# Elections and Strategic Positioning Games[¤]

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In the famous terms of Anthony Downs (1957: 28), parties "formulate policies in order to win elections, rather than win elections in order to formulate policies."

Abstract

We formalize the interplay between expected voting behavior and stragetic positioning behavior of candidates as a common agency problem in which the candidates (i.e., the principals) compete for voters (i.e., agents) via the issues they choose and the positions they take. A political situation is de…ned as a feasible combination of candidate positions and expected political payo¤s to the candidates. Taking this approach, we are led naturally to a particular formalization of the candidates' positioning game, called a political situation game. Within the context of this game, we de…ne the notion of farsighted stability (introduced in an abstract setting by Chwe (1994)) and apply Chwe's result to obtain existence of farsightedly stable outcomes. We compute the farsightedly stable sets for several examples of political situations games, with outcomes that conform to real-world observations.

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# 1 Introduction

Overview

In an election, voters' preferences over candidates depend in part upon the positions taken by the candidates on the issues. In turn, the issues which emerge during the campaign and the positions taken by the candidates depend jointly on the electoral system in place, the expected voting behavior of the electorate, and the strategic positioning behavior of candidates. In this paper, we assume that the electoral system in place selects a winning candidate via a simple plurality rule, and we focus on the interplay between expected voting behavior and strategic positioning behavior of candidates. We formalize this interplay as a common agency problem in which the candidates (i.e., the principals) compete for voters (i.e., agents) via the issues they choose and the positions they take. A political situation is de…ned as a feasible combination of positions by the candidates and expected political payo¤s to the candidates. The framework naturally leads to a particular formalization of the candidates' positioning game, called a political situation game. Within the context of this game, we de…ne the notion of farsighted stability, introduced by Chwe (1994) in an abstract setting, and, based on Chwe's existence result, show existence of farsightedly stable political situations. Stated informally, a political situation is farsightedly stable if no candidates (acting individually or collusively) have incentives to alter their positions on issues (and hence possibly their political payo¤s) for fear that such alterations might induce further position changes (or deviations) by other candidates that, in the end, leave some or all of the initially deviating candidates in a political situation where they are not better o¤ - and perhaps worse o¤. The notion of farsighted stability captures, in a way not possible with the myopic Nash equilibrium notion, the farsighted nature of political strategizing and position taking by candidates in political campaigns. Moreover, unlike pure strategy Nash equilibria, farsightedly stable political situations always exist.

In order to illustrate the notion of farsighted stability within the context of a political situation game, we present several examples. In all of our examples, two candidates compete for a single political o¢ce in a campaign in which the candidates can take (or not take) positions on two issues. Moreover, in all of our examples, we assume that each candidate's expected payo¤ is given by the candidate's relative expected voting share (i.e., the fraction of all votes caste, caste for the candidate). The simplicity of the examples allows us, in each case, to compute the farsightedly stable set of political situations. In all but one of the examples, no pure strategy Nash equilibrium exists, but in all the examples, the farsightedly stable set of political situations is nonempty. In all but one of our examples, a single candidate emerges as the expected winner in all political situations contained in the farsightedly stable set. In these examples, the farsightedly stable set predicts a winner. However, in one example (the example corresponding to Table 5 below), the farsightedly stable set consists of two political situations. In one farsightedly stable political situation candidate 1 is the expected winner, while in the other farsightedly stable political situation, candidate 2 is the expected winner. Thus, in this example the election is too close to call in a strategic sense.

Related Literature

In our model of voting (Section 2.2), voter's are assumed to vote for a particular candidate (or abstain from voting) based on incentives created by the candidates' positions on the issues, without strategic regard for how their vote might in‡uence the outcome of the election as expressed via pivot probabilities. In this sense, our paper is related to the literature on spatial voting (Hotelling (1929), Downs (1957), and Enelow and Hinich (1990) - see Mueller (1989) for an overview) and the various extensions of spatial voting to probabilistic voting theory (see, for example, Coughlin (1992) and Lin, Enelow, and Dorussen (1999)) - rather than the literature on strategic voting and the seminal work of Ledyard (1984), Myerson and Weber (1993), and Myerson (1998).[1] However, the essential details of our model of voter preferences and voter choice are more in the tradition of the principal-agent literature rather than in the tradition of the spatial and probabilistic voting literature. Notably, unlike the case in spatial voting models, our descriptions of candidates' positions can be quite general and are not required to be representable as points on a line or points in the plane. We think of candidates' positions as playing the role of contracts which, along with the voter's type and the state of nature, determine the voter's incentives for a subsequent action choice - the choice a particular candidate as expressed via the act of voting. Thus, in our model the candidates acting as the principals compete for voters via the positions they take on the issues (i.e., via the positions they o¤er to the electorate).

In our model of the candidates' positioning game, farsighted stability replaces the Nash equilibrium notion found in, say, Osborne (1993) and McKelvey and Patty (1999). Our move away from the Nash equilibrium notion to the notion of farsighted stability, as well as our move from spatial-type positioning games to political situation games have several advantages, especially in modeling elections with more than two candidates. For example, within the context of a spatial-type positioning game, Osborne (1993) …nds that in elections involving more than two candidates, if each potential candidate has an option of not entering the campaign rather than to enter and loose, then for almost any distribution of the voters' ideal points (i.e., most preferred positions) the candidates' positioning game has no Nash equilibrium in pure strategies. Osborne (1993) also …nds that if each potential candidate prefers to enter and loose rather than to stay out of the campaign and chooses a position to maximize his plurality, then the game has no pure strategy Nash equilibrium for almost any single-peaked distribution over voters' ideal points. No such nonexistence problem arises for the farsightedly stable set of a political situation game: for any political

[1] If we include as part of each voter's type description, a vector of subjective pivot probabilities, then it is possible to specialize our model of voter choice to a model of voter choice similar in spirit to the model developed by Myerson and Weber (1993).

situation game in which candidates' can choose from finitely many positions on each of finitely many issues, the farsightedly stable set of political situations is always nonempty. Moreover, if in such a political situation game there is a strict strong Nash equilibrium, then it is automatically contained in the farsightedly stable set. Farsighted stability has another advantage. As noted by Osborne (1993), the Nash equilibrium notion of the simultaneous move candidates' positioning game fails to capture the strategic reasoning of candidates competing in a political race. Farsighted stability provides one way in which such strategic reasoning can be captured in the equilibrium notion.

To further position our research in the literature, we remark that unlike recent work by Besley and Coate (1997) and Osborne and Slivinski (1996), the set of candidates is not endogenous. One possibility we allow, however, is that a candidate may take the position "No position" on every issue. (There is no guarantee, however, that a candidate following this strategy would not win the election.)

