Subjective Evaluations with Performance Feedback*

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Abstract

This paper models two key roles of subjective performance evaluations: their incentive role and their feedback role. The latter is important in real world firms but largely overlooked by economists. The paper shows that in contrast to received wisdom, interaction between these two roles makes subjective pay feasible even in a finite horizon relationship. The optimal contract depends critically upon the existence and quality of a verifiable performance measure. Furthermore, when commitment to a forced distribution of evaluations is possible, it is valuable only if the firm employs many workers or if the verifiable measure is poor.

Keywords: Subjective Evaluations, Performance Feedback, Optimal Incentive Contracts

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1. Introduction

Most workers are regularly evaluated by their superiors and these evaluations typically include the supervisor's subjective judgement about the worker's performance. For example, Gibbs et al (2007) document the use of subjective performance evaluations in the compensation packages of auto dealership managers; Levin (2003) cites survey evidence of subjective performance pay in law firms; and Eccles and Crane (1988) describe how investment bankers' pay depends on such subjective measures as the quality of their deals and customer satisfaction. Even the pay of CEOs often depends on the board of directors' subjective assessment of the CEO's performance (Bushman et al, 1996; Hayes and Schaefer, 1997), despite the fact that a top executive's contribution to the company value is typically easier to capture by objective performance measures, such as stock price and accounting profits, than the contributions of lower level employees.

The human resources management literature has documented that in most companies, performance evaluations serve multiple goals (Cleveland et al, 1989). Two of the most important purposes are provision of incentives and performance feedback. For example, Cleveland et al (1989) report that 69% of their survey respondents considered salary administration and 53% considered performance feedback to be among the three main purposes of performance appraisals. In the economics literature, the incentive role of subjective evaluations has been studied extensively (e.g. MacLeod and Malcomson, 1989; Baker et al, 1994; Levin, 2003), but their feedback role, although recognized (Milgrom and Roberts, 1992, p. 407; Prendergast, 2002), has been largely missing from formal models.[1]

This paper spotlights the feedback role of evaluations by incorporating it into a two-period principal-agent model in which subjective evaluations both motivate workers and provide them with information about their productive abilities. The paper shows that the feedback and the incentive roles of evaluations can interact in an important way and that absent reputational concerns, performance feedback is key to making the incentive part of the evaluations operational.

A well recognized problem with subjective pay, which seriously undermines its incentive effects, is that supervisors are tempted to underreport the workers' performance in order to save on labor costs. Gibbons (2005) argues that this reneging/commitment problem is at least as important for understanding real world contracts as the more frequently studied trade-off between incentives and insurance. Accordingly, the focus in the related economics literature has been on exploring how firms can make subjective pay functional despite this trust problem. Most of the existing work follows Bull (1987) in emphasizing the supervi-

[1] A recent exception is Ederer's (2009) model of feedback in tournaments, discussed in greater detail later.

sors' reputational concerns in infinitely repeated games. The received wisdom is that the reneging problems make stationary contracts with subjective pay ineffective (Levin, 2003; Prendergast, 2002).

I show that the feedback role of performance assessments mitigates the reneging problem and makes truthful subjective schemes feasible even without infinite interaction. Central to this conclusion are two ideas. The first is that ability and effort are complementary in production. Although in some jobs (say, janitorial jobs) ability may have no bearing on the productivity of one's effort, most jobs are likely to exhibit some degree of complementarity between effort and ability. The second central idea is that feedback from a supervisor allows a worker to update his belief about his productive ability. Because under complementarity the worker's subsequent performance depends on this belief, the employer has an incentive to give the worker a good evaluation, in order to boost his effort. A properly designed reward scheme then balances the supervisor's desire to inflate the worker's self-assessment against her temptation to save on labor costs by under-reporting. This makes honest evaluations feasible without resorting to infinite interaction arguments.

The starting point of the model is the observation that firms use subjective evaluations for the lack of perfect objective measures of performance.[2] A worker's contribution to firm value is frequently complex and hard to capture by an objective measure. Consequently, when objective measures are imperfect and could lead to dysfunctional behavior, firms complement them with subjective schemes in which a worker's salary, bonus, or promotion depend upon his superior's perception of his performance. This point has been recognized by many writers (e.g., Baker et al., 1994; Prendergast, 1999) and goes back at least to Alchian and Demsetz's (1972) classic theory of the firm.[3]

In line with the above, I assume that an objective (verifiable) performance measure is available, but it is imperfect. The exact source of contracting imperfections is not important for the paper's main conclusions, but for concreteness, I assume that they are due to multitasking problems similar to those studied in Feltham and Xie (1994), Datar et al (2001), and Baker (2002).[4] Specifically, the objective performance measure distorts an agent's allocation of effort across tasks because it aggregates his individual efforts in a manner that differs from his contribution to the firm value. In this setting, subjective evaluations are useful because

[2] In their study of auto dealership manager incentive systems, Gibbs et al (2007) find that the dealerships use implicit evaluations and rewards as a response to flaws in the available performance measures.

[3] Alchian and Demsetz argue that the lack of good objective measures for workers' individual contributions is the very reason firms exist. The firm, in their view, is a device that allows some individuals to specialize in observing workers' performance and in rewarding them according to their marginal contribution to joint output.

[4] An alternative way to introduce contractual frictions would be to assume that the objective measure is distorted a la Baker (1992) and Baker et al (1994).

they are based on an undistorted measure; in fact, the principal would ideally like all of the agent's incentives to derive from subjective pay. I show that this is in general not possible, as the principal's freedom to design the contract is constrained by the need to ensure that the evaluations are truthful. Nevertheless, at least some incentives derived from subjective pay are always feasible, as long as the objective measure is not completely worthless. If the objective measure is worthless, then truthful subjective evaluations are feasible, but they do not provide any incentives.

An optimal contract in this environment arises from a mechanism design problem in which the principal faces her own, rather than the agent's, truthtelling constraint. Consequently, the contract is not shaped by the standard trade-off between rent extraction and allocation efficiency. Rather, the trade-off is between the efficiency of the incentives provided by the objective measure in the second period and the efficiency of the incentives provided by subjective pay in the first period. In particular, for the subjective evaluations to provide any incentives, the objective part of the second period contract must necessarily be distorted away from the optimal shape it would have in the absence of subjective pay. This trade-off limits the usefulness of the subjective evaluations, preventing the optimal contract from achieving full efficiency in the first period.

The above conclusion is dramatically changed when one allows the firm to pre-commit to a specific distribution of evaluations, a scheme that is often referred to as a "forced distribution." Such forced distributions are common in real world firms; one well known example is GE's "vitality curve." In the present setting, the advantage of a forced distribution is that it relaxes the principal's truthtelling constraint by making the size of the wage bill independent of individual evaluations.[5] This eliminates the above tradeoff and allows the principal to achieve full efficiency in the first period by completely replacing the objective measure with subjective pay.

The truthtelling benefit of forced distributions does not come for free, however. A forced distribution limits the amount of information that the evaluations convey about the workers' productive capacities, which impedes the workers' ability to tailor their second period efforts to their productivities. Crucially, this constraint gets more stifling the smaller is the number of workers. The number of workers is therefore of central importance in the choice between subjective evaluations with and without a forced distribution: When the workforce is large, a forced distribution can closely approximate the true distribution of the workers' productivities and is therefore very informative. In this case, the efficiency gain from improved first period incentives outweighs the loss from the misallocation of the second period effort.

[5] This is similar to the benefit of tournaments pointed out by Malcomson (1984).

In contrast, when the number of workers is small, the choice between the two types of subjective pay schemes turns on the quality of the objective measure. If the objective measure is good, the main benefit of subjective evaluations is to inform the workers about their productive abilities, which favors subjective evaluations without a forced distribution. If the objective measure is poor, then it provides very inefficient incentives even if the workers are fully informed about their abilities. In this case, the main goal is to improve the workers' incentives, which is best achieved via evaluations with a forced distribution.

Related literature. In its focus on the interaction between the incentive effects of objective and subjective measures, this paper is related to Baker et al (1994), who were the first to formally model such an interaction. In Baker et al, however, subjective pay is sustained through infinite interaction in a repeated game, whereas mine is a finite horizon model. Furthermore, the feedback function of subjective evaluations, so prominent in the current model, plays no role in their analysis.

MacLeod (2003) has generalized the logic of the repeated game models by showing that subjective pay schemes can be feasible even without infinite interaction if workers can punish a deviation from the implicit contract by imposing on the employer some type of socially wasteful cost, say, through quitting or sabotage at the firm. The optimal contract then trades off ex post socially wasteful conflict against ex ante performance incentives. This model was further developed by Fuchs (2007), who extended it to a more dynamic environment. The present paper complements the MacLeod/Fuchs theory by suggesting an alternative mechanism for sustaining subjective evaluations, that does not require ex post destruction of surplus. Further, it points to the availability of an objective measure as a crucial determinant of feasibility of subjective pay and to the number of workers as a determinant of whether the subjective pay scheme will include a forced distribution. Finally, where Fuchs concludes that in his setting it is important for incentive purposes not to give the agent interim feedback about his performance, the current paper provides a framework in which interim feedback is vital. This accords well with the evidence that providing feedback to workers is one of the most important reasons companies use subjective evaluations.

One recent paper that models the feedback role of evaluations in a setting with heterogeneous agents is Ederer (2009), who studies the effects of performance feedback on the strategic behavior of contestants in a multi-stage tournament. In Ederer's model, feedback does not always affect effort in a desirable way. Consequently, the main question in his analysis is whether or not feedback should be provided and the answer turns on the precise shape of the contestants' marginal costs of effort. Also, Ederer assumes that the principal has full commitment power, which eliminates the problem of inducing the principal to reveal the agents' performance truthfully. In the present model, feedback is always useful and the focus is on the feasibility of truthful subjective evaluations, on how subjective and objective measures interact in the optimal contract, and on the benefits and drawbacks of using a forced distribution of evaluations.

Finally, the model of this paper is also formally related to Hermalin (1998), who shows that a leader of a team can use a contract with side payments to credibly communicate to the other team members her superior information about the productive state of the world. However, other issues central to the present paper, such as the quality of the objective measures of performance, the effects of the messages on the efforts in previous periods, and the usefulness of forced distributions of messages do not arise in Hermalin's setting.

The plan for the rest of the paper is as follows. Section 2 describes the model and the main assumptions. Section 3 contains an analysis of the case without a forced distribution of evaluations. It provides conditions under which subjective pay is feasible, a result regarding the efficiency of subjective pay schemes, and a characterization of the optimal contract. This section also discusses the role of commitment for the feasibility of subjective pay, as well as the possibility that supervisors dislike giving bad evaluations. Section 4 allows for evaluations with a forced distribution and compares the benefits and disadvantages of the two types of subjective schemes. Section 5 concludes.

2. The Model

Production technology. A principal (she) supervises an agent/worker (he) over two periods, $t = 1, 2$. The worker's output in period t is $y_t \in \{0, 1\}$. The probability of the high output $y_t = 1$ is given by $q_t = a\mathbf{e}_t{\cdot}\mathbf{f}$, where a is the worker's innate time-invariant ability, $\mathbf{e}_t \in \mathbb{R}^K_+$, $\mathbf{e}_t = (e_{1t}, e_{2t}, ..., e_{Kt})$, is his K-dimensional vector of efforts provided in period t, and $\mathbf{f} = (f_1, f_2, ..., f_K) \in \mathbb{R}^K_+$ is the vector of marginal contributions of the worker's efforts to firm value. A key feature of this specification is that ability and effort are complements in the production function. As already noted, in many settings this seems to be a natural assumption about the interaction between ability and effort.

The worker's ability is initially unknown. Both the worker and the principal only know that the ability is drawn from an interval $[0, \bar{a}]$ according to a distribution function $H(a)$ with density $h(a)$, which is positive and twice differentiable at each a.[6]

Output measures. Neither the worker's expected contribution to firm value, q_t, nor its realization, y_t, are contractible. Instead, the worker's incentives come from two alternative sources.

[6] It would be easy to adapt the model so that a represents human capital that the worker develops during the first period.

Objective measures. First, as in Feltham and Xie (1994), Datar et al (2001), and Baker (2002), there are imperfect but contractible measures of the worker's first and second period performance, $z_t \in \{0,1\}$, $t = 1,2$. The probability that $z_t = 1$ is equal to $p_t = a\mathbf{e}_t{\cdot}\mathbf{g}$, where $\mathbf{g} = (g_1, g_2, ..., g_K) \in \mathbb{R}_+^K$ is the vector of marginal impact of the worker's efforts on the measures z_1 and z_2. The verifiable measures z_t are imperfect in the sense that $\mathbf{g} \neq \alpha\mathbf{f}$ for any constant α. This makes it impossible for a contract based on z_t to induce the vector of efforts that maximizes the firm's value. To ensure that p_t and q_t can be interpreted as probabilities, assume $\max\{\bar{a}f_k, \bar{a}g_k\} < 1$ for all $k = 1, 2, ..., K$.

Subjective measure. At the end of period t, the principal privately observes the worker's expected contribution to the firm's period-t value, $q_t = a\mathbf{e}_t{\cdot}\mathbf{f}$.[7] This specification captures the idea, long present in the economics literature, that by the nature of her job, a supervisor has superior information about the worker's contribution to the firm's value: "The employer, by virtue of monitoring many inputs, acquires special superior information about their productive talents" (Alchian and Demsetz, 1972, p. 793).[8] Alternatively, one could think of a as representing the quality of the principal's project and q_t as the principal's private signal about this quality.

To ease the exposition, it will be assumed that z_1, z_2, y_1, and y_2 only become observable at the end of period 2. If z_1 and/or y_1 were observable at the end of period 1, the worker would use them to update his belief about his ability, but this would not allow him to fully infer a. Since the analysis will not depend on the exact functional form of the worker's prior belief $H(a)$, such an updating plays no important role and can be ignored.

Subjective evaluations and contracting. After privately observing the worker's first period performance, the supervisor provides him with a subjective evaluation, which consists of a message $m \in \mathbb{R}^+$ about the value of the worker's ability a.[9] This message is contractible, so that the worker's wage, w, can be written as $w = w(z_1, z_2, m)$. The possibility that m is not contractible will be discussed in Subsection 3.5.

In the first part of the paper, m will be the only contractible part of the subjective

[7] This stark informational structure is adopted for its simplicity. The paper's main conclusions would continue to hold if both the principal and the worker received imperfect signals about the worker's ability, as long as the principal's signal contains some additional information not contained in the worker's signal; i.e., as long as the worker's signal is not a sufficient statistic for the principal's signal with respect to a.

[8] Early formalizations of the idea that employers have private information about their workers' productivities include Hart (1983) and Hall and Lazear (1984). The informative value of a supervisor's feedback is also recognized in the human resources management literature (e.g., Ashford, 1986).

