Communication with Two-sided Asymmetric Information*

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Abstract

Even though people routinely ask experts for advice, they often have private information as well. I study strategic communication when both the expert and the decision maker have private information. In one-way communication, non-monotone equilibria may arise (i.e., the expert conveys whether the state is extreme or moderate instead of low or high), even if preferences satisfy the single-crossing property. In two-way communication, the decision maker *cannot* credibly reveal her information when communicating first to the expert and hence benefits little from two-way sequential communication. This result provides another explanation for the "bottom up" arrangement of information flow in organizations.

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1 Introduction

Even though people routinely ask experts for advice when making decisions, they often have their own private information as well. For example, homeowners consult real estate agents to decide for what price to sell their houses, but they often do independent research to find out market conditions; congressional representatives hold hearings to gather information on the consequences of certain policies, but they may have experience with similar issues in past legislation; mangagers ask their subordinates to evaluate workers to help with compensation and promotional decisions, but they could have their own assessment of workers from occasional interaction with them.

Because the expert's and the decision maker's interests are typically not perfectly aligned, information transmission is a non-trivial problem. Although many papers in the economics literature have analyzed the problem of strategic information transmission using sender-receiver games,[1] the standard model typically used assumes that only the sender has private information. So it leaves out one crucial aspect common in the examples above – that the decision maker may be privately informed as well.

Interesting questions arise when the DM is privately informed. For example, how does the decision maker's private information affect the expert's incentive to communicate? Does the transmission of information take a qualitatively different form? Can the decision maker elicit more information from the expert by communicating to him first? What are the implications for the arrangement of information flow?

To answer these questions, I introduce a simple model that incorporates two-sided asymmetric information into communication. In my model, both sides – the expert and the decision maker – have private information. In particular, I assume that the expert privately observes the state of the world (t) and the decision maker privately observes a noisy signal (s) of the state. I also assume that when the decision maker observes a high (low) signal, she believes with a higher probability that the state is high (low). (Formally, the random variables t and s are affliated.) The players' conflict of interest in paramerized by the expert's bias (b). Without loss of generality, I assume that the expert has an upward bias ($b > 0$), which implies that the expert always prefers a higher action than the decision maker does.

[1] The classic model of strategic information transmission by Crawford and Sobel (1982) has applications in many areas. Examples include Matthews (1989) and Austen-Smith (1990) in political economy, Stein (1989) and Moscarini (2007) in macroeconomics and Morgan and Stoken (2003) in financial economics.

I start by looking at a simple game (Γ_I) of one-way communication from the expert to the decision maker when the decision maker cannot communicate to the expert and hence keeps her signal private (this happens, for example, when the decision maker's signal arrives only *after* the expert's report). A well-known result in the literature of sender-receiver games is that if the players' preferences satisfy the single-crossing property, then all equilibria are "monotone." That is, higher types of the sender induce higher actions in equilibrium and only types next to one another pool together. Strikingly, some equilibria lose such monotonicity when the receiver is privately informed: it can happen in equilibrium that high and low types pool together but are separated from middle types. So instead of conveying whether the state is low or high through his messages, the expert conveys to the decision maker whether the state is extreme or moderate. To apply this non-monotonicity result to concrete situations of strategic communication, towards the end of section 4, I discuss an example of the relation between a real estate agent (the expert) and a homeowner (the decision maker) and illustrate when and how communication may be non-monotone.

Both the expert's uncertainty over the decision maker's information and the correlation between the two players' signals are essential to generate non-monotone equilibria. Since the decision maker's action depends on both the expert's message and her own private signal, the expert's message induces a *distribution* of actions by the decision maker. In a non-monotone equilibrium, the high and low types send a message that induces a distribution of "extreme" (either very low or very high) actions and the middle types send a message that induces a distribution of moderate actions. The expert's incentive constraints are satisfied for the following reason. The high and the low types have relatively skewed beliefs over the decision maker's signal. So they believe that with sufficiently high probability, the signal realization will be in their favor, i.e., the decision maker will choose a favorable action. Hence they are willing to induce a distribution of extreme actions. The middle types, on the other hand, have more diffuse beliefs and it is in their interest to induce a distribution of moderate actions rather than a distribution of extreme actions.

The simple one-round game is appropriate for analyzing situations in which the decision maker has no way to communicate, but there are settings in which the decision maker has an opportunity to communicate to the expert first, before the expert reports. For example, a manager can discuss a worker's performance with the worker's supervisor before the supervisor submits his evaluations. To study two-way sequential communication like this, I introduce a game (Γ_{II}) in section 5. In this game, after the decision maker privately observes her signal, she sends a cheap-talk message to the expert. After receiving the message and observing the state, the expert reports back to the decision maker, who then chooses an action.

The central question of the analysis of two-way communication is whether the decision maker can strategically exploit the communication opportunity. That is, can talking to the expert first help her elicit more information from the expert? Note that to elicit more information from the expert, the decision maker must reveal some of her information in the first stage. My result finds that under some mild conditions, no equilibrium exists in which the decision maker truthfully reveals her signal in the first stage.

To gain some intuition, let's imagine that the decision maker reveals her signal truthfully in the first stage. Then, the decision maker no longer has any private information in the second stage. In the continuation, the players will play a canonical sender-receiver game *a la* Crawford and Sobel (1982) with appropriately updated beliefs. If the decision maker reveals her signal to be Low (High), the players will play a Crawford-Sobel game with common prior $L(t)$ $(H(t))$. Under the assumption on the information structure, the players' belief on the state following the decision maker's revelation of a High signal is a monotone likelihood ratio (MLR) improvement of the players' belief following the decision maker's revelation of a Low signal.

The decision maker's incentive for truth telling in the first stage depends on the information transmitted by the expert in the second stage. By comparing equilibria in Crawford-Sobel (CS) games under different priors ranked by the MLRP, I find that the decision maker's preference over them is clear. If the decision maker has belief H, then her expected payoff is higher in the most informative CS equilibrium under prior H than under prior L. Under some mild conditions, the decision maker who has belief L also has a higher expected payoff in the most informative equilibrium under prior H than under L. So no matter what the realization of her private signal is, the decision maker would rather the expert believe that her signal is High. This immediately implies that it is impossible for the decision maker to reveal her signal credibly in the first stage.

Although this result does not completely rule out the possibility that the first round of communication can be partially informative,[2] it illustrates that the benefit

[2] In section 5.3, I provide an example that shows that under certain restrictive conditions, the decision maker partially reveals her information in the first stage and gets a higher expected payoff than if she keeps her signal private.

the decision maker gets from communicating to the expert first is limited. One application of this result on two-way communication is the organizational structure within firms. Scharfstein and Stein (1990) argue that "bottom up," rather than "top down" organization of information flow is advantageous because it lessens herding behavior of managers who are motivated by reputational concerns. My result points to another disadvantage of "top down" communication: when different levels of an organization have different objectives, it may be difficult for managers at the top to elicit information from lower levels by passing down their own ideas first. To the extent that communication has cost, a "bottom up" arrangement results in higher efficiency. (For simplicity my model assumes that communication is costless, but it is easy to modify the model by adding a fixed cost of communication and the result continues to hold.)

My result on two-way communication complements the findings in a small but growing literature on multiple-stage communication. One main finding in this literature is that more elaborate communication often help improve information transmission than the one-way, one-shot protocol. For example, Krishna and Morgan (2004) consider a simple two-stage game between an informed expert and an uninformed decision maker. Strikingly, they find that adding only one round of simultaneous cheap talk improves information transmission. Both Matthews and Postlewaite (1995) and Aumann and Hart (2003) consider pre-play communication that can potentially last for infinite rounds. Both papers find equilibrium outcomes with longer cheap talk that improve information transmission than what is achievable by a single message. A recent paper by Golosov, Skreta, Tsyvinski and Wilson (2008) extend the Crawford-Sobel model to a dynamic setting: the expert and the decision maker interact repeatedly – the expert's information does not change over time, but the decision maker chooses an action in each period. They also find that more information is revealed by the expert in the dynamic setting than the static one. In contrast to these papers, my result shows in a natural setting that communcation from the partially-informed decision maker to the expert is often ineffective and at best limited.

As mentioned earlier, only a few papers in the literature have explicitly modeled informed receivers. An early reference is Seidmann (1990), who gives examples to illustrate how the receiver's private information facilitates communication. In Watson (1996), the sender's and the receiver's private information are complementary. In Olszewski (2004), the sender is concerned with his reputation of being honest. Both papers find conditions on the information structure under which a fully revealing equilibrium exists. A recent paper by Lai (2008) looks at communication from an expert to an "amateur." The amateur can tell whether the state is "low" or "high" depending on the true state and a cutoff point that is his private information. Lai (2008) shows that because the expert may become less helpful in providing information, being partially informed does not necessarily benefit the amateur. Although not the focus of this paper, a similar result on the value of the decision maker's information holds in my model as well. I discuss it in Remark 3 in section 4.

2 The Model

There are two players in the game, the expert and the decision maker (DM).[3]

The expert privately observes the state of the world, or his type, t, which is a random variable distributed on the interval $[0,1]$. The common prior on t has distribution function $G(\cdot) \in \mathcal{C}^1$ and density function $g(\cdot)$. The DM privately observes a signal $s \in S = \{s_L, s_H\}$ with $s_H > s_L$. Let the conditional distribution functions of t, $G(t|s = s_H)$ and $G(t|s = s_L)$ be denoted by $H(t)$ and $L(t)$. Suppose they have continuous density functions $g(t|s = s_H)$ and $g(t|s = s_L)$, denoted by $h(t)$ and $l(t)$. Suppose they satisfy the condition that $\frac{h(t)}{l(t)}$ is strictly increasing in t, i.e., the monotone likelihood ratio property (MLRP) holds. (Or, the random variables t and s are affiliated.) Statitiscally, when the DM sees the signal s_H, she believes that t is more likely to be high than when she sees the signal s_L. Assume also that $h(t) > 0$, $l(t) > 0$ for all $t \in [0,1]$, which implies that the support of the DM's belief does not change with the realization of her signal. If $H(t) = L(t)$ for $t \in [0,1]$, we are back to the standard model in which the decision maker is uninformed. (Assuming that the DM's signal has two realizations is only for notational simplicity. The results will go through even if s has more than two realizations, as long as the assumptions of full support and MLRP hold.)

