = \mu_1(\varepsilon_2, t)$ yields

$$t_1(\varepsilon_2, \varepsilon_3) = -\frac{1}{2}\,\frac{7\varepsilon_3^2 - 97\varepsilon_2^2 + 54\varepsilon_2\varepsilon_3 + 14\varepsilon_3 + 22\varepsilon_2 - 9}{45\varepsilon_3^2 + 13\varepsilon_2^2 - 54\varepsilon_2\varepsilon_3 - 18\varepsilon_3 + 14\varepsilon_2 + 1}, \tag{21}$$

which leads to an alternative characterization for the stability of Δ_1 given below:

Corollary 4: *Let* $\varepsilon_2 \in [1/11, 113/316]$. *Then,* Δ_1 *with* $\lambda(t)$ *is stable* $\Leftrightarrow t \ge t_1(\varepsilon_2, \varepsilon_3)$.

Hence, internal cooperation represented by the share t is a key determinant of merger stability. Region I in Figures 2a, 2b and 3 represent, respectively, the set of markets in which Δ_1, Δ_2 and Δ_3 are stable. By the two preconditions, the feasible region is bounded by θ_0 (or θ_2) from above and the 45^0 line and θ_6 (or θ_4) from below.[13] The stability of Δ_1 is also illustrated by Example 2 below:

Example 2: *Let* $(a;c;z)=(6;0.5,1.05,2.46;3,3,3)$, *then* $\varepsilon_2=0.1$, $\varepsilon_3=0.356$, $(\pi_1,\pi_2,\pi_3)=(4.01,2.11,0.002)$, $(\tilde{\pi}_1,\tilde{\pi}_{23})=(4.067,2.152)$, $t_1(\varepsilon_2,\varepsilon_3)=0.12$. *Hence,* $t=0.1$ *or* $\lambda(0.1)=(4.067, 2.114, 0.038)$ *is unstable, and* $t=0.2$ *or* $\lambda(0.2)=(4.067,2.118,0.034)$ *is stable. If* c_3 *rises so* $\varepsilon_3=0.358$, *then* $t_1(\varepsilon_2,\varepsilon_3)$ *becomes* $t_1(0.1,0.358)=0.45$, *so* $t=0.2$ *or* $\lambda(0.2)$ *is now unstable.*

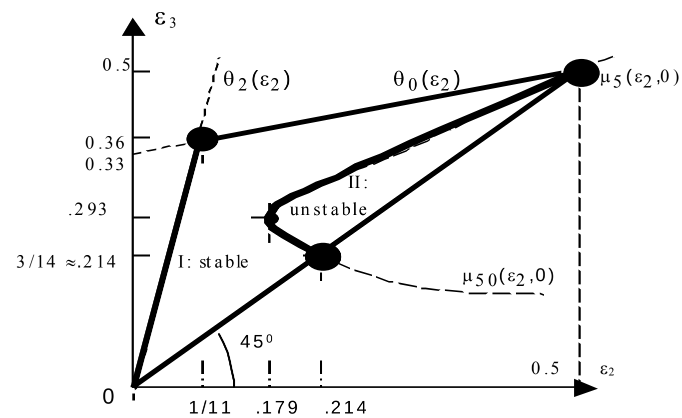

Figure 3. The Stability of Δ_3: feasible region is $\varepsilon_2 \le \varepsilon_3 \le \mathrm{Min}\{\theta_0,\theta_2\}$, t is set at 0, and $\mu_3(\varepsilon_3,0)$ is represented by $\mu_5(\varepsilon_2,0)$ and $\mu_{50}(\varepsilon_2,0)$.

Finally, one now knows that the original Cournot structure $\Delta_0=\{1;\ 2;\ 3\}$ is stable if and only if none of the four mergers are profitable, which is given by the corollary below:

Corollary 5: $\Delta_0=\{1;\ 2;\ 3\}$ *is stable if and only if the following* (22) *holds:*

13 Figure 3 shows the two solutions of ε_3 in $\varepsilon_2=\mu_3(\varepsilon_3,t)$: $\varepsilon_3=\mu_5(\varepsilon_2,t)\ge\varepsilon_3=\mu_{50}(\varepsilon_2,t)$, and the minimum of $\mu_3(\varepsilon_3,t)$ is $\varepsilon_2^*(t)=\mathrm{Min}\{\mu_3(\varepsilon_3,t)|\varepsilon_3\}=\mu_3(\varepsilon_3^*(t),t)$, where $\varepsilon_2^*(0)=0.179$ and $\varepsilon_3^*(0)=0.293$. Then, part (iii) becomes: $\lambda(t)$ is stable if (a) $\varepsilon_2\le\varepsilon_2^*(t)$, or (b) $\varepsilon_2>\varepsilon_2^*(t),\varepsilon_3\le\varepsilon_3^*(t),\varepsilon_3\le\mu_{50}(\varepsilon_2,t)$; or (c) $\varepsilon_2>\varepsilon_2^*(t)$, $\varepsilon_3>\varepsilon_3^*(t)$, $\varepsilon_3\ge\mu_5(\varepsilon_2,t)$.

