# Signaling by Refund in Auctions

Very preliminary; please do not quote

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**Abstract**

In this paper, we analyze refund policies as a signaling device in first-price and second-price auctions. When the seller's value follows a two-point distribution, we characterize all equilibria of the auction game. A strong intuitive criterion selects the separating equilibria and they all yield the same (completely revealing) equilibrium outcome. When the seller's value follows a continuous distribution, we show that a completely separating equilibrium does not exist, but characterize all fully pooling and one partially pooling equilibria.

Key words: auctions, refund policies, pooling, separating, intuitive criterion

JEL classification: D44, D72, D82

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# 1 Introduction

(To be expanded)

Auctions have been used for trading for thousands of years. Due to the rapid growth of internet commerce, online auctions have become extremely popular. These auctions create a problem for both the buyers and the sellers. As a buyer is unable to closely examine the good being auctioned, he may find the good not exactly of the quality he expects when he receives it. Even though this could also happen in store purchases, it is inarguably a more common problem in online auctions. If the sellers allow buyers to return the goods, how would the refund policy signal the quality of the object being auctioned? This is the central issue we will investigate in this paper.

Each day, there are millions of objects being auctioned on the internet through many online auction sites. We observe that many of the sellers provide very generous return policies in the auctions. The NHL (National Hockey League) online auctions, for example, provide a 7-Day, 100% Money-Back Guarantee. Most sellers in Amazon.com and eBay.com provide at least partial return policies; buyers may need to pay some fees (usually less than 15% of the transaction prices) if they would like to return the merchandises.

In this paper, we analyze refund policies as a signaling device in first-price and second-price auctions. A seller first posts a refund policy, and observing this, buyers compete in a first-price or second-price sealed-bid auction. The winning bidder in the auction pays for the object according to the respective auction rule, discovers the value (quality) of the object, and decides whether or not to return the object for refund.

When the seller's value follows a two-point distribution, we characterize all equilibria of the auction game. A strong intuitive criterion selects the separating equilibria and they all yield the same (completely revealing) equilibrium outcome. When the seller's value follows a continuous distribution, we show that a completely separating equilibrium does not exist, but characterize all fully pooling and one partially pooling equilibria. (To be expanded.)

There is a huge literature on auctions. The equilibria of common value auctions have already been extensively examined (see Goeree and Offerman (2003), Hausch (1987), Milgrom and Weber (1982a), Reece (1978), and Wilson (1977)). Milgrom and Weber (1982b) consider a very general model which includes the private values and common values as special cases. However, none of the above papers consider return policies. To our best knowledge, our paper is the first one including return policies in auctions.

The rest of this paper is organized as follows. In Section 2, we describe the model. In Section 3, we characterize the separating and pooling equilibria in second-price auctions. In Section 4, we characterize the separating and pooling equilibria in first-price auctions. In Section 5, we extend the model to continuous seller values. In Section 6, we conclude.

## 2 The Model

Suppose that there are $n$ bidders, $i = 1, 2, ..., n$, competing in a first- or second-price, sealed-bid auction with no reserve price. The value of the object is $vx_i$ for bidder $i$. Here $x_i$ is the private information of bidder $i$ and $v$ is the seller's private information. As is common in the literature, we assume that $v$ follows a binomial distribution. It is equal to $v_H$ with probability $\mu_H$ and $v_L$ with probability $\mu_L = 1 - \mu_H$. Nevertheless, $x_i$ is drawn from a common uniform distribution with support $[0, 1]$. The value of $v$ will be observed by the auction winner after the auction. This captures the fact that a buyer will discover more information about the object, such as whether or not a painting is a fake, whether or not a product is defective, etc, after he takes the object home. We assume that all random variables are independently distributed.

We define a refund policy in an auction as the percentage of the transaction price that a seller will refund to the buyer if the object is returned. Suppose that $\gamma$ is the percentage of refund and $B$ is the transaction price. If the buyer returns the object, then $\gamma B$ would be refunded to him. Therefore, the buyer suffers a net loss of $(1 - \gamma)B$ when he returns the object.

The timing of the game is as follows.

1. Nature randomly draws values for $v$ and $x_i$, $i = 1, 2, ..., n$.
2. The seller learns $v$ and buyer $i$ learns $x_i$ privately, $i = 1, 2, ..., n$.
3. The refund policy $\gamma$ is announced by the seller.
4. An auction is held and transaction takes place. (We will consider both first and second price auctions.)
5. The winning bidder observes $v$ and decides whether or not to return the object.

We will characterize the Perfect Bayesian Nash Equilibria in these games. These equilibria can be formulated as follows in our model. Each of them consists of a pair of refund policies for the seller, $\gamma_H$ and $\gamma_L$, and a belief function for the buyers $\mu_H(\gamma)$. This belief function specifies the buyers' common belief that the seller is of type $v_H$ when the refund policy $\gamma$ is observed. Buyers bid according to this belief. The following conditions are satisfied for a PBNE:

- The seller's refund policy is optimal given buyers' beliefs and bidding strategies.
- The believe function $\mu_H(\gamma)$ is derived from the seller's strategy using the Bayes' rule whenever possible.
- Buyers' bids optimally in the auction given the common belief $\mu_H(\gamma)$.

Denote $\mu_L(\gamma) = 1 - \mu_H(\gamma)$. We will examine the second-price auction and the first-price auction separately.

# 3 Second-Price Auctions

We first examine second-price, sealed-bid auctions. The transaction price will be the second highest bid in the auction. We will characterize all the pure strategy Perfect Bayesian Equilibria, and then apply the intuitive criterion (as well as a stronger version of it) to refine the equilibria since there are many of them.

## 3.1 Perfect Bayesian Nash Equilibrium

We begin our analysis by first analyzing the buyers' bidding strategy in the auction. After seeing refund policy $\gamma$, buyers believe that with probability $\mu_H(\gamma)$ the seller is of type $v_H$. Let $x_1$ and $x_2$ be the highest and the second highest $x_i$ among the bidders. The following lemma characterize the equilibrium in the second-price auction with refund.

