University of Toronto
Department of Economics

Working Paper 426

Search Intermediaries

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April 05, 2011

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March 14, 2011

Abstract

In frictional matching markets with heterogeneous buyers and sellers, sellers incur discrete showing costs to show goods to buyers who incur discrete inspection costs to assess the suitability of the goods on offer. This paper studies how brokers can help reduce these costs by managing the level and mix of goods in their inventory. We find that intermediaries emerge and improve social welfare when there is sufficient heterogeneity in the types of goods and preferences. Our analysis highlights how learning and inventory management enable search intermediaries to internalize information externalities generated in unintermediated private search.

*This is a substantial revision of our earlier paper circulated under the title "Information Externalities and Intermediaries in Frictional Search Markets." We thank Ettore Damiano, Jean Guillaume Forand, Robert McMillan, Carolyn Pitchik, Shouyong Shi, Matt Turner, Asher Wolinsky, Andriy Zapechelnyuk, and seminar participants at Peking University, Queen's University, University of Toronto, and Econometric Society world congress at Shanghai for helpful comments and suggestions. We also thank Lucas Siow for research assistance. The first author is grateful to the Alexander von Humboldt Foundation for financial support and to the Chair of Economic Theory II at University of Bonn for its hospitality where part of this work was completed. Both authors also thank SSHRC for financial support.

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1 Introduction

In many frictional matching markets, heterogenous buyers and heterogenous sellers (characterized by different types of goods) search to match and trade with each other. Upon meeting a seller, a buyer has to inspect the good on offer to see if it matches his preference. If there is no trade, both the buyer and seller will separate and continue to search for other trading partners. While a buyer's preference and the characteristics of a good are persistent over time, buyers and sellers exit the market after transacting and they seldom return. Intermediaries are active in some of these markets and not others. What is their role?

The leading example of such a market with intermediaries is the residential housing resale market where the majority of transactions are brokered by real estate agents. Houses are different and a description in an online listing service, such as the Multiple Listing Service, does not describe a house completely. Thus a buyer must incur a costly personal inspection of the house in order to investigate all its attributes. Similarly, a seller also has to incur a discrete showing cost to show the house to interested buyers.[1] Therefore, both buyers and sellers in the housing market would like to reduce inspection or showing costs by avoiding unnecessary but costly home inspections.

Another example is the market for corporate executives where headhunters, as intermediaries, play an important role in matching workers with vacancies. Based on interviews with headhunters, Finlay and Coverdill (2002) conclude that the success of a placement in this market depends on both tangible information about candidates and employer preferences revealed through job advertisements and resumes, and intangible ones revealed through subsequent costly interviews. In other labor markets such as retail sales, workers and employers often match directly without using employment agencies. Two potential differences between labor markets which use intermediaries in hiring versus those which do not have to do with the degree of heterogeneity across workers and across employers, and the screening costs involved to ascertain the match between a worker and an employer.

The objective of this paper is to investigate the role of intermediaries in reducing inspection and showing costs in the standard framework of two-sided sequential search. We study how intermediaries use both the level and the mix of inventory to reduce these costs. Although the model is not specific to a particular market, we will use the housing market as an ongoing example to provide context for the model.

To see the advantage an intermediary may have, first consider a market without brokers. Both houses and buyer preferences are fixed and horizontally differentiated. Suppose a buyer values a house only if the characteristics of the house (seller type) fit his preferences

[1] The assumption of imperfect advertising of homes for sale is a common one in search models of housing market (e.g., Wheaton (1990)).

(buyer type). Both house characteristics and buyer preferences are difficult to articulate or describe completely a priori. Each period buyers and sellers in the market search for trading opportunities. When a type A buyer randomly meets a type b seller, the buyer incurs an inspection cost and the seller incurs a showing cost, the buyer finds out that the house is type b and decides not to buy it. The information that the house is type b has no value to this buyer for his own future search, and similarly, the information that the buyer is type A has no value to the future search of this seller. The information, however, is valuable for other buyers and sellers to avoid incurring unnecessary search costs. Since communication with other potential trading partners is costly, neither the type b seller nor the type A buyer has incentive to pass the information on to others. As a result, some socially useful information generated in private search is lost and not efficiently utilized.

Now consider such a market with sellers' brokers. Suppose these brokers do not have any inspection or showing costs advantage over buyers and sellers. A broker has to look for sellers to represent. Upon meeting a potential client, the broker has to pay an inspection cost and the seller has to incur a showing cost for the broker to inspect the good to determine its type. After an agreement to represent the seller, the broker also has to pay a showing cost to show the good to any potential buyer. Thus employing a broker to complete a transaction incurs additional resource costs which are absent without such a broker.

A broker's advantage in the market comes from the possibility that a broker can represent more than one type of sellers. In this case, it is advantageous for a buyer to contact a broker because after the broker learns of the buyer's type through a first showing, the broker can economize on further showing costs and inspection costs by not showing other goods that the buyer will not be interested in. For example, suppose a type A buyer randomly meets a broker who represents two different houses, say type b and c. After showing house b to the buyer, the broker learns that the buyer is type A. Then the broker can tell the buyer that there is no need for him to see the second house because it does not fit, saving search costs for both parties.

Note that this advantage can be materialized only when the broker has additional goods in his inventory which are of potential interest to the buyer. Otherwise, the seller may as well show the good herself. That is, each seller hires a costly broker because the broker has other types of goods in his inventory to attract potential buyers.[2] Thus brokers have to actively manage both the level and mix of goods that they represent.

We first study the perfect communication and learning case where the broker learns the

[2]The argument is reminiscent of Wolinsky's (1983) argument for why competing retailers locate in shopping malls in spite of more intense price competition. By inspecting different goods in the same location, shopping malls help consumers save on travel costs. Different from his model, the goods in our model cannot be easily relocated to a centralized location.

buyer's type perfectly after a costly first showing and inspection for both parties. We then extend the analysis to the case where the communication and learning of the buyer's type is imperfect. That is, after the buyer turns down the first good, the broker is imperfectly informed about the buyer's type. We restrict our brokers to represent at most two sellers at a time. This is the minimal size of inventory needed for brokers to exist. In order to tease out the intermediaries' role in reducing discrete search costs from their other roles, our analysis primarily focuses on the limit equilibrium as the discount rate goes to zero. In the limit equilibrium, (1) brokers will search and choose to have two different types of goods in their inventory before they search for buyers; (2) there has to be sufficient heterogeneity in buyers and goods types for brokers to exist; (3) imperfect communication and learning raise the amount of heterogeneity needed for brokers to exist; (4) brokers reduce the expected total inspection and showing costs by all parties needed to complete a transaction.

