University of Toronto
Department of Economics
ECO 2061H
Economic Theory - Macroeconomics (MA)
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Answers to Practice Set 2

1. Exercise 2.10 from Romer (pp. 95-96).

   (a) The two key equations of the Ramsey-Cass-Koopmans model are the Euler and the law of motion for capital per unit of effective labor. When we have taxes on investment income, that alters the real rate of return that households face, $r(t) = (1-\tau)f'(k(t))$, and thus distorts the household's choices. In particular the Euler equation now becomes,

   $$\frac{\dot{c}(t)}{c(t)} = \frac{(1-\tau)f'(k(t)) - \rho - \theta g}{\theta}$$

   The law of motion for capital is not affected. The reason is that tax revenues are rebated to households in a lump-sum fashion, and thus the government does not take any resources from the economy,

   $$\dot{k}(t) = f(k(t)) - c(t) - (n+g)\,k(t)$$

   How does the $\dot{c}(t) = 0$ locus change? In the presence of taxes when $\dot{c} = 0$, the steady state level of k is given by, $f'(k_T^*) = \frac{\rho+\theta g}{1-\tau}$, where the subscript T is for "tax". With no taxes the steady state level of k is given by, $f'(k_{NT}^*) = \rho + \theta g$, where the subscript NT stands for "no-tax". Since $\tau < 1$, it follows that $\frac{\rho+\theta g}{1-\tau} > \rho + \theta g$ and thus $f'(k_T^*) > f'(k_{NT}^*)$. Since we have a diminishing marginal product of capital this implies that $k_T^* < k_{NT}^*$. In other words, the steady state with taxes lies below the steady state without taxes. Intuitively, the more you tax capital income the less capital you will have in the long-run, because such a tax reduces the incentive to accumulate capital. Fig.5 shows the shifted $\dot{c} = 0$ locus.

   (b) Given that at the time of the change k is pre-determined (and thus given), c must jump to place the economy on the new saddle path that will deliver the economy to the new BGP. See Fig. 5.

(c) See Fig.5. Both are lower in the new BGP. Intuitively, the tax reduces your capital stock in the long run, which reduces you capacity to consume.

(d) (i) Show that the saving rate on the BGP is decreasing in the tax, i.e., we want to show that $s^* \equiv \frac{y^*-c^*}{y^*}$ is decreasing in τ or $\frac{\partial s^*}{\partial \tau} < 0$. Note that by definition of the production function $y^* = f(k^*)$ and by setting $\dot{c} = \dot{k} = 0$, in the steady state, $f'(k^*) = \frac{\rho+\theta g}{1-\tau}$ and $c^* = f(k^*) - (n+g)k^*$ respectively. Plug (y^*, c^*) in s^* to get,

$$s^* = \frac{y^* - c^*}{y^*} = \frac{f(k^*) - f(k^*) + (n+g)k^*}{f(k^*)} = \frac{(n+g)k^*}{f(k^*)}$$

Now take the derivative with respect to τ. Note that from $f'(k^*) = \frac{\rho+\theta g}{1-\tau}$, k^* is implicitly a function of τ.

$$\frac{\partial s^*}{\partial \tau} = \frac{(n+g)\frac{\partial k^*}{\partial \tau}}{f(k^*)} - \frac{(n+g)k^*}{[f(k^*)]^2} f'(k^*)\frac{\partial k^*}{\partial \tau} =$$

$$= \frac{(n+g)}{f(k^*)}\left[1 - \frac{f'(k^*)k^*}{f(k^*)}\right]\frac{\partial k^*}{\partial \tau} = \frac{(n+g)}{f(k^*)}\left[1 - \alpha_K(k^*)\right]\frac{\partial k^*}{\partial \tau}$$

