Information and Intertemporal Signal Garbling in Relational Contract

Preliminary Draft

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Abstract

We investigate the use of information in repeated principal-agent relationships with imperfect public monitoring and report three results. First, if we only consider information garbling within each period, then, consistent with Kandori (1992), efficiency of the relationship is increasing in the informativeness of the signal in the sense of Blackwell. Second, contrary to Abreu, Milgrom, and Pearce (1991), bundling signals across periods and then fully revealing them hurts efficiency of the relational contract. Third, and most importantly, we construct an alternative intertemporal signal garbling process that transforms the repeated relationship into one with private monitoring. The main finding of the paper is that in the transformed game, there exists a belief-based pure-strategy equilibrium that can be more efficient than the optimal equilibrium in the original game with imperfect public monitoring.

1 Introduction

The prevalence and importance of relational contracts, contracts enforced not by the rule of the court but rather by the self-interests of the participating parties in concern of future contracts, have been emphasized both inside and outside the economics literature. The existing theoretical literature on relational contracts, see for example MacLeod and Malcomson (1988), Baker, Gibbons, and Murphy (2002), Levin (2003), and Fuchs (2007), has focused on the efficiency of the relational contract taking the information structure as fixed. Less attention has been focused on how the efficiency of relational contract is affected by the information structure.

In contrast, in the literature of repeated-game (without transfers), some studies have focused on how the use of information affects the efficiency of repeated game. The two most influential papers in this literature are Abreu, Milgrom, and Pearce (1991) and Kandori (1992).[1] Kandori (1992) shows that in a repeated game with imperfect public monitoring, the efficiency of the game is weakly increased if the commonly observed public signal of the output becomes more informative in the sense of Blackwell. Kandori (1992) also provides conditions under which the efficiency of the game can be strictly increased. Abreu, Milgrom, and Pearce (1991) (AMP hereafter) show that, when the players play strongly-symmetric strategies and their discount factors approach 1, the efficiency of the game can be enhanced through bundling signals across several consecutive periods and then fully revealing them at the end of these periods.

In this paper, we examine the efficiency of relational contracts in repeated principal-agent relationships with imperfect public monitoring under different information structures. We show that the logic of Kandori (1992) developed in repeated game without transfers carries through to repeated principal-agent relationship and the efficiency of the relational contract is enhanced if the signals are more informative in the sense of Blackwell. On the other hand, contrasting AMP's finding, we show that bundling signals across periods and fully revealing them every T periods decreases the efficiency of the relational contract. While these two results seem to suggest the efficiency of the relational contract increases when the signals become more informative and are revealed more frequently, our main result shows that this is not true.

[1] Kandori and Obara (2006) show that when the discount factor is close to one, reducing observability allows for asymmetric punishment in repeated games with imperfect public monitoring. This can expand the equilibrium payoff set.

In our main result, we construct an signal garbling process according to which information is linked intertemporally and is revealed partially. Since the distribution of future (garbled) signals depends on the entire history of the past outputs yet these outcomes were only piratically revealed, when the agent deviates from the equilibrium strategy, he will have a different belief about the future signals from the principal. This information structure transforms the repeated principal-agent relationship (with transfers) of public monitoring into one with private monitoring. With this information structure, we identify a belief-based pure-strategy equilibrium which is strictly more efficient than any Pareto dominant equilibrium under imperfect public monitoring when the likelihood of success is low.

Our setup can be viewed is a simplified version of Levin's (2003), except that we allow the public signal in each period to potentially depend on the entire history of previous outputs. A principal and an agent trade repeatedly until at least one of them decides to terminate the relationship. Every period the principal offers a contract specifying some court enforceable fixed wage and a relational bonus. Production technology is that if the agent exerts a fixed positive effort, then with a positive probability the output is positive, and if the agent does not exert effort, then the probability of a positive output is lower (or for simplicity, zero in part of our analysis).

If we restrict attention to information garbling within each period, then Kandori's (1992) result that more precise signals in the sense of Blackwell lead to a larger equilibrium payoff set continues to hold in the context of relational contracts. The intuition is that noisier signals are less indicative of effort, so the principal has to pay a larger bonus upon observing a good signal in order to induce effort. Requiring the principal to pay a larger bonus leads to a stronger incentive for her to renege, rendering the relational contract harder to sustain.

AMP's result that the equilibrium payoff set of a repeated game may be expanded when public signals are pooled and revealed once every multiple periods does not generalize to relational contracts with public monitoring for the following reason.[2] The idea behind their finding is that pooling signals across periods allows players to coordinate on punishment more efficiently, punishing only when the worst possible signals

[2] Fuchs (2007) shows that with private monitoring, similar signal pooling helps reduce the probability of inefficient termination (a form of money burning). One way to see why such signal pooling does not enhance efficiency in repeated principal-agent relationships with public monitoring is that there is no inefficient termination when monitoring is public.

are realized and punishing harshly in these realizations. Such arrangement lowers the overall inefficiency due to punishment because the likelihood ratio in the test for deviation is highest at the worst signals. Applying AMP's insight to a relational contract with T-period signal pooling would be to pay a bonus to the agent at the end of every T periods except when the worst outcomes are realized. To make the punishment in the worst possible outcomes harsh, the bonus has to be large. This is problematic, however, because the maximal bonus the principal is willing to give is constrained by the present discount value of the future surplus of the relationship which remains unchanged. In summary, reducing the frequency of signal revelation necessarily increases the maximum bonus required to induce effort from the worker, and this increase makes the principal's incentive to pay out the bonus harder to sustain.

Nevertheless, a closer investigation suggests that an alternative form of signal garbling can address a limitation of relational contract under full revelation of (imperfect) signal. Here we describe the limitation and explain how it can be addressed. When the signal is perfectly indicative of the output (but not the effort) and it is revealed in each period, the bonus required to induce effort from the worker is decreasing in the probability of success. In other words, when a success is highly unlikely even if the agent puts in effort, the bonus needs to be very large to induce effort. But such large bonus hurts the incentive of the principal, who will not find it incentive compatible to pay out the bonus to the agent if the bonus exceeds the future surplus of the relationship. Therefore, a relational contract is hard to sustain when the probability of success is small.

An alternative way to provide incentive to the worker we propose is to break the total reward for success into two parts: a) a lowered bonus to be paid out immediately and b) a higher future continuation payoff for the agent. In this way, the principal's incentive to renege on the bonus is weakened (as long as he does not know about the agent's higher continuation payoff which implies more bonus will be required in the future). For the agent, a success not only brings bonus in the immediate future, but also increases the likelihood that future bonus will be paid out. In particular, we construct an equilibrium in which the bonus is paid out based on a garbled signal which is generated with the following garbling process, as illustrated in Figure 1.

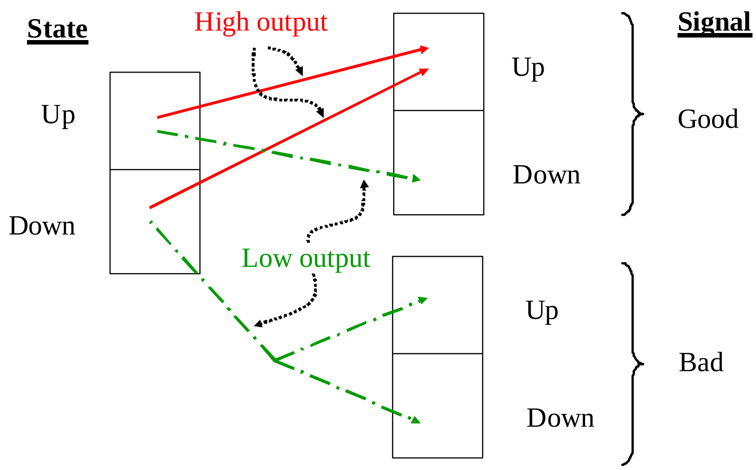

Figure 1: Signal Garbling Process

In each period, the garbled signal may be *good* or *bad*, but given any signal, there are two secrete states: *up* and *down*. Players only observe the garbled signal but do not know the state within the signal. If the output is a success, a *good* signal is publicly observed and the state is *up*. If the output is a failure, there are two cases. If the state is *up* in the previous period, then a *good* signal is publicly observed and the state is *down*. If the state is *down* in the previous period, then a *bad* signal is publicly observed and the *up* state is generated with a fixed probability. This signal garbling process may be interpreted as the consequence of a specific exogenous signal generating process in which the information about the output is gradually released. Alternatively, this can be viewed as the evaluation of the agent's performance written by a supervisor and given to the principal who does not directly observe the worker's production.

Moving the state to *up* following a high output regardless of the current state in the garbling process is certainly intended to provide additional incentive for the agent to put in effort. The disincentivizing forces of such scheme are, however, that a) when the agent is in the *up* state, he will be rewarded a bonus regardless of the outcome of production, and b) when the state is low, the agent will be moved to the *up* state with a positive probability even when the output is low. It will become clear in our formal analysis that these disincentivizing parts of the signal garbling process are needed for maintaining the stationarity of the process and ensuring that both the principal and agent never know which state they are in on the equilibrium path. One part of our equilibrium construction is to show that when the probability of success in production becomes small, the disincentivizing forces become less important will be eventually dominated by the incentivizing force. The basic intuition is that when the probability of success is small, then the (stationary) probability that players are in the *up* state is small. Therefore, the probability that disincentivizing bonus reward in (a) is given out in a small probability and the increased chance of being in the *up* state in (b) is also small.