**Lemma 1** *(a) Suppose that* $\gamma > \frac{v_L}{v_L\mu_L(\gamma)+v_H\mu_H(\gamma)}$. *Buyers bid according to*

$$b(x) = \frac{\mu_H(\gamma)v_H x}{1-\gamma\mu_L(\gamma)}. \tag{1}$$

*The winning bidder returns the object if* $v = v_L$ *and* $x_2 > \frac{[1-\gamma\mu_L(\gamma)]v_L}{\gamma\mu_H(\gamma)v_H}x_1$, *and keeps the object otherwise. The type* $v_H$ *seller's revenue is* $R_H = \frac{n-1}{n+1}\frac{\mu_H(\gamma)v_H}{[1-\gamma\mu_L(\gamma)]}$, *and the type* $v_L$ *seller's revenue is* $R_L = \frac{n-1}{n+1}\left\{\frac{(1-\gamma)\mu_H(\gamma)v_H}{[1-\gamma\mu_L(\gamma)]} + \frac{[1-\gamma\mu_L(\gamma)]^{n-1}v_L^n}{\gamma^{n-1}\mu_H(\gamma)^{n-1}v_H^{n-1}}\right\}$.

*(b) Suppose that* $\gamma \leq \frac{v_L}{v_L\mu_L(\gamma)+v_H\mu_H(\gamma)}$. *Buyers bid according to*

$$b(x) = [\mu_H(\gamma)v_H + \mu_L(\gamma)v_L]\, x. \tag{2}$$

*The winning bidder does not return the object. The type* $v_H$ *seller's revenue is* $R_H = \frac{n-1}{n+1}[\mu_H(\gamma)v_H + \mu_L(\gamma)v_L]$, *and the type* $v_L$ *seller's revenue is* $R_L = \frac{n-1}{n+1}[\mu_H(\gamma)v_H + \mu_L(\gamma)v_L]$.

**Proof:** We assume that all buyers adopt the same strictly increasing bidding function $b^S(\cdot)$ in the auction. First, note that no one will bid more than $v_H x_i$ since that is the highest possible value to buyer $i$. Consider bidder 1 with signal $x_1$. Suppose that he bids $b^S(\tilde{x}_1)$, wins the auction, and pays $b^S(Y_1)$, where $Y_1$ is the highest signal among the rest of the buyers. The winner returns the object iff $vx_1 < \gamma b^S(Y_1)$. If it turns out that $v = v_H$, he will not return the object since his payment is less than $v_H x_1$. If $v = v_L$, he may keep or return the object depending on the value of $Y_2$. If $v_L x_1 < \gamma b^S(Y_1)$, he returns it; if otherwise, he will keep it.

Given the optimal behavior regarding return above, we can formulate bidder 1's optimization problem in the auction as follows:

$$\max_{\tilde{x}_1} \mu_H \int_0^{\tilde{x}_1} [v_H x_1 - B(x_2)] dx_2 + \mu_L \int_0^{\tilde{x}_1} max \left\{ v_L x_1 - b^S(x_2), -(1-\gamma) b^S(x_2) \right\} dx_2$$

The FOC, evaluating $\tilde{x}_1 = x_1$, gives us:

$$\mu_H [v_H x_1 - b^S(x_1)] f(x_1) + \mu_L max \left\{ v_L x_1 - b^S(x_1), -(1-\gamma) b^S(x_1) \right\} = 0.$$

If $v_L x_1 - b^S(x_1) > -(1-\gamma) b^S(x_1)$, which is equivalent to $\gamma > \frac{v_L}{v_L \mu_L(\gamma) + v_H \mu_H(\gamma)}$, then $b^S(x_1) = (\mu_H v_H + \mu_L v_L) x_1$. If $v_L x_1 - b^S(x_1) \leq -(1-\gamma) b^S(x_1)$, which is equivalent to $\gamma \leq \frac{v_L}{v_L \mu_L(\gamma) + v_H \mu_H(\gamma)}$, then $b^S(x_1) = \frac{\mu_H v_H}{\mu_H v_H + \mu_L v_L} x_1$. The seller's revenue can be obtained by utilizing the buyers' bidding function. **Q.E.D.**

There are some important features of the revenue function, which are summarized in the following corollary.

**Corollary 1** *Fixing $\mu_H(\gamma)$ and $\mu_L(\gamma)$, the type $v_H$ seller's revenue is increasing in $\gamma$, and the type $v_L$ seller's revenue is decreasing in $\gamma$.*

### 3.1.1 Separating equilibrium

In a separating equilibrium, let $\gamma_H^*$ and $\gamma_L^*$ be the seller's equilibrium refund policies as a function of her type, and let $b^*(x;\gamma)$ be buyers' equilibrium bidding function as a function of the seller's refund policy. We first establish two useful lemmas.

**Lemma 2** *In any separating equilibrium, $\mu_H(\gamma_H^*) = 1$ and $\mu_H(\gamma_L^*) = 0$; $b^*(x;\gamma_H^*) = v_H x$ and $b^*(x;\gamma_L^*) = v_L x$.*

**Proof:** In any PBNE, beliefs on the equilibrium path must be correctly derived from the equilibrium strategies using the Bayes' rule. Here this implies that upon seeing refund policy $\gamma_H^*$, buyers must assign probability one to the seller being type $v_H$. Likewise, upon seeing refund policy $\gamma_L^*$, buyers must assign probability one to the seller being type $v_L$. From (2), the resulting bidding functions are exactly $v_H x$ and $v_L x$, respectively. **Q.E.D**

From the above lemma, if the type $v_L$ seller does not deviate and provides $\gamma_L^*$, she gets revenue $\frac{(n-1)v_L}{n+1}$. If she deviates to $\gamma_H^*$, buyers will believe that she is of type $v_H$ with probability one, and from (2) they will bid $v_H x$. The winning bidder returns the object iff

$\gamma_H^* v_H x_2 > v_L x_1$. The seller's revenue becomes

$$R_L = \begin{cases} \frac{n-1}{n+1}\left[(1-\gamma_H^*)v_H + \frac{v_L^n}{{\gamma_H^*}^{n-1} v_H^{n-1}}\right], & \text{if } \gamma_H^* > \frac{v_L}{v_H}; \\ \frac{n-1}{n+1} v_H, & \text{if } \gamma_H^* \leq \frac{v_L}{v_H}. \end{cases}$$

If $\gamma_H^* \leq \frac{v_L}{v_H}$, then the type $v_L$ seller will deviate for sure. To make this deviation non-profitable, we need $\gamma_H^* > \frac{v_L}{v_H}$, as well as

$$\frac{n-1}{n+1}\left[(1-\gamma_H^*)v_H + \frac{v_L^n}{{\gamma_H^*}^{n-1} v_H^{n-1}}\right] \leq \frac{n-1}{n+1} v_L$$
$$\Leftrightarrow \quad (1-\gamma_H^*)v_H + \frac{v_L^n}{{\gamma_H^*}^{n-1} v_H^{n-1}} \leq v_L \qquad (3)$$

This is a necessary condition for the PBNE. It is easy to prove that Condition (3) is also a sufficient condition for PBNE. To see this, we assign all out-of-equilibrium belief as $v = v_L$ with probability one if a buyer sees any off-equilibrium refund policy. Then the low type seller has no incentive to deviate to any out-of-equilibrium refund policy, since it provides him with the same revenue $\frac{n-1}{n+1} v_L$. Given Condition (3), the low type seller does not have incentive to mimic the high type seller as well. In addition, the high type seller has no incentive to deviate, since his equilibrium revenue is $\frac{n-1}{n+1} v_H$, which is higher than the revenue in any deviation, $\frac{n-1}{n+1} v_L$.