The number of houses seen by a buyer is a good proxy for the buyer search duration while the number of showings by a seller is a good proxy of the time on the market. Therefore, our model predicts that the real-estate brokers reduce the expected time for a seller to sell a house and buyers to buy a house, which is a robust finding of empirical studies on the role of real estate brokers (see for example, Baryla and Zumpano (1995), Elder, et. al. (2000), Hendel, et. al. (2009), Bernheim and Meer (2008), among others).

The literature on intermediaries in frictional search markets starts with Rubinstein and Wolinsky (1987) where intermediaries meet potential trading partners at a faster rate than potential trading partners can meet each other directly.[3] The role of inventory for intermediaries has been investigated by other researchers. Johri and Leach (2001) show that intermediaries can improve match quality and reduce delay costs if they can carry two units in a setting with heterogeneous goods and tastes. The matching quality in their model is assumed to be idiosyncratic between any pair of buyer and seller, while in our model the preferences of buyers and types of goods are persistent.[4] Therefore, the information externalities arising in private search identified in this paper are absent in their setting. In a model with heterogenous agents, Shevchenko (2004) assumes that an intermediary can increase the level of inventory with a convex cost function. The probability of a match with a potential buyer increases with the level of inventory. He studies both the optimal level of inventory and the equilibrium price distribution of goods. Our paper is complimentary to his. He assumes a reduced form specification of the cost of holding inventory.[5] While we fix

[3]A similar assumption is made in Yavas (1992) who builds a one-period search model of intermediaries with endogenous search intensity.

[4]There are a large literature of frictional matching with vertically differentiated persistent types (e.g. Burdett and Coles (1997); Smith and Shimer (2000)). The informational inefficiency identified here is also present there.

[5]Another strand of literature studies the role of centralized matching agencies in a decentralized matching

maximum inventory capacity, we explicitly derive the cost and benefit of holding inventory from the structure in which the market operates.

Finally, informational externalities in frictional matching markets without intermediaries underlies the social learning literature (see a survey by Bikhchandani, et. al., 1998). But to our best knowledge, the role of intermediaries in internalizing information externalities through learning and inventory management has not been explored.

The remaining of the paper is organized as follows. Section 2 first studies a two-sided search model without brokers, which will serve as a benchmark for our analysis of brokers. Section 3 introduces brokers into the search model. We show that, under the assumption of perfect learning, brokers improves social welfare by reducing expected total numbers of showings and inspections necessary to complete an transaction. Section 4 relaxes the assumption of perfect learning, and demonstrates that the emergence of brokers still improves social welfare as long as there is sufficient heterogeneity in goods and buyer preferences. Section 5 discusses a few extensions of the model and concludes.

2 Search without Brokers

To aid exposition, we present the model in the familiar housing market setting. Time is discrete with a period length $\triangle$ which is assumed to be small. All participants discount the future with a common discount rate r. Following the standard two-sided search literature, we assume that two parties, buyers and sellers, simultaneously search for trading opportunities in the market. Each seller has one house for sale, and each buyer wants to buy one house. Buyers and sellers in the market are matched according to a random matching technology. Specifically, at a point of time, if there are B buyers and S sellers in the market, then in each instant, $M(B, S)$ buyers will randomly match with the same number of sellers. The flow of contacts or the matching function, $M(B, S)$, is assumed to be increasing in both arguments and has constant return to scale. If we use $\theta = B/S$ to denote the market tightness, then we can write the arrival rate of a match for a seller as $m(\theta) \equiv M(B, S)/S = M(\theta, 1)$ and the arrival rate of a match for a buyer as $M(B, S)/B = m(\theta)/\theta$. Therefore, when $\triangle$ is small, each period a buyer randomly meets a seller with probability $m(\theta)\triangle/\theta$, and a seller meets a buyer with probability $m(\theta)\triangle$.

There are n types of buyers and sellers, with equal fraction of each type in the population. When a randomly chosen buyer meets a randomly chosen seller, the value of the match is either 1 or 0. If the buyer's type (preferences) matches the type of the house for sale, the value of the house to the buyer is 1. Otherwise, the house has value 0 to the buyer. Before the buyer sees the house, both the buyer and the seller do not know the match value which

markets (see for example, Bloch and Ryder (2000) and references therein).

can be found out only through costly house inspection. Every time a buyer inspects a house, the seller has to pay a showing cost c_s and the buyer has to pay an inspection cost c_b.

If the seller has the type of the house that the buyer wants to buy and they are able to negotiate a sale, both parties leave the market permanently; otherwise, both of them will return to the market. We assume that there is an incoming flow of new buyers and new sellers such that the stocks and distributions of buyers and sellers do not change over time.

The goal of the paper is to investigate the role of search intermediaries in internalizing information externalities by reducing discrete search costs. In order to tease out intermediaries' role in reducing discrete search costs, we will primarily focus on the limit (steady-state) equilibrium when r approaches 0.[6]

2.1 Equilibrium Welfare

The continuation payoff V for a seller who remains in the market at the end of period $\triangle$, when $\triangle$ is small, is given by

$$V = \frac{1}{1 + r\triangle} \left[m(\theta) \triangle \left(-c_s + \frac{1}{n} t \right) + \left(1 - \frac{1}{n} m(\theta) \triangle \right) V \right]. \tag{1}$$

To understand the formula, note that if the seller randomly meets a buyer next period (which happens with probability $m(\theta)\triangle$), she incurs a showing cost c_s to show the house to the buyer, and if after costly inspection the buyer likes the house (which happens with probability $1/n$) she sells the house at a negotiated price t; if the seller does not meet any buyer or if the seller meets a buyer but the match value turns out to be 0 after costly inspection, the seller remains in the market and receives continuation value V, which happens with probability $\left(1 - \frac{1}{n} m(\theta) \triangle\right)$.

Similarly, the continuation payoff U for a buyer who remains in the market at the end of period $\triangle$, when $\triangle$ is small, is given by

$$U = \frac{1}{1 + r\triangle} \left[\frac{m(\theta)}{\theta} \triangle \left(-c_b + \frac{1}{n} (1 - t) \right) + \left(1 - \frac{1}{n} \frac{m(\theta)}{\theta} \triangle \right) U \right]. \tag{2}$$

If the buyer randomly meets a seller next period (which happens with probability $m(\theta)\triangle/\theta$), he incurs an inspection cost c_b to inspect the house, and if he likes the house (which happens with probability $1/n$) he will buy the house at the negotiated price t. If the buyer does not meet any seller, or if the buyer meets a seller but the match value turns out to be 0 after costly inspection, the buyer remains in the market and receives U, which happens with probability $(1 - \frac{1}{n} \frac{m(\theta)}{\theta} \triangle)$.

