University of Toronto

Department of Economics

ECO 2061H

Economic Theory - Macroeconomics (MA)

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Answers to Sample Final

1. (a) A resource constraint tells you how the economy's total resources in a given period are allocated across different uses. In this case the total resources are the sum of the endowments of the young and the old, and the total uses are consumption of the young and the old.

$$c_{1t}L + c_{2t}L = y_{t-1}(1+\phi)L + y_{t-1}(1+\gamma)L$$

and since the size of the population is constant this simplifies to,

$$c_{1t} + c_{2t} = y_{t-1}(2+\phi+\gamma)$$

(b) The first period budget constraint of an individual born at time t, when young, is,

$$c_{1t} + S_t = y_t$$

The budget constraint for the same individual when old, in the second period of his life, is,

$$c_{2t+1} = (1+r)S_t + y_t(1+\gamma)$$

Solve the first period constraint for S_t and substitute into the second to derive the individual's lifetime budget constraint,

$$c_{1t} + \frac{c_{2t+1}}{1+r} = y_t\left[1 + \frac{1+\gamma}{1+r}\right]$$

The Lagrangian for the individual's problem,

$$\text{L} = \ln(c_{1t}) + \frac{1}{1+\rho}\ln(c_{2t+1}) + \lambda\left\{y_t\left[1+\frac{1+\gamma}{1+r}\right] - c_{1t} - \frac{c_{2t+1}}{1+r}\right\}$$

Taking the first order conditions you can derive the Euler equation (standard), $c_{2t+1} = \frac{1+r}{1+\rho}c_{1t}$. Substitute the Euler into the lifetime budget constraint to solve for the individual's choice of first period consumption,

$$c_{1t} = \left(\frac{1+\rho}{2+\rho}\right)\left(\frac{2+r+\gamma}{1+r}\right)y_t$$

The individual's savings are, $S_t = y_t - c_{1t}$. The individual's saving rate is then the fraction of first period income (endowment) saved,

$$s_t = \frac{S_t}{y_t} = \frac{y_t - c_{1t}}{y_t} = 1 - \frac{c_{1t}}{y_t} = 1 - \left(\frac{1+\rho}{2+\rho}\right)\left(\frac{2+r+\gamma}{1+r}\right)$$

(c) To see the effect of lifetime income growth (or decline) on an individual born in time t, calculate the derivative of s_t with respect to γ, $\frac{\partial s_t}{\partial \gamma} = -\frac{(1+\rho)}{(1+r)(2+\rho)} < 0$. Intuitively, when I expect income growth over my lifetime I will save less when I am young. On the other hand, aggregate saving in the economy, at time t, is sy_tL since the saving rate is constant. Then aggregate saving in period $t+1$ is $sy_{t+1}L$. The growth rate of aggregate savings is $\frac{sy_tL}{sy_{t+1}L} = \frac{y_{t+1}}{y_t} = (1+\phi)$. This implies that as total income grows, the aggregate amount saved by this economy grows too.

2. The household's problem is,

$$\max_{c_t,\ell_t,I_t} E_0 \sum_{t=0}^{\infty} e^{-\rho t} \left\{ \frac{\left[c_t^{1-\theta}(1-\ell_t)^{\theta}\right]^{1-\sigma} - 1}{1-\sigma} \right\}$$

s.t.

$$c_t + I_t = w_t \ell_t (1-\tau_t) + R_t K_t + T_t$$

$$K_{t+1} = (1-\delta) K_t + I_t$$

where T_t are lump-sum taxes, and $R_t = r_t + \delta$. Set-up the Lagrangian,

$$\pounds = E_0 \left\{ \sum_{t=0}^{\infty} e^{-\rho t} \left[\frac{\left[c_t^{1-\theta}(1-\ell_t)^{\theta}\right]^{1-\sigma} - 1}{1-\sigma} \right] \right\} +$$

$$+E_0 \left\{ \sum_{t=0}^{\infty} \lambda_t \left[w_t \ell_t (1-\tau_t) + R_t K_t + T_t + (1-\delta) K_t - c_t - K_{t+1} \right] \right\}$$