$$\theta_2 < \varepsilon_3 \leq \theta_4,\ and\ (\pi_m\text{-}MC_N) < (\pi_1 + \pi_2 + \pi_3). \tag{22}$$

As shown in Figure 4, Δ_0 will always be unstable if $\varepsilon_2 \geq 1/14$, because $\theta_4 < \theta_2$ (or $d_{13} > 0$) holds for all $\varepsilon_2 \in [1/14,\ 1/2]$.

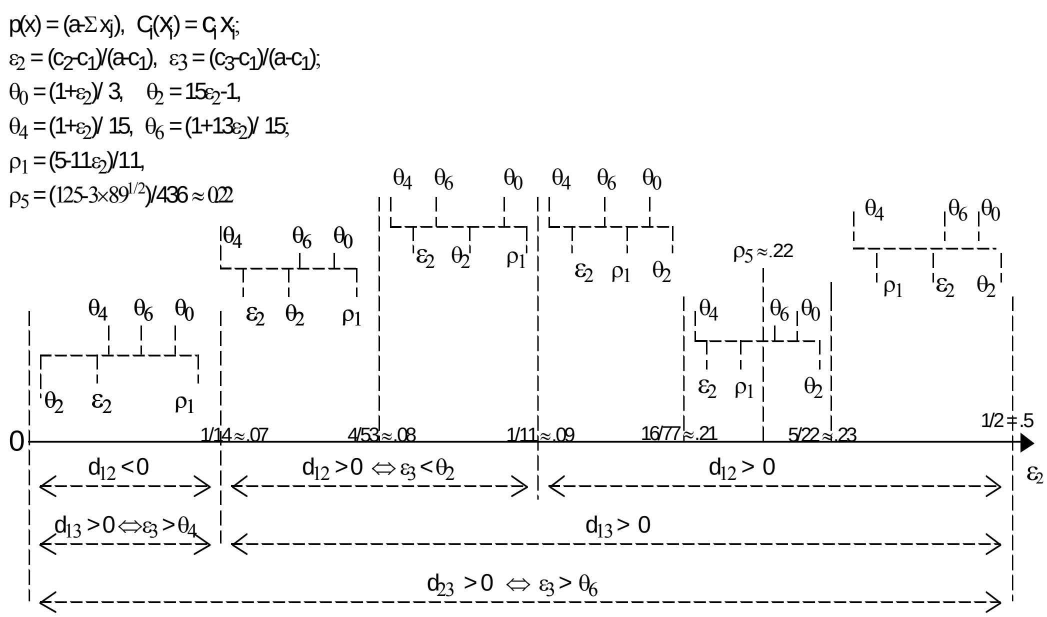

Figure 4. Merger profitability for $S = 12$, 13 and 23, and the relations among six intermediate variables.

To summarize, monopoly will be *the solution* if it is profitable and the two small firms are sufficiently small (i.e., ε_2 is large). When monopoly is ruled out by high merging costs, a profitable duopoly will be *the solution* if cost differentials are small (i.e., ε_3 is small) and two other technical conditions hold. Finally, Cournot equilibrium will be *the solution* if none of the four mergers is profitable.

5. Conclusion and discussion

The above analysis has shown that the most reasonable oligopoly solution is one of the solutions for its partition function game (i.e., one of its stable hybrid solutions) or a set of simultaneous mergers that are free of subsequent takeovers or spin-offs.

With three firms, a δ-stable partition based on loyal belief is indisputably the most reasonable solution. Applying the MNBP method for core existence, it has characterized how this solution is determined by cost differentials and by merging costs: monopoly is *the solution* if its merging cost is low and cost differentials are large. When monopoly is ruled out by high merging costs, a profitable two-member merger is *the solution* if its larger member's share of the merger's gain are large, and cost differentials are small. The original Cournot equilibrium is *the solution* if none of the four mergers is profitable.

Readers are encouraged to apply our technique to study the most reasonable solutions in more general oligopolies. In particular, readers are reminded that previous results in symmetric oligopolies need to be extended to asymmetric oligopolies, because "*Cournot's procedure may be excused by invoking the privileges of the pioneer. But those who dealt with the problem after him should realize that they did not gain but lose something by making the same* [symmetric] *assumption*" (Schumpeter 1954, chapter 7, part IV).