In both games Γ_I and Γ_{II} analyzed in this paper, only the DM takes an action that affects the players' payoffs directly. Both players maximize their expected utilities. The DM's twice continuously differentiable von Neumann-Morgenstern utility function is denoted by $U^{DM}(a,t)$, where $a \in \mathbb{R}$ is the action taken by the DM. The expert's twice continuously differentiable von Neumann-Morgenstern utility function is denoted by $U^E(a,t,b)$. Assume $U^{DM}(a,t) = U^E(a,t,0)$. So b measures the divergence of interests between the players. (For simplicity, when it is clear that b is fixed,

[3] I use the pronoun "he" for the expert and the pronoun "she" for the decision maker.

sometimes I just write $U^E(a,t)$.) Without loss of generality, assume that $b>0$.[4] Also assume that, for each t and for $i=E,DM$, denoting partial derivatives by subscripts in the usual way, $U_1^i(a,t)=0$ for some a, and $U_{11}^i(a,t)<0$, so that U^i has a unique maximum in a for each t. Assume $U^i(a,t)$ is supermodular in (a,t), i.e., $U_{12}^i(a,t)>0$. (This implies that the single crossing property holds.) For each t and $i=E,DM$, $a^i(t)$ denotes the unique solution to $\max_a U^i(a,t)$. Assume $U_{13}^E(a,t,b)>0$. Since $b>0$, this implies that $a^E(t)>a^{DM}(t)$ for all t. So the expert's ideal action is always higher than the DM's. Fix a distribution function F. For $0\le t'<t''\le 1$, let $\bar{a}_F(t',t'')$ be the unique solution to $\max_a \int_{t'}^{t''} U^{DM}(a,t)dF(t)$. So $\bar{a}_F(t',t'')$ is the DM's optimal action when he believes that t has support on $[t',t'']$ with distribution F. By convention, $\bar{a}_F(t,t)=a^{DM}(t)$.[5]

I analyze the following two games.

Γ_I : The expert and the DM privately observe their signals. The expert sends a message to the DM while the DM keeps her signal private. After receiving the expert's message, the DM chooses an action. Call this one-way communication.

Γ_{II} : The expert and the DM privately observe their signals. The DM sends a message to the expert before the expert reports to her. Then the DM chooses an action. Call this two-way communication.

Throughout the analysis, I use m to denote the message that the expert sends to the DM and z to denote the message that the DM sends to the expert in Γ_{II}. Without loss of generality, I assume that the expert's message space is the same as his type space: $M=T=[0,1]$ and the DM's message space is the same as her signal space: $Z=S$. Both m and z are cheap talk.

Because the DM's payoff function is strictly concave in a, she never mixes over actions in equilibrium. I will also restrict attention to pure strategies for the players' communication strategies.[6]

In Γ_I, the expert does not observe s when sending a message to the DM. So the expert's strategy is $m_I : T \to M$. The DM's action depends on both the expert's message and her signal. So the DM's strategy is $a_I : M\times S\to\mathbb{R}$.

In Γ_{II}, the DM's strategy has two parts: communication and action. Let $z_{II} : S\to Z$ denote her communication strategy. The expert sends a message to the DM after observing his type t and receiving the DM's message z. So the expert's strategy

[4] I preclude the degenerate case in which $b=0$, i.e., the two players' interests coincide.

[5] The leading example of the Crawford-Sobel model, the uniform-quadratic case, satisfies these assumptions. In that case, $U^E=-(a-t-b)^2$ and $U^{DM}=-(a-t)^2$.

[6] Similar to Crawford and Sobel (1982), this restriction does not change the results.

is $m_{II} : T \times Z \to M$. The DM's action can depend on her signal s, her message z and the message sent by the DM m. So her action strategy is $a_{II} : S \times Z \times M \to \mathbb{R}$.

The solution concept I use is Perfect Bayesian Equilibrium (PBE).

3 Benchmark: Uninformed Decision Maker

For comparison, let us first review briefly the equilibrium characterization in the Crawford-Sobel game in which the DM is uninformed. The setup is the same as described in section 2, except that the DM does not observe an informative signal.

Suppose the players' common prior is that t has distribution function F and density f. Suppose $m(t)$ is the expert's strategy and $a(m)$ is the DM's strategy in a Perfect Bayesian Equilibrium.

Crawford and Sobel (1982) find that all equilibria take a simple form: an equilibrium is characterized by a partition of the set of types, $\mathbf{t}(N) = (t_0(N), \ldots, t_N(N))$ with $0 = t_0(N) < t_1(N) < \ldots < t_N(N) = 1$, and messages m_i, $i = 1, \ldots, N$. The types in the same partition element send the same message, i.e., $m(t) = m_i$ for $t \in (t_{i-1}, t_i])$. The DM best responds, i.e., $a(m_i) = \bar{a}_F(t_{i-1}, t_i)$. The boundary types are indifferent between pooling with types immediately below or immediately above. So the following "arbitrage" condition holds: for all $i = 1, ..., N-1$,

$$U^E(\bar{a}_F(t_i, t_{i+1}), t_i)) - U^E(\bar{a}_F(t_{i-1}, t_i), t_i)) = 0, \qquad (A)$$

Crawford and Sobel (1982) make a regularity assumption that allows them to derive certain comparative statics. For $t_{i-1} \leq t_i \leq t_{i+1}$, let

$$V(t_{i-1}, t_i, t_{i+1}) \equiv U^E(\bar{a}_F(t_i, t_{i+1}), t_i) - U^E(\bar{a}_F(t_{i-1}, t_i), t_i).$$

A (forward) solution to (A) of length K is a sequence $\{t_0, \ldots, t_K\}$ such that $V(t_{i-1}, t_i, t_{i+1}) = 0$ for $i = 1, .., K-1$.

Definition 1 *The Monotonicity (M) Condition is satisfied if for any two solutions to (A), $\hat{\mathbf{t}}$ and $\tilde{\mathbf{t}}$ with $\hat{t}_0 = \tilde{t}_0$ and $\hat{t}_1 > \tilde{t}_1$, we have $\hat{t}_i > \tilde{t}_i$ for all $i \geq 2$.*

Note that an equilibrium partition of size K satisfies (A) with $t_0(K) = 0$ and $t_K(K) = 1$. Crawford and Sobel prove that if Condition (M) is satisfied, then there is exactly one equilibrium partition for each $N = 1, \ldots, N^*$. The equilibrium with the highest number of steps, N^*, is commonly referred to as the "most informative" equilibrium. Chen, Kartik and Sobel (2008) provide a condition ("No Incentive to

Separate") that selects the equilibrium with N^* steps when condition (M) holds. For the rest of this paper, I assume that (M) holds and focus on the equilibrium with the highest number of steps in a Crawford-Sobel game.

4 One-way Communication (Γ_I): the Decision Maker Keeps Her Signal Private

Suppose the DM privately observes an informative signal and her signal is kept private when the expert reports. This happens, for example, when the DM's private signal arrives *after* the expert reports. Since the DM's action depends on her signal as well as the expert's message, the expert is not certain what action the DM will choose in response to his message. So the expert's message induces a *distribution* of actions by the DM. Since the expert's type t is correlated with the DM's signal s, the expert's belief over the distribution of actions that a particular message induces varies with the expert's own type t.[7]

The correlation gives rise to equilibria that are qualitatively different from equilibria in games where the DM is uninformed. As we have seen in the Crawford-Sobel model, the single-crossing property of the players' payoff functions implies monotonicity in equilibrium outcome: higher types induce higher actions and the set of types that send the same equilibrium message forms an interval. In such an equilibrium, the boundary types are indifferent between the actions induced in the intervals immediately above and immediately below. These indifference conditions are necessary and sufficient for the expert's message strategy to be a best response.

When the DM is privately informed, however, the indifference conditions of the boundary types (now between *distributions* of actions) are no longer sufficient for the message strategy to be a best response. Furthermore, equilibria exist in which the expert of types t_1 and t_2 send the same message but some type $t \in (t_1, t_2)$ sends a different message. I will refer to these equilibria as non-monotone equilibria.

Let's first look at the failure of sufficienty. Take a partition of size K: $(t_0 = 0, t_1, ..., t_{K-1}, t_K = 1)$, $t_{i-1} < t_i$ for $i = 1, ..., K$. Suppose the expert's strategy is $m_I(t) = m_i$ for $t \in (t_{i-1}, t_i]$, $i = 1, ..., K$ and the DM's strategy $a_I(m, s)$ is a best response to $m_I(t)$. That is, $a_I(m_i, s_L) = \bar{a}_L(t_{i-1}, t_i)$ and $a_I(m_i, s_H) = \bar{a}_H(t_{i-1}, t_i)$. Also, suppose the boundary type t_i satisfy the following indifference condition:

[7] In Seidmann (1990), the sender's and the receiver's private signals are independent. So the results derived in this section do not apply in his setting.

$$
\begin{aligned}
& p\left(s_L|t_i\right) U^E\left(\bar{a}_L\left(t_{i-1}, t_i\right), t_i\right)+p\left(s_H|t_i\right) U^E\left(\bar{a}_H\left(t_{i-1}, t_i\right), t_i\right) \\
= \ & p\left(s_L|t_i\right) U^E\left(\bar{a}_L\left(t_i, t_{i+1}\right), t_i\right)+p\left(s_H|t_i\right) U^E\left(\bar{a}_H\left(t_i, t_{i+1}\right), t_i\right).
\end{aligned} \tag{1}
$$

To simplify notation, let $p_L(t) = p(s_L|t)$, $p_H(t) = p(s_H|t)$, $x(t) = U^E(\bar{a}_L(t_i, t_{i+1}), t)$ $-U^E(\bar{a}_L(t_{i-1}, t_i), t)$ and $y(t) = U^E(\bar{a}_H(t_i, t_{i+1}), t) - U^E(\bar{a}_H(t_{i-1}, t_i), t)$. So type t_i's indifference condition is $p_L(t_i)\, x(t_i) + p_H(t_i)\, y(t_i) = 0$.

