Moral Hazard and Equilibrium Matchings in a Market for Partnerships*

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Abstract

This paper embeds a repeated partnership game with imperfect monitoring into a matching environment. We show that even though the underlying technology of production exhibits no complementarities with respect to types of the partners, the presence of imperfect monitoring leads to non-trivial matching predictions. In particular, if the agents' effort is complementary to their own and their partners' types (marginal products of effort are increasing in types), equilibrium matching structure is negative (i.e., the high-type agents are matched with the low-type partners). If, on the other hand, effort and type are (sufficiently) substitutable, the types are matched positively in the equilibrium.

1 Introduction

The question we ask in this paper is how equilibrium matchings are influenced by the presence of moral hazard in the productive relationship ensuing the formation of a match. To answer this question, we consider a model of a matching market, the participants of which are heterogeneous with respect to their levels of productivity. The productivity of each agent is public information. Once a match is formed, the partners repeatedly choose unobservable effort levels, which affect the probability of success in the current period. The outcome is publicly observable. After the outcome is observed, the partners have the option to make transfers to each other.

To isolate the affect of moral hazard, we assume that the underlying production technology exhibits no complementarity. As is well-known (Becker (1973)), complementarity, by itself, has implications for equilibrium matchings. Our assumption ensures that our model delivers no

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matching predictions when effort choices of partners are observable.

When imperfect monitoring (i.e. moral hazard) is introduced, it becomes necessary to engage in inefficient punishments to induce effort by the partners. In Pareto optimal equilibria, both the size and the frequency of these punishments are effected by the types of the matched partners. This creates a non-additive interaction between the types and leads to complementarities in the amount of "efficiency loss" due to moral hazard. Since equilibrium matchings maximize the total surplus and, hence, minimize the efficiency loss, this leads to non-trivial matching predictions.

To better understand what drives the said complementarities, observe that in Pareto optimal equilibria punishments occur only after failure (low output). Therefore, it is always the case that more productive agents (the ones who are more likely to produce successful output) induce less frequent punishments. At the same time, the size of the punishment is related to how much the deviation by one of the partners affects the probability of success: if the change in this probability is small, the punishment should be big to deter the deviation. It is important to note that this change in the probability of success measures the marginal product of effort of the deviator. If it is increasing with the deviator's productivity (i.e. the agent's effort and productivity are complementary), the matches of more productive agents (who lead to smaller punishments) with the less productive ones (who punish more frequently) would decrease the loss in the total surplus due to imperfect monitoring. This mechanism, pushing towards negative assortative matching, would be reinforced further if the deviator's effort is also complementary to the partner's productivity. This is because the less productive partners, who, while playing the role of 'monitors', induce bigger punishments, should indeed be matched with more productive agents, who punish more frequently, in order to minimize inefficient punishments across the agents. Therefore, when agents' effort is complementary to their own as well as their partners' types, equilibrium matching is negative assortative. Notice that such complementarity would be a natural assumption if, for instance, 'type' stands for the capital stock that an agent brings to a partnership, and 'effort' is the amount of (unobservable) work hours put into operating the partnership's capital.

Alternatively, it is possible that effort can be a substitute for the agent's own as well his partner's type. This could occur if, for example, the 'type' measures experience and the more experienced a partner the less important is his intensity of effort. In this case, higher type agents induce larger size punishments (both as 'monitors' as well as potential deviators) and, therefore, matching them with other higher type agents who punish less frequently, would help reduce inefficient punishments across matches.

On the other hand, matching two high type agents together would increase the size of the pair's punishment in case of failure, thus making positive assortative matching less desirable. We show that, when the noise is sufficiently small, the second effect is weak and substitutability between effort and type leads to positive assortative matching. Intuitively, this is because the effect on the size of the punishment alone would be important only if such punishment occurs frequently. The smaller the noise, the less frequent the occurrence of inefficient punishment is and therefore the less important its size is.

The rest of this paper is organized as follows: Subsection 1.1 discusses the related literature, Section 2 introduces the model, Section 3 characterizes the Pareto frontier of the repeated partnership game, Section 4 presents our main results concerning the matching implications of moral hazard, Section 5 discusses the role of some our assumptions and possible alternatives to them.

1.1 Related literature

Our paper contributes to the literature exploring the effects of frictions on patterns of matching. This literature considers various departures from frictionless markets. For instance, it is well-understood that in the presence of search frictions (introduced in Shimer and Smith (2000)) complementarity in the underlying production technology is not sufficient for all equilibrium matchings to be assortative. Similarly, coordination frictions (Shimer (2005)) weaken Becker's result (Becker (1973)) that complementarity in production technology leads to perfect correlation between the types of matched pairs. In the presence of such frictions, there is positive but imperfect correlation. As studied in Legros and Newman (2002) market imperfections such as borrowing constraints may also have an impact on the matching patterns.

Informational frictions also play a role in determining the equilibrium matching patterns. In a dynamic model, Andersen and Smith (forthcoming) study the effect of symmetric but incomplete information about productivity types on the patterns of matching and predict failure of assortative matching under certain conditions in spite of the complementarities in the underlying production technology. Kaya (2008) shows that assortative matching fails in a static two-sided matching market with two-sided asymmetric information. Another paper that considers effect of private information on matching patterns is Damiano and Li (2007). Even though the latter paper considers a mechanism design problem, similar forces lead the mechanism designer to induce a matching that involves a "coarser" assortative matching while the underlying technology would imply that perfect assortative matching is optimal in the absence of private information.

Another informational friction that may have an impact on equilibrium matching patterns is unobservability of effort. This is the friction we focus on in this paper. A particularly related paper is recent work by Franco et al. (2008) which studies the effect of moral hazard on how a principal optimally allocates heterogenous agents into teams. Even though the details of the two models differ[1] the main message is similar: in an environment without inherent com-

[1]We are able to consider a wider range of technologies due to the simplification of having discrete effort choice while they consider a specific technology with a continuum of effort levels. Also in Franco et al. (2008), exogenous heterogeneity is in the cost of effort, while in our model, it is crucially in productivity. Since differing

plementarities, moral hazard alone may lead to matching predictions. In both environments, either positive or negative matching may occur, and both papers analyze which properties of the underlying technology lead to each matching structure. One notable contrast between the two papers is that the principal's optimal matching pattern in Franco et al. (2008) differs from the equilibrium matchings in our paper. In fact, it is easy to see that, in a given environment where sufficient conditions for all equilibrium matchings to be positive assortative are satisfied, the principal's optimal matching is negative assortative and vice versa. This is due to the fact that the principal, in choosing the matching pattern, minimizes the surplus accruing to the agents which is the quantity that is maximized in an equilibrium in our sense.

Two other papers study the effects of moral hazard on matching patterns in a very different environment, where the entities matched together are not symmetric, and only one of the matched partners exerts unobservable effort. Thiele and Wambach (1999) and Newman (2007) study a problem of assigning heterogeneous in wealth, risk averse entrepreneurs to the projects with different amount of risk. In the absence of frictions, wealthier (and hence less risk averse) entrepreneurs would be assigned to riskier projects because they would require lower compensation (which is independent of the project's outcome when the effort is observable). If, however, the entrepreneur's effort cannot be observed, the opposite matching pattern may arise if the utility is linear in effort. On the one hand, riskier projects - which reduce the probability of success - should now offer higher additional compensation for successful outcome in order to induce a particular effort level. This extra compensation becomes even bigger if the entrepreneur assigned to the riskier project has more wealth because richer agents have lower marginal utility of income. Thus assigning richer entrepreneurs to riskier projects may become too costly, and the frictionless assignment would not any more be sustained in the environment with unobservable effort.[2] While our paper also emphasizes the role of moral hazard, our question, as well as modeling environment, is very different from the ones studied in these papers. In addition (and, perhaps, most importantly), the mechanism outlined in these papers does not play any role in generating our results.

2 Model

We consider a one-to-one matching market for partnerships with N participants. The participants are heterogeneous with types from $\Theta \subset \mathbb{R}$. Types are public. Once a match is formed, the participants engage in a repeated partnership game which is described below. We assume

effort costs lead to differing effort levels in Franco et al. (2008), the realized marginal productivities endogenously vary across types. This connects the results of the two papers.

[2] Thiele and Wambach (1999) and Newman (2007) show that the properties of the utility function (namely, the relationship between prudence and risk aversion) determine whether or not the frictionless assignment remains optimal in the moral mazard environment.

that once a matching is formed it never breaks up, therefore the matching stage of the game is static. We consider the stable matchings of the market where the payoff possibilities of each match is given by the equilibrium payoffs of the ensuing partnership game. Formal description of the environment and definitions follow.

2.1 Technology

The partnership game played by a matched pair of types $n, m \in \Theta$ is the infinite repetition of the following stage game: at the beginning of each period each partner chooses an effort level $e_i \in \{E, S\}$, $i = 1, 2$, where E stands for "effort" and S stands for "shirk". Then an output $y \in Y = \{\overline{y}, 0\}$ is realized, with $\overline{y} > 0$. The choice of $e_i = 1$ entails a cost c while choosing $e_i = S$ is costless.

Effort choices and output probabilities are linked as follows:

$$Prob(y = \overline{y}) = \begin{cases} p(n,m) & \text{if } e_1 = e_2 = E \\ q_1(m,n) & \text{if } e_1 = E, e_2 = S \\ q_2(m,n) & \text{if } e_1 = S, e_2 = E \\ 0 & \text{if } e_1 = e_2 = S \end{cases} \tag{1}$$

We assume $p(n,m) = p(m,n)$ and $q_1(m,n) = q_2(n,m)$. We also assume that $p(\cdot,\cdot)$, and $q_i(\cdot,\cdot)$ are twice continuously differentiable.

We make the following assumptions:

A 1 For any n and m, $(p(m,n) - q_i(m,n))\overline{y} > c$ and $p(m,n)\bar{y} > 2c$, $i = 1, 2$.