With such signal generating process, we construct an equilibrium in which the principal pays out a bonus whenever the garbled signal is good. The agent's strategy is to put in effort whenever he believes that his probability of being in the up state is weakly small than a threshold. On the equilibrium path, the agent always puts in effort and he believes his probability of being in the up state is exactly equal to the threshold. For all discount factors, this equilibrium performs better than the imperfect public equilibrium without signal garbling when the probability of success is small. And its degree of improvement is increasing in the discount factor of the agent.

The key to this construction is to specify the action of the agent off the equilibrium path. In the above-constructed signal generating process, the signal in each period can depend on the entire history of past outputs, and thus it may depend on the entire private history of past actions of the agent. In other words, while the principal always forms one (equilibrium) belief, the agent's belief of the probability he is in an up state can depend on his entire private history of efforts. When it is possible for the principal and the agent to form different beliefs (following the agent's deviation), checking one-stage deviation no longer guarantees that a strategy profile constitutes an equilibrium. To check that the agent's strategy is an optimal response to the principal's strategy, one needs to check multi-stage deviations as well. Since this is an infinitely repeated game, checking such multi-stage deviations can be difficult[3].

In our construction, conditional on the current period's output, the signal is completely determined by the previous period's state the agent was in. In other words,

[3]The need to check multi-stage deviation also appears in Abreu, Milgrom, and Pierce (1991), and Fuchs (2007). These two papers consider T-period review strategies, so there is no need to check deviations that exceed T-stages. In contrast, there is a priori no upper bound in the number of stages of deviation.

with this two-state construction, the only payoff relevant belief of the agent is the probability that he is in an up state. This allows for a recursive formulation of the agent's value function with the state variable being the (privately known) probability that the agent is in an up state. With this recursive representation, we show that the optimal action of the agent is to put in effort if and only this probability falls weakly below a threshold.

In standard relational contracts with imperfect public monitoring, one way to maximize the enforceable bonus payment from the principal is to give the agent his individually rational continuation payoff after each bonus payout. Such arrangement will not be sustainable in our current setup with intertemporally garbled signals. This is because the agent can shirk and after privately knowing being punished by placed on the low state, quits the game, rendering the punishment of low state ineffective. We show that this incentive problem can be resolved by always postponing the bonus payment to be paid out as part a higher base wage offered by the principal in the following period. Since now the base wage is made contingent of the previous period's signal, it suggests that the optimal relational contract with intertemporal signal garbling may be necessarily nonstationary.

For the rest of the paper, we set up the model in Section 2. We analyze the model in Section 3, with our main result presented in Subsection 3.3 and some generalization relegated to the Appendix. In our discussion in Section 4, we show that the "belief-free" approach does not enhance efficiency and that signal garbling helps enhance efficiency only when the probability of success in production is not equal to half. Section 5 concludes.

2 Setup

Time is discrete and indexed by $t \in \{1, 2, ..., \infty\}$.

2.1 Players

There's one principal and one agent. Both are risk neutral, infinitely lived, and have a common discount rate of δ. The agent's per period outside option is $\underline{u}$; the principal's per period outside option is $\underline{v}$.

2.2 Production

If the principal and the agent engages in production together, there is one task that has two outcomes. If the agent puts in effort, the outcome Y is y with probability p and 0 with probability $1-p$. When no effort is put in, the outcome is y with probability $q < p$. The effort costs c.

We assume that effort is efficient. And moreover, the relationship is less efficient than the outside options if the effort is not put in.

$$py - c > \underline{u} + \underline{v} \geq \underline{v} > qy.$$

2.3 Timing

At the beginning of each period t, the principal decides whether to offer a contract to the agent, $d_t^P \in \{0,1\}$. If the principal chooses not to offer the contract ($d_t^P = 0$), then the two parties receive their outside options.

If the contract is offered, it specifies a base wage $w_t \in R$ and a performance bonus $b_t \in R^{+}$[4]. We assume that w_t is legally enforceable and is paid out as soon as the contract is accepted. The bonus b_t depends on public information to be described in the next section. This information is observable to both parties but not verifiable to the outside world, so the bonus cannot be contracted upon. The bonus is paid out at the end of the period.

The agent chooses $d_t^A \in \{0,1\}$, and if he rejects the contract ($d_t^A = 0$), the two parties receive their outside options. Otherwise, the relationship starts. The agent chooses effort $e_t \in \{0,1\}$, and the output Y_t is realized.

2.4 Information Set

We assume that it is publicly observed in each period whether the principal offers the contract, whether the agent accepts the contract, and whether the bonus is paid out. We also assume that the action of the agent's effort is his private information.

The key element of the information structure is the observability of the outputs. In the standard analysis of relational contract model with imperfect public monitoring,

[4]It is without loss of generality to specify a nonnegative bonus in this model.

it is assumed that the output per period (y_t) is publicly observed. Here we assume that the parties observe at the end of the period a signal s_t that contains information about the entire past history of outputs.

In particular, define the set of signals as S. In each period, there is a signal distribution function S_t that maps the history of outputs to a probability distribution on the set of signals, i.e.

$$S_t : \prod_{j=1}^{t} Y_j \rightarrow \Delta S,$$

where ΔS is the set of probability distribution on S.

The signal distribution function incorporates several special cases. First, we give two examples that are commonly used in the literature.

Example 1 (Output perfectly observable)

In the standard relational contract model, see for example Malcomson and Macleod (1988), Levin (2003), it is assumed that the output each period can be perfectly observed. To incorporate this information structure, we can have the set of signals be $S = \{0, Y\}$, and in period t the signal $s_t = y_t$, where recall that y_t is the output in period t. More formally, the signal distribution function is given by

$$\Pr(S_t(y_1, ..., y_t) = y_t) = 1, \text{ for all } \{y_1, ..., y_t\}.$$

Example 2: (T-period Revelation)

One type of information structure that has received considerable attention from the literature is the T-period revelation, see for example Abreu, Milgrom, Pearce (1991) and Fuchs (2007). This information structure specifies that the information becomes public every T periods and no information is revealed in public in between. To incorporate these cases, we let $S = \{0, Y\}^T \cup \{N\}$, where N stands for no information. When $t \neq nT$ for each $n \in N$, the signal $s_t = N$. When $t = nT$, $s_t = (y_{(n-1)T+1}, ..., y_{nT})$. More formally, when $t \neq nT$, the signal distribution function is given by

$$\Pr(S_t(y_1, ..., y_t) = N) = 1.$$

When $t = nT$,

$$\Pr(S_t(y_1, ..., y_t) = (y_{(n-1)T+1}, ..., y_{nT})) = 1, \text{ for all } \{y_1, ..., y_t\}.$$

In addition to allowing for the standard information structures in the literature, the signal distribution function also incorporates cases that are relevant to real life but are less studied. We give two examples below.

Example 3: (Delayed Information Revelation)

In many economic situations, information of the outcome is not readily available. The time it takes to collect the information is an obvious reason why information is delayed. But even if information collection can be done in real time, information about the "right outcome" may not be known right after the actions are taken. For example, computer system can track the sales of each item, but the real sales figure (that the firm should care about) should account for the returns from the customers, and it will not be known right after the salesperson made the sale.

The simplest case of modelling delay in this model is to have the set of signals be $S = \{0, Y\} \cup \{N\}$, where again N stands for no information. In period $t = 1$, the signal $s_1 = N$, and in period $t > 1$, the signal $s_t = y_{t-1}$. More formally, the signal distribution function is given by

$$\begin{aligned} \Pr(S_1(y_1) &= N) = 1, \text{ for } t = 1 \\ \Pr(S_t(y_1, ..., y_t) &= y_{t-1}) = 1, \text{ for } t > 1 \text{ and all } \{y_1, ..., y_t\}. \end{aligned}$$

It is straightforward to model information structure that has more than one period of delays.

Example 4: (Partial Information Revelation)

Information about outputs is often only revealed partially. For example, when a building or bridge is finished, information about its reliability can take years or decades to be revealed. That how well the building stands up in an earthquake is, in most cases, not known. Costs of collecting information also prevents the information from being completely revealed. For instance, when senior executives decide the bonus of a worker at the end of the year, it is difficult and very costly to know the performance of the worker on each single day. Instead, the senior executives may base the bonus of the worker on a crude performance evaluation measure (often supplied by some middle-level manager).

One example that captures partial information revelation is the following. Let the set of the signal be $S = \{Success, Failure\}$. In period t, the signal $s_t = Success$ if more than half of the previous outcomes $y = Y$, and $s_t = Failure$ otherwise. More formally,

$$\begin{aligned}
\Pr(S_t(y_1, ..., y_t) &= Success) = 1, \text{ if } \sum_{j=1}^{t} y_j > \frac{ty}{2} \\
\Pr(S_t(y_1, ..., y_t) &= Success) = 0, \text{ if } \sum_{j=1}^{t} y_j \leq \frac{ty}{2}
\end{aligned}$$

Note that in this example, the signal in each period only captures the average of the existing outputs. Consequently, the exact output in many periods may not be publicly known. More interestingly and importantly here, even if the output y_t in period t may not be known, it has an impact on all future signals. While this seems to suggest that the signal generating process is very complicated and may require an infinite memory, some simplification is actually possible by having the right "state variable". In the example above, even if the signal s_t depends on all of the previous t outputs, it can be determined by the output in period t (y_t) and the average output in all previous periods (the state variable).