[9] Alternatively, the message could be about the agent's first period contribution q_1. The current formulation simplifies the exposition of some of the proofs.

Although the worker's ability remains the same in both periods, the supervisor could, in principle, provide a subjective evaluation also in the second period. However, for standard reasons, truthful subjective evaluation is not feasible in the last period.

evaluation scheme. The second part of the paper will consider subjective evaluations with a forced distribution, where not only m but also the resulting distribution of m is contractible.

Preferences. Both the principal and the worker are risk neutral and do not discount future income. The principal's goal is to maximize the firm's expected profit. The worker's per period reservation utility from not working is u, normalized to $u = 0$, and his lifetime utility from being employed by the firm is $w - \Psi(\mathbf{e}_1) - \Psi(\mathbf{e}_2)$, where $\Psi(\mathbf{e}_t) = \sum_{k=1}^{K} \psi(e_{kt}) = \sum_{k=1}^{K} e_{kt}^2/2$ is the worker's disutility from effort in period $t = 1, 2$. It will be assumed that the worker's participation constraint only needs to be satisfied at the beginning of the relationship, when the contract is signed.

Timing. At the beginning of the first period, the principal and the worker sign a contract that specifies the wage function $w(z_1, z_2, m)$. Subsequently, the worker chooses his first period effort levels, $\mathbf{e}_1$. At the end of the first period, the principal observes the worker's first period input q_1 and provides the performance evaluation m. At the beginning of the second period, the worker updates his belief about his own ability and exerts second period efforts $\mathbf{e}_2$. At the end of the second period, z_1 and z_2 are observed and the worker is paid $w(z_1, z_2, m)$.

3. The Analysis

The first goal of the analysis is to explore the conditions under which truthful subjective evaluations that provide incentives are feasible. In this inquiry, it will be convenient to write the worker's expected pay in terms of a base salary and "bonuses." Let $w_{ij}(m) \equiv w(i, j, m)$, i.e. $w_{ij}(m)$ is the worker's wage as a function of m, conditional on $z_1 = i$ and $z_2 = j$, where $i, j \in \{0, 1\}$. Then at the beginning of period 1, the worker's expected wage is

$$\begin{aligned} E(w) &= p_1[p_2 w_{11}(m) + (1 - p_2) w_{10}(m)] + (1 - p_1)[p_2 w_{01}(m) + (1 - p_2) w_{00}(m)] \\ &= s(m) + b_1(m) p_1 + b_2(m) p_2 + b_3(m) p_1 p_2 \\ &= s(m) + b_1(m) p_1 + \beta(m) p_2, \end{aligned}$$

where $s(m) = w_{00}(m)$ is the worker's base salary, and $b_1(m) \equiv w_{10}(m) - w_{00}(m)$, $b_2(m) \equiv w_{01}(m) - w_{00}(m)$, and $b_3(m) \equiv w_{11}(m) - w_{01}(m) - w_{10}(m) + w_{00}(m)$ are the bonuses. Note that b_2 and b_3 matter only through the composite bonus $\beta(m) \equiv b_2(m) + b_3(m) p_1$.

3.1. The worker's problem

Let $x(m)$ be the worker's posterior belief about his ability based on his subjective evaluation m. The worker's second period problem is then

$$\max_{e_2} \beta(m)x(m)\mathbf{e}_2{\cdot}\mathbf{g} - \Psi(\mathbf{e}_2),$$

which determines his second period efforts $e_{k2}(\beta, x)$ through

$$\beta(m)x(m)g_k = \psi'(e_{k2}), \;\; k = 1, 2, ..., K.$$

Although the principal does not observe the worker's first period efforts, she makes a conjecture about them, $\tilde{\mathbf{e}}_1$. She then combines this conjecture with her observation of $q_1 = a\mathbf{e}_1{\cdot}\mathbf{f}$ to infer the worker's ability as $a = \frac{q_1}{\tilde{\mathbf{e}}_1 \cdot \mathbf{f}}$. Using the revelation principle to focus on truthtelling contracts, the principal's equilibrium message will be $m = \frac{q_1}{\tilde{\mathbf{e}}_1 \cdot \mathbf{f}}$, which will allow the worker to infer his true ability via $x(m) = m\frac{\tilde{\mathbf{e}}_1 \cdot \mathbf{f}}{\mathbf{e}_1 \cdot \mathbf{f}}$. In equilibrium, the principal's conjecture will be correct, $\tilde{\mathbf{e}}_1 = \mathbf{e}_1$. The first set of incentive compatibility constraints for the principal's general optimization problem is thus obtained as

$$\beta(m)mg_k = \psi'(e_{k2}), \;\; k = 1, 2, ..., K. \tag{ICW$_2$}$$

The key thing to notice here is that, holding $\beta(m)$ fixed, each component of the worker's vector of efforts increases in his belief $x(m)$ and hence in the principal's evaluation m.

In period 1, the worker chooses his efforts so as to maximize his expected lifetime utility

$$E_a[s + b_1(m)a\mathbf{e}_1{\cdot}\mathbf{g} + \beta(m)a\mathbf{e}_2{\cdot}\mathbf{g} - \Psi(\mathbf{e}_2)] - \Psi(\mathbf{e}_1),$$

taking into account the effect of $\mathbf{e}_1$ on the principal's report m. In particular, for any first period effort vector $\hat{\mathbf{e}}_1$, the worker expects evaluation $m(\hat{\mathbf{e}}_1) = \frac{a\hat{\mathbf{e}}_1 \cdot \mathbf{f}}{\tilde{\mathbf{e}}_1 \cdot \mathbf{f}}$. This yields the second incentive compatibility constraint for the worker:

$$\mathbf{e}_1 \in \arg\max_{\hat{\mathbf{e}}_1} E_a[s(\frac{a\hat{\mathbf{e}}_1{\cdot}\mathbf{f}}{\tilde{\mathbf{e}}_1{\cdot}\mathbf{f}})+b_1(\frac{a\hat{\mathbf{e}}_1{\cdot}\mathbf{f}}{\tilde{\mathbf{e}}_1{\cdot}\mathbf{f}})a\hat{\mathbf{e}}_1{\cdot}\mathbf{g}+\beta(\frac{a\hat{\mathbf{e}}_1{\cdot}\mathbf{f}}{\tilde{\mathbf{e}}_1{\cdot}\mathbf{f}})a\mathbf{e}_2{\cdot}\mathbf{g}-\Psi(\mathbf{e}_2)]-\Psi(\hat{\mathbf{e}}_1) \tag{ICW$_1$}$$

3.2. The principal's problem

The principal's subjective evaluation m maximizes her expected second period profit subject to the worker's incentive compatibility constraint (ICW$_2$). Combined with the requirement of truthtelling, this yields the following incentive compatibility constraint for the principal:

$$\begin{aligned} a &\in \arg\max_m a\mathbf{e}_2\cdot\mathbf{f} - \beta(m)a\mathbf{e}_2\cdot\mathbf{g} - s(m) - b_1(m)a\mathbf{e}_1\cdot\mathbf{g} \\ \beta(m) &= b_2(m) + b_3(m)a\mathbf{e}_1\cdot\mathbf{g}. \end{aligned} \tag{ICP}$$

The principal's general problem is then to maximize the total surplus from the employment relationship according to the following program:

$$\text{(P)} \qquad \max_{s(m),b_1(m),b_2(m),b_3(m)} E_a[a\mathbf{e}_1\cdot\mathbf{f} - \Psi(\mathbf{e}_1) + a\mathbf{e}_2\cdot\mathbf{f} - \Psi(\mathbf{e}_2)]$$

subject to (ICW_1), (ICW_2), and (ICP).

The first result provides conditions under which truthful subjective evaluations with incentive effects are feasible.

Proposition 1. *Let θ be the angle between $\mathbf{f}$ and $\mathbf{g}$.*

(i) *If $\cos\theta = 0$, no subjective evaluation scheme with incentive effects is feasible.*

(ii) *If $\cos\theta > 0$, then a subjective scheme that is both truthful and provides incentives for first period effort is feasible.*

Proof: All omitted proofs are in Appendix A.

In the existing literature, subjective evaluations with incentive effects are feasible only if the principal and the agent are engaged in a repeated interaction (e.g., Baker et al, 1994) or if agents can take ex post inefficient actions that destroy surplus (MacLeod, 2003; Fuchs, 2007). Proposition 1 shows that neither repeated interaction nor surplus destruction are needed for subjective evaluations to have incentive effects.

What drives this result is that, in contrast to the earlier models of subjective pay, the subjective measure of the worker's performance depends not only on the worker's actions, but also on his underlying type (ability). This presents the principal with two countervailing temptations. On the one hand, she is tempted to give the worker a bad evaluation, in order to save on the wage bill. This is the standard consideration, extensively studied in the previous literature. On the other hand, the principal knows that as long as the worker provides some effort in the second period, this effort will increase in m. This tempts her to boost the worker's self-assessment through a good evaluation. A truthtelling wage scheme then balances these two temptations in such a way that they offset each other.

Critically, the second effect is only present if the worker's output in period 2 depends on m. It is therefore important that the objective measure is not completely useless — otherwise, the worker provides no valuable effort in period 2 and there is no point to influencing his belief. In such a case, evaluations can be truthful only if they do not affect the worker's pay, which means that they cannot have any incentive effect. This is why the result in Proposition 1 depends on $\cos\theta$. As shown by Baker (2002), $\cos\theta$ captures the degree of congruence between the performance measure and the firm value: the larger is $\cos\theta$, the less distorted is the measure. In one extreme, $\cos\theta = 1$ and the measure z_t is perfectly aligned with y_t. This case is precluded here by the assumption that $\mathbf{g} \neq \alpha\mathbf{f}$ for any constant α.

In the other extreme, $\cos\theta = 0$ and the performance measure elicits no valuable effort. This can be seen most clearly from the expression for the firm's second period expected revenue: After substituting from (ICW$_2$), this revenue is $E(TR_2) = a\mathbf{e}_2^*{\cdot}\mathbf{f} = m\beta(m)a\,\|\mathbf{g}\|\,\|\mathbf{f}\|\cos\theta$, where $\|\mathbf{g}\|$ and $\|\mathbf{f}\|$ represent the lengths of the vectors $\mathbf{g}$ and $\mathbf{f}$ respectively.[10] Hence, $E(TR_2) = 0$ if $\cos\theta = 0$. Consequently, (ICP) can hold only if the worker's expected wage, $E(w)$, is independent of m, which leads to part (i) in the proposition.

3.3. Limits on efficiency

Constraint (ICW$_2$) makes it clear that full efficiency cannot be achieved with respect to the second period efforts. This is because $\mathbf{e}_2$ is induced only by the objective measure z_2, which provides distorted incentives: actual efforts are proportional to $\mathbf{g}$, whereas efficient efforts are proportional to $\mathbf{f}$. The best the principal can possibly do in period 2 is to set $\beta = \beta^{SB}$, where $\beta^{SB} \equiv \frac{\mathbf{f}\cdot\mathbf{g}}{\|\mathbf{g}\|^2} = \frac{\|\mathbf{f}\|}{\|\mathbf{g}\|}\cos\theta$ is the second best bonus, i.e. the bonus that maximizes the one-period surplus $a\mathbf{e}_t{\cdot}\mathbf{f} - \Psi(\mathbf{e}_t)$ subject to (ICW$_2$).[11] The benchmark bonus β^{SB} will prove useful later, in characterizing the optimal contract. For future reference, note that β^{SB} does not depend on a.

What about the first period efforts? One benefit of subjective evaluations is that in period 1, incentives from the distorted measure z_1 are at least partly replaced by incentives from the undistorted measure m. Does this imply that the optimal contract will elicit the first best vector of efforts $\mathbf{e}_1^{FB}$?[12] As the next result shows, the answer is No.

Proposition 2. *The optimal contract elicits vectors of efforts $\mathbf{e}_1^*$ and $\mathbf{e}_2^*$ such that $\mathbf{e}_1^* \neq \mathbf{e}_1^{FB}$ and $\mathbf{e}_2^* \neq \mathbf{e}_2^{FB}$.*

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[10] That is, $\|\mathbf{g}\| = \sqrt{\sum_{k=1}^{K} g_k^2}$ and $\|\mathbf{f}\| = \sqrt{\sum_{k=1}^{K} f_k^2}$.

[11] As is known from the multitasking literature (e.g., Baker, 2002), the term $\frac{\|\mathbf{f}\|}{\|\mathbf{g}\|}$ in β^{SB} reflects scaling, i.e., it accounts for the fact that the length of the vector $\mathbf{g}$ in general differs from the length of the vector $\mathbf{f}$. The term $\cos\theta$ reflects the quality of the performance measure, as discussed earlier.

[12] Vector $\mathbf{e}_2^{FB}$ is defined by $\psi'(e_{k2}^{FB}(a)) = af_k$. Because a is not known when $\mathbf{e}_1$ is chosen, $\mathbf{e}_1^{FB}$ is defined by $\psi'(e_{k1}^{FB}) = E(a)f_k$.

Even though a subjective scheme that elicits the efficient efforts in period 1 might be feasible, Proposition 2 says that the principal will not find such a scheme optimal. This is because $\mathbf{e}_1^{FB}$ does not depend on $\mathbf{g}$. Consequently, any scheme that elicits $\mathbf{e}_1^{FB}$ requires $b_1(m) = 0$, which places a constraint on the shape of the wage scheme. The principal is thus left with two functions she can control, $s(m)$ and $\beta(m)$, tied down by two constraints, (ICP) and $\mathbf{e}_1 = \mathbf{e}_1^{FB}$. The functions that satisfy these two constraints do not in general optimize the worker's second period incentives. Consequently, a small change in the contract that moves $\beta(m)$ towards β^{SB} increases the principal's overall payoff, as it generates a first order improvement in period 2 incentives, but only a second order loss due to period 1 deviation from $\mathbf{e}_1^{FB}$.

Proposition 2 will play an important role in Section 4, where the current setting will be compared with a setting in which the principal commits to a specific distribution of subjective evaluations.