A 2 $q_i(m,n)\frac{\overline{y}}{2} < c$, $i = 1, 2$.

A 3 $\frac{c}{p(m,n) - q_1(m,n)} + \frac{c}{p(m,n) - q_2(m,n)} > \overline{y}$.

A 4 $p(n,m)$, $q_i(m,n)$ are (weakly) increasing in m and n, $i = 1, 2$.

Assumption A1 implies that exerting effort is socially optimal for each agent in any match. Assumption A2 implies that without transfers, it is not incentive compatible for any partner to unilaterally exert effort. Under assumption A3 the output cannot be split in such a way that in the stage game it is incentive compatible for both partners to exert effort. Assumption A4 says that an increase in the type of a partner raises the probability of high output for any combination of efforts and that high types have lower cost of exerting effort.

2.2 The partnership game

In the stage game, after the realization of the output $\bar{y}$, the partners simultaneously make transfers $t_1, t_2 \in [0, \bar{y}]$. Here, t_i represents the net transfer that player i receives. We assume that no transfers are made when $y = 0$. This assumption is equivalent to limited liability: transfers cannot exceed the total output a partner is entitled to in one period. We assume that each partner is entitled to half the output. Therefore, when the output is $\bar{y}$, the total payment to player i is $\frac{\bar{y}}{2} + t_i$. We also assume that $t_1 + t_2 = 0$. That is, the sum of the ex-post payments to each of the players is equal to the total output. Throughout e refers to an effort profile (e_1, e_2) and t refers to a profile of contingent transfers t_1, t_2.

In the repeated game, each player discounts the future with a common discount factor δ. Each period, the realized output y and the transfers t_i are publicly observable while the choice of effort by each player is not. The public outcome $h(\tau)$ in period τ consists of the realized output $y(\tau) \in Y$ and transfers $t_1(\tau), t_2(\tau) \in [-\frac{\overline{y}}{2}, \frac{\overline{y}}{2}] \times [-\frac{\overline{y}}{2}, \frac{\overline{y}}{2}]$. A public history of length τ is therefore $h^\tau = (h(1), ..., h(\tau))$. Let $\mathcal{H}^\tau$ represent the set of all public histories of length τ and $\mathcal{H} = \bigcup_{\tau=1}^{\infty} \mathcal{H}^\tau \bigcup \{h_0\}$ represent the set of all public histories. Here h_0 is the null history. A pure public strategy for player i is a map $\sigma_i : \mathcal{H} \to \{E, S\} \times \mathbb{R}^2$ that maps each public history to a stage game strategy of player i, consisting of effort choice and output-contingent transfers. We focus on the pure strategy public perfect equilibria (PSPPE) of this game.[3] A PSPPE is a public strategy profile $\sigma = (\sigma_1, \sigma_2)$ such that σ_1 is a best response to σ_2 and vice versa.

Let $W_{mn}(\delta)$ represent the set of PSPPE payoff vectors of the repeated partnership game for discount factor δ when the partners have types m and n, respectively. Also define the Pareto frontier of $W_{mn}(\delta)$ by

$$\mathcal{W}^{\delta}_{mn}(v) = \sup\{w | \exists v' \geq v \text{ such that } (v', w) \in W_{mn}(\delta)\}$$

Let $W_{mn} = \lim_{\delta \to 1} W_{mn}(\delta)$ where the limit is with respect to the Hausdorff distance. Finally, define the Pareto frontier of $\mathcal{W}_{mn}$ by

$$\mathcal{W}_{mn}(v) = \sup\{w | \exists v' \geq v \text{ such that } (v', w) \in W_{mn}\}$$

That is, $\mathcal{W}_{mn}(v)$ is the maximum payoff that player 2 can get among equilibria where player 1's payoff is at least v as $\delta \to 1$.

[3] Allowing for mixed strategies complicates the characterization of the equilibrium payoff set because if $q_i > p/2$ then a randomization between effort 0 and 1 eases the incentive constraints of partner $-i$, while reducing the expected output. How these effects balance out depends on the parameters.

2.3 The matching game

The description of the matching game follows Legros and Newman Legros and Newman (2007). As also described at the beginning of the section, the economy includes N agents who are heterogeneous with types from a compact set $\Theta \subset \mathbb{R}$. Let $\kappa : N \to \Theta$ be the type assignment; i.e. $\kappa(i)$ is the type of agent i. The type of each agent is publicly known. With an abuse of notation we use N to refer to the set of agents as well as the number of agents. In our context, a matching is a one-to-one map $M : N \to N$ such that for any $i, j \in N$, $i = M(j)$ if and only if $j = M(i)$. Each matching induces a "matching correspondence" $\mathcal{M} : \Theta \rightrightarrows \Theta$ defined by

$$\mathcal{M}(m) = \{n | \exists i, j \in N \text{ with } \kappa(i) = m, \kappa(j) = n, i \in M(j)\}$$

By positive assortative matching (PAM) we mean a matching M that induces a matching correspondence $\mathcal{M}$ that satisfies:

$$\forall m, n, m', n' : \text{ if } m > n; \; m' \in \mathcal{M}(m); \text{ and } n' \in \mathcal{M}(n) \text{ implies } m' \geq n'.$$

By negative assortative matching (NAM) we mean a matching M that induces a matching correspondence $\mathcal{M}$ that satisfies:

$$\forall m, n, m', n' : \text{ if } m > n; \; m' \in \mathcal{M}(m); \text{ and } n' \in \mathcal{M}(n) \text{ implies } m' \leq n'.$$

The achievable utility pairs of a match between two agents of type n and m, respectively, is described by the function $\mathcal{W}^{\delta}_{mn}(\cdot)$ introduced in the previous subsection. Therefore, the payoff pair (v, w) is feasible for a pair i, j if $w \leq \mathcal{W}^{\delta}_{\kappa(i)\kappa(j)}(v)$.

An equilibrium of the matching game is a matching M and a payoff assignment $v^* : N \to \mathbb{R}$ such that (1) for all i, j with $i \in M(j)$, $(v^*(i), v^*(j))$ is feasible given their types; and (2) the matching is stable: for any $i, j \in N$, there exists no feasible payoff vector w such that $w(i) > v^*(i)$ and $w(j) > v^*(j)$.

2.4 Complementarity and matching patterns

It is well-known that complementarity properties of the payoff possibilities of matches have implications for the patterns of matching that can obtain in equilibrium. In this subsection, we review some of these results that we cite in the rest of the paper.

Firstly, assume that for all m, n, the Pareto frontier of the achievable payoffs is of the form

$$\mathcal{W}_{mn}(v) = -v + f(m, n)$$

This is the "transferable utility" environment where the sum of payoffs to a matched pair is constant along the Pareto frontier. In this case, for all equilibrium matchings to be positive (negative) assortative it is sufficient that the function $f(\cdot,\cdot)$ satisfies increasing (decreasing) differencesBecker (1973):

Definition 1 *A function $f : \Theta \times \Theta \to \mathbb{R}$ exhibits increasing (decreasing) differences if for all $m > m'$, $n > n'$:*

$$f(m,n) + f(m',n') - f(m,n') - f(m',n) > \quad (<) \;\; 0$$

The following is a convenient characterization of increasing (decreasing) differences property.

Remark 1 Topkis (1978) proves the following result: A twice continuously differentiable function $f : \Theta \times \Theta \to \mathbb{R}$ exhibits increasing (decreasing) differences if and only if $\frac{\partial^2 f(m,n)}{\partial m \partial n} > (<) 0$.

In a more general environment, where the sum of payoffs to a matched pair may vary along the Pareto frontier of achievable payoffs (non-transferable utility case), Legros and Newman (2007) introduce the following generalization of increasing (decreasing) differences property. Analogously, in such an environment, this property is sufficient for all equilibrium matchings to be positive (negative) assortative.

Definition 2 *A type-dependent utility possibilities frontier $\mathcal{W}^{\delta}_{mn} : \mathbb{R} \to \mathbb{R}$ satisfies generalized increasing (decreasing) differences if for all $m > m'$ and $n > n'$*

$$\forall v, v' : \mathcal{W}^{\delta}_{m,n'}(v) = \mathcal{W}^{\delta}_{m',n'}(v') \Rightarrow \mathcal{W}^{\delta}_{m,n}(v) > \mathcal{W}^{\delta}_{m',n}(v')$$

3 Characterization of equilibrium payoffs of the partnership game

Once a matching is formed, the partners in a match play a repeated partnership game described in the previous section. As is well-known in this setting, the set of equilibrium payoff vectors is difficult to characterize. However, it is possible to bound the equilibrium payoff set using techniques introduced in Fudenberg et al. (1994). The following proposition is a direct application of Fudenberg et al. (1994)'s result to our setting:

Proposition 1 *Define*

$$\eta(m,n) = \bar{y} - \frac{c}{p(m,n) - q_2(m,n)} - \frac{c}{p(m,n) - q_1(m,n)} \tag{2}$$

For any $m \geq n$, let $\mathcal{W}_{mn}(v) = -v + S(m,n)$ where

$$S(m,n) = p(m,n)\bar{y} - 2c - \min\{(1-p(m,n))\eta(m,n), (p(m,n)-q_1(m,n))\bar{y} - c\} \tag{3}$$

For any $\varepsilon > 0$ there exists $\bar{\delta}_{mn}(\varepsilon) < 1$ such that for any $\delta > \bar{\delta}_{mn}(\varepsilon)$ and for all v:

$$\mathcal{W}^{\delta}_{mn}(v) \in (\mathcal{W}_{mn}(v) - \varepsilon, \mathcal{W}_{mn}(v)]$$

Proof: See appendix.

Proposition 1 first introduces the Pareto frontier ($\mathcal{W}(\cdot)$) of a set that bounds the equilibrium payoff vectors and states that the true Pareto frontier of the equilibrium payoff vectors converges to this bound as the discount factor δ approaches 1. The derivation of this frontier is included in the Appendix. The proof of the convergence result follows from Fudenberg et al. (1994) with minor modifications.