2.5 Strategy and Equilibrium Concept

2.5.1 History

We denote $h_t = \{d_t^P, w_t, d_t^A, s_t, b_t\}$ as public events that happens in period t. Denote $h^t = \{h_n\}_{n=0}^{t-1}$ as a public history path at the beginning of period t. $h^1 = \Phi$. Let $H^t = \{h^t\}$ be the set of public history paths till time t. Finally, define $H = \cup_t H^t$ as the set of public histories. For the principal, the public history is all that he observes. For the agent, at the beginning of period t, he also observes his past actions $e^t = \{e_j\}_{j=1}^{t-1}$. Denote $H_A^t = H^t \cup \{e^t\}$ as the set of agent's private history at the beginning of period t.

2.5.2 Strategy

In period t, the principal chooses an offering action D_t^P from H^t to $\{0,1\}$; wage action W_t from H^t to R; and bonus action B_t from $H^t \cup \{s_t\}$ to R. The strategy of the principal is $\{D_t^P, W_t, B_t\}_{t=1}^{\infty}$. We assume that the principal may use mixed strategy, and we denote σ^P as the mixed public strategy of the principal. We define $\sigma_{h^t}^P$ as the principal's mixed strategy following public history h^t.

In period t, the agent chooses an accepting decision D_t^A from $H_A^t \cup \{w_t, b_t\}$ to $\{0,1\}$, effort action e_t from $H_A^t \cup \{w_t\}$ to $\{0,1\}$. And the strategy of the agent is $\{D_t^A, A_t, \}_{t=1}^{\infty}$. We also assume that the agent may use mixed strategy, and we denote σ^A as the mixed public strategy of the agent. We define $\sigma_{h_A^t}^A$ as the agent's mixed strategy following agent's private history h_A^t.

2.5.3 Payoff and Equilibrium Concept

Suppose the principal chooses strategy σ^P and the agent chooses strategy σ^A. Following the private history of the agent h_A^t, the agent's expected continuation payoff is given by

$$\begin{aligned} & U(h_A^t, \sigma_{h_A^t}^A, \sigma_{h^t}^P) \\ = \quad & E[\sum_{\tau=t}^{\infty} \delta^{\tau-t}\{\underline{u} + 1_{\{D_\tau^P D_\tau^A = 1\}}(-ce_\tau + w_\tau + B_\tau - \underline{u})\} | h_A^t, \sigma_{h_A^t}^A, \sigma_{h^t}^P]. \end{aligned}$$

The principal's continuation payoff following the private history of the agent h_A^t is given by

$$\begin{aligned} & v(h_A^t, \sigma_{h_A^t}^A, \sigma_{h^t}^P) \\ = \quad & E[\sum_{\tau=t}^{\infty} \delta^{\tau-t}\{\underline{v} + 1_{\{D_\tau^P D_\tau^A = 1\}}(y(q + (p-q)e_\tau) - w_\tau - B_\tau - \underline{v})\} | h_A^t, \sigma_{h_A^t}^P, \sigma_{h^t}^P]. \end{aligned}$$

Of course, the principal does not know that agent's private history, his expected payoff following the public history h^t is given by

$$V(h^t, \sigma^P, \sigma^A) = E[v(h_A^t, \sigma_{h_A^t}^A, \sigma_{h^t}^P) | h^t]$$

The solution concept we use in this setting is Perfect Bayesian Equilibrium (PBE). In this setting, when the principal uses strategy σ^{*P} and the agent uses strategy σ^{*A}, the PBE requires that following any private history of the agent h_A^t, the agent's expected continuation payoff

$$U(h_A^t, \sigma_{h_A^t}^{*A}, \sigma_{h^t}^{*P}) \geq U(h_A^t, \sigma_{h_A^t}^{A}, \sigma_{h^t}^{*P}) \quad \text{for all } \sigma_{h_A^t}^{A}.$$

Following any public history h^t, the principal's expected payoff

$$V(h^t, \sigma^{*P}, \sigma^{*A}) \geq V(h^t, \sigma^{P}, \sigma^{*A}) \quad \text{for all } \sigma^{P}.$$

And the beliefs of the principal (on the probability distribution of h_A^t) is updated through Bayes rule.

Note that while the principal and the agent will share the same belief along the equilibrium path, this is not true if the agent deviates. When the agent deviates, his belief of the output distribution in the past is different from the equilibrium belief of the principal. Since the future signals depend on the realization of past outputs, the agent's belief of the signal distribution in the future will be different from the principal as well. This difference in beliefs imply that checking one-stage deviation is satisfied will no longer be sufficient to guarantee that a strategy profile is a PBE.

3 Analysis

In this section, we study how the information structure affects the efficiency of the relational contract. In Section 3.1, we show that garbling of information within a period worsens the efficiency of relational contract. This result is consistent with a related result in repeated game; see Kandori (1992). In Section 3.2, we show that the T-period review contracts, in which the signals of performance becomes public every T periods, is strictly worse than the contract in which information about output is fully revealed in each period. This result contrasts with the finding in the repeated game literature in which bundling information across periods can expand the equilibrium payoff set; see Abreu, Milgrom, and Pearce (1991). While the preceding results seem to suggest that the efficiency of the relational contract increases with the informativeness of the signal and the frequency at which the signal is disseminated, we show in Section 3.3 that the efficiency of the relationship can be enhanced through some form of intertemporal garbling of signals that transforms the game between the principal and agent into one of private monitoring.

3.1 Within-Period Information Garbling

In this subsection, we restrict attention to garbling information within each period and we study how the efficiency of the relationship is affected by the informativeness of the signal. We focus on the two-signal case, which eases the analysis and helps highlight the intuition of the result. The general case with multiple action and multiple signals is analyzed in the appendix.

Let the set of signals be $S = \{0, y\}$. And we start with the case in which the signal is perfectly informative of the output, i.e. $s_t = y_t$. Note that even if the signal is perfectly informative, the output still does not correspond one-to-one to the effort level. In particular, this information structure is a special case of relational contract with imperfect public monitoring studied by Levin (2003). Levin (2003) shows that the optimal relational contract with public monitoring can be implemented by a sequence of stationary contracts.

In the stationary contract, the principal pays out a base wage w in each period, and he also pays out a bonus b if the high output is realized. Note that to induce the effort from the worker, we need the bonus to be big enough such that

$$\begin{aligned} w - c + pb &\geq w + qb \\ b &\geq \frac{c}{p-q}. \end{aligned} \tag{1}$$

With the bonus big enough so that the agent will put in effort per period, the principal can lower the base wage of the worker to his outside option, i.e.

$$w - c + pb = \underline{u}.$$

In this way, the principal can capture the entire surplus of the relationship.

Finally, since the bonus is non-contractible, we need to check that the principal is willing to pay the bonus. It is incentive compatible for the principal to pay the bonus if the future gain of doing so exceeds the short-term loss of paying the bonus. We may assume without loss of generality that if the principal fails to pay the bonus the two parties will receive their outside options forever. This implies that for the principal to pay the bonus, we need

$$b \leq \frac{\delta(py - c - \underline{u} - \underline{v})}{1 - \delta}, \tag{2}$$

where $\frac{\delta(py-c-\underline{u}-\underline{v})}{1-\delta}$ is the discounted expected future surplus of the relationship, which is completely captured by the principal.

Note that equation (1) and (2) combined implies that an relational contract can induce effort in this setting if and only if

$$\frac{c}{p - q} \leq \frac{\delta(py - c - \underline{u} - \underline{v})}{1 - \delta}. \tag{3}$$

In other words, the incentive cost should be smaller than the discounted expected future surplus.

Now suppose that instead of having signal as being a perfect indicator of the output, the signal is noisy instead. In particular, we assume that

$$\begin{aligned}
\Pr(s_t &= y|Y_t = y) = \theta_1 \\
\Pr(s_t &= 0|Y_t = y) = 1 - \theta_1 \\
\Pr(s_t &= 0|Y_t = 0) = \theta_2 \\
\Pr(s_t &= y|Y_t = 0) = 1 - \theta_2,
\end{aligned}$$

where $\theta_1 > \frac{1}{2}$ and $\theta_2 > \frac{1}{2}$ so that the signal is indicative of the true output. This information structure is a garbling of the perfect signal. In other words, the garbled signal is less informative than the perfect signal in the sense of Blackwell.

With this information structure, it can be shown that the optimal contract can again be implemented by a sequence of stationary contracts. In the stationary contract, a bonus b' is paid out when a signal $s_t = y$ is realized. To induce the agent to put in effort, we need

$$\begin{aligned} -c + (p\theta_1 + (1-p)(1-\theta_2))b' &\geq (q\theta_1 + (1-q)(1-\theta_2))b' \\ b' &\geq \frac{c}{(p-q)(\theta_1+\theta_2-1)}. \end{aligned}$$

Now the principal can again set the wage to capture the entire surplus of the relationship. In this case, the incentive constraint of the principal to pay the bonus is again given by

$$b' \leq \frac{\delta(py - c - \underline{u} - \underline{v})}{1-\delta}.$$

Combining the two equations above, we have that, with noisy signals, the necessary and sufficient condition to induce effort in a relational contract is given by

$$\frac{c}{(p-q)(\theta_1+\theta_2-1)} \leq \frac{\delta(py - c - \underline{u} - \underline{v})}{1-\delta}.$$

It is clear from the expression above that, as along as $\theta_1 < 1$, or $\theta_2 < 1$, i.e. the signal does not reflect the output perfectly, we have

$$\frac{c}{(p-q)(\theta_1+\theta_2-1)} > \frac{c}{(p-q)},$$

so the condition for sustaining effort is strictly more stringent here.

The intuition for this result is straightforward. When the signals are noisy, they are less indicative of effort, so it requires a larger bonus to induce effort. But the larger bonus makes the principal more likely to renege, and it follows that efforts are harder to sustain in equilibrium.