where $\alpha_K(k^*) \equiv \frac{f'(k^*)k^*}{f(k^*)}$ is the elasticity of output with respect to capital. Since $\alpha_K < 1$, the sign of $\frac{\partial s^*}{\partial \tau}$ is determined by the sign of $\frac{\partial k^*}{\partial \tau}$. The sign of $\frac{\partial k^*}{\partial \tau}$ can be determined by the Euler equations with $\dot{c} = 0$, i.e., from $f'(k^*) = \frac{\rho+\theta g}{1-\tau}$. Differentiating this you get: $\frac{\partial k^*}{\partial \tau} = \frac{f'(k^*)}{(1-\tau)f''(k^*)} < 0$. Thus we have that $\frac{\partial s^*}{\partial \tau} < 0$.

(ii) Citizens in low τ, high k^* countries would not like to invest elsewhere. They would if the net return, r, was higher. Here they would not though, because in the long run the real interest rate is pinned down by preferences, $r = \rho + \theta g$. Regardless of the tax policy (τ, k^*) offset each other, i.e., k adjusts so that r is the same across countries.

(e) The subsidy (financed by lump sum taxes) would not raise welfare because it would be distortionary. The competitive equilibrium is already Pareto efficient (recall first welfare theorem). The policy free allocation is the same as what a social planner would have chosen. With a subsidy, even though you would move to an allocation with both higher c^* and k^* it would not be welfare improving: the cost of the initial decrease in consumption outweighs the long run benefit of the increase in long run consumption and capital.

(f) If revenues from the tax were used to finance government expenditures (rather than rebates) then not only would the Euler equation change as in part (a) but the law of motion for capital per unit of effective labor would too. Reason: the government is now taking resources from the economy. The new law of motion is,

$$\dot{k}(t) = f(k(t)) - c(t) - G(t) - (n + g)\, k(t)$$

In this case $\dot{c} = 0$ locus would shift left as before, but also the $\dot{k} = 0$ locus would shift down. In the new long run equilibrium both c^* and k^* would be lower than in the case without taxes. Whether initially c will jump up or down depends on whether the saddle path intersects the original $\dot{c} = 0$ locus above or below the original BGP equilibrium (point E). See Fig.6.