Note that for any pair of types m, n, the limit frontier $\mathcal{W}_{mn}$ has slope -1. Therefore, the sum of payoffs for the two players for any payoff vector on the Pareto frontier is fixed. The quantity $S(m,n)$ introduced in (3) is the limit as $\delta \to 1$ of the maximum surplus obtainable in a match between two partners of types m and n.

Each value pair in the set of equilibrium payoff vectors, including on the Pareto frontier, is associated with a current action profile and outcome contingent continuation values. In general, the associated current action profile may vary along the frontier. In our case, the current action profile consists of an effort profile $(e_1, e_2) \in \{EE, ES, SE, SS\}$ and a transfer profile t_1, t_2. It turns out that, in our case, the current effort profile used to achieve the value pairs on the Pareto frontier is fixed while the transfers vary. In fact, it is the possibility of making transfers that leads to a Pareto frontier with slope -1.[4] For a given matched pair, which effort profile in the current period will be associated with value pairs on the frontier depends on the monitoring technology which is associated with the conditional (on effort) probability distribution of output.

Firstly, for comparison, note that if there were no moral hazard (perfect monitoring) it would be possible to use trigger strategies to implement effort by both agents at no cost and therefore all payoffs would be obtained with the current effort profile EE. In that case, the total surplus would be given by:

$$S^*(m,n) = p(m,n)\bar{y} - 2c \tag{4}$$

[4] Note that our limited liability assumption makes this outcome non-trivial. Without limited liability, the monetary transfers would immediately imply a Pareto frontier of slope -1.

When monitoring is imperfect, having EE as the first period effort profile entails an incentive cost in terms of continuation values. This is because the technology of production does not allow for separately identifying deviators from the effort profile EE and therefore giving incentives to both players to exert effort requires inefficient punishments. The size of the expected inefficient punishment is given by $(1 - p(m,n))\eta(m,n)$.

The expression $(1 - p(m,n))\eta(m,n)$ has a very intuitive interpretation, as suggested by Abreu et al. (1991). It is the "expected monitoring cost": inefficient punishments of size $\eta(m,n)$ occur when low output is realized, i.e. with frequency $(1 - p(m,n))$. The size of the punishment $\eta(m,n)$ is the sum of the punishments required to satisfy the incentive constraints of each agent. For instance, if there are no transfers, the size of the punishment for agent of type m is

$$\eta_1(m,n) = \frac{c - (p(m,n) - q_2(m,n))\bar{y}/2}{p(m,n) - q_2(m,n)}$$

The expression in the numerator is the instantaneous gain obtained by agent type m from deviating. On the other hand, the denominator measures by how much the frequency of punishment increases in case of deviation. Therefore, the optimal punishment is such that the expected cost of punishment is equalized with the expected gain. Therefore, the highest surplus $S(m,n)^{EE}$ that can be achieved with effort profile EE in the first period is

$$S^{EE}(m,n) = S^*(m,n) - (1 - p(m,n))\eta(m,n)$$

On the other hand, if ES is the first period effort profile, incentives to the partner that is supposed to exert effort can be given via transfers between the partners without reducing the total surplus. However, in this case, the effort profile in the first period is not efficient and therefore leads to a reduction of $(p(m,n) - q_1(m,n))\bar{y} - c$ in expected surplus relative to the first best. Therefore, the highest surplus $S^{ES}(m,n)$ that can be obtained, when the first period effort level is ES is

$$S^{ES}(m,n) = S^*(m,n) - (p(m,n) - q_1(m,n))\bar{y} - c$$

And similarly, the analogous expression for SE is

$$S^{SE}(m,n) = S^*(m,n) - (p(m,n) - q_2(m,n))\bar{y} - c$$

Clearly, if $m \geq n$ then $S^{ES}(m,n) \geq S^{SE}(m,n)$. Then, the level of the Pareto frontier is $S(m,n) = \max\{S^{EE}(m,n), S^{SE}(m,n)\}$, which is the expression given in (3).

4 Equilibrium matching patterns

4.1 Convergence

While the limit frontier $\mathcal{W}_{mn}(\cdot)$ of equilibrium payoffs of the repeated partnership game has slope -1 (and, therefore, utility is transferable), this is not necessarily the case for the Pareto frontiers when $\delta < 1$. In this Section we first establish that if the limit frontier $\mathcal{W}_{mn}(\cdot)$ satisfies increasing (decreasing) differences, then the frontier $\mathcal{W}^{\delta}_{mn}(\cdot)$ satisfies generalized increasing (decreasing) differences for large enough δ.

Lemma 1 *Assume $S(m,n)$ exhibits (decreasing) differences. Then there exists $\bar{\delta} < 1$ such that for all $\delta > \bar{\delta}$, $\mathcal{W}^{\delta}_{mn}(\cdot)$ exhibits generalized increasing (decreasing) differences.*

Proof: Let $T = \{\kappa(i)|i \in N\}$. That is T is the set of types of the N agents in the economy. First assume increasing differences, i.e.

$$(S(m,n) - S(m,n')) - (S(m',n) - S(m',n')) > 0$$

Let $\varepsilon = \inf\{S(m,n) + S(m',n') - S(m,n') - S(m',n')|m > m', n > n' \in T\}$ and $\varepsilon^* = \frac{1}{4}\varepsilon$. For each m, n there exists $\delta_{mn}(\varepsilon^*)$, such that for all $\delta > \delta_{mn}(\varepsilon^*)$, $\sup_v |\mathcal{W}^{\delta}(v) - \mathcal{W}(v)| < \varepsilon^*$. Now, let $\bar{\delta} = \max\{\bar{\delta}_{mn}(\varepsilon^*)|m, n \in T\}$. Since T is finite, $\varepsilon^* > 0$ and $\bar{\delta} < 1$.

Take $\delta > \bar{\delta}$, $m > m', n > n'$ and v, v' such that

$$\mathcal{W}^{\delta}_{mn'}(v) = \mathcal{W}^{\delta}_{m'n'}(v')$$

Then,

$$\mathcal{W}_{mn'}(v) - \mathcal{W}_{m'n'}(v') > -2\varepsilon^*$$

Moreover,

$$\mathcal{W}^{\delta}_{mn}(v) - \mathcal{W}^{\delta}_{m'n}(v') > \mathcal{W}_{mn}(v) - \mathcal{W}_{m'n}(v') - 2\varepsilon^*$$

Also,

$$(\mathcal{W}_{mn}(v) - \mathcal{W}_{m'n}(v')) - (\mathcal{W}_{mn'}(v) - \mathcal{W}_{m'n'}(v')) = (S(m,n) - S(m'n)) - (S(m,n') - S(m',n')) \geq 4\varepsilon^*$$

The last three inequalities together imply

$$\mathcal{W}^{\delta}_{mn}(v) - \mathcal{W}^{\delta}_{m'n}(v') > 0$$

which establishes generalized increasing differences. The proof for generalized decreasing differences is analogous.□

4.2 Matching predictions

In what follows, to ensure that there is no inherent positive or negative complementarity in the production technology which would bias equilibrium outcomes towards assortative matching, we make the following assumption:

A 5 $\frac{\partial^2 p(m,n)}{\partial m \partial n} = 0$, $\frac{\partial^2 q_1(m,n)}{\partial m \partial n} = 0$ and $\frac{\partial^2 q_2(m,n)}{\partial m \partial n} = 0$

Remark 2 Recall that in the model with perfect monitoring the total surplus is given by (4). Therefore, Assumption A5 implies that the model with perfect monitoring would not generate any matching predictions; i.e. in the model without moral hazard any matching can be an equilibrium matching.

In the light of Lemma 1, it is sufficient to analyze whether $S(m,n)$ has increasing or decreasing differences in order to understand the equilibrium matching patterns. Recall that, when $m \geq n$

$$\begin{aligned} S(m,n) = & p(m,n)\bar{y} - 2c - \\ & \min\left\{(1 - p(m,n))\eta(m,n), (p(m,n) - q_1(m,n))\bar{y} - c\right\} \end{aligned}$$

where

$$\eta(m,n) = \bar{y} - \frac{c}{p(m,n) - q_2(m,n)} - \frac{c}{p(m,n) - q_1(m,n)}$$

When characterizing the equilibrium matching patterns, it is useful to distinguish between two cases[5]:

1. (low precision monitoring) $\forall m,n$: $(1 - p(m,n))\eta(m,n) > (p(m,n) - q_1(m,n)$
2. (high precision monitoring) $\forall m,n$: $(1 - p(m,n))\eta(m,n) < (p(m,n) - q_1(m,n)$

As discussed in Section 3, under the first case the first period effort profile for any matched pair will be ES, while under the second case, it will be EE. The first case will occur when $\frac{1-p(m,n)}{p(m,n)-q_i(m,n)}$ is large enough for each m,n and $i = 1,2$. Notice that $1 - p(m,n)$ is the probability of inefficient punishment if both partners follow their first period equilibrium effort level E. Also, $p(m,n) - q_i(m,n)$ is by how much this probability increases if partner $-i$ shirks.

[5]We do not analyze the cases where different pairs of types may fall under different cases.

Therefore, $\frac{1-p(m,n)}{p(m,n)-q_i(m,n)}$ is the inverse of the percentage change in the probability of inefficient punishment in case of deviation by one of the partners from EE. When this change is small, i.e. $\frac{1-p(m,n)}{p(m,n)-q_i(m,n)}$ is large, the partners are willing to deviate from EE and therefore, incentive costs associated with EE in the first period (i.e. size of the inefficient punishment) has to be large. When it is large enough, this loss of efficiency is larger than the loss of efficiency from choosing the sub-optimal effort profile ES in the first period. By the same token, when $\frac{1-p(m,n)}{p(m,n)-q_i(m,n)}$ is large enough for each m, n and $i = 1, 2$, the second case will occur.