APPENDIX

The proof of Proposition 1 follows from $v_\delta(S) \geq v_\gamma(S) \geq v_{\alpha*}(S) \geq v(S)$. Lemmas 1 and 2 below provide the MNBP and profitability in Propositions 2-4.

Let $\Delta_0 = \{1; 2; 3\}$, $\Delta_1 = \{1; 23\}$, $\Delta_2 = \{13; 2\}$, $\Delta_3 = \{12; 3\}$, $\Delta_m = \{(1, 2, 3)\}$, and $\pi_i = \pi_i(\hat{x}) = v_i$ and $\tilde{\pi}(\Delta) = \{\tilde{\pi}_S(\Delta) | S \in \Delta\}$ be pre- and postmerger profits. For $(a; c_1, c_2, c_3; z_1, z_2, z_3) \in \mathbf{R}^7_{++}$, let ε_2 and ε_3 be given by (13), θ_i (*i = 0, 2, 4* and *6*) be defined as

$$\theta_0 = \frac{1+\varepsilon_2}{3},\ \theta_2 = 15\varepsilon_2 - 1,\ \theta_4 = \frac{1+\varepsilon_2}{15},\ \theta_6 = \frac{1+13\varepsilon_2}{15}, \text{ and} \tag{A1}$$

$$d_{12} = \tilde{\pi}_{12}(\Delta_3) - (\pi_1 + \pi_2),\ d_{13} = \tilde{\pi}_{13}(\Delta_2) - (\pi_1 + \pi_3),\ d_{23} = \tilde{\pi}_{23}(\Delta_1) - (\pi_2 + \pi_3) \tag{A2}$$

be the gains of a merger S for S = 12, 13 and 23.

Lemma 1: *(I) $d_{12} > 0 \Leftrightarrow \varepsilon_3 < \theta_2$; (II) $d_{13} > 0 \Leftrightarrow \varepsilon_3 > \theta_4$; and (III) $d_{23} > 0 \Leftrightarrow \varepsilon_3 > \theta_6$.*

As shown in Figure 4, a merger is profitable$\Leftrightarrow$ its cost savings are sufficiently large (note a larger ε_2 or ε_3 represents larger cost saving). Let ρ_i, v_i and y_i be given by

$$\rho_0 = -1+\sqrt{2-2\varepsilon_2+5\varepsilon_2^2},\ \rho_1 = (5-11\varepsilon_2)/11,\ \rho_2 = -1+27\varepsilon_2-4\sqrt{-3\varepsilon_2+42\varepsilon_2^2},$$

$$\rho_3 = \frac{19+27\varepsilon_2+4\sqrt{17-125\varepsilon_2+218\varepsilon_2^2}}{89},\ \rho_4 = \frac{19+27\varepsilon_2-4\sqrt{17-125\varepsilon_2+218\varepsilon_2^2}}{89}, \tag{A3}$$

$$\rho_5 = \frac{125-3\sqrt{89}}{436} \approx 0.22;$$

$$v_1 = \pi_1 = (a-c_1)^2(1+\varepsilon_2+\varepsilon_3)^2/16,\ v_2 = \pi_2 = (a-c_1)^2(1-3\varepsilon_2+\varepsilon_3)^2/16,$$
$$v_3 = \pi_3 = (a-c_1)^2(1-3\varepsilon_3+\varepsilon_2)^2/16; \tag{A4}$$

$$v_{12} = (a-c_1)^2(1+\varepsilon_3)^2/9,\ v_{13} = v_1^\delta = \tilde{\pi}_1(\Delta_1) = (a-c_1)^2(1+\varepsilon_2)^2/9,$$
$$v_{23} = v_2^\delta = \tilde{\pi}_2(\Delta_2) = (a-c_1)^2(1-2\varepsilon_2)^2/9,\quad v_3^\delta = \tilde{\pi}_3(\Delta_3) = (a-c_1)^2(1-2\varepsilon_3)^2/9, \text{ and} \tag{A5}$$
$$v_{123} = v(N) = (a-c_1)^2/4;$$

$$y_1 = (v_{12}+v_{13}-v_{23})/2,\ y_2 = (v_{12}+v_{23}-v_{13})/2,\ y_3 = (v_{13}+v_{23}-v_{12})/2. \tag{A6}$$

