**University of Toronto**
**Department of Economics**
**ECO 2061H**
**Economic Theory - Macroeconomics (MA)**
**Winter 2012**

**Professor Tasso Adamopoulos**

**Sample Final**
Total Time: 3 hours
Total Marks: 100

1. Consider an overlapping generations economy that goes on to infinity, but in which individuals live only for two periods each. The size of each cohort remains constant over time. There is no production in this economy. Individuals receive exogenous endowments in each period of life. In particular, let $y_t$ denote the endowment of an individual born at time $t$. The second period endowment of the same individual (born at time $t$) is $y_t(1+\gamma)$, where $\gamma$ can be negative. Individuals can save through a storage technology, which pays a fixed return $1+r$ next period for every unit of the endowment saved this period. An individual born at time $t$ has preferences over consumption in the two periods of his life,

$$U_t = \ln(c_{1t}) + \frac{1}{1+\rho}\ln(c_{2t+1})$$

where $c_{1t}$ is the consumption of an individual born at time $t$ in the first period of his life, and $c_{2t+1}$ is the consumption of the same individual in the second period of his life. $(1+\rho)^{-1} > 0$ is the subjective discount factor. Finally, assume that first period endowments grow at rate $\phi$ between successive generations of young, so that $y_t = (1+\phi)y_{t-1}$.

   (a) Write down the resource constraint for this economy at time $t$.

   (b) Write down the budget constraints for an individual born at time $t$. Assuming that the individual maximizes utility, solve for his choice of first period consumption and savings rate.

   (c) How does the growth of income expected by one individual $\gamma$ affect his savings rate? Explain. How does an increase in $\phi$ affect the growth of aggregate savings in this economy? Explain.

2. Consider a real business cycle model with only technology shocks. The representative household is endowed with one unit of time each period and has expected lifetime utility

of,

$$E_0 \sum_{t=0}^{\infty} e^{-\rho t} \left\{ \frac{\left[c_t^{1-\theta} (1-\ell_t)^{\theta}\right]^{1-\sigma} - 1}{1-\sigma} \right\}$$

where $c$ is per capita consumption and $\ell$ is the amount of time spent working. Assume that $0 < \theta < 1$, where $\theta$ is the share parameter for leisure. The parameter $\sigma > 0$ determines the intertemporal elasticity of substitution $1/\sigma$. Capital depreciates at rate $\delta$. Denote the real interest rate by $r_t$, and the wage rate by $w_t$. Assume that labor income is taxed at rate $\tau_t$ per period, and the tax revenues are rebated in a lump-sum fashion to the household. Assume that output is produced according to a Cobb-Douglas production function $Y(t) = K(t)^{\alpha} [A(t)L(t)]^{1-\alpha}$, where the technology term follows: $\ln A_t = \overline{A} + gt + \widetilde{A}_t$. The stochastic component of technology $\widetilde{A}_t$ is assumed to follow an autoregressive process of order one. Markets are competitive.

(a) Write down the household's problem and derive its first order necessary conditions.

(b) Derive three relationships: (i) between consumption today and consumption tomorrow, (ii) between leisure today and leisure tomorrow, (iii) between consumption today and leisure today.

(c) Consider the deterministic version of the model (no uncertainty) and assume $\theta = 1$. How does the relative choice of labor supply between the two periods depend on the relative taxes between periods? Provide intuition for your answer.

3. Consider an infinitely lived agent economy with no uncertainty. The economy is populated by a continuum of agents of measure 1. Each agent is endowed with one unit of time, which is allocated between leisure and work, and with initial capital holdings $k_0$. Capital and labor income for each individual is taxed at the same rate $\tau_t < 1$. Tax revenues are used to make equal lump-sum transfers $T_t$ to individuals. The government is assumed to balance its budget each period. The preferences of the representative agent in the economy, over sequences of consumption-leisure bundles are described by,

$$max \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t^{\theta} \ell_t^{1-\theta}\right)^{1-\sigma}}{1-\sigma}$$

where $c$ and $\ell$ denote the representative agent's consumption and leisure respectively. $\beta < 1$ is the subjective discount factor. The parameter $\theta < 1$ determines the weight of consumption in the instantaneous utility function, while the parameter $\sigma$ determines the intertemporal elasticity of substitution. The capital holding of the representative individual evolve according to,

$$k_{t+1} = (1-\delta)k_t + i_t$$

where $\delta < 1$ is the rate of depreciation. Production in the economy takes place according to the constant returns to scale technology,

$$Y_t = F(K_t, L_t) = AK_t^{\alpha}L_t^{1-\alpha}$$

where $K$ is aggregate capital and $L$ is aggregate labor for this economy. $A$ measures TFP and is constant over time as is the total population size. Denote the wage rate and the rental price of capital in period $t$ by $w_t$ and $r_t$ respectively. Firms hire factors and sell their output in competitive markets.

(a) Derive the first order conditions from the problem of the representative form.

(b) Write down the representative agent's problem in dynamic programming form. Be careful in defining economy-wide and individual states. Define a recursive competitive equilibrium for this economy.

(c) Using the fact that the value function is differentiable derive the Euler equations for the representative individual.

4. Explain whether the following statements are true, false or uncertain.

   (a) "The Lucas incomplete information model provides a rationalization of why policy-makers should cause inflation."

   (b) "Discretionary monetary policy is better than rigid rules because it allows flexibility for the policymaker, against unforseen developments in the economy."

5. Consider the Ramsey-Cass-Koopmans model with government expenditures. Assume that government purchases affect the utility from private consumption. In particular, assume that lifetime utility in units of effective labor is,

$$U = B\int_{t=0}^{\infty} e^{-\beta t}\frac{[c(t)+G(t)]^{1-\theta}}{1-\theta}$$

where private consumption $c(t)$ and government purchases $G(t)$ (in units of effective labor) are perfect substitutes. Assume that government purchases are constant at a level, $G(t) = G_0$. The production function in intensive form is $y(t) = f(k(t)) = k(t)^{\alpha}$. There is no depreciation. The Euler equation for the household's maximization problem can be shown to be,

$$\frac{\dot{c}(t)}{c(t)+G_0} = \frac{f'(k(t)) - \rho - \theta g}{\theta}$$

Suppose the economy is initially in long-run equilibrium (on the balanced growth path).

(a) Using a carefully labelled diagram, show the economy's long-run equilibrium. Solve for the long-run equilibrium values of $c$ and $k$. What is the growth rate of output per worker?

(b) Suppose that at time $t_0$, government purchases increase unexpectedly and permanently to $G_1 > G_0$. Show the new long-run equilibrium on a graph. What happens to the capital stock, consumption, and the interest rate in the adjustment to the new balanced growth path.