(a) The first order conditions are,

$$E_t \left\{ e^{-\rho t} \left[c_t^{1-\theta}(1-\ell_t)^{\theta}\right]^{-\sigma} (1-\theta) c_t^{-\theta} (1-\ell_t)^{\theta} - \lambda_t \right\} = 0$$

$$E_t \left\{ -e^{-\rho t} \left[c_t^{1-\theta}(1-\ell_t)^{\theta}\right]^{-\sigma} \theta c_t^{1-\theta} (1-\ell_t)^{\theta-1} + \lambda_t w_t (1-\tau_t) \right\} = 0$$

$$E_t \left\{ -\lambda_t + \lambda_{t+1} [R_{t+1} + 1 - \delta] \right\} = 0$$

(b) Combining the first and the third, the second and the third and the first and the second you get respectively,

$$\left[c_t^{1-\theta}(1-\ell_t)^{\theta}\right]^{-\sigma} c_t^{-\theta} (1-\ell_t)^{\theta} = e^{-\rho} E_t \left\{ \left[c_{t+1}^{1-\theta}(1-\ell_{t+1})^{\theta}\right]^{-\sigma} c_{t+1}^{-\theta} (1-\ell_{t+1})^{\theta} (1+r_{t+1}) \right\}$$

$$e^{-\rho} E_t \left\{ (1+r_{t+1}) \left(\frac{c_{t+1}^{1-\theta}(1-\ell_{t+1})^{\theta}}{c_t^{1-\theta}(1-\ell_t)^{\theta}} \right)^{-\sigma} \left(\frac{c_{t+1}}{c_t} \right)^{1-\theta} \left(\frac{1-\ell_{t+1}}{1-\ell_t} \right)^{\theta-1} \frac{w_t}{w_{t+1}} \frac{(1-\tau_t)}{(1-\tau_{t+1})} \right\} = 1$$

$$(1-\ell_t)^{-1} \frac{\theta}{1-\theta} = \frac{w_t (1-\tau_t)}{c_t}$$

where I have used that $R_t = r_t + \delta$. The first of these equations reflects the trade-off between consumption today and consumption tomorrow. The second reflects the trade-off between leisure (labor supply) today and leisure (labor supply) tomorrow. The third reflects the trade-off between leisure and consumption today. See class notes for the intuition.

(c) Consider the deterministic version of the model (no uncertainty) and assume $\theta = 1$. In this case the second intertemporal condition from part (b) becomes,

$$\frac{1-\ell_t}{1-\ell_{t+1}} = \left\{ \frac{w_{t+1}}{w_t} \frac{(1-\tau_{t+1})}{(1-\tau_t)} \frac{1}{e^{-\rho}(1+r_{t+1})} \right\}^{\frac{1}{\sigma}}$$

Intuitively, if taxes are expected to be higher next period than this period ($\tau_{t+1} > \tau_t$), this will reduce my expected after tax income in $t+1$ relative to t. This implies that I would reduce my relative labor supply tomorrow, and therefore work harder today (take relatively less leisure today). See equation for mechanics of the sign of this effect.

Question 3

(a) Firm Problem:

$$\max \left\{ A K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - r_t K_t \right\}$$

FONC:

$$w_t = (1-\alpha) A \left(\frac{K_t}{L_t}\right)^{\alpha}$$

$$r_t = \alpha A \left(\frac{K_t}{L_t}\right)^{\alpha-1}$$

(b) To write down the representative agent's problem in recursive form we use the following notation

k - capital stock of individual

K - economy-wide average per capita capitalstock

K is the aggregate state

(K, k) is the indiv. state

Let $V(K, k)$ be the value function for the indiv.