Outline

# 2 The Model

## 2.1 Issues and Positions

Consider an election in which $K$ candidates compete for a single political office. In the campaign, there are potentially $M$ issues about which candidates can differ in their positions. We shall assume that,

(A-1) for each of finitely many issues, there is a finite number of positions that can be taken by the candidates including the position, "no position." Moreover, the set of possible positions on each issue is known to all candidates and voters.

The notion of a position can be broadly interpreted. For example, a candidate's position on the issue of gun control might be defined by a position statement (for example, that the private ownership of automatic and semi-automatic hand guns be strictly forbidden) as well as by a parameter measuring the intensity with which the candidate advertises his stated position. Thus, two candidates may have similar position statements (or messages) on a particular issue, but differ in their positions because they differ in the intensity with which they advertise their positions. Alternatively, a candidate may choose to take no position on a particular issue by choosing not to make and/or advertise his position statement. If both candidates choose not to take a position on an issue, then the issue is absent from the campaign. In this way campaign competition determines the issues in the campaign.

Negative advertising can also be captured by the notion of a candidate's position. For example, the list of campaign issues might include the issue of the character of a candidate's opponent, with the list of possible positions on the character issue including the position statement "go negative" along with a parameter measuring the intensity with which the candidate's negative advertising campaign his opponent is carried out.

Let

$$\begin{gathered} I_i := \text{ the finite set of all possible positions} \\ \text{that can be taken on issue } i = 1, 2, \dots, M; \\ \text{and let} \\ P := I_1 \times \cdots \times I_M. \end{gathered}$$

We shall denote by 0 the position, "no position." Thus, for each issue $i$, the set of possible positions $I_i$ includes the position 0, indicating that no position is being taken.

Each candidate $k = 1, 2, \dots, K$ can be described by the position type, $p_k = (p_{k1}, \dots, p_{kM}) \in P$, chosen by the candidate, that is, by the $M-$tuple of positions taken by the candidate. Thus, $p_{ki} \in I_i$ denotes the position taken by the $k^{th}$ candidate on issue $i$.[2] Let

$$P := P \times \cdots \times P$$

denote the $K$-fold Cartesian product of $P$. We shall refer to the set $P$ as the set of position profiles and we shall denote by

$$p := (p_1, p_2, \ldots, p_K) \in P \times \cdots \times P := P$$

a typical element of $P$.

We shall assume that

(A-2) each candidate $k$ $(= 1, 2, \ldots, K)$ is constrained to choose his position type from some subset $P_k$ of $P$.[3] Moreover, for $k = 1, 2, \ldots, K$, the position constraint set $P_k$ is known to all candidates and voters.

We shall denote by $P_c$ the $k$ - fold Cartesian product of the $P_k$. Thus,

$$P_c := P_1 \times \cdots \times P_k \times \cdots \times P_K.$$

## 2.2 Choice and Voter Preferences

The voter's choice set is given by

$$V = \{0, 1, 2, \ldots, K\}, \tag{1}$$

with typical element denoted by $v$. If the voter chooses $v \in V$, then the voter chooses, via his vote, candidate $v$. If the voter chooses $v = 0$, then the voter chooses not to vote.

Let

$$u(t, \omega, p, \cdot) : V \to R$$

be the utility function corresponding to a type $t \in T$ voter given state of nature $\omega \in \Omega$ and position profile $p \in P$. We shall maintain the following assumptions throughout:

(A-3) Voter types are drawn from a probability space $(T, \Sigma, \mu)$ and this probability space is known by all candidates. Here $T$ is an arbitrary set equipped with $\sigma$-field $\Sigma$ and $\mu$ is probability measure defined on $\Sigma$.

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[2] If the $k^{th}$ candidate is a special interest candidate then his position type is of the form

$$p_k = (p_{k1}, \ldots, p_{kM}) \text{ where } p_{ki} = 0 \text{ for all issues } i \neq i'$$

where $i'$ denotes the $k^{th}$ candidate's special interest.

[3] For example, if candidate $k'$ is the candidate representing the Christian Coalition, then $P_{k'}$ cannot contain an $M-$ tuple of positions $p_{k'} = (p_{k'1}, \ldots, p_{k'M})$ where the candidate's position on the issue of abortion, say issue $i'$, is pro choice (i.e., $p_{k'i'} \in I_{i'}$ cannot equal the pro choice position).

(A-4) States of nature are drawn from a probability space $(\Omega, \mathcal{F}, \lambda)$ and this probability space is known by all candidates. Here $\Omega$ is an arbitrary set equipped with $\sigma$-field $\mathcal{F}$ and $\lambda$ is probability measure defined on $\mathcal{F}$.

(A-5) States of nature and voter types are stochastically independent.

(A-6) At the time the voter makes his choice (i.e., castes his vote), the voter knows his type, the state of nature, and the position of each candidate.

(A-7) The utility function

$$u(\cdot,\cdot,\cdot,\cdot) : T \times \Omega \times P \times V \to R$$

is such that for each $(p, v) \in P \times V$, $u(\cdot,\cdot,p,v) : T \times \Omega \to R$ is $\Sigma \times \mathcal{F}$-measurable.[4]

(A-8) If for $(t, \omega) \in T \times \Omega$, position types $p = (p_1, p_2, \ldots, p_K) \in P$ are such that

$$u(t, \omega, p, v) = u(t, \omega, p, v')$$

for all $v$ and $v' = 1, 2, \ldots, K$, then

$$u(t, \omega, p, 0) > u(t, \omega, p, v) \text{ for all } v = 1, 2, \ldots, K$$

Assumption (A-8) reflects the fact that political participation (i.e., voting) is costly. Therefore, if candidates offer the voter no real choices (as expressed via their position types), then the voter is better off not voting.

The voter's choice problem can now be stated formally as follows:

$$\max \{u(t, \omega, p, v) : v \in V\}. \tag{2}$$

Because the voter can choose to abstain from voting by choosing $v = 0$, political participation is endogenous.