Fig.5: Tax vs. No-Tax BGP and Adjustment

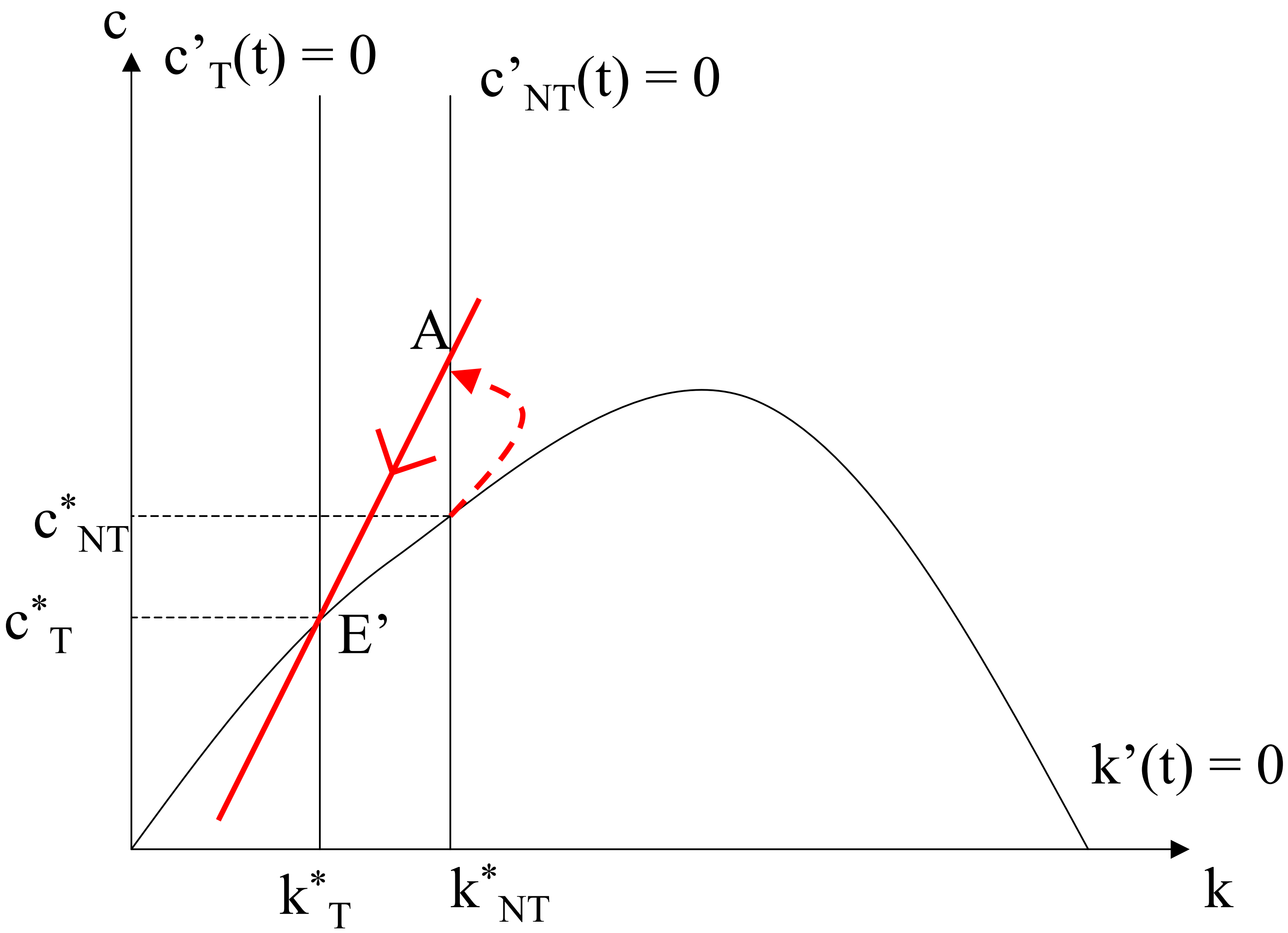