Proposition 1 *The indifference conditions of the boundary types t_i ($i = 1, ..., K-1$) are not sufficient for the expert's message strategy $m_I(t)$ to be a best response to $a_I(m, s)$.*

All proofs of propositions and lemmas are in the appendix.

Here is some intuition for Proposition 1. To see whether $m_I(t)$ is a best response, let's look at different types' preference over the distribution of actions induced. In particular, let $\Delta U^E(t) = p_L(t)\, x(t) + p_H(t)\, y(t)$. It measures the difference in type t's expected payoff by sending message m_{i+1} and by sending message m_i.

As t increases, there are two distinct contributions to the change in ΔU^E. One is the change in the preference over actions: as t increases, higher actions become more favorable to the expert, and this makes sending m_{i+1} more attractive relative to sending m_i. The other is the change in the expert's belief over the distributions of induced actions: as t increases, the expert believes with higher probability that the DM's private signal is s_H and this makes m_{i+1} less attractive relative to m_i. (This is because the indifference of type t_i implies that $x(t_i) > 0$ and $y(t_i) < 0$. Continuity implies that $x(t) > 0$ and $y(t) < 0$ for t close to t_i.) So $\Delta U^E(t)$ is not necessarily increasing in t. So roughly, if the expert's preference over actions changes little with t but his belief over the DM's signal changes with t dramatically, then the sufficiency of the boundary types' indifference conditions fails.[8]

The indifference condition between the actions induced in adjacent intervals are not necessary for equilibrium in Γ_I either. To illustrate, I construct a non-monotone equilibrium below.

Consider the following strategies. Let $0 < t_1 < t_2 < 1$. The expert's strategy satisfies $m_I(t) = m_1$ if $t \in [0, t_1) \cup (t_2, 1]$ and $m_I(t) = m_2$ if $t \in [t_1, t_2]$ ($m_1 \neq m_2$).

[8] Although not presented in the paper, one can easily construct an example in which the expert's strategy is not a best response although the indifference conditions of the boundary types hold.

The DM's strategy $a_I(m,s)$ satisfies $a_I(m_1,s_F) = \arg\max(\int_0^{t_1} U^E(a,t)\,dF(t) + \int_{t_2}^{1} U^E(a,t)\,dF(t))$ for $F = L,H$ and $a_I(m_2,s_F) = \arg\max \int_{t_1}^{t_2} U^E(a,t)\,dF(t)$ for $F = L,H$.[9] So $a_I(m,s)$ is a best response to the expert's strategy $m_I(t)$.

To simplify notation, let $a_i^F = a_I(m_i,s_F)$ for $i = 1,2$ and $F = L,H$. Also, let $x(t) = \left(U^E\left(a_2^L,t\right) - U^E\left(a_1^L,t\right)\right)$, $y(t) = \left(U^E\left(a_2^H,t\right) - U^E\left(a_1^H,t\right)\right)$ and $\Delta U^E(t) = p_L(t)\,x(t) + p_H(t)\,y(t)$. If type-$t_1$ and type-t_2 experts are indifferent between sending m_1 and m_2, then $\Delta U^E(t_1) = \Delta U^E(t_2) = 0$. If $\Delta U^E(t) < 0$ for $t \in [0,t_1) \cup (t_2,1]$ and $\Delta U^E(t) > 0$ for $t \in (t_1,t_2)$, then $m_I(t)$ is a best response to $a_I(m,s)$. These conditions can be satisfied for certain parameter values. Below is an example.

Example 1 *Suppose the common prior on t is uniform on $[0,1]$ and the conditional probabilities for the DM's signal are $prob(s = s_L|t) = \frac{3}{4} - \frac{1}{2}t$ and $prob(s = s_H|t) = \frac{1}{4} + \frac{1}{2}t$. So $g(t|s_L) = \frac{3}{2} - t$ and $g(t|s_H) = \frac{1}{2} + t$ and the conditional distribution functions are $L(t) = \frac{3}{2}t - \frac{1}{2}t^2$ and $H(t) = \frac{1}{2}t + \frac{1}{2}t^2$. Suppose the players' payoff functions are $U^{DM}(a,t) = -(a-t)^2$ and $U^E(a,t,b) = -(a-t-b)^2$. Let $b = 0.15$.*

Using the indifference conditions $\Delta U^E(t_1) = \Delta U^E(t_2) = 0$, I find that $t_1 = 0.109$, $t_2 = 0.905$. Simple calculation shows that $a_1^L = 0.276, a_1^H = 0.679$, $a_2^L = 0.454$, $a_2^H = 0.56$.

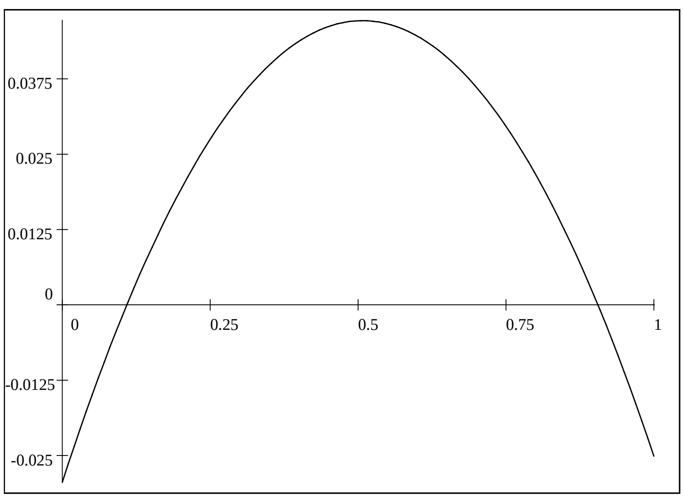

Figure 1: Difference in type t's payoff

To check whether the incentive constraints for every type is satisfied, I plot $\Delta U^E(t) = p_L(t)\,x(t) + p_H(t)\,y(t)$ in figure 1. When $\Delta U^E(t) < 0$, type t gets a higher payoff by sending m_1; when $\Delta U^E(t) > 0$, type t gets a higher payoff by sending m_2.

[9] For $m \neq m_1, m_2$, let $a_I(m,s_F) \in \{a_I(m_1,s_F), a_I(m_2,s_F)\}$, $F = L,H$.

The inverse-U shape of the plot shows that $\Delta U^E(t) < 0$ for $t \in [0, t_1) \cup (t_2, 1]$ and $\Delta U^E(t) > 0$ for $t \in (t_1, t_2)$. So for $t \in [0, t_1) \cup (t_2, 1]$, it is a best response for the expert to send m_1 and for $t \in (t_1, t_2)$, it is a best response for the expert to send m_2.

How are the incentive constraints satisfied in a non-monotone equilibrium? By sending m_1, the expert induces a distribution over actions a_1^L and a_1^H; by sending m_2, the expert induces a distribution over actions a_2^L and a_2^H. As the example above shows, $a_1^L < a_2^L < a_2^H < a_1^H$. So message m_1 induces actions that are "extreme" – either low or high depending on the realization of the DM's signal. Message m_2, on the other hand, induces intermediate actions. For a low-type expert, a_1^L is the best and a_1^H is the worst among the actions that she can possibly induce the DM to choose. If the expert believes with sufficiently high probability that the DM's signal realization is s_L (this happens when $t < t_1$), sending m_1 (and inducing a_1^L with sufficiently high probability) is better than sending m_2 and inducing the intermediate actions. Conversely, for a high-type expert, the action a_1^H is the best and the action a_1^L is the worst among the actions that she can possibly induce the DM to choose. If the expert believes with sufficiently high probability that the DM's signal realization is s_H (this happens when $t > t_2$), sending m_1 is better than sending m_2. For a middle-type expert ($t_1 < t < t_2$), his belief of the DM's signal distribution is more diffuse. Because of the concavity of his payoff function, inducing a distribution of intermediate actions is better than inducing a distribution of extreme actions.

The non-monotonicity in the expert's reporting strategy has applications in real life communication. For instance, homeowners often hire real estate agents when selling houses because the agents have superior information on the local market conditions and hence the value of the house. Because the agent's commission is only a fraction of the selling price, however, he has an incentive to persuade the owner to sell too cheaply and too quickly than is ideal for the owner herself (see Levitt and Syverson (2008)). So one can model the communication between the agent and the owner as a sender-receiver game with partially aligned incentives. If the homeowner has no private information on how much the house could sell for, then communication from the agent to the homeowner must be monotone in equilibrium. For example, the agent suggests "accept" if an offer is above a cutoff (which depends on the agent's private information) and "reject" if it is below the cutoff. If the owner has private information, however, it may emerge in equilibrium that the agent follows a non-monotone strategy. For example, if an offer is especially low or high (below a low cutoff or above a high cutoff with the cutoffs depending on the agent's private information), then the agent may say "it is up to you," (or choose to be "silent,"[10]) expecting the owner to use her own information to make the right choice. If an offer is in the intermediate range (between the cutoffs), then the agent tries to persuade the owner to sell, and the owner decides to accept, or reject, or make a counter-offer, using the information conveyed by the agent (that the offer is neither especially low or high) and her own private information on the value of the house.

