University of Toronto
Department of Economics
ECO 2061H
Economic Theory - Macroeconomics (MA)
Winter 2012

Professor Tasso Adamopoulos

Answers to Sample Test

1. (a) Following the steps we did in class you should derive the following law of motion for capital per unit of effective labor (in an exam you would have to go through those steps).

$$\dot{k}(t) = sk(t)^\alpha - (n + g + \delta)\, k(t)$$

The law of motion tells you how capital per unit of effective labor evolves over time. In particular, it says that when actual investment ($sk(t)^\alpha$) is higher that required investment ($(n + g + \delta)\, k(t)$), capital per unit of effective labor would be rising. Actual investment is the new investment in the economy. Required investment (or break-even) is the investment that has to be made to keep k at its existing level. This investment has to be made because part of the capital stock wears out (δ) and because AL is growing at a rate $n + g$.

(b) First you have to solve for the steady state level of k, which occurs when $sk^\alpha = (n + g + \delta)k$,

$$k^* = \left(\frac{s}{n + g + \delta}\right)^{\frac{1}{1-\alpha}}$$

Then you solve for the golden rule level of k, which is the k that maximizes $c = k^\alpha - (n + g + \delta)k$,

$$k_{GR} = \left(\frac{\alpha}{n + g + \delta}\right)^{\frac{1}{1-\alpha}}$$

Comparing these two solutions it is clear that $k^* = k_{GR}$ when $\alpha = s$.

(c) The Solow model predicts conditional convergence, i.e., that after controlling for country characteristics, there should be convergence. The reason is that according to the Solow model countries converge to their own balanced growth paths, and not necessarily a common one. Divergence is not consistent with the Solow model. In the long-run, once countries have converged to their steady-states income levels will be different if they have different characteristics. But these differences will persist, they will not grow larger. Further in the short run, countries further away from their steady states grow faster than countries closer to their steady states.

(d) At time t_0 after the permanent one time increase in the level of efficiency, $k = \frac{K}{AL}$ will fall below the BGP value k^*. Note however that the BGP will not change, since no characteristic of the economy changed. Suppose, we call the value of k after the rise in A, k_0. Then at k_0, we have that actual investment is higher than required investment: $sf(k_0) > (n + g + \delta)k_0$. By the law of motion for capital this implies that $\dot{k} > 0$, and thus k will rise until it returns to the original steady state. See Fig.1.

2. (a) Substitute the firm's first order condition with respect to capital into the household's Euler to get the economy's first key dynamic equation,

$$\frac{\dot{c}(t)}{c(t)} = \frac{f'(k(t)) - \delta - \rho - \theta g}{\theta}$$

The Euler equation tells you how this economy is going to choose consumption/saving over time. On the margin the decision of giving up consumption today for more consumption tomorrow, depends on the economy's net return to capital (market return) and on preference parameters. The second key dynamic equation of the model is the law of motion for capital per unit of effective labor,

$$\dot{k}(t) = f(k(t)) - c(t) - (n + g + \delta)\, k(t)$$

This is different from the one in class because now we have depreciation. Required investment is higher, i.e., the level of investment required just to keep k constant is now higher. The reason is that now you have to account for the replacement of worn out capital.

(b) See Fig.2. Suppose the economy starts off from a level of capital per unit of effective labor below the steady state: $k(0) < k^*$. At time 0, $k(0)$ is pre-determined and cannot be adjusted. Consumption per unit of effective labor at time 0, $c(0)$, is a "control" variable, i.e., it is to be chosen. There is a whole continuum of possible choices for $c(0)$. Each one of these places the economy on a given trajectory. However, not all of these trajectories will take the economy to the steady state. In fact there is only one trajectory that takes the economy to the steady state: the saddle path. This is the only trajectory that satisfies, household optimization, law of motion for capital, the household intertemporal budget constraint and the No-Ponzi-Game condition. Thus if the economy is at $k(0)$ it chooses a level of consumption $c(0)$ that places it on the saddle path. Once the economy is on the saddle path it will go to the steady state. For any point above this level of $c(0)$ the economy will diverge up and to the left, while for any point below it, it will diverge down and to the right.