HH Problem in recursive form:

$$V(K, k) = \max \left\{ \frac{\left[c^{\theta} \ell^{1-\theta}\right]^{1-\sigma}}{1-\sigma} + \beta V(K', k') \right\} \quad (FE)$$

s.t.

$$c + k' = \left[r(1-\tau) + (1-\delta)\right] k + w(1-\tau)(1-\ell) + T$$

$$K' = \Gamma(K)$$

$$r = r\left(\frac{K}{L}\right), \quad w = w\left(\frac{K}{L}\right)$$

since functions of capital-labor ratio from (a)

$$L = G(K)$$

where $\Gamma(K)$ and $G(K)$ reflect the expectations of the representative agent of how aggregate capital and labor will evolve respectively as functions of agg. state K.

- indiv. and aggregate laws of motion are consistent:

$$g^{k'}(K,K) = \Gamma(K)$$

$$1 - g^{\ell^*}(K,K) = G(K).$$

(c)

$$V(K,k) = \max\left\{ \frac{\left[\left\{[r(1-\tau)+(1-\delta)]k + w(1-\tau)(1-\ell) + T - k'\right\}^{\theta} \ell^{1-\theta}\right]^{1-\sigma}}{1-\sigma} + \beta V(K',k') \right\}$$

FONC:

$$k': \quad \theta c^{\theta-1} \ell^{1-\theta} \left[c^{\theta}\ell^{1-\theta}\right]^{-\sigma} (-1) + \beta V_2(K',k') = 0 \qquad ①$$

$$\ell: \quad (1-\theta) c^{\theta} \ell^{-\theta} \left[c^{\theta}\ell^{1-\theta}\right]^{-\sigma} + \theta c^{\theta-1} \ell^{1-\theta} \left[c^{\theta}\ell^{1-\theta}\right]^{-\sigma} w(1-\tau)(-1) = 0 \qquad ②$$

Envelope Condition

$$V_2(K,k) = \theta c^{\theta-1} \ell^{1-\theta} \left[c^{\theta}\ell^{1-\theta}\right]^{-\sigma} \left[r(1-\tau)+(1-\delta)\right] \qquad ③$$

from ① and ③ : (for clarity I switch to time subscript notation):

$$\cancel{\theta} c_t^{\theta-1} \ell_t^{1-\theta} \left[c_t^{\theta} \ell_t^{1-\theta}\right]^{-\sigma} = \beta \cancel{\theta} c_{t+1}^{\theta-1} \ell_{t+1}^{1-\theta} \left[c_{t+1}^{\theta} \ell_{t+1}^{1-\theta}\right]^{-\sigma} \left[r_{t+1}(1-\tau_{t+1})+1-\delta\right]$$

$$\Rightarrow \boxed{\frac{\left[c_t^{\theta} \ell_t^{1-\theta}\right]^{1-\sigma}}{c_t} = \beta \frac{\left[c_{t+1}^{\theta} \ell_{t+1}^{1-\theta}\right]^{1-\sigma}}{c_{t+1}} \left[r_{t+1}(1-\tau_{t+1}) + 1-\delta\right].} \qquad ④$$

from ② :

$$(1-\theta) \frac{\cancel{\left[c_t^{\theta} \ell_t^{1-\theta}\right]^{1-\sigma}}}{\ell_t} = \theta \frac{\cancel{\left[c_t^{\theta} \ell_t^{1-\theta}\right]^{1-\sigma}}}{c_t} w_t(1-\tau_t)$$

$$\Rightarrow \boxed{\frac{(1-\theta)}{\theta} \frac{c_t}{w_t(1-\tau_t)} = \ell_t} \qquad ⑤$$

The solution to this problem takes the form of decision rules:

$$k' = g^{k'}(K, k) \qquad c = g^c(K, k)$$
$$\ell = g^{\ell}(K, k) \qquad i = g^i(K, k)$$

Definition of RCE:

A RCE consists of a set of decision rules for individuals $g^{\ell}(K, k)$, $g^{k'}(K, k)$, g^c, g^i, a set of per capita decision rules $G(K)$, $\Gamma(K)$ and a value function $V(K, k)$, price functions $w(\frac{K}{L})$, $r(\frac{K}{L})$ output function $f(\frac{K}{L})$, transfer function $T(\frac{K}{L})$ such that:

- given $w(\frac{K}{L})$, $r(\frac{K}{L})$, $T(\frac{K}{L})$ and aggregate decision rules $\Gamma(K)$, $G(K)$ the value function $V(K, k)$ solves the HH (FE), where $g^{k'}(K, k)$, $g^{\ell}(K, k)$, g^c, g^i are the optimal decision rules.

