Engines of Liberation

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Abstract

Electricity was born at the dawn of the last century. Households were inundated with a flood of new consumer durable goods. What was the impact of this consumer durable goods revolution? It is argued here that the consumer goods revolution liberated women from the home. To analyze this hypothesis, a Beckerian model of household production is developed. Households must decide whether to adopt the new technologies or not, and whether a married woman should work. Can such a model explain the rise in married female labor-force participation that occurred in the last century? Yes.

Keywords: The second industrial revolution, technology adoption, household production theory, female labor-force participation.

Subject Area: Macroeconomics.

JEL Classification Numbers: E1, J2, N1.

"The housewife of the future will neither be a slave to servants or herself a drudge. She will give less attention to the home, because the home will need less; she will be rather a domestic engineer than a domestic laborer, with the greatest of all handmaidens, electricity, at her service. This and other mechanical forces will so revolutionize the woman's world that a large portion of the aggregate of woman's energy will be conserved for use in broader, more constructive fields."

Thomas Alva Edison, as interviewed in *Good Housekeeping Magazine*, LV, no. 4 (October 1912, p. 436)

1 Introduction

1.1 Facts

The dawning of the last century ushered in the second industrial revolution: the rise of electricity, the internal combustion engine, and the petrochemical industry. As this was happening, another technological revolution was beginning to percolate in the home: the housework revolution. The impact of the housework revolution was no less than the industrial revolution.

Durable Goods: The housework revolution was spawned by massive investmentspecific technological progress in the production of household capital. This era saw the rise of central heating, dryers, electric irons, frozen foods, refrigerators, sewing machines, washing machines, vacuum cleaners, and other appliances now considered fixtures of everyday life. The spread of electricity, central heating, flush toilets and running water, through the U.S. economy is shown in Figure 1.¹ Likewise, Figure 2 plots the diffusion of some common electrical appliances through American

 $^{^{1}}$ The data for this figure is taken from Lebergott (1976, p. 100) and Lebergott (1993, Tables II.14 and II.15).



Figure 1: Spread of Basic Facilities Through the U.S. Economy

households.²

Investment in household appliances as a percentage of GDP has almost doubled over the last century. It represented about 0.5% of GDP in 1988, which was about 2.9% of total investment spending. Likewise, the stock of appliances as a percentage of GDP has also risen continuously, as Figure 3 illustrates.³

To understand the impact of the housework revolution, try to imagine the tyranny of household chores at the turn of the last century. In 1890 only 24 percent of houses had running water, none had central heating. So, the average household lugged around the home, 7 tons of coal and 9,000 gallons of water per year. The simple

²Sources: Survey of Current Business and Fixed Reproducible Tangible Wealth in the United States, 1925-89. Washington, D.C., U.S. Department of Commerce.

³Sources: (i) Dishwashers, refrigerators, and vacuum cleaners – *Electrical Merchandising*; (ii) Dryers and microwaves – Burwell and Swezey (1990, Figures 11.8 and 11.10); (iii)Washers – Lebergott (1993, Table II.20)



Figure 2: Diffusion of Electrical Appliances in the Household Sector

task of laundry was a major operation in those days. While mechanical washing machines were available as early as 1869, this invention really took off only with the development of the electric motor. Ninety-eight percent of households used a 12 cent scrubboard to wash their clothes in 1900. Water had to be ported to the stove, where it was heated by burning wood or coal. The clothes were then cleaned via a washboard or mechanical washing machine. They had to be rinsed out after this. The water then needed to be wrung out, either by hand or by using a mechanical wringer. The clothes were dried using a clothes line. Then, the oppressive task of ironing began, using heavy flatirons that had to be heated continuously on the stove.

The electric iron was first patented in 1882 by Henry W. Seely. Westinghouse launched an advertising campaign to acquaint the public with the benefits of the iron in 1906. The iron diffused quickly with the spread of electricity. Seventy-one percent



Figure 3: Household Appliances

of wired homes had them in 1926.⁴ The first electric washing machine surfaced in 1908. It was invented by Alva J. Fisher and sold by the Hurley Machine Company. As with many inventions, the initial incarnation of the idea was crude. Clothes were spun around on a drum driven by an uncovered chain attached to an electric motor. Maytag introduced an electric washing machine with an agitator in 1922. It was a great success and by 1927 the company had produced a million of them. Thirty-six percent of wired homes had an electric washing machine in 1926.⁵ The early machines really just replaced the scrubboard. Homeowners still had to use a wringer. The electric water heater arrived around this time, too. Fully automatic washing machines with a spin-cycle didn't appear until about 1940. The clothes dryer didn't catch on until the beginning of the 1950s.

⁴Source: *Electrical Merchandising*, January 1926, pp. 6002.
⁵Source: Ibid.

Time Savings: The amount of time freed by modern appliances is somewhat speculative. Controlled engineering studies documenting the time saved on some specific task by the use of a particular machine would be ideal. Unfortunately, these studies seem hard to come by. The Rural Electrification Authority supervised one such study based on 12 farm wives during 1945-46. They compared the time spent doing laundry by hand to that spent using electrical equipment. The women also wore a pedometer. One subject, Mrs. Verett, was reported on in detail.⁶ Without electrification, she did the laundry in the manner described above.⁷ After electrification Mrs.Verett had an electric washer, dryer and iron. A water system was also installed with a water heater. They estimated that it took her about 4 hours to do a 38 lb load of laundry by hand, and then about 4 and 1/2 hours to iron it using old-fashioned irons. By comparison it took 41 minutes to do a load of the laundry using electrical appliances and 1 and 3/4 hours to iron it. The woman walked 3,181 feet to do the laundry by hand, and only 332 feet with electrical equipment. She walked 3,122 feet when ironing the old way, and 333 the new way.

In 1900 the average household spent 58 hours a week on housework – meal preparation, laundry and cleaning – Lebergott (1993, Table 8.1) estimates. This compares with just 18 in 1975. Sociologists suggest that modern appliances have had little effect on the total time allocated to housework. This is based on time diary evidence. They feel that with the mechanization of the household, societal standards for good housekeeping have risen to keep women enslaved. Vanek (1973, Tables 14.4 and 14.5) reports that total amount of time spent on housework in a family with an employed or non-employed mother seems remarkably constant over the last 70 years or so, about 26 hours a week for the former and 55.4 hours for the later in 1965-1966. Even taking this pessimistic attitude, this implies that the average amount of total time spent on housework (ignoring part-time work) is shown

⁶This study is reported in *Electrical Mechandising*, March 1, 1947: pp. 38-39.

⁷She actually had a gas-powered washing machine that replaced the scrubboard.



Figure 4: Housework

in Figure 4. At the same time the number of paid domestic workers declined, presumably in part due to the labor-saving nature of household appliances.⁸ A reasonable conclusion is that the time spent on the more onerous household chores, such as those associated with cooking, cleaning, doing the laundry, etc., declined considerably in the last century.