(c) The increase in δ means that break-even investment increases. From the law of motion the amount of consumption possible is now lower than before. Further, the level of extra investment required is higher at higher levels of k. Thus the $\dot{k} = 0$ locus shifts down but not in a parallel fashion, i.e., it shifts down more at higher levels of k. The $\dot{c} = 0$ locus will shift to the left. To see this note that when $\dot{c} = 0$, we have that $f'(k^*) = \rho + \delta + \theta g$. Then $\delta_{new} > \delta$ implies that $\rho + \delta_{new} + \theta g > \rho + \delta + \theta g$, and thus $f'(k^*_{new}) > f'(k^*)$. Since we have a diminishing marginal product of capital this implies $k^*_{new} < k^*$. See Fig.3. Both (c^*, k^*) will be lower in the new steady state. At the time of the change in δ k is predetermined and cannot change. Thus c must jump to place the economy on the new saddle path. Whether c will jump up or down depends on whether the new saddle path crosses the original $c\dot{c} = 0$ locus, above or below E.

3. (a) This type of government expenditures constitute a resource cost and affect the law of motion for capital: the more the government takes away from the economy the fewer resources are available to the private sector. When government expenditures are high at level G_0, before time t_0, the law of motion for k is,

$$\dot{k}(t) = f(k(t)) - c(t) - G_0 - (n+g)k(t)$$

After time t_0, when government expenditures are low, the law of motion becomes,

$$\dot{k}(t) = f(k(t)) - c(t) - G_1 - (n+g)k(t)$$

The Euler equation is the same in both cases,

$$\frac{\dot{c}(t)}{c(t)} = \frac{f'(k) - \rho - \theta g}{\theta}$$

At time t_0 the $\dot{k} = 0$ locus shifts up by $G_1 - G_0$. At that point k is pre-determined at k^* - the original BGP. Thus, c must jump up by $G_1 - G_0$ to place the economy on the new saddle path at point E' (see Fig.4). With a permanent decrease in government expenditures and thus taxes, lifetime wealth increases for households. Because this change in permanent the households will not alter the time pattern of their consumption but instead will take a permanent rise in how much they consume.

Thus we go directly from E to E' and k is not affected (remains at k^* throughout), while c^* rises.

(b) Before time t_0 and after time t_1 the law of motion for k is,

$$\dot{k}(t) = f(k(t)) - c(t) - G_0 - (n+g)k(t)$$

Between times (t_0, t_1) the law of motion is,

$$\dot{k}(t) = f(k(t)) - c(t) - G_1 - (n+g)k(t)$$

Again the Euler in both cases is,

$$\frac{\dot{c}(t)}{c(t)} = \frac{f'(k) - \rho - \theta g}{\theta}$$

In this case at time t_0, c will not rise by the full amount $G_1 - G_0$. If it did, then consumption would have to drop discontinuously at t_1, and this marginal utility would increase discontinuously. But since the return to G_0 is anticipated, so will the discrete jumps in consumption and marginal utility. This however is not optimal for rational individuals who like smooth consumption patterns. So at time t_0, c will increase less than $G_1 - G_0$, such that the dynamics implied by the Euler and the law of motion with $G(t) = G_1$ will push the economy to the original saddle path as time $t_1$arrives. After that the economy rides the original saddle path to the original BGP. So k first increases and then decreases. See Fig.5.

4. (a) The capital accumulation equation is,

$$\dot{k}(t) = Ak(t) - c(t) - \delta k(t)$$

$$\dot{k}(t) = (A - \delta)\, k(t) - c(t)$$

(b) Real interest rate:

$$r(t) = (1 - \tau)\, A - \delta$$

Growth of consumption per worker:

$$\frac{\dot{c}(t)}{c(t)} = \frac{(1 - \tau)\, A - (\delta + \rho)}{\theta}$$

Consider the capital accumulation equation from part (a). Divide both sides by $k(t)$, and use the fact that $\frac{\dot{k}(t)}{k(t)}$ will be constant. Let $\frac{\dot{k}(t)}{k(t)} \equiv b$. Then we have,

$$b = (A - \delta) - \frac{c(t)}{k(t)}$$

This implies that $\frac{\dot{k}(t)}{k(t)} = \frac{\dot{c}(t)}{c(t)}$, i.e., the growth rate of capital per worker is the same as that of consumption per worker. From the production function $y(t) = Ak(t)$, you can see that also $\frac{\dot{y}(t)}{y(t)} = \frac{\dot{k}(t)}{k(t)}$. The savings rate $s = \frac{y-c}{y}$, can be shown to be,

$$s = \frac{Ak - c}{Ak} = 1 - \frac{1}{A}\frac{c}{k} = 1 - \frac{1}{A}\left[A - \delta - \frac{(1 - \tau)\, A - (\delta + \rho)}{\theta}\right]$$

$$= 1 - \left[\frac{(\theta - 1 + \tau)\, A + \rho + \delta\,(1 - \theta)}{\theta A}\right]$$