It is easy to verify that $\gamma_H^* > \frac{v_L}{v_H}$ is implied by Condition (3). The left hand side (LHS) of the condition is a strictly decreasing function of $\gamma_H^*$. When $\gamma_H^* = \frac{v_L}{v_H}$, $LHS = v_H$; when $\gamma_H^* = 1$, $LHS = v_L(\frac{v_L}{v_H})^{n-1}$. Thus, there exists a unique cutoff $\hat{\gamma}_H^* \in (\frac{v_L}{v_H}, 1)$ such that Condition (3) holds with equality. Therefore, we have the following proposition.

**Proposition 1** *All separating PBNE in the signaling by refund in the second-price auction can be characterized as follows. The high type seller provides refund policy $\gamma_H^* \geq \hat{\gamma}_H^*$, and the low type seller provide refund policy $\gamma_L^* \neq \gamma_H^*$. A buyer's out-of-equilibrium belief is $v = v_L$ with probability one if he sees any off-equilibrium refund policy.*

### 3.1.2 Pooling equilibrium

We now consider pooling equilibria. Both types of sellers choose the same refund policy, i.e., $\gamma_H^* = \gamma_L^* = \gamma^*$. Since buyers' beliefs must be correctly derived from the equilibrium strategy and the Bayes' rule when possible, they must assign probability $p_H$ to type $v_H$ when they see $\gamma^*$. Thus, in any pooling equilibrium, from (1) and (2), we must have

$$b(x) = \begin{cases} \frac{\mu_H v_H x}{1-\gamma^* \mu_L}, & \text{if } \gamma^* > \frac{v_L}{v_L\mu_L + v_H\mu_H}; \\ \\ [\mu_H v_H + \mu_L v_L]\, x, & \text{if } \gamma^* \le \frac{v_L}{v_L\mu_L + v_H\mu_H}. \end{cases}$$

The winning bidder returns the object when $v = v_L$, $\gamma^* > \frac{v_L}{v_L\mu_L + v_H\mu_H}$ and $x_2 > \frac{[1-\gamma^*\mu_L]v_L}{\gamma^*\mu_H v_H} x_1$, where $x_1$ is his own bid and $x_2$ is the highest bid among other bidders; otherwise, he keeps the object. From Lemma (1), the high type seller's revenue is

$$R_H = \begin{cases} \frac{n-1}{n+1} \frac{\mu_H v_H}{[1-\gamma^*\mu_L]}, & \text{if } \gamma^* > \frac{v_L}{v_L\mu_L + v_H\mu_H}; \\ \\ \frac{n-1}{n+1} [\mu_H v_H + \mu_L v_L], & \text{if } \gamma^* \le \frac{v_L}{v_L\mu_L + v_H\mu_H}. \end{cases}$$

Similarly, the low type seller's revenue is

$$R_L = \begin{cases} \frac{n-1}{n+1} \left[ \frac{(1-\gamma^*)\mu_H v_H}{(1-\gamma^*\mu_L)} + \frac{(1-\gamma^*\mu_L)^{n-1} v_L^n}{\gamma^{*n-1}\mu_H^{n-1} v_H^{n-1}} \right], & \text{if } \gamma^* > \frac{v_L}{v_L\mu_L + v_H\mu_H}, \\ \\ \frac{n-1}{n+1} [\mu_H v_H + \mu_L v_L] & \text{if } \gamma^* \le \frac{v_L}{v_L\mu_L + v_H\mu_H}. \end{cases}$$

If the low type seller deviates to the no refund policy, i.e., $\gamma = 0$, it guarantees her revenue at least $\frac{n-1}{n+1} v_L$ regardless of the buyers' beliefs. Thus for a PBNE, a necessary condition is that the seller's equilibrium payoff is better than offering the no refund policy. If $\gamma^* \le \frac{v_L}{v_L\mu_L + v_H\mu_H}$, then this is always satisfied. If $\gamma^* > \frac{v_L}{v_L\mu_L + v_H\mu_H}$, then we need the following condition:

$$\begin{aligned} & \frac{n-1}{n+1} \left[ \frac{(1-\gamma^*)\mu_H v_H}{(1-\gamma^*\mu_L)} + \frac{(1-\gamma^*\mu_L)^{n-1} v_L^n}{\gamma^{*n-1}\mu_H^{n-1} v_H^{n-1}} \right] \ge \frac{n-1}{n+1} v_L \\ \Leftrightarrow \quad & \frac{(1-\gamma^*)\mu_H v_H}{(1-\gamma^*\mu_L)} + \frac{(1-\gamma^*\mu_L)^{n-1} v_L^n}{\gamma^{*n-1}\mu_H^{n-1} v_H^{n-1}} \ge v_L \end{aligned} \tag{4}$$

This is also a sufficient condition for a PBNE. A PBNE can be supported by out-of-equilibrium belief of $v = v_L$ with probability one for any off-equilibrium refund policy; neither types of sellers will deviate given this belief.

Note that the LHS of Condition (4) is strictly decreasing in $\gamma$. When $\gamma^* = \frac{v_L}{v_L\mu_L + v_H\mu_H}$, $LHS = \mu_H v_H + \mu_L v_L$. When $\gamma = 1$, $LHS = v_L (\frac{v_L}{v_H})^{n-1}$. Therefore, there must exist a unique solution $\hat{\gamma}^* \in (\frac{v_L}{v_L\mu_L + v_H\mu_H}, 1)$ for (4) to be binding. Thus, we have the following proposition.