While the analysis above was based on a two-action, two-output setting, the intuition carries over to more general settings. For example, the idea that noisy signals require larger bonus is directly related to the idea that larger prizes are required to induce effort in a tournament setting with continuous effort and outputs. We also perform a similar analysis in a multiple signal, multiple action setting in the appendix.

3.2 Bundling T Periods

In this subsection, we analyze how the efficiency of the relational contract is affected when the signals are not revealed in each period, but rather are bundled together and revealed once every several periods.

In particular, we assume that the information becomes public every T periods and no information is revealed in between. Let $S = \{0, Y\}^T \cup \{N\}$, where N stands for no information. When $t \neq nT$ for each $n \in \mathbb{N}$, the signal $s_t = N$. When $t = nT$, $s_t = (y_{(n-1)T+1}, ..., y_{nT})$. More formally, when $t \neq nT$, the signal distribution function is given by

$$\Pr(S_t(y_1, ..., y_t) = N) = 1.$$

When $t = nT$,

$$\Pr(S_t(y_1, ..., y_t) = (y_{(n-1)T+1}, ..., y_{nT})) = 1, \text{ for all } \{y_1, ..., y_t\}.$$

In this game, it is straightforward to show that the optimal contract can be implemented as a sequence of stationary contracts. In particular, the bonus will be paid out at the end of each T periods, and there is a bonus function $B(y_{(n-1)T+1}, ..., y_{nT})$ that maps $\{0, y\}^T$ to R^+ for all n.

Now define the maximum bonus the principal ever pays out as

$$B_{\max} = \max_{\{(y_{(n-1)T+1}, ..., y_{nT})\}} \{B(y_{(n-1)T+1}, ..., y_{nT})\}.$$

To induce the principal to pay out this bonus, we need that

$$B_{\max} \leq \frac{\delta(py - c - \underline{u} - \underline{v})}{1 - \delta}.$$

In other words, the bonus cannot be larger than the discounted expected future surplus.

Now consider the agent's incentive to exert effort. Let $\{e^*_{nT+1}, e^*_{nT+2}, ...e^*_{(n+1)T}\}$ be the equilibrium effort of the agent from period $nT + 1$ to $(n + 1)T$. For the agent to find it incentive compatible to exert effort in period $nT + 1$, it is necessary that he does not find it profitable to shirk in that period:[5]

$$\begin{aligned} c \leq E[\delta^{T-1}B(y_{nT+1}, ..., y_{(n+1)T})|e_{nT+1} &= 1, e^*_{nT+2}, ...e^*_{(n+1)T}] \\ -E[\delta^{T-1}B(y_{(nT+1}, ..., y_{(n+1)T})|e_{nT+1} &= 0, e^*_{nT+2}, ...e^*_{(n+1)T}]. \end{aligned}$$

Now

$$\begin{aligned} & E[B(y_{nT+1}, ..., y_{(n+1)T})|e_{nT+1} = 1, e^*_{nT+2}, ...e^*_{(n+1)T}] \\ = \ & pE[B(y, ..., y_{(n+1)T})|, e^*_{nT+2}, ...e^*_{(n+1)T}] + (1-p)E[B(0, ..., y_{(n+1)T})|, e^*_{nT+2}, ...e^*_{(n+1)T}]. \end{aligned}$$

Similarly,

$$\begin{aligned} & E[B(y_{nT+1}, ..., y_{(n+1)T})|e_{nT+1} = 0, e^*_{nT+2}, ...e^*_{(n+1)T}] \\ = \ & qE[B(y, ..., y_{(n+1)T})|, e^*_{nT+2}, ...e^*_{(n+1)T}] + (1-q)E[B(0, ..., y_{(n+1)T})|, e^*_{nT+2}, ...e^*_{(n+1)T}]. \end{aligned}$$

Therefore, the expected benefit of putting effort in period $nT+1$ while keeping other periods' efforts fixed is given by

$$\begin{aligned} & (p-q)\delta^{T-1}(E[B(y, ..., y_{(n+1)T})|, e^*_{nT+2}, ...e^*_{(n+1)T}] - E[B(0, ..., y_{(n+1)T})|, e^*_{nT+2}, ...e^*_{(n+1)T}]) \\ \leq \ & (p-q)\delta^{T-1}B_{\max}. \end{aligned}$$

It follows that a necessary condition to induce effort in period $nT+1$ is that

$$\frac{c}{p-q} \leq \delta^{T-1}B_{\max}.$$

It is also immediate that a necessary condition to induce effort in any period $nT+k$ is given by

$$\frac{c}{p-q} \leq \delta^{T-k}B_{\max}. \tag{4}$$

Since (4) is the easiest to satisfy for $k = T$, the necessary condition for some effort to be exerted in some period is $c/(p-q) \leq B_{\max}$. Combining with the incentive constraint of the principal, a necessary condition for inducing effort in any period is

[5] The sufficient condition would be that the optimal deviation, which potentially involves changing efforts in subsequent periods, is also not profitable.

given by

$$\frac{c}{p-q} \leq \frac{\delta(py - c - \underline{u} - \underline{v})}{1-\delta},$$

and this is exactly the necessary and sufficient condition for inducing effort in the case in which information is revealed in each period. In other words, bundling periods together cannot help sustain cooperation in the relational contract.

3.3 Intertemporal Garbling with Partial Revelation

In the previous tow subsections, the analysis seems to suggest that the relational contract is more efficient when the signals are more precise and are revealed more frequently. In this section, we show that when the success probability is low, the efficiency of the relationship can be enhanced through linking information intertemporally but not fully revealing them. To keep the analysis tractable, we look at the special case that $q = 0$.

To describe the signal generating process, we imagine that there is a supervisor who privately observes the output and then publicly announces a garbled signal of his observation. Every period, the supervisor reports whether the (garbled) signal is *good* or *bad*. The report of a *good* signal can be viewed as a recommendation for the principal to pay the agent a bonus although the principal will pay only if it is incentive compatible to do so. Apart from that, the supervisor is also required to *privately* keep tract of a state which may be *up* or *down*. It is important that the supervisor never discloses the state to the principal nor the agent. We also assume the supervisor has no interest in the game and can be asked to garble the signal in any way the principal would like him to, with the restriction that the agent is fully aware of the garbling process.

We further restrict our attention to a specific garbling process which is described as follows. If the output is high, then regardless of the state, the supervisor publicly announces the signal *good* (g) and privately move the state to *up*. Conditional of the state being *up*, if the output is low, he will publicly announce *good* but at the same time move the state to *down*. If both the state is *down* and the output is low, then he will announce *bad* ($\sim g$) and at the same time he will randomize and move to the *up* state with probability ρ^* and to the *down* state with probability $1-\rho^*$.

The following figure illustrates how different outputs and previous states lead to different signals and states. It is similar to Figure 1 except the probability of each path is labeled.

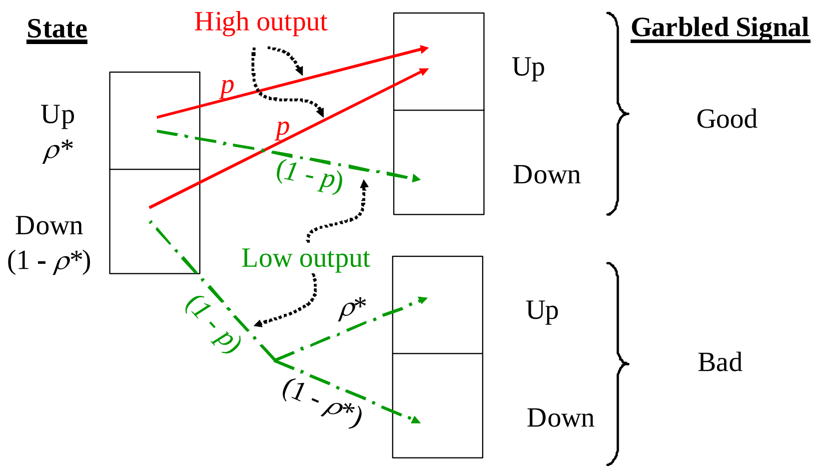

Figure 2: Signal Garbling Process with Details

Suppose ρ^* is chosen to satisfy

$$\rho^* = \frac{p}{p + (1-p)\rho^*}.$$

It can be verified that if the probability that the state is *up* is ρ^* and the agent puts in effort every period, leading to a probability p of high output, then the probability that the state is *up* is always maintained at ρ^*.

Garbling signal the way suggested here has three effects on the incentive to exert effort. If being in the up state is indeed valuable, then rewarding success by moving the agent to the up state on top of paying him bonus provides additional incentive to exert effort. However, (a) paying bonus regardless of outcome of production whenever the agent is in the up state, which happens with probability ρ^* in equilibrium, weakens incentives to exert effort. Similarly, (b) moving the agent to the up state with probability ρ^* following a failure in production in the down state also hurts effort incentives. Note that the disincentivizing forces vanish as ρ^* goes to zero but the incentivizing force does do. And ρ^* clearly goes to zero as we let p go zero (but at the same time also let B go to infinity so that pB is still comparable to the cost of effort c). This explains why intertemporally signal garbling enhances efficiency as p is sufficiently small.

Now we construct an equilibrium with positive effort under the garbled informational structure which we term as *Productive Garbled-Signal Relational Contract.*

Definition 1 (Productive Garbled-Signal Relational Contract) *At the beginning of the game, the supervisor sets $\rho = \rho^*$. In period t, the principal offers a contract with an contractible base wage $\omega_t = w$ if $t = 1$ or if $s_{t-1} = \sim g$. She offers an contractible base wage $\omega_t = w + \frac{B}{\delta}$ if $s_{t-1} = g$.* [6]

The agent accepts the principal's contract offer if the base wage is at least $w + \frac{B}{\delta}$ when the previous period's signal was good or if the base wage is at least w when the previous period's signal was bad. Any other contract will be rejected. After accepting a contract, the agent exerts effort if and only if he believes $\rho \leq \rho^*$.