4.2.1 Low precision monitoring

In this case,

$$S(m,n) = \max\{q_1(m,n), q_2(m,n)\}\bar{y}/2 - \min\{c, c\}$$

It is easy to see that in this case, positive assortative matching cannot be an equilibrium matching. To see this consider four participants of the matching market. If the top two types are matched with each other, one of them will be idle. Instead, one of the bottom two types will exert effort. On the other hand, if each of the top two types is matched with one of the bottom two types only the bottom two types will be idle. This arrangement will maximize the total surplus and hence will be stable, while positive assortative matching will not. In fact, it is possible to show that, any matching where the top half of the types are randomly matched with the bottom half of the types will be an equilibrium outcome.

4.2.2 High precision monitoring

Throughout this subsection we assume that

A 6 For all $m \geq n$:

$$\left[-\bar{y} + \frac{c}{p(m,n) - q_2(m,n)} + \frac{c}{p(m,n) - q_1(m,n)}\right] < p(m,n) - q_1(m,n) + c$$

In this case,

$$S(m,n) = \underbrace{p(m,n) - 2c}_{S^*(m,n)} - \underbrace{(1 - p(m,n))}_{\text{frequency}} \underbrace{\left[-\bar{y} + \frac{c}{p(m,n) - q_2(m,n)} + \frac{c}{p(m,n) - q_1(m,n)}\right]}_{\text{size}}$$

As remarked above, $S^*(m,n)$ exhibits no complementarities under our assumption. Therefore, the possibility of complementarity lies in the potential interaction of types of the two partners via the monitoring technology which determine the frequency and size of the inefficient punishment. In this subsection we take a closer look at this interaction.

First notice that by assumption **A4**, the frequency of inefficient punishment— which occurs only when the output is low—is decreasing in the types of both partners. The effect of a change in a partner's type on the size of the punishment is less straightforward. To see how this effect transpires, it is sufficient to analyze the term $\frac{c}{p(m,n)-q_2(m,n)}$. This term determines the size of the punishment that will accrue to partner 1 in case of low output, i.e. this is the surplus loss to ensure partner 1 has incentive to exert effort. Therefore, in relation to this term, partner 1 of type m is "being monitored" while partner 2 of type n is "monitoring" him. The case where partner 1 is a monitor and and partner 2 is being monitored can be analyzed analogously.

How this term responds to changes in the type of the partners is determined by how the term $p(m,n) - q_2(m,n)$ responds to such changes. Notice that, the latter term is the marginal product of effort for partner 1 when partner 2 is exerting effort. If this term is increasing in type we say that type and effort are complements. If it is decreasing we say that type and effort are substitutes. Next, we take up these two cases separately.

Effort and type are complements:

In this part we make the following assumption:

A 7 For any m, n, $\frac{\partial p(m,n)}{\partial n} - \frac{\partial q_1(m,n)}{\partial n} > 0$ and $\frac{\partial p(m,n)}{\partial m} - \frac{\partial q_1(m,n)}{\partial m} \geq 0$.

Under assumption A7, effort is complementary to type: it is more productive when the agent is of higher type or is matched with a higher type partner. In this case, one can interpret 'type' as capital that the agent brings to the match and effort is labor (where capital and labor have some degree of complementarity, as in Cobb-Douglas production function). In general, any production function (determining expected output as a function of effort and types) of the form $F_{mn}(e_1, e_2) = F_0(e_1, e_2) + mF_1(e_1, e_2) + nF_2(e_1 + e_2)$, where F_1 and F_2 are increasing in both arguments satisfies this assumption. In particular, a canonical technology that fits in this framework is where

$$p(m, n) = m + n + a_0; \qquad q_1(m, n) = m + a_1 \qquad q_2(m, n) = n + a_2$$

The following proposition establishes that when the first period effort profile is EE, the equilibrium matching is positive assortative under the additional assumption **A7**.

Proposition 2 *Suppose that assumptions A1-A7 hold. Then, there exists $\bar{\delta} < 1$ such that for all $\delta > \bar{\delta}$, all equilibrium matchings are negative assortative.*

Proof: By Lemma 1, it suffices to show that $\mathcal{W}_{mn}(v)$ has strictly decreasing differences in (m,n). First, recall that

$$\mathcal{W}_{mn}(v) = -v + p(m,n)\bar{y} - 2c - (1-p(m,n))\eta(m,n)$$

whenever $(p(m,n)-q_1(m,n))\bar{y}-c < (1-p(m,n))\eta(m,n)$. Thus, by Assumption A5, it suffices to verify that

$$\frac{\partial^2 (1-p(m,n))\eta(m,n)}{\partial m \partial n} > 0 \quad \text{for all } m \text{ and } n. \tag{5}$$

Recall that

$$\eta(m,n) = \frac{c(m) - (p(m,n) - q_2(m,n))\bar{y}/2}{p(m,n) - q_2(m,n)} + \frac{c(n) - (p(m,n) - q_1(m,n))\bar{y}/2}{p(m,n) - q_1(m,n)},$$

which can be simplified as

$$\eta(m,n) = \frac{c(m)}{p(m,n) - q_2(m,n)} + \frac{c(n)}{p(m,n) - q_1(m,n)} - \bar{y}$$

Since $p(m,n) = p(n,m)$ and $q_2(m,n) = q_1(n,m)$, (5) holds when $\frac{1-p(m,n)}{p(m,n)-q_1(m,n)}c(n)$ satisfies strict increasing differences. For brevity, drop the reference to m and n, the subindex in q_1 and denote the corresponding partial derivatives by p_m, p_n, q_m and q_n. Then

$$\begin{aligned}\frac{\partial^2 \frac{1-p}{p-q}c}{\partial m \partial n} =& \frac{1}{(p-q)^3}\left[p_m(p_n-q_n) + p_n(p_m-q_m) + 2(p_m-q_m)(p_n-q_n)\frac{1-p}{p-q}\right]c + \\ &+ \frac{1-p}{(p-q)^2}\left[q_m - p_m\frac{1-q}{1-p}\right]c'(n)\end{aligned} \tag{6}$$

The above expression is strictly positive since, $p_n - q_n > 0$ and $p_m - q_m \geq 0$ (by assumption A7), $c'(n) < 0$ (by assumption A4) and $p > q$ implying that $\frac{1-q}{1-p} > 1$. Hence, $\mathcal{W}_{mn}(v)$ has strictly decreasing differences in (m,n). $\square$

The intuition behind Proposition 2 is as follows. Equilibrium matching pattern maximizes the total surplus. Hence, it minimizes the sum of the costs of incentive provision $(1-p(n,m))\eta(m,n)$ across matched pairs. On the one hand, the frequency of inefficient punishments $1-p(n,m)$ is decreasing in the types of both partners. On the other hand, the size of the punishment $\eta(m,n)$ is inversely related to $p(m,n) - q_1(m,n)$, which is increasing in m and n by assumption A7. The monotonicity in n means that higher types are "better monitors" because the deviations of their partners have larger effects on the probability of success and, therefore, are more easily detectable. By the same token, the monotonicity of $p(m,n) - q_1(m,n)$ in m implies that higher types are easier to monitor.

Since the frequency and the size of the punishments are simultaneously influenced by the types of both partners, the effect of the interaction of the two types on $S(m, n)$ can be decomposed into four parts:

- First, the types that are harder to monitor (i.e. needing larger size punishments) should be matched with the types that lead to less frequent punishments. This is the effect of the monitored agent's own type on $\eta(m, n)$ and his partner's type on $(1 - p(m, n))$.
- Second, the types that are better monitors (i.e. who reduce the size of necessary inefficient punishment) should be matched with the types that lead to more frequent punishments. This is the effect of the monitored agent's own type on $(1 - p(m, n))$ and his partner's type on $\eta(m, n)$.
- Third, the types that are harder to monitor should be matched with better monitors. This is the effect of both types on $\eta(m, n)$.
- Fourth, the types that lead to more frequent punishments should be matched with types that lead to less frequent punishments. This is the effect of both types on $1 - p(m, n)$.

Under assumption A7, the first three effects simultaneously push the equilibrium matching towards negative assortative, because higher types are better monitors, are easier to monitor and lead to less frequent punishments. The fourth effect, however, does not play any role because, by assumption A5, there is no complementarity in $p(m, n)$.[6]

Effort and type are substitutes

In this part, we make the following assumption:

A 8 For any m, n, $\frac{\partial p(m,n)}{\partial n} - \frac{\partial q_1(m,n)}{\partial n} < 0$ and $\frac{\partial p(m,n)}{\partial m} - \frac{\partial q_1(m,n)}{\partial m} \leq 0$.

If assumption A8 holds, effort substitutes for the agent's type. This could happen, for instance, if 'type' represents the amount of accumulated knowledge, effort is exerted to generate more knowledge accumulation and the probability of success is a decreasing returns to scale function of total knowledge. Notice that for the result in Proposition 2 to obtain it is crucial that higher types are better monitors and are easier to monitor. If, however, this assumption is reversed (i.e., A7 is replaced with A8), the first and second effects push towards positive assortative matching while the third effect continues to push towards negative assortative

[6]In the proof of Proposition 2 the first three effects correspond (in the same order) to the three terms in the right hand side of (6).

matching. Therefore, if the degree of substitutability between effort and type is sufficiently large, positive assortative matching could obtain in equilibrium. Proposition 3 gives sufficient conditions for this to happen.