**Proposition 2** *All the pooling PBNE can be characterized as follows. Both types of sellers provide refund policy $\gamma^* \le \hat{\gamma}^*$; the out-of-equilibrium belief is $v = v_L$ with probability one for any off-equilibrium refund policy.*

### 3.2 Refinements by Intuitive Criterion

Since there are multiple separating and pooling equilibria, in this subsection, we intend to eliminate some of the equilibria. First note that from Proposition 2, pooling at the full return policy $\gamma = 1$ is not even a PBNE. Second, consider any pooling equilibrium. If the low type seller provides the full refund policy, the maximal revenue she can get is $\frac{n-1}{n+1}v_L$. To see this, suppose that a buyer's belief is $v = v_L$ with probability one for the full refund policy, then the low type seller gets payoff $\frac{n-1}{n+1}v_L$. Suppose that a buyer's belief is $v = v_L$ with probability less than one, then buyers will bid up to $v_H x$. The seller's revenue, according to Lemma 1, is equal to $\frac{n-1}{n+1}v_L(\frac{v_L}{v_H})^{n-1}$, which is less than $\frac{n-1}{n+1}v_L$. Therefore, no matter what the belief is, the low type sell's maximal revenue is $\frac{n-1}{n+1}v_L$ by providing full refund policy.

From Proposition 2, the equilibrium revenue is always strictly higher than $\frac{n-1}{n+1}v_L$ (except at $\hat{\gamma}^*$ where they are equal). Therefore, for all pooling equilibria (except $\hat{\gamma}^*$), providing the full refund policy is dominated by the equilibrium strategy for the low type seller. As a result, for those pooling equilibria, upon seeing the full refund policy, buyers should assign belief $v = v_H$ with probability one according to the intuitive criterion. Given this, the high type seller would deviate to the full refund policy and obtain her revenue of $\frac{n-1}{n+1}v_H$. This is strictly higher than her equilibrium revenue of $\frac{n-1}{n+1}(\mu_L v_L + \mu_H v_H)$. This means that these pooling equilibria do not survive the intuitive criterion.

The lone pooling equilibrium left is the one with refund policy $\hat{\gamma}^*$. Deviating to the full refund policy gives the seller the same revenue as the equilibrium revenue. Therefore, the intuitive criterion cannot eliminate this equilibrium. However, if we use a stronger version of the intuitive criterion, we can also eliminate this pooling equilibrium as well: upon seeing a deviation, bidders do not assign a positive probability for a type with maximum revenue weakly dominated by the equilibrium revenue. Given this criterion, we can eliminate this equilibrium similarly to other pooling equilibria.

Although we can not eliminate any separating equilibria using the intuitive criterion, all separating equilibria lead to the same outcome, which coincides with the outcome when the seller's type is publicly observed. Therefore, by introducing refund policies, the asymmetry seller's type information problem is resolved. We have the following proposition.

**Proposition 3** *In second-price auctions with refund, after applying the above stronger version of the intuitive criterion, the remaining equilibria are all separating equilibria as defined in Proposition 1. All these equilibria result in the same outcome.*

## 4 First-price auctions

We now examine refunds in the first-price, sealed-bid auctions. The transaction price is now equal to the winner's bid in the auction. We will characterize all pure strategy separating and pooling Perfect Bayesian Nash Equilibria, and then apply the intuitive criterion (and a stronger version of the intuitive criterion) to eliminate some of the equilibria similarly to what was done in the second-price auctions.

## 4.1 Perfect Bayesian Nash Equilibrium

We begin our analysis by characterizing the buyers' bidding strategy in the first-price, sealed-bid auction. Suppose that after seeing refund policy $\gamma$, buyers assign a probability of $\mu_H(\gamma)$ that the seller is of type $v_H$. The following lemma characterize the equilibrium in a first-price auction with refund.

**Lemma 3** *In a first-price auction with refund policy $\gamma$ and belief $\mu_H(\gamma)$, buyers will bid according to*

$$b(x) = \begin{cases} \frac{n-1}{n}[\mu_H(\gamma)v_H + \mu_L(\gamma)v_L]x, & \text{if } \gamma \leq \frac{nv_L}{(n-1)\mu_H(\gamma)v_H+(n-1)\mu_L(\gamma)v_L} \text{ (case a);} \\ \frac{(n-1)\mu_H(\gamma)v_H}{n[1-\mu_L(\gamma)\gamma]}x, & \text{if } \gamma > \frac{nv_L}{(n-1)\mu_H(\gamma)v_H+n\mu_L(\gamma)v_L} \text{ (case b).} \end{cases} \tag{5}$$

*In case a, no winner returns the object, and case b, the winner always returns the object if $v = v_L$.*

*The high type seller's revenue is*

$$R_H = \begin{cases} \frac{n-1}{n+1}[\mu_H(\gamma)v_H + \mu_L(\gamma)v_L], & \text{if } \gamma \leq \frac{nv_L}{(n-1)\mu_H(\gamma)v_H+(n-1)\mu_L(\gamma)v_L}; \\ \frac{(n-1)\mu_H(\gamma)v_H}{(n+1)[1-\gamma\mu_L(\gamma)]}, & \text{if } \gamma > \frac{nv_L}{(n-1)\mu_H(\gamma)v_H+n\mu_L(\gamma)v_L}. \end{cases} \tag{6}$$

*The low type seller's revenue is*

$$R_L = \begin{cases} \frac{n-1}{n+1}[\mu_H(\gamma)v_H + \mu_L(\gamma)v_L], & \text{if } \gamma \leq \frac{nv_L}{(n-1)\mu_H(\gamma)v_H+(n-1)\mu_L(\gamma)v_L}; \\ \frac{(n-1)(1-\gamma)\mu_H(\gamma)v_H}{(n+1)[1-\gamma\mu_L(\gamma)]}, & \text{if } \gamma > \frac{nv_L}{(n-1)\mu_H(\gamma)v_H+n\mu_L(\gamma)v_L}. \end{cases} \tag{7}$$

There are some important features of the revenue function, which are summarized in the following corollary.

**Corollary 2** *Fixing the buyers' belief, the high type seller's revenue is increasing in $\gamma$, and the low type seller's revenue is decreasing in $\gamma$.*

### 4.1.1 Separating equilibrium

We start by analyzing the separating equilibria in this game. Let $\gamma_H^*$ and $\gamma_L^*$ be the seller's equilibrium refund policies when she is of high type and low type, respectively, and let $b^*(x;\gamma)$ be the buyers' equilibrium bidding function conditional on seeing the seller's refund policy $\gamma$.

