University of Toronto
Department of Economics
ECO 2061H
Economic Theory - Macroeconomics (MA)
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Assignment 1

Due: Thursday, February 9, 2012 (9:00a.m. in class)

1. Briefly explain whether the following statement is true or false: "The Solow model cannot be a good description of reality because today there are large differences in income levels across countries."

2. Consider the baseline Ramsey-Cass-Koopmans model discussed in class. Explain what is the effect of a permanent one time increase in θ on, (a) the BGP values of c, k, (b) the transitional dynamics of $(c(t), k(t))$. Use a phase diagram to explain your answer.

3. Consider the standard Solow model. Assume that the production technology is of the Cobb-Douglas form,

$$Y(t) = K(t)^{\alpha}[A(t)L(t)]^{1-\alpha}$$

where Y is total output, K is the stock of capital, L is labor, and A is the level of efficiency. We assume that $0 < \alpha < 1$. Capital depreciates at a constant rate δ. Population and efficiency grow exogenously at constant rates n and g respectively. Lower case letters denote variables in units of effective labor, i.e., $y = \frac{Y}{AL}$, $k = \frac{K}{AL}$.

   (a) Show that the marginal products of capital and labor respectively are given by,

$$\frac{\partial Y(t)}{\partial K(t)} = \alpha k(t)^{\alpha-1}$$

$$\frac{\partial Y(t)}{\partial L(t)} = A(t)(1-\alpha)k(t)^{\alpha}$$

   (b) The evolution of aggregate capital in the economy is given by

$$\dot{K}(t) = sY(t) - \delta K(t)$$

   where s is a constant and exogenous savings rate. Re-write the law of motion for capital in terms of capital per unit of effective labor k.

(c) Solve for the values of k and y on the steady state. What are the growth rates of k, y, K, Y, and $K/L, Y/L$ on the balanced growth path.

(d) Suppose the economy starts at a level of capital stock per unit of effective labor $k(0)$ above the steady state. What are the growth rates of k, y, K, Y, and $K/L, Y/L$ in transition to the steady state. How do these growth rates compare to their balanced growth path values.

(e) The golden rule level of capital per unit of effective labor k_G is defined as the level of k that maximizes steady state consumption. Solve for k_G in this economy.

(f) Suppose the economy is initially on the steady state. At time t_0 there is a one time, permanent increase in the rate of technological progress g. How do the levels of k, y in the new steady state compare to the ones in the original steady state. How do the growth rates of k, y in the new steady state compare to the ones in the original steady state. What happens to these growth rates in transition between the two steady states.

4. Consider the augmented Solow model in which the economy's total output is produced according to the following production function

$$Y(t) = K(t)^{\alpha_K} H(t)^{\alpha_H} [A(t)L(t)]^{1-\alpha_K-\alpha_H} \tag{1}$$

where Y is total output, K is the stock of physical capital, H is the stock of human capital, L is raw labor, and A is the level of efficiency. We assume that $0 < \alpha_K, \alpha_H < 1$. Further assume that efficiency and labor grow at exogenous and constant rates g and n respectively.

(a) Does (1) exhibit constant returns to scale? Explain.

(b) Does (1) exhibit diminishing returns with respect to K and H together?

(c) Write (1) in growth accounting format, i.e., express the growth rate of output per worker in terms of the growth rate of physical capital per worker, human capital per worker, and a residual.

(d) Re-write (1) in intensive form, i.e., expressing aggregate variables, Y, K, H, in terms of units of effective labor, $y = \frac{Y}{AL}$, $k = \frac{K}{AL}$, $h = \frac{H}{AL}$ respectively.

(e) Let s_K be the fraction of income invested in physical capital, and s_H the fraction of income invested in human capital. Further assume that physical and human capital depreciate at the same rate, δ. Then the laws of motion for the two capital stocks are

$$\dot{K}(t) = s_K Y(t) - \delta K(t)$$

$$\dot{H}(t) = s_H Y(t) - \delta H(t)$$

Derive the laws of motion for k and h.