For each $(t, \omega, p) \in T \times \Omega \times P$ the voter's choice problem (2) has a solution. Let

$$u^*(t, \omega, p) := \max \{u(t, \omega, p, v) : v \in V\} \tag{3}$$

and

$$\Phi(t, \omega, p) := \{v \in V : u(t, \omega, p, v) \geq u^*(t, \omega, p)\}. \tag{4}$$

The function

$$u^*(t, \omega, \cdot) : P \to R$$

[4] Since the set $P \times V$ is finite, for each voter type $t \in T$ and each state of nature $\omega \in \Omega$, the utility function $u(t, \omega, \cdot, \cdot) : P \times V \to R$ is automatically continuous on $P \times V$.

gives a type $t$ voter's optimal level of utility as a function of position profiles given state of nature $\omega$. Thus, $u^*(t;\omega;\cdot)$ expresses a type $t$ voter's preferences over the set of position profiles $P$ given state of nature $\omega$. The set-valued mapping

$$(\omega;p) \to \Phi(t;\omega;p)$$

gives a type $t$ voter's best responses as a function of the state of nature and the position profile. For each voter type $t \in T$, each state of nature $\omega \in \Omega$, and each position profile $p \in P$,

$$\Phi(t;\omega;p) \text{ is a nonempty subset of } V.$$

Note that in a two candidate election (i.e., $V = \{0;1;2\}$), assumption (A-8) implies that

$$\begin{gathered}\text{for each } (t;\omega;p) \in T \times \Omega \times P, \\ \Phi(t;\omega;p) = \{v\} \text{ for some } v \in V.\end{gathered}$$

Thus, in a two candidate election, (A-8) implies that for each $(t;\omega;p) \in T \times \Omega \times P;$ $\Phi(t;\omega;p)$ is single-valued.

## 2.3 Election Mechanisms and Expected Voter Turnout

Given position profile $p = (p_1;\dots;p_K) \in P$, an election mechanism is a mapping from voter types and states of nature into the voter's choice set that specifies for each voter type and state of nature the voter's optimal candidate choice. Formally, an election mechanism is a

$$\begin{gathered}\Sigma \times z\text{-measurable function } \gamma_p(\cdot;\cdot) : T \times \Omega \to V \\ \text{such that} \\ \gamma_p(t;\omega) \in \Phi(t;\omega;p) \text{ for all } (t;\omega) \in T \times \Omega.\end{gathered}$$

Here, $\Sigma \times z$ denotes the product $\sigma$-field generated by the $\sigma$-fields $\Sigma$ and $z$.[5] Note that, if given the voter's and the state of nature, the voter is indifferent between two or more candidates, the election mechanism specifies how the tie will be broken.

We shall denote by

$$\Psi(p)$$

the set of all election mechanisms given position profile $p \in P$: Under assumptions (A-1)-(A-7), $\Psi(p)$ is nonempty. for each position profile $p \in P$. Moreover, if assumption (A-8) is added, then in a two candidate election

$$\begin{gathered}\text{for each } p \in P, \\ \Psi(p) = \{\gamma_p(\cdot;\cdot)\} \\ \text{for some } \Sigma \times z\text{-measurable function } \gamma_p(\cdot;\cdot) : T \times \Omega \to V.\end{gathered}$$

[5] A function $\gamma_p(\cdot;\cdot) : T \times \Omega \to V$ is $\Sigma \times z$-measurable if given any $v \in V$ the set

$$\{(t;\omega) \in T \times \Omega : \gamma_p(t;\omega) = v\}$$

is contained in $\Sigma \times z$.

Let

$$I_k(v) := \begin{cases} 1 & \text{if } v = k \\ 0 & \text{if } v \neq k. \end{cases}$$

Given position profile $p = (p_1, \ldots, p_K) \in P$ and election mechanism $\nu_p(\cdot,\cdot) \in \Upsilon(p)$, the expression

$$T_k(\nu_p(\cdot,\cdot)) := \int_{T\times\Omega} I_k(\nu_p(t,\omega))\,d\mu\times\lambda(t,\omega) = \mu\times\lambda\{(t,\omega) \in T\times\Omega : \nu_p(t,\omega) = k\}, \quad (5)$$

represents the $k^{th}$ candidate's expected voter share, while the expression

$$T(\nu_p(\cdot,\cdot)) := \sum_{k=1} \int_{T\times\Omega} I_k(\nu_p(t,\omega))\,d\mu\times\lambda(t,\omega) = \sum_{k=1} \mu\times\lambda\{(t,\omega) \in T\times\Omega : \nu_p(t,\omega) = k\}, \quad (6)$$

represents the corresponding expected voter turnout.[6] We shall assume that

(A-9) the set of feasible position profiles $P_c$ is such that for each $p \in P_c$ and each election mechanism $\nu_p(\cdot,\cdot) \in \Upsilon(p)$, $T(\nu_p(\cdot,\cdot)) > 0$, that is we shall assume that $P_c$ is such that expected voter turnout is positive.

## 2.4 Candidates' Expected Payoff Functions

The $k^{th}$ candidate's payoff function is given by,

$$g_k(\cdot,\cdot,\cdot,\cdot) : T\times\Omega\times P\times V \to R.$$

We shall maintain the following assumption throughout:

(A-10) For $k = 1, 2, \ldots, K$, the payoff function $g_k(\cdot,\cdot,\cdot,\cdot) : T\times\Omega\times P\times V \to R$ is such that for each position profile $p \in P$ and each election mechanism, $\nu_p(\cdot,\cdot) \in \Upsilon(p)$,

$$(t,\omega) \to g_k(t,\omega,p,\nu_p(t,\omega))$$

is $\S\times z$-measurable and $\mu\times\lambda$-integrable.[7]

[6] Thus, for $k = 0$,

$$\mu\times\lambda\{(t,\omega) \in T\times\Omega : \nu_p(t,\omega) = 0\},$$

is the expected share of voters not voting, where $\mu\times\lambda$ denotes the product measure defined on the product $\sigma$-field, $\S\times z$.

[7] The function $(t,\omega) \to g_k(t,\omega,p,\nu_p(t,\omega))$ is $\S\times z$-measurable if given any real number $\overline{g}$, the set

$$\{(t,\omega) \in T\times\Omega : g_k(t,\omega,p,\nu_p(t,\omega)) > \overline{g}\}$$

is contained in $\S\times z$. The function is $\mu\times\lambda$-integrable if

$$\int_{T\times\Omega} |g_k(t,\omega,p,\nu_p(t,\omega))|\,d\mu\times\lambda(t,\omega)$$

is finite.

