

# Search, Design, and Market Structure<sup>1</sup>

Heski Bar-Isaac  
NYU

Guillermo Caruana  
CEMFI

Vicente Cuñat  
LSE

October, 2009

## Abstract

The Internet has made consumer search much easier, with consequences for competition, industry structure and product offerings. We explore these consequences in a rich but tractable model that allows for strategic design choices. We find a polarized market structure, where some firms choose designs aiming for broad-based audiences, while others target narrow niches. Such an industry structure can arise even when all firms and consumers are ex-ante identical. We perform comparative statics and show the effect of a fall in search costs on the designs, market shares, prices, and profits of different firms. In particular, a fall in search costs, through its effect on product designs, can lead to higher industry prices and profits. In characterizing sales distributions, our analysis is related to discussions of how the Internet has led to the prevalence of niche goods and the long-tail and superstar phenomena.

## 1 Introduction

The Internet has changed the nature of demand and competition in numerous industries. A significant and growing literature has sought to examine this impact both theoretically and empirically, drawing on older models of consumer search.<sup>2</sup> Much of this work has

---

<sup>1</sup>Previous versions have circulated as “Costly Search and Design.” We thank for their helpful comments Michael Baye, Juanjo Ganuza, Avi Goldfarb, Maarten Janssen, George Mailath, Eric Rasmussen, Michael Rauh, Andrew Rhodes, and excellent seminar participants at the IIOC (Boston 2009), the North American meeting of the Econometric Society (Boston 2009), 2nd Workshop on the Economics of Advertising and Marketing (St-Germain en Laye), the Madrid Summer Workshop in Economic Theory 2009, University of Pennsylvania, and the Stern work-in-progress lunch. Financial support from the NET Institute (<http://www.NETinst.org>) is gratefully acknowledged. Guillermo Caruana acknowledges the financial support of the Spanish Ministry of Science and Innovation through the Consolider-Ingenio 2010 Project “Consolidating Economics.”

Contact info: Bar-Isaac: [heski@nyu.edu](mailto:heski@nyu.edu); Department of Economics, Stern School of Business, NYU, 44 West 4th street 7-73, NYC, NY 10012 USA; Caruana: [caruana@cemfi.es](mailto:caruana@cemfi.es); Casado del Alisal 5, 28014 Madrid, Spain; and Cuñat: [v.cunat@lse.ac.uk](mailto:v.cunat@lse.ac.uk), Room A450, Department of Finance, London School of Economics, Houghton Street, London WC2A 2AE, U.K.

<sup>2</sup>A relatively early and influential example is Bakos (1997). Baye, Morgan and Scholten (2006) provide an excellent overview with a particular focus on price dispersion.

focused on the persistence of price dispersion, with less attention devoted to the impact on market structure. However, recently, there has been considerable interest, both popular and academic, in how new production and search technologies have changed the pattern of sales and the market shares of the most popular goods as compared to fringe goods in the “long tail.”<sup>3</sup>

This paper maintains a focus on market structure but allows for a richer set of firm strategies than typically considered. Specifically, it considers firms that choose the “design” or marketing of their products from a broad set of options. Our starting point is that firms, through their choices of marketing and product design, have some ability to affect the nature of the demand that they face.

A growing literature, notably Johnson and Myatt (2006) and Lewis and Sappington (1994), has considered these choices. More recently, Bar-Isaac, Caruana and Cuñat (2008, 2009) put more emphasis on consumers’ information-gathering decisions and highlight that these are co-determined with the firm’s pricing and marketing strategies in equilibrium. This literature has focused on monopoly settings. This paper, instead, is one of the first to extend this analysis to a competitive environment. In order to do so, we incorporate the notion of product design into an established model that considers consumers who search both to obtain price-quotes and to learn about the extent to which differentiated goods suit them (Wolinsky, 1986; Bakos, 1997; Anderson and Renault, 1999). In particular, the model allows us to view the impact of search engines, the Internet, communication technologies and information technologies in general as a fall in search costs and to consider its consequences.<sup>4</sup> This approach leads to a wide variety of results that shed light on the coexistence of niche goods with mass-market strategies, the related “long-tail” phenomenon and how search affects the nature of competition, industry structure, and the kinds of product offered.

Formally, we consider firms that compete by choosing price and “design” along the lines of Johnson and Myatt’s (2006) model of a monopoly rotating demand: Here, competitive

---

<sup>3</sup>The phrase was coined in an article in *Wired* (Anderson, 2004) and later expanded and developed in Anderson (2006). It refers to the well-documented and dramatic increase in the market share for goods in the tail of the sales distribution. See Brynjolfsson, Hu and Smith (2006) for a discussion and references to academic work.

<sup>4</sup>There is a small related literature that considers firms that vary design in response to falling search costs. Larson (2008) studies horizontal differentiation in a model of sequential search with a particular emphasis on welfare considerations in what can be viewed as a special case of our model. Kuksov (2004) presents a duopoly model where consumers know the varieties available (but not their location) prior to search, and different designs come with different costs associated; and Cachon, Terwiesch and Xu (forthcoming) and Watson (2007) focus specifically on multi-product firms, where consumers search costlessly within a firm but at some cost between firms. Our model allows for a continuum containing a broad range of designs and a much more general demand specification; the paper has a different focus and results from these papers, which, for example, do not consider sales distributions explicitly.

firms can choose from a range of designs that vary between broad market designs that are inoffensive to all consumers and more niche or quirky designs that consumers either love or loathe.<sup>5</sup> Consumers search among firms in a way that is standard in models of costly sequential search: Each consumer can pay a small cost to obtain a price-quote from an additional firm and learn about the extent to which that firm’s product suits his tastes.

The model generates a number of simple and interesting results. First, firms choose extremal product designs—that is, either a most-broad-based design or a most-niche design. Second, more-advantaged firms choose most-broad designs, while disadvantaged firms prefer most-niche designs.

Moreover, by allowing for an endogenous choice of product design, we are able to analyze both the direct effect of lower search costs on prices and the indirect effect through changes to the offered designs. We show that lower search costs induce a larger fraction of firms to choose niche designs.

The effect of more-niche designs on price can overcome the direct effect of competition, thus leading to prices and profits being non-monotonic in search costs. There is a clear intuition: With low search costs, and consumers visiting many stores, firms have to offer consumers something very attractive not only in terms of price, but also in terms of the utility that the good provides. This latter consideration leads firms to choose niche designs, but these niche designs effectively differentiate firms and so soften price competition.

Reduced search costs and endogenous designs also have interesting effects on sales distributions. Lower search costs allow consumers to find “better” firms. This, in turn, leads better firms to be even more successful. Thus, reduced search costs allow for superstar effects.<sup>6</sup> However, lower search costs also allow consumers to search longer for better-suited products. As a consequence, more firms provide niche products. This results in a long-tail effect where niche products have higher sales.<sup>7</sup> Thus, in our model, lower search costs can explain both superstar and long-tail effects, which can arise simultaneously (as shown empirically in Elberse and Oberholzer-Gee, 2006).<sup>8</sup> Moreover, the mechanisms and results do not rely on ex-ante firm heterogeneity, in contrast to this previous literature: These effects can arise even when all firms and consumers are ex-ante identical.

---

<sup>5</sup>Note that one does not need a physical design interpretation to induce demand rotations. Firms might similarly induce demand rotations through providing more or less information: In an e-commerce application, this might take the form of more- or less-detailed product descriptions.

<sup>6</sup>Goldmanis et al. 2009 consider better firms to be low-cost (rather than high-quality) and find such a superstar effect both theoretically and empirically.

<sup>7</sup>This is only a partial intuition insofar as consumer and firm behavior are co-determined in equilibrium.

<sup>8</sup>Further evidence on simultaneous long-tail and superstar effects appears in Osterreicher-Singer and Sundarajan (2008) and Tucker and Zhang (2007). Hervas-Drane (2009) provides further references and a model that contrasts two different channels (sequential search and ex-ante recommendations) through which the Internet might generate superstar and long-tail effects.

## 2 Model

There is a continuum of firms of measure 1. Each firm produces a single product. There is a continuum of consumers of measure  $m$ . Each consumer,  $l$ , has tastes described by a conditional utility function (not including any search costs) of the form

$$u_{li}(p_i) = -p_i + v_i + \varepsilon_{li} \quad (1)$$

if she buys product  $i$  at price  $p_i$ . The term  $v_i$  can be thought of as the natural quality of firm  $i$ . Meanwhile,  $\varepsilon_{li}$  can be interpreted as a match value between consumer  $l$  and product  $i$  and is the realization of a random variable with distribution  $F_i$ . We assume that realizations of  $\varepsilon_{li}$  are independent across firms and individuals.<sup>9</sup> These match components are intended to capture the fact that some products might be better suited to some consumers than to others. Note that we assume that consumers are risk-neutral.

A consumer incurs a search cost  $c$  to learn the price  $p_i$  and the match value  $\varepsilon_{li}$  for the product offered by any particular firm  $i$ . Consumers search sequentially. The utility of a consumer  $l$  is given by

$$u_{lk}(p_k) - kc, \quad (2)$$

if she buys product  $k$  at price  $p_k$  at the  $k$ th firm she visits. From now on, and for simplicity, we will omit the firm and consumer subscripts, unless there is ambiguity.

Firms cannot affect  $v$ , which we assume to be the exogenously given quality of the good, distributed according to some continuously differentiable distribution  $H(v)$  with support  $(\underline{v}, \bar{v})$ .

We introduce the notion of design by supposing that the firm can affect the distribution of the match-specific component of consumer tastes  $F_s$  by picking a design  $s \in S = [B, N]$ . That is, designs range from a most-broad ( $B$ ) to a most-niche ( $N$ ) design. A design  $s$  leads to  $\varepsilon_{li}$  distributed according to  $F_s(\theta)$  with support on some bounded interval  $(\underline{\theta}_s, \bar{\theta}_s)$  and logconcave and positive densities  $f_s(\theta)$ . Regardless of design and intrinsic quality, the firm produces goods at a marginal cost of 0.