Remark 1 *There is an interesting link between the non-monotone equilibrium found in* Γ_I *and the counter-signaling equilibrium in Feltovich, Harbaugh and To (2002). They look at a costly signaling model in which the receiver has private and noisy information on the sender's type and find that "counter-signaling" equilibria emerge: the medium types acquire costly signals to separate from the low types, but the high types, like the low type, choose not to signal (or counter-signal).*

Remark 2 *Proposition 1 points out the failure of the indifference conditions to guarantee best response. It does not, however, imply that monotone equilibrium does not exist when the DM is privately informed. Indeed, in Example 1, there exists a monotone equilibrium with the partition* $(0, 0.183, 1)$.[11]

Remark 3 *Although the DM directly benefits from having an informative signal, the welfare implication is ambiguous in a strategic setting. It is straightforward to show that because the information transmitted from the expert to the DM can be less valuable when the DM is known to be privately informed, the DM may be worse off overall.*

[10] As we know, silence can speak volumes in equilibrium. It is especially likely that the expert uses silence in equilibrium when communication is not free of cost. For simplicity, my model assumes that communication is costless, but it is easy to incorporate a fixed cost of communication and it does not change the results.

[11] Like other cheap-talk games, Γ_I has multiple equilibria. One selection criterion is Farrell's (1993) "neologism-proofness." A well-known problem with this criterion is that it may result in non-existence. In fact, it is straightforward to show that neither the non-monotone equilibrium found in Example 1 or the monotone equilibrium with the partition $(0, 0.183, 1)$ is "neologism-proof."

One can also adapt the "no incentive to separate" condition (Chen, Kartik and Sobel (2008)) to Γ_I. The condition requires that the type-0 expert's equilibrium payoff is at least as high as the payoff he would get if the DM knew that he was type 0 and responded optimally. It is easy to verify that the non-monotone equilibrium found in Example 1 violates the condition, while the monotone equilibrium with the partition $(0, 0.183, 1)$ satisfies it. However, in general, the "no incentive to separate" condition does not necessarily rule out a non-monotone equilibrium in Γ_I.

For instance, in Example 1, if the DM is uninformed, then the corresponding Crawford-Sobel game has an informative equilibrium with partition $(0, 0.2, 1)$ *and the DM has a higher expected payoff in this equilibrium than in either the monotone or the non-monotone equilibrium found when the DM is informed. This implies that if the DM can choose to acquire a private signal before the expert reports and if this information acquisition decision cannot be covert, then the DM may optimally choose to be "ignorant."*

5 Two-way Sequential Communication (Γ_{II})

Is it possible for the DM to exploit her private information strategically? Can she extract more information from the expert by communicating to him first? In this section, I enrich the communication environment by allowing communication to go in both direction.

In Γ_{II}, after the DM privately observes s, she sends a message z to the expert. After receiving z and privately observing t, the expert sends a message m to the DM, who then chooses an action a. Both z and m are cheap-talk messages.

Of course, an equilibrium exists in which the DM babbles in the first stage and in effect keeps her signal private. If the DM were to extract more information from the expert than if she keeps s private, she must reveal some of her information through her messages. The main question is whether she can do so credibly in equilibrium.

To answer this question, we'd like to first see whether an equilibrium exists in which the DM truthfully reveals her signal to the expert in the first stage. Suppose such an equilibrium exists. Then, after the first round of communication, the DM no longer has any private information. In the continuation, the players will play a Crawford-Sobel game with appropriately updated beliefs. So it is useful to first study the comparative statics of the Crawford-Sobel equilibria with respect to the players' prior.

5.1 Comparative Statics of the Crawford-Sobel Equilibria w.r.t. the Prior

If the DM reveals that $s = s_L (s = s_H)$, the players play a CS game with common prior $L(t)$ $(H(t))$ in the continuation. Recall that $H(t)$ is a monotone likelihood ratio (MLR) improvement of $L(t)$. The following lemma is a standard result in monotone comparative statics under uncertainty. (See, for example, Ormiston and Schlee (1993).)

Lemma 1 $\bar{a}_H(t', t'') > \bar{a}_L(t', t'')$, $\forall\ 0 \leq t' < t'' \leq 1$.

This lemma says that if the DM believes that $t \in (t', t'')$, then her optimal action under belief H is higher than her optimal action under belief L.

Let $\mathbf{t}^F(K) = (t_i^F(K))_{i=0,...,K}$ with $t_i^F(K) < t_{i+1}^F(K)$ for $i = 0, ..., K-1$ be a partial partition of size K satisfying the "arbitrage" condition (A) (page 8) when the players' prior over t is F. I will sometimes use the notation $\mathbf{t} = (t_i)$ when it is clear what the players' prior is and what the size of the partition is.

Lemma 2 *If* $t_0^H(K) = t_0^L(K)$ *and* $t_K^H(K) = t_K^L(K)$, *then* $t_i^H(K) > t_i^L(K)$ *for* $i = 1, 2, ..., K-1$.

Lemma 2 applies to all (partial) partitions that have the same endpoints. If $t_0^L = t_0^H = 0$ and $t_K^L = t_K^H = 1$, then $\mathbf{t}^L(K)$ is an equilibrium partition of size K under prior L and $\mathbf{t}^H(K)$ is an equilibrium partition of size K under prior H. So Lemma 2 implies that for a fixed equilibrium size, the boundary types in the equilibrium partition under prior H are to the right of those under L, pointwise.

To gain some intuition, let's look at the simple case of an equilibrium partition of size two. Suppose $(0, t_1^L, 1)$ is an equilibrium partition under prior L, and $\bar{a}_L\left(0, t_1^L\right)$ and $\bar{a}_L\left(t_1^L, 1\right)$ are the DM's best responses. The expert of type t_1^L is indifferent between $\bar{a}_L\left(0, t_1^L\right)$ and $\bar{a}_L\left(t_1^L, 1\right)$ where $\bar{a}_L\left(0, t_1^L\right)$ is lower than his ideal point and $\bar{a}_L\left(0, t_1^L\right)$ is higher than his ideal point. If we keep the partition but change her belief to H, then, by Lemma 1, the DM's best responses will shift to the right. That is, $\bar{a}_H\left(0, t_1^L\right) > \bar{a}_L\left(0, t_1^L\right)$ and $\bar{a}_H\left(t_1^L, 1\right) > \bar{a}_L\left(t_1^L, 1\right)$. Since his payoff function is single peaked in a, the expert of type t_1^L strictly prefers $\bar{a}_H\left(0, t_1^L\right)$ to $\bar{a}_H\left(t_1^L, 1\right)$. So t_1^L cannot be an equilibrium boundary type under H. The regularity condition (M) implies that the equilibrium boundary type under H must be to the right of t_1^L. Induction on equilibrium size shows that the result holds for partitions of larger sizes as well.

Let $N^*(F)$ be the maximum number of steps in an equilibrium when the players' prior on t is F. Combined with the condition (M), Lemma 2 also implies the following.

Corollary 1 $N^*(H) \geq N^*(L)$.

Corollary 1 says that the most informative equilibrium under prior H has a weakly higher number of steps than the most informative equilibrium under prior L.

Remark 4 *It is instructive to compare this section's comparative statics result with respect to the players' prior and Crawford and Sobel's (1982) comparative statics result with respect to the players' preferences. Crawford and Sobel find that for equilibrium partitions of the same size, the partition associated with the players' preferences closer together (i.e., smaller b) begins with larger steps (Lemma 6) and that the maximum possible equilibrium size is nonincreasing in b (Lemma 5). So the two sets of comparative static results are parallel to each other. The following discusses how they are related.*

Take an equilibrium partition of size K under prior F and bias b. If we fix F but lower b, the DM's optimal actions associated with the steps in the original equilibrium partition remain the same but the expert's preference changes. The indifference conditions of the boundary types no longer hold because with a lower b, a boundary type now strictly prefers the action associated with the step immediately below to the action associated with the step immediately above. Under condition (M), in the new equilibrium partition the boundary types must all shift to the right.

Alternatively, if we fix b but change F with an MLR improvement, the expert's preference remains the same but the DM's optimal actions change. With the MLR improvement of her belief, the DM's optimal actions associated with the steps in the original equilibrium partition all shift to the right. The indifference conditions for the boundary types no longer hold because a boundary type now prefers the action associated with the step immediately below to the action associated with the step immediately above. An analyous change in the equilibrium partition follows. That is, all boundary types shift to the right in the new equilibrium partition.

Next, I use the comparative statics results to show that under mild conditions, the DM cannot truthfully reveal s through cheap talk.

5.2 The Decision Maker Cannot Reveal Her Signal Truthfully in Equilibrium

Suppose the DM reports s truthfully, i.e., $z_{II}(s_L) = z_1$ and $z_{II}(s_H) = z_2$ with $z_1 \neq z_2$. Then, following the message z_1 (z_2), the expert believes that the DM's belief on t is $L(t)$ ($H(t)$). Whether the DM has an incentive to deviate from $z_{II}(\cdot)$ depends on her preference over the CS equilibrium paritions associated with the priors $L(t)$ and $H(t)$.

Fix the DM's belief F. Take a partial partition of size K, $\mathbf{t} = (t_i)_{i=0,\dots,K}$. The DM's expected payoff on $[t_0, t_K]$ when she faces the partition $(t_i)_{i=0,\dots,K}$ is given by $EU^{DM} = \sum_{i=1}^{K} \int_{t_{i-1}}^{t_i} U^{DM}(\bar{a}_F(t_{i-1}, t_i), t)\, dF(t)$.

The following lemma will be useful. Fix the end points t_0 and t_K. Let $(t_i(x))_{i=0,\dots,K}$ be a partition that satisfies (A) for $i = 2, ..., K$ with $t_{K-1}(x) = x$. So the partition satisfies (A) except for (possibly) $i = 1$. We want to look at the DM's expected payoff on $[t_0, t_K]$ when she faces the partition $(t_i(x))_{i=0,\dots,K}$ as x moves to the right.

Let y be the type that satisfies $t_1(y) = t_0$. So the first step of the partition $(t_i(y))_{i=0,\dots,K}$ is degenerate: the partition has size $(K-1)$. Let y' be the type such that the partition $(t_i(y))_{i=0,\dots,K}$ satisfies (A) for $i = 1$ as well as $i = 2, ..., K$. (That is, $U^{DM}(\bar{a}_F(t_0, t_1(y')), t_1(y')) = U^{DM}(\bar{a}_F(t_1(y'), t_2(y')), t_1(y'))$). Note that (M) implies that for $x \in (y, y')$, $U^{DM}(\bar{a}_F(t_0, t_1(x)), t_1(x)) > U^{DM}(\bar{a}_F(t_1(x), t_2(x)), t_1(x))$.