(c) You can see that the interest rate, the savings rate, and the growth rate are all decreasing in the tax, by calculating the derivatives: $\frac{\partial r}{\partial \tau} = -A < 0$, $\frac{\partial s}{\partial \tau} = -\frac{1}{\theta} < 0$, $\frac{\partial(\frac{\dot{y}}{y})}{\partial \tau} = -\frac{A}{\theta} < 0$.

(d) Since the economies have the same tax rates their growth rates will be the same. However the two economies will never converge. Income differences will persist forever.

Fig.1: One Time Increase in A

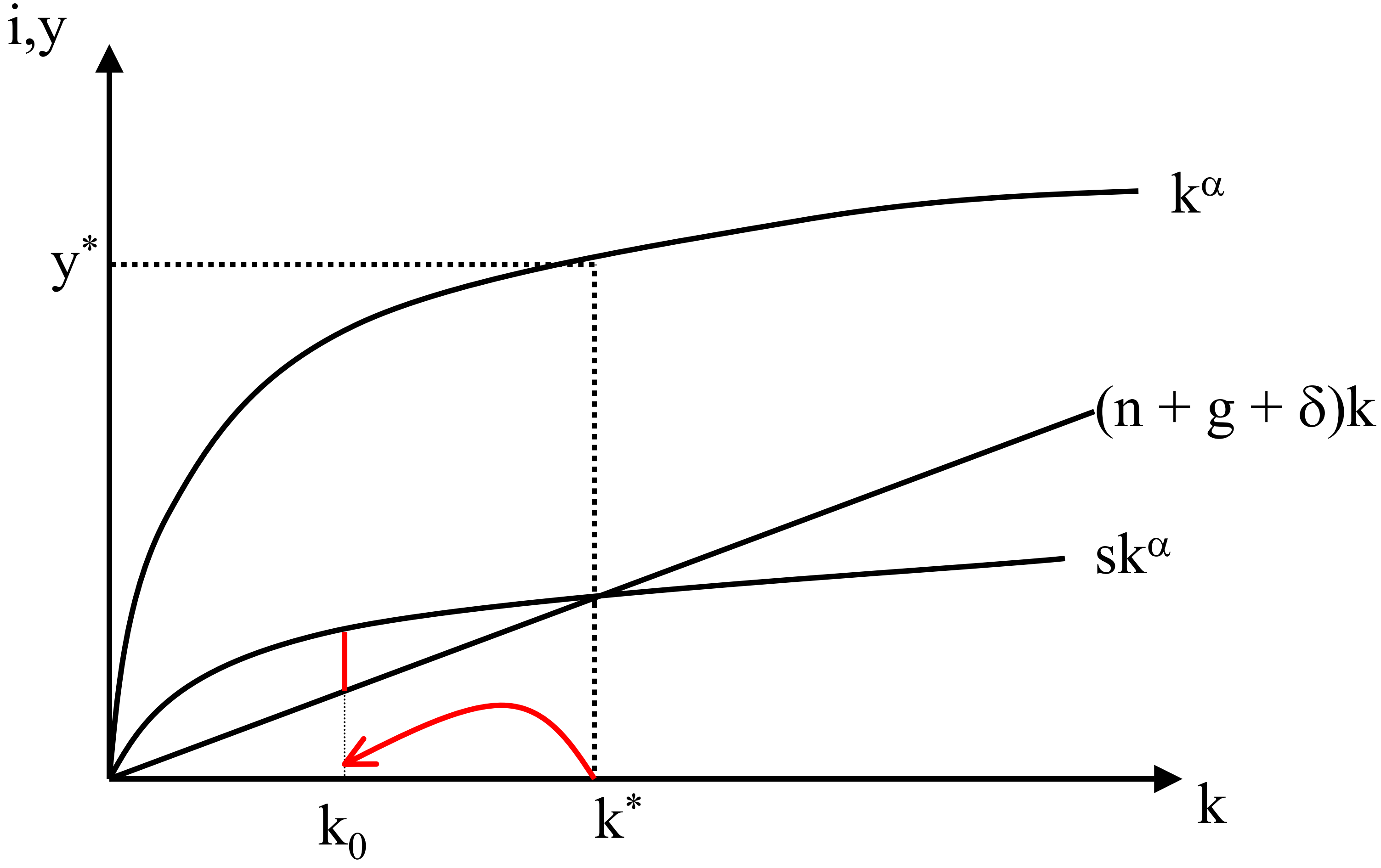

Fig.2: Adjustment to BGP Equilibrium

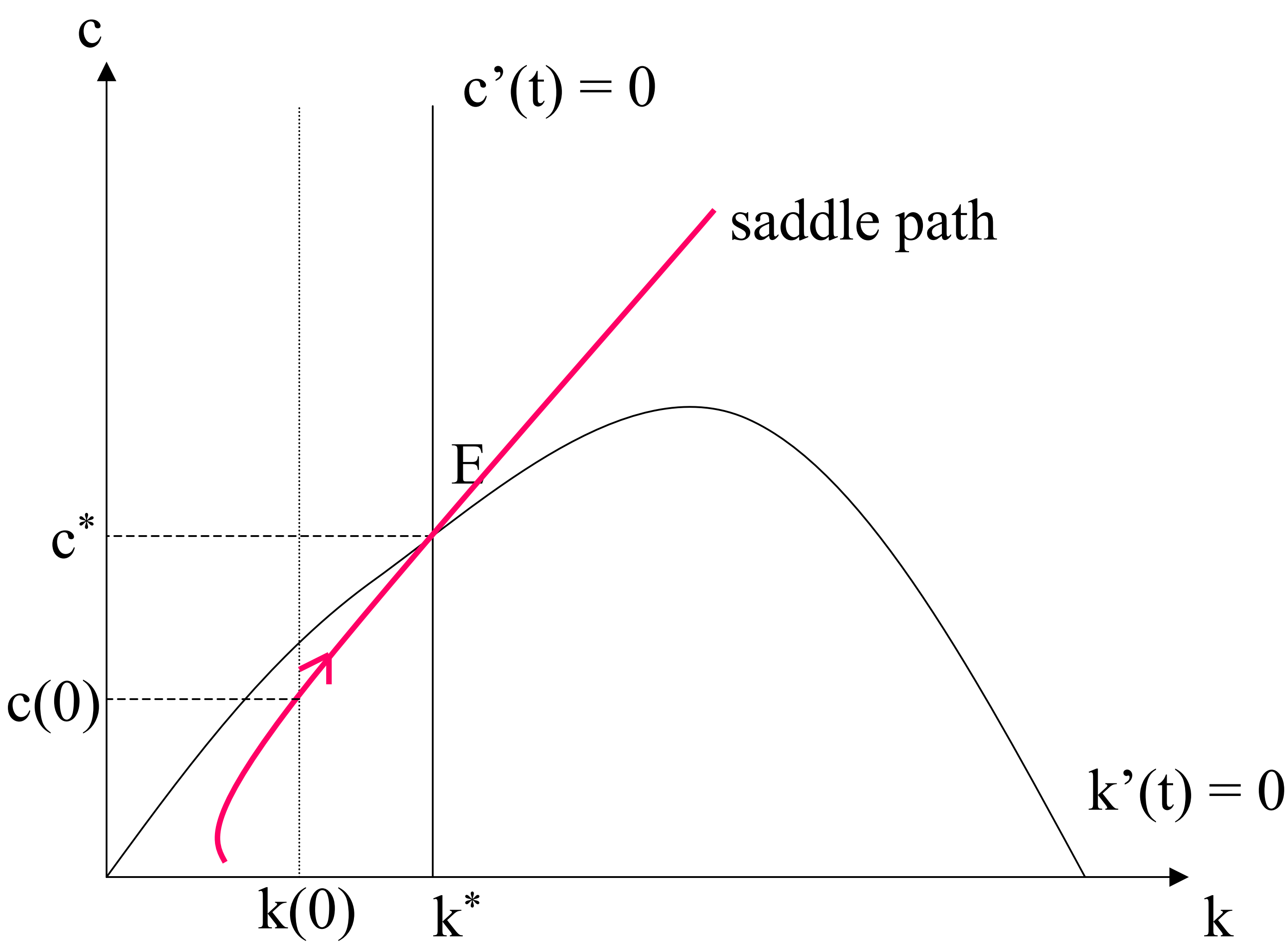