Proposition 3 *Suppose that assumption A1-A6 and A8 hold. Then, there exists $\bar{\delta} < 1$ such that for all $\delta > \bar{\delta}$, all equilibrium matchings are positive assortative if and only if the following two condition holds for any m and n:*

$$2\frac{\partial(p(m,n)-q_1(m,n))}{\partial m}\frac{\partial(p(m,n)-q_1(m,n))}{\partial n}\frac{1-p(m,n)}{p-q_1(m,n)} < -\frac{\partial p(m,n)}{\partial m}\frac{\partial(p(m,n)-q_1(m,n))}{\partial n} - \frac{\partial p(m,n)}{\partial n}\frac{\partial(p(m,n)-q_1(m,n))}{\partial m}$$

Proof: By Lemma 1, it suffices to verify that $\mathcal{W}_{mn}(v)$ has strictly increasing differences in (m, n), which is true when $(1-p(m,n))\eta(m,n)$ has strictly decreasing differences. Condition (ii) in the Proposition stipulates that $\frac{\partial^2 \frac{1-p}{p-q}}{\partial m \partial n}$ defined in (6) is strictly negative, implying strict decreasing differences of $(1-p(m,n))\eta(m,n)$. $\square$

Observe that condition (3) in Proposition 3 holds if the ratio $\frac{1-p(m,n)}{p-q_1(m,n)}$ is sufficiently close to 0, i.e. when the amount of noise is sufficiently small.

Remark 3 If the cost of effort is type-dependent, with lower types having higher costs of effort, the results of the case when type and effort are complementary are strengthened. This is because the size of the punishment in this case decreasing even faster with the type of the monitored partner. On the other hand, when type and effort are substitutes, the effect of differing costs in this manner goes the opposite direction. In this case, for the results to continue to hold it is necessary that the cost is decreasing sufficiently slowly with type.

5 Discussion

Our model features dynamic interaction and allows the agents to transfer utility either through transfers of the current-period output or through implementation of asymmetric effort. In this section we investigate how each of these features affects matching predictions. In short, we find that (a) all our results would hold in a static model, in which the partners can commit to money burning, (b) if the utility cannot be transferred at all (i.e. neither through output sharing nor through asymmetric effort assignment) then the equilibrium matching structure is positive, (c) if the utility can be transferred only using the transfers of instantaneous output then all our results hold, implying that the matching predictions in our model are not driven by the possibility of exerting different efforts (neither in the current nor in the future periods), and (d) if instantaneous output cannot be transferred from one agent to another but asymmetric actions can be taken, then the equilibrium matching structure can switch from negative to positive, implying that non-transferability of output favors positive matching.

5.1 The role of dynamic interaction

As emphasized repeatedly, the limit Pareto frontier of the payoff set of the repeated partnership game has a very simple form, namely $\mathcal{W}_{mn}(v) = -v + S(m, n)$. In this section, we introduce an alternative game, which leads to the same Pareto frontier of payoffs (and therefore same matching predictions) while also preserving the underlying intuition regarding the monitoring technology.

Consider a model in which once a match is formed the partners play a static game. In this game, they simultaneously choose to exert effort or shirk. The production technology is as in the stage game of the repeated partnership game. Assume also that the agents *can commit to output-contingent compensation profiles*, but effort choices are not contractible. Finally assume that the compensation schedules need not be "budget balancing": the partners can make transfers to a third party (in other words "burn money").

It is worth making a couple remarks before establishing the equivalency of this model to the repeated partnership game:

- Firstly, if the agents cannot commit to any transfers, under assumptions **A2** and **A3**, the unique equilibrium of this game involves both partners choosing to shirk. Therefore, in that case there are no matching predictions.
- Secondly, if the agents can commit to transfers but cannot commit to money burning, then the equilibrium set expands. However, due to assumption **A3**, independent of the monitoring technology, effort combination EE is never part of an equilibrium. In this case, the Pareto frontier for this game is identical to the Pareto frontier of the repeated partnership game with "imprecise monitoring" technology.

Let $\bar{r}_i, \underline{r}_i$ (for $i = 1, 2$) be the total payment (output he is entitled to and transfers) to partner i when the output is $\bar{y}$ and 0, respectively. First, note that in the one-shot game, it is obvious that the Pareto frontier has slope -1. What remains to do is to compute the maximal total payoff.

Consider the following problem:

$$
\begin{aligned}
S^{EE}(m,n) = \max_{\underline{r}_1,\underline{r}_2,\overline{r}_1,\overline{r}_2} & \; p(m,n)(\overline{r}_1+\overline{r}_2)+(1-p(m,n))(\underline{r}_1+\underline{r}_2)-2c \\
\text{s.t.} \quad & (p(m,n)-q_1(m,n))(\overline{r}_2-\underline{r}_2) \geq c \\
& (p(m,n)-q_2(m,n))(\overline{r}_1-\underline{r}_1) \geq c \\
& \overline{r}_1+\overline{r}_2 \leq \overline{y} \\
& \underline{r}_1+\underline{r}_2 \leq 0
\end{aligned}
\tag{7}
$$

Here the first and second constraints are the incentive constraints for both partners, and the last two conditions state that the total compensation cannot exceed the realized output.
It is straightforward to verify that the solution to (7) satisfies the first three constraints with equality, which implies that

$$\underline{r}_1+\underline{r}_2 = \overline{y} - \frac{c}{p(m,n)-q_1(m,n)} - \frac{c}{p(m,n)-q_2(m,n)} < 0$$

where the last inequality holds by A3. Therefore the last constraint in (7) remains slack, meaning that it is necessary for the partners to commit to burning money. It follows that the maximal surplus obtained from (7) is given by

$$S^{EE}(m,n) = \overline{y} - 2c - \frac{1-p(m,n)}{p(m,n)-q_1(m,n)}c - \frac{1-p(m,n)}{p(m,n)-q_2(m,n)}c,$$

which obviously coincides with $S^{EE}(m,n)$ derived for the repeated partnership game..

Similarly, if only the more productive partner works ($m \geq n$) then the maximum sum of the equilibrium payoffs is given by

$$
\begin{aligned}
S^{ES}(m,n) = \max_{\underline{r}_1,\underline{r}_2,\overline{r}_1,\overline{r}_2} & \; q_1(m,n)(\overline{r}_1+\overline{r}_2)+(1-q_1(m,n))(\underline{r}_1+\underline{r}_2)-c \\
\text{s.t.} \quad & (p(m,n)-q_1(m,n))(\overline{r}_2-\underline{r}_2) \leq c \\
& q_2(m,n)(\overline{r}_1-\underline{r}_1) \geq c \\
& \overline{r}_1+\overline{r}_1 \leq \overline{y} \\
& \underline{r}_1+\underline{r}_2 \leq 0
\end{aligned}
\tag{8}
$$

Obviously, in this case the incentive constraints will be slack, but the last two conditions will be satisfied with equality. Therefore, the value function of this problem is

$$S^{ES}(m,n) = q_1(m,n)\overline{y} - c$$

which coincides with the corresponding surplus derived for the repeated partnership game.

This observation is arguably not very surprising. The money burning in this model corresponds to moving to sub-optimal equilibria as a punishment in the repeated partnership game. Also, the dynamics of the repeated partnership game provides incentives for the partners to make transfers, which would not be possible in the one-shot version without the assumption of explicit commitment. It is also important to note that the frontier obtained for the game introduced in this section is identical only to the limiting frontier of the repeated partnership game as $\delta \to 1$. For $\delta < 1$ the frontier for the repeated partnership game will be different due to the impossibility of giving incentives to the partners to make transfers when their promised value is low enough.

5.2 The role of transferability

So far, we have been considering the case where asymmetric effort choices are possible and monetary transfers between partners are allowed. Both of these aspects can be considered as contributing to the degree of transferability of utility. On the one hand, asymmetric effort choices allows utility to be transferred through moving to equilibria that has one partner working more than the other. On the other hand, with quasilinear preferences as here, monetary transfers are obvious channels for utility transfers.

We have so far been assuming full transferability through both of these channels. In this section we explore how the results are effected when transferability is restricted. We first consider the extreme case where no transfers are allowed and the partners play a strongly symmetric equilibrium (after any history they choose identical effort levels). Then we consider the cases where only transfers through output sharing are allowed and where only transfers through asymmetric effort levels are allowed.

5.2.1 Non-transferable utility

First, consider a team of partners of the same type. The equilibrium is obtained by trigger strategies: the agents play (E, E) and if low output is realized, they switch to (S, S) forever with probability α. Identical partners obtain the same equilibrium payoff v^* which satisfies

$$v^* = p\frac{\overline{y}}{2} - c - \frac{\delta}{1-\delta}(1-p)\alpha v^* \tag{9}$$

The incentive constraint for each partner is

$$v^* \geq q\frac{\overline{y}}{2} - \frac{\delta}{1-\delta}(1-q)\alpha v^* \tag{10}$$

Obviously, α is chosen to make the incentive constraints bind, and the equilibrium value v^* is found by substituting this value of α into (9):

$$\text{(11)} \qquad v^* = \frac{\overline{y}}{2} - c - \frac{1-p}{p-q}c \quad \text{and} \quad \alpha = \frac{1-\delta}{\delta} \frac{c - (p-q)\frac{\overline{y}}{2}}{(p-q)\frac{\overline{y}}{2} - (1-q)c}$$

Clearly, v^* is decreasing in q and positive value v^* cannot be sustained in the equilibrium if q is sufficiently close to p. Also, the probability of inefficient punishment α increases with q (as monitoring technology becomes weaker).

Similarly, in a team where partners have different types, the equilibrium values (v^*, w^*) must satisfy:

$$\text{(12)} \qquad \begin{aligned} v^* &= p\frac{\overline{y}}{2} - c - \frac{\delta}{1-\delta}(1-p)\alpha v^* \\ w^* &= p\frac{\overline{y}}{2} - c - \frac{\delta}{1-\delta}(1-p)\alpha w^* \end{aligned}$$

subject to the incentive constraints

$$\text{(13)} \qquad \begin{aligned} v^* &\geq q_2\frac{\overline{y}}{2} - \frac{\delta}{1-\delta}(1-q_2)\alpha v^* \\ w^* &\geq q_1\frac{\overline{y}}{2} - \frac{\delta}{1-\delta}(1-q_1)\alpha w^* \end{aligned}$$

Note that in this case one of the incentive constraints would be left slack. For example, if $q_1 < q_2$ then the second agent is a weaker monitor, hence the incentive constraint for the first agent binds, and the incentive constraint for the second agent remains slack. In this case,

$$v^* = w^* = \frac{\overline{y}}{2} - c - \frac{1-p}{p - \min(q_1, q_2)}c$$

Correspondingly, the value of the high productivity agent decreases if he is matched with the less productive partner, implying that the equilibrium matching is positive assortative.