- given $w(\frac{K}{L})$, $r(\frac{K}{L})$ the output function $f(\frac{K}{L})$ solves the firm's problem.

- markets clear

$$L = 1 - \ell$$
$$K = k$$
$$g^c(K, K) + g^i(K, K) = f(\tfrac{K}{L})$$

- govt. budget constraint satisfied

$$T(\tfrac{K}{L}) = \tau f(\tfrac{K}{L})$$

4. (a) False. The purpose of the Lucas model is to provide a rationalization of why prices are sticky and therefore how surprises in the money supply can possibly affect output. The implication is that unexpected money growth can increase both output above its normal and inflation above what is expected. That is, if the policymaker surprises the public with unexpected increase in money it can get output gains. It will appear then that when inflation is high output is high (this is the positive reduced form correlation found in the Phillips Curve). However, if the policymaker switches to a policy of permanently higher money growth, the public will figure this out and then there will be no output gains (only price changes). According to the Lucas critique, once the policymaker tries to exploit a reduced form correlation that correlation can disappear.

   (b) False. Discretionary monetary policy is subject to the time inconsistency problem. Once the time of implementation arrives the government has the incentive to cheat and deviate from its previously announced low inflation policy, in order to get output gains (by causing high inflation). But then you will just end up with high inflation without output gains because the public will figure it out. With binding rules the government cannot cheat but the problem is that it does not allow for flexibility in responding to unexpected circumstances. This does not mean that the answer is discretion. The policymaker has to somehow tie its own hands by building credibility (reputation) or delegating authority for the conduct of monetary policy to a body that does not share the same preferences as the policymaker (delegation).

5. (a) The two dynamic equation of this model are the Euler equation and the capital accumulation equation (in units of effective labor),

$$\frac{\dot{c}(t)}{c(t)+G_0} = \frac{\alpha k(t)^{\alpha-1}) - \rho - \theta g}{\theta}$$

$$\dot{k}(t) = k(t)^{\alpha} - c(t) - G_0 - (n+g)k(t)$$

   In long-run equilibrium (steady state or BGP) we have that: $\dot{c} = \dot{k} = 0$. The $\dot{c} = 0$ schedule implies that $k^* = \left(\frac{\alpha}{\rho+\theta g}\right)^{\frac{1}{1-\alpha}}$. The $\dot{k} = 0$ schedule then implies that, $c^* = (k^*)^{\alpha} - (n+g)k^* - G_0$. And the production function implies, $y^* = (k^*)^{\alpha}$. The BGP is depicted as point E in Fig.3. Since y is constant on the BGP the growth rate of output per worker is, $\frac{\dot{Y/L}}{Y/L} = \frac{\dot{y}}{y} + \frac{\dot{A}}{A} = g$.

   (b) When government purchases increase unexpectedly and permanently to $G_1 > G_0$ the $\dot{k} = 0$ schedule shifts down just as in the standard textbook case with wasteful government expenditures. The reason is that the government is now taking away more resources from the economy. See Fig.4. The $\dot{c} = 0$ schedule does not shift. The reason is that the increase in G leads to a one-for-one decrease in consumption (note that c and G are perfect substitutes here). So changes in G will affect only the level of c but will leave its growth rate unaffected. So at the time of the change (t_0) c jumps down by the full amount of the increase in G, to place the economy on the new BGP - point E' in Fig.4. Thus k is not affected and consequently $r = \alpha k^{\alpha-1}$ is unaffected.