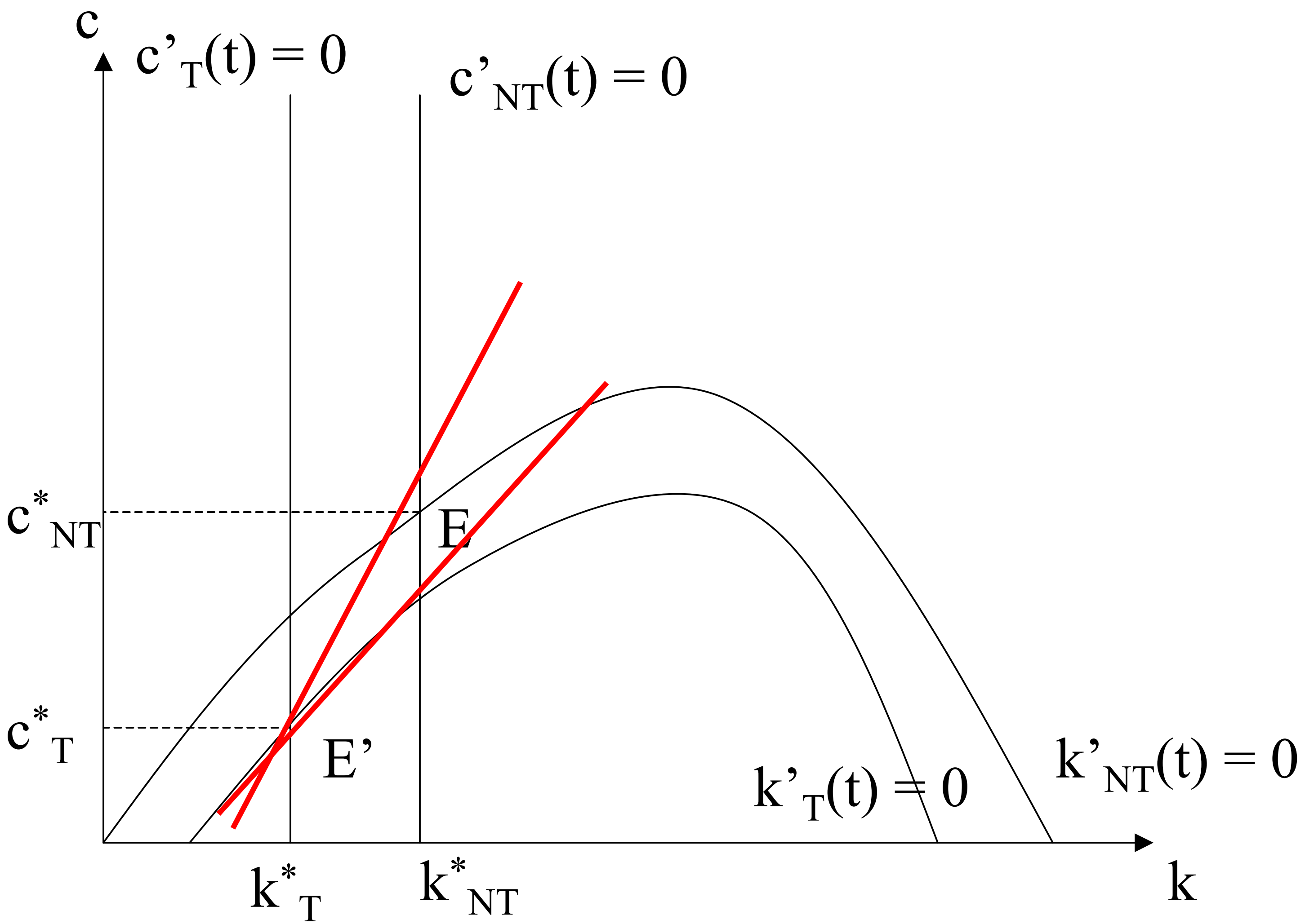