(f) Solve for the steady state levels of k, h, y. What is the growth rate of aggregate output Y, and output per worker Y/L in the steady state.

(g) Using (1) express output per worker Y/L in terms of physical capital intensity K/Y, and human capital intensity H/Y. Suppose the elasticity of output with respect to each capital stock is 1/3. Further suppose that the disparity in K/Y between two countries i and j is 3, while the disparity in H/Y is 2. A is the same in both countries. What is the predicted disparity in Y/L between these two countries, if the augmented Solow model is true?

5. Consider an economy populated by firms and households. Assume that all firms face the same production technology, which is of the Cobb-Douglas form,

$$Y(t) = K(t)^{\alpha}[A(t)L(t)]^{1-\alpha}$$

where Y is total output, K is the stock of capital, L is labor, and A is the level of efficiency, and $0 < \alpha < 1$. There is no capital depreciation. Population and efficiency grow exogenously at constant rates n and g respectively. Lower case letters denote variables in units of effective labor, i.e., $y = \frac{Y}{AL}$, $k = \frac{K}{AL}$. Each of the H identical households in the economy have preferences described by the following intertemporal utility function,

$$U = \int_{t=0}^{\infty} e^{-\rho t} \ln\left(C(t)\right) \frac{L(t)}{H} dt$$

where C is consumption per household member. Assume that $\rho - n > 0$. Consider the competitive (or decentralized) equilibrium of this economy.

(a) Assuming that firms maximize profits, derive the representative firm's first order conditions with respect to capital and labor. Express them in units of effective labor.

(b) Assume that each household has initial capital holdings $\frac{K(0)}{H}$. Also, define the interest rate factor from time 0 to time t, as $e^{R(t)} \equiv e^{\int_{\tau=0}^{t} r(\tau)d\tau}$. Write down the household's intertemporal budget constraint.

(c) Households maximize intertemporal utility subject to their intertemporal budget constraint. Re-write the household's problem in units of effective labor. In particular show that lifetime utility can be re-written as,

$$U = B_0 + B_1 \int_{t=0}^{\infty} e^{-(\rho-n)t} \ln(c(t))dt$$

where $c(t) = \frac{C(t)}{A(t)}$ and

$$B_0 = \int_{t=0}^{\infty} e^{-(\rho-n)t} gt \frac{L(0)}{H} dt + \int_{t=0}^{\infty} e^{-(\rho-n)t} \ln(A(0)) \frac{L(0)}{H} dt$$

$$B_1 = \frac{L(0)}{H}$$

B_0 is bounded (you do not need to show this). Derive the Euler equation from the household's maximization problem.

(d) Write down the economy's law of motion for capital per unit of effective labor.

(e) Solve for the steady state levels of c, y, k of this economy. Along this balanced growth path, what are the growth rates of the aggregate variables K, Y and the variables per worker $K/L, Y/L$?

(f) Assume the economy starts off below the steady state level of k. Will the economy reach the steady state? Draw a phase diagram to show the dynamics of $c(t)$ and $k(t)$.

(g) Is the competitive equilibrium you solved for socially optimal?

6. For this part of the assignment use data from the Penn World Table, version 6.3.

(a) Why is this data appropriate for making cross-country comparisons of output and its components?

(b) Download real GDP per capita and real GDP per worker (constant prices, chain series) in 2005 for all countries in the data set. Order countries by their real GDP per capita. Calculate average real GDP per capita and average real GDP per worker for each of the deciles of the world income distribution. What is the disparity in (average) real GDP per capita and (average) real GDP per worker between the richest 10% of countries and each other decile of the income distribution.

(c) Obtain real GDP per capita in 1960 for all countries available. Rank them according to real GDP per capita. Calculate the income disparity for each decile compared to the richest 10% of countries. How does the income disparity between the richest and poorest 10% of countries in 1960 compare to that in 2005?

(d) Focus on Canada and the United States. For each country plot real GDP per capita from 1950 to 2007. Calculate each country's average annual growth rate over this period. In 2007 what accounts for the income disparity between the countries, labor productivity or labor force participation? (Hint: use other information from the Penn World Table). Has that changed since 1950?