Given position profile $p \in P_c$ and election mechanism $\nu_p(\cdot;\cdot) \in \Upsilon(p)$, the $k^{th}$ candidate's expected payoff is

$$\Pi_k(p;\nu_p(\cdot;\cdot)) = \int_{T\times\Omega} g_k(t;\omega;p;\nu_p(t;\omega))\, d\mu\times\lambda(t;\omega). \tag{7}$$

One possible specification for a candidate's expected payoff is relative expected voter share (REVS) given by

$$\Pi_k(p;\nu_p(\cdot;\cdot)) = \frac{T_k(\nu_p(\cdot;\cdot))}{T(\nu_p(\cdot;\cdot))},\ k = 1;2;\ldots;K. \tag{8}$$

By assumption (A-9), REVS is well-defined. Moreover, if all that matters to a candidate is winning the election, and therefore, if all that matters to a candidate is his expected voter share relative to other candidates; then assuming that each candidate's expected payoff is given by REVS is appealing. Note that a candidate, in considering a particular change in position, must take into account the possibility that while the contemplated change may induce some abstaining voters to enter and vote for him; it may also induce other voters to enter and vote for other candidates. By taking as the candidate's expected payoff relative expected voter share, this possibility is measured and taken into account. What assumptions must be made on primitives in our model to ensure that each candidates expected payoff is given by REVS? Suppose the $k^{th}$ candidate's payoff function is given by

$$g_k(t;\omega;p;v) := \frac{I_k(v)}{T(\nu_p(\cdot;\cdot))};$$

Then, given position profile $p \in P_c$ and election mechanism $\nu_p(\cdot;\cdot) \in \Upsilon(p)$; we have for each candidate $k = 1;2;\ldots;K$

$$\begin{aligned}
\Pi_k(p;\nu_p(\cdot;\cdot)) &= \textstyle\int_{T\times\Omega} g_k(t;\omega;p;\nu_p(t;\omega))\, d\mu\times\lambda(t;\omega) \\
&= \textstyle\int_{T\times\Omega} \frac{I_k(\nu_p(t;\omega))}{T(\nu_p(\cdot;\cdot))}\, d\mu\times\lambda(t;\omega) \\
&= \frac{T_k(\nu_p(\cdot;\cdot))}{T(\nu_p(\cdot;\cdot))}.
\end{aligned}$$

## 2.5 Election Mechanisms, Position Profiles, and Political Situations

Each candidate's expected payoff is determined by the positions chosen by the other candidates as well as by the election mechanism that emerges as a result of the optimizing behavior of voters.

**Definition 1** (Political Situations) We shall refer to a pair $(\pi, p)$, where $\pi = (\pi_1, \ldots, \pi_K) \in R^K$ and $p = (p_1, \ldots, p_K) \in P_c$, as a political situation if there exists an election mechanism $\nu_p(\cdot,\cdot) \in \Upsilon(p) := \Upsilon(p_1, \ldots, p_K)$ such that

$$\pi = \Pi(p, \nu_p(\cdot,\cdot)) := (\Pi_1(p, \nu_p(\cdot,\cdot)), \ldots, \Pi_K(p, \nu_p(\cdot,\cdot))).$$

We shall denote by

$$\mathbb{P} := \{(\pi, p) \in R^K \times P_c : \pi = \Pi(p, \nu_p(\cdot,\cdot)) \text{ for some } \nu_p(\cdot,\cdot) \in \Upsilon(p)\}, \tag{9}$$

the set of all political situations.

Thus, a political situation is a 2-tuple $(\pi, p)$ where $\pi = (\pi_1, \ldots, \pi_K) \in R^K$ is a vector of expected payoffs which might result if candidates choose positions (i.e., strategies) given by position profile $p = (p_1, \ldots, p_K) \in P_c$. Thus, the $k^{th}$ component, $p_k$, of position profile vector, $p = (p_1, \ldots, p_K) \in P_c$, represents the $k^{th}$ candidate's strategy choice. Given assumptions (A-1)-(A-7) and (A-10), the set of political situations $\mathbb{P}$ is nonempty. and finite.

# 3 Strategic Positioning Games and Farsightedly Stable Political Situations

Consider two political situations,

$$(\pi^0, p^0) \text{ and } (\pi^1, p^1),$$
$$\text{such that } \pi_k^1 > \pi_k^0 \text{ for candidates } k \in S,$$
$$S \text{ a nonempty subset of } N := \{1, 2, \ldots, K\}.$$

From the perspective of candidates $k \in S$, political situation $(\pi^1, p^1)$ is preferred to political situation $(\pi^0, p^0)$. Three questions now arise: (i) Is it within the power of candidates $k \in S$ acting collusively or acting independently to change the political situation from $(\pi^0, p^0)$ to $(\pi^1, p^1)$ by changing their political positions? (ii) Will such a change trigger further position changes, and thus further changes in expected payoffs, that leave some or all candidates $k \in S$ not better off and possibly worse off? (iii) Is there a political situation which is stable in the sense that no candidate or subset of candidates has incentives to change their positions for fear that such changes might trigger a sequence of changes which makes the initially deviating candidates not better off and possibly worse off? These are the questions we now address.

## 3.1 Credible Improvements in Political Situations

We begin with some definitions. Throughout we shall denote by $S$ a nonempty subset of $N := \{1, 2, \ldots, K\}$.

**Definition 2** (Credible Change and Improvement) Let $(\pi^0; p^0)$ and $(\pi^1; p^1)$ be two political situations (i.e., pairs contained in P), and let $S \subseteq N$.

(1) (Credibly Change) We say that candidates $k \in S$ can credibly change the political situation from $(\pi^0; p^0)$ to $(\pi^1; p^1)$, denoted

$$(\pi^0; p^0) \rightarrow_S (\pi^1; p^1),$$

$$\text{if } p^0_k = p^1_k \text{ for all candidates } k \in N \backslash S \text{ (i.e, } k \text{ not contained in } S).$$

(2) (Improvement) We say that political situation $(\pi^1; p^1)$ is an improvement over political situation $(\pi^0; p^0)$ for candidates $k \in S$, denoted

$$(\pi^1; p^1) \succ_S (\pi^0; p^0),$$

$$\text{if } \pi^1_k > \pi^0_k \text{ for candidates } k \in S.$$

(3) (Credible Improvement) We say that political situation $(\pi^1; p^1)$ is a credible improvement over political situation $(\pi^0; p^0)$ for candidates $k \in S$, denoted

$$(\pi^1; p^1) \triangleright_S (\pi^0; p^0),$$

$$\begin{gathered} \text{if } (\pi^0; p^0) \rightarrow_S (\pi^1; p^1), \text{ and} \\ (\pi^1; p^1) \succ_S (\pi^0; p^0). \end{gathered}$$