The strategy for each firm, therefore, consists not only of a choice of price  $p$ , but also (in a departure from Wolinsky (1986), Bakos (1997) and Anderson and Renault (1999)) of a choice of a product design  $s \in D$ . We suppose that there are no costs associated with

---

<sup>9</sup>Taking these realizations to be independent, while consistent with the previous literature on search (Wolinsky (1986) and Anderson and Renault (1999)), is not without loss of generality. It does not permit modeling that different firms might attempt to target different niches. That is, there is no spatial notion of differentiation or product positioning. However, given that we assume a continuum of firms and no ability for consumers to determine location in advance, this assumption may be more reasonable. Some of the outcomes are similar to the ones of a spatial model (see Bakos, 1997).

choosing different designs  $s$ .

We follow Johnson and Myatt (2006) in supposing that different product designs induce demand rotations. Formally, there is a family of rotation points  $\theta_s^\dagger$  such that  $\frac{\partial F_s(\theta)}{\partial s} < 0$  for  $\theta > \theta_s^\dagger$  and  $\frac{\partial F_s(\theta)}{\partial s} > 0$  for  $\theta < \theta_s^\dagger$ ; further  $\theta_s^\dagger$  is increasing in  $s$ . The concept of a demand rotation is a formal approach to the notion that some designs lead to a wider spread in consumer valuations than others. In particular, a higher value of  $s$  should be interpreted as a more “quirky” product that appeals more to some consumers and less to others; the bounds on  $s$  correspond to most broad ( $B$ ) styles of design and most niche ( $N$ ) styles of design. This definition is general enough to accommodate a wide range of concepts of product design. It can accommodate differences in physical characteristics of the product that make it more or less appealing to particular customers. It can also be interpreted as the level of information provided to consumers before purchase. For the latter interpretation, the rotations must be mean-preserving spreads.

Our notion of equilibrium is Nash in consumer and firm strategies.<sup>10</sup> As is standard in the search literature (and will be shown below), a consumer’s search and purchase behavior can be described by a threshold rule  $U$ : She buys the current product obtaining  $u_i(p_i)$  if this is more than or equal to  $U$ , and continues searching otherwise. Therefore, in equilibrium, consumers choose a threshold  $U$ , while firms choose a pair  $(p, s)$  that depends on  $v$ .<sup>11</sup> One advantage to this notation is that  $U$  also represents the consumer surplus from participating in the market.

Finally, note that there always exist equilibria where consumers do not search and firms choose prohibitively high prices. We do not consider such equilibria if others exist.

### 3 Equilibrium

#### 3.1 Consumer behavior

Suppose that a consumer expects a firm of type  $v$  to choose the strategy  $(p_v, s_v)$ .<sup>12</sup> When the consumer currently holds a best alternative with utility  $u$ , then if the consumer samples an additional firm of type  $v$  she will prefer to buy its product if  $-p_v + v + \varepsilon > u$ . In this case, the additional utility obtained is  $v + \varepsilon - (u + p_v)$ , and so the expected incremental utility from searching one more firm that is expected to have design  $s_v$  and price  $p_v$  and to be of quality  $v$  is

---

<sup>10</sup>In particular, this implies passive beliefs: That is, if a consumer observes an off-equilibrium price or design, it does not affect her search and purchase rule.

<sup>11</sup>More broadly, we can allow firms to mix, so that each firm chooses an element  $\sigma_v \in \Delta(\mathbb{R} \times [B, N])$ .

<sup>12</sup>With a continuum of firm types and no atoms in the distribution, it is without loss of generality to assume that each type of firm chooses a pure strategy in design and price.

$$g_v(u) \equiv E_\varepsilon(v + \varepsilon - p - u | v + \varepsilon - p > u) \Pr(v + \varepsilon - p > u) = \int_{u+p-v}^{\infty} (v + \varepsilon - u - p) f_s(\varepsilon) d\varepsilon. \quad (3)$$

Finally, it is worth searching exactly one more firm if and only if the expected value of a search is worth more than the cost, where the final expectation is taken over  $v$  (with an implicit firm strategy for both price and design); that is, as long as  $E[g_v(u)] \geq c$ , or, equivalently, if  $u < U$  where  $U$  is implicitly defined by:

$$E[g_v(U)] = \int_{-\infty}^{\infty} \left( \int_{U+p_v-v}^{\infty} (v + \varepsilon - U - p_v) f_{s_v}(\varepsilon) d\varepsilon \right) h(v) dv = c. \quad (4)$$

Note that there is, at most, one solution to (4) since the left-hand side is strictly decreasing in  $U$  (as the integrand is decreasing in  $U$  and the lower limit of the inner integral is increasing in  $U$ ). For  $c$  large enough, there is no feasible positive  $U$  that satisfies (4): No consumer would ever continue searching and firms would have full monopoly power (as in Diamond, 1971). In other words, the consumer initiates search if and only if  $U \geq 0$ .

### 3.2 Firm profit maximization

Suppose that consumers are using a  $U$ -threshold strategy. Consider now the problem of a firm of type  $v$  maximizing profits by choosing  $(p, s)$ . Consumers who visit the firm would choose to buy as long as they receive a match  $\varepsilon$  such that  $v - p + \varepsilon > U$ . Thus, the probability of sale is  $1 - F_s(p + U - v)$ .

We define  $\rho$  as the expected probability that a consumer who visits a random firm buys from that firm; this is exogenous from the perspective of firm  $v$ . The expected number of consumers who visit firm  $v$  as a first visit is  $m$ , a further  $m(1 - \rho)$  visit the firm as a second visit,  $m(1 - \rho)^2$  as a third visit, and so on. Thus, the total number of visits is  $\frac{m}{\rho}$ . Each time a consumer visits firm  $v$ , she purchases with probability  $1 - F_s(p + U - v)$ . We can, therefore, write demand for firm  $v$  that chooses a design  $s$  and price  $p$  as

$$D_v(p, s) \equiv \frac{m}{\rho} (1 - F_s(p + U - v)). \quad (5)$$

and its profits as

$$\Pi = \frac{m}{\rho} p (1 - F_s(p + U - v)). \quad (6)$$

It is useful to define  $p_{vs}(U)$  as firm  $v$ 's profit-maximizing price when the consumer's threshold is  $U$  and the design strategy is  $s$ :

$$p_{vs}(U) \equiv \arg \max p (1 - F_s(p + U - v)). \quad (7)$$

This price is implicitly determined as

$$p_{vs}(U) = \frac{1 - F_s(p_{vs}(U) + U - v)}{f_s(p_{vs}(U) + U - v)}. \quad (8)$$

Now we present our first result. Note that all proofs in the paper are in the Appendix.

**Lemma 1** *The profit-maximizing price  $p_{vs}(U)$  associated with a design  $s$  for a firm of type  $v$ , when a consumer's stopping rule is given by  $U$ , is uniquely defined and is continuously decreasing in the consumers' reservation threshold  $U$ , and continuously increasing in the firm's quality,  $v$ . Further  $p_{vs}(U) + U$  is continuous and increasing in  $U$ .*

These properties are intuitive. A higher-quality firm charges a higher price, and firms charge lower prices when they face consumers with a higher reservation utility.

Given the definition of  $p_{vs}(U)$ , we can write profits as

$$\Pi = \frac{m}{\rho} p_{vs}(U) (1 - F_s(p_{vs}(U) + U - v)). \quad (9)$$

The firm's problem is to maximize this with respect to its remaining strategic variable  $s$ . Note that neither the optimal price nor the optimal design choice depends on  $m$  or  $\rho$ , as these are just constant factors in profits.<sup>13</sup>

Johnson and Myatt (2006) have shown in a monopoly model that, when designs are rotation-ordered, then profits are quasi-convex in design, and, so, a monopoly firm would always choose an extremal design. In our environment, taking the behavior of all other firms as given, the residual demand that a firm faces is still determined through a demand rotation. Since every firm is, in effect, a monopolist on the residual demand that it faces, the result still applies.

**Proposition 1** *Firms choose extremal designs, that is, every firm chooses either the most-niche ( $s = N$ ) or the most-broad ( $s = B$ ) design.*

To gain some intuition for this result, first consider the case when the optimal price at a given design  $s$  is below the point of rotation, so that the profit-maximizing quantity is greater than the quantity at the point of rotation  $1 - F_s(\theta_s^\dagger)$ . Then, decreasing  $s$  (and so “flattening” out demand) will lead to a greater quantity sold even if the price is kept fixed. Therefore, decreasing  $s$  must lead to higher profits. A similar argument applies when the optimal price is above the point of rotation.

---

<sup>13</sup>This highlights that search costs play a qualitatively different role from that of scale effects, which is, of course, a central point of Wolinsky (1986). As discussed by Anderson and Renault (1999), the limits when search costs tend to 0 and when the ratio of firms to consumers increases are quite different.

Using Proposition 1, we can restrict attention to equilibrium strategies in which firm  $v$  either chooses a broad design  $(p_{vB}, B)$ , or a niche one  $(p_{vN}, N)$ , where  $p_{vB}$  and  $p_{vN}$  are defined by (8) for  $s = B, N$  respectively.

Next, we show that high-quality firms are more likely than low-quality ones to adopt a broad strategy. We prove this as a corollary of a more general result: The more severe the competition that a firm faces (either because consumers are pickier and require more utility in order to purchase, or because the firm faces a disadvantage as compared to other firms), the more likely it is to choose the niche strategy. Loosely, the intuition here is that a firm in a disadvantageous position needs the consumer to “love” the good in order to buy it. The chances of this happening increase with a design that leads to dispersed valuations—a niche design.