3.4. Optimal contract

In general, the worker's first period effort depends on b_1 and b_3 directly and on all of s, b_1, b_2, and b_3 indirectly, through the effects of $\mathbf{e}_1$ on the evaluation m. Furthermore, $\mathbf{e}_2$ can depend on $\mathbf{e}_1$ through β. A wage scheme that allows for all of s, b_1, b_2, and b_3 to depend on m therefore has complicated effects on both $\mathbf{e}_1$ and $\mathbf{e}_2$, which could preclude a tractable characterization of the optimal contract. Fortunately, Lemma 1 below simplifies the problem significantly. It shows that if the optimal contract is piecewise differentiable (assumed for the rest of this section[13]), then the only parts of the contract that need to be allowed to depend on m are $s(m)$ and $b_2(m)$. Specifically, consider the problem

$$\text{(P')} \qquad \max_{b_1,\beta(m),\mathbf{e}_1} E_a \left[a\mathbf{e}_1 \cdot \mathbf{f} - \Psi(\mathbf{e}_1) + \|\mathbf{g}\|^2 a^2 \beta(a) \left[\beta^{SB} - \frac{\beta(a)}{2} \right] \right]$$

subject to

$$\begin{aligned} \mathbf{e}_{1k} &= E_a \left[ab_1 g_k + \frac{\|\mathbf{g}\|^2 f_k}{\mathbf{e}_1 \cdot \mathbf{f}} a^2 \left[\beta^{SB} - \beta(a) \right] \left[\beta(a) + a\beta'(a) \right] \right] && \text{(ICW}_1\text{')} \\ \beta(m) &= b_2(m). \end{aligned}$$

[13] Piecewise differentiability is a common assumption in optimal control problems. In many mechanism design problems, the optimal contract can be shown to be monotonic, which ensures that it is differentiable almost everywhere. In the current setting, monotonicity of the optimal contract cannot be ascertained ex ante.

Problem (P') was obtained from problem (P) by (i) setting $b_3(m) = 0$ and $b_1(m) = b_1$, where b_1 is a constant, (ii) substituting (ICW$_2$) into the objective function and into the remaining constraints, and (iii) substituting $s'(m)$ from the first order condition for (ICP) into (ICW$_1$) (see the proof of Lemma 1 for details).

Lemma 1. *Suppose b_1^*, $\beta^*(.)$, $s^*(.)$, $\mathbf{e}_1^*$, and $\mathbf{e}_2^*$ solve the amended problem (P'). If the term $a\beta^*(a)\left[\beta^{SB} - \beta^*(a)\right]$ is non-decreasing in a, then b_1^*, $\beta^*(.)$, $s^*(.)$, $\mathbf{e}_1^*$, and $\mathbf{e}_2^*$ also solve the original problem (P).*

The following assumption will be sufficient to guarantee that the condition in the lemma is satisfied, so that the solution to the simplified problem (P') also solves (P). Let $\varepsilon(a) \equiv \frac{ah'(a)}{h(a)}$ be the elasticity of the density function $h(.)$ at a.

ASSUMPTION 1. (a) $\varepsilon(a) = 0$ for some $a \in [0, \bar{a}]$; (b) $|\varepsilon'(a)| \leq M$ for each a, where $M \geq 0$ is small.

Assumption 1 requires that the density function $h(a)$ does not vary "too much." This assumption is satisfied, for example, by the uniform distribution.

The optimal control program (P') is formally analyzed in Appendix B. The analysis yields the following characterization of the optimal bonuses b_1^* and $\beta^*(m)$.

Proposition 3. *Under Assumption 1, the optimal contract sets (i) $b_1^* > 0$ and (ii) $0 < \beta^*(m) \leq \beta^{SB}$ for all m. Specifically, $\beta^*(m) = \beta^{SB}$ for $m = \bar{a}$ and*

$$\beta^*(m) = \beta^{SB}\left[1 - \frac{\lambda}{1 - \lambda - \lambda\varepsilon(m)}\right] < \beta^{SB} \quad \text{for } m < \bar{a}, \tag{1}$$

where $\lambda \in (0, \frac{1}{2})$ is a constant.

Proposition 3 reveals that in the presence of subjective evaluations the principal optimally weakens the agent's formal second period incentives ($\beta^* < \beta^{SB}$ for $m < \bar{a}$). This is not because subjective pay has second period incentive effects that substitute for formal contracts. Rather, the rationale for weakening the second period formal contract is to ensure that the evaluations induce *first period* effort. To see this, recall that β^{SB} maximizes the second period surplus; hence, there is no further benefit to increasing $\mathbf{e}_2$ when $\beta = \beta^{SB}$. Therefore, if β were equal to β^{SB} the principal would have no desire to induce a higher $\mathbf{e}_2$ through a good evaluation, in which case the only way to ensure truthful evaluation would be to make $s(m)$ independent of m. But then the evaluations would have no incentive effects, as they would not affect the agent's pay. Thus, to ensure that the evaluations provide incentives, the principal scales back the second period formal contract. This makes it desirable for her to boost $\mathbf{e}_2$ by providing good evaluations, which in turn allows $s(m)$ to depend on m and thus provide first period incentives. However, since incentives in period 2 come solely from β, it is not optimal to scale β all the way back to zero. Consequently, $\beta^* > 0$.

Observe that the optimal contract retains some formal incentives also in the first period, $b_1^* > 0$, even though subjective evaluations, being undistorted, provide more efficient incentives. The logic is the same as that behind Proposition 2: The first period incentive effects of subjective evaluations come at the cost of further muting second period effort. Because this is costly, first period incentives are supplemented by incentives from the objective measure.

Note also that, under Assumption 1, $e_{k2}^*(m) = m\beta^*(m)g_k$ increases in m. A potentially testable implication is that good evaluations are followed by good performance and bad evaluations are followed by poor performance.

3.4.1. Optimal contract under uniform distribution

To gain further insights into the economic forces that govern the relationship between formal contracts and subjective evaluations, it is instructive to consider the case where the distribution of abilities is uniform. When $H(.)$ is uniform, $\varepsilon(a) = 0$ and (1) becomes $\beta^* = \beta^{SB}\frac{1-2\lambda}{1-\lambda}$. Thus, β^* is independent of m in this case[14], and the optimal contract is effectively separated into a subjective part, consisting of $s(m)$, and an objective part, consisting of the fixed bonuses b_1 and β. This makes it possible to characterize the optimal subjective scheme $s(.)$.

With b_1 and β constant, the principal's truthtelling problem reduces to

$$q_1 \in \arg\max_m E_a[a\mathbf{e}_2\cdot\mathbf{f} - \beta a\mathbf{e}_2\cdot\mathbf{g}|q_1] - s(m), \tag{ICP}$$

subject to

$$\psi'(e_{k2}^*) = e_{k2}^* = \beta m g_k \ , \quad k = 1, 2, ..., K, \tag{ICW$_2$}$$

for which the first order condition yields

$$s'(m) = a\beta\left(\mathbf{g}\cdot\mathbf{f} - \beta\left\|\mathbf{g}\right\|^2\right). \tag{2}$$

The proof of Proposition 4 below verifies that (2) describes the principal's optimum. Imposing truthtelling then yields the differential equation that implicitly defines the optimal

[14] Except at $m = \bar{a}$.

subjective scheme $s^*(m)$:

$$s^{*\prime}(m) = m\beta\left(\mathbf{g}\cdot\mathbf{f} - \beta\left\|\mathbf{g}\right\|^2\right).$$

This leads to the following result.

Proposition 4. *When $H(.)$ is uniform, it is optimal to set*

$$\beta^* = \beta^{SB}\frac{1-2\lambda}{1-\lambda} \text{ for } m < \bar{a}, \quad \beta^* = \beta^{SB} \text{ for } m = \bar{a}, \quad \text{and} \tag{3a}$$

$$s^*(m) = \frac{m^2}{2}\left\|\mathbf{g}\right\|^2\beta^*\left(\beta^{SB} - \beta^*\right) + D \text{ for all } m, \tag{3b}$$

where $\lambda \in (0, \frac{1}{2})$ and $D \in \mathbb{R}$ are constants.

Proposition 4 provides several insights into the economics of the model. First, (3b) reveals that in order to be truthful, the wage scheme $s(m)$ must be not only increasing but also strictly convex in the subjective evaluation m. This skewness of the subjective pay reflects the production complementarities between ability and effort, which induce convexity in the worker's marginal output as a function of ability. Intuitively, the more productive is the worker, the bigger is the principal's potential gain from boosting the worker's effort by inflating his belief $x(m)$ through a good evaluation. To balance this temptation, the "price" for increasing the evaluation must be higher for higher ability workers, which leads to the convexity of the pay scheme.

Second, Proposition 4 confirms more directly the result of Proposition 1, which says that subjective evaluations can simultaneously be truthful and have incentive effects only if some formal contract is in place, no matter how weak. In the absence of an objective measure ($\beta = 0$), telling the worker that he is of a high type entails no benefit to the principal. In this case, evaluations can be truthful only if they do not affect the worker's pay, as can be seen from (3b), which reduces to $s^*(m) = D$. Such a scheme, however, has no incentive value.

Third, the expression for $s^*(m)$ illustrates the point that the second period formal contract must be weakened compared to its constrained efficient level if subjective evaluations are to induce effort. In particular, if it were $\beta = \beta^{SB}$, then $s^*(m)$ would again be constant and hence without incentive effects.

Finally, it is worth noting that the mechanism behind the incentive effects of subjective pay is reminiscent of the Holmström's (1999) model of career concerns. As in Holmström (1999), the agent exerts effort in order to influence his employer's perception about his ability. The difference is that in Holmström's model, the employer does not face the problem of truthfully revealing the agent's performance, because this performance is observed by multiple prospective employers, who are forced through competition to bid up the agent's wage to equal his true expected productivity.

3.5. Discussion and extensions

This subsection briefly discusses two potentially important considerations left out from the main model. The first is the possibility that the principal is unable to make s, b_1, b_2, and b_3 contractually contingent on the evaluations. The second is the possibility that supervisors dislike giving poor evaluations, which might affect the optimal subjective scheme.

3.5.1. No commitment

The assumption that the firm can make the worker's pay contractually contingent on the evaluations is not unrealistic. Subjective evaluation schemes are often well defined in advance and adherence to such schemes might be verifiable. Furthermore, Lemma 1 showed that a dependence of b_1 and b_3 on m is unnecessary. Nevertheless, commitment of this sort may not always be possible. This does not mean, however, that subjective evaluations are not feasible. Even without committing to it ex ante, the principal may have an incentive to convey her information by paying more to higher ability workers, as long as the workers interpret this signal correctly.

Thus, without commitment, the setting is formally a signaling game and as such can have multiple equilibria, including an uninformative equilibrium in which the workers never update their beliefs and the principal always pays the lowest possible wage. More interesting for the purposes of this analysis, there can also exist a separating Perfect Bayesian Equilibrium (PBE), in which the principal reveals her information truthfully. Specifically, consider for simplicity the uniform distribution case (so that β is optimally constant) and suppose that upon being paid a wage s, the agent's belief x is given by $x = m$ if $s = s^{*-1}(m)$ for some $m \in [0, \bar{a}]$, and by $x = 0$ otherwise. Then Proposition 4 tells us that if faced with a worker of ability a, the principal prefers to give him the evaluation $m = a$ and wage $s^*(a)$ to giving him any other evaluation $m = a'$ and wage $s^*(a')$. The only deviation one therefore needs to worry about is where the principal decides to pay a wage that is not in the image of $s^*(.)$. This, however, can be prevented by setting $D = s^*(0) = 0$[15,16] — any deviation from $s^*(a)$

[15]It seems natural to restrict attention here to $s(.) \geq 0$, i.e., to assume that at this stage the agent cannot be forced to transfer money to the principal.

[16]Setting $D = 0$ is always feasible: While in the previous analysis D was lumped together with the agent's base salary and hence determined by the agent's participation constraint, conceptually, these two wage components can be separated. The base salary is then specified in the initial contract so as to meet the agent's participation constraint when $D = 0$.

then necessarily involves paying the agent more than $s^*(\bar{a})$. Given the specified beliefs, this is dominated by paying $s^*(\bar{a})$. Thus, the above beliefs, together with the wage scheme (3b) and with $D = 0$, support a separating PBE of this signalling game.

3.5.2. Leniency bias

Some observers suggest that an important problem with subjective evaluations is that supervisors suffer from a leniency bias, i.e. a preference for giving good evaluations. Such a bias can be readily incorporated into the present model without changing its main insights. Specifically, suppose the principal derives utility $v(m)$ from giving an evaluation m, where the preference for good evaluations is captured by $v'(m) > 0$. Then the only part of the optimization problem (P) that needs to be adjusted to account for the leniency bias is the principal's truthtelling constraint (ICP), which is now written as

$$a \in \arg\max_m v(m) + a\mathbf{e}_2 \cdot \mathbf{f} - \beta(m) a\mathbf{e}_2 \cdot \mathbf{g} - s(m) - b_1(m) a\mathbf{e}_1 \cdot \mathbf{g}.$$

But this can be transformed into the original constraint by defining $\gamma(m) \equiv s(m) - v(m)$. Function $\gamma(m)$ then plays the same role here as $s(m)$ played in the original problem. Hence, if $s^*(m)$ was the optimal salary function in the original problem without a leniency bias, then the optimal $\gamma(.)$ in the current problem is $\gamma^*(m) = s^*(m)$. This means that the optimal salary function in the problem with a leniency bias, denoted by $s^B(m)$, is $s^B(m) = s^*(m) + v(m)$. All the other parts of the optimal contract remain unaffected.

Thus, the only effect of a leniency bias in this model is to make the optimal salary function $s(.)$ steeper. This is intuitive: Even without a leniency bias, the principal likes to give good evaluations, because they induce the agent to work harder. Truthtelling therefore requires that the salary function $s(.)$ is increasing, so as to make good evaluations costly. A leniency bias merely magnifies this effect.

4. Forced distributions

Some companies adopt forced distributions of subjective evaluations (FDSE), where they commit ex ante to a pre-specified distribution of evaluations. This section considers the benefits and disadvantages of such forced distributions in the present framework. The main conclusion will be that whether an FDSE improves upon the subjective evaluation scheme studied in the previous section depends critically on the number of workers the firm employs and on the quality of the objective measures z_1 and z_2. These points will be investigated in two alternative settings: In the first one, the firm employs a continuum of workers; in the second, the firm employs a finite number of workers n.

4.1. Continuum of workers

Under a forced distribution, the firm's total wage bill associated with the evaluations is always constant, whether the evaluations are truthful or not. Misreporting can therefore affect the firm's profit only through the effects it has on the workers' actions.

4.1.1. Truthtelling

The main benefit of an FDSE is that it allows the principal to eliminate the truthtelling constraint (ICP). To see this, suppose the firm employs a measure one of workers whose abilities are drawn independently from $[0, \bar{a}]$ according to the cumulative distribution $H(a)$ with density $h(a)$. Suppose also that the firm pre-commits to an FDSE under which a fraction $h(m)$ of the workers get evaluation m. Then the principal has no incentive to misreport because misreporting does not affect the wage bill, but hurts her second period expected profit by preventing the workers from tailoring their efforts to their abilities.