Fig.3: Effect of change in δ

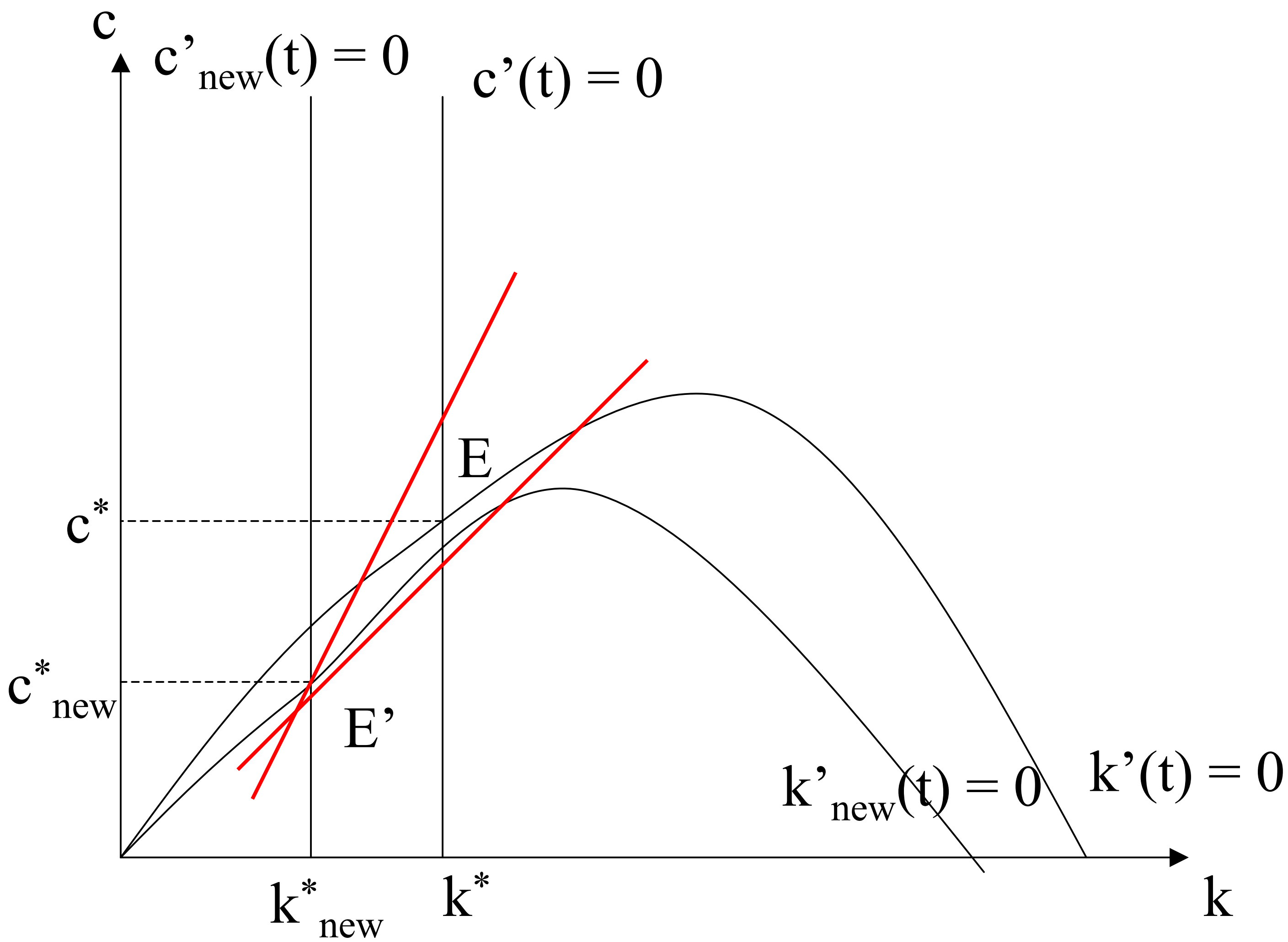

Fig.4: Permanent Fall in Government Expenditures

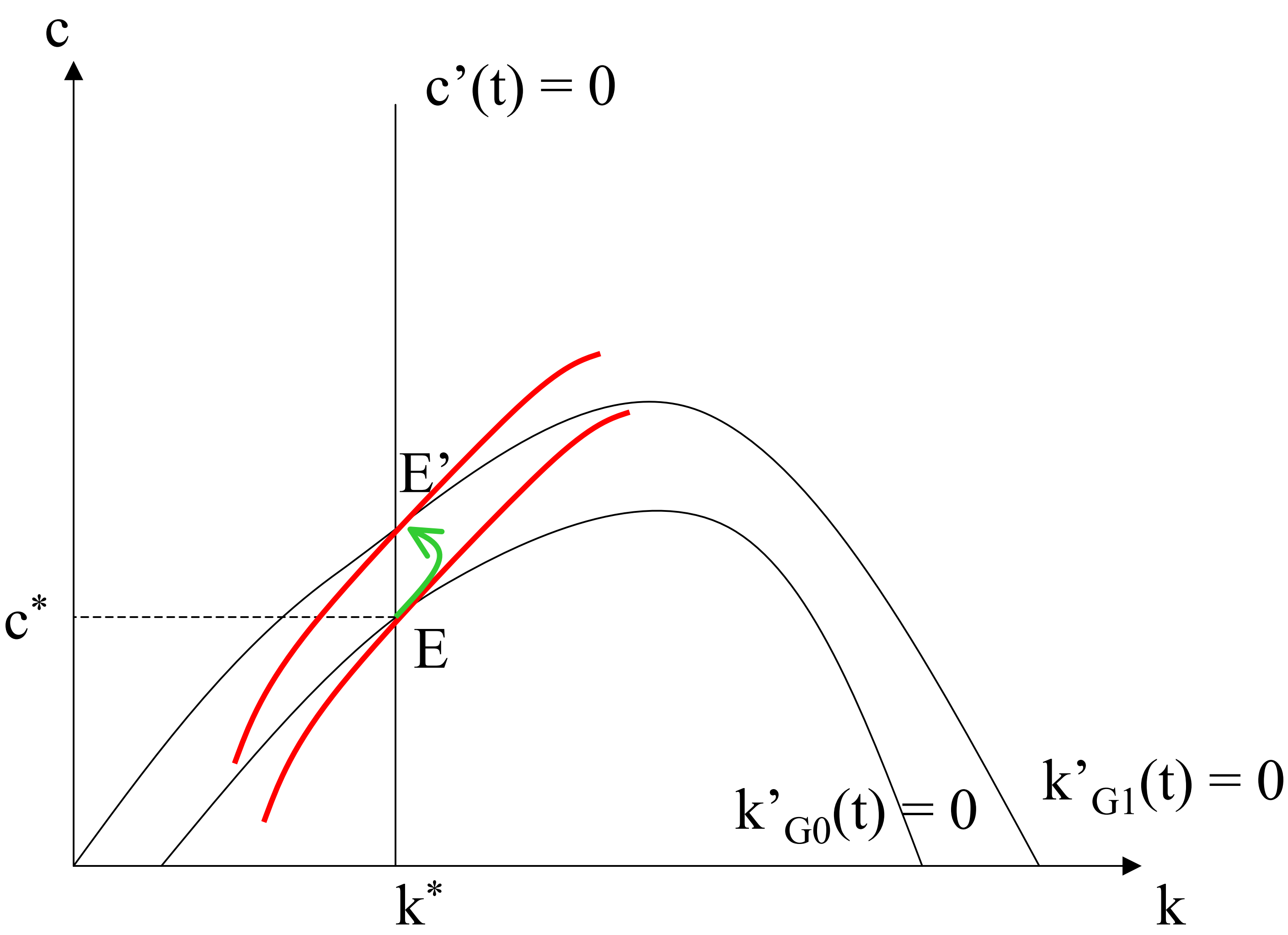

Fig.5: Temporary Fall in Government Expenditures