(4) (Farsightedly Credible Improvement) We say that political situation $(\pi^*; p^*)$ is a farsightedly credible improvement over political situation $(\pi; p)$ (or equivalently, we say that political situation $(\pi; p)$ is farsightedly dominated by political situation $(\pi^*; p^*)$), denoted

$$(\pi^*; p^*) \triangleright\triangleright (\pi; p);$$

if there exists a finite sequence of political situations,

$$(\pi^0; p^0); \ldots; (\pi^N; p^N);$$

and a corresponding sequence of sets of candidates,

$$S^1; \ldots; S^N;$$

such that

$$\begin{gathered} (\pi; p) := (\pi^0; p^0) \text{ and } (\pi^*; p^*) := (\pi^N; p^N); \text{ and} \\ \text{for } n = 1; 2; \ldots; N; \\ (\pi^{n-1}; p^{n-1}) \rightarrow_{S^n} (\pi^n; p^n) \text{ and} \\ (\pi^N; p^N) \succ_{S^n} (\pi^{n-1}; p^{n-1}). \end{gathered}$$

Thus, political situation $(\pi^*; p^*)$ is a farsighted credible improvement over political situation $(\pi; p)$ if (i) there is a finite sequence of credible changes in political situations starting with situation $(\pi; p)$ and ending with situation $(\pi^*; p^*)$, and if (ii) the expected payoff $\pi^*$ in ending political situation $(\pi^*; p^*)$ is such that for each n and each candidate $k \in S^n$, the expected political payoff in the ending situation is greater than the expected political payoff in the situation $(\pi^{n-1}; p^{n-1})$ that candidates $k \in S^n$ changed - that is, $\pi^*_k := \pi^N_k > \pi^{n-1}_k$ for each candidate $k \in S^n$.

## 3.2 Farsightedly Stable Political Situations

Again we begin with a definition.

**Definition 3** (Farsighted Stability) A subset $F$ of political situations is said to be farsightedly stable if for each political situation $(\pi^0; p^0) \in F$ the following is true: given any $(\pi^1; p^1) \in P$ such that

$$(\pi^0; p^0) \rightarrow_S (\pi^1; p^1) \text{ for candidates } S \subseteq N,$$

there exists another political situation $(\pi^2; p^2) \in F$ with

$$\text{either } (\pi^2; p^2) = (\pi^1; p^1) \text{ or } (\pi^2; p^2) \gg (\pi^1; p^1)$$

such that,

$$(\pi^2; p^2) \not\succ_S (\pi^0; p^0):$$

A subset $F^*$ of political situations is said to be the largest far sightedly stable set if for any far sightedly stable set $F$ it is true that $F \subseteq F^*$.

In words, a set $F$ of political situations is farsightedly stable, if given any political situation $(\pi^0; p^0)$ in $F$ and any credible $S$-deviation to political situation $(\pi^1; p^1) \in P$, there exists another political situation $(\pi^2; p^2)$ in $F$ such that either $(\pi^2; p^2) = (\pi^1; p^1)$ or $(\pi^2; p^2)$ farsightedly dominates $(\pi^1; p^1)$ and such that $(\pi^2; p^2)$ is not an $S$ -improvement over $(\pi^0; p^0)$. Thus, $F$ is farsightedly stable if, given any political situation $(\pi^0; p^0)$ in $F$; any credible $S$ -deviation to another political situation $(\pi^1; p^1)$ in $P$ carries with it the possibility of further credible deviations which end in a political situation that is not preferred. That is, credible deviations may continue and reach a political situation $(\pi^2; p^2) \in F$ in which all or some of the initially deviating candidates in $S$ are not better off and are possibly worse off. [8]

# 4 Political Situation Games

We may think of the set of political situations $P$ equipped with binary relation $\gg$ as describing a political situation game. We say that a political situation $(\pi^*; p^*) \in P$ is an equilibrium of the game $(P; \gg)$ if $(\pi^*; p^*)$ is contained in the largest farsightedly stable set, that is, if

$$(\pi^*; p^*) \in F^*.$$

Chwe (1994) has shown that for all games, such as the political situation game $(P; \gg)$, there exists a unique, largest farsightedly stable set (see Chwe (1994), Proposition 1). However, like the core, the largest farsightedly stable set $F^*$ may be empty. What guarantees that $F^* \neq \emptyset$?

[8] In Chwe (1994) a farsightedly stable set $F$ is called a consistent set.

## 4.1 Nonemptiness of the Farsightedly Stable Set

**Theorem 1** (Nonemptiness of $F^*$ for political situation games $(P, \gg)$) Suppose assumptions (A-1)-(A-7) and (A-10) hold. The political situation game $(P, \gg)$ has a nonempty, unique, largest farsightedly stable set $F^*$. Moreover, $F^*$ is externally stable, that is, for all $(\pi, p) \in P \backslash F^*$, there exists $(\pi^*, p^*) \in F^*$, such that $(\pi^*, p^*) \gg (\pi, p)$.

**Proof.** First, recall that under assumptions (A-1)-(A-7) and (A-10), the set of political situations $P$ is nonempty and finite. Second, note that for all $S \subseteq N$, the relation $\succ_S$ defined on $P$ is irreflexive (i.e., for all $(\pi, p) \in P$, $(\pi, p) \nsucc_S (\pi, p)$). The proof of the Theorem now follows immediately from the Corollary to Proposition 2 in Chwe (1994). ■

If $(\pi^*, p^*)$ is a farsightedly stable political situation, then we shall refer to the position profile

$$p^* = (p_1^*, p_2^*, \ldots, p_K^*) \in P_c$$

as being farsightedly stable.

## 4.2 Strict Strong Nash Equilibrium in a Political Situation Game

We say that situation $(\pi^\eta, p^\eta) \in P$ is a strict strong Nash equilibrium of the political situation game $(P, \gg)$ if $(\pi^\eta, p^\eta)$ is such that for any situation $(\pi, p) \in P$ where $(\pi^\eta, p^\eta) \rightarrow_S (\pi, p)$ for some nonempty subset of candidates $S \subseteq \{1, 2, \ldots K\}$,

$$(\pi^\eta, p^\eta) \succ_S (\pi, p).$$

**Theorem 2** (Strict strong Nash equilibria are contained in $F^*$; Chwe (1994)) Suppose assumptions (A-1)-(A-7) and (A-10) hold. If $(\pi^\eta, p^\eta) \in P$ is a strict strong Nash equilibrium of the political situation game $(P, \gg)$, then $(\pi^\eta, p^\eta)$ is contained in the farsightedly stable set, that is,

$$(\pi^\eta, p^\eta) \in F^*.$$

# 5 Examples: Two Candidate, Two Issue Elections

Consider an election model satisfying assumptions (A-1)-(A-9) in which two candidates compete for a single office and assume that for each possible position profile $p \in P_c$, the $k^{th}$ candidate's expected payoff is given by

$$\Pi_k(p, \nu_p(\cdot, \cdot)) = \frac{T_k(\nu_p(\cdot, \cdot))}{T(\nu_p(\cdot, \cdot))},$$

relative expected voter share(REVS). Recall that under assumption (A-8), in a two candidate race the election mechanism, $\gamma_p(\cdot;\cdot) \in \Delta(p)$, is unique. Moreover, given assumption (A-9), assumption (A-10) holds automatically.