**Proposition 2** *Suppose that a firm of type  $\hat{v}$  makes positive sales when facing consumers whose threshold rule is given by  $\hat{U}$  and is indifferent between choosing a broad design and a niche design. Then, if consumer behavior is characterized by  $U$ , any firm,  $v$ , of sufficiently low quality with  $U - v > \hat{U} - \hat{v}$  prefers a niche strategy, and any firm of sufficiently high quality with  $v > U - \hat{U} + \hat{v}$  prefers a broad strategy.*

Note that firms that make no sales are indifferent about the design they choose. However, it is convenient for the statement of results (while having no effect on equilibrium transactions) to assume that such firms respect the design choices implied by Proposition 2.

Next, define  $V$  as the solution to

$$p_{vB}(U)(1 - F_B(p_{vB}(U) + U - V)) = p_{vN}(U)(1 - F_N(p_{vN}(U) + U - V)). \quad (10)$$

If  $V$  lies in the feasible range  $[\underline{v}, \bar{v}]$ , then  $V$  captures the firm that is indifferent between choosing the broad or the niche strategy. If  $V$  falls outside this range, with some abuse of notation, we redefine it to be the appropriate extreme of the range.<sup>14</sup> As a direct corollary of Proposition 2, we can see that  $V$  captures the cut-off rule that determines firms’ design strategy:

**Corollary 1** *All firms with  $v < V$  choose a niche design, and all firms with  $v > V$  choose a broad one.*

Intuitively, a low  $v$  firm needs to compete harder to overcome its disadvantage in terms of the innate quality, and so is more likely to adopt a strategy that, while unappealing to

---

<sup>14</sup>Mathematically, we redefine  $V$  to be  $\max\{\underline{v}, \min\{\bar{v}, \cdot\}\}$  of the solution to (10).



many consumers, has a chance at providing a great match and being appealing to some consumers. Instead, a high-value firm can, to a greater extent, try to appeal to many consumers by adopting the broad strategy.

This result is economically rich and appealing. For example, consider five-star hotels competing in a city. Although they are in the same category, they differ in an important dimension: location. Our model predicts that hotels that are well located (center of the city, close to the airport or other facilities) are more likely to deliver standard services. Meanwhile, those with less-desirable locations are more likely to be specialized—for example, as boutique hotels with distinctive styling or catering to minority groups, such as customers with pets.

### 3.3 Equilibrium Summary

Given all the analysis above, we can express an equilibrium as a pair  $(U, V)$ , where  $U$  summarizes the search and purchase behavior of consumers and  $V$  determines which firms choose the broad or the niche strategy. These two parameters have to satisfy the following conditions. First, rearranging (4), consumers optimize their behavior when

$$c = \int_{-\infty}^V \left( \int_{U+p_{vN}(U)-v}^{\infty} (\varepsilon - U - p_{vN}(U) + v) f_N(\varepsilon) d\varepsilon \right) h(v) dv + \int_V^{\infty} \left( \int_{U+p_{vB}(U)-v}^{\infty} (\varepsilon - U - p_{vB}(U) + v) f_B(\varepsilon) d\varepsilon \right) h(v) dv. \quad (11)$$

Second, as explained above, firms' maximizing behavior is summarized by the indifference of  $V$  as in (10). Third, associated with broad and niche designs are profit-maximizing prices  $p_{vB}(U)$  and  $p_{vN}(U)$  as determined in (8). Finally, it must be worthwhile for a consumer to initiate search; that is,  $U \geq 0$ .

It is convenient to maintain notation for the expected probability that a consumer will buy when she visits a random firm. This is given by

$$\rho(U, V) \equiv \int_{-\infty}^V (1 - F_N(U + p_{vN}(U) - v)) h(v) dv + \int_V^{\infty} (1 - F_B(U + p_{vB}(U) - v)) h(v) dv. \quad (12)$$

## 4 Further Characterization

Next, we consider a series of general results and properties of the equilibria. A full characterization of the equilibria requires further structure on the distributions of matches  $f_s(\cdot)$

and quality  $h(\cdot)$ . Thus, we continue later by exemplifying the model in Sections 5 and 6 for particular choices of these distributions.

It is useful to consider firms' and consumers' reactions functions. Note that, prices are determined by (8) once  $V$  and  $U$  are established. Thus, with some abuse of notation, we focus on the strategic choices of  $V$  and  $U$ , while letting prices adjust in the background. In other words, we characterize the consumers' and firms' best response functions, which we write as  $U(V; c)$  and  $V(U; c)$ , respectively.

First, as an immediate consequence of Corollary 1, we obtain the following result.

**Lemma 2** *The firms' best response  $V(U)$  is a well-defined continuous function. It is independent of  $c$  and non-decreasing in  $U$ .*

That is, the higher the utility that consumers require for purchase, the more likely a firm is to choose a niche design. Next, consider the consumers' best response.

**Lemma 3** *The consumers' best response  $U(V; c)$  is a well-defined continuous function that is decreasing in  $c$ .*

That is, fixing firm design choices, the higher the search cost, the more willing a consumer is to purchase.

Note that Lemma 3 is silent about whether  $U$  increases or decreases with  $V$ . Indeed, both cases may arise. A slight change in  $V$  shifts some firms from one design to the other (and their corresponding change in prices). As one can see in equation (11), fixing  $U$ , the only change to a consumer's well-being comes from these firms. Now, depending on the particular elasticity configurations of  $F_N(\cdot)$  and  $F_B(\cdot)$ , the consumer might or might not like such a change; this, in turn, could make it either more or less valuable to continue search, so that  $U$  can adjust in either direction. However, we argue below that it is natural to focus on the case where  $U$  decreases in  $V$ .

Given the firms' and consumers' best response functions, and abstracting from  $c$ , we can characterize equilibria as  $(U, V(U))$  that satisfy  $U(V(U)) = U$ . In general, there might be multiple equilibria satisfying this. For a given search cost, there may be equilibria where many firms choose the broad design and consumers' search threshold is relatively low, which, following Lemma 2, is consistent with relatively many firms choosing the broad design. Alternatively, firm and consumer expectations may be aligned so that, in equilibrium, many firms choose a niche strategy and the consumer threshold is relatively high.

Note, however, that some of these equilibria are better behaved than others. Here, we propose disregarding unstable equilibria.<sup>15</sup> The fact that we are later interested in

---

<sup>15</sup>Stability refers to the adaptive best-response dynamics, as in Echenique (2002).

comparative statics with respect to search costs makes them even less appealing. Thus, we concentrate only on equilibria with the property that the function  $U(V(\cdot))$  has a slope  $< 1$  at the equilibrium value  $U$ . As we show in the proof to the following proposition, this is the case when  $\frac{\partial U}{\partial V}(\cdot)$  is not too positive. But, more importantly, we obtain the following result:

**Proposition 3** *Consider local comparative statics around any stable equilibrium; then, decreasing  $c$  raises consumer surplus (higher  $U$ ) and makes the fraction of niche firms (weakly) greater (higher  $V$ ).*

As mentioned in the Introduction, there has been much discussion of the long tail of the Internet. Proposition 3 provides a first theoretical result that speaks to the issue by demonstrating that, for stable equilibria, lower search costs bring more niche firms. Thus, the Internet makes a wider variety of products available. In itself, of course, this result need not mean that niche products sell more (as the discussions of the long tail suggest); nor does it address the consequences for profitability. It seems reasonable, however, that a greater fraction of niche firms can soften price competition in such a way that firm profits increase, and that firms that had been niche face less severe competition and sell more. Indeed, Sections 5 and 6, where we impose distributional assumptions that allow us to provide a full characterization, show that both these outcomes arise.

First, however, it is instructive to consider the extreme cases where all firms choose a broad or a niche design, which we can characterize without imposing specific distributional assumptions.

### All-broad and all-niche equilibria

We first define some search cost and utility values that are useful to characterize the equilibria in which all firms choose either a broad or a niche design, which we refer to as *all-broad* and *all-niche* equilibria.

Consider first a situation in which consumers use a  $U = 0$  search rule. Firms would react using a  $V(0)$  strategy. Now, using (11), one can compute the searching cost  $c_0$  that delivers  $(0, V(0))$  as an equilibrium:

$$c_0 = \int_{-\infty}^{V(0)} \left( \int_{p_{vN}(0)-v}^{\infty} (\varepsilon - p_{vN}(0) + v) f_N(\varepsilon) d\varepsilon \right) h(v) dv + \int_{V(0)}^{\infty} \left( \int_{p_{vB}(0)-v}^{\infty} (\varepsilon - p_{vB}(0) + v) f_B(\varepsilon) d\varepsilon \right) h(v) dv. \quad (13)$$

Next, consider the consumer stopping rule  $U_B$  that makes all firms prefer the broad strategy and the lowest-quality firm indifferent. Given Lemma 2, this is the highest level

of consumer search compatible with all firms offering a broad product. This value  $U_B$  is characterized by:

$$p_{\underline{v}B}(U_B)(1 - F_B(p_{\underline{v}B}(U_B) + U_B - \underline{v})) = p_{\underline{v}N}(U_B)(1 - F_N(p_{\underline{v}N}(U_B) + U_B - \underline{v})). \quad (14)$$

Using (11), we can compute the search cost  $c_B$  that results in an equilibrium with firm and consumer behavior of  $(U_B, \underline{v})$ :

$$c_B := \int_{U_B + p_{\underline{v}B}(U_B) - \underline{v}}^{\bar{\theta}_B} (\varepsilon - U_B - p_{\underline{v}B}(U_B) + \underline{v}) f_B(\varepsilon) d\varepsilon. \quad (15)$$

We can now characterize the range of searching costs in which all-broad equilibria arise:

**Proposition 4** *There exists an equilibrium where all firms choose the broad design if and only if  $U_B > 0$  and  $c \in [c_B, c_0)$ .*

Thus, search costs need to be high enough for all-broad equilibria to exist. But if they are too high—specifically, higher than  $c_0$ —then no consumer would initiate search. Note that if  $c_0 \leq c_B$ , then  $U_B \leq 0$  and no all-broad equilibria exist.