To make this argument more formally, suppose the principal's evaluation strategy upon inferring that a worker has ability a is to report $m \in [0, \bar{a}]$ according to the probability density function $\sigma(m|a)$. A worker who receives evaluation m then uses the Bayes' rule to form a posterior belief $h(a|m)$ about the distribution of his true ability,

$$h(a|m) = \frac{\sigma(m|a)h(a)}{\int_0^{\bar{a}} \sigma(m|\tau)h(\tau)d\tau} = \frac{\sigma(m|a)h(a)}{h(m)}, \tag{4}$$

where the second equality used the fact that the evaluations must adhere to the forced distribution. The worker's expected ability conditional on evaluation m, $x(m)$, is then

$$x(m) = \int_0^{\bar{a}} ah(a|m)da, \tag{5}$$

and his optimal second period effort $\mathbf{e}_2(m)$ is given by the first order condition

$$e_{2k}(m) = \beta(m)x(m)g_k.$$

Ignoring $s(m)$, the firm's second period expected profit from a worker of ability a is therefore

$$\begin{aligned} E_\sigma \pi_2(a) &= \int_0^{\bar a} \left[a\mathbf{e}_2 \cdot \mathbf{f} - \beta(m) a \mathbf{e}_2 \cdot \mathbf{g}\right] \sigma(m|a) dm \\ &= \int_0^{\bar a} a \left\|\mathbf{g}\right\|^2 x(m)\beta(m) \left[\beta^{SB} - \beta(m)\right] \sigma(m|a) dm, \end{aligned}$$

so that its total expected second period profit is

$$E\pi_2 \equiv E_a E_\sigma \pi_2(a) = \int_0^{\bar a} \int_0^{\bar a} a \left\|\mathbf{g}\right\|^2 x(m)\beta(m) \left[\beta^{SB} - \beta(m)\right] \sigma(m|a) h(a) dm da.$$

As will become apparent shortly, $\beta(m)$ can be set constant here w.l.o.g. Thus, let $\beta(m) = \beta$, where $\beta \leq \beta^{SB}$ is a constant. Using (4) and (5), $E\pi_2$ can then be written as

$$\begin{aligned} E\pi_2 &= \left\|\mathbf{g}\right\|^2 \beta \left(\beta^{SB} - \beta\right) \int_0^{\bar a} \left[\int_0^{\bar a} a h(a|m) da\right] x(m) h(m) dm \\ &= \left\|\mathbf{g}\right\|^2 \beta \left(\beta^{SB} - \beta\right) E_m \left[x(m)\right]^2. \end{aligned}$$

Now, any improvement in the informativeness of the principal's reporting strategy σ, in particular a switch to truthful reporting, induces a mean-preserving spread of the agents' posteriors x (Marschak and Miyasawa, 1968). This increases the principal's expected second period profit, because $E\pi_2$ is an expectation of a convex function of the posteriors. The principal therefore finds it optimal to provide truthful evaluations.[17]

4.1.2. First period incentives

Observe that the principal's incentives to provide truthful evaluations under the FDSE depend neither on $b_1(m)$ nor on the exact shape of $s(m)$; the only constraint on the contract is that $\beta \leq \beta^{SB}$. The principal can therefore choose $s(.)$, b_1, and $\beta \leq \beta^{SB}$ so as to optimize the workers' incentives. In the second period, the best one can do is to set $\beta = \beta^{SB}$. I will now show that in the first period, it is possible to achieve the efficient efforts $\mathbf{e}_1^{FB}$.

Because $\mathbf{e}_1^{FB}$ does not depend on $\mathbf{g}$, start by setting $b_1(m) = 0$ for all m. Given that β is constant, $\mathbf{e}_1$ then depends solely on $s(m)$. Now, recall that when evaluations are truthful, a worker who provides effort $\hat{\mathbf{e}}_1$ expects his evaluation m to be equal to $\frac{a\hat{\mathbf{e}}_1 \cdot \mathbf{f}}{\tilde{\mathbf{e}}_1 \cdot \mathbf{f}}$, where $\tilde{\mathbf{e}}_1$ is the principal's conjecture about $\mathbf{e}_1$. This suggests that the first best outcome in period 1

[17] Being similar to a cheap talk game, an FDSE game has multiple equilibria, including a babbling equilibrium in which the subjective evaluations are completely uninformative. Note, however, that an FDSE is formally not a cheap talk game because the individual workers' payoffs depend directly on the messages.

is obtained by setting $s(m) = m\mathbf{e}_1^{FB}\cdot\mathbf{f}$. The worker's first period expected utility is then $E_a\left[a\hat{\mathbf{e}}_1\cdot\mathbf{f}\frac{\mathbf{e}_1^{FB}\cdot\mathbf{f}}{\tilde{\mathbf{e}}_1\cdot\mathbf{f}} - \Psi(\mathbf{e}_1)\right]$, so that his optimal effort is given by the first order condition

$$\psi'(e_{1k}) = E(a)f_k\frac{\mathbf{e}_1^{FB}\cdot\mathbf{f}}{\tilde{\mathbf{e}}_1\cdot\mathbf{f}}.$$

The equilibrium requirement $\tilde{\mathbf{e}}_1 = \mathbf{e}_1$ then yields $\mathbf{e}_1 = \mathbf{e}_1^{FB}$ as an equilibrium outcome.

The analysis of the case with a continuum of workers is summed up as follows.

Proposition 5. *Suppose the firm employs a continuum of workers and uses a forced distribution of subjective evaluations (FDSE) according to which the fraction $h(m)$ of workers get evaluation m. Then there always exists a Perfect Bayesian Equilibrium in which the evaluations are truthful. An optimal contract sets $b_1 = b_3 = 0$, $b_2 = \beta^{SB}$, and $s(m) = m\mathbf{e}_1^{FB}\cdot\mathbf{f}$ and achieves the first best outcome in $t = 1$ and the second best outcome in $t = 2$. This contract strictly dominates the optimal contract without FDSE.*

Proposition 5 shows that when the firm employs a lot of workers, a forced distribution improves efficiency by ensuring that the subjective pay scheme is incentive compatible. The subjective scheme can then be used solely to shape the workers' first period incentives. This has two effects on the optimal contract. First, it allows the firm to provide fully efficient incentives in period 1 by completely removing from the contract the distortive objective measure z_1 and replacing it with the undistorted incentives from subjective evaluations. Second, it eliminates the dependence of $s(.)$ on $\beta(.)$, thus freeing $\beta(.)$ to be used solely for the purpose of second period incentives, which improves efficiency in period 2.

4.2. Finite number of workers

When the number of workers is finite, it is obviously not possible for a forced distribution to replicate the true distribution of abilities $H(.)$. Nevertheless, much of the above analysis goes through. Specifically, an FDSE relaxes the principal's truthtelling constraint and allows for the first best to be achieved in period 1.

Suppose the firm employs $n \geq 2$ workers. As will be shown shortly, it will again be possible to find an $s(m)$ such that $\mathbf{e}_1 = \mathbf{e}_1^{FB}$ for each worker. Hence, $\beta(m)$ will be optimally set to maximize the second period surplus: $\beta(m) = \beta^{SB}$. Furthermore, $\mathbf{e}_1 = \mathbf{e}_1^{FB}$ requires $b_1(m) = 0$. A forced distribution of subjective evaluations then entails (i) n possible evaluations, $m_1 \leq m_2 \leq ... \leq m_n$, (ii) the corresponding salaries $s_j \equiv s(m_j)$, $j = 1, 2, ..., n$, and (iii) a commitment by the firm to assign each evaluation to exactly one worker.[18] Clearly, for n

[18] Note that this allows for the possibility that multiple workers get the same evaluation. For example, if

finite, this scheme cannot fully reveal the workers' true abilities. However, arguments similar to those behind Proposition 5 imply that the principal will assess the workers truthfully in the sense that she will assign evaluation m_n to the highest ability worker, evaluation m_{n-1} to the second highest, and so on.[19]

To see that one can find $\{s_j\}_{j=1}^n$ that elicit $\mathbf{e}_1^{FB}$, suppose the FDSE entails only two possible salaries, s^H and $s^L < s^H$, and define $\triangle s \equiv s^H - s^L$. Suppose further that salary s^L is attached to the first r lowest evaluations $m_1, m_2, ..., m_r$, whereas salary s^H is attached to the high evaluations $m_{r+1}, ..., m_n$. Consider a worker i of ability a and denote by $H_{(r)}(a)$ the cumulative distribution function of the event that at least r workers other than worker i have abilities less than a and let $h_{(r)}(a)$ be the corresponding probability density function.[20] Then if worker i anticipates that all the other workers provide the first best efforts $\mathbf{e}_1^{FB}$ while his effort is $\mathbf{e}_1^i$, the probability with which he expects the high salary s^H is

$$\Pr\{m_i \geq m_r\} = \begin{cases} \int_0^{\bar{a}\frac{\mathbf{e}_1^i\cdot\mathbf{f}}{\mathbf{e}_1^{FB}\cdot\mathbf{f}}} \left[1 - H\left(\frac{a\mathbf{e}_1^{FB}\cdot\mathbf{f}}{\mathbf{e}_1^i\cdot\mathbf{f}}\right)\right] h_{(r)} da & \text{for } \mathbf{e}_1^i\cdot\mathbf{f} \leq \mathbf{e}_1^{FB}\cdot\mathbf{f} \\ \int_0^{\bar{a}} \left[1 - H\left(\frac{a\mathbf{e}_1^{FB}\cdot\mathbf{f}}{\mathbf{e}_1^i\cdot\mathbf{f}}\right)\right] h_{(r)} da & \text{for } \mathbf{e}_1^i\cdot\mathbf{f} \geq \mathbf{e}_1^{FB}\cdot\mathbf{f}. \end{cases}$$

Worker i's maximization problem is therefore

$$\max_{\mathbf{e}_1^i} s^L + \triangle s \Pr\{m_i \geq m_r\} - \Psi(\mathbf{e}_1^i),$$

with the corresponding first order condition

$$\psi'(e_{1k}^i) = \begin{cases} \triangle s \int_0^{\bar{a}\frac{\mathbf{e}_1^i\cdot\mathbf{f}}{\mathbf{e}_1^{FB}\cdot\mathbf{f}}} a\frac{f_k\mathbf{e}_1^{FB}\cdot\mathbf{f}}{\left(\mathbf{e}_1^i\cdot\mathbf{f}\right)^2} h\left(\frac{a\mathbf{e}_1^{FB}\cdot\mathbf{f}}{\mathbf{e}_1^i\cdot\mathbf{f}}\right) h(a)da & \text{for } \mathbf{e}_1^i\cdot\mathbf{f} \leq \mathbf{e}_1^{FB}\cdot\mathbf{f} \\ \triangle s \int_0^{\bar{a}} a\frac{f_k\mathbf{e}_1^{FB}\cdot\mathbf{f}}{\left(\mathbf{e}_1^i\cdot\mathbf{f}\right)^2} h\left(\frac{a\mathbf{e}_1^{FB}\cdot\mathbf{f}}{\mathbf{e}_1^i\cdot\mathbf{f}}\right) h(a)da & \text{for } \mathbf{e}_1^i\cdot\mathbf{f} \geq \mathbf{e}_1^{FB}\cdot\mathbf{f}. \end{cases}$$

Setting $\mathbf{e}_1^i = \mathbf{e}_1^{FB}$, this shows that $e_{1k}^i = e_{1k}^{FB}$ if

$$\triangle s \frac{f_k}{\mathbf{e}_1^{FB}\cdot\mathbf{f}} \int_0^{\bar{a}} ah(a)h_{(r)}(a)da = E(a)f_k,$$

$m_1 = m_2 = ... = m_n$, then all workers effectively receive the same evaluation. In this particular case, the evaluations do not convey any information.

[19] Two or more workers having the same ability is a zero probability event and will be ignored.

[20] In other words, $H_{(r)}(a)$ is the cdf of the rth order statistic; hence, it is given by $H_{(r)}(a) = \sum_{i=r}^{n-1} \binom{n-1}{i} H^i(a)[1 - H(a)]^{n-1-i}$.

which is achieved by letting $\triangle s = \frac{E(a)\mathbf{e}_1^{FB}\cdot\mathbf{f}}{\int_0^{\bar{a}} ah(a)h_{(r)}(a)da}$.

Proposition 6 below summarizes the analysis of the case with a finite number of workers.

Proposition 6. *Suppose the firm employs $n \geq 2$ workers and uses an FDSE with n different evaluations and with two salary levels, s^L and $s^H = s^L + \triangle s$. Then there exists a Perfect Bayesian Equilibrium in which the evaluations are truthful in the sense that the highest ability worker receives the highest evaluation, the second highest ability worker receives the second highest evaluation, and so on. An optimal contract sets $b_1 = b_3 = 0$, $b_2 = \beta^{SB}$, and $\triangle s = \frac{E(a)\mathbf{e}_1^{FB}\cdot\mathbf{f}}{\int_0^{\bar{a}} ah(a)h_{(r)}(a)da}$, and achieves full efficiency in $t = 1$.*

The above analysis suggests that the main difference between the cases with n workers and a continuum of workers, and, similarly, between the cases with n workers and with $n' > n$ workers, is in how precise is the information the workers have about their abilities in the second period. When there is a continuum of workers, each worker learns his exact ability. When the number of workers is finite, the workers' information remains coarse in the second period, because they only learn their rank out of n workers. This coarseness of beliefs is a source of a second period inefficiency, as it prevents the workers from fully tailoring their efforts to their abilities. However, the information content of the evaluations increases with the number of workers, since it is more informative to know how one ranks among $n + 1$ workers than to know how one ranks among n workers. In particular, as $n \to \infty$, each worker's estimate of his ability converges to his true ability a. These observations, combined with the last claim in Proposition 5, lead to the following result.

Proposition 7. *The (per worker) efficiency of FDSE increases with the number of workers. Furthermore, for any given $H(.)$ and $\cos\theta$, there exists an $n^* \geq 2$ such that for all $n \geq n^*$, FDSE dominates subjective evaluations without a forced distribution.*

One implication of the above proposition that is worth noting is that if formal performance appraisal systems are costly to administer, then, all else equal, larger companies should have an advantage in adopting them. This is consistent with the evidence that larger organizations are more likely to use performance appraisal than smaller organizations (Murphy and Cleveland, 1995, p. 4).

4.3. The role of the contractible measures

As shown above, FDSE is always optimal if the firm employs sufficiently many workers, but this leaves open the question whether subjective evaluations without a forced distribution can be optimal when the number of workers is small. Proposition 7 implies that to answer this question, it is enough to consider $n = 2$. Thus, for the remainder of this section, I will concentrate on a setting with two workers.