Labor-Force Participation: What was the effect of this massive technological advance in the production of household capital on labor force participation? A case can be made that it liberated women from the home. As can be seen from Figure 5, female labor-force participation rose steadily since 1890. At the same time the number of homemakers continuously declined. Real income per full-time female worker grew five fold over this period. In 1890 a female worker earned about 50 percent of a what a male did, and by 1970 this number had risen to only 60 percent. It seems

⁸Source: Oppenheimer (1969, Table 2.5)



Figure 5: Female Labor-Force Statistics

unlikely that the small rise in the relative income of a female worker could explain the dramatic rise in labor force participation. It is more likely that the rise in overall real wages, in conjunction with the introduction of labor-saving household appliances, explains the rise in female labor-force participation.⁹

A Wrinkle: Now, the data suggests that women from poor households entered the labor force first. As can be seen, the higher a woman's husband's income, the less likely she was to work. At the same time, the poorer a family, the slower they were to purchase durable goods. The relative price of new goods tends to fall rapidly after their introduction. Perhaps poor women went to work in order to save up the funds required to buy the new durable goods at some later date, when their prices would fall.

⁹Sources: (i) Female earnings, ratio of female to male earnings, and participation – Goldin (1990, Table 5.1); (ii) Homemakers – Vanek (1973, Table 1.22).

	No Children under 10		With Children under 10			
	White	Nonwhite	White	Nonwhite		
\$1-199	30.4%	44.5	15.4	23.2		
200-399	25.5	37.5	12.2	21.0		
400-599	24.0	37.2	11.0	19.2		
600-999	23.2	33.7	11.2	18.6		
1,000-1,499	21.3	25.5	8.7	14.1		
1,500-1,999	16.8	20.9	5.5	16.3		
$2,\!000$ and up	11.1	20.3	3.1	12.1		

Labor Force Participation by Husband's Income: US, April 1940

Source: Durand (1948, Table 17, p. 92)

	Working				Not Working			
	All	Family Income			All	Family Income		
		\$3,000	\$5,000	\$8,000		\$3,000	\$5,000	\$8,000
		- 4,999	- 7,999	and up		- 4,999	- 7,999	and up
Running Water	100%	100	100	100	99	100	99	100
Hot Water	99	100	99	98	98	99	97	100
Flush Toilet	98	96	98	100	98	99	97	100
Bath or Shower	98	91	99	98	98	98	97	100
Furnace	88	74	86	98	87	81	93	93
Telephone	90	65	94	94	92	88	99	100
Refrigerator	100	100	100	100	100	100	100	100
Freezer	14	4	13	19	12	8	17	14
Electric or Gas Stove	100	100	100	100	98	98	99	100
Washing Machine	90	78	92	92	98	97	99	100
– Nonautomatic	52	61	60	31	63	73	55	29
– Automatic	40	17	33	63	39	26	51	79
Dryer	60	43	56	75	57	45	70	86
Iron	100	100	100	100	100	100	100	100
Vacuum Cleaner	87	78	88	90	89	80	97	100
Sewing Machine	63	48	67	60	68	58	75	100
Source: Vanek (1973, Table 4.21, p. 155)								

Durable Goods Ownership by Socio-Economic Class: US, 1965

	\$2,000-\$2,999	\$3,000-\$3,999	\$4,000-\$4,999	\$5,000-\$5,999	\$6,000-\$7,999	
Refrigerator	79%	93	93	95	96	
Stove	57	76	82	88	92	
Washer	85	90	90	90	92	
Television	83	88	90	91	91	
Freezer	7	4	5	8	5	
Vacuum Cleaner	40	61	67	80	83	
Floor Polisher	21	29	42	51	51	
Sewing Machine	53	64	61	65	62	
Radio	88	91	92	93	95	

Durable Goods Ownership by Income: Canada, 1957

Source: Day (1992, Table 8, p. 319)

1.2 The Analysis

To address the question at hand, Becker's (1965) classic notion of household production is introduced into a dynamic general equilibrium model. In particular, household capital and labor can be combined to produce home goods, which yield utility. This isn't the first time that household production theory has been embedded into the neoclassical growth model. Benhabib, Rogerson and Wright (1991) have done so to study the implication of the household sector for business cycle fluctuations.¹⁰ Parente, Rogerson and Wright (2000) apply a similar framework to see whether household production can explain cross-country income differentials. Home production has also been introduced into an overlapping generations model by Rios-Rull (1993) to examine its impact on the time allocations of skilled versus unskilled labor.

The analysis undertaken here differs significantly, though, from the above work. It assumes that over the last century there has been tremendous investment-specific technological progress in the production of household capital. These new and im-

¹⁰This line of research has been recently been extended by Gomme, Kydland, and Rupert (2000).

proved capital goods allow household production to be undertaken using less labor. The formalization of the labor shedding nature of the new technologies is reminiscent of Goldin and Katz's (1998) description of the effect that continuous-process and batch methods had on labor demand during the second industrial revolution. It also resembles Krusell *et al* 's (2000) analysis of the impact that biased technological progress had on the postwar skill premium. At the heart of the developed framework are two interrelated decisions facing each household. First, they must choose whether or not to adopt the new technology at the going price. Second, they must decide whether the woman in the family should work in the market or not. The upshot of the analysis is that household production theory is a powerful tool for explaining the rise in U.S. female labor-force participation over the last century.

2 The Economic Environment

The world is made up of overlapping generations. Each generation lives n periods. Hence, in any period there are n generations around.

Tastes: Let tastes for an age-j household be given by

$$\sum_{i=j}^{n} \beta^{i-j} [\mu \ln m^{i} + \nu \ln n^{i} + (1 - \mu - \nu) \ln l^{i}],$$

where m^i and n^i are the consumptions of market and non-market produced goods and l^i is the household's leisure.

Income: A household is made up of a male and a female. They are endowed with two units of time, which they split up between market work, home work, and leisure. Work in the market is indivisible. Set market time at ω . It will be assumed that males always work in the market. The household can choose whether or not the female will work in the market. Each household is indexed by an ability level, λ , shared by both members. This is drawn at the beginning of their life. They make all decisions knowing the value of λ . Let ability λ be drawn from a lognormal distribution. Normalize the mean of λ at unity. Therefore, assume that $\ln \lambda \sim N(-\sigma^2/2, \sigma^2)$. Denote the distribution function for ability by $L(\lambda)$. The market wage for an efficiency unit of male labor is given by w. A woman earns the fraction ϕ of what a man does. Hence, in a given period, a family of efficiency level λ will earn the amount $w\lambda\omega$ if the female stays at home and the amount $w\lambda\omega + w\phi\lambda\omega$ if she works. The family may also have assets. Denote these by a and let the gross interest rate be r.

Household Production: Home goods are produced according to the following Leontief production function. Specifically,

$$n = \min\{d, \zeta h\},\$$

where d represents the stock of household durables and h proxies for the time spent on housework. The variable ζ represents labor-augmenting technological progress in the household sector. Durables goods are assumed to be lumpy. All housework is done by women.

The Durable Goods Revolution, A Preview: A household technology is defined by the triplet (d, h, ζ) . Recall that household capital, d, is lumpy, and assume that housework, h, is indivisible. Let the time price of the technology be q – this is set in terms of hours of work (at the mean skill level). The cost of the technology is equal to price of the durable goods, d, needed to operated it. Before the arrival of electricity suppose that $d = \delta$, $h = \rho \eta$, and $\zeta = \delta/(\rho \eta)$, where $0 < \rho \eta < 1 - \omega$ and $\rho > 1$. Using this old technology, $n = \min\{d, \zeta h\} = \delta$ units of non-market goods can be produced. The price of the old household technology will be set to zero. Now, imagine that electricity comes along together with a new set of durable goods. Define this new technology by the triplet (d', h', ζ') . Here $d' = \kappa \delta$, $h' = \eta$, and $\zeta' = \kappa \delta/\eta$, where $\kappa > 1$. Note that $\zeta' = \kappa \rho \zeta$, so that technological progress can be broken down into the additional amount of capital services provided and the amount of household labor freed up. That is, with the new technology capital services rise by a factor of κ . The old technology requires more labor, a factor ρ more. The new technology produces $n' = \min\{d', \zeta'h'\} = \kappa \delta > \delta$ units of non-market goods. Should a household adopt the new technology? This will depend on its price, q', of course.