Fig.6: Tax and Wasteful Government Expenditures



Romer 2.6

(a) Equation describing dynamics of the capital stock per unit of effective labor (law of motion):

$$\dot{k}(t) = f(k(t)) - c(t) - (n+g)k(t)$$

For a given k, the level of c that implies $\dot{k}=0$ is given by $c = f(k) - (n+g)k$. Thus a falling g makes the level of c consistent with $\dot{k}=0$ higher for a given k.

⇒ the $\dot{k}=0$ locus shifts up.

Intuitively: a lower g makes break-even investment lower at any given k and thus allows for more resources to be devoted to cons. and still maintain a given k.

Since $(n+g)k$ falls proportionately more at higher levels of k the $\dot{k}=0$ curve shifts up more at higher levels of k.

(b) The equation describing the dynamics of cons. per unit of effective labor (Euler) is:

$$\frac{\dot{c}(t)}{c(t)} = \frac{f'(k(t)) - \rho - \theta g}{\theta}$$

The condition required for $\dot{c}=0$ is given by $f'(k) = \rho + \theta g$. After the fall in g, $f'(k)$ must be lower in order for $\dot{c}=0$. Since $f''(k) < 0$ this means that the k needed for $\dot{c}=0$ rises.

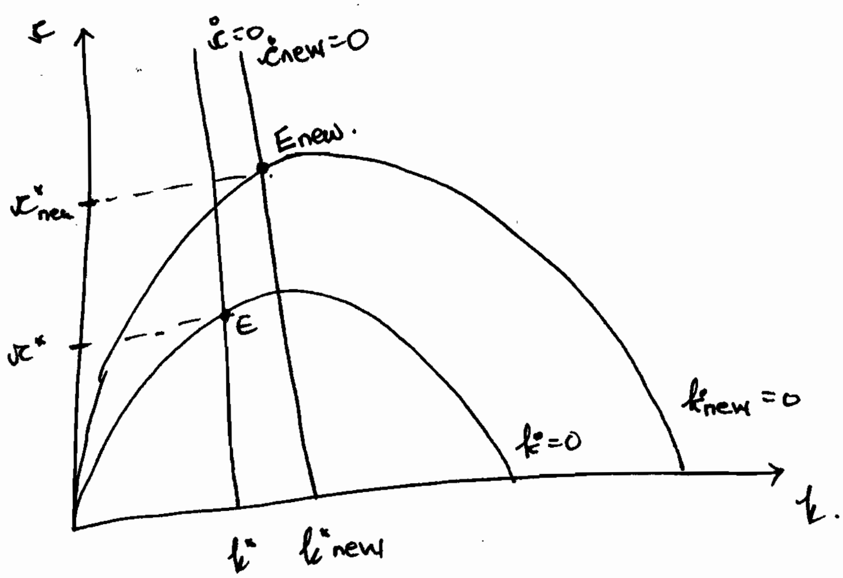
⇒ $\dot{c}=0$ locus shifts to the right.

(c) At the time of the change in g, k is pre-determined and cannot change discontinuously. It remains equal to k^* on old BGP.

On the other hand, c, the rate at which HHs are consuming in units of effective labor. can jump up at the time of the shock.

In order for the econ. to reach the new BGP, χ must jump at the instant of the change so that the economy is on the new saddle path.

However, we cannot tell whether the new saddle path passes above or below the original point E. So we cannot tell whether χ jumps up or down and in fact, if the new saddle path passes right through point E, χ might even remain the same at the instant that g falls. Thereafter χ, k rise gradually to their new BGP values which are higher than their values in the initial BGP.



d) Ignore this part.

Romer 2.7

The two key equations of motion are

$$\frac{\dot{c}(t)}{c(t)} = \frac{f'(k(t)) - \rho - \theta g}{\theta} \qquad \text{(Euler)}$$

$$\dot{k}(t) = f(k(t)) - c(t) - (n+g)k(t) \qquad \text{(Law of motion for } k\text{)}$$

(a) Rise θ ⇒ Fall in the elasticity of substitution $\frac{1}{\theta}$ ⇒

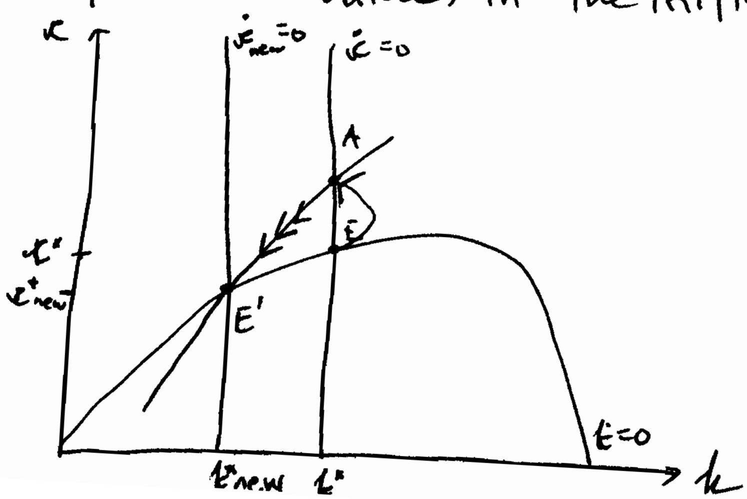
⇒ households become less willing to substitute consumption between periods.

The marginal utility of consumption falls off more rapidly as consumption rises.

If the economy is growing, this makes households value present cons. more than future consumption.