If, on the other hand, there are no differences in productivity (i.e. $q_1 = q_2$ and p is the same across matches), but there is heterogeneity in costs of effort, then the incentive constraint for the agent with higher cost binds, and the one for the agent with the lower effort cost holds with strict inequality. Correspondingly, if $c_1 > c_2$ then v^* is exactly the same as the one obtained in a homogeneous partnership of type c_1, but w^* is lower than what the partners of type c_2 would have achieved had they formed a match with each other. In this case in the equilibrium agents are also sorted positively, thereby avoiding unnecessary losses in the value for c_2-type agents.

To sum up, if the utility cannot in any way be transferred from one partner to another (as in a strongly symmetric equilibrium above), the positive assortative matching would be the

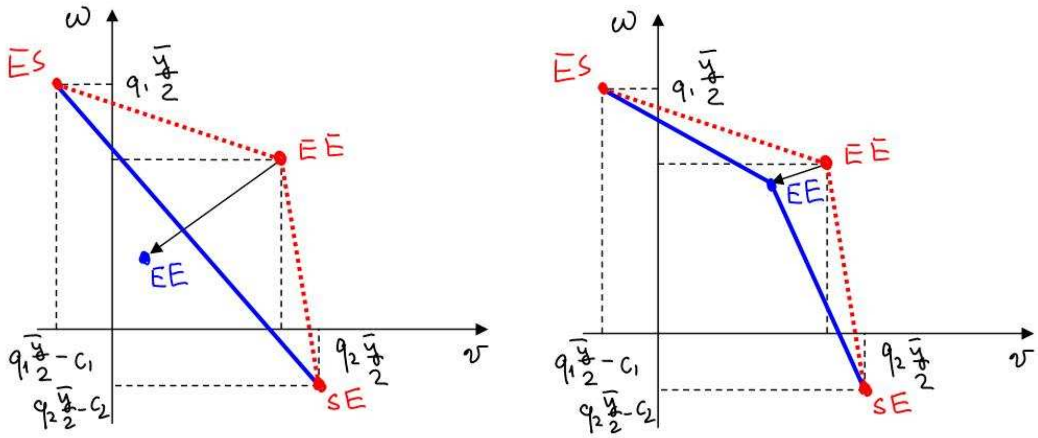

Figure 1: The shape of the limit Pareto frontier $\mathcal{W}_{mn}(v)$ without monetary transfers under imprecise (left) and precise (right) monitoring

property of the equilibrium.

5.2.2 Utility transferable through asymmetric effort assignment

Now suppose that the agents cannot implement monetary transfers (i.e. the total output must be equally shared in every period) but can choose asymmetric effort assignments. In this case, we can use the methodology in Fudenberg, Levine and Maskin (1994) to characterize the limit Pareto frontier $\mathcal{W}_{mn}(v)$ of the dynamic game.

Figure 1 illustrates the possible shapes of $\mathcal{W}_{mn}(v)$. First, observe that in the absence of moral hazard any convex combination of the stage game payoffs could be achieved in the equilibrium. Thus the set of equilibria would be given by a subset of a positive quadrangle bounded by the red dashed lines. When monitoring is imperfects and δ is large, (E, S) and (S, E) can still be sustained in the equilibrium because they require incentive provision for only one of the partners. However, the payoffs associated with the current action (E, E) would move inside the first best equilibrium set. Namely, the payoffs in this case are given by

$$v_{EE} = \underbrace{p\frac{\overline{y}}{2} - c}_{\text{first best}} - (1-p)\left(\frac{c}{p-q_2} - \frac{y}{2}\right)$$

$$w_{EE} = \underbrace{p\frac{\overline{y}}{2} - c}_{\text{first best}} - (1-p)\left(\frac{c}{p-q_1} - \frac{y}{2}\right)$$

When monitoring is imprecise (either $p - q_1$ or $p - q_2$ is sufficiently small), the cost of providing incentives to both agents is too large, and in all the equilibria on the limit Pareto frontier only one of the partners exerts effort in the current period (see left plot of Figure 1). Note that the slope of the limit frontier in this case is different from -1 if the partners are of different types, implying that the utility is not fully transferable. It turns out, however, that the generalized increasing differences condition (introduced in Legros and Newman 2007) is satisfied if there are only two possible types of agents (*high* and *low*).[7] This implies that the high types would be matched with each other.[8] In contrast, in our main model where monetary transfers are possible, the high types are matched with the low types if the monitoring is sufficiently noisy.

If however, the monitoring is sufficiently precise (as on the right plot of Figure 1), the equilibria on the limit Pareto frontier are obtained by randomizing between (E, S) and (E, E) (the left branch) or between (E, E) and (S, E) (the right branch) in the current period. Unfortunately, in this case we were not able to derive the set of conditions under which the limit frontier satisfies generalized increasing/decreasing differences. Thus we performed a series of numerical exercises and found that non-transferability of monetary payoffs may change the equilibrium matching structure from negative to positive. For example, this occurs in the model with two types and additive productivity ($q_1(m, n) = \alpha_m$, $q_2(m, n) = \alpha_n$ and $p(m, n) = \alpha_m + \alpha_n$) for all the parameter values we tried. At the same time, we were not able to construct an example for which positive matching occurs in the benchmark model, but the generalized increasing differences condition is violated after the monetary transfers are removed. This suggests that shutting down monetary transfers in our main model makes positive assortative matching more likely, though of course more rigorous analysis is required to claim it with certainty.

5.2.3 Utility transferable through output sharing

In this subsection we characterize the frontier of the game when attention is restricted to strategies that lead to the same effort level by both partners after any history, while still allowing monetary transfers. We show that this frontier also converges to a limiting frontier with slope -1 with level equivalent to the level $S^{EE}(m, n)$ derived for the repeated partnership game with asymmetric strategies. Recall that this is the maximum level of surplus achievable

[7] So far we have not been able to prove that the generalized increasing differences condition holds for more than two types, though we were not able to find a counterexample in which it is not satisfied.

[8] Notice also that, under imprecise monitoring, the frontier becomes steeper as the type of the first partner (whose value v is on the horizontal axis) increases. This means that higher types are more capable to transfer utility to their partners. Legros and Newman (2007) derive the set of conditions sufficient for generalized increasing differences to hold. One of them requires that the degree of transferability is increasing in type. Thus it is not surprising that we find that generalized increasing differences is satisfied in this case.

when the first period effort combination is EE.

We do this by establishing that for each $\kappa > 0$ the set

$$\Omega(\kappa) = \{v, w | v + w < S^{EE}(m,n)\} \bigcup \{v, w | v + w = S^{EE}(m,n); v, w \geq \kappa\}$$

is self-generating for large enough δ. Moreover, we show that the set can be obtained using symmetric effort levels only.

To see this take $v, w \in \Omega$. Choose continuation values $\bar{x}_1, \underline{x}_1, \bar{x}_2, \underline{x}_2$ such that the following incentive compatibility and promise keeping constraints are satisfied with equality. These will fully determine the continuation values as a function of t:

PK1 $v = (1-\delta)[\frac{1}{2}\bar{y} - t - c] + \delta[p\bar{x}_1 + (1-p)\underline{x}_1]$

PK2 $w = (1-\delta)[\frac{1}{2}\bar{y} + t - c] + \delta[p\bar{x}_2 + (1-p)\underline{x}_2]$

IC1 $v = (1-\delta)[\frac{1}{2}\bar{y} - t] + \delta[q_2\bar{x}_1 + (1-q_2)\underline{x}_1]$

IC2 $w = (1-\delta)[\frac{1}{2}\bar{y} + t] + \delta[q_1\bar{x}_2 + (1-q_1)\underline{x}_2]$

The continuation values are:

$$\begin{array}{rcl} \underline{x}_1 & = & \frac{v}{\delta} + \frac{1-\delta}{\delta}[c - \frac{p}{p-q_2}c] \\ \bar{x}_1 & = & \frac{v}{\delta} + \frac{1-\delta}{\delta}[c + \frac{1-p}{p-q_2}c - \frac{\bar{y}}{2} - t] \\ \underline{x}_2 & = & \frac{w}{\delta} + \frac{1-\delta}{\delta}[c - \frac{p}{p-q_1}c] \\ \bar{x}_2 & = & \frac{w}{\delta} + \frac{1-\delta}{\delta}[c + \frac{1-p}{p-q_1}c - \frac{\bar{y}}{2} + t] \end{array}$$

It remains to show that for some t these continuation values are from the set $\Omega(\kappa)$. First we note that

$$\begin{array}{rcl} \bar{x}_1 - \underline{x}_1 & = & \left[\frac{1-\delta}{\delta}\frac{c}{p-q_2} - \frac{\bar{y}}{2} + t\right] \\ \bar{x}_2 + \underline{x}_2 & = & \left[\frac{1-\delta}{\delta}\frac{c}{p-q_1} - \frac{\bar{y}}{2} - t\right] \end{array}$$

Therefore,

$$[\bar{x}_1 + \bar{x}_2] - [\underline{x}_1 - \underline{x}_2] = \frac{1-\delta}{\delta}\left[\frac{c}{p-q_2} + \frac{c}{p-q_1} - \bar{y}\right]$$

By assumption **A3**, this quantity is strictly positive. Second we note that

$$\begin{array}{rcl} \bar{x}_1 + \bar{x}_2 & = & \frac{w+v}{\delta} - \frac{1-\delta}{\delta}\left[\bar{y} - 2c - \frac{1-p}{p-q_1}c - \frac{1-p}{p-q_2}c\right] \\ & = & \frac{v+w}{\delta} - \frac{1-\delta}{\delta}K \leq K \end{array}$$

Therefore, if $v + w < K$, then we are done. Assume that $v + w = K$. Then $\bar{x}_1 + \bar{x}_2 = K$. Therefore, we need to argue that for some t, $\bar{x}_1 \geq \kappa$ and $\bar{x}_2 \geq \kappa$. But this follows from the fact that $v, w \geq \kappa$ and by appropriate choice of t $\bar{x}_1 \geq v$ and $\bar{x}_2 \geq w$.