Let the MNBP against α^*-and δ-deviations from Δ_m be given by

$$MNBP_{\alpha^*} = \{\text{Min } \Sigma x_i \mid x \in \boldsymbol{R}_+^n;\ \Sigma_{i\in S} x_i \geq \phi(S, \mathcal{B}_{\alpha^*}(S)), \text{ all } S \neq N\}, \text{ and} \tag{A7}$$
$$MNBP_\delta = \{\text{Min } \Sigma x_i \mid x \in \boldsymbol{R}_+^n;\ \Sigma_{i\in S} x_i \geq \phi(S, \mathcal{B}_\delta(S,\Delta_m)), \text{ all } S \neq N\}, \tag{A8}$$

where $\phi(S,\Delta)$, $\mathcal{B}_{\alpha^*}(S) = \{S,(i_1), \ldots,(i_{n\text{-}k})\}$ and $\mathcal{B}_\delta(S,\Delta_m) = \{S, N/S\}$ are given in (7)-(10).

Lemma 2: *Under parts (ii) and (iii) of A0, (A7) and (A8) are given by:*

$$MNBP_\delta = \begin{cases} v_{13} + v_{23} + v_3^\delta & \text{if } \varepsilon_3 \leq \rho_0 \\ v_{12} + v_3^\delta & \text{if } \varepsilon_3 > \rho_0; \end{cases} \tag{A9}$$

$$\text{For } \varepsilon_2 \leq \frac{1}{14},\quad MNBP_{\alpha^*} = \begin{cases} v_1+v_2+v_3 & \text{if } \varepsilon_3 < \theta_4 \\ v_2+v_{13} & \text{if } \varepsilon_3 \geq \theta_4; \end{cases} \tag{A10}$$

$$\text{For } \frac{1}{14} \leq \varepsilon_2 \leq \frac{1}{11},\quad MNBP_{\alpha^*} = v_2+v_{13}; \tag{A11}$$

$$For\ \varepsilon_2 \ge \frac{1}{11},\ \varepsilon_3 \le \theta_6,\ MNBP_{\alpha^*} = \begin{cases} v_2+v_{13} & if\ \frac{1}{11} \le \varepsilon_2 \le \frac{16}{77} \\ v_2+v_{13} & if\ \varepsilon_3 \le \rho_1;\ \frac{16}{77} \le \varepsilon_2 \le \frac{5}{22} \\ v_3+v_{12} & if\ \varepsilon_3 > \rho_1;\ \frac{16}{77} \le \varepsilon_2 \le \frac{5}{22} \\ v_3+v_{12} & if\ \varepsilon_2 \ge \frac{5}{22}; \end{cases} \quad \text{(A12)}$$

$$For\ \varepsilon_2 \ge \frac{1}{11},\ \varepsilon_3 > \theta_6,\ MNBP_{\alpha^*} = \begin{cases} \begin{cases} v_2+v_{13} & if\ \varepsilon_3 \le \rho_2 \\ y_1+y_2+y_3 & if\ \rho_2 < \varepsilon_3 < \rho_3 \\ v_3+v_{12} & if\ \varepsilon_3 \ge \rho_3 \end{cases} & for\ \varepsilon_2 \le \frac{16}{77} \\ \begin{cases} y_1+y_2+y_3 & if\ \rho_4 \le \varepsilon_3 \le \rho_3 \\ v_3+v_{12} & if\ \varepsilon_3 < \rho_4\ or\ \varepsilon_3 > \rho_3 \end{cases} & for\ \frac{16}{77} \le \varepsilon_2 \le \rho_5 \\ v_3+v_{12} & for\ \varepsilon_2 > \rho_5 \approx 0.22. \end{cases} \quad \text{(A13)}$$

The following expressions are used in proofs for Lemmas 1-2 and Propositions 2-4, where $\tilde{\pi}_{12}(\Delta_3) = v_{12}$, $\tilde{\pi}_{13}(\Delta_2) = v_{13}$, and $\tilde{\pi}_{23}(\Delta_1) = v_{23}$ (see (18) or (A5)).

$$\begin{aligned} &v_1^{\alpha*} = \pi_1 = (a-c_1)^2(1+\varepsilon_2+\varepsilon_3)^2/16,\quad v_2^{\alpha*} = \pi_2 = (a-c_1)^2(1-3\varepsilon_2+\varepsilon_3)^2/16, \\ &\qquad v_3^{\alpha*} = \pi_3 = (a-c_1)^2(1-3\varepsilon_3+\varepsilon_2)^2/16; \\ &v_1^{\delta} = (a-c_1)^2(1+\varepsilon_2)^2/9,\ v_2^{\delta} = (a-c_1)^2(1-2\varepsilon_2)^2/9,\ v_3^{\delta} = (a-c_1)^2(1-2\varepsilon_3)^2/9; \\ &v_{12} = v_{12}^{\alpha*} = v_{12}^{\delta} = (a-c_1)^2(1+\varepsilon_3)^2/9,\ v_{13} = v_{13}^{\alpha*} = v_{13}^{\delta} = v_1^{\delta} = (a-c_1)^2(1+\varepsilon_2)^2/9, \\ &\qquad v_{23} = v_{23}^{\alpha*} = v_{23}^{\delta} = v_2^{\delta} = (a-c_1)^2(1-2\varepsilon_2)^2/9. \end{aligned} \quad \text{(A14)}$$