[6] The sequential search literature worries about delay cost and is unconcerned with inspection and showing costs (e.g. Rogerson, Shimer and Wright 2005). A notable exception is Atakan (2006).

The price t is negotiated by the two trading parties. Different bargaining protocols may result in different transaction prices, but the total social welfare is independent of prices. Since we primarily concern about the total welfare implication of search intermediaries, we do not assume a particular bargaining protocol. Instead, we only impose a weak requirement that under the transaction price all market participants must be willing to participate in the market. In the end of this section, we use Nash bargaining as an example to illustrate how a specific bargaining protocol can determine the expected equilibrium payoff for each market participant.

Definition 1 *A transaction price t is feasible if all trading partners are willing to participate.*

We complete the model by imposing a free entry condition for sellers. Let K be the cost to a home builder to build a house. The free entry of sellers implies $V = K$. In order for the market to exist, we assume throughout of the paper that

$$1 - nc_b - nc_s > K. \tag{3}$$

Definition 2 *A steady-state search equilibrium without intermediaries is defined by the stocks of market participants (B, S), continuation payoffs (U, V), and market price t such that*

(a) steady-state conditions (1) and (2) hold;

(b) price t is feasible;

(c) free entry condition $V = K$ holds.

We are primarily interested in the limit equilibrium with $r \to 0$ where delay cost is negligible. Since the expected number of inspections for producing a successful match is n, the total welfare $(U+V)$ in the limit equilibrium for a pair of buyer and seller is $1-nc_b-nc_s$.

Proposition 1 *In the limit equilibrium without intermediaries, the total social welfare for a pair of seller and buyer is*

$$U + V = 1 - nc_b - nc_s. \tag{4}$$

2.2 Nash Bargaining

If we assume that the transaction price t is negotiated through Nash bargaining as in the literature, then we can completely solve the model and study how the surplus is divided between buyers and sellers. To avoid the hold-up problem and a non-existence of equilibrium with trade when r approaches zero, we assume that the price t is negotiated before inspection,[7]

$$t = \arg\max_{p} \left[-c_b + \frac{1}{n}(1-p) + \frac{n-1}{n}U - U \right] \left[-c_s + \frac{1}{n}p + \frac{n-1}{n}V - V \right]. \tag{5}$$

To understand the formula, note that if a buyer agrees to price p and proceeds to inspect the house by incurring cost c_b, with probability $1/n$ he likes the house and obtains net payoff $(1-p)$, and with probability $(n-1)/n$ the house does not match his preferences, so he returns to the market. If a buyer does not agree, he has to return to the market with continuation value U. The intuition for the seller's part is similar. It follows that the transaction price t is set at

$$t = \frac{1}{2}(1 + V - U + nc_s - nc_b). \tag{6}$$

The two functional equations (1) and (2) can be rearranged into

$$rV = m(\theta)\left[-c_s + \frac{1}{n}(t - V) \right],$$
$$rU = \frac{m(\theta)}{\theta}\left[-c_b + \frac{1}{n}(1 - t - U) \right].$$

Substituting t into the functional equations, we can solve U and V as

$$U = \frac{m(\theta)}{m(\theta) + \theta m(\theta) + 2n\theta r}(1 - nc_b - nc_s),$$
$$V = \frac{\theta m(\theta)}{m(\theta) + \theta m(\theta) + 2n\theta r}(1 - nc_b - nc_s).$$

Letting $r \to 0$, we have

$$U = \frac{1}{1+\theta}(1 - nc_b - nc_s), \tag{7}$$
$$V = \frac{\theta}{1+\theta}(1 - nc_b - nc_s). \tag{8}$$

Intuitively, the social surplus is shared according to market tightness θ: the seller gets a larger share if the market condition is more favorable to the seller (i.e., a higher θ). The market tightness θ is recovered from equation (8) and the free entry condition $V = K$.

[7] Both the hold-up problem in this class of models (see Spulber 2009) and the non-existence problem of the equilibrium with trade as r approaches zero are well known (e.g. Camera and Delacriox 2003). Dealing with these problems detract from our concern here.

3 Search with Brokers: Perfect Learning

Again suppose there are n types of houses and buyers with equal proportion. We add seller-brokers (henceforth brokers) who first contact sellers to seek exclusive representation and then sell houses on the sellers' behalf to buyers. There are two physically distinct markets where trade occurs. In the sellers' market, brokers search for sellers to represent them. In the buyers' market, brokers meet buyers to arrange transactions on the sellers' behalf. Buyers, sellers and brokers can visit either market at any time. Each participant can visit only one market at a time. We say a broker completes a "transaction" after he picks up a seller representation in the sellers' market and then successfully sells the house on the seller's behalf to a buyer in the buyers' market.

A broker can represent at most two sellers. We assume for now that a broker wants to have two different types of houses in her inventory before going to the buyers' market. Upon selling one house, the broker will return to the sellers' market to find another seller to represent whose house is different from the one that the broker has already represented. After the broker has obtained representation of two different types of houses, the broker returns to the buyers' market and so on.

We assume that brokers have *no cost advantage* over sellers and buyers. First, when a seller and a broker randomly meet in the sellers' market, a costly inspection is carried out. After the seller incurs a showing cost c_s and the broker incurs an inspection cost c_b, the broker learns the type of the house. Second, when a buyer and a broker randomly meet in the buyers' market, the broker incurs a showing cost to show a randomly chosen house to the buyer who incurs an inspection cost to see the house. After costly inspection, the buyer figures out if he likes the house, and the broker also learns the buyer's preferred type of house. The next section will relax the assumption of perfect learning.

We also assume that brokers have *no matching advantage* over sellers and buyers: the matching technology between brokers and sellers (or buyers) is the same as the one in the market without intermediaries. Let A_s denote the number of brokers with one house in the sellers' market, and A_b denote the number of brokers with two houses in the buyers' market. Let θ_s denote the market tightness of the sellers' market where brokers pick up houses: $\theta_s = A_s/S$. Then in the sellers' market, the arrival rate of a match for a seller is given by $m(\theta_s) \equiv M(A_s, S)/S = M(\theta_s, 1)$, and the arrival rate of a match for a broker is $M(A_s, S)/A_s = m(\theta_s)/\theta_s$. Similarly, define $\theta_b = B/A_b$ as the market tightness of the buyers' market where brokers sell houses to buyers. Then in the buyers' market, the arrival rate of a match for a broker is $m(\theta_b)$, and the arrival rate of a match for a buyer is $m(\theta_b)/\theta_b$. To simplify notation, in what follows we write $m_s = m(\theta_s)$ and $m_b = m(\theta_b)$.