Note that in a Productive Garbled-Signal Relational Contract, effort is exerted every period because ρ stays at ρ^*. The main result of this section in that this equilibrium is sustainable for a wider range of discount factors compared to the case when the (imperfect) signal fully revealed every period.

Define $B(p)$ as the infimum of the maximal bonus the principal needs to pay to the agent to induce effort. If fully revealing the output signal per period is the optimal information structure, then $B(p) \geq c/p$. However, next theorem says that we can do better.

Theorem 1: *As $p \to 0$,*

$$\lim_{p \to 0} \frac{c}{pB(p)} \geq 1 + \delta.$$

Since the maximum bonus the principal is willing to pay is $B = \frac{\delta(y-c-\underline{u}-v)}{1-\delta}$, Theorem 1 implies that as $p \to 0$, a Productive Garbled-Signal Relational Contract exists

[6] Here B is interpreted as a bonus payment for the good signal but it is paid out as part of the larger base wage in the following period. Note that in Levin (2003), it is unimportant whether a bonus is paid out at the end of a period or at the beginning of the following period as part of the base wage (now contingent on previous period's output signal) because the agent does not have a private history in his setting. Here postponing the bonus payment helps ensures that even after the agent shirks and privately knows that $\rho < \rho^*$, he still chooses to accept the principal's contract. The importance of this point will become clear in the proof of Theorem 1.

if and only if

$$\frac{c}{p(1+\delta)} \leq \frac{\delta(y-c-\underline{u}-\underline{v})}{1-\delta}.$$

This condition is obviously easier to satisfy than

$$\frac{c}{p} \leq \frac{\delta(y-c-\underline{u}-\underline{v})}{1-\delta},$$

which is the condition for sustainability of positive effort under imperfect public monitoring with fully revealed output signal every period. Intertemporal signal garbling cuts down the requirement on the size of the surplus by a factor of $1/(1-\delta)$ which goes to 50 percent as δ approaches 1.

Before formally proving the theorem, here we explain part of the idea behind the theorem. Let β be the benefit of being in the up state instead of the down state. The payoff from exerting effort can be viewed as

$$[p+\rho^*(1-p)]\,B+\delta[p+(1-p)(1-\rho^*)\rho^*]\beta$$

and the payoff from not exerting effort can be viewed as

$$\rho^* B+\delta(1-\rho^*)\rho^*\beta.$$

In other words, the benefit of exerting effort or the difference in these payoffs is $p(1-\rho^*)B+\delta p\,[1-(1-\rho^*)\,\rho^*]\,\beta$. As p goes to zero, ρ^* also goes to zero. In other words, both the equilibrium probability of receiving a bonus and the equilibrium probability of being in the up state are close to zero. On the other hand, once the agent is in the up state, he receives a bonus with probability one, as compared to p if he is in the down state. As p goes to zero, being in the *up* state increases the probability of getting a bonus from almost zero to one. Therefore, β goes to B as p goes to zero. This explain why the benefit of exerting effort can reach $(1+\delta)\,pB$, as compared to pB when the signal is not garbled. In other words, with intertemporal signal garbling, the same amount of bonus can provide a stronger incentive to put in effort.

The formal proof of the theorem is complicated by the fact that the signal garbling process transforms the repeated principal-agent relationship into one of private monitoring . To see this, consider an agent's deviation. If the agent deviates, then he will privately know the actual probability that he is in the *up* state. As a result, following such reporting rule, the belief and action of the agent can depend on the entire past history of private actions he takes. In other words, the one-stage deviation principle will not hold in this setting. To show that with this signal garbling process, there exists some appropriately chosen $B \in (\frac{c}{p(1+\delta)}, \frac{c}{p})$ such that the agent puts in effort in each period along the equilibrium path, it remains to that any arbitrary multi-stage deviation is unprofitable.

Proof. An important observation is that in this two-state example, there is a recursive formulation. Let $V(\rho)$ denote the agent's value function conditional on his believing that he is in the up state with probability ρ. In order to use the same value function following different signal announcements, suppose the previous period's signal is good and the agent's continuation payoff at the beginning of this period is $\tilde{V}(\rho, g)$, then $V(\rho) := \tilde{V}(\rho, g) - B/\delta$. However, if the previous period's signal is bad and the agent's continuation payoff at the beginning of this period is $\tilde{V}(\rho, \sim g)$, then $V(\rho) := \tilde{V}(\rho, \sim g)$. Recognizing that the agent can choose between exerting and not exerting effort given every ρ, the value function has the following recursive representation:

$$\begin{aligned} V(\rho) &= \max\{w - c + (p + (1-p)\rho)(B + \delta V(\frac{p}{p + (1-p)\rho})) \\ &\quad +(1 - (p + (1-p)\rho))\delta V(\rho^*), w + \rho(B + \delta V(0)) + (1-\rho)\delta V(\rho^*)\}. \end{aligned}$$

In the above expression, we implicitly assume that the agent does not choose his outside option. This will be verified.

Now note that the operator on the right hand side satisfies the Blackwell Sufficiency Conditions, so it is a contraction mapping. Therefore, there is a unique value function V that satisfies this equation. Moreover, it is clear that changing the value of w will only affect the value function by a constant. Therefore, we can choose w so that $V(\rho^*) = \underline{v}$. In particular, we let

$$w - c + (p + (1-p)\rho^*)(B + \delta\underline{v}) + (1 - (p + (1-p)\rho^*))\delta\underline{v} = \underline{v}$$

$$\begin{aligned} w &= c + (1-\delta)\underline{v} - (p + (1-p)\rho^*)B \\ &= c + (1-\delta)\underline{v} - \frac{p}{\rho^*}B. \end{aligned}$$

This normalization would indeed be valid if we can show that

$$w + \rho^*(B + \delta V(0)) + (1 - \rho^*)\delta \underline{v} = \underline{v}$$

We now conjecture that in equilibrium the agent will not put in effort if $\rho > \rho^*$ and will put in effort if $\rho \leq \rho^*$ and will later show that this indeed happens in equilibrium.

This implies that for $\rho > \rho^*$,

$$\begin{aligned} V(\rho) &= w + \rho(B + \delta V(0)) + (1 - \rho)\delta \underline{v} \\ &= \underline{v} - (\rho^*(B + \delta V(0)) + (1 - \rho^*)\delta \underline{v}) + \rho(B + \delta V(0)) + (1 - \rho)\delta \underline{v} \\ &= \underline{v} + (\rho - \rho^*)[B + \delta(V(0) - \underline{v})] \end{aligned}$$

For $\rho < \rho^*$,

$$\begin{aligned} V(\rho) &= w - c + (p + (1 - p)\rho)(B + \delta V(\frac{p}{p + (1 - p)\rho})) + (1 - (p + (1 - p)\rho))\delta \underline{v} \\ &= w - c + (p + (1 - p)\rho)(B + \delta \left[\underline{v} + (\frac{p}{p + (1 - p)\rho} - \rho^*)[B + \delta(V(0) - \underline{v})]\right]) \\ &\quad + (1 - (p + (1 - p)\rho))\delta \underline{v} \end{aligned}$$

We first show that the agent has the incentive to accept the principal's contract for all ρ. First, it is clear this is the case when $\rho \geq \rho^*$ because $V(\rho) \geq V(\rho^*) = \underline{v}$

$$\begin{aligned} \underline{v} + (\rho - \rho^*)[B + \delta(V(0) - \underline{v})] &\geq \underline{v} \\ \frac{B}{\delta} + V(0) &\geq \underline{v}. \end{aligned}$$

Notice that $\frac{B}{\delta} + V(\rho)$ is the agent's continuation payoff if he accepts a contract with $\omega_t = w + \frac{B}{\delta}$ after a good signal is reported, privately knowing ρ. Therefore, the agent always accepts the contract after a good signal is reported. After a bad signal is reported, ρ is reset to ρ^*. Since $V(\rho^*) = \underline{v}$, the agent will also have the incentive to accept the contract.

In the rest of the proof, we make the simplifying assumption that $\underline{v} = 0$.

This allows us to simply the value function as follows.

For $\rho > \rho^*$,

$$V(\rho) = (B + \delta V(0))(\rho - \rho^*).$$

For $\rho < \rho^*$,

$$\begin{aligned}
V(\rho) &= w - c + [p + (1-p)\rho][B + \delta V(\frac{p}{p + (1-p)\rho})] \\
&= w - c + [p + (1-p)\rho] \left[B + \delta(B + \delta V(0))(\frac{p}{p + (1-p)\rho} - \rho^*)\right] \\
&= w - c + [(p + (1-p)\rho][B - \delta\rho^*(B + \delta V(0))] + \delta(B + \delta V(0))p \\
&= (1-p)(B - \delta\rho^*(B + \delta V(0)))(\rho - \rho^*),
\end{aligned}$$

where the last equality follows from the fact that $V(\rho)$ is affine in ρ with slope $(1-p)(B - \delta\rho^*(B + \delta V(0)))$, and its value at ρ^* is 0.

For the value function defined above to be the real value function, we need to check that a) the functional equation is satisfied given the agent's actions; and b) The actions specified are optimal.

We first check a). Here, we need to choose B and $V(0)$ to make sure that the functional equation is satisfied.