Market Production: Market production is undertaken according to the standard neoclassical production function

$$\mathbf{y} = \mathbf{k}^{\alpha} (z\mathbf{I})^{1-\alpha},\tag{1}$$

where y is output, k represents the aggregate market capital stock, and l is the aggregate labor input. Labor-augmenting technological progress is captured by the variable z. Market output can be used for the consumption of market goods, m, gross investment in business capital, l, and for gross investment in household capital, d. Hence

$$\mathsf{m} + \mathsf{i} + \mathsf{d} = \mathsf{y}.\tag{2}$$

The law of motion for business capital is

$$\mathbf{k}' = \chi \mathbf{k} + \mathbf{i},\tag{3}$$

where χ factors in physical depreciation.

3 The Household's Decision Problem

Asset Accumulation and Labor-Force Participation Decisions: Consider the dynamic programming problem facing an age-j household. Suppose that the household has already made its decision about whether or not to adopt the new technology for the current period. Then the household's state of the world is summarized by the triplet (a, τ, λ) . Here $\tau \in \{0, 1, 2\}$ is an indicator function giving the state of the household technology. When $\tau = 0$ the household does not use the new technology in the current period. When $\tau = 1$ the household purchases and uses the new technology in the current period. Last, $\tau = 2$ denotes the case where the household has adopted previously. The lifetime utility for an age-j household with assets, a, state of technology τ , and ability level λ is represented by $V^{j}(a, \tau, \lambda)$. It is easy to see that the decisions regarding female labor-force participation and asset accumulation are summarized by

$$V^{j}(a,0,\lambda) = \max\{\max_{a'}\{\mu \ln(w\lambda\omega + \phi w\lambda\omega + ra - a') + \nu \ln(\delta) + (1 - \mu - \nu) \ln(2 - 2\omega - \rho\eta) + \beta \max[V^{j+1}(a',0,\lambda), V^{j+1}(a',1,\lambda)]\}, \\ \max_{a'}\{\mu \ln(w\lambda\omega + ra - a') + \nu \ln(\delta) + (1 - \mu - \nu) \ln(2 - \omega - \rho\eta) + \beta \max[V^{j+1}(a',0,\lambda), V^{j+1}(a',1,\lambda)]\}\},$$

$$V^{j}(a, 1, \lambda) = \max\{\max_{a'}\{\mu \ln(w\lambda\omega + \phi w\lambda\omega + ra - a' - wq) + \nu \ln(\kappa\delta) + (1 - \mu - \nu)\ln(2 - 2\omega - \eta) + \beta V^{j+1}(a', 2, \lambda)\},$$
$$\max_{a'}\{\mu \ln(w\lambda\omega + ra - a' - wq) + \nu \ln(\kappa\delta) + (1 - \mu - \nu)\ln(2 - \omega - \eta) + \beta V^{j+1}(a', 2, \lambda)\}\}.$$
 P(2)

and

$$V^{j}(a, 2, \lambda) = \max\{\max_{a'}\{\mu \ln(w\lambda\omega + \phi w\lambda\omega + ra - a') + \nu \ln(\kappa\delta) + (1 - \mu - \nu)\ln(2 - 2\omega - \eta) + \beta V^{j+1}(a', 2, \lambda)\},$$
$$\max_{a'}\{\mu \ln(w\lambda\omega + ra - a') + \nu \ln(\kappa\delta) + (1 - \mu - \nu)\ln(2 - \omega - \eta) + \beta V^{j+1}(a', 2, \lambda)\}\}.$$
 P(3)

Denote the female labor-force participation decision that arises from these problems by the indicator function $P^{j}(a, \tau, \lambda)$. Here $P^{j}(a, \tau, \lambda) = 1$ denotes the event where the woman works. Likewise, the household's asset decision is represented by $A^{j}(a, \tau, \lambda)$.

The Adoption Decision: Now, suppose that a household currently does not own the new technology. The household faces a choice about whether to adopt the new technology in the current period or not. The decision problem facing an age-j household is

$$\max_{\tau \in \{0,1\}} V^j(a,\tau,\lambda).$$
 P(4)

Let $T^{j}(a, \lambda)$ represent the indicator function that summarizes the decision to adopt the new technology or not. The solution to this problem is simple:

$$T^{j}(a,\lambda) = \begin{cases} 1, & \text{if } V^{j}(a,1,\lambda) > V^{j}(a,0,\lambda), \\ 0, & \text{if } V^{j}(a,1,\lambda) \le V^{j}(a,0,\lambda). \end{cases}$$

It only applies to those agents who haven't adopted previously. The law of motion for technology must specify that $\tau^{j+1} = 2$ if either $\tau^j = 1$ or $\tau^j = 2$.

Decision Rules: Consider generation j. Denote an age-j household's current asset holdings by a^j and their state of technology by τ^j . Now, note that for the first generation $a^1 = 0$. This implies that a^{j+1} and τ^{j+1} can be represented by $a^{j+1} =$ $A^j(\lambda)$ and $\tau^{j+1} = T^j(\lambda)$. To see that this is the case, suppose that $a^j = A^{j-1}(\lambda)$ and $\tau^j = T^{j-1}(\lambda)$. If $\tau = 2$ then either $\tau^{j+1} = T^{j-1}(\lambda)$ or $\tau^{j+1} = T^{j-1}(\lambda) + 1$, and if $\tau \neq 2$ then $\tau^{j+1} = T^j(A^{j-1}(\lambda), \lambda)$. Hence, write $\tau^{j+1} = T^j(\lambda)$. Likewise, $a^{j+1} = A^j(A^{j-1}(\lambda), T^j(\lambda), \lambda) \equiv A^j(\lambda)$. To start the induction, let $T^1(0, \lambda) \equiv T^1(\lambda)$ and $A^1(0, T^1(\lambda), \lambda) \equiv A^1(\lambda)$. Similarly, an age-j household's participation decision can be written as $P^j(\lambda)$. Now, at any point in time some households will have adopted the new technology while others won't have. Let λ^j be the threshold level of λ such that all households of generation j with $\lambda > \lambda^j$ will have adopted, while those with $\lambda \leq \lambda^j$ won't have.

4 Competitive Equilibrium

Market-Clearing Conditions: At each point in time all factor markets must clear. This implies that the market demand for labor must equal the market supply of labor. Therefore,

$$I = n\omega \int \lambda L(d\lambda) + \phi\omega \sum_{j=1}^{n} \int \lambda \mathsf{P}^{j}(\lambda) L(d\lambda).$$
(4)

The market supply of labor is obtained by summing males' and females' labor supplies across the varying ability levels and generations. Likewise, next period's market capital stock must equal today's purchases of assets so that

$$\mathsf{k}' = \sum_{j=1}^{n} \int \mathsf{A}^{j}(\lambda) L(d\lambda).$$
⁽⁵⁾

Since the market sector is competitive, factor prices are given by marginal products. Hence,

$$w = (1 - \alpha)z(z\mathsf{I}/\mathsf{k})^{-\alpha},\tag{6}$$

and

$$r' = \alpha (z \mathsf{I}'/\mathsf{k}')^{1-\alpha} + \chi. \tag{7}$$

It is time to define the competitive equilibrium under study.

