

University of Toronto
Department of Economics
ECO 2061H
Economic Theory - Macroeconomics (MA)
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Sample Test

Total Time: 100 minutes

Total Marks: 100

1. Consider the standard Solow model. Assume that the production technology is of the Cobb-Douglas form,

$$Y(t) = K(t)^\alpha [A(t)L(t)]^{1-\alpha}$$

where Y is total output, K is the stock of capital, L is labor, and A is the level of efficiency, all at time t . We assume that $0 < \alpha < 1$. Capital depreciates at a constant rate δ . Population and efficiency grow exogenously at constant rates n and g respectively. Denote with lower case letters the variables in units of effective labor, i.e., $y = \frac{Y}{AL}$, $k = \frac{K}{AL}$. The evolution of aggregate capital in the economy is given by

$$\dot{K}(t) = sY(t) - \delta K(t)$$

where s is a constant and exogenous savings rate.

- (a) Derive the law of motion for capital in terms of capital per unit of effective labor k . What is the economic interpretation of this equation?
- (b) Under what condition(s) would the steady state capital per unit of effective labor equal the golden rule level of capital per unit of effective labor in this economy?
- (c) Explain, in the context of the Solow model, whether the following statement is true, uncertain, or false: "Divergence in cross-country income levels is inconsistent with the predictions of the Solow model."
- (d) Consider a country that is initially on its balanced growth path. At time t_0 there is a permanent one time increase in the level of efficiency. Evaluate the following statement in the context of the Solow model: "In the short run and the long-run this economy will have a higher growth rate of output per worker."

2. Consider the Ramsey-Cass-Koopmans model. Assume that output is produced according to a Cobb-Douglas production function, $Y(t) = K(t)^\alpha [A(t)L(t)]^{1-\alpha}$. Aggregate capital depreciates at a rate $\delta < 1$. From the firm's profit maximization problem this implies that the real interest rate at time t is $r(t) = f'(k(t)) - \delta$.

The household's instantaneous utility from consumption is given by, $u(C(t)) = \frac{C(t)^{1-\theta}}{1-\theta}$, where $C(t)$ is consumption per household member. Household optimization implies the following Euler equation, $\frac{\dot{c}(t)}{c(t)} = \frac{r(t) - \rho - \theta g}{\theta}$.

- (a) What are the two equations that govern the economy's dynamics? Explain your answer.
- (b) Draw a carefully labelled phase diagram, to show (i) the balanced growth path equilibrium of this economy, and (ii) the transitional dynamics of this economy if the initial level of capital per unit of effective labor is below the balanced growth path level, $k(0) < k^*$.
- (c) Suppose that the economy is initially on the balanced growth path. At time t_0 there is a permanent one time increase in the rate of depreciation δ . How does the increase in δ affect the balanced growth path values of (c, k) ? Using a phase diagram explain what the dynamics of $(c(t), k(t))$ are for this economy.
3. Consider the Ramsey-Cass-Koopmans model with a government. Let $G(t)$ denote wasteful government expenditures per unit of effective labor at time t . Assume the expenditures are financed by lump-sum taxes each period such that the government runs a balanced budget. Initially the level of government expenditures is constant at G_0 for every period. At time t_0 the government expenditures fall unexpectedly to the new constant level of $G_1 < G_0$.
- (a) If the change in government expenditures is permanent explain what happens to the balanced growth path levels of (c, k) . What are the dynamics of adjustment to the new balanced growth path equilibrium. Use a phase diagram to explain your answer.
- (b) Assume that this unexpected decrease in G is known to be temporary instead of permanent. In other words, it is expected that at some future time $t_1 > t_0$, G will return to its original level G_0 . What is the effect on the balanced growth path values of (c, k) ? What are the transition dynamics of this economy? Use a phase diagram to explain your answer.

4. Consider a continuous time growth model, in which output per worker (y) is produced according to the following production function, $y = f(k) = Ak$, where k is capital per worker and A is a constant efficiency parameter. Labor is constant and normalized to 1. Capital depreciates at rate δ . The representative household has preferences described by the following intertemporal utility function,

$$U = \int_{t=0}^{\infty} e^{-\rho t} \frac{c(t)^{1-\theta}}{1-\theta} dt$$

where c is consumption per worker and ρ is the rate of time preference. Each period the government levies a proportional tax on investment income τ and uses the revenues to make lump-sum rebates to households, T_t . The Euler equation from the household's maximization problem is, $\frac{\dot{c}(t)}{c(t)} = \frac{r(t)-\rho}{\theta}$. Assume that the parameters of the model are such that the right hand side is always positive.

- (a) What is the capital accumulation equation for this economy?
- (b) What is the real interest rate in a competitive equilibrium? Calculate the growth rates of consumption per worker, capital per worker, and output per worker. What is the economy's savings rate?
- (c) Show what the effect of an increase in the tax rate is on the growth rate of output per worker. Explain.
- (d) Consider two economies i and j , which are identical, including their tax rates. In particular: $\tau_i = \tau_j$. Will the two economies converge in the long-run if economy j starts with a lower level of output per worker?