Note that given $V(0)$ and the action profile, the functional equation is satisfied immediately for $\rho > \rho^*$.

For $\rho \leq \rho^*$, since the equation is linear, we essentially need to satisfy the following two equations:

$$w + \rho^*(B + \delta V(0)) = 0.$$

$$\begin{aligned}
V(0) &= w - c + p(B + \delta V(1)) \\
&= -\frac{p}{\rho^*}B + p(B + \delta(B + \delta V(0))(1 - \rho^*)).
\end{aligned}$$

The second equation implies that

$$V(0) = \frac{p(1 - \frac{1}{\rho^*} + \delta(1-\rho^*))}{1 - p\delta^2(1-\rho^*)}B. \tag{5}$$

The first equation implies that

$$\begin{aligned}
& c + \frac{p}{\rho^*}B + \rho^*(B + \delta\frac{p(1 - \frac{1}{\rho^*} + \delta(1-\rho^*))}{1 - p\delta^2(1-\rho^*)}B) \\
=\ & c + (-\frac{p}{\rho^*} + \rho^* + \delta\frac{p\rho^*(1 - \frac{1}{\rho^*} + \delta(1-\rho^*))}{1 - p\delta^2(1-\rho^*)})B \\
=\ & 0.
\end{aligned}$$

Or equivalently,

$$B = \frac{c/p}{(\frac{1}{\rho^*} - \frac{\rho^*}{p} + \delta\frac{(1-\rho^*-\delta(1-\rho^*)\rho^*)}{1-p\delta^2(1-\rho^*)})}. \tag{6}$$

With B and $V(0)$ so chosen as above, the functional equation is satisfied.

Now we check that the actions specified are optimal. To ensure that the actions are optimal, we first need to make sure that for $\rho \leq \rho^*$,

$$\begin{aligned} & w + \rho(B + \delta V(0)) \\ \leq\ & w - c + [(p + (1-p)\rho)(B + \delta V(\frac{p}{p + (1-p)\rho})) \\ =\ & (1-p)(B - \delta\rho^*(B + \delta V(0)))(\rho - \rho^*), \end{aligned}$$

Note that the above is satisfied if

$$B + \delta V(0) \geq (1-p)(B - \delta\rho^*(B + \delta V(0))).$$

Let $x = B + \delta V(0)$, and define $T(x) = (1-p)(B - \delta\rho^* x)$, then the above can be rewritten as

$$T(x) \leq x.$$

We also want to make sure that for $\rho > \rho^*$, we have

$$\begin{aligned} & w + \rho(B + \delta V(0)) \\ \geq\ & w - c + [(p + (1-p)\rho)(B + \delta V(\frac{p}{p + (1-p)\rho})) \\ =\ & w - c + [(p + (1-p)\rho)(B + \delta(1-p)(B - \delta\rho^*(B + \delta V(0)))(\frac{p}{p + (1-p)\rho} - \rho^*))] \\ =\ & (1-p)(B - \delta\rho^*(1-p)(B - \delta\rho^*(B + \delta V(0))))(\rho - \rho^*). \end{aligned}$$

If we again have $x = B + \delta V(0)$ and $T(x) = (1-p)(B - \delta\rho^* x)$, then the slope of ρ in the expression above is given by $T(T(x))$, and we need

$$T(T(x)) \leq x.$$

Now note that $T(x)$ is an affine function of x with slope $-\delta\rho^*(1-p) > -1$. Let

x^* be such that $T(x^*) = x^*$, then

$$\begin{aligned}(1-p)(B - \delta\rho^* x^*) &= x^* \\ x^* &= \frac{(1-p)B}{1+\delta\rho^*(1-p)}.\end{aligned}$$

Now note that if $x \geq x^*$, then

$$T(x) \leq x.$$

Moreover, since the slope of $T(x)$ is equal to $-(1-p)\delta\rho^* > -1$, this implies that, for $x > x^*$,

$$\frac{T(x^*) - T(x)}{x - x^*} = \frac{x^* - T(x)}{x - x^*} < 1.$$

By the linearity of T, it follows that,

$$\frac{T(T(x)) - T(T(x^*))}{T(x^*) - T(x)} = \frac{T(x^*) - T(x)}{x - x^*} < 1$$

so that

$$T(T(x)) - T(T(x^*)) \leq x - x^*,$$

or

$$T(T(x)) \leq x.$$

The discussion above implies that, as long as

$$B + \delta V(0) = x \geq x^* = \frac{(1-p)B}{1+\delta\rho^*(1-p)},$$

the action profile is optimal. In other words, we need

$$V(0) \geq -\frac{1}{\delta}(\frac{p+\delta\rho^*(1-p)}{1+\delta\rho^*(1-p)})B.$$

Recalling from (5) that

$$V(0) = \frac{p(1 - \frac{1}{\rho^*} + \delta(1-\rho^*))}{1 - p\delta^2(1-\rho^*)}B.$$

Now

$$
\begin{aligned}
& \frac{p(1-\frac{1}{\rho^*}+\delta(1-\rho^*))}{1-p\delta^2(1-\rho^*)}+\frac{1}{\delta}(\frac{p+\delta\rho^*(1-p)}{1+\delta\rho^*(1-p)}) \\
= & \frac{1}{\delta\rho^*}\frac{\delta((\rho^*)^2(1-p)-p(1-\rho^*))+p\rho^*}{(1-p\delta^2(1-\rho^*))(1+\delta\rho^*(1-p))}
\end{aligned}
$$

Now note that

$$(\rho^*)^2(1-p)=p(1-\rho^*),$$

so the expression above is always positive. And so the actions are optimal. This shows that the proposed value function is truly the value function.

Following directly from (6),

$$
\begin{aligned}
\frac{c}{Bp} &= \frac{1}{\rho^*}-\frac{\rho^*}{p}+\delta\frac{(1-\rho^*-\delta(1-\rho^*)\rho^*)}{1-p\delta^2(1-\rho^*)} \\
&= \frac{\rho^*-p}{(1-p)\rho^*}+\delta\frac{(1-\rho^*-\delta(1-\rho^*)\rho^*)}{1-p\delta^2(1-\rho^*)},
\end{aligned}
$$

where we have used $(\rho^*)^2(1-p)=p(1-\rho^*)$ in simplifying $\frac{1}{\rho^*}-\frac{\rho^*}{p}$.

Now since

$$
\begin{aligned}
(1-p)\rho^{*2}+p\rho^*-p &= 0, \\
\rho^* &= \frac{-p+\sqrt{4p-3p^2}}{2}
\end{aligned}
$$

In other words, when p is small, ρ^* is roughly in the order of $\sqrt{p}$.

It is clear that as p goes to 0, both $\frac{\rho^*-p}{(1-p)\rho^*}$ and $\frac{(1-\rho^*-\delta(1-\rho^*)\rho^*)}{1-p\delta^2(1-\rho^*)}$ go to 1, so

$$\lim_{p\to 0}\frac{c}{Bp}=1+\delta.$$

Since the equilibrium is only one of the many possible equilibria, $B(p)\le B$, and the proof is complete.

Now we verify that the agent finds it optimal not to exercise the outside option for all ρ and we also show that this is independent of the normalization of $V(\rho^*)=0$.

Instead, we restate the value function setting $V(\rho^*) = \underline{v}$:

$$\begin{aligned} V(\rho) &= \max\{w - c + (p + (1-p)\rho)(B + \delta V(\frac{p}{p + (1-p)\rho})) + (1 - (p + (1-p)\rho))\delta V(\rho^*), \\ &\quad w + \rho(B + \delta V(0)) + (1-\rho)\delta V(\rho^*)\}. \end{aligned}$$

for $\rho > \rho^*$ This implies that for $\rho > \rho^*$,

$$\begin{aligned} V(\rho) &= w + \rho(B + \delta V(0)) + (1-\rho)\delta\underline{v} \\ &= \underline{v} - (\rho^*(B + \delta V(0)) + (1-\rho^*)\delta\underline{v}) + \rho(B + \delta V(0)) + (1-\rho)\delta\underline{v} \\ &= \underline{v} + (\rho - \rho^*)[B + \delta(V(0) - \underline{v})] \end{aligned}$$

$$\begin{aligned} \underline{v} + (\rho - \rho^*)[B + \delta(V(0) - \underline{v})] &> \underline{v} \\ \frac{B}{\delta} + V(0) &> \underline{v} \end{aligned}$$

Note that $B + \delta V(0)$ is the agent's continuation payoff from accepting a contract with wage $\omega_t = w + \frac{B}{\delta}$ following a period with a good signal. Recall that for $\rho > \rho^*$, $V(\rho) = (B + \delta V(0))(\rho - \rho^*) > V(\rho^*) = 0$. Therefore,

$$B + \delta V(0) > \frac{V(\rho^*)}{(\rho - \rho^*)} > V(\rho^*) = 0.$$

■

The reason that this type of intertemporal garbling can do better than fully revealing the information every period, especially when the success probability is small, is the following. Under relational contract with perfect signals, the principal will be required to pay a big bonus (c/p) to the agent when the probability of success is small. Consequently, relational contract is hard to sustain because the bonus cannot exceed the expected discounted future surplus of the relationship.

By linking information intertemporally, the reward for high output is decomposed into two parts: the bonus at the end of this period, and a higher continuation payoff in the future. This decomposition of reward reduces the immediate bonus to be paid out and helps softens the incentive constraint of the principal. On the other hand, delaying the reward does have a cost: the absolute amount of total bonus paid out will be larger due to discounting and the higher continuation payoff of the agent makes it difficult to induce effort. Therefore, this intertemporal garbling can more easily outperform perfect signal when the success probability is extreme and the discount factor is high.