- Definition: A stationary competitive equilibrium consists of a set of n threshold values $\Lambda = \{\lambda^1, ..., \lambda^n\}$, a set of allocation rules $\mathsf{A}^j(\lambda)$, $\mathsf{P}^j(\lambda)$, and $\mathsf{T}^j(\lambda)$ for j = 1, ..., n, and a set of wage and rental rates, w and r, such that
 - The allocation rules A^j(λ) and P^j(λ) solve problem P(1) to P(3), given w, r, and q.
 - 2. The allocation rule $\mathsf{T}^{j}(\lambda)$ solves problem $\mathsf{P}(1)$ to $\mathsf{P}(3)$, given w, r, and q.
 - 3. Factor prices clear all markets, implying that (4) to (7) hold.

Balanced Growth: Represent the pace of labor-augmenting technological progress by γ so that $\gamma = z'/z$. Let $z_0 = 1$ so that $z_t = \gamma^t$. Conjecture that y, m, i, d, and k all grow at this rate too. Also, posit that along a balanced-growth path the aggregate stock of labor, l, is constant. This conjecture is consistent with the forms of (1) to (3). This implies from (6) and (7) that r is constant over time, while wgrows at rate γ . It remains to be shown that $A_{t+1}^j(\lambda) = \gamma A_t^j(\lambda)$, $P_{t+1}^j(\lambda) = P_t^j(\lambda)$, and $T_{t+1}^j(\lambda) = T_t^j(\lambda)$. Observe that this solution will be consistent with the factor market-clearing conditions (4) and (5). Lemma 1 Along a balanced growth path, $\mathsf{A}_{t+1}^j(\lambda) = \gamma \mathsf{A}_t^j(\lambda)$, $\mathsf{P}_{t+1}^j(\lambda) = \mathsf{P}_t^j(\lambda)$, and $\mathsf{T}_{t+1}^j(\lambda) = \mathsf{T}_t^j(\lambda)$.

Proof. Suppose that along a balanced-growth path $V^{j+1}(\gamma a, \tau, \lambda; \gamma w) = V^{j+1}(a, \tau, \lambda; w) + [(1 - \beta^{n-j})/(1 - \beta)] \ln \gamma$.¹¹ Now, by eyeballing problems P(1) to P(3) it is easy to see that if $A^j(a, \tau, \lambda; w)$ and $P^j(a, \tau, \lambda; w)$ are the solutions to these problems when the state of world is $(a, \tau, \lambda; w)$, then $A^j(\gamma a, \tau, \lambda; \gamma w) = \gamma A^j(a, \tau, \lambda; w)$ and $P^j(\gamma a, \tau, \lambda; \gamma w) = P^j(a, \tau, \lambda; w)$ are the solutions when the state of the world is given by $(\gamma a, \tau, \lambda; \gamma w)$. It then follows that $V_t^j(\gamma a, \tau, \lambda; \gamma w) = V_t^j(a, \tau, \lambda; w) + [(1 - \beta^{n-j+1})/(1 - \beta)] \ln \gamma$. Finally, note from problem P(4) that $T^j(a, \lambda) = T^j(\gamma a, \lambda; \gamma w)$. Therefore, if $A^j(a, \tau, \lambda; w)$, $P^j(a, \tau, \lambda; w)$ and $T^j(\gamma a, \lambda; \gamma w)$ solve problems P(1) to P(3) today – when the state is $(a, \tau, \lambda; w)$ – then $\gamma A^j(a, \tau, \lambda; w)$, $P^j(a, \tau, \lambda; w)$ and $T^j(a, \lambda; w)$ will solve them tomorrow – when the state will be $(\gamma a, \tau, \lambda; \gamma w)$.

5 Findings

5.1 Some Preliminary Analysis

Calibration: Take the period for the model to be 5 years. Let n = 10 so that a household has a working life of 50 years. Clearly, ω , η , and ρ can be pinned down from time-use data. For instance, in a week there are 112 non-sleeping hours available per adult. If full-time work involves a 40 hour workweek, then $\omega = 0.36$. In 1900 about 58 hours a week were spent on housework, while in 1965 roughly 15 were. So, set $\eta = 0.13$ and $\rho\eta = 0.52$. Next, pick $\phi = 0.60$, approximately the ratio of female to male earnings in 1980. This leaves the parameters κ , μ , and ν . In 1929 the stock of appliances was about \$5,272 mil. (in 1982\$) while in 1959 it was \$32,882 mil. Hence, $\kappa = 4.1$. The lognormal distribution for λ was discretized so that

¹¹Note that the household's problem is a function of w and r. Hence, these factor prices should be entered into the value functions. Since r is constant along a balanced-growth path it has been suppressed in the value function.

 $\lambda \in \Lambda \equiv \{\lambda_1, ..., \lambda_{100}\}$. The skill distribution was parameterized by setting $\sigma = 0.70$. Now, the two utility parameters μ and ν are set so the model's balanced growth displays the two features discussed momentarily. This required picking $\mu = 0.33$ and $\nu = 0.20$.

The Household Sector, circa 1900: Imagine that the time is 1900. Virtually no one owns an electrical appliance. This situation can be obtained in the model by setting the price of durables high, say q = 20 – this implies that the median person would have to work 100 years to earn the income required to purchase modern durables. The annual interest rate for the model is 7.1 percent. This lies above the rate of time preference. Associated with this interest rate is a market investment to output ratio of 0.15.

At this time in history, almost all married women stayed at home. In 1890 Goldin (1987, Table 1) reports that only 5 percent of all married women worked. Surprisingly, female labor-force participation is *not* a function of income when nobody adopts the new technology. Hence, female labor-force participation must be some fraction contained in the set $\{1/n, 2/n, ..., 1\}$. With the adopted calibration, 10 percent $(1/10 \times 100\%)$ of women work. The standard deviation (for the ln) of household income in the model is 0.70.

Lemma 2 If $T^{j}(\lambda) = 0$ for all j = 1, ..., n and $\lambda \in \Lambda$, then $P^{j}(\lambda) = \pi^{j}$ for all j and λ .

Proof. Take a household of type λ . It's market consumption decision must satisfy the Euler equation

$$\frac{1}{m^j(\lambda)} = \beta r \frac{1}{m^{j+1}(\lambda)},\tag{8}$$

Using the household budget constraint, this implies that

$$m^{j}(\lambda) = (\beta r)^{j-1} \frac{1-\beta}{1-\beta^{n}} \Omega(\lambda),$$

where $\Omega(\lambda)$ is the present-value of the household's income – at age 1 – net of the cost of purchasing consumer durables. Since the household doesn't adopt, $\Omega(\lambda)$ is given by

$$\Omega(\lambda) = \sum_{j=1}^{n} \frac{w\lambda\omega + \phi w\lambda\omega\mathsf{P}^{j}(\lambda)}{(r)^{j-1}} = w\lambda\omega[\frac{1-1/r^{n}}{1-1/r} + \phi\sum_{j=1}^{n} \frac{\mathsf{P}^{j}(\lambda)}{(r)^{j-1}}].$$