In equilibrium, a low type seller provides $\gamma_L^*$, the buyers' belief is $\mu_L(\gamma) = 1$. According to (7), the seller gets revenue $\frac{(n-1)v_L}{n+1}$. If she deviates to $\gamma_H^*$, buyers will believe that $\mu_H(\gamma) = 1$. Her revenue, according to (7) with $\mu_H(\gamma) = 1$, is

$$R_L = \begin{cases} \frac{n-1}{n+1} v_H, & \text{if } \gamma \leq \frac{nv_L}{(n-1)v_H}; \\ \\ \frac{n-1}{n+1}(1-\gamma)v_H, & \text{if } \gamma > \frac{nv_L}{(n-1)v_H}. \end{cases}$$

Thus, if $\gamma \leq \frac{nv_L}{(n-1)v_H}$, a low type seller will deviate. If $\gamma > \frac{nv_L}{(n-1)v_H}$, we need an additional condition:

$$\begin{aligned} & \frac{n-1}{n+1}(1-\gamma)v_H \leq \frac{n-1}{n+1}v_L \\ \Leftrightarrow \quad & \gamma \geq 1 - \frac{v_L}{v_H}. \end{aligned}$$

Combining these two conditions, we have a necessary condition for a separating PBNE in this game:

$$\gamma \geq \max\{1 - \frac{v_L}{v_H}, \frac{nv_L}{(n-1)v_H}\} \tag{8}$$

It is easy to prove that it is also a sufficient condition; we just need to specify the out-of-equilibrium belief to be $\mu_H(\gamma) = 0$ for all off-equilibrium refund policy $\gamma$. We summarize these results in the following proposition:

**Proposition 4** *All separating equilibria in a first-price auction with refund are characterized by: a type $v_H$ seller providing a refund policy $\gamma_1$, where $\gamma_1$ satisfies (8); a type $v_L$ seller providing a refund policy $\gamma_L \neq \gamma_H$; a bidder observing $\gamma_H$ believes that the seller is of type $v_H$ and bids according to $b_1(x) = \frac{n-1}{n} v_H x$; a bidder observing any refund policy other than $\gamma_H$ believes that the seller is of type $v_L$ and bids according to $b_L(x) = \frac{n-1}{n} v_L x$.*

### 4.1.2 Pooling equilibrium

Now we consider the pooling equilibria, in which the two types of sellers choose the same refund policy, $\gamma_H^* = \gamma_L^* = \gamma^*$. Since buyers' beliefs must be correctly derived from the equilibrium strategy and the Bayes' rule when possible, they must assign probability $p_H$ to the seller being type $v_H$ when they see refund policy $\gamma^*$. Therefore, in any pooling equilibrium, from (6), the high type seller's revenue is

$$R_H = \begin{cases} \frac{n-1}{n+1}[p_H v_H + p_L v_L], & \text{if } \gamma^* \leq \frac{n v_L}{(n-1)p_H v_H + (n-1)p_L v_L}; \\ \frac{(n-1)p_H v_H}{(n+1)[1-\gamma^* p_L]}, & \text{if } \gamma^* > \frac{n v_L}{(n-1)p_H v_H + n p_L v_L}. \end{cases}$$

From (7), the low type seller's revenue is

$$R_L = \begin{cases} \frac{n-1}{n+1}[p_H v_H + p_L v_L], & \text{if } \gamma^* \leq \frac{n v_L}{(n-1)p_H v_H + (n-1)p_L v_L}; \\ \frac{(n-1)(1-\gamma^*)p_H v_H}{(n+1)[1-\gamma^* p_L]}, & \text{if } \gamma^* > \frac{n v_L}{(n-1)p_H v_H + n p_L v_L}. \end{cases}$$

If the low type seller deviates to the no refund policy, it guarantees her revenue at least $\frac{n-1}{n+1} v_L$ regardless the buyers' beliefs. Thus for a PBNE, a necessary condition is that there is no incentive to deviate to the no refund policy. If

$$\gamma^* \leq \frac{n v_L}{(n-1)p_H v_H + (n-1)p_L v_L}, \tag{9}$$

then this condition is always satisfied.

If $\gamma^* > \frac{n v_L}{(n-1)p_H v_H + n p_L v_L}$, then we require

$$\begin{aligned} \frac{(n-1)(1-\gamma^*)p_H v_H}{(n+1)[1-\gamma^* p_L]} &\geq \frac{n-1}{n+1} v_L \\ \Leftrightarrow \quad \gamma^* &\leq \frac{p_H v_H - v_L}{p_H v_H - p_L v_L} \end{aligned} \tag{10}$$

Combining these two conditions, we have

$$\frac{n v_L}{(n-1)p_H v_H + n p_L v_L} < \gamma^* \leq \frac{p_H v_H - v_L}{p_H v_H - p_L v_L} \tag{11}$$

This is also a sufficient condition for a pooling equilibrium; we just need to specify the buyers' out-of-equilibrium belief to be $v = v_L$ with probability one for any off-equilibrium refund policy; neither the high type or low type seller will deviate given this belief.

**Proposition 5** *All pooling equilibria in a first-price auction with refund belong to either a non-returning equilibrium or a returning equilibrium. In a non-returning equilibrium, both types of sellers provide a refund policy $\gamma^*$, where $\gamma^*$ satisfies (9); a bidder seeing $\gamma^*$ believes that the seller is of type $v_H$ with probability $p_H$ and bids according to $b(x) = \frac{n-1}{n}[p_H v_H + p_L v_L]\, x$; a bidder seeing a refund policy other than $\gamma^*$ believes that the seller is of type $v_L$ and bids according to $b(x) = \frac{n-1}{n} v_L x$; no winning bidder returns the object in this equilibrium.*

*In a returning equilibrium, both types of sellers provide a refund policy $\gamma^*$, where $\gamma^*$ satisfies (11); a bidder seeing $\gamma^*$ believes that the seller is of type $v_H$ with probability $p_H$ and bids according to $b(x) = \frac{n-1}{n} \frac{p_H v_H x}{p_H + p_L(1-\gamma^*)}$; a bidder seeing a refund policy other than $\gamma^*$ believes that the seller is of type $v_L$ and bids according to $b(x) = \frac{n-1}{n} v_L x$; a winning bidder returns the object if the seller is found to be of type $v_L$ in this equilibrium.*

## 4.2 Intuitive Refinement

Again, the Cho-Krep intuitive criterion cannot eliminate any separating equilibrium. A type $v_H$ seller is already getting the best possible payoff in the equilibrium and has no incentive to deviate to any other refund policy. Upon seeing any off-equilibrium refund policy, the equilibrium belief that the seller is of type $v_L$ survives the intuitive criterion. Therefore, all separating equilibria satisfy the intuitive criterion.