The advantage of FDSE is that it allows the firm to achieve full efficiency in period 1 and to increase the strength of the second period incentives (as measured by β). The downside is that the workers' information about their abilities remains coarse in the second period, which distorts each worker's effort choice from the level appropriate for his ability. When the number of workers is small, this trade-off determines whether the firm prefers subjective evaluations with or without a forced distribution.

Proposition 8. *Suppose the firm employs two workers. Then there exist c^* and c^{**}, $0 < c^* \leq c^{**} < 1$, such that FDSE is optimal if $\cos\theta \leq c^*$, whereas subjective evaluations without a forced distribution are optimal if $\cos\theta \geq c^{**}$.*

Proposition 8 says that the relative benefits of FDSE depend on the quality of the objective measure z. This result is intuitive. When the objective measure is poor ($\cos\theta \leq c^*$), a contract based solely on this measure provides poor incentives. The additional incentives from subjective evaluations are therefore highly valuable. This favors FDSE, as FDSE induces fully efficient effort in the first period and hence improves efficiency substantially. Moreover, because the second period incentives are severely distorted (as $\cos\theta$ is small), giving the workers somewhat more precise information about their abilities would not do much to improve efficiency in this period. This limits the efficiency loss from adopting FDSE.

In contrast, when the objective measure is good ($\cos\theta \geq c^{**}$), the formal contract provides quite efficient incentives in both periods even without subjective pay. The main benefit of subjective evaluations is then in informing the workers about their abilities, which is better achieved through evaluations without a forced distribution than through FDSE. Thus, when the contractible measure is good, subjective evaluations without a forced distribution are optimal.

5. Conclusion

Most firms that use subjective performance evaluations use them with multiple goals in mind. Economists have traditionally focused on the incentive effects of subjective evaluations, overlooking their other functions. This paper brings to forefront the feedback role of evaluations, which appears to be of equal, if not greater, importance to real world firms as their incentive role. The main insight from the model is that the feedback and the incentive roles of subjective evaluations are complementary in the optimal contract: when both are present, subjective evaluations are feasible where they could not be sustained otherwise.

The feedback from the evaluations improves efficiency by informing workers about their abilities, which allows them to better choose their optimal actions. Because higher ability workers optimally provide more effort, the principal has a motivation to give good evaluations, which makes truthful evaluations possible. The paper shows that truthful subjective evaluations are always feasible if there exists some, albeit imperfect, verifiable measure of performance. However, the need to ensure that the evaluations are truthful means that the optimal contract never fully replaces the imperfect objective measure with subjective pay. Instead, subjective and objective pay are intertwined in the optimal contract, and the contract's exact shape depends upon the quality of the objective measure.

The paper also shows that the ability to commit to a wage scheme that goes with the subjective evaluations is not essential for the existence of truthful evaluations, but the ability to commit to a specific distribution to which the evaluations must adhere may be helpful. In particular, a forced distribution of subjective evaluations is better than a subjective scheme without a forced distribution when the number of employees is sufficiently large or when the objective performance measure is poor.

Although it expands the view of subjective evaluations beyond that in traditional economic models, the model of this paper is far from capturing the variety of purposes for which subjective appraisals are used in practice. Building a more comprehensive economic model of performance evaluations that would incorporate additional reasons real world firms find performance evaluations useful (such as improved job matching or ensuring the employees' ongoing development) could be a fruitful topic for future research.

A. Appendix A: Proofs

Proof of Proposition 1. (i) The principal's period 2 expected revenue from a worker of ability a is $ETR_2 = a\mathbf{e}_2{\cdot}\mathbf{f}$. Using $e_{k2} = \beta(m)mg_k$ from (ICW$_2$) yields $ETR_2 = am\beta(m)\mathbf{g}\cdot\mathbf{f} = am\beta(m)\left\|\mathbf{g}\right\|\left\|\mathbf{f}\right\|\cos\theta$. If $\cos\theta = 0$, then $ETR_2 = 0$, so that truthtelling requires

$$[b_2(q_1) + b_3(q_1)a\mathbf{e}_1{\cdot}\mathbf{g}]\, a\mathbf{e}_2{\cdot}\mathbf{g} + s(q_1) + b_1(q_1)p_1 \leq [b_2(q_1') + b_3(q_1')a\mathbf{e}_1{\cdot}\mathbf{g}]\, a\mathbf{e}_2'{\cdot}\mathbf{g} + s(q_1') + b_1(q_1')p_1 \quad \text{(A1)}$$

for all a and a', where $q_1 = a\mathbf{e}_1{\cdot}\mathbf{f}$ and $q_1' = a'\mathbf{e}_1{\cdot}\mathbf{f}$.

Now, for subjective pay to provide incentives, the marginal effect of the agent's effort on his expected pay must be (at least for some effort levels) higher when his pay depends on m than when it does not. Formally, consider two first period effort vectors $\mathbf{e}_1$ and $\mathbf{e}_1'' \leq \mathbf{e}_1$, i.e. $e_{1k}'' \leq e_{1k}$ for all k, with $e_{1k}'' < e_{1k}$ for at least some k. Let $w(m) = (s(m), b_1(m), b_2(m), b_3(m))$ be a contract that depends on subjective evaluations and let $w'' = (\bar{s}'', \bar{b}_1'', \bar{b}_2'', \bar{b}_3'')$ be a contract where $\bar{s}'', \bar{b}_1'', \bar{b}_2''$, and $\bar{b}_3''$ are all constant, with $q_1'' = a\mathbf{e}_1''{\cdot}\mathbf{f}$, $\bar{s}'' = s(q_1'')$, $\bar{b}_1'' = b_1(q_1'')$, $\bar{b}_2'' = b_2(q_1'')$, and $\bar{b}_3'' = b_3(q_1'')$. The evaluations can have a positive incentive effect only if there exists some $\mathbf{e}_1'' \leq \mathbf{e}_1$ such that the increase in effort from $\mathbf{e}_1''$ to $\mathbf{e}_1$ induces a larger increase in expected pay for the worker under contract $w(m)$ than under w'' :

$$\begin{aligned} & E_a\left[[b_2(q_1) + b_3(q_1)a\mathbf{e}_1{\cdot}\mathbf{g}]\, a\mathbf{e}_2{\cdot}\mathbf{g} + s(q_1) + b_1(q_1)p_1\right] \\ & -E_a\left[[b_2(q_1'') + b_3(q_1'')a\mathbf{e}_1''{\cdot}\mathbf{g}]\, a\mathbf{e}_2''{\cdot}\mathbf{g} + s(q_1'') + b_1(q_1'')p_1''\right] \\ > \; & E_a\left[\left[\bar{b}_2 + \bar{b}_3 a\mathbf{e}_1{\cdot}\mathbf{g}\right] a\mathbf{e}_2{\cdot}\mathbf{g} + \bar{s} + \bar{b}_1 p_1\right] - E_a\left[\left[\bar{b}_2 + \bar{b}_3 a\mathbf{e}_1''{\cdot}\mathbf{g}\right] a\mathbf{e}_2{\cdot}\mathbf{g} + \bar{s} + \bar{b}_1 p_1''\right], \end{aligned}$$

where $p_1'' = a\mathbf{e}_1''{\cdot}\mathbf{g}$ and $e_{k2}'' = [b_2(q_1'') + b_3(q_1'')a\mathbf{e}_1''{\cdot}\mathbf{g}]\, ag_k$. Rearranging, this condition yields

$$E_a\left[[b_2(q_1) + b_3(q_1)a\mathbf{e}_1{\cdot}\mathbf{g}]\, a\mathbf{e}_2{\cdot}\mathbf{g} + s(q_1) + b_1(q_1)p_1\right] > E_a\left[[b_2(q_1'') + b_3(q_1'')a\mathbf{e}_1{\cdot}\mathbf{g}]\, a\mathbf{e}_2''{\cdot}\mathbf{g} + s(q_1'') + b_1(q_1'')p_1\right],$$

which contradicts (A1). Hence, when $\cos\theta = 0$, subjective pay cannot induce effort.

(ii) This part will be proven by constructing a contract with subjective evaluations that are truthful and improve incentives whenever $\cos\theta > 0$. In particular, let $b_1 \geq 0$ and $b_2 \in (0, \beta^{SB})$ be independent of m, let $b_3 = 0$ (so that $\beta = b_2$), and let $s(m) = \frac{1}{2}m^2 \|\mathbf{g}\|^2 b_2\left(\beta^{SB} - b_2\right) + D$, where D is a constant. Now, plug the above $s(m)$ into the principal's second period profit $\pi_2 = a\mathbf{e}_2{\cdot}\mathbf{f} - \beta a\mathbf{e}_2{\cdot}\mathbf{g} - s(m)$ and use $e_{k2} = \beta m g_k$ to get

$$\pi_2 = am\|\mathbf{g}\|^2 b_2\left(\beta^{SB} - b_2\right) - \frac{1}{2}m^2\|\mathbf{g}\|^2 b_2\left(\beta^{SB} - b_2\right) - D.$$

The first order condition for maximization with respect to m yields

$$a\|\mathbf{g}\|^2 b_2\left(\beta^{SB} - b_2\right) = m\|\mathbf{g}\|^2 b_2\left(\beta^{SB} - b_2\right),$$

which demonstrates that this contract induces truthful evaluations.[21]

To see that this subjective pay scheme improves first period incentives, observe that for any first period effort vector $\hat{\mathbf{e}}_1$, the worker expects evaluation $m(\hat{\mathbf{e}}_1) = \frac{a\hat{\mathbf{e}}_1\cdot\mathbf{f}}{\mathbf{e}_1\cdot\mathbf{f}}$. Hence, $E_a\frac{\partial s(m)}{\partial e_{1k}} = E_a[a\frac{f_k}{\mathbf{e}_1\cdot\mathbf{f}}s'(m)] = b_2\left(\beta^{SB} - b_2\right)\frac{f_k}{\mathbf{e}_1\cdot\mathbf{f}}\|\mathbf{g}\|^2 E_a[am(\hat{\mathbf{e}}_1)] > 0$. ■

Proof of Proposition 2. From (ICW$_2$), the second period efforts are $e_{2k} = a\beta(m)g_k$, whereas the efficient efforts are $e_{2k}^{FB} = af_k$. Thus, $\mathbf{e}_2^* = \mathbf{e}_2^{FB}$ is possible only if $\beta(m)g_k = f_k$ for all k and m. This is precluded by the assumption that $\mathbf{g} \neq \alpha\mathbf{f}$ for any constant α.

[21] The second order condition is satisfied because $\frac{\partial^2\pi_2}{\partial m^2} = -\frac{\|\mathbf{g}\|^2}{(\mathbf{e}_1\cdot\mathbf{f})^2}b_2\left(\beta^{SB} - b_2\right) < 0$.

Next consider $\mathbf{e}_1$. Because $\mathbf{g} \neq \alpha \mathbf{f}$ for any constant α, $\mathbf{e}_1 = \mathbf{e}_1^{FB}$ requires that $b_1(m) = 0$ almost everywhere. Suppose that a $\beta(m)$ and $s(m)$ that elicit $\mathbf{e}_1^{FB}$ exist (if not, then we are done) and denote them as $\hat{\beta}(m)$ and $\hat{s}(m)$. Assume first that $\hat{\beta}(m)$ maximizes the expected second period surplus $ETS_2 = E_a[a\mathbf{e}_2^* \cdot \mathbf{f} - \Psi(\mathbf{e}_2^*)]$ subject to (ICW$_2$), so that $\hat{\beta}(m) = \beta^{SB} = \frac{\mathbf{g} \cdot \mathbf{f}}{\|\mathbf{g}\|^2} = \frac{\|\mathbf{f}\|}{\|\mathbf{g}\|} \cos\theta$. The principal's truthtelling constraint (ICP) then becomes

$$\begin{aligned} a &\in \arg\max_m a\mathbf{e}_2 \cdot \mathbf{f} - \hat{\beta}(m) a\mathbf{e}_2 \cdot \mathbf{g} - b_1(m) a\mathbf{e}_1 \cdot \mathbf{g} - \hat{s}(m) \\ &= \arg\max_m a \|\mathbf{g}\|^2 m\hat{\beta}(m) \left[\beta^{SB} - \hat{\beta}(m)\right] - b_1(m) a\mathbf{e}_1 \cdot \mathbf{g} - \hat{s}(m) \\ &= \arg\max_m -\hat{s}(m). \end{aligned}$$

This can only hold if $\hat{s}(m) = \hat{s}$, where $\hat{s}$ is a constant. Hence, the whole contract is independent of m in this case, so that (ICW$_1$) reduces to

$$\mathbf{e}_1^* \in \arg\max_{\mathbf{e}_1} E_a[\hat{s} + \beta^{SB} a\mathbf{e}_2 \cdot \mathbf{g} - \Psi(\mathbf{e}_2^*)] - \Psi(\mathbf{e}_1).$$

This yields $e_{1k}^* = 0$ for all k, contrary to the assumption that $\mathbf{e}_1 = \mathbf{e}_1^{FB}$.