It is then easy to calculate that lifetime utility is given by

$$V^{1}(0,0,\lambda) = \mu \{ \frac{1-\beta^{n}}{1-\beta} [\ln(\frac{1-\beta}{1-\beta^{n}}) + \ln\Omega(\lambda)] + \sum_{j=1}^{n} (j-1)\beta^{j-1}\ln(\beta r) \} + \nu \frac{1-\beta^{n}}{1-\beta} \ln\delta (9) + (1-\mu-\nu) \sum_{j=1}^{n} \beta^{j-1} \{\mathsf{P}^{j}(\lambda)\ln(2-2\omega-\rho\eta) + [1-\mathsf{P}^{j}(\lambda)]\ln(2-\omega-\rho\eta) \}.$$

Now, there are $2^n - 1$ other possible work combinations. Let $\mathsf{P}^{*j}(\lambda)$ denote some other arbitrary work profile and $V^{*1}(0,0,\lambda)$ represent the lifetime utility associated with this particular participation sequence. To obtain $V^{*1}(0,0,\lambda)$ replace $\mathsf{P}^j(\lambda)$ with $\mathsf{P}^{*j}(\lambda)$ in (9). For $\mathsf{P}^j(\lambda)$ to be optimal it must happen that $V^1(0,0,\lambda) \ge V^{*1}(0,0,\lambda)$. Observe that $V^1(0,0,\lambda) - V^{*1}(0,0,\lambda)$ is not a function of λ , however.¹² Hence, $\mathsf{P}^j(\lambda)$ cannot be a function of λ .

The Household Sector, circa 1980: Now, move ahead to 1980. Almost everybody owns electrical appliances. This situation transpires in the model when q = 0.04. If a period is 5 years then there are about 8,800 working hours (5 yrs. × 11 mths. × 4 wks. × 40 hrs.) per adult. Hence, the median male would only need to work about 350 hours to purchase modern appliances. The steady-state interest rate is 7.01 percent, which lies above the rate of time preference. At this interest rate, the investment-to-output ratio is 0.15. This is not far off from the postwar average of 0.11. Appliance investment amounts to 0.64 percent of GDP.

About one half of married women work now, in line with Goldin (1987, Table 1). When everybody adopts the new technology immediately, female-labor participation is a decreasing function of household income. This is proved in the lemma below.

¹²Note that $\ln \Omega(\lambda) = \ln(w\lambda\omega) + \ln[(1-1/r^n)/(1-1/r) + \phi \sum_{j=1}^n \mathsf{P}^j(\lambda)/r^{j-1}].$



Figure 6: Female Labor-Force Participation

Figure 6 illustrates the situation for the case when q = 0.04. Women from more wellto-do households retire earlier. (Just multiply the participation rate by 10 to get the period a women retires in.) The standard deviation for (the ln of) household income is 0.71.

Lemma 3 The present value of labor-force participation, $\sum_{j=1}^{n} p^{j}/r^{j-1}$, is nonincreasing in type, λ , holding fixed the date of adoption, m. Similarly, $\sum_{j=1}^{n} p^{j}/r^{j-1}$ is nonincreasing in m, holding fixed λ .

Proof (by contradiction). Consider two types of households, λ^* and λ^{**} with $\lambda^* < \lambda^{**}$. Let p^{*j} denote the optimal participation policy associated with λ^* , and p^{**j} represent the corresponding policy linked with λ^{**} . Analogously, let $B^{*1}(\lambda)$ and $B^{**1}(\lambda)$ be the period-1 lefthand sides of the Bellman equations connected with the policies. These can be obtained by replacing $\mathsf{P}^{j}(\lambda)$ in (9) with p^{*j} and p^{**j} , respectively, and adding $[(\beta^{m-1} - \beta^{n})/(1 - \beta)]\nu \ln \kappa$.

Suppose that the hypothesis is not true. Then, there exists a λ^* and λ^{**} such that $B^{*1}(\lambda^*) > B^{**1}(\lambda^*)$, $B^{*1}(\lambda^{**}) < B^{**1}(\lambda^{**})$, and $\sum_{j=1}^n p^{*j}/r^{j-1} < \sum_{j=1}^n p^{**j}/r^{j-1}$. Now, observe from the analogue to (9) that, $B^{*1}(\lambda) - B^{**1}(\lambda) = \mu[(1 - \beta^n)/(1 - \beta)]\{\ln[\Omega^*(\lambda)/\ln \Omega^{**}(\lambda)]\}$. Here $\Omega^*(\lambda)$ and $\Omega^{**}(\lambda)$ are the levels of permanent income (net of adoption cost) associated with the p^{*j} and p^{**j} policies. Now,

$$\frac{d[B^{*1}(\lambda) - B^{**1}(\lambda)]}{d\lambda} = \mu \frac{1 - \beta^n}{1 - \beta} \frac{q/r^{m-1}\phi\lambda\omega^2 \sum_{j=1}^n [p^{**j} - p^{*j}]/(r)^{j-1}}{\{\sum_{j=1}^n [w\lambda\omega + \phi w\lambda\omega p^{*j}/(r)^{j-1}] - q/r^{m-1}\}^2} \frac{\Omega^{**}(\lambda)}{\Omega^{*}(\lambda)} > 0.$$

Consequently, if $B^{*1}(\lambda^*) > B^{**1}(\lambda^*)$, then $B^{*1}(\lambda^{**}) > B^{**1}(\lambda^{**})$. The desired contradiction obtains. The proof for the second part of the hypothesis parallels the first, *mutatis mutandis.*

Pursuing Happiness: The second industrial revolution made women worse off, or so you might believe from reading the sociology literature. The lot of families in the artificial economy can be examined. Compare the 1900 and 1980 steady states. As a result of new, more productive household capital, GDP rises by 24 percent. It may be tempting to conclude that the gain in welfare must be less than this. After all, the increase in GDP occurs because more women are working. In fact, welfare increases by 50 percent, when measured in terms of market consumption. The new technology leads to a 23 percent increase in market consumption, a 50 percent increase in nonmarket consumption, and a 21 percent increase in leisure.¹³ Now, take a family living in 1980. They will reside at some percentile in the income

¹³Parente, Rogerson and Wright (2000) show that when household production is incorporated into the standard neoclassical growth model, cross-country differences in welfare are smaller than cross-country differences in GDP. In their framework cross-country income differentials are due to policy distortions. A tax on market activity reduces GDP. Welfare drops by less than GDP, though, because there is an increase in nonmarket activity. In the current paper, technological progress leads to a rise in all items in the utility function.

distribution and have an associated level of utility. At what spot in the 1900 income distribution would a family have to be located in order to realize this same level of utility? Figure 7 gives the answer. For instance, a poor family at the 20th percentile in 1980 is as well off as someone living in the 55th percentile in 1900. Trivially, (in a distortion-free competitive equilibrium) a family wouldn't adopt a new technology if it made them worse off; after all, they are pursuing happiness.¹⁴ The analysis here models the household from the perspective that members share a common set of preferences. Suppose they didn't. It seems likely that any reasonable bargaining model would predict that both males and females would share in the gain from the second industrial revolution.

The Effect of Declining Prices: Between 1900 and 1980 the prices for household appliances must have dropped dramatically. The time path of prices has a big impact on adoption and participation decisions. To see this, hold the interest rate fixed at 7 percent and imagine that prices fall 5 percent a year starting from an initial value of 2.0. Figure 8 tells the story. About 5 percent of households adopt immediately.