Proof of Lemma 1: Consider first S = 12. Let $d_{12} = \tilde{\pi}_{12}(\Delta_3) - (\pi_1+\pi_2)$. (A14) leads to

$$d_{12}(\varepsilon_3) = (a-c_1)^2 (1+\varepsilon_3-3\varepsilon_2)(15\varepsilon_2-1-\varepsilon_3)/72.$$

By A0, $(1+\varepsilon_3-3\varepsilon_2) > 0$. Hence, $d_{12} > 0 \Leftrightarrow \varepsilon_3 < \theta_2$, which is given by

$$\theta_2 = 15\varepsilon_2-1. \quad \text{(A15)}$$

Now consider S = 13. Let $d_{13} = \tilde{\pi}_{13}(\Delta_2) - (\pi_1+\pi_3)$. (A14) leads to

$$d_{13}(\varepsilon_3) = (a-c_1)^2 (1+\varepsilon_2-3\varepsilon_3)(15\varepsilon_3-1-\varepsilon_2)/72.$$

By $(1+\varepsilon_2-3\varepsilon_3) > 0$, $d_{13} > 0 \Leftrightarrow \varepsilon_3 > \theta_4$, which is given by

$$\theta_4 = (1+\varepsilon_2)/5. \quad \text{(A16)}$$

Finally, consider S = 23. Let $d_{23} = \tilde{\pi}_{23}(\Delta_1) - (\pi_2+\pi_3)$. By (A14), one has

$$d_{23}(\varepsilon_3) = \frac{5(a-c_1)^2}{8}\left(\frac{1+\varepsilon_2}{3} - \varepsilon_3\right)\left(\varepsilon_3-\frac{1+13\varepsilon_2}{15}\right) = \frac{5(a-c_1)^2}{8}(\theta_0-\varepsilon_3)(\varepsilon_3-\theta_6).$$

By $(\theta_0-\varepsilon_3) > 0$, $d_{23} > 0 \Leftrightarrow \varepsilon_3 > \theta_6$, where θ_0 and θ_6 are given by

$$\theta_0 = (1+\varepsilon_2)/3; \ \ \theta_6 = (1+13\varepsilon_2)/15. \qquad \text{(A17)}$$

This completes the proof for Lemma 1. **Q.E.D**

The following relations (see Figure 4) are useful in proving Lemma 2.

$$\varepsilon_3 \geq \theta_2 \ \text{ if } \varepsilon_2 \leq 1/14; \ \varepsilon_3 < \theta_2 = 15\varepsilon_2-1 \text{ if } \varepsilon_2 > 1/11; \qquad \text{(A18)}$$

$$\varepsilon_3 > \theta_4 = (1+\varepsilon_2)/5 \ \text{ if } \varepsilon_2 > 1/14; \qquad \text{(A19)}$$

$$\theta_4 < \theta_6 \leq \theta_0; \ \ \varepsilon_2 \leq \varepsilon_3 \leq \theta_0; \text{ and } \varepsilon_2 \leq \theta_6. \qquad \text{(A20)}$$

Proof of Lemma 2: I first compute $MNBP_\delta$. There are six constraints: $x_1 \geq v_1^\delta$, $x_2 \geq v_2^\delta$, $x_3 \geq v_3^\delta$; $x_1+x_2 \geq v_{12}$, $x_1+x_3 \geq v_{13}$, $x_2+x_3 \geq v_{23}$. By $v_1^\delta = v_{12}$, $v_2^\delta = v_{23}$, the problem becomes:

$$MNBP_\delta = \text{Min } \{ x_1+x_2+x_3 \,|\, x_1 \geq v_1^\delta, x_2 \geq v_2^\delta, x_3 \geq v_3^\delta; \ x_1+x_2 \geq v_{12}\}, \qquad \text{(A21)}$$

of which the minimum value is equal to

$$v_3^\delta + \text{Max } \{v_1^\delta + v_2^\delta, v_{12}\}. \qquad \text{(A22)}$$