The argument so far ignores the incentives to make the observable transfers t. However, for large enough δ any such transfer will be incentive compatible.

6 Appendix: Characterizing the Pareto frontier of the repeated partnership game payoffs

In what follows, for economy of notation we drop reference to types m, n when obvious from context.

6.1 Bounding the equilibrium payoff set: methodology

In the Fudenberg et al. (1994) characterization, the bounding set that is shown to be the limit of the set of equilibrium value vectors is the intersection of the largest half-spaces in each direction whose boundary values can be decomposed on these hyperplanes. A half space $H(\lambda, k)$ with direction $(\lambda, 1-\lambda)$ and level k is the set $\{v \in \mathbb{R}^2 | \lambda v_1 + (1-\lambda)v_2 \leq k\}$. Also for brevity, define $u_i(e,t) = -c_i e_i + E\{\frac{y}{2} + t_i(y)|e\}$ where $E\{\cdot|e\}$ represents the expectation taken with respect to y using the distribution induced by the effort profile e.

The following definition is adopted from Abreu et al. (1990) and Fudenberg et al. (1994) for our setting:

Definition 3 *A value vector $v = (v_1, v_2)$ is decomposable on a set $W \in \mathbb{R}^2$ if there exists an effort profile e, transfers t and continuation value vectors $\gamma(y) \in W$ for each $y \in \{\bar{y}, \underline{y}\}$ such that*

(PK) $v_i = (1-\delta)[u_i(e,t) + \delta E\{\gamma(y)|e\}$

(IC) $v_i \geq (1-\delta)[u_i(e_i', e_{-i}, t) + \delta E\{\gamma_i(y)|e_i', e_{-i}\}$ *for any* $e_i' \in \{0,1\}$

The first condition is the promise keeping condition: it guarantees that the current payoff and the expected continuation payoff average to v. The second condition is the standard incentive compatibility condition. Strictly speaking, the definition should also include the conditions stipulating that the transfers are also incentive compatible. That is,

(14) $$\forall y : (1-\delta)t_i(y) + \delta\gamma_i(y) \geq 0$$

The constraint takes this form because the transfers are observable and deviations can be punished by switching to the worst equilibrium with payoffs $(0, 0)$. Notice that as $\delta \to 1$, (14) becomes $\gamma_i(y) \geq 0$. Since we are characterizing the limit case, in what follows, we ignore this constraint keeping in mind that $v, w \geq 0$.

The largest half-space in direction λ whose boundary values can be decomposed on itself by effort profile e and transfers t is $H(\lambda, k^*(\lambda, e, t))$ where $k^*(\lambda, e, t)$ is characterized by the following linear programming problem:

$$(15) \qquad k^*(\lambda, e, t) = \max_x \quad \lambda[u_1(e,t) + E_y\{x_1(y)|e\}] + (1-\lambda)[u_1(e,t) + E_y\{x_2(y)|e\}]$$

subject to

$$u_1(e,t) + E_y\{x_1(y)|e\}] \geq u_i(e_i', e_{-i}, t) + E_y\{x_i(y)|e_i', e_{-i}\}$$
$$\lambda x_1(y) + (1-\lambda)x_2(y) \leq 0$$

This can be seen by noting that if the continuation values $x_i(y)$ are obtained via the normalization $x_i(y) = (\gamma_i(y) - v)\frac{\delta}{1-\delta}$ from unnormalized continuation values $\gamma_i(y)$, the first constraint is equivalent to the condition (IC), the objective function is nothing but $\lambda v_1 + (1-\lambda)v_2$ for some $v = (v_1, v_2)$ for which (PK) is satisfied. Finally, the last constraint guarantees that the unnormalized continuation values γ come from the hyperplane $H(\lambda, k^*(\lambda, e, t))$. Define

$$k^*(\lambda) = \max_{e,t} k^*(\lambda, e, t)$$

Therefore, $H(\lambda, k^*(\lambda))$ is the largest half space in direction λ. Now, define the set

$$W = \bigcap_{\lambda} H(\lambda, k^*(\lambda))$$

The rest of this section is devoted to characterizing the set W for our repeated partnership game.

6.2 Characterizing $k^*(\lambda, EE, t)$

The linear programming problem described in the previous section becomes:

$$\begin{aligned} k^*(\lambda, EE, t) = & \max_{x_1, x_2} \lambda \left(\frac{p\bar{y}}{2} - c + pt_1 + px_1(\bar{y}) + (1-p)x_1(\underline{y})\right) + ... \\ & ... + (1-\lambda)\left(\frac{p\bar{y}}{2} - c + pt_2 + px_2(\bar{y}) + (1-p)x_2(\underline{y})\right) \\ \text{subject to:} \quad & \frac{1}{p-q_2}\left(c - \frac{(p-q_2)\bar{y}}{2}\right) - t_1 \leq x_1(\bar{y}) - x_1(\underline{y}) \\ & \frac{1}{p-q_1}\left(c - \frac{(p-q_1)\bar{y}}{2}\right) - t_2 \leq x_2(\bar{y}) - x_2(\underline{y}) \\ & \lambda x_1(y) + (1-\lambda)x_2(y) \leq 0 \quad \text{for all} \quad y \in Y \end{aligned}$$

Denote the left hand side of the IC constraint for agent i by L_i. That is,

$$L_1 = \frac{1}{p-q_2}\left(c_1 - \frac{(p-q_2)\bar{y}}{2}\right) - t_1.$$

$$L_2 = \frac{1}{p-q_1}\left(c_n - \frac{(p-q_2)\bar{y}}{2}\right) - t_2.$$

To characterize the solution to this problem, it is convenient to distinguish between two separate cases,

$$\lambda L_1 + (1-\lambda)L_2 \leq 0 \tag{16}$$

and

$$\lambda L_1 + (1-\lambda)L_2 > 0. \tag{17}$$

If λ and t satisfy the first inequality then it is possible to choose $x_1(\cdot), x_2(\cdot)$ in such a way that $\lambda x_1(\bar{y}) + (1-\lambda)x_2(\bar{y})$=$\lambda x_1(\underline{y}) + (1-\lambda)x_2(\underline{y}) = 0$ and incentive constraints are satisfied. If (16) holds with strict inequality, at least one of the incentive constraints will be slack. Therefore, in this case

$$k^*(\lambda, EE, t) = \lambda\left(\frac{p\bar{y}}{2} - c + pt_1\right) + (1-\lambda)\left(\frac{p\bar{y}}{2} - c + pt_2\right) \tag{18}$$

Thus (16) implies that orthogonal implementation is possible.

On the other hand, if λ and t are such that (17) holds, then

$$\lambda x_1(\underline{y}) + (1-\lambda)x_2(\underline{y}) < \lambda x_1(\bar{y}) + (1-\lambda)x_2(\bar{y})$$

and it would be optimal to choose a combination of $x_1(\bar{y})$ and $x_2(\bar{y})$ that satisfies $\lambda x_1(\bar{y}) + (1-\lambda)x_2(\bar{y}) = 0$. Obviously, $\lambda x_1(\underline{y}) + (1-\lambda)x_2(\underline{y}) = 0$ is not feasible any more, and in the optimal solution both incentive constraints must bind. Therefore

$$(19) \qquad k^*(\lambda, EE, t) = \lambda\left[\frac{p\bar{y}}{2} - c + pt_1 - (1-p)L_1\right] + (1-\lambda)\left[\frac{p\bar{y}}{2} - c + pt_2 - (1-p)L_2\right].$$

Note that if (16) holds with equality then both incentive constraints must bind and therefore equations (19) and (18) deliver the same value.

The next step is to choose the transfer profile $t^*(\lambda)$ that maximizes the level of the hyperplane in the direction λ when EE is the effort profile. This is done in the following Lemma.

Lemma 2 *Define*

$$(20) \qquad t_1^*(y) = -t_2^* = \begin{cases} \frac{\bar{y}}{2} & \textit{if } \lambda \geq \frac{1}{2} \\ -\frac{\bar{y}}{2} & \textit{if } \lambda < \frac{1}{2} \end{cases}$$

Then $t^* \in \mathbf{argmax}_t k^*(\lambda, EE, t)$.

Proof: The result follows from observing that t^* defined in (2) maximizes (18) and minimizes the left hand side of (16). □

This Lemma implies that for large δ and when v, w is on the frontier, one of the agents receives all of the current output if both agents work. Therefore, $k^*(\lambda, EE)$ can be expressed as follows:

$$(21) \qquad k^*(\lambda, EE) = \begin{cases} \lambda(p\bar{y} - c) + (1-\lambda)(-c) - (1-p)\max\{0, \eta_1(\lambda)\} & \text{if } \lambda > \frac{1}{2} \\ \lambda(-c) + (1-\lambda)(p\bar{y} - c) - (1-p)\max\{0, \eta_2(\lambda)\} & \text{if } \lambda \leq \frac{1}{2} \end{cases}$$

where

$$\eta_1(\lambda) = \lambda\frac{c - (p-q_2)\bar{y}}{p - q_2} + (1-\lambda)\frac{c}{p - q_1}$$

and

$$\eta_2(\lambda) = \lambda\frac{c}{p - q_2} + (1-\lambda)\frac{c - (p-q_1)\bar{y}}{p - q_1}$$

Note that $\eta_1(\lambda)$ and $\eta_2(\lambda)$ are the values of $\lambda L_1 + (1-\lambda)L_2$ evaluated at the corresponding optimal t's. The term $\max\{0, \eta_1(\lambda)\}$ in equation (21) becomes positive when the condition (17) holds: ICs are binding and orthogonal implementation is not possible.