¹⁴This isn't the view held by many social historians. On the one hand, they believe that "the change from the laundry tub to the washing machine is no less profound than the change from the hand loom to the power loom; the change from pumping water to turning on a water faucet is no less destructive of traditional habits than the change from manual to electric calculating" – Cowan (1976, pp. 8-9). On the other hand, they feel that a change in societal tastes, generated by commercial interests, enslaved women into undertaking new tasks by making them feel guilty if "their infants had not gained enough weight, embarrassed if their drains clogged, guilty if their children went to school in soiled clothes, guilty if all the germs behind the bathroom sink are not eradicated, guilty if they fail to notice the first sign of an oncoming cold, embarrassed if accused of having body odor, guilty if their sons go to school without good breakfasts, guilty if their daughters are unpopular because of old-fashioned, or unironed, or – heaven forbid – dirty dressed" — Cowan (1976, p. 16). To most producing clean clothes for school is a utility generating activity. So, these sociologists must believe that society has been duped by advertising campaigns, etc. into having these tastes. Cowan (1976, p. 23) ends with "how long ... can we continue to believe that we will have orgasms while waxing the kitchen floor." Well according to Mark Twain, "It isn't what we don't know that kills us, it's everything we know that ain't so."



Figure 7: A Utilitarian View of Economic Development

The higher a household's type, the earlier they adopt. In order to acquire consumer durables the woman in a household may have to go to work. That is, in line with the lemma, adoption goes hand in hand with increased labor effort.¹⁵ In most cases the woman goes to work *before* the durables are purchased. As λ rises from λ_1 to λ_{64} labor effort increases continuously as households adopt at successively earlier dates. Holding fixed the adoption date, however, labor effort is decreasing in type – as was proved in the previous lemma. For example, from λ_{78} to λ_{100} all households adopt immediately; hence, the adoption date is fixed here. Labor effort decreases continuously as the lemma states.

It may seem that theoretically the date of adoption should be a nonincreasing function of λ . This is difficult to establish, though, given the lumpy nature of the adopt and work decisions. The lumpy nature of these decisions can be partially smoothed out by increasing the number of periods that a household lives while holding fixed its lifespan; i.e., by shortening the length of a period.

Lemma 4 Along a balanced growth path the date of adoption is a nonincreasing function in type, at least when the length of a period is sufficiently short.

Proof. See Appendix. \blacksquare

5.2 Some More Evidence

U.S. Evidence: So, what is the relationship between the adoption of appliances and female labor-force participation? The evidence is scant. Still, here it is. Some state-by-state data on the quantity of appliances sold is available from *Electrical Merchandising* (1957). From this a crude measure of the stock of household appliances

¹⁵Take the case where the type distribution is continuous. Consider some threshold value of λ and a local neighborhood around it. Suppose that above this value of λ the household adopts at some date ς , while below it they adopt at some later date, say $\varsigma + j$ where the integer $j \ge 1$. As the threshold is crossed the adoption date jumps forward, but λ remains more or less fixed. Hence, the lemma applies.



Figure 8: The Effect of Prices on Adoption and Participation



Figure 9: The State-by-State Relationship between Female Labor-Force Participation and Appliance Ownership

per family can be constructed for each state. In particular, this source presents data for washers, dryers, refrigerators, electric stoves, freezers, ironers, and electric water heaters. For each appliance the total numbers of units sold over the twelve year period 1946-1957 is reported. The resulting numbers can then be summed across appliances, after multiplying each figure by the price of the appliance. These figures can be normalized by the number of families in the state, as reported in the 1950 U.S. Decennial Census, to obtain a measure of household capital per family. Stateby-state female labor-force participation numbers can be computed from the 1950 census. The relationship between these two series is plotted in Figure 9. There is no question that, visually, appliance ownership is positively associated with female labor-force participation.

To test the robustness of this relationship, female labor-force participation (part)

will be regressed against appliance ownership (appl), plus some additional control variables. The control variables are the ratio of females with some high school education in the state (edu), per-capita income in the state (inc), the extent of urbanization (urb), and some regional dummies.¹⁶ The result obtained is

$$part = -0.01 + 0.000040 \times appl + 0.365869 \times edu + 0.043235 \times \log(inc) + 0.000629 \times urb + dummies,$$

with $r^2 = 0.60, \sigma = 0.024, obs = 48, d.f. = 40.$

All coefficients have the expected signs. All are significant, except for the level of percapita income in a state. As can be seen, appliance ownership is positively associated with female labor-force participation. The coefficient, if interpreted literally, implies that a \$1,000 increase in the per-capital stock of appliances in a state will be associated with a 4.0 percentage point rise in female labor-force participation.

International Evidence: Countries where new durable goods are expensive tend to have low levels of female labor-force participation. Here is the evidence. The Penn World Table (Version 5.6) presents some national income account statistics for a number of countries. By dividing the price of investment goods through by the GDP deflator, a measure of the relative price of durables can be obtained. Strictly speaking one would like a measure of the price of household equipment. This isn't available. The price of new equipment in the business sector is probably a reasonable proxy for the price of new equipment in the household sector. The behavior of the relative price of automobiles, computers, refrigerators, stoves, etc. used in the business sector across time and space is likely to be similar to the relative price of these goods used in the household sector. A measure of female labor-force participation can be computed using data available from the Economically Active Population, 1950-2010, a publication of the International Labor Office. In particular, the ratio of female employees to total employees in industry and services will be taken as proxy for

¹⁶Regions are classified in line with Barro and Sala-i-Martin (1995, Table 10.4)

female labor-force participation. This excludes the agricultural sector. Handling the agricultural sector is a tricky issue. First, the ILO data treats any woman who works more than one hour a week in agriculture as being employed there. For less-developed countries this weak restriction will be satisfied by most rural women. The tendency to count rural adults as agricultural workers, irrespective of the time they allocate to this sector, has been noted by Parente, Rogerson and Wright (2000). Second, in less-developed countries a lot of agriculture is really household production, at least for the purposes here. A panel data set of 128 countries can be complied from these two data sources. The time interval is a decade, starting with 1950 and ending in 1990. For some countries an observation was not available for each decade.

To judge the significance of the relationship, female labor-force participation (part) can be regressed against the relative price of durables (price) and some control variables. The control variables are GDP per worker (inc) and dummy variables for decades and regions.¹⁷ The result is:

part =
$$0.27 - 0.011 \times \text{price} + 0.022 \times \text{inc} + \text{dummies},$$

(0.0036) $r^2 = 0.46, \sigma = 0.08, \text{obs} = 512, \text{d.f.} = 499.$

The price and income variables take the expected sign. Both are significant¹⁸.

The cross-sectional relationship between durable goods prices and female laborforce participation for 1990 in plotted Figure 10. Here, the relative price of durables is deflated by GDP per worker to obtain a measure of the time price of durables. It's easy to see that higher durable goods prices are associated with a lower level of female labor-force participation. The figure also graphs the steady-state relationship between household appliance prices and female labor-force participation predicted by

¹⁷The seven regions are Africa (except North Africa), Asia (sans the Middle East), Europe, Middle East (plus North Africa), North and Central America, Pacific Region, and South America. This classification is taken from Barro and Sala-i-Martin (1995, Table 10.3).

¹⁸For 1990 there is data on the fraction of the population with a primary or secondary education. These variables turned out to be insignificant.



Figure 10: The World-Wide Relationship between the Price of New Durables and Female Labor-Force Participation, 1990

the model.¹⁹ The relationship generated by the model is not dissimilar to the one found in the international data. This exercise heroically assumes that each country was resting in its steady-state in the last decade. Don't take this seriously. The mapping generated by the model is shown just to illustrate how the model can be used to shed light on some historical and geographical facts.