Thus, if $\mathbf{e}_1 = \mathbf{e}_1^{FB}$ then $\hat{\beta}(m) \neq \beta^{SB}$, which means that the first period surplus, ETS_1, is maximized with respect to β, but ETS_2 is not. The standard variational argument therefore implies that a small change in β can increase the total surplus, because its positive effect on ETS_2 is of first order magnitude, but its negative effect on ETS_1 is of second order magnitude. Consequently, $\hat{\beta}(m)$ cannot be optimal. Thus, it must be $\mathbf{e}_1 \neq \mathbf{e}_1^{FB}$. ■

Proof of Lemma 1. Plugging $e_{k2} = \beta(m) m g_k$ into the objective function and into (ICW$_1$) and (ICP), and using $\mathbf{g} \cdot \mathbf{f} = \|\mathbf{g}\| \|\mathbf{f}\| \cos\theta = \|\mathbf{g}\|^2 \beta^{SB}$, problem (P) can be written as

$$\max_{s(m), b_1(m), b_2(m), b_3(m)} E_a \left[a\mathbf{e}_1^* \cdot \mathbf{f} - \Psi(\mathbf{e}_1^*) + \beta(a) a^2 \|\mathbf{g}\|^2 \left[\beta^{SB} - \frac{\beta(a)}{2}\right]\right] \tag{A3}$$

subject to

$$\mathbf{e}_1 \in \arg\max_{\hat{\mathbf{e}}_1} E_a \left[s(\hat{m}) + b_1(\hat{m}) a\hat{\mathbf{e}}_1 \cdot \mathbf{g} + \frac{1}{2}\beta^2(\hat{m}) a^2 \|\mathbf{g}\|^2\right] - \Psi(\hat{\mathbf{e}}_1) \tag{A4}$$

$$a = \arg\max_m a \|\mathbf{g}\|^2 m\beta(m) \left[\beta^{SB} - \beta(m)\right] - b_1(m) a\mathbf{e}_1 \cdot \mathbf{g} - s(m) \tag{A5}$$

$$\beta(m) = b_2(m) + b_3(m) a\mathbf{e}_1 \cdot \mathbf{g} \quad ; \quad \hat{m} = \frac{a\hat{\mathbf{e}}_1 \cdot \mathbf{f}}{\tilde{\mathbf{e}}_1 \cdot \mathbf{f}}.$$

Given piecewise differentiability in m, the first order condition for (A5) is

$$s'(m) + b_1'(m)a\mathbf{e}_1{\cdot}\mathbf{g} = a\|\mathbf{g}\|^2\left[\beta(m)\left[\beta^{SB} - \beta(m)\right] + m\beta'(m)\left[\beta^{SB} - 2\beta(m)\right]\right], \qquad \text{(A6)}$$

except for the points of non-differentiability. Imposing truthtelling, $m = a$, and substituting (A6) into the first order condition for (A4) reduces the problem to

$$\max_{s(m),b_1(m),b_2(m),b_3(m)} E_a\left[a\mathbf{e}_1^*{\cdot}\mathbf{f} - \Psi(\mathbf{e}_1^*) + \beta(m)a^2\|\mathbf{g}\|^2\left[\beta^{SB} - \frac{\beta(m)}{2}\right]\right] \qquad \text{(A7)}$$

subject to

$$\begin{aligned} e_{1k}^* &= E_a\left[[b_1(a)a + \beta(a)b_3(a)a^3]g_k + \frac{\|\mathbf{g}\|^2 f_k}{\mathbf{e}_1{\cdot}\mathbf{f}}a^2\left[\beta^{SB} - \beta(a)\right]\left[\beta(a) + a\beta'(a)\right]\right] \quad \text{(A8)} \\ \beta(m) &= b_2(m) + b_3(m)a\mathbf{e}_1{\cdot}\mathbf{g} \quad ; \quad \beta'(m) = b_2'(m) + b_3'(m)a\mathbf{e}_1{\cdot}\mathbf{g}. \end{aligned}$$

Observe that because neither (A7) nor (A8) depend on $b_1'(a)$, one can without loss of generality set $b_1(m) = b_1$, where b_1 is a constant such that $b_1E(a) = E(b_1(a)a)$. Next, observe that $b_3(m)$ enters only through the term $\beta(a)b_3(a)a^3g_k$ in (A8). It is therefore again w.l.o.g. to replace $b_3(a)$ with $\hat{b}_3 = 0$, while replacing b_1 with $\hat{b}_1 \equiv b_1 + \frac{E_a[\beta(a)b_3(a)a^3]}{E(a)}$ and replacing $b_2(m)$ with $\hat{b}_2(m)$ such that $E_a\left[a\left[\beta^{SB} - \hat{b}_2(a)\right]\left[\hat{b}_2(a) + a\hat{b}_2'(a)\right]\right] = E_a\left[a\left[\beta^{SB} - \beta(a)\right]\left[\beta(a) + a\beta'(a)\right]\right]$. This converts the problem (A7)-(A8) into problem (P') in the text.

Now, by construction, any solution to problem (A7)-(A8) also solves (A3)-(A5) if it induces truthtelling. The last step in the proof is thus to show that it is possible to find a salary function $s(m)$ such that the truthtelling constraint (A5) holds. Let $\Phi(m) \equiv \frac{\partial\left[m\beta(m)\left[\beta^{SB} - \beta(m)\right]\right]}{\partial m}$ and let $s(m)$ be given by $s'(m) = m\|\mathbf{g}\|^2\Phi(m)$. Then for $b_1 = const$ and $b_3 = 0$, the first order condition for (A5) is

$$a\|\mathbf{g}\|^2\Phi(m) - s'(m) = \|\mathbf{g}\|^2\Phi(m)(a - m) = 0,$$

which yields $m = a$. Moreover, $m = a$ is the maximum if $\Phi(m) \geq 0$ for all m, because then $\|\mathbf{g}\|^2\Phi(m)(a - m) \leq 0$ for all $m < a$ and $\|\mathbf{g}\|^2\Phi(m)(a - m) \geq 0$ for all $m > a$. On the other hand, if $\Phi(m') < 0$ for some m', then $\frac{\partial[\Phi(m)(a-m)]}{\partial m}|_{a=m'} = -\Phi(m') > 0$, which means that the local second order condition does not hold at m'. To sum up, one can find a salary function $s(m)$ such that (A5) holds if and only if $a\beta(a)\left[\beta^{SB} - \beta(a)\right]$ is non-decreasing in a. ■

Proof of Proposition 3. Part (i) is established in the analysis of Problem (2), and part

(ii) in the analysis of Problem (1), in Appendix B. ■

Proof of Proposition 4. When $H(.)$ is uniform, then $\varepsilon(a) = 0$ and the expression for β^* follows immediately from (1). Now, the only role of the salary function $s(m)$ is to ensure truthtelling. To see that $s^*(m) = \frac{1}{2}m^2 \|\mathbf{g}\|^2 \beta^* \left(\beta^{SB} - \beta^*\right) + D$ induces truthtelling, rewrite the (ICP) constraint as in the proof of Lemma 1 to get

$$a = \arg\max_m a \|\mathbf{g}\|^2 m\beta^* \left[\beta^{SB} - \beta^*\right] - b_1^* a\mathbf{e}_1 \cdot \mathbf{g} - s^*(m). \tag{A9}$$

Plugging into (A9) the expression for $s^*(m)$, the objective function in (A9) becomes

$$\|\mathbf{g}\|^2 \beta^* \left[\beta^{SB} - \beta^*\right] \left(am - \frac{m^2}{2}\right) - D - b_1^* a\mathbf{e}_1 \cdot \mathbf{g}. \tag{A10}$$

Because (by Proposition 3) $\beta^* < \beta^{SB}$ for all $a < \bar{a}$, (A10) is strictly maximized at $m = a$ when $a < \bar{a}$. When $a = \bar{a}$, then $\beta^* = \beta^{SB}$, so that $m = \bar{a}$ is weakly optimal. ■

Proof of Proposition 5. All of the claims in the proposition, except for the last one, follow from the analysis in the text. The claim that the optimal FDSE contract strictly dominates contracts without FDSE follows from the fact, established in Proposition 2, that the optimal contract in the absence of FDSE entails $\mathbf{e}_1^* \neq \mathbf{e}_1^{FB}$ and $\mathbf{e}_2^* \neq \mathbf{e}_2^{FB}$. ■

Proof of Proposition 8. Denote the two workers as A and B and consider first FDSE. With two workers, there are two possible evaluations, m_L and $m_H > m_L$. The expected ability of the worker who got the evaluation m_H (say, worker A) is $x_H \equiv x(m_H) = E(a_A|a_A > a_B) = 2\int_0^{\bar{a}} aH(a)h(a)da$. Similarly, the expected ability of the worker with evaluation m_L (worker B) is $x_L \equiv x(m_L) = E(a_B|a_A > a_B) = 2\int_0^{\bar{a}} a[1 - H(a)]h(a)da$. Worker i's second period effort is then $e_{2k}^i = \beta^{SB} x_i g_k$, $i = L, H$; $k = 1, 2, ..., K$. In $t = 1$, both workers provide efforts $e_{1k}^{FB} = E(a)f_k$. The total surplus under FDSE, TS^{FDSE}, is then

$$\begin{aligned}
TS^{FDSE} &= 2E_a[a\mathbf{e}_1^{FB} \cdot \mathbf{f} - \Psi(\mathbf{e}_1^{FB})] + x_H \mathbf{e}_2^H \cdot \mathbf{f} + x_L \mathbf{e}_2^L \cdot \mathbf{f} - \Psi(\mathbf{e}_2^H) - \Psi(\mathbf{e}_2^L) \\
&= 2E_a\left[aE(a)\mathbf{f}\cdot\mathbf{f} - \frac{1}{2}[E(a)]^2\,\mathbf{f}\cdot\mathbf{f}\right] + \beta^{SB}\left(x_H^2 + x_L^2\right)\mathbf{g}\cdot\mathbf{f} - \left[\beta^{SB}\right]^2 \frac{x_H^2 + x_L^2}{2}\mathbf{g}\cdot\mathbf{g} \\
&= [E(a)]^2 \|\mathbf{f}\|^2 + \beta^{SB}\left(x_H^2 + x_L^2\right)\|\mathbf{f}\|\,\|\mathbf{g}\|\cos\theta - \left[\beta^{SB}\right]^2 \frac{x_H^2 + x_L^2}{2}\|\mathbf{g}\|^2 \\
&= [E(a)]^2 \|\mathbf{f}\|^2 + \left[\beta^{SB}\right]^2 \frac{x_H^2 + x_L^2}{2}\|\mathbf{g}\|^2,
\end{aligned}$$

so that $\lim_{\theta\to 0} TS^{FDSE} = [E(a)]^2 \|\mathbf{f}\|^2$ (because $\lim_{\theta\to 0}\beta^{SB} = \lim_{\theta\to 0}\frac{\|\mathbf{f}\|}{\|\mathbf{g}\|}\cos\theta = 0$).

Without FDSE, $e_{2k}^{*} = a\beta^{SB} g_k$, and the total surplus, TS^0, is

$$\begin{aligned} TS^0 &= 2E_a[a\mathbf{e}_1^{*}\cdot\mathbf{f} - \Psi(\mathbf{e}_1^{*}) + a\mathbf{e}_2^{*}\cdot\mathbf{f} - \Psi(\mathbf{e}_2^{*})] \\ &= 2E_a[a\mathbf{e}_1^{*}\cdot\mathbf{f} - \Psi(\mathbf{e}_1^{*}) + \beta^{SB} a^2 \mathbf{g}\cdot\mathbf{f} - \frac{1}{2}\left[\beta^{SB}\right]^2 a^2 \mathbf{g}\cdot\mathbf{g}] \\ &= 2E_a[a\mathbf{e}_1^{*}\cdot\mathbf{f} - \Psi(\mathbf{e}_1^{*})] + \left[\beta^{SB}\right]^2 a^2 \left\|\mathbf{g}\right\|^2. \end{aligned} \tag{A11}$$

Now, using the expression for $\delta^{*}(a)$ in Proposition 3, $\lim_{\theta\to 0}\beta^{SB} = 0$ implies that both $\delta^{*}(a)$ and $\delta^{*\prime}(a)$ converge to zero as $\theta \to 0$. Consequently, the left hand side of (B3) also converges to zero, and so does $C^{\max}$ in constraint (B20) in Appendix B. This in turn implies $\lim_{\theta\to 0} C^{*} = 0$, where C^{*} solves Problem (2) in Appendix B. (B22) and (B24) then yield $\lim_{\theta\to 0} b_1^{*} = \lim_{\theta\to 0}\beta^{SB} = 0$. Hence, from (B19), it must be $\lim_{\theta\to 0} e_{1k}^{*} = E(a) g_k \lim_{\theta\to 0} b_1^{*} + \left\|\mathbf{g}\right\|^2 f_k \lim_{\theta\to 0}\frac{C^{*}}{\mathbf{e}_1^{*}\cdot\mathbf{f}} = 0$ for each k, so that $\lim_{\theta\to 0}\mathbf{e}_1^{*}\cdot\mathbf{f} = \lim_{\theta\to 0}\mathbf{e}_1^{*}\cdot\mathbf{e}_1^{*} = 0$. This shows that $\lim_{\theta\to 0} TS^0 = 0 < \lim_{\theta\to 0} TS^{FDSE}$. Therefore, there exists a $\theta^{*} > 0$ such that $TS^0 < TS^{FDSE}$ for all $\theta \leq \theta^{*}$. Setting $c^{*} \equiv \cos\theta^{*}$ concludes the proof of the first claim in the proposition.

To obtain the second claim, let $\theta \to 1$. Then $\lim_{\theta\to 1}\beta^{SB} = \frac{\|\mathbf{f}\|}{\|\mathbf{g}\|}$, so that $\lim_{\theta\to 1} TS^{FDSE} = [E(a)]^2 \left\|\mathbf{f}\right\|^2 + \frac{x_H^2 + x_L^2}{2}\left\|\mathbf{f}\right\|^2$. As for TS^0, set $C = 0$ and optimize over b_1 and $\mathbf{e}_1$. Denote the solution as b_1^0 and $\mathbf{e}_1^0$ and observe that, by definition, $b_1^0 = \beta^{SB}$, so that $\lim_{\theta\to 1}\mathbf{e}_1^0 = \mathbf{e}_1^{FB}$. Then, using (A11), the fact that $\mathbf{e}_1^{*}$ maximizes TS^0 implies $\lim_{\theta\to 1} TS^0 \geq \lim_{\theta\to 1} 2E_a[a\mathbf{e}_1^0\cdot\mathbf{f} - \Psi(\mathbf{e}_1^0)] + \left[\beta^{SB}\right]^2 a^2 \left\|\mathbf{g}\right\|^2 = [E(a)]^2\left\|\mathbf{f}\right\|^2 + E(a^2)\left\|\mathbf{f}\right\|^2$. Because the true distribution of a is a mean-preserving spread of the beliefs $x(m_H)$ and $x(m_L)$, it must be $E(a^2) > \frac{x_H^2 + x_L^2}{2}$, which yields $\lim_{\theta\to 1} TS^0 > \lim_{\theta\to 1} TS^{FDSE}$. Therefore, there exists a $\theta^{**} < 1$ such that $TS^0 \geq TS^{FDSE}$ for all $\theta \geq \theta^{**}$. Setting $c^{**} \equiv \cos\theta^{**}$ concludes the proof. ■

B. Appendix B: Analysis of problem (P')

It will prove useful to restate the problem in terms of $\delta(a) \equiv a\beta(a)$:

$$\text{(P')} \qquad \max_{b_1,\delta(m),\mathbf{e}_1} \int_0^{\bar{a}} \left[a\mathbf{e}_1\cdot\mathbf{f} - \Psi(\mathbf{e}_1) + \left\|\mathbf{g}\right\|^2 \delta(a)\left[a\beta^{SB} - \frac{\delta(a)}{2}\right]\right] h(a)da$$

subject to

$$\begin{aligned} e_{1k} &= \int_0^{\bar{a}} \left[ab_1 g_k + \frac{\left\|\mathbf{g}\right\|^2 f_k}{\mathbf{e}_1\cdot\mathbf{f}} a\left[a\beta^{SB} - \delta(a)\right]\delta'(a)\right] h(a)da, \quad k = 1, 2, ..., K; \qquad \text{(B1)} \\ \delta(a) &\geq 0; \quad \delta(0) = 0. \end{aligned}$$

Constraint (B1) can be rearranged as follows:

$$\int_0^{\bar{a}} a\left[a\beta^{SB}-\delta(a)\right]\delta'(a)h(a)da=\frac{\mathbf{e}_1\cdot\mathbf{f}}{\|\mathbf{g}\|^2}\frac{[e_{1k}-E(a)b_1g_k]}{f_k},\quad k=1,2,...,K. \tag{B2}$$

Recalling that $b_3(m)=0$, so that $\delta(a)=ab_2(a)$ is independent of $\mathbf{e}_1$, (B2) shows that (P') is separable into two self-contained problems: (1) Optimization over $\delta(m)$, taking b_1 and $\mathbf{e}_1$ as given, and (2) optimization over b_1 and $\mathbf{e}_1$, taking into account the effect on $\delta(m)$.