Analogously, one can consider all firms choosing the niche design, so that  $V = \bar{v}$ , together with the consumer stopping rule that makes the highest-quality firm indifferent in its design choice,  $U_N$ , and the associated search cost,  $c_N$ . These are defined by:

$$p_{\bar{v}B}(U_N)(1 - F_B(p_{\bar{v}B}(U_N) + U_N - \bar{v})) = p_{\bar{v}N}(U_N)(1 - F_N(p_{\bar{v}N}(U_N) + U_N - \bar{v})), \text{ and} \quad (16)$$

$$c_N = \int_{U_N + p_{\bar{v}N}(U_N) - \bar{v}}^{\bar{\theta}_N} (\varepsilon - U_N - p_{\bar{v}N}(U_N) + \bar{v}) f_N(\varepsilon) d\varepsilon. \quad (17)$$

We obtain a characterization of all-niche equilibria:

**Proposition 5** *There exists an equilibrium where all firms choose the niche design if and only if  $c < \underline{c}$ , where  $\underline{c} = \begin{cases} c_N & \text{if } U_N > 0 \\ c_0 & \text{if } U_N \leq 0 \end{cases}$ .*

Given that  $c_0 > 0$  and that  $c_N > 0$  when  $U_N > 0$ , Proposition 5 proves the existence of all-niche equilibria for sufficiently low positive search costs.

### Long tails and superstars

Previously, we derived the comparative statics of firms' consumers' behavior. It is also of interest to consider how the distribution of prices and sales change. In particular, to assess long-tail and superstar effects, we define these in our model.

**Definition 1** *We say that a superstar effect is present if the sales distribution of the firm with the highest sales captures an increasing market share as search costs fall.*

**Definition 2** *We say that a long-tail effect is present if the sales distribution of the firm with the lowest sales captures an increasing market share as search costs fall.*

Our definitions of long-tail and superstar effects may seem somewhat extreme in focusing only on one firm. But in this model, because of continuity, if the extreme firm behaves in a certain way, so do so adjacent ones. Thus, our definitions imply a mass of firms at the tails gaining market share.

We later study distributional changes in the case in which different designs coexist in equilibrium. But we start, here, by arguing that when all firms choose the same design, there are always superstar effects but never long-tail effects.

**Proposition 6** *Suppose that all firms choose the same design  $s$ , and the distribution of consumer valuations  $F_s(\cdot)$  is not too concave; the superstar effect arises, but the long-tail effect does not.*

A sufficient condition for the proposition to hold is that  $F_s(\cdot)$  is convex or, equivalently, that  $1 - F_s(\cdot)$ , the demand function for a monopolist firm, is concave. Note that the assumption that  $F_s(\cdot)$  is log-concave already limits how convex  $1 - F_s(\cdot)$  can be (and thereby ensures, as in Lemma 1, that a firm has a unique profit-maximizing price).<sup>16</sup>

Proposition 6 suggests that the documented long-tail effect cannot solely be a consequence of a fall in the cost of search. If firms continued delivering the same type of products, we should see low-quality firms losing market share. It is through a change towards more-niche designs that the long tail arises. Note, also, that holding design constant, and following Lemma 1 and Proposition 3, firm profits decrease as search costs decrease, which appears counterfactual to the rise of new firms on the Internet.

While, it is plausible that the Internet has reduced fixed costs of entry of firms, we demonstrate that when firms' designs are strategic choices, the long-tail effect arises naturally and that as search costs fall, firm profits can increase, leading to new firm entry.

---

<sup>16</sup>Note that this result is related to the comparative statics results in the related, but somewhat different, model of Goldmanis et al. (2009). Their model considers firms heterogeneous in the marginal cost of production (logconcave distribution), selling a homogenous product to heterogeneous consumers with uniformly distributed search costs.

We show these effects clearly by adding some further structure to the model: In Section 5, we assume ex-ante symmetry of all firms, and in Section 6, we allow for heterogeneous types but suppose that they are uniformly distributed types and that the distributions of consumer valuations are uniform.

## 5 Homogeneous Firms

We consider the case where all firms are ex-ante identical. Without any loss of generality, we assume that  $v = 0$  for all firms. To simplify notation, we drop the  $v$  subscripts throughout this section.<sup>17</sup>

In the analysis above, we considered a continuous distribution of firm types. Thus, it was without loss of generality to ignore mixed strategies. Instead, with all firms homogeneous, we must allow for the possibility of mixed-strategy equilibria. In particular, we denote  $\lambda$  as the proportion of firms that choose a niche rather than a broad design. Analogous to the characterization of Section 3.3, equilibria can be summarized by  $(U, \lambda)$ , and conditions (10)-(12) can be adapted as:

$$c = \lambda \int_{U+p_N(U)}^{\infty} (\varepsilon - U - p_N(U)) f_N(\varepsilon) d\varepsilon + (1 - \lambda) \int_{U+p_B(U)}^{\infty} (\varepsilon - U - p_B(U)) f_B(\varepsilon) d\varepsilon. \quad (18)$$

$$\lambda \in \arg \max \{ \lambda(1 - F_B(p_B(U) + U)) + (1 - \lambda)p(U)(1 - F_N(p_N(U) + U)) \}. \quad (19)$$

$$\rho(U) = \lambda \int_{U+p_N(U)}^{\infty} f_N(\varepsilon) d\varepsilon + (1 - \lambda) \int_{U+p_B(U)}^{\infty} f_B(\varepsilon) d\varepsilon. \quad (20)$$

Note that the characterization of prices, given by (8), and the consumer's participation constraint ( $U \geq 0$ ) still apply.

Given that all firms are identical,  $U_B$  and  $U_N$  as defined in (14) and (16), are identical. We write  $\bar{U} = U_B = U_N$ . For  $U > \bar{U}$ , therefore, all firms prefer a niche design, whereas, for  $U < \bar{U}$ , all firms prefer a broad design. These equilibria have been characterized in Propositions 4 and 5. It is only at  $U = \bar{U}$  that firms might mix. However, a mixed-strategy equilibrium can exist over a wide range of search costs. This is immediate, by noting that at  $U = \bar{U}$  expression (18) can be rewritten as

$$c = \lambda c_N + (1 - \lambda) c_B, \quad (21)$$

---

<sup>17</sup>For example, we write  $p_N(U)$  instead of  $p_{vN}(U)$ .

where, using (15) and (17),  $c_B$  and  $c_N$  can be written as

$$c_B = \int_{\bar{U} + p_B(\bar{U})}^{\bar{\theta}_B} (\varepsilon - (\bar{U} + p_B(\bar{U}))) f_B(\varepsilon) d\varepsilon, \text{ and} \quad (22)$$

$$c_N = \int_{\bar{U} + p_B(\bar{U})}^{\bar{\theta}_N} (\varepsilon - (\bar{U} + p_N(\bar{U}))) f_N(\varepsilon) d\varepsilon. \quad (23)$$

Note that each of these has an interpretation as the expected consumer surplus from visiting a broad or a niche firm, respectively, when the reservation utility  $\bar{U}$  is such that a firm makes identical profits whether choosing a broad or a niche design.

If  $c_N < c_B$ , then the mixed-strategy equilibrium exactly fills the gap between the regions where all-broad and all-niche exist and  $\lambda$  is linear and decreasing in  $c$ . If  $c_N > c_B$ , then in this region there are, in principle, three equilibria: one all-broad, one all mixed and one all-niche. However, note that the mixed equilibrium in this case is unstable. Thus, for  $c \in (c_B, c_N)$ , only two pure equilibria remain.

Finally, if  $c_N = c_B$ , the mixed-strategy equilibrium has no mass. This is the case when demands are linear (or equivalently  $f_s(\cdot)$  is uniform). Then the ratio of consumer surplus to firm profits for a monopolist is constant at  $\frac{1}{2}$ , regardless of the level of the constant marginal costs (which, for a monopolist, play a similar role to the reservation utility  $U$  in our model of monopolistic competition). Therefore, two firms facing linear demands (regardless of their slopes) who earn the same profits must generate the same consumer surplus. This proves that if  $F_N(\cdot)$  and  $F_B(\cdot)$  are uniform, then  $c_N = c_B$ . This suggests that it is easy to find cases where either uniqueness or multiplicity arise. For example, if demand is convex, the ratio of consumer surplus to profits is always higher than it would be in the linear case. Thus, if  $F_B$  is linear and  $F_N$  is concave, then  $c_N > c_B$  and multiplicity arises, whereas in the opposite case, with  $F_B$  concave and  $F_N$  linear, a unique equilibrium exists.

In a mixed-strategy equilibrium, given the demand (5) expression and substituting for the probability of a sale (20), we can write the sales for a broad and a niche firm, respectively, as

$$m \frac{(1 - F_B(\bar{U} + p_B(\bar{U})))}{\lambda(1 - F_B(\bar{U} + p_B(\bar{U}))) + (1 - \lambda)(1 - F_N(\bar{U} + p_N(\bar{U})))}, \text{ and} \quad (24)$$

$$m \frac{(1 - F_N(\bar{U} + p_N(\bar{U})))}{\lambda(1 - F_B(\bar{U} + p_B(\bar{U}))) + (1 - \lambda)(1 - F_N(\bar{U} + p_N(\bar{U})))}. \quad (25)$$

Note that when  $\lambda = 0$  or  $\lambda = 1$ , sales are simply given by  $m$ . This is intuitive: Since all consumers buy and since all firms are symmetric in their behavior, they share out the market and each firm gets the same sales volume  $m$ .

### Comparative statics on search costs

As shown below, local comparative statics of all-broad and all-niche equilibria are all monotone and straightforward. Given this, the interesting and rich case to analyze is the one in which mixed-strategy equilibria arise. Thus, we concentrate our analysis on the case in which  $c_0 > c_B > c_N$ . In this case, using Propositions 4 and 5 and the analysis above we know that: (i) for  $c > c_0$ , the market breaks down; (ii) for  $c \in [c_B, c_0)$ , the unique equilibrium is all-broad; (iii) for  $c \in (c_N, c_B)$ , there is a mixed-strategy equilibrium with a positive mass of firms going broad and niche; and (iv) for  $c \leq c_N$ , all firms choose a niche design. We characterize market outcomes in all these cases.