5.3 The Durable Goods Revolution

The Computational Experiment: The time is 1900. The age of electricity has just dawned. This era herds in many new household goods: the dryers, frozen foods, hot water, refrigerators, washing machines, etc. What will be the effect in the artificial

¹⁹The price series inputted into the model has been normalized (multiplied by scalar) to make the price for U.S. in 1990 equal to 0.04.

economy? To answer this, the transition path from the 1900 steady state to the 1980 one will be analyzed. This experiment is in the spirit of King and Rebelo's (1993) analysis of the inability of the standard neoclassical growth model to explain the rise of postwar Japan, Hansen and Prescott's (1999) study of the pickup in growth from 1800 on, and Caselli and Coleman's (forth.) research on the catch up of the southern states.

To do this, a time path for durable goods prices must be inputted into the model. Hard numbers are hard to come by, but Figure 11 plots several price series for appliances. The NIPA measure for appliances declines at about 2.2 percent a year, relative to the GDP deflator. This is likely to underestimate the decline in prices because it doesn't control for quality improvement in goods. The figure also shows Gordon's (1990, Table 7.23) quality-adjusted price index for eight appliances, *viz* refrigerators, air conditioners, washing machines, clothes dryers, TV sets, dishwashers, microwaves, and VCR's. This series drops at 8.5 percent a year versus only 3.5 percent for the standard PPI measure. Note that time prices will decline at an even faster clip, since wages have risen over time. Quality-adjusted time prices for some select appliances are shown in Figure 12.²⁰ Assume, then, that time prices decline on average at about 5 percent a year for the first 100 years. The analysis also presumes that agents have perfect foresight; a heroic assumption, for sure.

Adoption and Labor-Force Participation: The new appliances catch on slowly at first. This can be seen from Figure 13, which plots the diffusion curve. It takes 50 years for half of the population to own the durables. Female labor-force participation rises along with adoption. This is shown in Figure 13, too. Only wealthy households – high types – can afford to buy when prices are high. The diffusion curves for three age-averaged types of households, λ_1 , λ_{50} , and λ_{100} , are graphed in Figure 14.

Pursuing Happiness, Again: The increase in GDP due to the durable goods revolution is shown in Figure 15. This rise occurs solely because of the increase in

²⁰These are based on series contained in Gordon (1990, Tables 7.4, 7.12, 7.15, and 7.22)



Figure 11: Decline in the Relative Price of Appliances



Figure 12: Prices for Some Select Appliances



Figure 13: Transitional Dynamics – Diffusion and Female Labor-Force Participation



Figure 14: Diffusion by Type of Household



Figure 15: The Gain in GDP and Welfare

female labor-force participation. Other than the durable goods revolution there is no technological progress. The associated rise in welfare – for the flow of new households into the economy – is also plotted in this figure. Again, the gain in welfare is greater than the increase in GDP.

Between 1929 and 1999 per-capita real GDP grew at 2.2 percent per annum. Take this as reflective of the average rate of growth over the last century. In the model over a 50 year period GDP grows by about 0.35 percent per annum, while for an 80 year period it grows by about 0.22 percent. Therefore, according to the model, the durable goods revolution can be thought of as accounting for about 15 percent of growth, say, between 1920 and 1970, or about 10 percent of growth between 1900 to 1980.

6 Conclusions

"For ages woman was man's chattel, and in such condition progress for her was impossible; now she is emerging into real sex independence, and the resulting outlook is a dazzling one. This must be credited very largely to progression in mechanics; more especially to progression in electrical mechanics.

Under these new influences woman's brain will change and achieve new capabilities, both of effort and accomplishment."

Thomas Alva Edison, as interviewed in *Good Housekeeping Magazine*, LV, no. 4 (October 1912, p. 440)

Did technological progress unlock the manacles chaining women to the home? That's the question posed here. Some may argue that the increase in female laborforce participation was due to a change in social norms, say spawned by the women's liberation movement. After reviewing public opinion poll evidence, Oppenheimer (1970, p. 51) concludes "it seems unlikely that we can attribute much of the enormous postwar increases in married women's labor force participation to a change in attitudes about the propriety of their working."²¹ Besides without the labor-saving household capital ushered in by the second industrial revolution it simply would not have been feasible for many women to spend more time outside of the home, notwith-standing any shift in societal attitudes. While sociology may have acted as a fertilizer, the seed of women's liberation came from economics.

A Appendix

Growth Transformation: Consider the consumption decision for an age-1 household. It must satisfy the Euler equation

$$\frac{1}{m^j} = \beta r \frac{1}{m^{j+1}},\tag{10}$$

where m^{j} is the household's consumption at age j. The household's budget constraint will read

$$m^{1} + \frac{m^{2}}{r} + \frac{m^{3}}{r^{2}} + \dots + \frac{m^{n}}{r^{n-1}} = \Omega,$$

where Ω is the household's permanent income, net of the cost of purchasing the consumer durables. The Euler equation (10) implies that $m^{j+1} = \beta r m^j = (\beta r)^j m^1$. Therefore,

$$m^1 = \frac{1-\beta}{1-\beta^n}\Omega.$$

²¹Unfortunately the questions asked are different both across polling organizations and years – see Oppenheimer (1970, Table 2.10). In 1960 only 34 percent of respondents answered approvingly to the following question: "There are many wives who have jobs these days. Do you think it is a good thing for a wife to work or a bad thing, or what? Why do you say so?" A poll in 1937 asked the question "Do you approve a married women earning money in business or industry if she has a husband capable of supporting her?" Eighteen percent of respondents approved. The same percentage answered favorably to a similar question in 1945. When questions were qualified to indicate some sort of financial need – support for children, a new marriage, etc. – the percentage of favorable responses went up.

Adding growth would not seem to change this equation much. All variables that grow along a balanced path should be transformed to obtain a stationary representation. Define $\hat{a}_t^j = a_t^j/\gamma^t$, $\hat{m}_t^j = m_t^j/\gamma^t$, $\hat{w}_t = w_t/\gamma^t$, and $\hat{\Omega}_t = \Omega_t/\gamma^t$. Then, the Euler equation would appear as

$$\frac{1}{\widehat{m}_t^j} = \beta(r/\gamma) \frac{1}{\widehat{m}_{t+1}^{j+1}}.$$

The household's budget constraint now reads

$$\widehat{m}_{t}^{1} + \frac{\widehat{m}_{t+1}^{2}}{(r/\gamma)} + \frac{\widehat{m}_{t+2}^{3}}{(r/\gamma)^{2}} + \dots + \frac{\widehat{m}_{t+n-1}^{n}}{(r/\gamma)^{n-1}} = \widehat{\Omega}_{t}.$$

Therefore,

$$\widehat{m}^1 = \frac{1-\beta}{1-\beta^n}\widehat{\Omega}.$$

Here

$$\widehat{\Omega}_t = \sum_{j=1}^n \frac{\widehat{w}\lambda\omega + \phi\widehat{w}\lambda\omega P^j(\lambda) - \widehat{w}qT^j(\lambda)}{(r/\gamma)^{j-1}},$$

where

$$\widehat{w} = (1 - \alpha) \left[\frac{r/\gamma - \chi}{\alpha} \right]^{\alpha/(\alpha - 1)},\tag{11}$$

and

$$r/\gamma = \alpha (\mathsf{I}/\widehat{\mathsf{k}})^{1-\alpha} + \chi.$$

Last, the market clearing condition for capital would appear as

$$\widehat{\mathsf{k}}' = \sum_{j=1}^n \int \widehat{\mathsf{A}}^j(\lambda) L(d\lambda).$$

Transitional Dynamics: Imagine that the economy is resting in some initial steadystate. Since the electric age hasn't emerged yet, all households are using primitive durables at home. Now, suppose that suddenly new household durables are invented. At the time of the durable goods revolution, the initial state of the economy is described by $s = (a^2(\lambda), a^3(\lambda), ..., a^n(\lambda))$. The system will eventually converge to a new steady state represented by $s = (a^{2*}(\lambda), a^{3*}(\lambda), ..., a^{n*}(\lambda))$, where an asterisk attached to a variable signifies its value in the new steady state. Assume that this convergence will take place within e + 1 periods. The time path of prices for these goods is given by $q_1 > q_2 > ... \ge q_{e-m} = q_{e+1} = q^*$. The algorithm used to compute the model's transitional dynamics can now be outlined.