B.1. Problem (1): Optimization with respect to $\delta(m)$.

Step 1. Problem setup. Note that with respect to Problem (1), (B2) is just a single constraint: Because the left hand side of (B2) does not depend on k, the right hand side cannot depend on k either, i.e. it must be $\frac{\mathbf{e}_1\cdot\mathbf{f}}{\|\mathbf{g}\|^2}\frac{[e_{1k}-E(a)b_1g_k]}{f_k}=C$ for all k, where C is a constant. The choice of C is analyzed in Problem (2) below; here, C is treated as exogenous. Thus, (ignoring the constant $\|\mathbf{g}\|^2$) the problem of optimizing with respect to $\delta(m)$ can be stated as

$$\text{(P1)}\qquad \max_{\delta(a)}\int_0^{\bar{a}}\delta(a)\left[a\beta^{SB}-\frac{\delta(a)}{2}\right]h(a)da$$

subject to

$$\int_0^{\bar{a}} a\left[a\beta^{SB}-\delta(a)\right]\delta'(a)h(a)da=C. \tag{B3}$$

It will be proven in Problem (2) and taken here as given that $C>0$.

Program (P1) is an isoperimetric optimal control problem, i.e., an optimal control problem with an integral constraint. To formulate it as a proper optimal control problem, define a new control variable $u(a)=\delta'(a)$ and a new state variable $y(a)=\int_0^a t\left[t\beta^{SB}-\delta(t)\right]u(t)h(t)dt$. This transforms the problem to

$$\text{(P1')}\qquad \max_{\delta,y,u}\int_0^{\bar{a}}\delta(a)\left[a\beta^{SB}-\frac{\delta(a)}{2}\right]h(a)da$$

subject to

$$y'(a) = a\left[a\beta^{SB}-\delta(a)\right]u(a)h(a); \tag{B4}$$
$$\delta'(a) = u(a); \tag{B5}$$
$$y(0) = 0;\quad y(\bar{a})=C; \tag{B6}$$
$$\delta(a) \geq 0;\quad \delta(0)=0, \tag{B7}$$

where u is a control variable and y and δ are state variables.

Step 2. Necessary conditions. Let $\lambda(a)$ and $\mu(a)$ be the multiplier functions that go with the state variables y and δ respectively and $\eta(a)$ the multiplier that goes with (B7). The Hamiltonian for this problem is then

$$H(a, y, \delta, u, \lambda, \mu) = \delta\left(a\beta^{SB} - \frac{\delta}{2}\right)h + \lambda a\left(a\beta^{SB} - \delta\right)uh + \mu u + \eta\delta.$$

Pontryiagin's maximum principle says that any solution to (P1'), denoted by $\delta^*(a)$, $y^*(a)$, $u^*(a)$, $\lambda^*(a)$, $\mu^*(a)$, must satisfy (B4)-B(7), plus

$$\begin{aligned}
u &= \arg\max H = \arg\max \delta\left(a\beta^{SB} - \frac{\delta}{2}\right)h + \lambda a\left(a\beta^{SB} - \delta\right)uh + \mu u + \eta\delta && \text{(B8)}\\
\lambda' &= -H_y = 0 && \text{(B9)}\\
\mu' &= -H_\delta = -\left(a\beta^{SB} - \delta\right)h + \lambda a u h - \eta && \text{(B10)}\\
\eta\delta &= 0, && \text{(B11)}
\end{aligned}$$

and the transversality condition

$$\mu(\bar{a}) = 0. \qquad \text{(B12)}$$

Given that H is affine in u, the above is a singular control problem with an unbounded control. This suggests that the solution entails a singular arc on some interval $I \subset [0, \bar{a}]$. Along this arc, the solution must lie on the singular surface defined by $H_u = 0$, $\frac{d}{da}H_u = 0$, ..., and $\frac{d^r}{da^r}H_u = 0$, where r is the order of the singular arc, i.e., the smallest positive integer r such that $\frac{\partial}{\partial u}\left(\frac{d^r}{da^r}H_u\right) \neq 0$ (see, e.g. Chachuat, 2007, pp. 145-6). Noting that (B9) implies that λ is a constant, we have

$$H_u = \lambda a\left(a\beta^{SB} - \delta\right)h + \mu = 0. \qquad \text{(B13)}$$

Using (B5) and (B10), this yields

$$\begin{aligned}
\frac{d}{da}(H_u) &= \lambda\left(2a\beta^{SB} - \delta - a\delta'\right)h + \lambda a\left(a\beta^{SB} - \delta\right)h' + \mu'\\
&= \lambda\left(2a\beta^{SB} - \delta - au\right)h + \lambda a\left(a\beta^{SB} - \delta\right)h' - \left(a\beta^{SB} - \delta\right)h + \lambda a u h - \eta\\
&= \lambda\left(2a\beta^{SB} - \delta\right)h + \lambda a\left(a\beta^{SB} - \delta\right)h' - \left(a\beta^{SB} - \delta\right)h - \eta = 0, && \text{(B14)}
\end{aligned}$$

which does not depend on u. Next,

$$\frac{d^2}{da^2}(H_u) = \lambda\left(2\beta^{SB}-u\right)h+\lambda\left(4a\beta^{SB}-2\delta-au\right)h'+\lambda a\left(a\beta^{SB}-\delta\right)h'' \\ -\left(\beta^{SB}-u\right)h-\left(a\beta^{SB}-\delta\right)h', \tag{B15}$$

so that $\frac{\partial}{\partial u}\left(\frac{d^2}{da^2}H_u\right)=(1-\lambda)(h+ah')\neq 0$ as long as $\lambda\neq 1$ (which will be verified shortly). Thus, the singular arc is given by (B13)-(B15).

If $1-\lambda(1+\varepsilon)\neq 0$ (again verified shortly), solving for δ from (B14) yields

$$\delta^*(a)=a\beta^{SB}\left[1-\frac{\lambda}{1-\lambda(1+\varepsilon)}\right]+\frac{\eta}{h(a)\left[1-\lambda(1+\varepsilon)\right]},$$

where $\varepsilon(a)\equiv\frac{ah'(a)}{h(a)}$ is the elasticity of the distribution function $h(.)$ at a. Combined with constraint (B11), this means that

$$\delta^*(a)=\max\left\{0,a\beta^{SB}\left[1-\frac{\lambda}{1-\lambda(1+\varepsilon)}\right]\right\}. \tag{B16}$$

The optimal λ^* is then determined by plugging (B16) to constraint (B3) and solving for λ. Appendix C shows that a solution to (B3) exists if and only if $C\in[0,C^{\max}]$, where $C^{\max}>0$, and that $\lambda^*\in\left(0,\frac{1}{3}+\phi_2(M)\right)$, where M is as in Assumption 1 and $\phi_2(M)>0$, with $\lim_{M\to 0}\phi_2(M)=0$. Note that $\lambda^*<\frac{1}{3}+\phi_2(M)$ implies $1-\lambda(1+\varepsilon)>0$ for all ε when M is small, verifying that the denominator in (B16) is non-zero.

Step 3. End points. Equation (B16) satisfies the requirement that $\delta(0)=0$, but not the transversality condition (B12): Substituting (B12) to (B13) and using the fact that u is unbounded implies that $\delta(\bar{a})=\bar{a}\beta^{SB}$. This, however, is generically not compatible with (B16). Consequently, the optimal solution has an impulse at $a=\bar{a}$: The optimal δ is given by (B16) for $a\in I=[0,\bar{a})$ and is then transported via an impulse to $\delta(\bar{a})=\bar{a}\beta^{SB}$ at $a=\bar{a}$.[22]

Step 4. Sufficiency. It is straightforward to check that the Hessian of H is indefinite. Consequently, the Mangasarian sufficiency conditions for global maximum are not satisfied and an alternative way of proving that (B16) solves the problem is needed. This will be done by showing that (B16) cannot be a minimum. Since any solution must satisfy (B16), the above must be a maximum. Thus, suppose, as a way of contradiction, that the above singular arc is a global minimum. Then it is also a local minimum, which requires that the

[22] On impulses in singular optimal control problems see e.g. Bryson and Ho (1975).

generalized Legendre-Clebsch condition for minimum holds, i.e., $-\frac{\partial}{\partial u}\left(\frac{d^2}{da^2}H_u\right) > 0$, or

$$-(1-\lambda)(1+\varepsilon) > 0. \tag{B17}$$

Given that $1+\varepsilon > 0$ for M small, (B17) holds only if $\lambda > 1$, which for M small contradicts $\lambda^* < \frac{1}{3} + \phi_2(M)$ established in Appendix C. Therefore, the above arc must be a maximum. (Observe that the generalized Legendre-Clebsch condition for local maximum is the reverse of (B17), which holds for M small.)

Step 5. The condition imposed by Lemma 1. Lemma 1 says that for the solution to problem (P1) to be a part of the solution to the original problem (P), $a\beta(a)\left[\beta^{SB} - \beta(a)\right]$ must be non-decreasing in a. Using (B16) and $\delta(a) = a\beta(a)$, we have

$$a\beta(a)\left[\beta^{SB} - \beta(a)\right] = a\beta^{SB}\frac{\lambda\left[1-\lambda\left(2+\varepsilon\right)\right]}{\left[1-\lambda\left(1+\varepsilon\right)\right]^2},$$

which is non-decreasing in a if and only if

$$\beta^{SB}\left[1-\lambda\left(2+\varepsilon\right)\right] + \lambda\varepsilon'\left[2\delta(a) - a\beta^{SB}\right] \geq 0. \tag{B18}$$

Given that $\lambda^* < \frac{1}{3} + \phi_2(M)$ and $\lim_{M\to 0}\phi_2(M) = \lim_{M\to 0}\varepsilon(a) = \lim_{M\to 0}\varepsilon'(a) = 0$, it must be that $\lim_{M\to 0} LHS$(B18)> 0. Thus, (B18) holds for M small.

B.2. Problem (2): Optimization with respect to e_1, b_1, and C.

Step 1. Problem setup. Let $V(\delta^*, \lambda^*, C)$ be the principal's optimal value function from problem (P1), i.e., $V(\delta^*, \lambda^*, C) \equiv \int_0^{\bar{a}} \delta^*(a)\left[a\beta^{SB} - \frac{\delta^*(a)}{2}\right]h(a)da$, and denote by π her total expected profit over the two periods. Problem (2) can then be written as

(P2) $$\max_{b_1,\mathbf{e}_1,C} \pi = \max_{b_1,\mathbf{e}_1,C} E(a)\mathbf{e}_1\cdot\mathbf{f} - \Psi(\mathbf{e}_1) + \|\mathbf{g}\|^2 V(\delta^*, \lambda^*, C)$$

subject to

$$\begin{aligned} e_{1k} &= E(a)b_1g_k + \frac{C\|\mathbf{g}\|^2}{\mathbf{e}_1\cdot\mathbf{f}}f_k, \quad k = 1,2,...,K; \qquad \text{(B19)} \\ C &\geq 0; \end{aligned}$$

subject to $\delta(a)$ being determined by (B3) and B(6); and subject to C being feasible (i.e., such that the set of $\delta(a)$ that satisfy (B3) is non-empty). Appendix C shows that the feasibility constraint for C can be expressed as

$$C \leq C^{\max}, \tag{B20}$$

where $C^{\max} > 0$.

Step 2. First order conditions. The dynamic Envelope Theorem for the fixed endpoint class of optimal control problems (e.g., Theorem 9.1 in Caputo, 2005, p. 232) implies that $\frac{\partial V(\delta^*,\lambda^*,C)}{\partial C} = -\lambda^*$. Let μ_C be the Lagrange multiplier associated with constraint (B20). The first order conditions for problem (P2) are then

$$\frac{\partial \pi}{\partial C} = \sum_{k=1}^{K} \frac{\partial e_{1k}}{\partial C} \left[E(a) f_k - e_{1k}\right] - \lambda^* \|\mathbf{g}\|^2 + \mu_C = 0, \tag{B21}$$

$$\frac{\partial \pi}{\partial b_1} = \sum_{k=1}^{K} \frac{\partial e_{1k}}{\partial b_1} \left[E(a) f_k - e_{1k}\right] = 0, \quad \text{and} \tag{B22}$$

$$C\mu_C = 0, \quad \mu_C \geq 0, \tag{B23}$$

where, from (B19),

$$\frac{\partial e_{1k}}{\partial C} = \frac{\|\mathbf{g}\|^2 f_k}{\mathbf{e}_1 \cdot \mathbf{f} + \|\mathbf{g}\|^2 \frac{C f_k^2}{\mathbf{e}_1 \cdot \mathbf{f}}} > 0 \quad \text{and} \quad \frac{\partial e_{1k}}{\partial b_1} = \frac{E(a) g_k \mathbf{e}_1 \cdot \mathbf{f}}{\mathbf{e}_1 \cdot \mathbf{f} + \|\mathbf{g}\|^2 \frac{C f_k^2}{\mathbf{e}_1 \cdot \mathbf{f}}} > 0. \tag{B24}$$

Step 3. $C^* > 0$. To see this, suppose $C = 0$. Then $\frac{\partial e_{1k}}{\partial C} = \frac{\|\mathbf{g}\|^2 f_k}{\mathbf{e}_1 \cdot \mathbf{f}}$, $\frac{\partial e_{1k}}{\partial b_1} = E(a) g_k$, $e_{1k} = E(a) b_1 g_k$, and (from (B3) and (B16)) $\lambda^* = 0$, so that (B22) yields $b_1 = \beta^{SB} = \frac{\|\mathbf{f}\|}{\|\mathbf{g}\|} \cos\theta$ and $\frac{\partial \pi}{\partial C}$ becomes

$$\begin{aligned}
\frac{\partial \pi}{\partial C}|_{C=0} &= E(a) \frac{\|\mathbf{g}\|^2}{\mathbf{e}_1 \cdot \mathbf{f}} \sum_{k=1}^{K} \left[f_k^2 - \beta^{SB} f_k g_k\right] + \mu_C \\
&= E(a) \frac{\|\mathbf{g}\|^2}{\mathbf{e}_1 \cdot \mathbf{f}} \left[\|\mathbf{f}\|^2 - \beta^{SB} \|\mathbf{f}\| \|\mathbf{g}\| \cos\theta\right] + \mu_C \\
&= E(a) \frac{\|\mathbf{g}\|^2 \|\mathbf{f}\|^2}{\mathbf{e}_1 \cdot \mathbf{f}} \left(1 - \cos^2\theta\right) + \mu_C > 0.
\end{aligned}$$

Hence, it must be $C^* > 0$.