First, consider consumer surplus  $U$ . In the pure strategy regions, where all firms choose the same design, the value of  $V$  does not change, and one can use Lemma 2 to conclude that a reduction in  $c$  decreases  $U$ . It follows that for values of  $c$  where all firms choose the same design, consumer surplus is strictly decreasing in  $c$ . In the mixed-strategy area,  $U$  is constant at  $\bar{U}$ . Thus, consumer surplus is monotonically decreasing overall, consistent with the general result provided by Proposition 3.

Next, consider prices. Within the pure strategy areas,  $p$  is increasing in  $c$ , as shown in Lemma 1. In the mixed-strategy area, broad firms charge a price  $p_B(\bar{U})$ , while niche firms charge  $p_N(\bar{U})$ . We can write the average price of an item sold as:

$$\frac{\lambda(1 - F_B(\bar{U} + p_B(\bar{U}))p_B(\bar{U}) + (1 - \lambda)(1 - F_N(\bar{U} + p_N(\bar{U}))p_N(\bar{U}))}{\lambda(1 - F_B(\bar{U} + p_B(\bar{U})) + (1 - \lambda)(1 - F_N(\bar{U} + p_N(\bar{U})))}. \quad (26)$$

This is a convex combination of  $p_N(\bar{U})$  and  $p_B(\bar{U})$ . Given that  $p_N(\bar{U}) \geq p_B(\bar{U})$ , as argued in the proof of Proposition 2, and that  $\lambda$  is decreasing in  $c$ , the average price of an item sold is also decreasing in  $c$ . In sum, average prices are non-monotonic in  $c$ : For small and large values of  $c$  they are increasing, while for intermediate ones they are decreasing.

The same qualitative comparative statics arises for industry profits, as these are total sales,  $m$ , times the average price charged per sale.

Total welfare in the pure strategy areas can be written as  $mU + mp_s(U)$ , where  $s \in \{B, N\}$ . As shown in the proof of Lemma 1, this expression increases as  $c$  decreases. In the mixed-strategy area, welfare is  $m\bar{U} + m\bar{p}$ , which also increases as  $c$  decreases.

Finally, we focus on the distribution of sales across firms. In the pure strategy area, every single firm sells  $m$ . Meanwhile, in the mixed-strategy area, both the composition of firms and the sales by type of firm change. First note that  $1 - F_B(p_B(\bar{U}) + \bar{U}) >$



$1 - F_N(p_N(\bar{U}) + \bar{U})$  from the proof of Proposition 2. Therefore, sales by type of firm, as in (24) and (25), are increasing in  $\lambda$  (through the effect on  $\rho$ ) and, consequently, decreasing in  $c$ . At the same time, the proportion of broad firms is increasing in  $c$ . These results are illustrated in Figure 1 below.

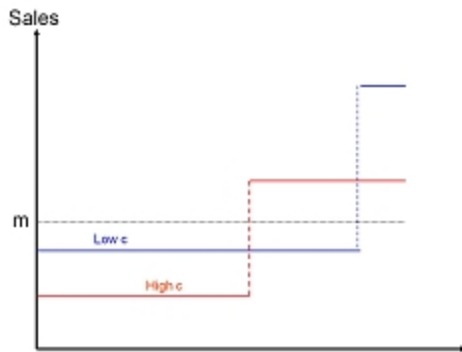


Fig 1: Distribution of sales at different search costs.

To summarize, in a stable mixed-strategy region, consistent with “long-tail” stories, as search costs fall, there are more niche firms and each niche firm sells more. Since the total volume of sales is constant, it follows that the niche firms account for a greater proportion of overall sales. Note, also, that throughout this range of  $c$ , superstar effects are present. The “top” firm is broad and sells more as  $c$  goes down. The tail is niche throughout and also sells more as  $c$  goes down. The middle region, where the mix of broad and niche is changing, is the one that loses sales to both the head and the tail of the sales distribution.

## 6 Uniformly distributed quality and linear demands

We return to consider heterogeneous firms, but impose further structure that allows us to derive additional analytic results. These highlight that the results of Section 5 with homogeneous firms extend naturally to more general settings. We analyze the case where the distribution of firm quality is uniform  $v \sim U[L, H]$ , and the distributions  $F_s(\cdot)$  are uniform, leading to linear demand functions. In particular, the niche and broad product designs are, respectively,  $\varepsilon \sim U[\underline{\theta}_N, \bar{\theta}_N]$  and  $\varepsilon \sim U[\underline{\theta}_B, \bar{\theta}_B]$ . We impose that  $\underline{\theta}_N < \underline{\theta}_B$  and  $\bar{\theta}_N > \bar{\theta}_B$ . This ensures that these are demand rotations (i.e., the demand curves cross once).

The following proposition demonstrates that comparative statics and qualitative results similar to those in Section 5 arise in this environment. In particular, part (iii) of

the proposition demonstrates that long-tail and superstar effects can arise simultaneously. Note that if firms' types are very dispersed then a low quality firm must be forced out of the market when search costs are sufficiently low; following our definition, trivially, in such circumstances, long tail effects cannot arise. The proposition, therefore, focuses on parameter ranges where all firms remain active even for low values of  $c$  (as we show in the example below, this is a non-trivial region).

**Proposition 7** *Under the assumptions above, when all firms are active then (i)  $c_B > c_N$ . (ii) There is a unique equilibrium  $(U, V)$  for each search cost  $c$ . When different firms choose different design strategies then (iii) As the search cost decreases,  $V$  decreases and  $U$  increases. (iv) Both long-tail and superstar effects are present.*

We illustrate the results of Proposition 7, the non-monotonicity of prices and profits, and the superstar and long-tail effects through a numeric example.

#### Comparative statics on search costs

Consider the following firm and consumer distributions  $f_N(x) = \frac{1}{16}$  on  $[-12, 4]$ ,  $f_B(x) = \frac{1}{6}$  on  $[-3, 3]$  and  $h(x) = 1$  on  $[0, 1]$ .

Figure 2 illustrates how prices vary with search costs for a particular firm (at  $v = 0.9$ ). As one would anticipate, in general, prices increase with search costs. However, when the firm changes design from niche to broad, prices drop substantially, leading to prices that are non-monotonic in search costs. The price pattern for other  $v$ 's is qualitatively the same.

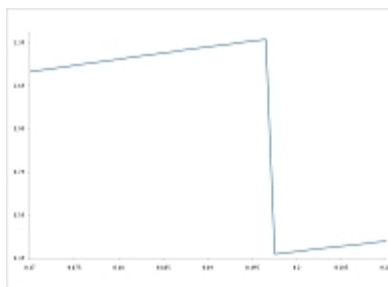


Fig 2: Prices against search costs at  $v = 0.9$ .

Next, consider average firm profits, as illustrated in Figure 3. Equivalently, since there is a mass 1 of firms, the graph represents total industry profits. Note the two points where the derivative is discontinuous. These are the search cost thresholds at which the equilibrium changes from all-niche to mixed to all-broad: Below  $c_N = 0.09$ , all firms are niche, but as search costs increase, the high-quality firms gradually start switching to a

broad design. At  $c_N = 0.18$  and beyond, all firms choose a broad design. Figure 2 also illustrates that profits may be non-monotonic. The intuition is the by now familiar one that as search costs fall in the intermediate region, more firms choose a niche design. This softens price competition and raises prices for the industry as a whole.

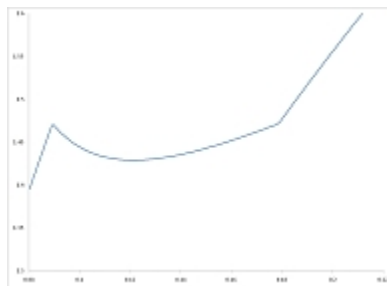


Fig 3: Average firm profits  
against search costs.

Finally, we consider sales distributions. Figure 4 plots the distribution of sales. Naturally, higher-quality firms sell more than low-quality firms, regardless of the search costs. Comparing sales at different search costs, both the highest- and lowest-quality firm sell more at the lower level of search costs, illustrating the superstar and long-tail effects.

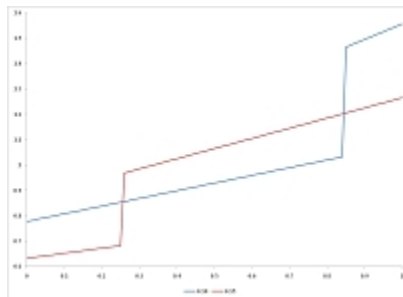


Fig 4: Sales against quality ( $v$ )  
at two different search costs  
( $c = 0.1$  and  $c = 0.15$ ).

## 7 Conclusions

There has been considerable attention on the influence of the Internet on the kind of products offered and the distribution of their sales. In particular, academic and popular

commentators have highlighted both long-tail and superstar effects. This paper presents a simple and tractable model integrating consumer search and firms’ strategic product-design choices that is useful to analyze these phenomena.

We show that, in equilibrium, different product designs coexist. More-advantaged firms prefer “broad-market” strategies, seeking a very broad design and choosing a relatively low price, while less-advantaged firms take a niche strategy with quirky products priced high to take advantage of the (relatively few) consumers who are well-matched to the product. Such design diversity arises even when all firms are homogeneous.

The contrast between broad-market and niche strategies has been explored elsewhere, notably in Johnson and Myatt (2006), in the earlier work of Lewis and Sappington (1994) and, more recently, in Bar-Isaac, Caruana and Cuñat (2009); however, these models focus on monopolies. Here we present a competitive model in a market with search frictions where these different strategies can coexist.