4 Discussion

The equilibrium constructed in Theorem 1 uses a two-state representation. It is natural to ask whether one can do better with more states. We think that the answer is yes, but proving those strategies with more than two states are equilibrium is difficult. This is because one needs to check more than one-stage deviation in this setting, and the recursive structure in Theorem 1 becomes unwieldy when there are multiple states. One possible way in the literature to deal with this problem is the "belief-free" approach, in which the marginal benefit of having a high output is independent of which state the agent is in. Unfortunately, such approach will be unable to work here, as the next proposition shows.

Proposition 2 *If an information set has n states, and the difference in payoffs (between a high to low output) is independent of which state the agent is in, such information structure can do no better than perfect signals.*

Proof. Suppose that there are n states within an information set, with value $k_1 > k_2 > .... > k_n$. (The argument extends naturally to the case where n is infinity or represents a continuum.)

Let x_s^H be expected payoff following a high output when the agent is in state s. We define x_s^L accordingly.

Belief free requires that there exists a D such that

$$x_s^H - x_s^L = D \quad \text{for all } s.$$

Now note that

$$k_s = p x_s^H + (1-p) x_s^L.$$

This implies that

$$\begin{aligned} x_s^H &= k_s + (1-p)D \\ x_s^L &= k_s - pD \end{aligned}$$

Now note that

$$x_1^H = k_1 + (1-p)D \leq b + \delta k_1,$$

where b is the per period bonus. This implies that

$$k_1 \leq \frac{b-(1-p)D}{1-\delta}$$

Also note that

$$x_n^L = k_n - pD \geq \delta k_n$$

This implies that

$$k_n \geq \frac{pD}{1-\delta}.$$

Since $k_1 > k_n$,

$$\begin{aligned} \frac{b-(1-p)D}{1-\delta} &\geq \frac{pD}{1-\delta} \\ b &\geq D. \end{aligned}$$

■

Since the intertemporal garbling creates improvement through exploiting the extremeness of the information, such linkage is less likely to be useful when the information content on the equilibrium path of perfect signal is more even. In fact, when $p = \frac{1}{2}$, revealing information perfectly is optimal.

Theorem 2: *When $p = 1/2$, the optimal information structure is given by $s_t = Y_t$ for all t.*

Proof. First recall that when $s_t = Y_t$ for all t, the necessary and sufficient condition for sustaining cooperation is given by

$$\frac{c}{p-q} \leq \frac{\delta}{1-\delta}(py - c - \underline{u} - \underline{v}).$$

Let $b = \frac{c}{p-q}$, then we need to show that in any equilibrium that induces the effort, the maximum bonus is at least b.

Now suppose that information of the past outputs is not perfect. Suppose the value of a state x within an information set is $V(x)$. Now for each x, denote $x_i \in \{x_y, x_0\}$ as the value from state x after a new outcome (but before a bonus is paid out).

Since effort is induced, we have

$$V(x) = V(x) + p(V(x_H) - V(x)) + (1-p)(V(x_L) - V(x)).$$

Note that whether a path becomes H or L is independent of which state x the agent is in. It follows that the variance of the values following the y or 0 becomes

$$\begin{aligned} Var(V(x_i)) &= Var(V(x)) + Var(V(x_i) - V(x)) \\ &= Var(V(x)) + E[Var(V(x_i) - V(x))|Y] + Var(E[V(x_i) - V(x)|Y]) \\ &\geq Var(V(x)) + E[Var(V(x_i) - V(x))|Y] + p(1-p)b^2 \\ &\geq Var(V(x)) + p(1-p)b^2. \end{aligned}$$

where the first inequality follows from the requirement to induce effort, namely, $V(x_H) - V(x_L) \geq b$.

Now let J be the set of information sets after the outcome is realized, b_j be the bonus associated with information set j. Note that for the principal to be willing to pay out the bonus in information set j, we must have

$$b_j \leq \frac{\delta}{1-\delta}(py - c - \underline{u} - \underline{v}) \equiv S,$$

so

$$\max b_j \leq S,$$

and

$$Var(b_j) \leq \frac{1}{4}S^2.$$

Now note that $V(x_i)$ must be an element in some information set $j \in J$:

$$V(x_i) = b_j + \delta V_j(x_{ij}),$$

where we denote b_j and $V_j(x_{ij})$ as the bonus and the continuation payoff of the agent following the output of x_i and being in the information set j.

Using the formula of variance decomposition on each information set j,

$$\begin{aligned} Var(V(x_i)) &= Var(E[b_j + \delta V_j(x_{ij})|j]) + E[Var(b_j + \delta V_j(x_{ij})|j)] \\ &\leq \frac{1}{4}S^2 + \delta^2 E[Var(V_j(x_{ij})|j)], \end{aligned}$$

where the inequality follows because

$$\max_j E[b_j + \delta V_j(x_{ij})|j] - \min_j E[b_j + \delta V_j(x_{ij})|j] \leq S,$$

which follows from the principal's reneging constraint, so $Var(E[b_j + \delta V_j(x_{ij})|j]) \leq \frac{1}{4}S^2$.

The two inequality together implies that

$$\frac{1}{4}S^2 + \delta^2 E[Var(V_j(x_{ij})|j)] \geq Var(V(x_i)) \geq Var(V(x)) + p(1-p)b^2.$$

Now note that within each information set, the variance of values must be bounded. We will show that this requirement will be violated when $p = 1/2$.

When $p = 1/2$, the above inequality becomes

$$\delta^2 E[Var(V_j(x_i)|j)] \geq Var(V(x)) + \frac{1}{4}(b^2 - S^2).$$

Now if $b > S$, or equivalently,

$$\frac{c}{p-q} > \frac{\delta}{1-\delta}(py - c - \underline{u} - \underline{v}),$$

then the equation implies that for some information set j, we must have the variance of values in that information set

$$Var(V_j(x_{ij})|j) \geq \frac{1}{\delta^2} Var(V(x)),$$

so the variance will blow up along some equilibrium play path, and this cannot be an equilibrium.

Therefore, the only way this equilibrium is sustainable is when

$$\frac{c}{p-q} \leq \frac{\delta}{1-\delta}(py - c - \underline{u} - \underline{v}),$$

which is the condition under which cooperation can be sustained when $s_t = Y_t$. $\blacksquare$

5 Conclusion

When probability of success is low, it is difficult to sustain efficient production with a relational contract because, to motivate the agent, it requires the principal to pay a large bonus payment upon observation of successful production, leading to a stronger incentive for her to renege. Our analysis showed that intertemporal garbling of the signals of the agent's success/failure can restore efficient production by reducing amount of bonus needed to be paid out by the principal. We believe this not only of theoretical interest.

There are many situations in which senior managers have to rely on supervisors/mid-level managers to monitor and evaluate employees' performances. Our result implies that there are situations in which it is suboptimal for an organization to require supervisors to write the most accurate year-end evaluations for their subordinates. In fact, it is better for the supervisor to privately keep track of the employee's performance in previous years and use that information to determine the employee's performance evaluation in the current year.

Due to intractability of relational contract with private monitoring, characterization of the optimal signal garbling process remains an open question and will be the focus of future research.

Appendix A: Extension to General Production Function and Signals

In this section, we generalize the production function and the signal structure to show that the results in Subsections 3.1-3.2 hold generally.

If the principal and the agent engage in production together, the agent chooses effort $e \in [0, \overline{e}]$, incurring an effort cost of $c(e)$. Assume that $c(0) = 0$, $c' > 0$ and $c'' > 0$. The outcome Y is a random variable distributed with the c.d.f. $F(\cdot|e)$, where $f(\cdot|e)$ exists, and the support of Y is independent of e.

We assume that there exists an effort level such that if this effort level can be induced in the relationship, then it is efficient to form the relationship. However, forming the relationship is less efficient than each player receiving his/her outside option if no effort can be induced in the relationship. In other words, there exists

$e \in [0, \bar{e}]$ such that

$$\int_y yf(y|e)dy - c(e) > \underline{u} + \underline{v},$$

where $\underline{u}$ and $\underline{v}$ are respectively the agent and the principal's per-period outside option, and

$$\int_y yf(y|0)dy < \underline{u} + \underline{v}.$$

Define

$$u(e|Y) = (1-\delta)[w_0 - c(e) + \int \tilde{b}(y)f(y|e)dy] + \delta \int \tilde{u}(y)f(y|e)dy.$$

$$\begin{aligned}
v(y) &= -(1-\delta)[-y + w_0 + \tilde{b}(y)] + \delta\tilde{v}(y) \\
v(e|Y) &= \int -(1-\delta)[-y + w_0 + \tilde{b}(y) + \delta\tilde{v}(y)]f(y|e)dy
\end{aligned}$$

Note that every feasible payoff set is characterized by an upper bound on the total surplus s^* and can be written as the following:

$$W = \{(u, v) : u \geq \underline{u},\ v \geq \underline{v} \text{ and } u + v \leq s^*\}.$$

For any set $W \in R^2$, a vector $(e, b, \tilde{u}, \tilde{v})$ is called admissible with respect to W under Y if

(1) $(\tilde{u}(y), \tilde{v}(y)) \in W$ for all y in the support, and

(2) $u(e|Y) \geq u(e'|Y)$ for all $e' \in [0, \bar{e}]$ (agent's IC)

(3) $-(1-\delta)\tilde{b}(y) + \delta\tilde{v}(y) \geq \delta\underline{v}$ for all y in the support (principal's IC)

Let $B(W|Y)$ be defined by

$$B(W|Y) = \{(u, v)|(e, b, \tilde{u}, \tilde{v}) \text{ is admissible w.r.t. } W \text{ under } Y\}.$$

A payoff set W is self-generating under Y if $W \subseteq B(W|Y)$.