Let

$$d(\varepsilon_3) = v_1^\delta + v_2^\delta - v_{12} = (a-c_1)^2[(1+\varepsilon_2)^2+(1-2\varepsilon_2)^2-(1+\varepsilon_3)^2]/9.$$

By $d'' < 0$, d is $\cap$-shaped. $d(\varepsilon_3) = 0$ has two roots: $\mu_1 < 0 < \mu_2$, where μ_2 is given by

$$\mu_2 = \rho_0 = -1+\sqrt{2-2\varepsilon_2+5\varepsilon_2^2}.$$

Hence, $\text{Max}\{v_1^\delta+v_2^\delta, v_{12}\} = v_1^\delta+v_2^\delta$ if $\varepsilon_3 \leq \rho_0$, v_{12} if $\varepsilon_3 > \rho_0$. Then, (A14), (A21)-((A22) lead to (A9).

I only provide an outline for calculating $MNBP_{\alpha*}$, because complete computation like those for (A9) would make the paper too long. Figure 4 illustrates all the sub-cases.

Case 1. $\varepsilon_2 \in [0, \frac{1}{14}]$. By Lemma 1, $d_{12} \leq 0$, so only five constraints are left:

$$x_1 \geq v_1 = v_1^\alpha, \ \ x_2 \geq v_2 = v_2^\alpha, \ \ x_3 \geq v_3 = v_3^\alpha; \ \ \ x_1+x_3 \geq v_{13}, \ \ \ x_2+x_3 \geq v_{23}.$$

Let $h_1 = v_{13}-v_1$, $h_2 = v_{23}-v_2$, one has $d(\varepsilon_2,\varepsilon_3) = \max\{h_1, h_2\} = h_1$, and $v_3 \geq d(\varepsilon_2,\varepsilon_3) \Leftrightarrow \varepsilon_3 \leq \theta_4$.

By $MNBP_{\alpha*} = v_1 + v_2 + \max\{v_3, d(\varepsilon_2, \varepsilon_3)\}$, $MNBP_{\alpha*} = v_1 + v_2 + v_3$, if $\varepsilon_3 < \theta_4$, and $MNBP_{\alpha*} = v_2 + v_{13}$ if $\varepsilon_3 \geq \theta_4$. This proves (A10).

Case 2. $\varepsilon_2 \in [\frac{1}{14}, \frac{4}{53}]$. One has $\theta_4 \leq \varepsilon_2 \leq \theta_2 < \theta_6$. If $\varepsilon_3 \geq \theta_2$, then $d_{12} \leq 0$. By Case 1, $MNBP_{\alpha*} = v_2 + v_{13}$. If $\varepsilon_3 < \theta_2 < \theta_6$, then $d_{23} \leq 0$. So the constraint $x_2+x_3 \geq v_{23}$ can be removed. Using similar steps as in Case 1, one can show $MNBP_{\alpha*} = v_2 + v_{13}$.

Case 3. $\varepsilon_2 \geq \frac{4}{53}$, and $\varepsilon_3 \leq \theta_6$. By $d_{23} \leq 0$, $x_2+x_3 \geq v_{23}$ is removed. Similar to Case 2, and by $d_{13} > 0$, one can show $MNBP_{\alpha*} = v_2 + v_{13}$ if $\varepsilon_3 \leq \rho_1$, and $= v_3+v_{12}$ if $\varepsilon_3 > \rho_1$. One can also show that $\frac{4}{53} \leq \varepsilon_2 \leq \frac{16}{77}$ implies $\varepsilon_3 \leq \rho_1$, and $\varepsilon_2 \geq \frac{5}{22}$ implies $\varepsilon_3 > \rho_1$.

Case 4. $\varepsilon_2 \geq \frac{4}{53}$, and $\varepsilon_3 \geq \theta_2 > \theta_6$. This can only occur for $\varepsilon_2 \in [\frac{4}{53}, \frac{1}{11}]$. By Case 1, $d_{12} \leq 0$, and $\varepsilon_3 \geq \theta_4$, $MNBP_{\alpha*} = v_2 + v_{13}$. By Cases 2-3, one gets (A11) and (A12).