We also assume for now that (i) buyers and sellers will not trade directly, (ii) sellers will not pretend to be brokers with two houses, and (iii) buyers will not pretend to be brokers with one house. We also assume earlier that (iv) a broker wants to have two different types of houses in his inventory before going to the buyers' market. Later we will specify conditions under which, in the search equilibrium with brokers, these incentive conditions (i)-(iv) are indeed satisfied.

3.1 Equilibrium Welfare

We derive the continuation values for a seller, a broker with one house, a broker with two houses, and a buyer. A seller in the market could be in one of the following three possible states: not contracted with a broker, contracted with a broker who has another client, and contracted with a broker who has no other client.

First, the functional equation for V, the continuation value of a seller without a broker at the end of period $\triangle$, is

$$V = \frac{1}{1+r\triangle}\left[m_s\triangle\left(-c_s+\frac{n-1}{n}\left(V_a-\phi\right)\right)+\left(1-m_s\triangle\frac{n-1}{n}\right)V\right]. \tag{9}$$

Here $m_s\triangle$ is the probability for a seller to meet a broker in the sellers' market. Once they meet, the seller incurs cost c_s to show the house to the broker who incurs a cost c_b. With probability $\left(n-1\right)/n$, the house type is different from the type of the other house that the broker already represents. In this case, the seller pays a commission ϕ to the broker who will show the house on the seller's behalf. Otherwise, the seller continues to search for a broker in the sellers' market.

Second, the functional equation for V_a, the continuation value of a seller contracted with a broker who has another client, is

$$V_a = \frac{1}{1+r\triangle}\left[m_b\triangle\left(\frac{1}{n}\left(t-c_s\right)+\frac{1}{n}V_b\right)+\left(1-m_b\triangle\frac{2}{n}\right)V_a\right]. \tag{10}$$

The seller's broker meets a buyer in the buyers' market with probability $m_b\triangle$. Once the broker and the buyer meet, the broker incurs a cost c_s and the buyer incurs a cost c_b through a costly inspection to find out the buyer's preferences. If the buyer's type is a perfect match with one of the broker's house (which happens with probability $2/n$, with probability $1/n$ for each seller), the broker asks the owner of the house to show it to the buyer. Consider seller 1 who is contracted with the broker. If her house matches the buyer preferences (with probability $1/n$), she incurs cost c_s to show her house to the buyer and obtains price t; if the other house of the broker matches the buyer preferences (with probability $1/n$), then her broker has to go back to the sellers' market to pick up another house, that is, her continuation value becomes V_b which is defined below. If none of the above happens (with probability $1-m_b\triangle\frac{2}{n}$), the seller retains continuation value V_a.

Third, the functional equation for V_b, the continuation value of a seller who is contracted with a broker but her broker has no other client and thus has to return to the sellers' market to pick up another seller before selling her house in the buyers' market, is

$$V_b = \frac{1}{1+r\triangle}\left[\frac{m_s}{\theta_s}\triangle\frac{n-1}{n}V_a + \left(1 - \frac{m_s}{\theta_s}\triangle\frac{n-1}{n}\right)V_b\right]. \tag{11}$$

The term $\frac{m_s}{\theta_s}\triangle$ is the probability that the broker meets another seller in the sellers' market and $\frac{n-1}{n}$ is the probability that the second house is different from the type of the first house that the broker already represents. That is, $\frac{m_s}{\theta_s}\triangle\frac{n-1}{n}$ is the probability for the broker to successfully pick up another seller, and in this case the continuation value for the seller becomes V_a. Otherwise, the broker has to continue search in the sellers' market for another period.

Let W_1 and W_2 be the continuation value of a broker with one house and a broker with two houses at the end of period $\triangle$, respectively. Then the functional equation for a broker with one house is

$$W_1 = \frac{1}{1+r\triangle}\left[\frac{m_s}{\theta_s}\triangle\left(-c_b + \frac{n-1}{n}(W_2+\phi)\right) + \left(1 - \frac{m_s}{\theta_s}\triangle\frac{n-1}{n}\right)W_1\right]. \tag{12}$$

Here $\frac{m_s}{\theta_s}\triangle$ is the probability for a broker with one house to meet another seller in the sellers' market. Once they meet, the broker performs a costly inspection. With probability $\frac{n-1}{n}$, the second house type is different from the type of the first house represented by the broker. In this case, the broker will agree to represent the second house with commission ϕ, and the continuation value for the broker becomes W_2. Otherwise the broker has to continue search in the sellers' market.

The functional equation for a broker with two houses is

$$W_2 = \frac{1}{1+r\triangle}\left[m_b\triangle\left(-c_s + \frac{2}{n}(W_1 - \frac{1}{2}c_s)\right) + \left(1 - m_b\triangle\frac{2}{n}\right)W_2\right]. \tag{13}$$

The term $m_b\triangle$ is the probability for the broker to meet a buyer in the buyers' market. Upon a meeting, the broker shows a first house to the buyer at a cost of c_s. After showing the house, the broker will learn whether the buyer likes the first or second house, or other types of houses. If the buyer likes the first house (with probability $1/n$), the buyer will next deal with the seller to buy the house. If the broker learns that the buyer likes the second house (with probability $1/n$), he will incur another showing cost c_s to show the second house to the buyer. After seeing the second house, the buyer will deal with the seller to buy the house. After showing the first house, if the broker learns that the buyer does not like either house, they separate and the broker has to continue search in the buyers' market for another period.

Finally, let U be the continuation value of a buyer who remains in the market at the end of period $\triangle$. Then we have

$$U = \frac{1}{1+r\triangle}\left[\frac{m_b}{\theta_b}\triangle\left(-c_b + \frac{2}{n}\left(1 - t - \frac{3}{2}c_b\right)\right) + \left(1 - \frac{m_b}{\theta_b}\triangle\frac{2}{n}\right)U\right]. \tag{14}$$

The term $\frac{m_b}{\theta_b}\triangle$ is the probability for a buyer to meet a broker with two houses in the buyers' market. Once a buyer meets a broker, the buyer incurs an inspection cost c_b to see a house. If the first house fits (with probability $1/n$), the buyer will incur another inspection cost to deal with the seller and buy the house for price t. If the first house does not fit and the second house fits (with probability $1/n$), the buyer will incur another inspection cost to see the second house, and a further inspection cost to deal with the seller and buy the house for price t. If neither house fits, the buyer separates from the broker and continue to search in the buyers' market for another period.

As in the previous section, we defer specifying the bargaining protocol that determines the transaction price t and the commission ϕ. We only require that the price and the commission are such that both sellers and buyers are willing to participate in the markets and the payoffs for brokers are bounded in the limit equilibrium.