Similarly to the second-price auction, the Cho-Krep intuitive criterion eliminates all of the non-returning pooling equilibria and all but one returning pooling equilibria. Using a stronger version of the intuitive criterion, we can eliminate the remaining returning pooling equilibrium. Therefore, we have a similar proposition.

**Proposition 6** *In first-price auctions with refund, after applying the stronger version of the intuitive criterion, the remaining equilibria are all separating equilibria as defined in Proposition 4. All these equilibria result in the same outcome.*

# 5 Extension to Continuous Values

In this section, we relax the assumption of discrete seller type and instead assume that $v$ follows a uniform distribution on $[0, 1]$, with p.d.f. $g(v) = 1$ and c.d.f. $G(v) = v$ for $v \in [0, 1]$. For simplicity, we focus on second-price auctions. The first-price auctions have similar properties.

Let us first calculate the equilibrium bidding function when there is no signaling and no refund. Suppose that $v$ is common knowledge. Then in the symmetric bidding function, bidder $i$ bids $B(x_i) = vb(x_i)$, where $b(x_i)$ is independent of $v$, and is determined in the usual second-price, sealed-bid auction with no reserve price when $x_i$ follows p.d.f. $f(\cdot)$ and c.d.f. $F(\cdot)$. That is, $b(x_i) = x_i$. Therefore, when there is no signaling and no refund, the bidding function is given by $B(x_i) = vx_i$.

Now suppose that $v$ is common knowledge and there is refund. Because a bidder always pays less than his valuation in a second price auction, he will never return the object for refunds. As a result, he bids the same as if there were no refund policy, i.e., $B(x_i) = vx_i$.

## 5.1 Separating equilibria

Now suppose that $v$ is signaled by the refund policy function $\gamma = \gamma(v)$. Consider a completely separating equilibrium $\gamma(v)$.[1] From $\gamma(v)$, the bidders infer the value of $v$ correctly in an equilibrium, and therefore bidder $i$ bids the same as before, $B(x_i) = vx_i$. In equilibrium, no bidder will return the object because they pay lower than their "expected" valuation.

When the seller deviates, however, the winning bidder may return the object upon discovering the true $v$. Note that a seller does not benefit from signaling a value lower than the true value. This is because the bidders will bid lower, and when they discover the true value, they are happier. There is return in this case. Since the buyers bid lower, the seller's revenue is lower. If the seller signals a higher $v$, then there could be some returns and some benefits to the seller.

We shall argue that $\gamma(v)$ is discontinuous everywhere. Suppose not, and it is continuous in the interval $(v', v'')$. Consider the seller with type $v \in (v', v'')$. Suppose that the seller provides $\gamma(\tilde{v})$, where $\tilde{v}$ is locally higher than $v$. Then, according to the Bayes' rule, all buyers believe that the seller is of type $\tilde{v}$ and bid $\tilde{v}x_i$. The winner, say $i$, will return the object if $\gamma(\tilde{v})\tilde{v}y_1 \geq vx_i$, where $y_1$ denotes the highest $x$ among the rest of the bidders. Since this is a marginal deviation by the seller and $\gamma(v)$ is continuous on $(v', v'')$, $\tilde{v}$ is slightly larger than $v$. If $\gamma(\tilde{v})$ is bounded away from 1 (i.e. strictly less than 1), then NO ONE will return the object. This is because $y_1 < x_i$ and $\tilde{v}$ is very close to $v$. As a result, no return will happen and the seller's revenue is higher. However, since the type space is continuous, $\gamma(v)$ can not be discontinuous everywhere. This IMPLIES that there is no separating equilibrium in this signaling game!

We next argue that no equilibrium could be continuous and strictly monotone in any interval. Suppose not and assume that $\gamma(v)$ is continuous and strictly monotone in the interval $(v', v'')$. We have proved that for the valuations in this interval, it cannot be separating. This means there must exist other disjointed intervals of valuations which pool with some subset $(v'_0, v''_0) \subseteq (v', v'')$. Denote those intervals by $(v'_1, v''_1), (v'_2, v''_2), \cdots, (v'_T, v''_T)$. Since all the intervals, $\{(v'_i, v''_i)\}_{i=0}^{T}$, are disjoint, we can select the interval with the highest valuation, say $(v'^*, v''^*)$. Now pick one point from the interval, $\tilde{v} \in (v'^*, v''^*)$ and another point $\tilde{\tilde{v}}$, which is locally higher than $\tilde{v}$. If the seller with valuation $\tilde{v}$ truthfully provides a refund policy according to her type, no return will happen since $\tilde{v}$ is the highest possible valuation for refund policy $\gamma(\tilde{v})$. However, there is incentive for her to provide a refund policy assigned for type $\tilde{\tilde{v}}$ for the same reason as in the previous paragraph. We have the following proposition:

**Proposition 7** *There exists no completely separating equilibrium in the continuous-value second-price auction with refund. Furthermore, no refund signaling function $\gamma = \gamma(v)$ can be continuous and strictly monotone in any interval of $v$.*

[1] Here, we do not assume this function is increasing or decreasing.

## 5.2 Pooling Equilibria

We now consider pooling equilibria. Define $K(\gamma) = \frac{[b-(1-\gamma)a]-\sqrt{[b-(1-\gamma)a]^2-\gamma^2 b^2}}{\gamma^2}$, which is increasing in $\gamma$ with range $[a, b]$. The following lemma characterizes the equilibrium of a second-price auction with a refund policy. It will be used many times when we characterize the pooling equilibria.

**Lemma 4** *Suppose that the seller provides refund policy $\gamma$, and that a buyer's valuation is i.i.d. according to c.d.f. $F(\cdot)$, and that buyers believe that the seller's $v$ is uniformly distributed on $[a, b]$, with c.d.f. $H(v) = \frac{v}{b-a}$. If $\gamma \leq \frac{2a}{a+b}$, then all buyers adopt the same bidding function below:*

$$B(x) = \frac{a+b}{2}x,$$

*and no return happens.*

*If $\gamma \geq \frac{2a}{a+b}$, then all buyers adopt the same bidding function below:*

$$B(x) = K(\gamma)x,$$

*and the winning bidder, say buyer 1, returns the object iff $v < \gamma K(\gamma)\frac{y_1}{x_1}$, where $y_1$ denotes the highest $x$ among all other bidders.*