Suppose now that in the campaign, there are two issues:

(1) the character of the opponent,
(2) the environment, and in particualr global warming.

On issue (1), the character issue, there are two positions,

$$I_1 = \{0; -1\}:$$

Here, $-1$ indicates that the candidate is going negative with regard to his position on his opponent's character.[9]

On issue (2), the issue of global warming, there are three positions,

$$I_2 = \{-1; 0; +1\}:$$

Here, $-1$ indicates that the candidate is taking the position (the negative position) that, thus far, the scientific evidence does not indicate that global warming is a serious problem, and that to the extent that it is a problem, the solution is best left to the market place to work out. Alternatively, $+1$ indicates that the candidate is taking the position (the positive position) that global warming is a serious problem, that an international body should be established to monitor green house gases, and that an international pollution voucher market should be established.

For candidate 1, the following positions are possible:

$$P_1 = \{(0; +1); (-1; +1); (-1; 0)\}:$$

While for candidate 2, the following positions are possible:

$$P_2 = \{(0; -1); (-1; -1); (-1; 0)\}:$$

Note that on the issue of global warming candidate 1 is constrained to take either no position or a positive position, while candidate 2 is constrained to take either no position or a negative position. We can summarize the candidates' possible position types via the following table:

| Changes in 1's Positions $\downarrow$ Changes in 2's Positions $\rightarrow$ | | |
|---|---|---|
| $((0; +1); (0; -1))_{1;1}$ | $((0; +1); (-1; -1))_{1;2}$ | $((0; +1); (-1; 0))_{1;3}$ |
| $((-1; +1); (0; -1))_{2;1}$ | $((-1; +1); (-1; -1))_{2;2}$ | $((-1; +1); (-1; 0))_{2;3}$ |
| $((-1; 0); (0; -1))_{3;1}$ | $((-1; 0); (-1; -1))_{3;2}$ | $((-1; 0); (-1; 0))_{3;3}$ |

Table 1: Position Profiles

[9] Recall that 0 indicates that no position is being taken.

As we move from northwest to southeast in Table 1, candidates' position types move from purely substantive types (taking positions on the substantive issue of global warming) to purely negative types (taking positions only on the character issue). For example, in cell $1,1$ of the table, the position profile is given by

$$(p_1, p_2) = ((0, +1), (0, -1)),$$

indicating that candidates are taking opposing positions on global warming, while taking no positions on the character issue. Alternatively, in cell $3,3$ of the table, the campaign's position profile is

$$(p_1, p_2) = ((-1, 0), (-1, 0)),$$

indicating that candidates are taking no positions on global warming, while taking negative positions on the character issue. Thus, the position profile has moved from substantive positions to nonsubstantive positions on the character issue.

## 5.1 Demographics: Position Profiles and Expected Voter Shares

Which position profiles in the table are farsightedly stable? This depends on the demographics summarizing the outcomes (i.e., expected voter shares) generated by the underlying unique election mechanism. Table 2 below summarizes the demographics.[10] The upper portion of each cell in Table 2 consists of a 3-tuple, while the lower portion of each cell gives candidates' corresponding position profile. The first entry in the 3-tuple is candidate 1's expected voter share, while the second entry is candidate 2's expected share. The third entry is the expected voter share abstaining from participation (i.e., not voting) in the election. Thus, for example cell $2,3$ in the Table 2, given by

$$\boxed{\begin{array}{c}(.254, .246, .50)\\((-1, +1), (-1, 0))\end{array}}_{2,3}$$

indicates that the voter shares corresponding to position profile $((-1, +1), (-1, 0))$ are

$$25.4\% \text{ for candidate } 1, 24.6\% \text{ for candidate } 2, \text{ and } 50\% \text{ not voting.}$$

[10] In principle, each candidate's expected voter share could be estimated using polling data.

| Changes in 1's Positions $\downarrow$ Changes in 2's Positions $\rightarrow$ | | |
|---|---|---|
| $(.234, .216, .55)$ $((0, +1), (0, -1))_{1,1}$ | $(.234, .296, .47)$ $((0, +1), (-1, -1))_{1,2}$ | $(.234, .286, .48)$ $((0, +1), (-1, 0))_{1,3}$ |
| $(.30, .25, .45)$ $((-1, +1), (0, -1))_{2,1}$ | $(.304, .266, .43)$ $((-1, +1), (-1, -1))_{2,2}$ | $(.254, .246, .50)$ $((-1, +1), (-1, 0))_{2,3}$ |
| $(.20, .25, .55)$ $((-1, 0), (0, -1))_{3,1}$ | $(.214, .266, .52)$ $((-1, 0), (-1, -1))_{3,2}$ | $(.214, .236, .55)$ $((-1, 0), (-1, 0))_{3,3}$ |

Table 2: Demographics

Table 3 below contains all possible political situations and is constructed from the information in the demographics table, Table 2. For example, cell $2,3$ in Table 3, given by

$$\boxed{\begin{array}{c}(.51, .49)\\((-1, +1), (-1, 0))\end{array}}_{2,3}$$

contains, in the upper portion, the 2-tuple $(.51, .49)$ of relative expected voter shares for candidates 1 and 2, and in the lower portion, the position profile

$$(p_1, p_2) = ((-1, +1), (-1, 0)).$$

Thus, if candidate 1 takes positions $(-1, +1)$ on the issues, while candidate 2 takes positions $(-1, 0)$, then of the expected voter turnout of 50% (= 25.4% + 24.6%, see Table 2), 51% are expected to vote for candidate 1, while 49% are expected to vote for candidate 2.[11] Thus, if candidates' position profile is $(p_1, p_2) = ((-1, +1), (-1, 0))$, then candidate 1 is expected to win the election, carrying 51% of the voter turnout to candidate 2's 49%. Thus, cell $2,3$ of Table 3 displays the political situation $(\pi, p)$ given by

$$(\pi, p) = ((\pi_1, \pi_2), (p_1, p_2)) = ((.51, .49), ((-1, +1), (-1, 0))).$$

[11] Thus, in this case $.51 = \frac{T_1(\gamma_p(\cdot,\cdot))}{T(\gamma_p(\cdot,\cdot))}$, while $.49 = \frac{T_2(\gamma_p(\cdot,\cdot))}{T(\gamma_p(\cdot,\cdot))}$.