Fig.3

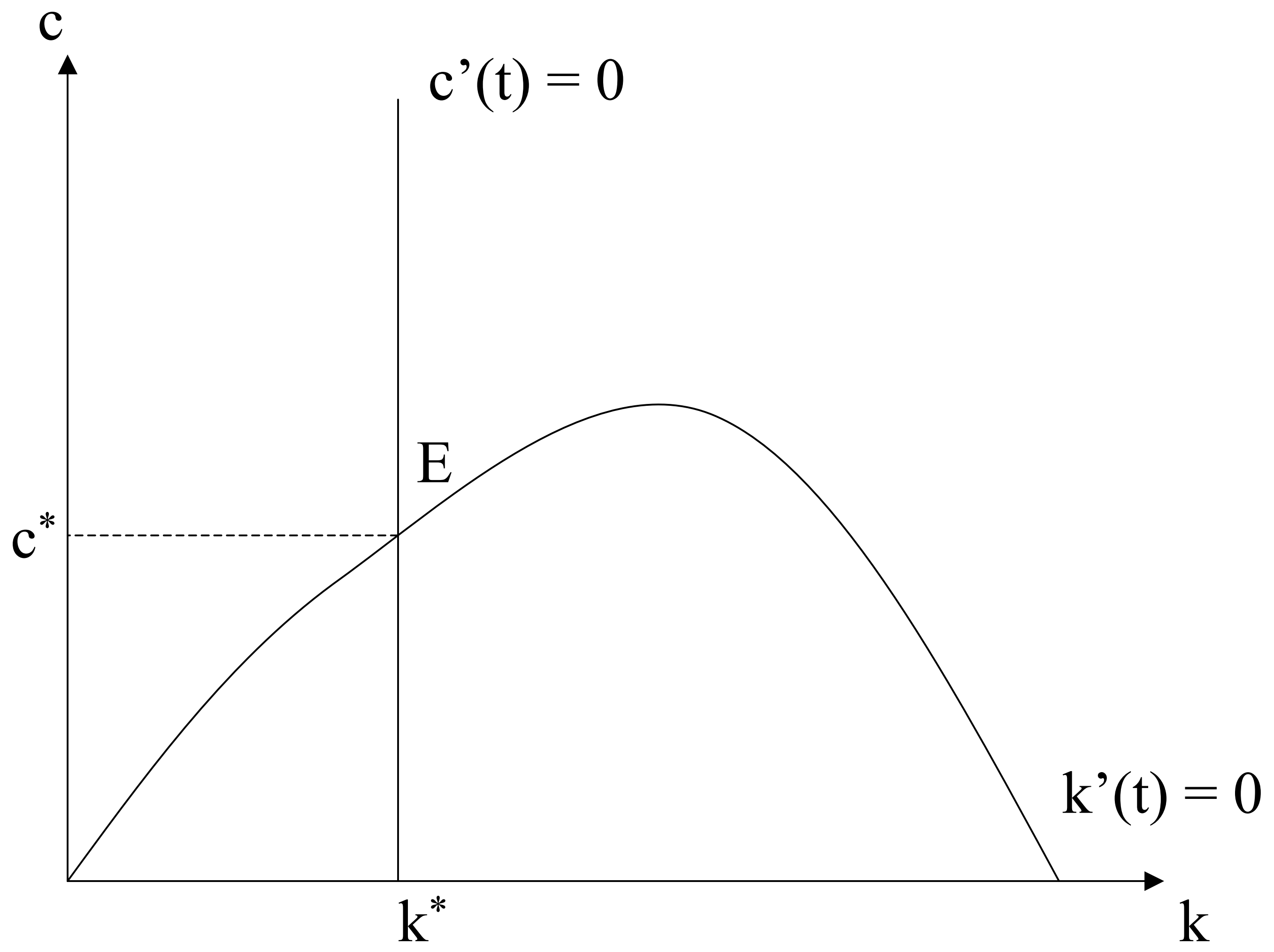

Fig.4