The comparative statics analysis presents a demand-side explanation of the long-tail effect. As search costs fall, a greater proportion of firms choose the niche strategy. Due, in part to the different industry structure, but also in part because it is cheaper for consumers to more easily seek better-suited products, niche firms account for a larger proportion of the industry’s sales. Moreover, lower search costs can simultaneously account for a superstar effect. Note that in contrast to much discussion surrounding scale or production cost effects, we assume that production technologies do not vary and are identical in terms of costs.

In addition, the comparative statics results highlight that prices (and profits) can be non-monotonic in consumer search costs. There is an intuitive rationale: As search costs fall, then as long as the product designs remain unchanged, prices fall. However, at ever lower prices, the broad-market strategy becomes less appealing to firms, some of whom adopt a niche strategy, charging a high price to the (few) consumers who are well-matched for the product. Moreover, the firms’ decision to adopt a niche strategy effectively imposes a positive externality on other firms since this choice of a niche strategy acts as a form of differentiation that softens price competition.

One aspect that our model did not consider is the entry of new firms into the market. This was done for simplicity, but could be easily accommodated. One could endogenize the proportion of consumers per firm  $m$  by assuming a fixed entry cost and imposing a firm free entry condition. Qualitatively, the general results and intuition would be identical. In particular, in the same way that average profits in the case of exogenous entry can be non-monotonic in search costs, the number of firms can be non-monotonic in search costs when entry is endogenous. This is consistent with anecdotal evidence on the effect

of the Internet (consider, for example, countless online stores created on eBay) and hard to reconcile if one uses a model with exogenous product design, unless one also imposes an alternative complementary mechanism, such as reduction in the fixed costs of entry.

In our model, we have assumed that firms take their actions separately. Given that their choices have consequences for all other firms in the industry, there is a rationale for industry coordination. In particular, since profits can be non-monotonic in search costs, as search costs fall exogenously, the industry might benefit from further reducing them. Thus an industry response to the appearance of the Internet may be to provide additional technologies (such as industry-sponsored comparison sites) that further reduce search costs for consumers.

## References

- [1] Anderson, Chris (2004) "The Long Tail," *Wired*, Issue 12.10, Oct.
- [2] Anderson, Chris (2006). *The Long Tail: Why the Future of Business Is Selling Less of More*. New York: Hyperion. ISBN 1-4013-0237-8.
- [3] Anderson, Simon P. and Régis Renault (1999) "Pricing, product diversity and search costs: a Bertrand-Chamberlin-Diamond model," *RAND Journal of Economics*, Vol. 30 No. 4, Winter, pp. 719-735.
- [4] Bagnoli, Mark and Ted Bergstrom (2005) "Log-concave probability and its applications," *Economic Theory*, Vol. 26, pp. 445-469.
- [5] Bakos, Yannis "Buyer Search Costs: Implications for Electronic Marketplaces," (1997) *Management Science*, Vol. 43 No. 12, Frontier Research on Information Systems and Economics Dec. pp. 1676-1692.
- [6] Bar-Isaac, Heski, Guillermo Caruana and Vicente Cuñat (2008) "Information gathering externalities in product markets," working paper.
- [7] Bar-Isaac, Heski, Guillermo Caruana and Vicente Cuñat (2009) "Information gathering and marketing," *Journal of Economics and Management Strategy*, forthcoming.
- [8] Baye, Michael R., John Morgan and Patrick Scholten (2006) "Information, Search, and Price Dispersion," in *Handbook of Economics and Information Systems* (T. Hendershott, ed.), Elsevier Press, Amsterdam.
- [9] Brynjolfsson, Erik, Hu Yu Jeffrey, and Michael D. Smith, (2006) "From Niches to Riches: Anatomy of the Long Tail," *Sloan Management Review*, Vol. 47, No. 4, pp. 67-71, Summer.
- [10] Brynjolfsson, Erik, Hu, Yu Jeffrey and Simester, Duncan, (2007) "Goodbye Pareto Principle, Hello Long Tail: The Effect of Search Costs on the Concentration of Product Sales" November. Available at SSRN: <http://ssrn.com/abstract=953587>

- [11] Cachon, Gérard, Christian Terwiesch and Ye Xu (2009) “On the Effects of Consumer Search and Firm Entry in a Multiproduct Competitive Market,” *Marketing Science*, forthcoming
- [12] Diamond, Peter (1971) “A Model of Price Adjustment,” *Journal of Economic Theory*, 3, 156-68.
- [13] Echenique, Federico (2002) “Comparative Statics by Adaptive Dynamics and the Correspondence Principle,” *Econometrica* Vol. 70, No. 2. March., pp. 833-844.
- [14] Elberse, Anita and Felix Oberholzer-Gee (2006) ‘Superstars and Underdogs: An examination of the Long Tail Phenomenon in Video Sales’, *Harvard Business School Working Paper* No. 07-015.
- [15] Maris Goldmanis, Ali Hortaçsu, Emre Onsel and Chad Syverson “E-commerce and the Market Structure of Retail Industries” at <http://home.uchicago.edu/~syverson/ecommerce.pdf>
- [16] Hervas-Drane, Andres, (2009) “Word of Mouth and Taste Matching: A Theory of the Long Tail” working paper UPF.
- [17] Johnson, Justin P. and David P. Myatt (2006) “On the Simple Economics of Advertising, Marketing, and Product Design,” *American Economic Review* 96(3): 756–784.
- [18] Kuksov, Dimitri (2004) “Buyer search costs and endogenous product design,” *Marketing Science*, 23(4), 490-499.
- [19] Larson, Nathan, (2008) “Endogenous Horizontal Differentiation and Consumer Search,” *University of Virginia, working paper*.
- [20] Lewis, Tracy R. and David E. M. Sappington (1994) “Supplying Information to Facilitate Price Discrimination,” *International Economic Review*, 95(2), pp. 309-327.
- [21] Oestreicher-Singer, Gal and Arun Sundararajan (2008) “The Visible Hand of Social Networks in Electronic Markets” at <http://ssrn.com/abstract=1268516>
- [22] Payne, John W., Bettman, James R. and Eric J. Johnson (1993) *The Adaptive Decision Maker* Cambridge University Press.
- [23] Stiglitz, Joseph E. 1989 “Imperfect Information in the Product Market,” in *Handbook of Industrial Organization Vol. 1*, eds. R. Schmalensee and R. Willig, Elsevier, Amsterdam. pp. 771-948.
- [24] Tucker, Catherine and Juanjuan Zhang (2007) “Long Tail or Steep Tail? A Field Investigation into How Online Popularity Information Affects the Distribution of Customer Choices,” *MIT Sloan School Working Paper 4655-07*
- [25] Watson, Randall (2007) “Search, Availability, and Competition in Product Ranges,” *University of Texas, working paper*.
- [26] Wolinsky, Asher “True Monopolistic Competition as a Result of Imperfect Informa-

tion,” *The Quarterly Journal of Economics*, Vol. 101 No. 3 (Aug., 1986), pp. 493-511.

## A Proofs

**Proof of Lemma 1** First note that since  $f_s(x)$  is logconcave, then  $\frac{1-F_s(x)}{f_s(x)}$  is strictly decreasing in  $x$ .<sup>18</sup>

Suppose (for contradiction) that at some value of  $U$ ,  $p_{vs}(U)$  is increasing in  $U$ ; then  $p_{vs}(U) + U$  is also increasing in  $U$  and so  $\frac{1-F_s(p_{vs}(U)+U-v)}{f_s(p_{vs}(U)+U-v)} = p_{vs}(U)$  is decreasing in  $U$ , which provides the requisite contradiction. A similar argument ensures that  $p_{vs}(U) + U$  is increasing in  $U$ , that  $p_{vs}(U)$  is increasing in  $v$ , and that  $p_{vs}(U) - v$  is decreasing in  $v$ . ■

**Proof of Proposition 1** The optimal design is chosen to maximize  $p_{vs}(U)(1 - F_s(p_{vs}(U) + U - v))$ . Now, given that  $p_{vs} - U + v$  is an affine transformation of  $p_s$ , it follows that  $D_v(p_{vs}, s)$ , as in (5), are rotation-ordered. The proof then follows immediately from Proposition 1 in Johnson and Myatt (2006), p. 761. ■

**Proof of Proposition 2** It is convenient to work directly in terms of  $W = U - v$  and  $\widehat{W} = \widehat{U} - \widehat{v}$  and write  $p_B(W) := \arg \max p(1 - F_B(p + W))$  and  $p_N(W) := \arg \max p(1 - F_N(p + W))$ . Then by definition  $\widehat{W}$

$$p_B(\widehat{W})(1 - F_B(p_B(\widehat{W}) + \widehat{W})) = p_N(\widehat{W})(1 - F_N(p_N(\widehat{W}) + \widehat{W})). \quad (27)$$

In principle, it is conceivable that there is more than one solution to this equation (we show later that this is not the case). Consider one such solution and notice that

$$\begin{aligned} p_B(\widehat{W})(1 - F_B(p_B(\widehat{W}) + \widehat{W})) &= p_N(\widehat{W})(1 - F_N(p_N(\widehat{W}) + \widehat{W})) \\ &\geq p_B(\widehat{W})(1 - F_N(p_B(\widehat{W}) + \widehat{W})). \end{aligned} \quad (28)$$

It follows that

$$1 - F_B(p_B(\widehat{W}) + \widehat{W}) \geq 1 - F_N(p_B(\widehat{W}) + \widehat{W}). \quad (29)$$

Similarly,

$$\begin{aligned} p_N(\widehat{W})(1 - F_N(p_N(\widehat{W}) + \widehat{W})) &\geq p_N(\widehat{W})(1 - F_B(p_N(\widehat{W}) + \widehat{W})), \text{ and so} \\ 1 - F_N(p_N(\widehat{W}) + \widehat{W}) &\geq 1 - F_B(p_N(\widehat{W}) + \widehat{W}) \end{aligned} \quad (30)$$

We use these facts to show that  $p_N(\widehat{W}) > p_B(\widehat{W})$  and  $1 - F_B(p_B(\widehat{W}) + \widehat{W}) > 1 - F_N(p_N(\widehat{W}) + \widehat{W})$ .

