

**University of Toronto**  
**Department of Economics**  
**ECO 2061H**  
**Economic Theory - Macroeconomics (MA)**  
**Winter 2012**

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**Assignment 2**

Due: Tuesday, March 20, 2012 (9:00a.m. in class)

1. Consider a real business cycle model with only technology shocks. The representative household is endowed with one unit of time each period and has expected lifetime utility of,

$$E_0 \sum_{t=0}^{\infty} e^{-\rho t} \left[ \ln(c_t) + \frac{(1 - \ell_t)^\theta}{\theta} \right]$$

where  $c$  is per capita consumption and  $\ell$  is the amount of time spent working. Assume  $0 < \theta < 1$ . Capital depreciates at rate  $\delta$ . Denote the real interest rate by  $r_t$ , and the wage rate by  $w_t$ . Assume that labor income is taxed at rate  $\tau_t$  per period, and the tax revenues are rebated in a lump-sum fashion to the household. Assume that output is produced according to a Cobb-Douglas production function  $Y(t) = K(t)^\alpha [A(t)L(t)]^{1-\alpha}$ , where the technology term follows:  $\ln A_t = \bar{A} + gt + \tilde{A}_t$ . The stochastic component of technology  $\tilde{A}_t$  is assumed to follow an autoregressive process of order one. Markets are competitive.

- (a) Write down the household's problem and derive its first order necessary conditions.
  - (b) Derive three relationships: (i) between consumption today and consumption tomorrow, (ii) between leisure today and leisure tomorrow, (iii) between consumption today and leisure today.
  - (c) How is the current level of consumption affected if the covariance between  $c_t$  and  $r_t$  is positive rather than negative. Explain.
2. Consider a standard growth model with the following features. There is only one sector that produces consumption and investment goods using the same constant returns to scale technology:

$$Y_t = F(K_t, L_t) = K_t^\alpha (A_t L_t)^{1-\alpha}$$

where  $A_t$  is a technology parameter that grows at the exogenous rate  $g$ . The price received by producers of consumption and investment goods is equal. Normalize this

price to one in each period. There is an infinitely lived representative household, with preferences:

$$\sum_{t=0}^{\infty} \beta^t \frac{(C_t/L_t)^{1-\sigma}}{1-\sigma} L_t$$

where  $L$  is the size of the working population, growing exogenously according to  $L_{t+1} = (1+n)L_t$ . The representative household owns the capital in the economy, rents labor and capital services to firms, and spends income on consumption and investment. Households pay a tax  $\theta$  for each unit of investment. Therefore,  $\theta$  represents the level of barriers to investment or distortions in the economy. We assume that all revenues are rebated to consumers as a lump-sum transfer  $T$ . The law of motion for capital is:

$$K_{t+1} = (1 - \delta)K_t + X_t$$

- (a) Write down the household's budget constraint and the economy's resource constraint.
- (b) Define a recursive competitive equilibrium for this economy. Make sure to specify the problems of households and firms clearly, as well as the government budget constraint. (Hint: before defining equilibrium transform all variables in the problem in terms of effective units of labor, denoting them by lower case letters. For example  $y_t = \frac{Y_t}{A_t L_t}$ , and similarly define  $c$ ,  $k$ ,  $x$  and transfers  $\tau$ . Show that the adjusted discounted factor of the transformed problem is,  $\tilde{\beta} = \beta(1+g)^{1-\sigma}(1+n)$ ).
- (c) Derive the Euler equation for the household using dynamic programming. Write down the first order conditions to the firm's problem.
- (d) Define a steady state equilibrium. Derive the investment rate (investment-output ratio) and output per working age person in the steady state of this economy (Hint: do this in terms of the transformed variables).

**For the next parts use Penn World Table version 6.1.**

- (e) Plot the relative investment rate in *international prices* (ratio of US to country  $i$ ) against relative real GDP per worker in 1996. What is the correlation coefficient between these two variables?
- (f) Plot the relative investment rate in *domestic prices* (ratio of US to country  $i$ ) against relative real GDP per worker in 1996. What is the correlation coefficient between these two variables?
- (g) Plot the relative price of investment to consumption (ratio of US to country  $i$ ) against the relative investment rate in international prices, in 1996.

- (h) Consider the average top 5% (denote group  $R$ ) and average bottom 5% (denote group  $P$ ) in real GDP per worker. Calculate the ratio between these two groups in the relative price of investment to consumption. Assume  $R$  and  $P$ , are identical in all other parameters except for the tax rate  $\theta$ . We will proxy the disparity in  $1 + \theta$  by the relative price of investment. In particular assume that  $\theta_R = 0$  and  $1 + \theta_P$  is equal to the ratio in the relative price of investment ( $P/R$ ). With  $\alpha = 1/3$ , what is the factor difference in investment rates between these two economies according to the above model? What is the disparity in output per worker between these two economies according to the model? How do these predicted disparities compare to the actual ones observed in the data?