- 1. Enter each iteration i with a guess for the interest rate path, or $\overrightarrow{r}_2 = \{r_t\}_{t=2}^{e+1}$. Denote this guess by $\overrightarrow{r}_2^i = \{r_t^i\}_{t=2}^{e+1}$. Using (11) this will imply a guess for wages $\overrightarrow{w}_2^i = \{w_t^i\}_{t=2}^{e+1}$. Note that by assumption $r_{e+1} = r^*$ and $w_{e+1} = w^*$.
- 2. Using this guess, solve out for $\overrightarrow{s}_1 = \{s_t\}_{t=1}^{e+1}$. This is done as in the manner below:
 - (a) Enter period t with state of world s_t , which was computed in the previous period t - 1. For each j and λ solve the household's decision problems to obtain $\hat{A}_t^j(\lambda)$, $\hat{P}_t^j(\lambda)$, and $\hat{T}_t^j(\lambda)$. Set $s_{t+1} = (\hat{A}_t^2(\lambda), \hat{A}_t^3(\lambda), ..., \hat{A}_t^n(\lambda))$. Move onto period t + 1 (unless t = e, in which case you're finished).
 - (b) For age-j agent, with skill level λ , permanent income in period t will be given by the formula²²

$$\widehat{\Omega}_{t}^{j}(\lambda) = r_{t}^{i}\hat{a}^{j} + \sum_{m=0}^{n-j} \frac{\widehat{w}_{t+m}[\lambda\omega + \phi\lambda\omega P^{j+m}(\lambda) - q_{t+m}T^{j+m}(\lambda)]}{\prod_{k=t+1}^{t+m}(r_{k}^{i}/\gamma)}$$

(c) The period-t market clearing wage can be obtained by finding the \hat{w}_t such that (4) holds. Set $\hat{w}_{t+m} = w_{t+m}^i$ for m > 0.

Compute a revised guess for the interest rate path \overrightarrow{r}_2 , denoted by $\overrightarrow{r}_2^{i+1}$, using the formula

$$r_{t+1}^{i+1}/\gamma = \alpha (\mathsf{I}_{t+1}/\widehat{\mathsf{k}}_{t+1})^{1-\alpha} + \chi.$$

²²In this formula, $\Pi_{n=t+1}^{t} \equiv 1$.

It may be better to set

$$r_{t+1}^{i+1}/\gamma = \vartheta[\alpha(\mathsf{I}_{t+1}/\widehat{\mathsf{k}}_{t+1})^{1-\alpha} + \chi] + (1-\vartheta)r_{t+1}^{i}/\gamma, \text{ for } 0 < \vartheta < 1.$$

Lemma 4: Along a balanced growth path the date of adoption is a nonincreasing function in type, at least when the length of a period is sufficiently short.

Proof. Consider the continuous-time analogue to the adopt/work problem framed by P(1) to P(4). Let the date of adoption chosen by the household be represented by α . The household will choose an interval $[\sigma, \varepsilon] \subseteq [0, n]$ over which to work. Here σ denotes the start date for working and ε denotes the end date. As an example of how things work, take the case where $\sigma = 0 < \alpha < \varepsilon < n$. Here the woman in a household starts working immediately, builds up some resources to purchase durables at age α , and then retires at ε . A type- λ household's decision problem is

$$\begin{split} &\max_{\alpha,\varepsilon} \{\mu \frac{1-e^{-\beta n}}{\beta} [\ln(\frac{\beta}{1-e^{-\beta n}}) + \ln \Omega(\lambda)] + \mu \int_{0}^{n} (r-\beta) j e^{-\beta j} dj \\ &+ \nu \frac{1-e^{-\beta n}}{\beta} \ln \delta + \nu \ln \kappa \int_{\alpha}^{n} e^{-\beta j} dj + (1-\mu-\nu) [\ln(2-2\omega-\rho\eta) \int_{0}^{\alpha} e^{-\beta j} dj \\ &+ \ln(2-2\omega-\eta) \int_{\alpha}^{\varepsilon} e^{-\beta j} dj + \ln(2-\omega-\eta) \int_{\varepsilon}^{n} e^{-\beta j} dj] \}, \end{split}$$

subject to

$$\Omega(\lambda) = w\lambda\omega \frac{1 - e^{-rn}}{r} + \phi w\lambda\omega \int_0^\varepsilon e^{-rj} dj - qw e^{-\iota\alpha}$$

Now, r represents the *net* interest rate and β is the *rate of time preference*.

The first-order conditions to this problem are:

$$\mu \frac{1 - e^{-\beta n}}{\beta} w q \iota e^{-(r-\beta)\alpha} = \left[(1 - \mu - \nu) \ln(\frac{2 - 2\omega - \eta}{2 - 2\omega - \rho \eta}) + \nu \ln \kappa \right] \Omega(\lambda),$$

and

$$\mu \frac{1 - e^{-\beta n}}{\beta} \phi w \lambda \omega e^{-(r-\beta)\varepsilon} = (1 - \mu - \nu) \ln(\frac{2 - \omega - \eta}{2 - 2\omega - \eta}) \Omega(\lambda).$$

Undertaking the requisite comparative statics exercise gives

$$\frac{d\alpha}{d\lambda} = \left[(1 - \mu - \nu) \ln(\frac{2 - 2\omega - \eta}{2 - 2\omega - \rho \eta}) + \nu \ln \kappa \right] (w\omega/r) \left[1 - e^{-rn} + \phi (1 - e^{-\varepsilon r}) \right] \\ \times (r - \beta) \frac{\mu (1 - e^{-rn}) \phi w \lambda \omega e^{-(r - \beta)\varepsilon}}{\beta \det(H)} < 0,$$

where H is the 2 × 2 Hessian associated with the maximum problem. To sign the above expression, note that the second-order conditions for a maximum necessitate that the matrix H is negative semidefinite. Necessary conditions for this to transpire are that det $(H) \ge 0$ and $H_{11}, H_{22} \le 0$, where H_{11} and H_{22} are the entries in upper left and lower righthand corner of H. When $r > \beta$ it is easy to see that $d\alpha/d\lambda < 0$. When $r < \beta$ it can be shown that a maximum cannot obtain, since $H_{11}, H_{22} < 0$ then imply det(H) < 0. There are many other cases to consider, but they all proceed in the same manner. (Basically, the rest of the proof is a boring taxonomy.)

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