University of Toronto
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Professor Tasso Adamopoulos

Answers to Assignment 3

1. (a) Under complete knowledge, the producer of good i in the Lucas model produces and supplies labor according to: $q_i = \ell_i = \frac{1}{\gamma-1}(p_i - p) = \frac{1}{\gamma-1} r_i$. Given the assumption of certainty equivalence, under imperfect information, the labor supply (quantity production) decision will be: $\ell_i = \frac{1}{\gamma-1} E(r_i|p_i)$, where $E(r_i \mid p_i)$ is the conditional expectation of the relative price (given that the own price is observed). Given the Bayesian updating of the aggregate price, $E(p \mid p_i) = (1-\theta) E(p) + \theta p_i$, we have:

$$E(r_i \mid p_i) = p_i - E(p \mid p_i) = p_i - (1-\theta) E(p) - \theta p_i =$$

$$= (1-\theta)(p_i - E(p)) = \frac{V_r}{V_p + V_r}(p_i - E(p))$$

where I have used the definition of θ in the last equality. Consequently the supply of labor (and output) of individual i is,

$$q_i = \ell_i = \frac{1}{\gamma - 1} E(r_i|p_i) = \frac{1}{\gamma - 1}\frac{V_r}{V_p + V_r}(p_i - E(p)) = b(p_i - E(p))$$

where $b \equiv \frac{1}{\gamma-1}\frac{V_r}{V_p+V_r}$. Take the average over all q_i's to get aggregate output in this economy. This implies, $y = b(p - E(p))$, which is the Lucas aggregate supply curve.

(b) The Lucas AS curve derived in part (a) and the AD curve given in the question imply that in equilibrium (equalize quantity demanded and supplied),

$$b(p - E(p)) = m - p \Rightarrow p = \frac{1}{1+b} m + \frac{b}{1+b} E(p)$$

Substitute this into the AD curve to get output,

$$y = \frac{b}{1+b} m - \frac{b}{1+b} E(p)$$

Take expectations through in the expression for the price level,

$$E(p) = \frac{1}{1+b} E(m) + \frac{b}{1+b} E(p) \Rightarrow E(p) = E(m)$$

Also note that we can re-write m as $m = E(m) + (m - E(m))$. Using these last two equations we can re-write the aggregate price level and the aggregate output level respectively as,

$$p = E(m) + \frac{1}{1+b}(m - E(m))$$
$$y = \frac{b}{1+b}(m - E(m))$$

Think of $E(m)$ as the observed component of monetary policy (or aggregate demand more generally), and $m - E(m)$ as the unobserved component. These equations say that observed changes in monetary policy affect only the price level but have no real effects (on output or labor supply). Intuitively, this occurs because agents in the economy *know* that it is an increase in economy-wide demand and not an increase in the demand for their particular product. If however, there are unobserved changes in monetary policy, then they can have real effects. For example, if $m - E(m)$ increases, i.e., if there is unexpected increase in the money supply, then aggregate output will increase. Intuitively, the reason this happens is that when individuals in the economy see an increase in p_i they do not know whether this is coming from an increase in p or r_i and therefore they end up each increasing their labor supply and output. Since all do this aggregate output increases. To sum up: only surprises in aggregate demand can increase aggregate output, known events do not.

(c) False. This statement ignores the Lucas critique. To see this, suppose that there is an unexpected increase in money. Then according to part (b) this will lead to an increase in both the aggregate price level and the aggregate level of output. In other words, they will appear positively correlated. However, this is only a reduced-form correlation. It occurred because people had given expectations and those expectations were falsified by actual policy. However, once individuals figure out that the change has occurred, then there is no surprise and output does not change. In other words, the statement is false because it does not take into account that people's expectations can change if policy changes. Thus reduced form correlations can disappear once policymakers try to exploit them because people's expectations change.

2. (a) If the policymaker can make a binding commitment about inflation then the public expectations about inflation will be equal to the announced and actual inflation rate: $\pi^e = \pi$. Then from the Lucas supply curve, this implies $y = \overline{y}$. Substitute these two things into the policymaker's objective function, and maximize with respect to

π

$$\min_{\pi} \frac{1}{2}(\overline{y}-y^*)^2+\frac{1}{2}a(\pi-\pi^*)^2$$

The first order condition from this problem implies, $a(\pi-\pi^*)=0$. This means that: $\pi=\pi^*$, i.e., the policymaker chooses the optimal rate of inflation if there is commitment.

(b) If the policymaker has discretion then he chooses policy taking expectation of inflation by the public (π^e) as given. The policymaker's objective in this case is,

$$\min_{\pi} \frac{1}{2}(\overline{y}+b(\pi-\pi^e)-y^*)^2+\frac{1}{2}a(\pi-\pi^*)^2$$

Taking the first order condition with respect to π produces,

$$(\overline{y}+b(\pi-\pi^e)-y^*)b+a(\pi-\pi^*)=0$$

Re-arrange this and solve for π as a function of expectations π^e,

$$\pi=\pi^*+\frac{b}{a+b^2}(y^*-\overline{y})+\frac{b^2}{a+b^2}(\pi^e-\pi^*)$$

Think of this as the policymaker's reaction function to the private sector's expectations. Since there is no uncertainty, a rational expectations equilibrium requires that expected inflation is equal to actual (agents correctly predict inflation): $\pi^e=\pi$. Substitute this into the last equation and solve for π to get the equilibrium rate of inflation,

$$\pi=\pi^*+\frac{b}{a}(y^*-\overline{y})>\pi^*$$

But since $\pi^e=\pi$, from the Lucas supply curve we end up having, $y=\overline{y}$. In other words, with discretion, you end up having more inflation but the same amount of output relative to the case of commitment.