Suppose (for contradiction) that  $p_N(\widehat{W}) < p_B(\widehat{W})$ . Note that since  $N$  and  $B$  are drawn from a family of demand rotations, it follows that there is some  $\tilde{x}$  such that  $1 - F_N(x) > 1 - F_B(x)$  if and only if  $x > \tilde{x}$ .

First suppose  $p_B(\widehat{W}) + \widehat{W} > \tilde{x}$ , then,  $1 - F_N(p_B(\widehat{W}) + \widehat{W}) > 1 - F_B(p_B(\widehat{W}) + \widehat{W})$  in contradiction to (29). If, instead,  $\tilde{x} \geq p_B(\widehat{W}) + \widehat{W} > p_N(\widehat{W}) + \widehat{W}$ , then (30) is contradicted.

It follows that  $p_N(\widehat{W}) > p_B(\widehat{W})$  and from (27), trivially,  $1 - F_B(p_B(\widehat{W})) > 1 - F_N(p_N(\widehat{W}))$ .

Next, returning to the maximization problem, we can rewrite  $p_B(\widehat{W})$  and  $p_N(\widehat{W})$  as the solutions

---

<sup>18</sup>See Corollary 2 of Bagnoli and Bergstrom (2005). More broadly, check this paper for functions that do and do not satisfy the logconcavity assumption.

to the maximization problems explicitly and so re-write(27) as:

$$\max_{p_B} p_B(1 - F_B(p_B + \widehat{W})) = \max_{p_N} p_N(1 - F_N(p_N + \widehat{W})). \quad (31)$$

A change of variable allows us to write the dual as

$$\max_{q_B} (P_B(q_B) - \widehat{W})q_B = \max_{q_N} (P_N(q_N) - \widehat{W})q_N.$$

Then, by the envelope theorem,  $\frac{d\pi_B}{dU}|_{\widehat{W}} = -q_B$  and  $\frac{d\pi_N}{dU}|_{\widehat{W}} = -q_N$  but, as argued above,  $q_B|_{\widehat{W}} = 1 - F_B(p_B(\widehat{W})) > q_N|_{\widehat{W}} = 1 - F_N(p_N(\widehat{W}))$ . Thus,  $\frac{d(\pi_B - \pi_N)}{dU}|_{\widehat{W}} < 0$ , which ensures that  $\pi_B - \pi_N = p_B(W)(1 - F_B(p_B(W) + W)) - p_N(W)(1 - F_N(p_N(W) + W))$  always crosses zero from above. This assures that the uniqueness of  $\widehat{W}$  follows trivially: since  $\pi_B - \pi_N$  is a continuous function there can be at most one such crossing. ■

**Proof of Lemma 2** This is a consequence of Proposition 2, which also delivers the monotonicity of  $V$  in  $U$  ■

**Proof of Lemma 3** Consumers' best response arise as the solution to

$$\int_{-\infty}^V \left( \int_{U+p_{vN}-v}^{\infty} (\varepsilon - U - p_{vN} + v) f_N(\varepsilon) d\varepsilon \right) h(v) dv + \int_V^{\infty} \left( \int_{U+p_{vB}-v}^{\infty} (\varepsilon - U - p_{vB} + v) f_B(\varepsilon) d\varepsilon \right) h(v) dv = c. \quad (32)$$

The lower limits of the integrals are increasing and the integrands are decreasing in  $U$ . As a result, the left-hand side above is decreasing in  $U$ . This is sufficient to show that  $U(V, c)$  is decreasing in  $c$ . Moreover, for  $U$  sufficiently negative this expression is bigger than  $c$ , while for  $U$  sufficiently high, it becomes zero, which assures the existence of a  $U$  ■

**Proof of Proposition 3** Consider a stable equilibrium  $(U, V(U))$ . Then,  $U(V(\cdot))$  has a slope  $< 1$ , or  $\frac{\partial U}{\partial V}(V(U)) \frac{\partial V}{\partial U}(U) < 1$ .

Next, denote the left-hand side expression of (32) as  $H(U, V)$ . Then,  $\frac{\partial U}{\partial V} = \frac{\frac{\partial H}{\partial V}}{-\frac{\partial H}{\partial U}}$ . Thus,

stability can be expressed as equilibria satisfying

$$\frac{\partial H}{\partial U}(V(U)) + \frac{\partial H}{\partial V}(V(U)) \frac{\partial V}{\partial U}(U) < 0 \quad (33)$$

We know that  $\frac{\partial H}{\partial U}(\cdot) < 0$  from Lemma 3, and that  $\frac{\partial V}{\partial U}(\cdot) > 0$  from Lemma 2. Thus, a stable equilibrium requires  $\frac{\partial H}{\partial V}(V(U))$  (or equivalently  $\frac{\partial U}{\partial V}$ ) to be small enough.

Finally, consider local comparative statics starting at the equilibrium  $(U, V(U))$ . We know that  $\bar{H}(U) \doteq H(U, V(U)) = c$ . If  $c$  decreases,  $\bar{H}(U)$  needs to decrease as well. Note that, because of expression (33), we can conclude that  $\frac{\partial \bar{H}}{\partial U}(U) < 0$ , which means that  $U$  needs to increase to restore equilibrium. Finally, using Lemma 2, we know that  $V(U)$  increases as well. ■

**Proof of Proposition 4** If  $U_B \leq 0$ , clearly there is no all-broad equilibrium. If  $U_B > 0$ , by the definition of  $c_0$ , there is no positive search equilibrium with  $c > c_0$ . Take, now,  $c \in [c_B, c_0]$ . Using the definitions of  $c_B$  and  $c_0$  and by looking at condition (11), one can easily see that there



exists a  $U \in (0, U_B]$  such that  $(U, \underline{v})$  constitutes an equilibrium. Finally, for  $c < c_B$ , there cannot be an all-broad equilibrium. By looking at (15), note that the induced  $U$  had to be bigger than  $U_B$ , but this would imply that the type  $\underline{v}$  firm prefers a niche strategy, providing a contradiction. ■

**Proof of Proposition 5** First, it is straightforward from the definitions of  $c_N$  and  $c_0$  that  $\underline{c}$  is the highest search cost value that supports an all-niche equilibrium. We argue now that an all-niche equilibrium exists for any  $c' < \underline{c}$ . Lemma 3 shows that consumers' best response  $U' = U(\bar{v}; c') > U$ . Now, Lemma 2 shows that in response to  $c'$  and  $U'$  firms would like to increase  $V$ , but this is already at its highest value  $\bar{v}$ . Thus,  $(U', \bar{v})$  is an all-niche equilibrium at  $c'$ .

**Proof of Proposition 6** As shown in Lemma 1,  $p_v(U) + U$  is increasing in  $U$ . Now, since design is fixed, analogous to the proof of Lemma 3, we can conclude that a fall in  $c$  implies an increase in  $U$ . Given that the only effect of a change of  $c$  is through  $U$ , we can study changes in  $U$  directly.

Since the total size of the market is constant (and given by  $m$ ), it follows that superstar effects arise if and only if

$$\frac{d}{dU} \frac{m}{\rho} [1 - F(p_{\underline{v}}(U)) + U - \underline{v}] = \frac{d}{dU} m \frac{[1 - F_s(p_{\underline{v}}(U) + U - \underline{v})]}{\int_{\underline{v}}^{\bar{v}} [1 - F(p_v(U) + U - v)] h(v) dv} > 0.$$

A sufficient condition, therefore, is that

$$\frac{d}{dU} \frac{1 - F(p_{\bar{v}}(U) + U - \bar{v})}{1 - F(p_v(U) + U - v)} > 0 \text{ for all } v < \bar{v}. \quad (34)$$

Similarly, a sufficient condition to ensure that no long-tail effect arises is

$$\frac{d}{dU} \frac{1 - F(p_{\underline{v}}(U) + U - \underline{v})}{1 - F(p_v(U) + U - v)} < 0 \text{ for all } v > \underline{v}. \quad (35)$$

Writing  $W = U - v$ , as in the proof of Proposition 2, we can write  $1 - F(p_v(U) + U - v) = q^*(W)$  then (34) is equivalent to  $\frac{d}{dU} \frac{q^*(U - \bar{v})}{q^*(U - v)} > 0$  or

$$q^*(U - v) \frac{d}{dU} q^*(U - \bar{v}) - q^*(U - \bar{v}) \frac{d}{dU} q^*(U - v) > 0. \quad (36)$$

First, note that, as a consequence of the proof of Lemma 1,  $q^*(U - \bar{v}) > q^*(U - v)$  and that  $\frac{d}{dU} q^*(U - v) < 0$ . Next, note (36) can be rewritten as

$$[q^*(U - v) - q^*(U - \bar{v})] \frac{d}{dU} q^*(U - v) + q^*(U - v) \left[ \frac{d}{dU} q^*(U - \bar{v}) - \frac{d}{dU} q^*(U - v) \right] > 0. \quad (37)$$

The first square bracket is negative and  $\frac{d}{dU} q^*(U - v) < 0$ , so the first term is positive. Since  $q^*(U - v)$  is positive, a sufficient condition for (34) and (35) is that  $\frac{d}{dU} q^*(U - \bar{v}) - \frac{d}{dU} q^*(U - v) > 0$  for all  $v$ , or

$$\frac{d^2}{dW^2} q^*(W) < 0. \quad (38)$$

It remains to verify this condition.