Step 4. $C^* < \frac{E(a)\mathbf{e}_1 \cdot \mathbf{f}}{\|\mathbf{g}\|^2}$. Suppose $C \geq \frac{E(a)\mathbf{e}_1 \cdot \mathbf{f}}{\|\mathbf{g}\|^2}$. Then (B19) implies $e_{1k} \geq E(a) b_1 g_k +$

$E(a)f_k,\ k=1,2,...,K$. Plugging this to $\frac{\partial\pi}{\partial C}$ in (21) yields

$$\frac{\partial \pi}{\partial C} \leq -\sum_{k=1}^{K}\frac{\partial e_{1k}}{\partial C}E(a)b_1 g_k - \lambda^* \left\|\mathbf{g}\right\|^2 + \mu_C.$$

Now suppose for the moment that (P2) is not constrained by (B20). Then $\mu_C=0$ and the above implies $\frac{\partial\pi}{\partial C}<0$ for all $C \geq \frac{E(a)\mathbf{e}_1\cdot\mathbf{f}}{\|\mathbf{g}\|^2}$, where the inequality follows from $\lambda^*>0$ for $C>0$, established in Appendix C. Therefore, it must be $C^* < \frac{E(a)\mathbf{e}_1\cdot\mathbf{f}}{\|\mathbf{g}\|^2}$ if C is unconstrained and hence also if C is constrained by (B20).

Step 5. $b_1^*>0$. Suppose $b_1=0$. Because $C<\frac{E(a)\mathbf{e}_1\cdot\mathbf{f}}{\|\mathbf{g}\|^2}$, (B19) then implies $e_{1k} < E(a)f_k,\ k=1,2,...,K$, so that, from (B22), $\frac{\partial\pi}{\partial b_1}|_{b_1=0}>0$. Hence, it must be $b_1^*>0$.

C. Appendix C: Technical details regarding λ^* and constraint (B20)

Define

$$V(\lambda) \equiv \int_0^{\bar{a}} \delta^*(a)\left[a\beta^{SB} - \frac{\delta^*(a)}{2}\right]h(a)da \tag{C1}$$

$$Z(\lambda) \equiv \int_0^{\bar{a}} a\left[a\beta^{SB} - \delta^*(a)\right]\delta^{*\prime}(a)h(a)da, \tag{C2}$$

where $\delta^*(a)$ is given by (B16). That is, $V(\lambda)$ is the principal's optimal value function from problem (P1) in Appendix B as a function of λ, and $Z(\lambda)$ is the left hand side of constraint (B3) evaluated at $\delta^*(a)$. Also, note that for M small,

$$\delta^*(a) = 0 \text{ if } \lambda \in [\frac{1}{2+\varepsilon}, \frac{1}{1+\varepsilon}) \tag{C3a}$$

$$\delta^*(a) > 0 \text{ otherwise.} \tag{C3b}$$

Step 1. Shape of $V(\lambda)$. Let $\hat{V}(\lambda,\varepsilon)\equiv \delta^*(a)\left[a\beta^{SB}-\frac{\delta^*(a)}{2}\right]$. By (C3), $\hat{V}(\lambda,\varepsilon)=0$ for $\lambda\in[\frac{1}{2+\varepsilon},\frac{1}{1+\varepsilon})$. Let $\varepsilon^{\max}\equiv \max_a\{\varepsilon(a)\}$ and $\varepsilon^{\min}\equiv\min_a\{\varepsilon(a)\}$. Then $\hat{V}(\lambda,\varepsilon)=0$ for all $\lambda\in[\frac{1}{2+\varepsilon^{\min}},\frac{1}{1+\varepsilon^{\max}})$, so that $V(\lambda)=0$ for $\lambda\in[\frac{1}{2+\varepsilon^{\min}},\frac{1}{1+\varepsilon^{\max}})$.

Next, suppose $\lambda\notin[\frac{1}{2+\varepsilon},\frac{1}{1+\varepsilon}]$. Then substituting (C3) into (C1) yields

$$\hat{V}(\lambda,\varepsilon)=\frac{1}{2}\left(\beta^{SB}\right)^2 a^2\left[1-\frac{\lambda^2}{\left[1-\lambda(1+\varepsilon)\right]^2}\right]=\frac{1}{2}\left(\beta^{SB}\right)^2 a^2\left[1-\frac{1}{\left[\frac{1}{\lambda}-(1+\varepsilon)\right]^2}\right],$$

so that $\frac{\partial \hat{V}(\lambda,\varepsilon)}{\partial \lambda} < 0$ if $0 < \lambda < \frac{1}{2+\varepsilon}$ and $\frac{\partial \hat{V}(\lambda,\varepsilon)}{\partial \lambda} > 0$ if $\lambda < 0$ or if $\lambda > \frac{1}{1+\varepsilon}$. Furthermore, we have $\hat{V}(\lambda,\varepsilon) = 0$ iff $\lambda^2 = [1-\lambda(1+\varepsilon)]^2$, i.e., iff either $\lambda = \lambda_1(\varepsilon) = \frac{1}{2+\varepsilon}$ or $\lambda = \lambda_2(\varepsilon) = \frac{1}{\varepsilon}$. Finally, note that $\hat{V}(\lambda,\varepsilon)$ is continuous in λ except for $\hat{\lambda}(\varepsilon) = \frac{1}{1+\varepsilon}$, and that $\lim_{\lambda \uparrow \hat{\lambda}(\varepsilon)} \hat{V}(\lambda,\varepsilon) = \lim_{\lambda \downarrow \hat{\lambda}(\varepsilon)} \hat{V}(\lambda,\varepsilon) = -\infty$ and $\hat{V}(0,\varepsilon) = \frac{1}{2}\left(\beta^{SB}\right)^2 a^2 > 0$ for all ε.[23] Hence,

$$\begin{aligned}
V'(\lambda) &> 0 \text{ for } \lambda < 0 && \text{(C4a)}\\
V'(\lambda) &< 0 \text{ for } 0 < \lambda < \frac{1}{2+\varepsilon^{\max}} && \text{(C4b)}\\
V'(\lambda) &\leq 0 \text{ for } \frac{1}{2+\varepsilon^{\max}} \leq \lambda \leq \frac{1}{2+\varepsilon^{\min}} && \text{(C4c)}\\
V'(\lambda) &= 0 \text{ for } \frac{1}{2+\varepsilon^{\min}} \leq \lambda < \frac{1}{1+\varepsilon^{\max}} && \text{(C4d)}\\
V'(\lambda) &\geq 0 \text{ for } \lambda \geq \frac{1}{1+\varepsilon^{\max}}, && \text{(C4e)}
\end{aligned}$$

and

$$\begin{aligned}
V(\lambda) &> 0 \text{ for } \lambda_0 \leq \lambda \leq \frac{1}{2+\varepsilon^{\max}} \text{ where } \lambda_0 < 0 && \text{(C5a)}\\
V(\lambda) &\leq 0 \text{ for } \frac{1}{2+\varepsilon^{\max}} \leq \lambda \leq \frac{1}{2+\varepsilon^{\min}} && \text{(C5b)}\\
V(\lambda) &= 0 \text{ for } \frac{1}{2+\varepsilon^{\min}} \leq \lambda < \frac{1}{1+\varepsilon^{\max}} && \text{(C5c)}\\
V(\lambda) &< 0 \text{ for } \frac{1}{1+\varepsilon^{\min}} < \lambda < \frac{1}{\varepsilon^{\max}} && \text{(C5d)}\\
V(\lambda) &> 0 \text{ for } \lambda > \frac{1}{\varepsilon^{\min}}. && \text{(C5e)}
\end{aligned}$$

Step 2. Shape of $Z(\lambda)$. Let $\hat{Z}(\lambda,\varepsilon) \equiv \delta^{*\prime}(a)\left[a\beta^{SB} - \delta^*(a)\right]$. Then (C3a) implies $\hat{Z}(\lambda,\varepsilon) = 0$ for $\lambda \in [\frac{1}{2+\varepsilon^{\min}}, \frac{1}{1+\varepsilon^{\max}})$, so that $Z(\lambda) = 0$ for all $\lambda \in [\frac{1}{2+\varepsilon^{\min}}, \frac{1}{1+\varepsilon^{\max}})$.

Next, suppose $\lambda \notin [\frac{1}{2+\varepsilon}, \frac{1}{1+\varepsilon}]$. Then substituting (C3) into (C2) yields

$$\hat{Z}(\lambda,\varepsilon) = \frac{\lambda\left[1-\lambda(2+\varepsilon)\right]}{\left[1-\lambda(1+\varepsilon)\right]^2} + \frac{a\lambda^3\varepsilon'}{\left[1-\lambda(1+\varepsilon)\right]^3}, \tag{C6}$$

so that $\frac{\partial \hat{Z}(\lambda,\varepsilon)}{\partial \lambda} = \frac{[1-\lambda(1+\varepsilon)][1-\lambda(3+\varepsilon)]+3a\lambda^2\varepsilon'}{[1-\lambda(1+\varepsilon)]^4}$. For M small, we thus have

[23] $\lambda \uparrow \hat{\lambda}(\varepsilon)$ indicates convergence of λ to $\hat{\lambda}(\varepsilon)$ from below; similarly, $\downarrow$ indicates convergence of from above.

$$
\begin{aligned}
Z'(\lambda) &> 0 \text{ if } \lambda < \frac{1}{3} - \phi_1(M) && \text{(C7a)}\\
Z'(\lambda) &\leq 0 \text{ if } \frac{1}{3} + \phi_2(M) < \lambda < \frac{1}{2+\varepsilon^{\min}} && \text{(C7b)}\\
Z'(\lambda) &= 0 \text{ if } \frac{1}{2+\varepsilon^{\min}} \leq \lambda < \frac{1}{1+\varepsilon^{\max}} && \text{(C7c)}\\
Z'(\lambda) &\gtrless 0 \text{ if } \frac{1}{1+\varepsilon^{\max}} \leq \lambda < \frac{1}{1+\varepsilon^{\min}} + \phi_3(M) && \text{(C7d)}\\
Z'(\lambda) &> 0 \text{ if } \lambda > \frac{1}{1+\varepsilon^{\min}} + \phi_3(M), && \text{(C7e)}
\end{aligned}
$$

where $\phi_i(M) > 0$ and $\lim_{M\to 0} \phi_i(M) = 0$, $i = 1, 2, 3$.

Now, from (C6), there exists a $\lambda^+ \in (0, \infty)$ such that $Z(\lambda) < 0$ for all $\lambda \geq \lambda^+$. Further, $Z(0) = 0$, $\lim_{M\to 0} Z(\frac{1}{3}) = \frac{1}{4}E(a) > 0$, and $\lim_{M\to 0} Z(\frac{1}{2}) = 0$. This, together with (C7), implies that, for M small,

$$
\begin{aligned}
Z(\lambda) &< 0 \text{ for } \lambda < 0 && \text{(C8a)}\\
Z(\lambda) &> 0 \text{ for } 0 < \lambda < \frac{1}{2+\varepsilon^{\max}} && \text{(C8b)}\\
Z(\lambda) &\leq 0 \text{ for } \frac{1}{2+\varepsilon^{\min}} \leq \lambda < \frac{1}{1+\varepsilon^{\max}} && \text{(C8c)}\\
Z(\lambda) &\gtrless 0 \text{ for } \frac{1}{1+\varepsilon^{\max}} \leq \lambda < \frac{1}{1+\varepsilon^{\min}} + \phi_3(M) && \text{(C8d)}\\
Z(\lambda) &< 0 \text{ for } \lambda > \frac{1}{1+\varepsilon^{\min}} + \phi_3(M). && \text{(C8e)}
\end{aligned}
$$

Step 3. Solution to (B3) and the form of constraint (B20). Conditions (C8) imply that for $C > 0$, (C2) can hold only if either $\lambda \in \left(0, \frac{1}{2+\varepsilon^{\min}}\right)$, or, possibly, $\lambda \in \left(\frac{1}{1+\varepsilon^{\max}}, \frac{1}{1+\varepsilon^{\min}} + \phi_3(M)\right)$. But $\lim_{M\to 0} V(\lambda = \frac{1}{1+\varepsilon^{\min}}) = \lim_{M\to 0} V(\lambda = \frac{1}{1+\varepsilon^{\max}}) = -\infty$ implies that $\lambda \in \left(\frac{1}{1+\varepsilon^{\max}}, \frac{1}{1+\varepsilon^{\min}} + \phi_3(M)\right)$ cannot be an optimum. Therefore, it must be $\lambda^* \in \left(0, \frac{1}{2+\varepsilon^{\min}}\right)$. Furthermore, $Z(\lambda)$ is continuous on $\left[0, \frac{1}{2+\varepsilon^{\min}}\right]$, with $Z(0) = 0 \geq Z(\frac{1}{2+\varepsilon^{\min}})$ and with $Z(\frac{1}{3}) > 0$. Hence, $\max Z(\lambda)$ on $\left[0, \frac{1}{2+\varepsilon^{\min}}\right]$ exists and is positive. Denote this maximum as $C^{\max}$. Then by continuity, for every $C \in [0, C^{\max}]$ there exists a $\lambda \in \left[0, \frac{1}{2+\varepsilon^{\min}}\right]$ such that (C2) holds, whereas if there is a λ such that (C2) holds for $C > C^{\max}$, this λ cannot be a part of the solution to (P1). Consequently, the feasibility constraint on C can be expressed as $0 \leq C \leq C^{\max}$.

Finally, (C4) says that $V'(\lambda) \leq 0$ on $\left(0, \frac{1}{2+\varepsilon^{\min}}\right)$. Hence, if multiple λ solve (B3), then λ^* is the smallest of them. But, from (C7), $Z(0) = 0$ and $Z'(\lambda) \leq 0$ for all $\lambda \in \left(\frac{1}{3} + \phi_2(M), \frac{1}{2+\varepsilon^{\min}}\right)$. The smallest λ that solves (B3) therefore cannot exceed $\frac{1}{3} + \phi_2(M)$. That is, $\lambda^* \leq \frac{1}{3} + \phi_2(M)$.

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