Case 5. $\varepsilon_2 \geq \frac{4}{53}$, and $\theta_2 \geq \varepsilon_3 > \theta_6$. One has $d_{12} > 0$, $d_{13} > 0$, $d_{23} > 0$. Note at most one of $x_1 \geq v_1$, $x_2 \geq v_2$, $x_3 \geq v_3$ can be binding. First solving each of the three cases: Case 5.1, $x_1 = v_1$; Case 5.2, $x_2 = v_2$; Case 5.3, $x_3 = v_3$. Now solve Case 5.4, $x_1>v_1$, $x_2>v_2$, $x_3>v_3$. In case 5.4, one must have $x_1+x_2= v_{12}$, $x_1+x_3= v_{13}$, and $x_2+x_3 = v_{23}$. Solving these equations, one gets y_1, y_2, y_3. By checking $y_i > v_i$, and using Cases 5.1-5.3, one can get (A13). **Q.E.D**

Proof of Proposition 2: Part (i). For each of the values of $MNBP_{\alpha*}$, one can show $v(N) > MNBP_{\alpha*}$. Now consider part (ii). If $\varepsilon_3 \leq \rho_0$, $d = v(N) - MNBP_\delta$ is given by

$$d(\varepsilon_3) = (a-c_1)^2 \left[\frac{1}{4} - \frac{(1+\varepsilon_2)^2+(1-2\varepsilon_2)^2+(1-2\varepsilon_3)^2}{9}\right].$$

Using $d'' < 0$ and solving the two roots $\mu_1 < \mu_2$ for $d(\varepsilon_3) = 0$, one has:

$$d > 0 \Leftrightarrow \omega_1 \leq \varepsilon_3 \leq \rho_0, \text{ where } \omega_1 = \mu_1 = \frac{1}{2} - \frac{\sqrt{1+8\varepsilon_2-20\varepsilon_2^2}}{4} \leq \rho_0. \tag{A23}$$

If $\varepsilon_3 > \rho_0$, d is given by

$$d(\varepsilon_3) = (a-c_1)^2 \left[\frac{1}{4} - \frac{(1+\varepsilon_3)^2+(1-2\varepsilon_3)^2}{9}\right] = \frac{(a-c_1)^2}{36}(1+10\varepsilon_3)(1-2\varepsilon_3) > 0.$$

Using (A23), one gets $d > 0 \Leftrightarrow \omega_1 \leq \varepsilon_3$, which completes the proof of part (ii). **Q.E.D**

Proof of Proposition 3: Since $\Delta_1 = \{1; 23\}$ and $\Delta_2 = \{13; 2\}$ have identical welfare, Δ_2 can be ignored. First, I evaluate six cases below.

(1) $\Delta_0 \to \Delta_3$. Let $d_1(\varepsilon_3) = W_3 - W_0$. Using $d_1(\varepsilon_3)'' < 0$ and solving $d_1 = 0$, one can show: $d_1(\varepsilon_3) \geq 0 \Leftrightarrow \varepsilon_3 \leq \sigma_1 = (-7+69\varepsilon_2)/31$, $\varepsilon_3 < \sigma_1$ if $\varepsilon_2 > 13/44$, and $\varepsilon_3 > \sigma_1$ if $\varepsilon_2 < 7/38$;

(2) $\Delta_0 \to \Delta_1$. $d_2(\varepsilon_3) = W_2 - W_0$ leads to: $d_2(\varepsilon_3) \geq 0 \Leftrightarrow \varepsilon_3 \geq \sigma_2$; $\varepsilon_3 > \sigma_2$ if $\varepsilon_2 > 7/38$, where

$$\sigma_2 = \omega_2 = (7+31\varepsilon_2)/69; \tag{A24}$$

(3) $\Delta_3 \to \Delta_1$. $d_3(\varepsilon_3) = W_1 - W_3$ leads to: $d_3(\varepsilon_3) \geq 0 \Leftrightarrow \varepsilon_3 \leq \sigma_3 = (-\varepsilon_2 + 8/11)$, $\varepsilon_3 \leq \theta_0 < \sigma_3$ if $\varepsilon_2 < 13/44$, and $\varepsilon_3 \geq \varepsilon_2 > \sigma_3$ if $\varepsilon_2 > 4/11$;

(4) $\Delta_1 \to \Delta_m$. $d_4(\varepsilon_2) = W_m - W_1$ leads to $d_4(\varepsilon_2) \geq 0 \Leftrightarrow \varepsilon_2 \geq \sigma_4 = 5/22$;

(5) $\Delta_0 \to \Delta_m$. $d_5(\varepsilon_3) = W_m - W_0$ leads to: $d_5(\varepsilon_3) \geq 0 \Leftrightarrow \sigma_5 \leq \varepsilon_3 \leq \sigma_6$, $d_5 < 0$ if $\varepsilon_2 < \sigma_7 = 5/14 - \sqrt{23}/28 \approx 0.19$, and $d_5 > 0$ if $\varepsilon_2 > 5/22$, where σ_5 and σ_6 are given by

$$\sigma_5 = (5+9\varepsilon_2 - 2\sqrt{-11+80\varepsilon_2-112\varepsilon_2^2})/23,\ \sigma_6 = (5+9\varepsilon_2+2\sqrt{-11+80\varepsilon_2-112\varepsilon_2^2})/23; \tag{A25}$$

(6) $\Delta_3 \to \Delta_m$. $d_6(\varepsilon_3) = W_m - W_3$ leads to $d_6(\varepsilon_3) \geq 0 \Leftrightarrow \varepsilon_3 \geq 5/22$.