The capital accumulation equation ($\dot{k}(t)$) is not affected. The condition required for $\dot{c}=0$ is given by $f'(k) = \rho + \theta g$. Since $f''(k) < 0$, the $f'(k)$ that makes $\dot{c}=0$ is now higher. Thus the value of k satisfying $\dot{c}=0$ is lower. The $\dot{c}=0$ locus shifts to the left.

The economy moves up to point A on the new saddlepath. People now consume more. Then the economy rides the new saddle path until it reaches the new BGP equilibrium at E'. At that point (c^*, k^*) are both lower than their equilibrium values in the initial BGP.



(b) We can assume that a downward shift of the production function means that for any given k both $f(k)$ and $f'(k)$ are lower than before.

The condition required for $\dot{k}=0$ is given by $c = f(k) - (n+g)k$. The $\dot{k}=0$ locus will shift down more at higher levels of k. Also since for a given k, $f'(k)$ is lower now, the golden rule k will be lower than before.

The condition required for $\dot{c}=0$ is given by $f'(k) = \rho + \theta g$. For a given k, $f'(k)$ is now lower. Thus we need a lower k to keep $f'(k)$ the same and satisfy the $\dot{c}=0$ equation. Thus the $\dot{c}=0$ locus shifts left.

The economy will eventually reach E' with lower (c^*, k^*). Whether initially c jumps up or down depends upon whether the new saddle path passes above or below E.

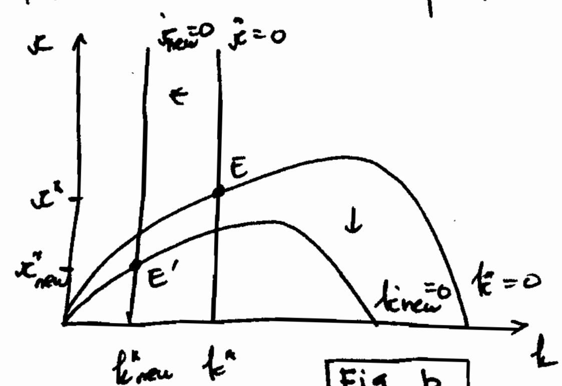
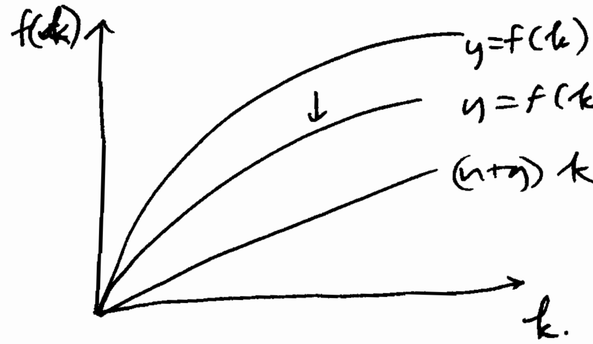


Fig. b



(c) With $\delta > 0$ the new law of motion is: $\dot{k}(t) = f(k(t)) - c(t) - (n+g)k(t)$.
The level of saving/investment required to keep any given k constant is now higher and thus the amount of consumption possible is now lower than in the case with no depreciation.
The level of extra investment required is also higher at higher levels of k. Thus the $\dot{k}=0$ locus shifts down more at higher levels of k. Also the real return to capital is now $f'(k) - \delta$ and the Euler becomes: $\dfrac{\dot{c}(t)}{c(t)} = \dfrac{f'(k(t)) - \rho - \theta g}{\theta}$.
If $\dot{c}=0 \Rightarrow f'(k) = \rho + \theta g + \delta$. Relative to the case with $\delta=0$ $f'(k)$ must be higher and k lower for $\dot{c}=0$. The $\dot{c}=0$ locus shifts left. Economy goes to E' in the long-run. with lower (c^*, k^*).
Again c can jump up or down. Fig. similar to Fig. b. applies here too.