Definition 3 *A pair of commission ϕ and price t are feasible if*

(a) both buyers and sellers are willing to participate in the market, and

(b) the expected profit for a broker from each transaction is zero as $r \to 0$.

Recall that each transaction consists of a purchase and a sale. If commission ϕ and price t are equilibrium commission and price, brokers must earn zero profit from each transaction. Otherwise, they will make infinite total profits because they are infinitely lived. In particular, when $r \to 0$, the commission ϕ just covers the expected search costs incurred by a broker to complete a transaction, that is,

$$\phi = \frac{n+1}{2}c_s + \frac{n}{n-1}c_b. \tag{15}$$

To see this, notice that a broker needs to meet $n/2$ buyers on average in order to find a buyer who will like one of the two houses, and the broker needs to meet $n/(n-1)$ sellers in order to re-stock a house. And when he finds a buyer who matches a house, the broker may still need to incur another showing cost with probability $1/2$.

In the steady state, the number of brokers picking up houses successfully in the sellers' market must be equal to the number of brokers selling houses successfully in the buyers' market. That is,

$$A_s\frac{n-1}{n}\frac{m_s}{\theta_s} = A_b\frac{2}{n}m_b. \tag{16}$$

Finally, we impose free entry conditions to complete the model. First, with free entry of home builders, sellers must get the same reservation utility as home builders, that is, $V = K$. Second, we impose free entry condition for brokers. Let L denote the broker's outside option. The broker without a client can go to the sellers' market to pick up a house by incurring cost c_b. But the continuation value for a broker with one client is W_1. Therefore, when $r \to 0$, we can write the free entry condition for the broker as:

$$W_1 = L + c_b - \phi.$$

Definition 4 *A steady-state search equilibrium with brokers is defined by the stocks of market participants (B, S), continuation payoffs $(V, V_a, V_b, W_1, W_2, U)$, commission ϕ and market price t such that*

(a) steady-state conditions (9)-(16) hold;
(b) commission ϕ and price t are feasible;
(c) free entry conditions for sellers and brokers hold;
(d) incentive conditions (i)-(iv) are satisfied.

The following proposition characterizes the equilibrium total welfare.

Proposition 2 *In the limit equilibrium with intermediaries and perfect learning, the total expected payoff for a pair of buyer and seller is*

$$U + V = 1 - \frac{n}{n-1}(c_b + c_s) - \frac{n+3}{2}(c_b + c_s) \tag{17}$$

Proof. Since the price and the commission are feasible, the brokers earn zero profit from each successful transaction. Therefore, the total expected payoff of buyers and sellers coincide with the total social welfare.

First, a successful transaction takes $\frac{n}{n-1}$ inspections on average in the sellers' market because a broker picks up a new seller with probability $\frac{n-1}{n}$. Second, a buyer needs to talk to $\frac{n}{2}$ brokers on average in order to find a good match because the probability that a buyer likes one of the two houses managed by a broker is $\frac{2}{n}$. Moreover, in case there is a match, the buyer needs to deal with the seller of the house one more time to verify the match. As a result, a successful transaction needs $\frac{n}{2} + 1$ inspections in the buyers' market. Finally there is an expected $\frac{1}{2}$ extra inspection when a broker meets a buyer because conditional on a match, the second house, rather than the first, fits the buyer with probability 1/2.

Therefore, the total expected search cost for a successful transaction is $\frac{n}{n-1}(c_b + c_s) + \frac{n+3}{2}(c_b + c_s)$. The claim then follows from the fact that the value of a good match is 1. ■

By comparing the social welfare with brokers (17) and without brokers (4), we obtain the welfare improvement due to brokers:

$$\Delta(U + V) = \frac{n^2 - 6n + 3}{2(n-1)}(c_b + c_s). \tag{18}$$

Therefore, the introduction of brokers improves social welfare as long as $n \geq 6$. Intuitively, introducing brokers into the market adds two extra rounds of screening costs from using brokers. In order to recover these additional screening costs, the degree of heterogeneity must be sufficiently large such that buyers can avoid inspecting houses that they will reject.

3.2 Incentive Conditions

In order to fully characterize the equilibrium, we need to find conditions under which the following incentive conditions hold: (i) buyers and sellers will not trade directly, (ii) sellers will not directly go to the buyers' market, (iii) buyers will not directly go to the sellers' market, and (iv) brokers will not go to the buyers' market unless they have picked up two seller representations with two different types of houses.

Recall that, when $n \geq 6$, the joint payoffs of buyers and sellers are higher in dealing with brokers compared to trading directly. By trading directly either the seller or the buyer will be worse off and thus at least one of the two parties will refuse to trade directly. Therefore, incentive condition (i) is satisfied if $n \geq 6$. It remains to find conditions for (ii)-(iv).

Seller's Incentives

A seller can pretend to be a broker with two houses and approach buyers in the buyers' market. If a buyer rejects the house on offer, the seller can say that the other house in her phantom inventory does not fit the buyer's preference either.

When a seller goes to the buyers' market directly by pretending to be a broker, she saves the commission but incurs an increase in expected showing cost equal to

$$nc_s - \left(\frac{n}{n-1} + 1\right) c_s,$$

where the first term is the expected showing cost without a broker and the second term is the expected showing cost with a broker.

Therefore, a seller will not pretend to be a broker if

$$\phi = \frac{n+1}{2} c_s + \frac{n}{n-1} c_b \leq nc_s - \left(\frac{n}{n-1} + 1\right) c_s,$$

which is equivalent to

$$\frac{c_b}{c_s} \leq \frac{n^2 - 6n + 3}{2n}. \tag{19}$$

Buyer's Incentives

A buyer can pretend to be a broker with one house and buy directly from a seller at price $(t - \phi)$ in the following way. If the house fits the buyer's preference, the buyer pays $(t - \phi)$

to the seller who will happily accept. If the house does not fit, the buyer tells the seller that the house coincides what he already has.

When a buyer goes to the sellers' market directly by pretending to be a broker, he gains from price reduction of ϕ but incurs an increase in expected inspection cost equal to

$$nc_b - \frac{n}{2}\left(1 + \frac{2}{n}\left(1 + \frac{1}{2}\right)\right) c_b$$

The first term is the expected inspection cost without a broker and the second term is the expected cost with a broker.