| Changes in 1's Positions $\downarrow$ Changes in 2's Positions $\leftrightarrow$ | | |
|---|---|---|
| $(.52, .48)$ $((0,+1),(0,-1))_{1,1}$ | $(.44, .56)$ $((0,+1),(-1,-1))_{1,2}$ | $(.45, .55)$ $((0,+1),(-1,0))_{1,3}$ |
| $(.55, .45)$ $((-1,+1),(0,-1))_{2,1}$ | $(.53, .47)$ $((-1,+1),(-1,-1))_{2,2}$ | $(.51, .49)$ $((-1,+1),(-1,0))_{2,3}$ |
| $(.44, .56)$ $((-1,0),(0,-1))_{3,1}$ | $(.45, .55)$ $((-1,0),(-1,-1))_{3,2}$ | $(.48, .52)$ $((-1,0),(-1,0))_{3,3}$ |

Table 3: Political Situations (REVS & Position Profiles)

## 5.2 Computing the Unique, Largest Set of Farsightedly Stable Political Situations

Let $2^P$ denote the collection of all subsets of $P$ (including the empty set), and define the mapping

$$*_P(\cdot) : 2^P \to 2^P,$$

as follows:

given a subset of political situations $H \in 2^P$,
a political situation $(\pi^0, p^0) \in P$ is contained in $*_P(H)$
if and only if
$\forall\, (\pi^1, p^1) \in P$ such that $(\pi^0, p^0) \to_S (\pi^1, p^1)$ for some nonempty $S \subseteq N$
$\exists$ a political situation $(\pi^2, p^2) \in H$ such that
(i) $(\pi^2, p^2) = (\pi^1, p^1)$ or $(\pi^2, p^2) \ll (\pi^1, p^1)$, and
(ii) $(\pi^2, p^2) \not\succ_S (\pi^0, p^0)$, that is, $\pi_j^2 \leq \pi_j^0$ for some $j \in S$.

Thus, if $(\pi^0, p^0) \in *_P(H)$, then any move away from $(\pi^0, p^0)$ by candidates $j \in S$ (to a political situation $(\pi^1, p^1)$ in $P$) can be undone by a sequence of credible moves to some other political situation $(\pi^2, p^2) \in H$ where some candidates $j \in S$ are not better off. As has been shown by Chwe (1994), a subset $F^*$ of $P$ is the unique, largest farsightedly stable set if and only if $F^*$ is a fixed point of the mapping $*_P(\cdot)$ (i.e., if and only if $F^* = *_P(F^*)$): Because the mapping $*_P(\cdot)$ is isotonic, that is, because $H \subseteq H'$ implies $*_P(H) \subseteq *_P(H')$, the mapping $*_P(\cdot)$ has a fixed point - but it may be empty. Here, however, since the relation $\succ_S$ defined on $P$ is irreflexive (i.e., $(\pi, p) \not\succ_S (\pi, p)$ for nonempty $S \subseteq N$ and $(\pi, p) \in P$), and since the set of political situations $P$ is finite, it follows immediately from the Corollary to Proposition 2 in Chwe (1994) that

$F^*$ is nonempty. More importantly, $F^*$ can be computed by iteratively applying the mapping $\Lambda_P(\cdot)$ as follows: step 1, compute $\Lambda_P(P)$; step 2, compute $\Lambda_P(\Lambda_P(P))$; step 3, compute $\Lambda_P(\Lambda_P(\Lambda_P(P)))$ ; ... ; etc. Since P is finite, for some finite $n$, $\Lambda_P^n(P) = \Lambda_P^{n+k}(P)$ for all $k = 1, 2, \ldots$.[12] Thus,

$$F^* = \Lambda_P^n(P) = \Lambda_P(F^*).$$

Applying the mapping $\Lambda_P(\cdot)$ to the entries in Table 3, we obtain after one iteration

$$\Lambda_P(\text{Table 3}) = \left( \begin{array}{|c|} \hline (.51, .49) \\ ((-1, +1), (-1, 0)) \\ \hline \end{array}_{\,2,3} \right).$$

In the expression above, the political situations missing from the table on the right hand side are those that are indirectly dominated (i.e., **BB**-dominated), and therefore those that are not candidates for membership in the farsightedly stable set. Thus, in this example, the far sightedly stable set is given by

$$F^* = \{((.51, .49), ((-1, +1), (-1, 0)))\},$$

Referring to Table 3, note that candidate 1 has a dominate strategy, namely position type $p_1 = (-1, +1)$. In particular, if candidate 1 takes positions given by $p_1 = (-1, +1)$, then candidate 1 is expected to win no matter what positions are taken by candidate 2. Moreover, note that the far sightedly stable position profile $(p_1, p_2) = ((-1, +1), (-1, 0))$ corresponding to the farsightedly stable political situation, $(\pi, p) = ((\pi_1, \pi_2), (p_1, p_2)) = ((.51, .49), ((-1, +1), (-1, 0)))$ is a strict strong Nash equilibrium.

## 5.3 Other Possibilities

Suppose now that the underlying demographics are such that the resulting table of political situations is given by Table 4 below.

[12] Here, $\Lambda_P^n(P) := \Lambda_P \cdots \Lambda_P(\Lambda_P(P)))$ (i.e., $\Lambda_P(\cdot)$ applied iteratively n times).

| Changes in 1's Positions l Changes in 2's Positions $ | | |
|---|---|---|
| (:45; :55) ((0; +1); (0; ¡1)) $_{1;1}$ | (:48; :52) ((0; +1); (¡1; ¡1)) $_{1;2}$ | (:53; :47) ((0; +1); (¡1; 0)) $_{1;3}$ |
| (:47; :53) ((¡1; +1); (0; ¡1)) $_{2;1}$ | (:53; :47) ((¡1; +1); (¡1; ¡1)) $_{2;2}$ | (:44; :56) ((¡1; +1); (¡1; 0)) $_{2;3}$ |
| (:44; :56) ((¡1; 0); (0; ¡1)) $_{3;1}$ | (:45; :55) ((¡1; 0); (¡1; ¡1)) $_{3;2}$ | (:52; :48) ((¡1; 0); (¡1; 0)) $_{3;3}$ |

Table 4: Political Situations (REVS & Position Pro…les)