Lemma 3 *For $x \in [y, y']$, the DM's expected payoff on $[t_0, t_K]$ when she faces the partition $(t_i(x))_{i=0,\dots,K}$ is increasing in x.*

Lemma 3 has important implications for the DM's preference over different partitions of T. As we will see in Lemma 4 and Lemma 5 below, if we fix the payoff functions and the prior and start with an equilibrium partition, then the DM *would not* prefer another partition with the boundary types shifted to the left. Moreover, the DM *would* prefer another partition with the boundary types shifted to the right, at least locally.

Here is some intuition. Recall that for each equilibrium boundary type, the expert is indifferent between the actions induced in the steps immediately below and immediately above. Since the DM prefers a lower action than the expert does, the DM must prefer the action induced in the lower step to the action induced in the higher step. So, roughly speaking, if the boundary types are shifted to the left, the partition becomes even more skewed to the left, making the DM worse off. When the boundary types are shifted locally to the right, the partition becomes more "balanced," making the DM better off.

From Lemma 2, we know that the boundary types of the equilibrium partition $t^L(K)$ are to the left of the boundary types of the equilibrium partition $t^H(K)$. Hence the preference of the DM with signal s_H (and hence belief H) follows.

Lemma 4 *For a fixed number of steps K, the DM with the belief H strictly prefers the equilibrium partition $\mathbf{t}^H(K)$ to the equilibrium partition $\mathbf{t}^L(K)$.*

The most informative equilibria under L and under H may have different sizes. We have seen in Corollary 1 that $N^*(H) \geq N^*(L)$. Theorem 3 in Crawford and Sobel (1982) shows that when the payoff functions and the prior are fixed, the DM prefers an equilibrium with a higher number of steps. Let $\mathbf{t}^L(N^*(L))$ be the most informative equilibrium partition under L and $\mathbf{t}^H(N^*(H))$ be the most informative equilibrium partition under H. We have the following proposition.

Proposition 2 *The DM with the belief H strictly prefers $\mathbf{t}^H(N^*(H))$ to $\mathbf{t}^L(N^*(L))$.*

Clearly, the DM who has observed $s = s_H$ would not want the expert to believe that she has observed $s = s_L$. What about the DM with signal s_L? Does she prefer the equilibrium partition under H as well? I have already argued that the DM benefits when the boundary types in an equilibrium partition shift to the right locally. As long as the the boundary types are not shifted "too far" to the right, the DM is better off. Lemma 5 and Proposition 3 below make it precise what "too far" means. Basically, as long as the DM still prefers the action induced in the lower step to the action induced in the higher step, she benefits from a shift of the boundary types to the right.

Lemma 5 *Fix the DM's prior F. Take two partial partitions of the same size K, $\mathbf{t} = (t_i)_{i=1,...,K}$ and $\hat{\mathbf{t}} = (\hat{t}_i)_{i=1,...,K}$. Suppose $t_0 = \hat{t}_0$, $t_K = \hat{t}_K$ and $\hat{t}_i > t_i$ for all $i = 1, ..., K-1$. If $U^{DM}(\bar{a}_F(t_{i-1}, t_i), t_i) \geq U^{DM}(\bar{a}_F(t_i, t_{i+1}), t)$ and $U^{DM}\left(\bar{a}_F\left(\hat{t}_{i-1}, \hat{t}_i\right), \hat{t}_i\right) \geq U^{DM}\left(\bar{a}_F\left(\hat{t}_i, \hat{t}_{i+1}\right), \hat{t}_i\right)$, then the DM strictly prefers the partition $\hat{\mathbf{t}}$ to $\mathbf{t}$.*

Equilibrium condition implies that $U^{DM}\left(\bar{a}_L\left(t_{i-1}^L, t_i^L\right), t_i^L\right) \geq U^{DM}\left(\bar{a}_L\left(t_i^L, t_{i+1}^L\right), t_i^L\right)$ always holds. Since the boundary types under H are to the right of the boundary types under L, Lemma 5 immediately implies that the DM with belief L prefers the equilibrium under H to the equilibrium under L with the same size, as long as $U^{DM}\left(\bar{a}_L\left(t_{i-1}^H, t_i^H\right), t_i^H\right) \geq U^{DM}\left(\bar{a}_L\left(t_i^H, t_{i+1}^H\right), t_i^H\right)$.

This result can be generalized even if the most informative equilibrium under H has more steps than the most informative equilibrium under L, i.e., if $N^*(H) > N^*(L)$.

Proposition 3 *If $U^{DM}\left(\bar{a}_L\left(t_{i-1}^H, t_i^H\right), t_i^H\right) \geq U^{DM}\left(\bar{a}_L\left(t_i^H, t_{i+1}^H\right), t_i^H\right)$ for $i = 1, ..., N^*(H)$, then the DM with belief L strictly prefers the equilibrium partition $\mathbf{t}^H(N^*(H))$ to the equilibrium partition $\mathbf{t}^L(N^*(L))$.*

Proposition 3 gives sufficient conditions under which the DM with belief L prefers the most informative equilibrium partition under H to the most informative equilibrium partition under L. Under these conditions, the DM with s_L has an incentive to deviate from reporting s truthfully.[12] So the main result regarding the DM's (failure of) communication follows.

Proposition 4 *(The DM cannot truthfully reveal her signal) If the conditions in Proposition 3 are met, no equilibrium exists in Γ_{II} such that the DM reveals s truthfully to the expert.*

Remark 5 *Crawford and Sobel (1982) have a related result on DM's preference over equilibrium partitions. Their Theorem 4 says that for a given size, the DM prefers the equilibrium associated with more similar preferences (i.e., a smaller b).*

Again, it is instructive to compare their result with mine. As we know, when b gets smaller, the boundary types shift to the right. This shift is never "too far" to the right to benefit the DM. That is, the conditions on the DM's payoffs given in Proposition 3 are always satisfied. To see this, note that the indifference condition of a boundary type t_i requires that $U^E\left(\bar{a}\left(t_{i-1},t_i\right),t_i,b\right)=U^E\left(\bar{a}\left(t_i,t_{i+1}\right),t_i,b\right)$. Since $U^{DM}\left(a,t\right)=U^E\left(a,t,0\right)$ and $U^E_{13}>0$, it follows that $U^{DM}\left(\bar{a}\left(t_{i-1},t_i\right),t_i\right)>U^{DM}\left(\bar{a}\left(t_i,t_{i+1}\right),t_i\right)$ for any $b>0$.

Remark 6 *I have assumed in this paper that the DM has private information on the state of the world. An alternative assumption is that the DM has private information on her preference. In particular, suppose the DM has private informtion on the divergence of interest between the two players, the parameter b.[13] A similar result holds in this setting: since the DM prefers the most informative equilibrium associated with a lower b, the DM with a high b has an incentive to lie. Intuitively, the DM*

[12] One may wonder what happens if the DM can make verifiable reports of her signal. Does Proposition 3 imply that the DM's information will be fully revealed through "unravelling," a la Milgrom and Roberts (1986)? The answer is not necessarily so. This is because sometimes both types of the DM may benefit from the expert's uncertainty over her signal. In particular, under certain parameter values, the only CS equilibrium is babbling even if the players have common prior H, but an informative non-monotone equilibrium exists when the expert is uncertain about what the DM's signal is. In this case, even if the DM can verifiably report her signal, an equilibrium exists in which the DM is "silent."

[13] Althought the original CS model specifies that b enters the expert's payoff function, one can change the assumption so that b enters the DM's payoff function instead. This change affects no result.

wants to convince the expert that their interests are closely aligned so that the expert would reveal more information subsequently, but this incentive prevents the DM from communicating truthfully.

With this alternative assumption that the DM has private information on her preference, it is plausible that her signal is independent of the expert's signal. Without correlation between the two players' signals, the non-monotone equilibrium such as the one constructed in section 4 fails to exist.

Below, I provide an example that illustrates that the DM prefers the equilibrium partition under H no matter what her signal realization is. The example also shows that the sufficient conditions given in Lemma 5 are not tight. In the second set of parameter configurations, the sufficient conditions in Lemma 5 are violated, but the DM with belief L still prefers the equilibrium partition under H.

Example 2 *Suppose the common prior on t is uniform on $[0,1]$ and the conditional probabilities for the DM's signal are prob $(s = s_L|t) = \frac{3}{4} - \frac{1}{2}t$ and prob $(s = s_H|t) = \frac{1}{4}+\frac{1}{2}t$. So $g(t|s_L) = \frac{3}{2}-t$ and $g(t|s_H) = \frac{1}{2}+t$ and $L(t) = \frac{3}{2}t-\frac{1}{2}t^2$ and $H(t) = \frac{1}{2}t+\frac{1}{2}t^2$. Suppose the players' payoff functions are $U^{DM}(a,t) = -(a-t)^2$ and $U^E(a,t,b) = -(a-t-b)^2$. Let $b = 0.15$.[14] (These assumptions are the same as in Example 1.)*

The most informative equilibria under $L(t)$ and $H(t)$ both have size two. The equilibrium partition under L is $\mathbf{t}^L = (0, 0.132, 1)$ and that under H is $\mathbf{t}^H = (0, 0.25, 1)$.

Proposition 4 says that the DM with s_H prefers $\mathbf{t}^H$ to $\mathbf{t}^L$. Indeed, straightforward calculation shows that for the DM with s_H, her expected payoff when facing $\mathbf{t}^L$ is -0.055 whereas her expected payoff when facing $\mathbf{t}^H$ is -0.039.