Consider the firm's maximization problem  $p[1 - F_s(p + U - v)]$ ; this is equivalent to maximiz-

ing  $(P - W)(1 - F(P))$  and  $q^*(W) = 1 - F(P)$ . It follows that we can write:

$$\frac{d^2 q}{dW^2} = -f \frac{d^2 P}{dW^2} - f' \left( \frac{dP}{dW} \right)^2. \quad (39)$$

By differentiating the firm's first-order condition with respect to  $W$ , and differentiating again, and rearranging both expressions, we obtain

$$\frac{dp}{dW} = \frac{1}{2 + \frac{1-F(P)}{f(P)} \frac{f'(P)}{f(P)}}, \text{ and} \quad (40)$$

$$\frac{d^2 p}{dW^2} = \frac{1 + 2 \frac{1-F(P)}{f(P)} \frac{f'(P)}{f(P)} - \frac{1-F(P)}{f(P)} \frac{f''(P)}{f'(P)}}{(2 + \frac{1-F(P)}{f(P)} \frac{f'(P)}{f(P)})^3} \frac{f'(P)}{f}. \quad (41)$$

Then, we can substitute these expressions into (39) and rearrange to obtain:

$$\frac{d^2 q}{dW^2} = -f(P)^4 \frac{(f'(P)^2 - f(P)f''(P))(1 - F(P)) + f'(P) \left( f'(P)^2 \frac{(1-F(P))^2}{f(P)^2} + 5(f'(P)(1 - F(P)) + f^2) \right)}{(f'(P)(1 - F(P)) + 2f(P)^2)^3} \quad (42)$$

Logconcavity of  $f(\cdot)$  implies that  $f'(P)^2 - f(P)f''(P) > 0$ , and that  $1 - F(\cdot)$  is logconcave. This, in turn implies that  $f'(P)(1 - F(P)) + f(P)^2 > 0$  and so also  $f'(P)(1 - F(P)) + 2f(P)^2 > 0$ . It follows that (38) is satisfied as long as

$$f'(P) > - \frac{(f'(P)^2 - f(P)f''(P))(1 - F(P))}{f'(P)^2 \frac{(1-F)^2}{f^2} + 5(f'(P)(1 - F(P)) + f^2(P))} \quad (43)$$

This is necessarily the case when  $f'(\cdot) > 0$  or, more generally, when  $F(\cdot)$  is not too concave.

■

**Proof of Proposition 7** We use the functional forms for  $F_N(\cdot)$ ,  $F_B(\cdot)$  and  $h(\cdot)$  to rewrite the equations in Section 3.3 that characterize equilibrium assuming that all firms are active (that is, they make positive sales).

First, consider prices. Condition (8) delivers

$$p_{vB}(U) = \frac{\bar{\theta}_B + v - U}{2}, \text{ and} \quad (44)$$

$$p_{vN}(U) = \frac{\bar{\theta}_N + v - U}{2}. \quad (45)$$

Next, we focus on the firm decision  $V$ . We rewrite condition (10) as:

$$\frac{(\bar{\theta}_B + V - U)^2}{b^2} = \frac{(\bar{\theta}_N + V - U)^2}{n^2}, \quad (46)$$

where we introduce the notation  $b^2 = \bar{\theta}_B - \underline{\theta}_B$  and  $n^2 = \bar{\theta}_N - \underline{\theta}_N$  for convenience. Note that  $n > b$ .

Recalling footnote 3.2 and doing some algebra on the previous expression, we obtain

$$V = \min\{H, \max\{U + K, L\}\}, \quad (47)$$

where  $K$ , defined as

$$K := \frac{\bar{\theta}_N b - \bar{\theta}_B n}{n - b}, \quad (48)$$

is a constant that depends on exogenous parameters.

Finally, we rewrite the consumer condition (11) as:

$$c = \int_L^V \left( \int_{\frac{\bar{\theta}_N - v + U}{2}}^{\bar{\theta}_N} \left( \varepsilon - \frac{\bar{\theta}_N - v + U}{2} \right) \frac{d\varepsilon}{n^2} \right) \frac{dv}{H - L} + \int_V^H \left( \int_{\frac{\bar{\theta}_B - v + U}{2}}^{\bar{\theta}_B} \left( \varepsilon - \frac{\bar{\theta}_B - v + U}{2} \right) \frac{d\varepsilon}{b^2} \right) \frac{dv}{H - L} \quad (49)$$

Suppose that there are some firms choosing both a niche and a broad design. Then, then we can write  $V = U + K \in (L, H)$  and simplify the previous expression to

$$c = \frac{1}{24} \left( \frac{V - L}{H - L} \frac{(V - L)^2 + 3(K + \bar{\theta}_N)(K + L + \bar{\theta}_N - V)}{n^2} + (H - V) \frac{(H - V)^2 + 3(K + \bar{\theta}_B)(K + H + \bar{\theta}_B - V)}{b^2 (H - L)} \right) \quad (50)$$

Note that the right hand-side is a polynomial in  $V$ . Denote it by  $A(V)$ .

Next, it follows that, here, trivially,  $U_B = L - K$  and  $U_N = H - K$  and so, after substituting for  $K$ , we obtain

$$c_B = \frac{1}{24} \frac{(H - L)^2 + 3\left(\frac{\bar{\theta}_N b - \bar{\theta}_B n}{n - b} + B\right)\left(\frac{\bar{\theta}_N b - \bar{\theta}_B n}{n - b} + H + B - L\right)}{b^2}, \text{ and} \quad (51)$$

$$c_N = \frac{1}{24} \frac{(H - L)^2 + 3\left(\frac{\bar{\theta}_N b - B n}{n - b} + \bar{\theta}_N\right)\left(\frac{N b - B n}{n - b} + L + \bar{\theta}_N - H\right)}{n^2}. \quad (52)$$

It follows that

$$c_B - c_N = \frac{1}{24} (b + n) (H - L) \frac{(H - L)(n - b)^2 + 3bn(\bar{\theta}_N - \bar{\theta}_B)}{b^2 n^2 (n - b)} > 0, \quad (53)$$

proving part (i) of the proposition.

Next, since  $A(V)$  is a cubic, it has at most three roots. Note that  $n > b$  so as  $V \rightarrow -\infty$  that  $A \rightarrow \infty$  and as  $V \rightarrow \infty$  then  $A \rightarrow -\infty$ .

Consider

$$\frac{dA}{dV} = \frac{1}{8} \frac{(Lb + \bar{\theta}_B n - Vb - Ln - \bar{\theta}_N n + Vn)^2}{n^2 (H - L) (n - b)^2} - \frac{1}{8} \frac{(\bar{\theta}_B b + Hb - \bar{\theta}_N b - Hn - Vb + Vn)^2}{b^2 (H - L) (n - b)^2} \quad (54)$$

and

$$\frac{d^2 A}{dV^2} = \frac{1}{4} \frac{Hn^2 - Lb^2 + bn(\bar{\theta}_N - \bar{\theta}_B)}{b^2 n^2 (H - L)} - \frac{1}{4} \frac{n^2 - b^2}{b^2 n^2 (H - L)} V. \quad (55)$$

Now  $V \in (\min\{K, L\}, H)$ . Note that  $\frac{d^2 A}{dV^2}|_{V=H} = \frac{1}{4} \frac{(H-L)b+n(\bar{\theta}_H-\bar{\theta}_B)}{bn^2(H-L)} > 0$  and since  $\frac{d^3 A}{dV^3} < 0$  this means that  $\frac{d^2 A}{dV^2} > 0$  throughout the relevant region.

Now, consider  $\frac{dA}{dV}|_H = -\frac{1}{8} \frac{2n(\bar{\theta}_N - \bar{\theta}_B) - (H-L)(n-b)}{n^2(n-b)}$ . If  $\frac{dA}{dV}|_H = -\frac{1}{8} \frac{2n(\bar{\theta}_N - \bar{\theta}_B) - (H-L)(n-b)}{n^2(n-b)} < 0$ ; then, since  $\frac{d^2 A}{dV^2} > 0$  through the region,  $\frac{dA}{dV} < 0$  and there can be at most one solution to  $A = 0$ .

This is the case if and only if

$$2n \frac{\bar{\theta}_N - \bar{\theta}_B}{n - b} > H - L. \quad (56)$$

Note that throughout we assumed that all firms are active. Consider, now, the limiting case where all firms choose niche designs and the marginal firm is indifferent, so that  $V = H$  (which we know must arise when  $c$  is sufficiently small, following Proposition 5). Then, the lowest-quality firm makes positive sales as long as  $p_{LN}(H - K) > 0$ . Note that

$$p_{LN}(H - K) = \frac{\bar{\theta}_N + L - H + K}{2} = \frac{\bar{\theta}_N + L - H + \frac{\bar{\theta}_N b - \bar{\theta}_B n}{n - b}}{2} = \frac{1}{2} \frac{n(\bar{\theta}_N - \bar{\theta}_B) - (H - L)(n - b)}{n - b}. \quad (57)$$

So,  $p_{LN}(H - K) > 0$  if and only if

$$n \frac{\bar{\theta}_N - \bar{\theta}_B}{n - b} > H - L \quad (58)$$

which, trivially, implies (56).

This shows that  $\frac{dA}{dV}|_H < 0$  and so also that  $\frac{dA}{dV} < 0$  for all  $V \in (\min\{K, L\}, H)$ ; thus, there is a unique solution to  $A = 0$  and, moreover, that  $V$  is decreasing in  $c$ . This proves (ii) and (iii) of the Proposition.

Finally, consider the sales of the highest-quality firm. Note that throughout this region, it chooses the broad strategy, following Proposition 2, so its sales are given by  $\frac{\bar{\theta}_B - p_{HB}(V - K)}{b^2} = \frac{\bar{\theta}_B - \frac{\bar{\theta}_B + H - V + K}{2}}{b^2} = \frac{1}{4}(\bar{\theta}_B - H - K + V)$

Note that this is increasing in  $V$  and  $V$  is decreasing in  $c$ , so sales for the highest-quality firm rise as search costs fall, proving the second half of part (iv) of the Proposition. The first half is analogous and so is omitted. ■