Correspondingly, the hyperplane associated with the optimal transfer schedule for $\lambda \geq \frac{1}{2}$ [the hyperplane $\lambda v + (1-\lambda)w = k^*(\lambda, EE)$] passes through either A_1 or B_1 defined below—namely the one which delivers a lower level in this direction λ.[9]

$$
\begin{aligned}
&A_1: \quad (p\bar{y} - c, -c) \\
&B_1: \quad \left(p\bar{y} - c - \frac{1-p}{p-q_2}(c - (p-q_2)\bar{y}), -c - \frac{1-p}{p-q_1}c\right)
\end{aligned}
\tag{22}
$$

For $\lambda \leq \frac{1}{2}$ the corresponding hyperplane passes through the lower one of the following two points:

$$
\begin{aligned}
&A_2: \quad (-c, p\bar{y} - c) \\
&B_2: \quad \left(-c - \frac{1-p}{p-q_2}c, p\bar{y} - c - \frac{1-p}{p-q_1}(c - (p-q_1)\bar{y})\right)
\end{aligned}
\tag{23}
$$

6.3 Characterizing $k^*(\lambda, ES)$ and $k^*(\lambda, SE)$

For the effort profile ES and transfers t, the linear programming problem (15) can be written as

$$
\begin{aligned}
k^*(\lambda, ES, t) = \mathbf{argmax}_x \quad & \lambda[q_1(\bar{y} + t_1) - c + q_1 x_1(\bar{y}) + (1-q_1)x_1(\underline{y})] + ... \\
& ...(1-\lambda)[q_1(\bar{y} + t_2) + (1-q_1)t_2 + q_n x_2(\bar{y}) + (1-q_n)x_2(\underline{y})]
\end{aligned}
\tag{24}
$$

subject to

$$
\begin{aligned}
x_1(\bar{y}) - x_1(\underline{y}) &\geq \frac{1}{q_1}\left(c - \frac{q_1\bar{y}}{2}\right) - t_1 \\
x_2(\bar{y}) - x_2(\underline{y}) &\leq \frac{1}{p-q_1}\left(c - \frac{(p-q_1)\bar{y}}{2}\right) - t_2 \\
\lambda x_1(y) + (1-\lambda)x_2(y) &\leq 0; \quad y \in \{\underline{y}, \bar{y}\}
\end{aligned}
$$

If there were no incentive constraints, it would always be possible to enforce (ES, t) orthogonally, which would deliver

$$
k^*(\lambda, ES, t) = \lambda[-c + q_1(\bar{y} + t_1)] + (1-\lambda)[q_1(\bar{y} + t_1)]. \tag{25}
$$

[9] By this we mean, the inner product $(\lambda, 1-\lambda) \times (v, w)$ is minimized.

Since the incentive constraints in (24) bound $x_1(\bar{y}) - x_1(\underline{y})$ from below and $x_2(\bar{y}) - x_2(\underline{y})$ from above, it is always possible to choose such x that $\lambda x_1(y) + (1-\lambda)x_2(y) = 0$ for any y and both incentive constraints are satisfied.[10] Thus (25) is also a solution to the linear program (24).

To maximize $k^*(\lambda, ES, t)$ with respect to t we need to set $t_1 = -t_2 = -\frac{\bar{y}}{2}$ if $\lambda < \frac{1}{2}$ and $t_1 = -t_2 = \frac{\bar{y}}{2}$ otherwise. Therefore,

$$k^*(\lambda, ES) = \begin{cases} \lambda[q_1\bar{y} - c] & \text{if } \lambda \geq \frac{1}{2} \\ \lambda(-c) + (1-\lambda)q_1\bar{y} & \text{otherwise} \end{cases}.$$

The level of the largest half-space in direction λ decomposed on itself by SE can be straightforwardly determined in a similar way:

$$k^*(\lambda, SE) = \begin{cases} \lambda q_2\bar{y} + (1-\lambda)(-c) & \text{if } \lambda \geq \frac{1}{2} \\ (1-\lambda)[q_2\bar{y} - c] & \text{otherwise} \end{cases}.$$

6.4 Characterizing $k^*(\lambda)$

For each λ the level of largest half space in the direction λ is found as

$$k^*(\lambda) = \max\{k^*(\lambda, EE), k^*(\lambda, ES), k^*(\lambda, SE)\} \tag{26}$$

It is convenient to first characterize $\max\{k^*(\lambda, ES), k^*(\lambda, SE)\}$ and then compare it with $k^*(\lambda, EE)$.

For $\lambda \geq \frac{1}{2}$ the hyperplane $\lambda v + (1-\lambda)w = k^*(\lambda, ES)$ passes through D_1 and hyperplane $\lambda v + (1-\lambda)w = k^*(\lambda, SE)$ passes through G_1 defined as follows:

$$\begin{aligned} D_1 &: \quad (q_1\bar{y} - c, 0)) \\ G_1 &: \quad (q_2\bar{y}, -c)) \end{aligned} \tag{27}$$

For $\lambda \leq \frac{1}{2}$ the corresponding hyperplanes pass through D_2 and G_2:

$$\begin{aligned} D_2 &: \quad (-c, q_1\bar{y})) \\ G_2 &: \quad (0, q_2\bar{y} - c)) \end{aligned} \tag{28}$$

It is easy to see that for $m > n$ the points G_1, G_2 lie below the line connecting the points D_1

[10] We only need to make sure that $x_1(\bar{y}) - x_1(\underline{y})$ and $x_2(\bar{y}) - x_2(\underline{y})$ are sufficiently far away from each other.

and D_2.[11] Therefore, for $\lambda \geq \frac{1}{2}$, the hyperplane passing through D_1 should be chosen and for $\lambda \leq \frac{1}{2}$ the hyperplane passing through D_2 should be chosen. This is intuitive because it implies that whenever only one of the agents works it is the more efficient one.

In the light of this discussion, we get that whenever $m > n$:

$$k^*(\lambda) = \max\{k^*(\lambda, EE), k^*(\lambda, ES)\}.$$

6.5 Characterization of $\mathcal{W}(\cdot)$

Recall that

$$\mathcal{W} = \bigcap_{\lambda} \{(v, w) | \lambda v + (1 - \lambda) w \leq k^*(\lambda)\}$$

Proposition 4 *Define*

$$\eta = \frac{c - (p - q_2)\bar{y}/2}{p - q_2} + \frac{c - (p - q_1)\bar{y}/2}{p - q_1}$$

Then,

$$\mathcal{W}(v) = -v + p\bar{y} - 2c - \min\{2(1 - p)\eta, (p - q_1)\bar{y} - c\}$$

Proof: First, notice that $\overline{A_1A_2}$, $\overline{B_1B_2}$ and $\overline{D_1D_2}$ all have slopes -1. Also, all points $A_1, A_2, B_1, B_2, D_1, D_2$ lie outside of the positive orthant. Next, observe that $\overline{A_1A_2}$ lies above $\overline{B_1B_2}$ by assumption A3, and by assumption A1, $\overline{A_1A_2}$ lies above $\overline{D_1D_2}$. Finally, observe that $p\bar{y} - 2c - 2\eta$ and $q_1\bar{y} - c$ are the levels of $\overline{B_1B_2}$ and $\overline{D_1D_2}$, respectively. $\Box$

6.6 Proof of Proposition 1

The following proposition reproduces the result of FLM Fudenberg et al. (1994) that $\mathcal{W}$ is the limit of the equilibrium payoff set as δ converges to 1.

Proposition 5 *(FLM Fudenberg et al. (1994)) Let $V \subset intW$ be smooth*[12] *and convex. Then there exists $\bar{\delta}$ such that for any $\delta > \bar{\delta}$, $V \subset W(\delta)$.*

[11] To see this note that the line connecting D_1 and D_2 has slope -1 and level $f(1, 0) - c$ while the lines through G_1 and G_2 with slope -1 have levels $f(0, 1) - c < f(1, 0) - c$.

[12] A smooth set is closed, with non-empty interior and its boundary is twice continuously differentiable.

Proof: We note that the Pareto frontier of the repeated partnership game is obtained using $(t_1, t_2) \in \{(-\bar{y}/2, \bar{y}/2), (\bar{y}/2, -\bar{y}/2)\}$. That is, restricting attention to equilibria that use only these transfers does not shrink the equilibrium set. Therefore, the proof directly follows from Fudenberg et al. (1994).□

Corollary 1 (Proposition 1) *Define*

$$\eta(m,n) = \frac{c - (p(m,n) - q_2(m,n))\bar{y}/2}{p(m,n) - q_2(m,n)} + \frac{c - (p(m,n) - q_1(m,n))\bar{y}/2}{p(m,n) - q_1(m,n)}$$

For any $m \geq n$, *let*

$$\begin{aligned}\mathcal{W}_{mn}(v) = &- v + p(m,n)\bar{y} - 2c \\ &- \min\{\ (1 - p(m,n))\eta(m,n)\ \ ,\ \ (p(m,n) - q_1(m,n))\bar{y} - c\ \}\end{aligned}$$

For any $\varepsilon > 0$ *there exists* $\bar{\delta}_{mn}(\varepsilon) < 1$ *such that for any* $\delta > \bar{\delta}_{mn}(\varepsilon)$ *and for all* v*:*

$$\mathcal{W}^{\delta}_{mn}(v) \in (\mathcal{W}_{mn}(v) - \varepsilon, \mathcal{W}_{mn}(v)]$$

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