**Proof:** Suppose that in the auction every bidder adopts the bidding function $B(\cdot)$. Then in the return stage, the winner, say buyer 1, keeps the object if and only if $vx_1 \geq \gamma B(y_1)$, or equivalently, $v \geq \frac{\gamma B(y_1)}{x_1}$. We can formulate a buyer's problem in the auction as follows giving the above best action in the return stage:

$$\begin{aligned} \Pi(x_1, \tilde{x}) &= \int_a^{\tilde{x}} \int_{\min\{b, \max\{a, \frac{\gamma B(y_1)}{x_1}\}\}}^b [vx_1 - B(y_1)] dH(v) dF(y_1) \quad (12) \\ &- \int_a^{\tilde{x}} \int_b^{\min\{b, \max\{a, \frac{\gamma B(y_1)}{x_1}\}\}} (1-\gamma) B(y_1) dH(v) dF(y_1). \quad (13) \end{aligned}$$

The first order condition implies that

$$\int_{\min\{b, \max\{a, \frac{\gamma B(x_1)}{x_1}\}\}}^b [vx_1 - B(x_1)] dH(v) = \int_a^{\min\{b, \max\{a, \frac{\gamma B(x_1)}{x_1}\}\}} (1-\gamma) B(x_1) dH(v). \quad (14)$$

If $\frac{\gamma B(x_1)}{x_1} < a$, then from (14), we have

$$\int_a^b [vx_1 - B(x_1)]dH(v) = \int_a^a (1-\gamma)B(x_1)dH(v) \tag{15}$$

$$\Rightarrow B(x_1) = \int_a^b vx_1 dH(v) = \frac{a+b}{2}x_1 \tag{16}$$

However, to ensure condition $\frac{\gamma B(y_1)}{x_1} < a$ is satisfied, we require

$$\frac{\gamma B(x_1)}{x_1} < a \Leftrightarrow \frac{\gamma \frac{a+b}{2} x_1}{x_1} < a \Leftrightarrow \gamma < \frac{2a}{a+b} \tag{17}$$

If $\frac{\gamma B(x_1)}{x_1} > b$, then from (14), we have

$$\int_b^b [vx_1 - B(x_1)]dH(v) = \int_a^b (1-\gamma)B(x_1)dH(v) \tag{18}$$

$$\Rightarrow B(x_1) = 0 \tag{19}$$

However, this contradicts the condition.

If $a \leq \frac{\gamma B(x_1)}{x_1} \leq b$, then from (14), we have

$$\int_{\frac{\gamma B(x_1)}{x_1}}^b [vx_1 - B(x_1)]dH(v) = \int_a^{\frac{\gamma B(x_1)}{x_1}} (1-\gamma)B(x_1)dH(v)$$

$$\Rightarrow \frac{x_1}{2}\left[b^2 - \frac{{\gamma^*}^2 B(x_1)^2}{x_1^2}\right] - B(x_1)\left[b - \frac{\gamma^* B(x_1)}{x_1}\right] = (1-\gamma^*)B(x_1)[\frac{\gamma^* B(x_1)}{x_1} - a]$$

$$\Rightarrow {\gamma^*}^2 B(x_1)^2 - 2[b-(1-\gamma)a]x_1 B(x_1) + b^2 x_1^2 = 0$$

$$\Rightarrow B(x_1) = \frac{[b-(1-\gamma)a] \pm \sqrt{[b-(1-\gamma)a]^2 - \gamma^2 b^2}}{\gamma^2} x_1.$$

Since the bidding function should be increasing in $\gamma$, the solution above should be the 'minus' root only.

However, to ensure condition $a \leq \frac{\gamma B(x_1)}{x_1} \leq b$ is satisfied, we require

$$a \leq \frac{\gamma B(x_1)}{x_1} \leq b \Leftrightarrow \gamma \geq \frac{2a}{a+b} \tag{20}$$

Note that the conditions for $\gamma$ in (17) and (20) do not overlap, and together, they constitute the entire support of $\gamma$. Therefore, the equilibrium is unique for any value of $\gamma$. **Q.E.D**

### 5.3 Fully Pooling Equilibria

We first consider the fully pooling equilibria. What conditions a fully pooling equilibrium must satisfy in this game? Suppose that the seller of all types pools at refund policy $\gamma^* \in [0,1]$. We first examine the equilibrium belief. After seeing refund policy $\gamma^*$, the buyers' common belief is the prior; that is, $v$ follows $U[0,1]$. A buyer's bidding strategy can be derived directly from Lemma 4 by replacing $\gamma = \gamma^*$, $a = 0$, $b = 1$. This fully pooling equilibrium can be sustained by the following out-of-equilibrium belief: if a buyer sees any off-equilibrium refund policy, he believes that $v = 0$ with probability one. With this belief, a buyer will bid zero when he sees $\gamma \neq \gamma^*$. In this case, the seller gets zero revenue. We have the following proposition:

**Proposition 8** *In a fully pooling equilibrium, all types of sellers provide a refund policy $\gamma^*$. Upon seeing this refund policy, buyers in the auction bid according to*

$$B(x) = \frac{1}{1+\sqrt{1-{\gamma^*}^2}}x,$$

*and the winning bidder returns the object if and only if $v < \frac{\gamma^*}{1+\sqrt{1-{\gamma^*}^2}}\frac{y_1}{x_1}$, $x_1$ is the winner's $x$ and $y_1$ is the highest $x$ among the rest of the bidders. Upon seeing any other refund policy, the buyers believe that the seller is of type $v = 0$, and bid 0. In equilibrium, the seller's expected revenue is*

$$R(v) = \begin{cases} \frac{2}{1+\sqrt{1-{\gamma^*}^2}} \int_{\underline{x}}^{\overline{x}} \xi\left[1-F(\xi)\right] dF(\xi)^{n-1}, & \text{if } v \geq \frac{\gamma^*}{1+\sqrt{1-{\gamma^*}^2}}; \\ \frac{2}{1+\sqrt{1-{\gamma^*}^2}} \int_{\underline{x}}^{\overline{x}} \xi\left[1-F(\xi)\right] dF(\xi)^{n-1} & \\ \quad - \frac{2\gamma^*}{1+\sqrt{1-{\gamma^*}^2}} \int_{\underline{x}}^{\overline{x}} \xi \left[F\left(\frac{\gamma^*}{v(1+\sqrt{1-{\gamma^*}^2})}\xi\right) - F(\xi)\right] dF(\xi)^{n-1}, & \text{if } v \leq \frac{\gamma^*}{1+\sqrt{1-{\gamma^*}^2}}. \end{cases} \tag{21}$$

The calculations for the above seller's revenue is straightforward. When the seller's $v$ is high enough, no return would happen; otherwise, some return will occur.