A.1 Within-period Information Garbling

Now, consider a modified setup in which the output is X. Let $X \sim G(\cdot|e)$, where

$g(\cdot|e)$ exists, such that

$$\int_y yf(y|e)dy = \int_x xg(x|e)dx \; \forall e \in [0, \bar{e}]. \tag{7}$$

This restriction is to preserve the overall productivity of the relationship. Furthermore, we assume that X is less informative than Y of the agent's effort in the sense of *quasi-garbling*. Following Kandori (1992), we impose that

$$\begin{aligned} \phi(x|y) &\geq 0 \text{ a.e. } x \text{ and } y \\ \int \phi(x|y)dx &= 1 \text{ a.e. } y \\ g(x|e) &= \int \phi(x|y)f(y|e)dy, \end{aligned} \tag{8}$$

and that the support of X is independent of e.

Proposition A (Kandori, 1992) Suppose X is a quasi-garbling of Y. Then if W is a compact self-generating set under X, it is also self-generating under Y.

Proof. Since W is self-generating under X, for any $w = (u, v) \in W$ there exists a vector $(e_X, \tilde{b}_X, \tilde{u}_X, \tilde{v}_X)$ which is admissible with respect to W under X and satisfies

$$u(e|X) = (1-\delta)[w_0 - c(e) + \int \tilde{b}_X(x)f(x|e)dy] + \delta \int \tilde{u}_X(x)f(x|e)dy.$$

$$\begin{aligned} v(x) &= -(1-\delta)[-x + w_0 + \tilde{b}_X(x)] + \delta\tilde{v}_X(x) \\ v(e|X) &= \int -(1-\delta)[-x + w_0 + \tilde{b}_X(x) + \delta\tilde{v}_X(x)]f(x|e)dx. \end{aligned}$$

Define $\tilde{u}_Y$ and $\tilde{b}_Y$ by

$$\begin{aligned} \tilde{u}_Y(y) &= \int \tilde{u}_X(x)\phi(x|y)dx \\ \tilde{v}_Y(y) &= \int \tilde{v}_X(x)\phi(x|y)dx \\ \tilde{b}_Y(y) &= \int \tilde{b}_X(x)\phi(x|y)dx. \end{aligned}$$

Then, for all $e \in [0, \overline{e}]$,

$$\begin{aligned}
\int_y \tilde{u}_Y(y) f(y|e) dy &= \int_y \int_x \tilde{u}_X(x) \phi(x|y) dx f(y|e) dy \\
&= \int_x \tilde{u}_X(x) [\int_y \phi(x|y) f(y|e) dy] dx \\
&= \int_x \tilde{u}_X(y) g(x|e) dx
\end{aligned}$$

and similarly

$$\int_y \tilde{v}_Y(y) f(y|e) dy = \int_x \tilde{u}_X(y) g(x|e) dx.$$

It is clear that $u(e|Y) = u(e|X)$ and $v(e|Y) = v(e|X)$. It follows that

(1) $(\tilde{u}(y), \tilde{v}(y)) \in coW$ for all y in the support, and

(2) $u(e|Y) \geq u(e'|Y)$ for all $e' \in [0, \overline{e}]$ (agent's IC)

(3) $-(1-\delta)\tilde{b}(y) + \delta \tilde{v}(y) \geq \delta \underline{v}$ for all y in the support (principal's IC).

Therefore, $u(e|Y)$and $v(e|Y)$ are admissible with respect to W under Y. Hence $W \subseteq B(W|Y)$. ■

A.2 T-period Bundling

Suppose signals (outputs) are released once every T periods. We call every T periods a stage. We reindex each period as $(i-1)T+\tau$ where $i \in \mathbb{N}$ and $\tau \in \{1, 2, ...T\}$, as the period in the τth period of the ith stage. We prove that if some efforts in the T periods of a stage, $\{\tilde{e}_1, \tilde{e}_2, ..., \tilde{e}_T\}$ are sustainable, then $\max\{\tilde{e}_1, \tilde{e}_2, ..., \tilde{e}_T\}$ can be supported by a fully revealing relational contract, i.e., when $T = 1$.

Suppose $\{\tilde{e}_1, \tilde{e}_2, ..., \tilde{e}_T\}$ are supported by $\tilde{b}(\mathbf{y}^T)$, where $\mathbf{y}^T \equiv \{y_1, y_2, ..., y_T\}$. Then $\max_{\mathbf{y}^T} \left\{\tilde{b}(\mathbf{y}^T)\right\}$ is no larger than the surplus of the relationship. Moreover, for these efforts to be sustainable, it requires that it is sequentially rational for the agent to exert the corresponding effort each period. In other words, $\tilde{e}_T$ solves

$$\max_{e_T} \int_{\mathbf{y}^T} \tilde{b}\left(\mathbf{y}^T\right) f(\mathbf{y}^T | \tilde{e}_1, \tilde{e}_2, ..., \tilde{e}_{T-1}, e_T) d\mathbf{y}^T + w_0 - c\left(e_T\right), \tag{9}$$

taking $\tilde{e}_1, \tilde{e}_2, ..., \tilde{e}_{T-1}$ as given.

Similarly, $\tilde{e}_{T-1}$ solves

$$\max_{e_{T-1}} \int_{\mathbf{y}^T} \delta\tilde{b}\left(\mathbf{y}^T\right) f(\mathbf{y}^T|\tilde{e}_1, \tilde{e}_2, ..., e_{T-1}, \tilde{e}_T(\tilde{e}_1, \tilde{e}_2, ..., e_{T-1}))d\mathbf{y}^T + w_0 - c\left(e_T\right) \tag{10}$$

taking $\tilde{e}_1, \tilde{e}_2, ..., \tilde{e}_{T-2}$ as given and anticipating $\tilde{e}_T$ to be the solution to (9).

More generally, $\tilde{e}_\tau$ solves for $\tau \in \{1, 2, ..., T-1\}$,

$$\max_{e_\tau} \int_{\mathbf{y}^T} \delta^{T-\tau}\tilde{b}\left(\mathbf{y}^T\right) f(\mathbf{y}^T|\tilde{e}_1, \tilde{e}_2, ..., e_\tau, \tilde{e}_{\tau+1}(\tilde{e}_1, ..., \tilde{e}_{\tau-1}, e_\tau), ..., \tilde{e}_T(\tilde{e}_1, ..., \tilde{e}_{\tau-1}, e_\tau)d\mathbf{y}^T + w_0 - c\left(e_T\right)$$

which is equivalent to solving

$$\max_{e_\tau} \int_{\mathbf{y}^T} \tilde{b}\left(\mathbf{y}^T\right) f(\mathbf{y}^T|\tilde{e}_1, \tilde{e}_2, ..., e_\tau, \tilde{e}_{\tau+1}(\tilde{e}_1, ..., \tilde{e}_{\tau-1}, e_\tau), ..., \tilde{e}_T(\tilde{e}_1, ..., \tilde{e}_{\tau-1}, e_\tau)d\mathbf{y}^T - \frac{c\left(e_T\right)}{\delta^{T-\tau}}. \tag{11}$$

Now, we define

$$\hat{b}(y_T) = \int_{\mathbf{y}^{-T}} \tilde{b}\left(\mathbf{y}^T\right) f(\mathbf{y}^T|\tilde{e}_1, \tilde{e}_2, ..., \tilde{e}_{T-1}, e_T)d\mathbf{y}^{-T}$$

and, for $\tau \in \{1, 2, ..., T-1\}$,

$$\hat{b}(y_\tau) = \int_{\mathbf{y}^{-\tau}} \tilde{b}\left(\mathbf{y}^T\right) f(\mathbf{y}^T|\tilde{e}_1, \tilde{e}_2, ..., e_\tau, \tilde{e}_{\tau+1}(\tilde{e}_1, ..., \tilde{e}_{\tau-1}, e_\tau), ..., \tilde{e}_T(\tilde{e}_1, ..., \tilde{e}_{\tau-1}, e_\tau)d\mathbf{y}^{-\tau}.$$

Now consider a one-period relational contract in which the principal pays the agent a bonus $\delta^{T-\tau}\hat{b}(y_\tau)$ in each period. Obviously, $\max_{y_\tau} \delta^{T-\tau}\hat{b}(y_\tau) \leq \max_{\mathbf{y}^T}\left\{\tilde{b}(\mathbf{y}^T)\right\}$. With such a contract, the agent solves

$$\delta^{T-\tau} \max_{y_\tau} \int_{y_\tau} \int_{\mathbf{y}^{-\tau}} \tilde{b}\left(\mathbf{y}^T\right) f(\mathbf{y}^T|\tilde{e}_1, \tilde{e}_2, ..., e_\tau, \tilde{e}_{\tau+1}(\tilde{e}_1, ..., \tilde{e}_{\tau-1}, e_\tau), ..., \tilde{e}_T(\tilde{e}_1, ..., \tilde{e}_{\tau-1}, e_\tau)d\mathbf{y}^{-\tau}dy_\tau - c(e_T).$$

Obviously, the solution is identical to that of (**??**). In other words, each of the efforts $\{\tilde{e}_1, \tilde{e}_2, ..., \tilde{e}_T\}$ can be induced in an unbundled relational contract. In particular, the unbundled relational contract inducing $\max\{\tilde{e}_1, \tilde{e}_2, ..., \tilde{e}_T\}$ every period weakly dominates the T-period relational contract.

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