Buyers will not enter the sellers' market directly if

$$\phi = \frac{n+1}{2}c_s + \frac{n}{n-1}c_b \leq nc_b - \frac{n}{2}\left(1 + \frac{2}{n}\left(1 + \frac{1}{2}\right)\right) c_b$$

That is

$$\frac{c_b}{c_s} \geq \frac{n^2 - 1}{n^2 - 6n + 3}. \tag{20}$$

Broker's Incentives

To characterize the broker's incentive conditions, we focus on the potential gain of the broker from each transaction by deviating from the assumed optimal strategy. Recall that a transaction for a broker consists of a successful representation for a seller and a successful sale to a buyer. If a broker cannot gain from deviation for one transaction, then it cannot gain by deviating for more than one transactions. Therefore, although in principle the broker can deviate for any number of transactions, it is sufficient to show that the broker cannot gain from deviation for one transaction.

We first find conditions under which a broker with one house will not immediately go to the buyers' market to look for a buyer. Since a broker's deviation cannot affect market prices, we only need to compare expected cost to complete a transaction. Consider a broker with one house. If he picks up another house of a different type before he goes to the buyers' market, his additional expected cost to complete a transaction is

$$\frac{n}{n-1}c_b + \frac{n+1}{2}c_s.$$

If a broker with one house goes to the buyers' market directly, his expected cost to completing a transaction is nc_s. Therefore, a broker with one house will not pretend to be a broker with two houses and search buyers directly if

$$\frac{n}{n-1}c_b + \frac{n+1}{2}c_s \leq nc_s$$

which reduces to

$$\frac{c_b}{c_s} \leq \frac{n^2 - 2n + 1}{2n}. \tag{21}$$

Now we look for conditions to insure that a broker will not go to the buyers' market with two identical houses. Suppose a broker with one house meets a seller to pick up a second house and finds out that it is a duplicate of the first. This broker's expected cost of completing a transaction is no different from that of a broker with one house. We have shown earlier that under condition (21), a broker with one house wants to find another house which is not the duplicate of the first. Since his inspection of the duplicate second house is already sunk, there is no additional cost to discarding it and searching for a different house compared with a broker with one house. Therefore, as long as condition (21) holds, the broker will reject the seller and continue search in the sellers' market rather than go to the buyers' market with two identical houses.

3.3 Summary

It is easy to see that the broker's incentive condition (21) are implied by the seller's incentive condition (19) for all $n \geq 2$. Therefore, all parties' incentive conditions are satisfied if $n \geq 6$ and

$$\frac{n^2 - 1}{n^2 - 6n + 3} \leq \frac{c_b}{c_s} \leq \frac{n^2 - 6n + 3}{2n}. \tag{22}$$

Notice that the set of cost ratio c_b/c_s that satisfies above constraints is non-empty as long as $n \geq 9$. Moreover, as n is large, the seller's IC constraint (19) is always satisfied, while the buyer's IC constraint is also satisfied if $c_b > c_s$.

The rationale for seller brokers is to hold inventory in order to economize on the buyers' expected inspection costs. Thus it should not be surprising that such equilibria exists only when inspection cost exceeds showing costs (see Section 5 for a discussion when the reverse is true). The following proposition summarizes the main result of this section.

Proposition 3 *Suppose the broker's learning of buyers' preferences is perfect. A search equilibrium with brokers exists and improves social welfare if* $n \geq 9$ *and condition (22) holds.*

3.4 Nash Bargaining

This section shows that a Nash bargaining protocol for determining the commission and the price of the house can support the above equilibrium with sellers' brokers. Moreover, the bargaining protocol provides unique individual equilibrium payoffs.

Suppose the commission ϕ between a seller and a broker is established by Nash bargaining before the broker inspects the house:

$$\phi = \arg\max_p \left(-c_b + \frac{n-1}{n}(p+W_2) + \frac{1}{n}W_1 - W_1\right)\left(-c_s + \frac{n-1}{n}(-p+V_a) + \frac{1}{n}V - V\right)$$
$$\Rightarrow \phi = \frac{1}{2}\left(V_a - V - W_2 + W_1 + \frac{n}{n-1}(c_b - c_s)\right)$$

The interpretation of the objective is analogous to (5) in the previous section.

We assume that after the broker meets with the buyer, the broker first negotiates a price t with the buyer on the sellers' behalf before performing costly inspections. Specifically, the transaction price t between a seller and a buyer is determined by Nash bargaining as follows:

$$t = \arg\max_p \left(\left[\frac{1}{n}(p-c_s) + \frac{1}{n}V_b + \frac{n-2}{n}V_a\right] - V_a\right)\left(\left[-c_b + \frac{2}{n}\left(1-p-\frac{3}{2}c_b\right) + \frac{n-2}{n}U\right] - U\right).$$

From a seller's perspective, if she agrees to price p, with probability $1/n$ her house may match the buyer's preferences in which case she gets $(p - c_s)$, and with probability $1/n$ the other house the broker is representing matches the buyer's preferences in which case she gets V_b. If none of the two houses match the buyer's preferences (which happens with probability $(n-2)/n$), her continuation value will be V_a. If she does not agree to price p, she gets her outside option V_a. From a buyer's perspective, if he agrees to price p and incurs cost c_b, with probability $1/n$ he will find out that the first of the two houses matches his preferences in which case he gets $(1-p-c_b)$, and with probability $1/n$ the second house (the one the broker didn't show) of the two houses matches his preferences in which case he gets $(1-p-2c_b)$. With probability $(n-2)/n$ neither house matches his preferences and he obtains continuation value U. If he rejects price p, he gets outside option U.

Therefore, the price is given by

$$t = \frac{1}{2}\left(1 + 2V_a - V_b - U + c_s - \frac{n+3}{2}c_b\right).$$

Substitute ϕ and t into the functional equations. We can show with some algebra that $V = \theta_s W_1$ and $V_a = \frac{1}{2}\theta_b U$. Taking $r \to 0$, we solve U and V in the limit equilibrium:[8]

$$\begin{aligned} U &= \frac{2}{\theta_b + 2}\left(1 - \frac{n+3}{2}c_b - c_s\right) \\ V &= \frac{\theta_b}{\theta_b + 2}\left(1 - \frac{n+3}{2}c_b - c_s\right) - \frac{n+1}{2}c_s - \frac{n}{n-1}(c_b + c_s) \end{aligned}$$

It follows that

$$U + V = 1 - \frac{n}{n-1}(c_b + c_s) - \frac{n+3}{2}(c_b + c_s)$$

[8] In limit equilibrium, we also obtain $W_1 = V/\theta_s$ and $W_2 = W_1 - \frac{n+1}{2}c_s$. Together with the free entry conditions and equation (16), one can pin down the market tightness θ, θ_b and θ_s.

which is consistent with Proposition 2. Moreover, one can easily verify that the equilibrium commission is indeed given by (15).