For the DM with s_L, one need to compare $\bar{a}_L\left(0, t_1^H\right)$ and $\bar{a}_L\left(t_1^H, 1\right)$ to apply Proposition 3. Since $\bar{a}_L\left(0, t_1^H\right) = \frac{\int_0^{t_1^H} x\left(\frac{3}{2}-x\right)dx}{\frac{3}{2}t_1^H-\frac{1}{2}\left(t_1^H\right)^2} = 0.121$ and $\bar{a}_L\left(t_1^H, 1\right) = \frac{\int_{t_1^H}^1 x\left(\frac{3}{2}-x\right)dx}{1-\frac{3}{2}t_1^H+\frac{1}{2}\left(t_1^H\right)^2} = 0.571$, $U^{DM}\left(\bar{a}_L\left(0, t_1^H\right), t_1^H\right) > U^{DM}\left(\bar{a}_L\left(t_1^H, 1\right), t_1^H\right)$. According to Proposition 3 , the DM with signal $s = s_L$ also prefers the partition under H to the partition under L. Straightforward calculation shows that for the DM with s_L, her expected payoff when facing $\mathbf{t}^L$ is -0.0475 whereas her expected payoff when facing $\mathbf{t}^H$ is -0.0307.

Now suppose the conditional probabilities are different: prob $(s = s_L|t) = 1 - t^4$ and prob $(s = s_H|t) = t^4$. Then $g(t|s_L) = \frac{5}{4}(1-t^4)$ and $g(t|s_H) = 5t^4$. The MLRP is satisfied. The conditional distributions are $L(t) = \frac{5}{4}t - \frac{t^5}{4}$ and $H(t) = t^5$. I keep

[14] One can verify that condition (M) is satisfied under the assumptions on the payoff functions and probability distributions.

the assumptions on the prior on t and the players' payoff functions but assume that $b = 0.1$.

Under the new information structure, the most informative equilibrium under L has size two: $\mathbf{t}^L = (0, 0.233, 1)$; the most informative equilibrium under H has size three: $\mathbf{t}^H = (0, 0.235, 0.565, 1)$. Again, Proposition 4 says that the DM with s_H prefers $\mathbf{t}^H$ to $\mathbf{t}^L$. As to the DM with s_L, it is easy to show that $\bar{a}_L\left(0, t_1^H\right) = 0.117$, $\bar{a}_L\left(t_1^H, t_2^H\right) = 0.397$ and $\bar{a}_L\left(t_2^H, 1\right) = 0.727$. So $U^{DM}\left(\bar{a}_L\left(t_1^H, t_2^H\right), t_1^H\right) < U^{DM}\left(\bar{a}_L\left(t_2^H, 1\right), t_2^H\right)$: the condition in Proposition 3 fails.

But the condition is sufficient, not necessary. I find that the DM with s_L has an expected payoff of (-0.0285) when the partition is $\mathbf{t}^L$ whereas she has an expected payoff of (-0.0084) when the partition is $\mathbf{t}^H$. Again, independent of her signal realization, the DM prefers the most informative equilibrium partition under H to the most informative equilibrium partition under L.

So, in both examples, no equilibrium exists in Γ_{II} that the DM reveals s truthfully to the expert through cheap talk.

The result on two-way communication may shed light on the organizational structure within firms, in particular, why a "bottom up", rather than "top down" arrangement may help an organization use information more efficiently.[15]

Scharfstein and Stein (1990) introduce a model in which managers care about their reputation of having informative signals. Because reputational concerns may lead to herding and thus information loss, Scharfstein and Stein argue that it helps a firm make better decisions to have those with stronger reputational concerns (usually younger memebers whose abilities are more uncertain) speak first. Hence a "bottom up" arrangement helps aggregate information. The result in this section suggests another explanation for the "bottom up" information flow within organizations: when different levels of an organization have different objectives, it may be difficult for decision makers at the top to elicit information from lower levels by communicating to their subordinates first. Since communication is usually not cost free, a "bottom up" arrangement results in higher efficiency.

[15] Another application of two-sided asymmetric information in organizations is by Harris and Raviv (2005). They assume that both a CEO and a division manager have private information regarding the profit maximizing investment level. They focus on the question of when the CEO prefers delegating the decision to making the decison himself with the division manager's report. They find that delegation is more likely when the division manager's information is more important relative to the CEO's.

5.3 Discussion: Partial revelation of the Decision Maker's Signal in Equilibrium

We have seen that under mild conditions, the DM cannot reveal s credibly in equilibrium. What about partial revelation? One can show, using essentially the same argument as in section 5.2, that if the expert follows a monotone strategy in the second round of communication, then it is impossible for the DM to even partially reveal her information in the first round.[16]

But, as we have seen in section 4, when the expert is uncertain what the DM's signal realization is, he may follow a non-monotone strategy in equilibrium and this may provide sufficient incentive for the DM to partially separate in the first round. In the example below, the DM partially reveals her information in the first round and the expert responds to a certain message by playing a non-monotone strategy in the second round.

Example 3 *Suppose the common prior on t is uniform on $[0,1]$ and the DM's signal s has three potential realizations: s_L, s_M, s_H with $s_L < s_M < s_H$. Assume the conditional probabilities are prob $(s = s_L|t) = \frac{2(0.55-0.1t)}{3}$ and prob $(s = s_H|t) = \frac{2(0.45+0.1t)}{3}$ and prob $(s = s_M|t) = \frac{1}{3}$. The conditional distribution functions are $L(t) = 1.1t - t^2$, $H(t) = 0.9t + 0.1t^2$ and $M(t) = t$ (i.e., observing s_M does not change the DM's prior on t). Suppose the players' payoff functions are $U^{DM}(a,t) = -(a-t)^2$ and $U^E(a,t,b) = -(a-t-b)^2$. Let $b = 0.2499$.*

Suppose in Γ_{II}, the DM plays the following reporting strategy $z_{II}(s_M) = z_1$ and $z_{II}(s_L) = z_{II}(s_H) = z_2$, ($z_1 \neq z_2$).

Then, after receiving z_1, the expert infers that $s = s_M$ and the players play a CS game with prior $M(t)$ subsequently. The most informative equilibrium in this CS game has a size-two partition: $(0, 0.000\,2, 1)$.[17]

[16] If a particular message induces the expert to believe that the DM has observed a high signal with a higher probability, then the DM has an incentive to always send this message, independent of her signal realization.

[17] The reader may notice that if $b \geq 0.25$, there exists no informative CS equilibrium under the uniform prior. In this example, $b(= 0.2499)$ is close to the threshold, and the partition $(0, 2 \times 10^{-4}, 1)$ is "not very informative." It is worth pointing out that the choice of b is deliberate. In fact, if b is a little lower, say $b = 0.249$, then the size-two partition under M is $(0, 2 \times 10^{-3}, 1)$ and the DM with either s_L or s_H prefers this partition to the non-monotone partition induced by z_2, violating equilibrium condition. This suggests that although one can find paramter values under which the first round of communication can be partially informative, they are highly limited.

After receiving z_2, the type-t expert infers that $s = s_L$ with probability $(0.55 - 0.1t)$ and $s = s_H$ with probability $(0.45 + 0.1t)$. There exists a non-monotone equilibrium in the continuation game. In this equilibrium, $m_{II}(z_2, t) = m_1$ if $t \in [0, 0.03604) \cup (0.9642, 1]$ and $m_{II}(z_2, t) = m_2$ if $t \in [0.03604, 0.9642]$ ($m_1 \neq m_2$) and $a_{II}(s_L, z_2, m_1) = 0.452$, $a_{II}(s_H, z_2, m_1) = 0.545$, $a_{II}(s_L, z_2, m_2) = 0.486$, $a_{II}(s_H, z_2, m_2) = 0.514$.

Next, I show that the DM has no incentive to deviate from her communication strategy in the first round. Imagine that the DM with s_M deviates, sends z_2 and induces the nonmonotone partition corresponding to $m_{II}(z_2, t)$. With prior $M(t)$, if the DM believes that $t \in [0, 0.03604) \cup (0.9642, 1]$, her optimal action is $0.499\,09$ and if the DM believes that $t \in [0.03604, 0.9642]$, her optimal action is $0.500\,12$. Note that with prior $M(t)$ and no additional information from the expert, the DM's optimal action is 0.5. So, to the DM with s_M, the value of information contained in the non-monotone partition is very low. He has a higher expected payoff by sending z_1 and inducing the monotone partition $(0, 0.0002, 1)$.

As to the DM with s_L or s_H, because these types have a more skewed belief than the DM with s_M, the information contained in the non-monotone partition is more valuable to them. Both types have higher expected payoff when facing the non-monotone partition than when facing the monotone partition $(0, 0.0002, 1)$. So the DM with s_L or s_H has no incentive to deviate either.

6 Conclusion

How information is transmitted from experts (information gatherers) to decision makers is a central question in both organizations and markets. While existing literature has focused on how the players' preferences affect the incentives of the expert and outcomes of communication, in this paper, I explore the implications for information transmission when the decision maker, as well as the expert, is privately informed.

I find that when the expert is uncertain about what the DM privately knows, information may be transmitted through communication in an interesting way, distinct from how information is transmitted when the DM has no private information. Instead of conveying whether the state is low or high, the expert may convey whether the state is extreme or moderate in equilibrium when the DM has private information.

By analyzing a simple game of two way communication in which the DM communicates to the expert first before the expert reports, I find that it is impossible for the DM to credibly reveal his information to the expert through cheap talk. This implies that the DM benefits little from two-way sequential communication. In the context of arrangement of information flow within organizations, this result on the "futility" of "top down" communication suggests that a "bottom up" arrangment may be more advantageous.

Appendix

Proof of Proposition 1

For the indifference condition 1 to hold, $x(t_i)$ and $y(t_i)$ must have different signs. In fact, it must be the case that $x(t_i) > 0$ and $y(t_i) < 0$. To see this, first note that $H(t)$ is a monotone likelihood ratio (MLR) improvement of $L(t)$. Lemma 1 says that $\bar{a}_H(t', t'') > \bar{a}_L(t', t'')$, $\forall\ 0 \leq t' < t'' \leq 1$.