Second, comparing cases 1-6 on [0, 0.5] and picking up the maximal W, one gets: $W^* = W_m$ if $\varepsilon_2 > 5/22$; $= W_1$ if $7/38 < \varepsilon_2 \leq 5/22$; $= W_1$ if $\varepsilon_2 \leq 7/38$ and $\varepsilon_3 \geq \omega_2$; $= W_0$ if $\varepsilon_2 \leq 7/38$ and $\varepsilon_3 < \omega_2$.

Q.E.D

Proof of Proposition 4: Part (i) Consider $\Delta_1 = \{1; 23\}$ and y with $y_1 = v_{13}$, $y_2 = v_2 + td_{23}$, and $y_3 = v_3 + (1-t)d_{23}$. By $d_{23} > 0$ (i.e., $\varepsilon_3 > \theta_6$) and the definition of y, $y \in Y(\Delta_1) = Y_{\alpha*}(\Delta_1) = Y_\delta(\Delta_1) = \{ y \mid y_1 \geq v_1^\delta = v_{13}, y_2 \geq v_2, y_3 \geq v_3, y_1 + y_2 \geq v_{12}, y_1 + y_3 \geq v_{13}, y_2 + y_3 \geq v_{23}\}$ is equivalent to

$$d(\varepsilon_3) = v_{13} + v_2 + t\, d_{23} - v_{12} \geq 0. \tag{A26}$$

Note $d'' < 0$ (i.e., *d* is $\cap$-shaped), and $d(\varepsilon_3) = 0$ has two roots:

$$\mu_1(\varepsilon_2,t) = \frac{-14 - 54\varepsilon_2 + 36t(1+3\varepsilon_2) + 8\sqrt{7+14\varepsilon_2+88\varepsilon_2^2+t(34-244\varepsilon_2+352\varepsilon_2^2)+9t^2(1-4\varepsilon_2+4\varepsilon_2^2)}}{2(7+90t)},$$
$$\mu_{10}(\varepsilon_2,t) = \frac{-14 - 54\varepsilon_2 + 36t(1+3\varepsilon_2) - 8\sqrt{7+14\varepsilon_2+88\varepsilon_2^2+t(34-244\varepsilon_2+352\varepsilon_2^2)+9t^2(1-4\varepsilon_2+4\varepsilon_2^2)}}{2(7+90t)}. \tag{A27}$$

It can be checked that the following three claims hold:

$$\mu_{10}(\varepsilon_2,t) < \theta_6;\ \theta_0 \leq \mu_1(\varepsilon_2,t) \Leftrightarrow \varepsilon_2 \leq 1/11;\ \text{and}\ \theta_6 \leq \mu_1(\varepsilon_2,t) \Leftrightarrow \varepsilon_2 \leq 113/316. \tag{A28}$$

Therefore, by (A28), by the $\cap$-shape of $d(\varepsilon_3)$, and by $\theta_6 \leq \varepsilon_3 \leq \theta_0$, one has

$$d(\varepsilon_3) = \begin{cases} > 0 & \text{if } \varepsilon_2 \leq 1/11 \\ \geq 0 \Leftrightarrow \varepsilon_3 \leq \mu_1(\varepsilon_2; t) & \text{if } 1/11 < \varepsilon_2 < 113/316 \\ < 0 & \text{if } \varepsilon_2 \geq 113/316; \end{cases}$$

which leads to part (i).

The above results have been confirmed by evaluating t = 1 and 0 separately. The proofs for parts (ii)-(iii) are similar. In particular, $\mu_2(\varepsilon_2, t)$ for Δ_2 in part (ii) is:

$$\mu_2(\varepsilon_2, t) = \frac{-14+18\varepsilon_2+36t(1+\varepsilon_2)+8\sqrt{7-28\varepsilon_2+37\varepsilon_2^2+t(34-256\varepsilon_2+430\varepsilon_2^2)+9t^2(1+2\varepsilon_2+\varepsilon_2^2)}}{2(7+90t)}. \tag{A29}$$

Note the formula for (iii) is given by that for (ii), after switching ε_2 and ε_3. **Q.E.D**

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