4 Search with Brokers: Imperfect Learning

In the previous section, we assume that after the costly first showing, the broker learns perfectly the buyer's type. This section analyzes the case where the broker's learning is imperfect. If the buyer likes the first house that he is shown by the broker, he will meet with the seller to buy it. If he does not like the first house, the broker is imperfectly informed as to what is his preferred type of house. Assume that the broker learns a set of houses, Θ, of cardinality k that the buyer's preferred house lies in. The set Θ contains the buyer's preferred house and each broker also randomly draws $k-1$ other types of houses from the $n-2$ house types (excluding the preferred type and the first house). The learning of the broker is more precise for a smaller k, and we have the special case of perfect learning if $k=1$. We assume that the set Θ is random and independent across buyer-broker pairs.

As in the previous section, we assume for now that (i) buyers and sellers will not trade directly, (ii) sellers will not pretend to be brokers with two houses, (iii) buyers will not pretend to be brokers with one house, and (iv) brokers will not go to the buyers' market unless they have two different types of houses in their inventory. Later we will specify conditions under which the incentive conditions (i)-(iv) hold.

4.1 Equilibrium Welfare

We use the same notations $(V, V_a, V_b, W_1, W_2, U)$ to denote continuation values of sellers, brokers, and buyers at the end of period $\triangle$. Since imperfect learning is relevant only in the buyers' market, the functional equations relating to the sellers' markets, V, V_b, and W_1 are the same as in the previous section.

The functional equations pertaining to the buyers' market, V_a, W_2 and U, may be different under imperfect learning. Consider a broker with two houses who search for buyers in the buyers' market. Once the broker meets a buyer, they incur inspection and showing costs to see the first house. If the buyer likes the first house, he will proceed to meet with the seller to buy the house. If he does not like the first house, the broker learns that the buyer type may be one of the k types in set Θ. If the broker's second house is in set Θ, she will show it to the buyer. If the buyer likes the second house, he will buy it. Otherwise they will separate. They will also separate if the second house is not in Θ.

First, we argue that the seller's functional equation when she has a broker, V_a, is unchanged as in the case of perfect learning. The reason is that she does not incur any extra showing cost associated with imperfect learning. Her only showing cost in the buyers' market occurs when the broker has found a match of her house with a buyer which is the same as in the perfect learning case. So as before, V_a is:

$$V_a = \frac{1}{1+r\triangle}\left[m_b\triangle\left(\frac{1}{n}(t-c_s)+\frac{1}{n}V_b\right)+\left(1-m_b\triangle\frac{2}{n}\right)V_a\right]$$

Next consider a buyer who randomly meets a broker with two houses. First, he incurs a cost c_b to talk to the broker and inspect the first house. If he likes the house, he meets with the seller and buys it. If he does not like the first house and the second house is in Θ, the buyer incurs another cost c_b to see the house. If he likes it, which happens with probability $1/k$, he will meet the seller and buy the house at a negotiated price t. Otherwise, they separate. Thus the functional equation for a buyer in the buyer's market, U, is:

$$U = \frac{1}{1+r\triangle}\left[\frac{m_b}{\theta_b}\triangle\left(-c_b+\frac{1}{n}(1-t-c_b)+\frac{n-1}{n}\frac{k}{n-1}(-c_b+\frac{1}{k}(1-t-c_b))\right)+\left(1-\frac{m_b}{\theta_b}\triangle\frac{2}{n}\right)U\right]$$

Compared with the perfect learning case, the buyer incurs an additional expected inspection cost of $\frac{k-1}{n}c_b$ where $\frac{k-1}{n}$ is the probability that he will see a second house which does not fit.

Finally, consider the problem of the broker with two houses who meets a buyer. He incurs c_s to show the first house. The house fits the buyer with probability $\frac{1}{n}$. With probability $\frac{n-1}{n}$, the first house does not fit. In this case, he shows the second house with probability $\frac{k}{n-1}$. The second showing will succeed with probability $\frac{1}{k}$. If the buyer does not buy a house from the broker's clients (which occurs with probability $\frac{n-2}{n}$), the broker has to look for another buyer. Thus the functional equation for the broker with two houses is:

$$W_2 = \frac{1}{1+r\triangle}\left[m_b\triangle\left(-(1+\frac{k}{n})c_s+\frac{2}{n}W_1\right)+\left(1-m_b\triangle\frac{2}{n}\right)W_2\right]$$

Compared with the perfect learning case, the broker incurs an additional expected showing cost of $\frac{k-1}{n}c_s$ where $\frac{k-1}{n}$ is the probability that he will show a second house which does not fit the buyer.

As in the previous section, when $r \to 0$, the brokers must be making zero expected profit per completed transaction. Thus the commission a broker receives to sell a house is equal to the expected search costs incurred by the broker. Assuming the broker already has the first house, the commission will be:

$$\phi = \frac{n}{n-1}c_b + \frac{n}{2}\left(1+\frac{n-1}{n}\frac{k}{n-1}\right)c_s = \frac{n}{n-1}c_b + \frac{n+k}{2}c_s \tag{23}$$

The intuition behind ϕ is as follows. The broker still needs to meet $n/(n-1)$ sellers in order to pick up the second house and each time he incurs an inspection cost c_b, which explains the first term. When the broker goes to the buyers' market, he needs to meet $n/2$

buyers on average in order to make a sale. When a broker meets a buyer, he incurs a showing cost c_s to show the first house, and if the first house does not fit but the second house lies in the set Θ (which incurs with probability $\frac{n-1}{n}\frac{k}{n-1}$), the broker incurs another showing cost to show the second house, which explains the second term.

In the steady state, the number of brokers picking up houses successfully must be equal to the number of brokers selling houses successfully. We also impose free entry conditions for sellers and brokers.

The following proposition characterizes the equilibrium total welfare when the broker's learning is imperfect.

Proposition 4 *In the limit equilibrium with intermediaries and imperfect learning, the total expected payoff for a pair of buyer and seller is*

$$U + V = 1 - \frac{n}{n-1}(c_s + c_b) - \frac{n+k+2}{2}(c_s + c_b) \tag{24}$$

Proof. As in the previous section, a successful transaction takes $\frac{n}{n-1}$ inspections on average in the sellers' market because a broker picks up a new seller with probability $\frac{n-1}{n}$. As we argue above when we calculate the commission (23), the expected number of searches with brokers for a buyer to find a house he likes is $\frac{n+k}{2}$. For each one of these searches the buyer and broker incurs a cost. Finally there is the final showing of house by the seller to the buyer, during which one more cost of each type is incurred. Thus, we arrive at the above equation. ■

By comparing the two expressions (24) and (4), we conclude that brokers with imperfect learning are welfare improving if

$$\begin{aligned} n &\geq \frac{n}{n-1} + \frac{n+k+2}{2} \\ &\Leftrightarrow n^2 - (k+5)\,n + k + 2 \geq 0. \end{aligned}$$

A sufficient condition is $n \geq k + 5$.