Since $U^E(a,t)$ is single-peaked in a and $\bar{a}_F(t_{i-1}, t_i) < a^E(t_i)$ for $F = H, L$, we must have $x(t_i) > 0$ and $y(t_i) < 0$. This can be shown by contradiction. Suppose $y(t_i) > 0$. Consider the following two cases. Case I: $\bar{a}_H(t_i, t_{i+1}) \leq a^E(t_i)$. Then, since $\bar{a}_L(t_i, t_{i+1}) < \bar{a}_H(t_i, t_{i+1})$, we have $\bar{a}_L(t_{i-1}, t_i) < \bar{a}_L(t_i, t_{i+1}) < a^E(t_i)$. But it follows from single-peakedness that $x(t_i) > 0$, which contradicts that $y(t_i)$ and $x(t_i)$ have different signs. Case II: $\bar{a}_H(t_i, t_{i+1}) > a^E(t_i)$. Since $\bar{a}_L(t_{i-1}, t_i) < \bar{a}_H(t_{i-1}, t_i) < a^E(t_i)$ and $\bar{a}_L(t_i, t_{i+1}) < \bar{a}_H(t_i, t_{i+1})$, it follows immediately from single-peakedness that $x(t_i) > 0$, again a contradiction. So if the DM's signal is s_H, sending m_{i-1} is better than sending m_i for the type-t_i expert and if the DM's signal is s_L, sending m_i is better than sending m_{i-1} for him. But under uncertainty, type t_i is indifferent.

Let $\Delta U^E = p_L(t)\,x(t) + p_H(t)\,y(t)$. It measures the difference in type t's expected payoff by sending message m_i and by sending message m_{i-1}. I will show that ΔU^E is *not* monotonically increasing in t, resulting in the failure of sufficiency. To see this, note that $\frac{d\Delta U^E}{dt} = p_L'(t)\,x(t) + p_L(t)\,x'(t) + p_H'(t)\,y(t) + p_H(t)\,y'(t)$. Since $U_{12}^E(a,t) > 0$, it follows that $x'(t) > 0$, $y'(t) > 0$ and hence $p_L(t)\,x'(t) > 0$ and $p_H(t)\,y'(t) > 0$. The MLRP implies that $p_L'(t) < 0$ and $p_H'(t) > 0$. Since $x(t_i) > 0$ and $y(t_i) < 0$ and $x(t)$, $y(t)$ are continuous, there exists $\delta > 0$ such that if $|t - t_i| < \delta$, we have $p_L'(t)\,x(t) < 0$ and $p_H'(t)\,y(t) < 0$. So $\frac{d\Delta U^E}{dt}$ is not necessarily positive – the indifference conditions of the boundary types do not guarantee that the other types are best responding.

Proof of Lemma 2

By induction on K.

Suppose $K = 2$. Condition (A) requires that $U^E(\bar{a}_L(t_0^L, t_1^L), t_1^L)) = U^E(\bar{a}_L(t_1^L, t_2^L), t_1^L))$ where $\bar{a}_L(t_0^L, t_1^L) < a^E\left(t_1^L\right) < \bar{a}_L(t_1^L, t_2^L)$. Since $U_{11}^E < 0$, and $\bar{a}_H(t_{i-1}^L, t_i^L) > \bar{a}_L(t_{i-1}^L, t_i^L)$ for $i = 1, 2$ by Lemma 1, it follows that $U^E(\bar{a}_H(t_0^L, t_1^L), t_1^L)) > U^E(\bar{a}_H(t_1^L, t_2^L), t_1^L))$. So there exists a $t \in \left(t_1^L, t_2^L\right)$ such that $U^E(\bar{a}_H(t_0^L, t_1^L), t_1^L)) = U^E(\bar{a}_H(t_1^L, t), t_1^L))$. Since $U^E(\bar{a}_H(t_0^H, t_1^H), t_1^H)) = U^E(\bar{a}_H(t_1^H, t_2^H), t_1^H))$, condition (M) implies that $t_1^H > t_1^L$.

Suppose the claim holds for all $i = 2, .., K-1$. Let $\mathbf{t}^L(K)$ and $\mathbf{t}^H(K)$ be two partial partitions of size K satisfying (A) with $t_0^L(K) = t_0^H(K)$ and $t_K^L(K) =$

$t_K^H(K)$. Then $\left(t_i^L(K)\right)_{i=0,K-1}$ is a partial partition of size $(K-1)$ satisfying (A). Let $\left(\hat{t}_i^H\right)_{i=0,K-1}$ be a partial partion of size $(K-1)$ satisfying (A) under distribution H with $\hat{t}_0^H = t_0^L(K)$ and $\hat{t}_{K-1}^H = t_{K-1}^L(K)$. Then by the induction hypothesis, $\hat{t}_i^H > t_i^L$ for all $i = 1, ..., K-2$. So $\bar{a}_H\left(\hat{t}_{K-2}^H, \hat{t}_{K-1}^H\right) > \bar{a}_L\left(\hat{t}_{K-2}^L, \hat{t}_{K-1}^L\right)$. Since $U^E(\bar{a}_L(t_{K-2}^L, t_{K-1}^L), t_{K-1}^L)) = U^E(\bar{a}_L(t_{K-1}^L, t_K^L), t_{K-1}^L))$ and U^E is single peaked, there exists a $t \in \left(t_{K-1}^L, t_K^L\right)$ such that $U^E(\bar{a}_H(\hat{t}_{K-2}^H, \hat{t}_{K-1}^H), \hat{t}_{K-1}^H)) = U^E(\bar{a}_H(\hat{t}_{K-1}^H, t), \hat{t}_{K-1}^H))$. Since $U^E(\bar{a}_H(t_{K-2}^H, t_{K-1}^H), t_{K-1}^H)) = U^E(\bar{a}_H(t_{K-1}^H, t_K^H), t_{K-1}^H))$, condition (M) implies that $t_i^H > \hat{t}_i^H$ for $i = 1, ..., K-1$. So $t_i^H(K) > t_i^L(K)$ for $i = 1, ..., K-1$.

Proof of Corollary 1

First, note that Lemma 2 and condition (M) imply that if $t_0^H(K) = t_0^L(K)$ and $t_1^L(K) = t_1^H(K)$, then $t_i^L(K) > t_i^H(K)$ for $i = 2, ..., K$.

Now suppose $\mathbf{t}^L\left(\overline{K}\right)$ is an equilibrium partition of size $\overline{K}$ under prior L. Let $\mathbf{t}^H\left(\overline{K}\right)$ be a partition satisfying (A) such that $t_0^H\left(\overline{K}\right) = t_0^L\left(\overline{K}\right)$ and $t_1^H\left(\overline{K}\right) = t_1^L\left(\overline{K}\right)$. Then $t_{\overline{K}}^H\left(\overline{K}\right) < t_{\overline{K}}^L\left(\overline{K}\right) = 1$. By (M) there exists an equilibrium partition of size $\overline{K}$ under prior H. So $N^*(H) \geq N^*(L)$.

Proof of Lemma 3

The arguments are similar to those in the proof of Theorem 3 in Crawford and Sobel (1982).

Note that $EU^{DM}(x) = \sum_{i=1}^K \int_{t_{i-1}(x)}^{t_i(x)} U^{DM}\left(\bar{a}_F\left(t_{i-1}(x), t_i(x)\right), t\right) dF(t)$. Since $t_0(x)$ and $t_K(x)$ are fixed and $\bar{a}_F\left(t_{i-1}(x), t_i(x)\right)$ is the DM's optimal action on $[t_{i-1}, t_i]$, the envelope theorem implies that

$$\begin{aligned}
\frac{dEU^{DM}(x)}{dx} &= \sum_{i=1}^{K-1} f\left(t_i(x)\right) \frac{dt_i(x)}{dx}(U^{DM}\left(\bar{a}_F\left(t_{i-1}(x), t_i(x)\right), t_i(x)\right) \\
&\quad -U^{DM}\left(\bar{a}_F\left(t_i(x), t_{i+1}(x)\right), t_i(x)\right).
\end{aligned}$$

Condition (M) implies that $\frac{dt_i(x)}{dx} > 0$ for all $i = 1, ..., K-1$. Also, since $\left(t_i(x)\right)_{i=0,...,K}$ satisfies (A) for $i = 2, ..., K$, we have $U^E\left(\bar{a}_F\left(t_{i-1}(x), t_i(x)\right), t_i(x)\right) - U^E\left(\bar{a}_F\left(t_i(x), t_{i+1}(x)\right), t_i(x)\right) = 0$ for $i = 2, ..., K-1$. It follows that $U^{DM}\left(\bar{a}_F\left(t_{i-1}(x), t_i(x)\right), t_i(x)\right) - U^{DM}\left(\bar{a}_F\left(t_i(x), t_{i+1}(x)\right), t_i(x)\right) > 0$ for $i = 2, ..., K-1$. Also, for $x \in (y, y')$, $U^{DM}\left(\bar{a}_F\left(t_0, t_1(x)\right), t_1(x)\right) > U^{DM}\left(\bar{a}_F\left(t_1(x), t_2(x)\right), t_1(x)\right)$. It follows that $\frac{dEU^{DM}(x)}{dx} > 0$.

Proof of Lemma 4

By induction on the step size K.

Suppose $K = 2$. Since $U^{DM}\left(\bar{a}_H\left(0, t_1^H\right), t_1^H\right) \geq U^{DM}\left(\bar{a}_H\left(t_1^H, 1\right), t_1^H\right)$ and $t_1^L < t_1^H$, the claim is true as immediately implied by Lemma 3 when $t_0 = 0$ and $t_K = 1$.

Suppose the claim holds for steps $i = 2, ..., K-1$. Below I show that it holds for steps K.