We can examine how a stronger equilibrium concept can eliminate some of the equilibria. Take the concept of sequential equilibria, for example. We argue that any fully pooling equilibrium other than fully pooled at the full refund policy cannot be a sequential equilibrium. This is because when full refund policy is provided, bidder $i$ will always bid up to $x_i$, as long as there is some chance for the seller to be type $v = 1$. Thus, by providing a full refund policy, the seller with type $v = 1$ earns expected revenue

$$\tilde{R}(1) = 2\int_{\underline{x}}^{\overline{x}} \xi\left[1-F(\xi)\right] dF(\xi)^{n-1} \tag{22}$$

Comparing (21) and (22), if $\gamma^* < 1$, we conclude that the seller with type $v = 1$ will deviate. Therefore, only $\gamma^* = 1$ can be supported. This result is summarized in the following proposition.

**Proposition 9** *All types of sellers pooling at the full refund policy $\gamma = 1$ is the unique fully pooling sequential equilibrium.*

### 5.3.1 Partially Pooling Equilibria

We now consider the partially pooling equilibria in this game. Since this kind of equilibrium could take many forms, we will focus on the following partially pooling equilibria: a seller of type $v \in [0, v^*]$ provides no refund, i.e. $\gamma = 0$; a seller of type $v \in [v^*, 1]$ provides full refund, i.e. $\gamma = 1$.

For $v \in [0, v^*]$, if the seller follows the equilibrium and provides $\gamma = 0$, the buyers then believe that his type is uniformly distributed on $[0, v^*]$, and therefore, from Lemma 4, bid according to $B(x) = \frac{v^*}{2}x$ and no return happens. The seller's revenue is given by

$$v^* \int_{\underline{x}}^{\overline{x}} \xi[1 - F(\xi)]dF(\xi)^{n-1}.$$

Note that the seller's revenue does not depend on the seller's true type $v$. This is because a bidder's bid does not depend on $v$. Furthermore, no return will occur.

If this seller deviates and provides full refund, then buyers believe that her type is uniformly distributed on $[v^*, 1]$ and bid according to $B(x) = x$. The winning bidder, say buyer 1, returns when $vx_1 \leq y_1$, where $y_1$ is the highest $x$ among other bidders. This deviated seller's revenue is given by

$$\begin{aligned} & 2\int_{\underline{x}}^{\overline{x}} \xi[1 - F(\xi)]dF(\xi)^{n-1} - 2\int_{\underline{x}}^{\overline{x}} \xi[F(\xi/v) - F(\xi)]dF(\xi)^{n-1} \\ = \; & 2\int_{\underline{x}}^{\overline{x}} \xi[1 - F(\frac{\xi}{v})]dF(\xi)^{n-1}, \end{aligned}$$

which is increasing in the seller's type $v$. Therefore, the seller with type $v^*$ has the highest incentive to deviate.

For $v \in [v^*, 1]$, if the seller follows the equilibrium and provides $\gamma = 1$, buyers believe that her type is uniformly distributed on $[v^*, 1]$. Therefore, from Lemma 4, the buyers bid according to $B(x) = x$, and the winning bidder returns the object when $vx_1 \leq y_1$. The seller's revenue is given by

$$2\int_{\underline{x}}^{\overline{x}} \xi[1 - F(\frac{\xi}{v})]dF(\xi)^{n-1},$$

which is increasing in the seller's type $v$.

If the seller deviates and provides no refund, then buyers believe that her type is uniformly distributed on $[0, v^*]$ and bid according to the function $B(x) = \frac{v^*}{2}x$. In this case, no return happens, and the seller's revenue is given by

$$v^* \int_{\underline{x}}^{\overline{x}} \xi[1 - F(\xi)]dF(\xi)^{n-1},$$

which does not depend on the seller's type $v$. Therefore, a seller of type $v^*$ has the highest incentive to deviate.

From the above analysis, we conclude that for an equilibrium to exist, the following condition must be satisfied:

$$2\int_{\underline{x}}^{\overline{x}} \xi[1 - F(\frac{\xi}{v^*})]dF(\xi)^{n-1} = v^* \int_{\underline{x}}^{\overline{x}} \xi[1 - F(\xi)]dF(\xi)^{n-1}. \tag{23}$$

To punish deviations to any other refund policy, we specify the following out-of-equilibrium belief: seeing any off-equilibrium refund policy, a buyer believes that the seller is of type $v = 0$ and bids 0. We have the following proposition:

**Proposition 10** *There exists a partially pooling equilibrium with the following properties. A seller of type $v \in [0, v^*]$ provides no refund, $\gamma = 0$; a seller of type $v \in [v^*, 1]$ provides full refund, $\gamma = 1$. This $v^*$ is determined by (23).*

*Seeing $\gamma = 0$, buyers bid according to $B(x) = \frac{v^*}{2}x$, and no return happens.*

*Seeing $\gamma = 1$, buyers bid according to $B(x) = x$, and the winning bidder returns the object if and only if $v < \frac{y_1}{x_1}$. Buyers' belief along the equilibrium path is implied by the equilibrium strategy, and their beliefs off the equilibrium path is that the seller is of type $v = 0$ and they bid 0.*

Example. When there are 2 buyers, $n = 2$, $x_i$ follows $U[0, 1]$, the cutoff $v^*$ determined by (23) becomes

$$\begin{aligned}
& 2\int_0^{v^*} \xi(1 - \frac{\xi}{v^*})d\xi = v^* \int_0^1 \xi[1 - \xi]d\xi \\
\Rightarrow \quad & 2(\frac{{v^*}^2}{2} - \frac{{v^*}^3}{3v^*}) = v^*(\frac{1}{2} - \frac{1}{3}) \\
\Rightarrow \quad & \frac{{v^*}^2}{3} = \frac{{v^*}^2}{6} \\
\Rightarrow \quad & v^* = \frac{1}{2}.
\end{aligned}$$

Therefore, in the partially pooling equilibrium we analyzed above, a seller of type $v$ lower than 0.5 will provide no refund, and a seller of type $v$ higher than 0.5 will provide full refund.

# 6 Conclusions

In this paper, we analyze refund policies as a signaling device in first-price and second-price auctions. When the seller's value follows a two-point distribution, we characterize all equilibria of the auction game. A strong intuitive criterion selects the separating equilibria and they all yield the same (completely revealing) equilibrium outcome. When the seller's value follows a continuous distribution, we show that a completely separating equilibrium does not exist, but characterize all fully pooling and one partially pooling equilibria. (To be expanded.)

# 7 References

To be added.