4.2 Incentive Conditions

As in the case of perfect learning, we need to check incentive compatibility conditions for sellers, buyers and brokers. Notice that when $n \geq k+5$ the joint payoffs of buyers and sellers are higher in dealing with brokers compared to trading directly. Therefore, either the seller or the buyer will be worse off by trading directly. Therefore, we only need to worry about incentive conditions (ii)-(iv).

Since the analysis here is analogous to the one in the previous section, we report the results directly and omit the details. A seller will not pretend to be a broker with two clients and approach buyers directly if

$$\frac{c_b}{c_s} \leq \frac{n^2 - (k+5)\,n + k + 2}{2n}. \tag{25}$$

A buyer will not pretend to be a broker with one client and search sellers directly if

$$\frac{c_b}{c_s} \geq \frac{(n+k)(n-1)}{n^2 - (k+5)\,n + k + 2}. \tag{26}$$

Finally, a broker with one house will not immediately go to the buyers' market to look for a buyer if

$$\frac{c_b}{c_s} \leq \frac{(n-k)\,(n-1)}{2n}. \tag{27}$$

4.3 Summary

Again, the broker's incentive condition (27) is implied by the seller's incentive condition (25) for all $n \geq 2$. Therefore, all parties' incentive conditions are satisfied if $n \geq k+5$ and

$$\frac{(n+k)(n-1)}{n^2 - (k+5)\,n + k + 2} \leq \frac{c_b}{c_s} \leq \frac{n^2 - (k+5)\,n + k + 2}{2n}. \tag{28}$$

The set of cost ratio c_b/c_s that satisfies above constraints is non-empty as long as n is large relative to k. When n is large relative to k, the second inequality in (28) is always satisfied, while the buyer's IC constraint is also satisfied if c_b is relatively higher than c_s. To summarize our analysis with imperfect learning, we have

Proposition 5 *Suppose the broker's learning is imperfect. A search equilibrium with brokers exists and improves social welfare if* $n \geq k+5$ *and condition (28) holds.*

It is intuitive that if the broker learns less from costly inspections (i.e., a higher k) we need a higher heterogeneity (i.e., a higher n) in order for brokers to exist. As is clear from our analysis, our results with imperfect learning are qualitatively similar to the case with perfect learning.

4.4 Nash Bargaining

As in the case with perfect learning, with Nash bargaining we can completely solve individual payoffs to market participants. The commission ϕ between the seller and the broker is established by Nash bargaining before inspection:

$$\phi = \arg\max_p \left(-c_b + \frac{n-1}{n}(p + W_2) + \frac{1}{n}W_1 - W_1\right)\left(-c_s + \frac{n-1}{n}(-p + V_a) + \frac{1}{n}V - V\right)$$

$$\Rightarrow \phi = \frac{1}{2}\left(V_a - V - W_2 + W_1 + \frac{n}{n-1}(c_b - c_s)\right)$$

Similarly, we assume that after the broker meets with the buyer, the broker first negotiates a price t with the buyer on the sellers' behalf before performing costly inspections. Specifically, the transaction price t between a seller and a buyer is determined by Nash bargaining as follows:

$$t = \arg\max_p \left(\frac{1}{n}(p - c_s) + \frac{1}{n}V_b + \frac{n-2}{n}V_a - V_a\right)\left(-c_b - \frac{k}{n}c_b + \frac{2}{n}(1 - p - c_b) + \frac{n-2}{n}U - U\right)$$

$$\Rightarrow t = \frac{1}{2}\left(1 - U - V_b + 2V_a - \frac{n+k}{2}c_b + (c_s - c_b)\right)$$

The seller's part is same as in the case with perfect learning. For the buyer's part, note that the expected search costs incurred by the buyer, as derived in the proof of Proposition 4, are subtracted from the expected surplus for the buyer.

By substituting ϕ and t into the functional equations, again we obtain that $V = \theta_s W_1$ and $V_a = \frac{1}{2}\theta_b U$. Taking $r \to 0$, we obtain the value of U and V in the limit equilibrium:

$$\begin{aligned} U &= \frac{2}{2+\theta_b}\left(1 - \frac{n+k}{2}c_b - (c_s + c_b)\right) \\ V &= \frac{\theta_b}{2+\theta_b}\left(1 - \frac{n+k}{2}c_b - (c_s + c_b)\right) - \frac{n+k}{2}c_s - \frac{n}{n-1}(c_b + c_s) \end{aligned}$$

Therefore, the total social welfare derived from each completed transaction is given by

$$U + V = 1 - \frac{n}{n-1}(c_s + c_b) - \frac{n+k+2}{2}(c_s + c_b)$$

which coincides with expression (24). Moreover, one can verify that the equilibrium commission charged by the brokers are indeed given by (23).

5 Concluding Remarks

We used a stylized model to illustrate how search intermediaries can internalize information externalities arising in the two-sided frictional matching market. There are several possible extensions.

In the paper, we model the imperfect learning as a specific learning process. Our results should generalize to other learning processes as long as they are memoryless across broker-buyer pairs in the sense that a buyer cannot communicate to a new broker the information he gathers in meetings with previous brokers. With memoryless learning processes, the additional expected number of inspections a buyer has to perform in order to complete a transaction relative to perfect learning is also equal to the additional number of showings a broker has to do. The zero profit constraint for brokers then implies that the broker's commission will pick up these additional expected showing costs. Therefore, we can proceed in the same way as in the paper to calculate welfare gains due to intermediation. The other results follow analogously.

The existence of a seller-broker equilibrium depends on inspection cost being larger than showing cost. If the reverse is true, we can demonstrate the existence of a buyer-broker equilibrium using a setup similar to what we have done here. In this case, brokers first search for different types of buyers. Then they go to the sellers market to search for goods which suit their clients. If they find suitable goods, they will show them to their clients, and the matched buyers and sellers will trade. Thus this class of models potentially rationalizes different kinds of intermediaries, if any, in different markets. Alternatively, one can relax the size two inventory capacity constraint to allow a broker to carry more than two units. A larger inventory size will amplify the advantage of learning and inventory management, allowing a seller-broker equilibrium to exist even when inspection cost is larger than showing cost.

We may also want to consider directed search rather than random sequential search. Consider a housing market in which there are neighborhoods and houses. Real estate agents specialize by neighborhoods. Buyers have to choose a neighborhood and a house within the neighborhood. Neighborhoods are fixed geographically. So a buyer can choose to search within specific neighborhoods. In this case, we have a hybrid model of directed and sequential search.

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