Consider two equilibrium partitions $\mathbf{t}^L(K) = \left(t_0^L = 0, t_1^L, ..., t_K^L = 1\right)$ under prior L and $\mathbf{t}^H(K) = \left(t_0^H = 0, t_1^H, ..., t_K^H = 1\right)$ under prior H. One can find a partition $\hat{\mathbf{t}}^H(K) = \left(\hat{t}_0^H = 0, \hat{t}_1^H, ..., \hat{t}_K^H = 1\right)$ such that $\hat{t}_1^H = t_1^L$ but the condition (A) holds for all $\hat{t}_i^H$ $(i = 2, ..., K-1)$ under distribution H. By Lemma 2, $\hat{t}_i^H > t_i^L$ for all $i = 2, ..., K-1$. By the induction hypothesis, the DM with belief H must strictly prefer partition $\hat{\mathbf{t}}^H(K)$ to $\mathbf{t}^L(K)$. All we need to show is that the DM with belief H prefers partition $\mathbf{t}^H(K)$ to $\hat{\mathbf{t}}^H(K)$. By (M), $U^{DM}\left(\bar{a}_H\left(0, \hat{t}_1^H\right), \hat{t}_1^H\right) \geq U^{DM}\left(\bar{a}_H\left(\hat{t}_1^H, \hat{t}_2^H\right), \hat{t}_1^H\right)$. Lemma 3 implies that the DM indeed prefers $\mathbf{t}^H(K)$ to $\hat{\mathbf{t}}(K)$.

Proof of Lemma 5

By induction on the step size K.

Step 1. Suppose $K = 2$. Lemma 3 implies that the claim is true.

Step 2. Suppose $K \geq 3$ and the claim holds for all $i = 2, ..., K-1$. Let's compare the partitions $(t_i)_{i=0,...,K}$ and $(t_0, t_1, ..., \hat{t}_{K-1}, t_K)$. There are two possibilities.

(1) Suppose $U^{DM}\left(\bar{a}_F\left(t_{K-2}, \hat{t}_{K-1}\right), \hat{t}_{K-1}\right) \geq U^{DM}\left(\bar{a}_F\left(\hat{t}_{K-1}, t_K\right), \hat{t}_{K-1}\right)$. Then by step 1, the DM prefers the partial partition $\left(t_{K-2}, \hat{t}_{K-1}, t_K\right)$ to (t_{K-2}, t_{K-1}, t_K). It follows that the DM prefers $(t_0, ..., t_{K-2}, \hat{t}_{K-1}, t_K)$ to $(t_i)_{i=0,...,K}$. Now compare the partitions $(t_0, t_1, ..., \hat{t}_{K-1})$ and $(\hat{t}_i)_{i=0,...,K-1}$. Since $\hat{t}_i \geq t_i$, by the induction hypothesis, the DM prefers $(\hat{t}_i)_{i=0,...,K-1}$ to $(t_0, t_1, ..., \hat{t}_{K-1})$. It follows that the DM prefers $(\hat{t}_i)_{i=0,...,K-1}$ to $(t_i)_{i=0,...,K}$.

(2) Suppose $U^{DM}\left(\bar{a}_F\left(t_{K-2}, \hat{t}_{K-1}\right), \hat{t}_{K-1}\right) < U^{DM}\left(\bar{a}_F\left(\hat{t}_{K-1}, t_K\right), \hat{t}_{K-1}\right)$. Compare the partitions $(t_i)_{i=0,...,K}$ and $(t_0, ..., t_{K-2}, \tilde{t}_{K-1}, t_K)$, where $\tilde{t}_{K-1}$ satisifies $U^{DM}\left(\bar{a}_F\left(t_{K-2}, \tilde{t}_{K-1}\right), \tilde{t}_{K-1}\right) = U^{DM}\left(\bar{a}_F\left(\tilde{t}_{K-1}, t_K\right), \tilde{t}_{K-1}\right)$. Note that $t_{K-1} \leq \tilde{t}_{K-1} < \hat{t}_{K-1}$. By step 1, the DM prefers the partition $\left(t_{K-2}, \tilde{t}_{K-1}, t_K\right)$ to (t_{K-2}, t_{K-1}, t_K) and hence the partition $(t_0, t_1, ..., t_{K-2}, \tilde{t}_{K-1}, t_K)$ to $(t_i)_{i=0,...,K}$. Now consider $\tilde{t}_{K-2}$ that satisfies $U^{DM}\left(\bar{a}_F\left(\tilde{t}_{K-2}, \hat{t}_{K-1}\right), \hat{t}_{K-1}\right) = U^{DM}\left(\bar{a}_F\left(\hat{t}_{K-1}, t_K\right), \hat{t}_{K-1}\right)$. Note that since $U^{DM}\left(\bar{a}_F\left(\hat{t}_{K-2}, \hat{t}_{K-1}\right), \hat{t}_{K-1}\right) \geq U^{DM}\left(\bar{a}_F\left(\hat{t}_{K-1}, \hat{t}_K\right), \hat{t}_{K-1}\right)$, we have $\tilde{t}_{K-2} \leq \hat{t}_{K-2}$. So $U^{DM}\left(\bar{a}_F\left(t_{K-3}, \tilde{t}_{K-2}\right), \tilde{t}_{K-2}\right) > U^{DM}\left(\bar{a}_F\left(\tilde{t}_{K-2}, \hat{t}_{K-1}\right), \tilde{t}_{K-2}\right)$. Since $\hat{t}_{K-1} > \tilde{t}_{K-1}$ and $\tilde{t}_{K-2} > t_{K-2}$, Lemma 3 implies that the DM prefers the partial partition $(t_{K-3}, \tilde{t}_{K-2}, \hat{t}_{K-1}, t_K)$ to $(t_{K-3}, t_{K-2}, \tilde{t}_{K-1}, t_K)$. It follows that the DM prefers the partial partition $(t_0, t_1, ..., t_{K-3}, \tilde{t}_{K-2}, \hat{t}_{K-1}, t_K)$ to $(t_0, t_1, ..., t_{K-2}, \tilde{t}_{K-1}, t_K)$ and hence to $(t_i)_{i=0,...,K}$. Now compare $(t_0, t_1, ..., t_{K-3}, \tilde{t}_{K-2}, \hat{t}_{K-1}, t_K)$ and $(\hat{t}_i)_{i=0,...,K}$. Since

$t_i \leq \hat{t}_i$ and $\tilde{t}_{K-2} \leq \hat{t}_{K-2}$, by the induction hypothesis, the DM prefers $(\hat{t}_i)_{i=0,...,K}$ to $(t_0, t_1, ..., t_{K-3}, \tilde{t}_{K-2}, \hat{t}_{K-1}, t_K)$. It follows that the DM prefers $(\hat{t}_i)_{i=0,...,K}$ to $(t_i)_{i=0,...,K}$.

Proof of Proposition 3

Suppose $N^*(H) = N^*(L)$. The result follows immediately from Lemma 5.

Suppose $N^*(H) > N^*(L)$. For notational convenience, order the boundary types in a partition from high to low. Lemma 2 and condition (M) imply that $t_i^H > t_i^L$ for $i = 1, ..., N^*(L) - 1$.

Consider a partition of size $N^*(L) + 1$, $\hat{\mathbf{t}}$, where $\hat{t}_1 = t_1^H(N^*(H))$ and $\hat{t}_i$ satisfies (A) under belief L for $i = 1, 2, ..., N^*(L) - 1$. Since $N^*(L)$ is the highest number of equilibrium steps under L, $U^E\left(\bar{a}_L\left(0, \hat{t}_{N^*(L)}\right), \hat{t}_{N^*(L)}\right) > U^E\left(\bar{a}_L\left(\hat{t}_{N^*(L)}, \hat{t}_{N^*(L)-1}\right), \hat{t}_{N^*(L)}\right)$ and hence $U^{DM}\left(\bar{a}_L\left(0, \hat{t}_{N^*(L)}\right), \hat{t}_{N^*(L)}\right) > U^{DM}\left(\bar{a}_L\left(\hat{t}_{N^*(L)}, \hat{t}_{N^*(L)-1}\right), \hat{t}_{N^*(L)}\right)$. By Lemma 3, the DM with belief L prefers the partition $\hat{\mathbf{t}}$ to $\mathbf{t}^L(N^*(L))$.

Suppose $N^*(H) = N^*(L) + 1$. Then the partitions $\hat{\mathbf{t}}$ and $\mathbf{t}^H(N^*(H))$ have the same size. By Lemma 2 and condition (M), $\hat{t}_i < t_i^H$ for $i = 2, ..., N^*(H) - 1$. So Lemma 5 implies that the DM with belief L prefers the partition $\mathbf{t}^H(N^*(H))$ to $\hat{\mathbf{t}}$. Hence the DM with belief L prefers the partition $\mathbf{t}^H(N^*(H))$ to $\mathbf{t}^L(N^*(L))$.

Suppose $N^*(H) > N^*(L) + 1$. Then construct a partition of size $N^*(L) + 2$, $\bar{\mathbf{t}}$, where $\bar{t}_i = t_i^H(N^*(H))$ for $i = 1, 2$ and $\bar{t}_i$ satisfies (A) under belief L for $i = 2, ..., N^*(L)$. Using argument similar as above, one can show that the DM with belief L prefers $\bar{\mathbf{t}}$ to $\hat{\mathbf{t}}$ by Lemma 3. If $N^*(H) = N^*(L) + 2$, then the partitions $\mathbf{t}^H(N^*(H))$ and $\bar{\mathbf{t}}$ have the same size and by Lemma 5, the DM with belief L prefers the partition $\mathbf{t}^H(N^*(H))$ to $\bar{\mathbf{t}}$. Hence the DM with belief L prefers the partition $\mathbf{t}^H(N^*(H))$ to $\mathbf{t}^L(N^*(L))$. If $N^*(H) > N^*(L) + 2$, then we can use similar argument as above, construct a partition of size $N^*(L) + 3$ (or larger as needed) and show that the DM with belief L prefers the partition $\mathbf{t}^H(N^*(H))$ to $\mathbf{t}^L(N^*(L))$.

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