Here candidate 2, rather than candidate 1, has a dominate strategy given by position type $p_2$ = (0; ¡1). Note also that there is no Nash equilibrium position pro…le. Computing the farsightedly stable set, we obtain after one application of the mapping ¤$_P$(¢) to Table 4 the following:

$$¤_P(\text{Table 4}) = \begin{pmatrix} \boxed{\begin{matrix}(:45; :55)\\((0; +1); (0; ¡1))\end{matrix}}_{1;1} & \\ \boxed{\begin{matrix}(:47; :53)\\((¡1; +1); (0; ¡1))\end{matrix}}_{2;1} & \\ \boxed{\begin{matrix}(:44; :56)\\((¡1; 0); (0; ¡1))\end{matrix}}_{3;1} & \boxed{\begin{matrix}(:45; :55)\\((¡1; 0); (¡1; ¡1))\end{matrix}}_{3;2} \end{pmatrix}:$$

Applying the mapping ¤$_P$(¢) again; we obtain

$$¤_P(¤_P(\text{Table 4})) = \begin{pmatrix} \boxed{\begin{matrix}(:47; :53)\\((¡1; +1); (0; ¡1))\end{matrix}}_{2;1} & \\ & \boxed{\begin{matrix}(:45; :55)\\((¡1; 0); (¡1; ¡1))\end{matrix}}_{3;2} \end{pmatrix}:$$

Thus, in this case the largest farsightedly stable set consists of two political situations,

$$F^¤ = f((:47; :53); ((¡1; +1); (0; ¡1))); ((:45; :55); ((¡1; 0); (¡1; ¡1)))g:$$

Note that in both the farsightedly stable political situations above, candidate 2 is expected to win. Thus, in this example, the farsightedly stable set predicts a win by candidate 2. This prediction is hardly surprising given that candidate 2 has a dominate strategy (i.e., position type $p_2 = (0, -1)$). What is surprising, is that in farsightedly stable situation

$$(\pi, p) = ((.45, .55), ((-1, 0), (-1, -1)));$$

candidate 2 chooses position type $p_2 = (-1, -1)$ – not a dominate strategy. However, because the largest farsightedly stable set is externally stable, candidate 1 can never turn this seemingly bad choice (i.e., $p_2 = (-1, -1)$) by candidate 2 to his advantage. In the end, candidate 2 can always move the political situation back to one that is farsightedly stable - and therefore one in which he (candidate 2) is the expected winner.

Does the largest farsightedly stable set always choose one particular candidate as the expected winner in all farsightedly stable political situations (i.e., does the largest farsightedly stable set always predict a winner) - even in the absence of a dominate strategy or a Nash equilibrium?[13] As our next two examples illustrate, no general conclusions can be drawn. Some elections are simply strategically too close to call. While in others, even elections in which no candidate has a dominate strategy, the largest farsightedly stable set does seem to predict a winner.

First, consider the "too-close-to-call" case. Table 5 below summarizes the political situations.

Changes in 1's Positions $\updownarrow$ Changes in 2's Positions $\leftrightarrow$

| | | |
|---|---|---|
| $(.52, .48)$ $((0, +1), (0, -1))_{1,1}$ | $(.44, .56)$ $((0, +1), (-1, -1))_{1,2}$ | $(.45, .55)$ $((0, +1), (-1, 0))_{1,3}$ |
| $(.44, .56)$ $((-1, +1), (0, -1))_{2,1}$ | $(.53, .47)$ $((-1, +1), (-1, -1))_{2,2}$ | $(.51, .49)$ $((-1, +1), (-1, 0))_{2,3}$ |
| $(.49, .51)$ $((-1, 0), (0, -1))_{3,1}$ | $(.45, .55)$ $((-1, 0), (-1, -1))_{3,2}$ | $(.48, .52)$ $((-1, 0), (-1, 0))_{3,3}$ |

Table 5: Political Situations (REVS & Position Profiles)

Computing the farsightedly stable set, we obtain after one application of the

[13] Predict a winner in the sense that one particular candidate is the expected winner in all farsightedly stable political situations.

mapping $\ast_P(\cdot)$ the following:

$$
\ast_P(\text{Table 5}) = \begin{pmatrix}
 & & \boxed{\begin{array}{c}(.45,.55)\\((0,+1),(-1,0))\end{array}}_{1,3} \\
 & & \boxed{\begin{array}{c}(.51,.49)\\((-1,+1),(-1,0))\end{array}}_{2,3} \\
\boxed{\begin{array}{c}(.49,.51)\\((-1,0),(0,-1))\end{array}}_{3,1} & \boxed{\begin{array}{c}(.45,.55)\\((-1,0),(-1,-1))\end{array}}_{3,2} & \boxed{\begin{array}{c}(.48,.52)\\((-1,0),(-1,0))\end{array}}_{3,3}
\end{pmatrix}
$$

Applying the mapping $\ast_P(\cdot)$ again, we obtain

$$
\ast_P(\ast_P(\text{Table 5})) = \begin{pmatrix}
 & \boxed{\begin{array}{c}(.51,.49)\\((-1,+1),(-1,0))\end{array}}_{2,3} \\
\boxed{\begin{array}{c}(.45,.55)\\((-1,0),(-1,-1))\end{array}}_{3,2} &
\end{pmatrix}.
$$

As in Table 4, the farsightedly stable set consists of two political situations,

$$
F^{\ast} = \{((.45,.55),((-1,0),(-1,-1))),((.51,.49),((-1,+1),(-1,0)))\}.
$$

But now there is no agreement as to the expected winner. In farsightedly stable political situation $((.45,.55),((-1,0),(-1,-1)))$, candidate 2 is the expected winner, while in farsightedly stable political situation $((.51,.49),((-1,+1),(-1,0)))$, candidate 1 is the expected winner.

In our final example, again no candidate has a dominate strategy and no Nash equilibrium exists, but the farsightedly stable set does predict a winner. Consider the political situations given in Table 6 below.

| Changes in 1's Positions $\updownarrow$ Changes in 2's Positions $\leftrightarrow$ | | |
|---|---|---|
| $\boxed{\begin{array}{c}(.52,.48)\\((0,+1),(0,-1))\end{array}}_{1,1}$ | $\boxed{\begin{array}{c}(.44,.56)\\((0,+1),(-1,-1))\end{array}}_{1,2}$ | $\boxed{\begin{array}{c}(.45,.55)\\((0,+1),(-1,0))\end{array}}_{1,3}$ |
| $\boxed{\begin{array}{c}(.49,.51)\\((-1,+1),(0,-1))\end{array}}_{2,1}$ | $\boxed{\begin{array}{c}(.53,.47)\\((-1,+1),(-1,-1))\end{array}}_{2,2}$ | $\boxed{\begin{array}{c}(.51,.49)\\((-1,+1),(-1,0))\end{array}}_{2,3}$ |
| $\boxed{\begin{array}{c}(.44,.56)\\((-1,0),(0,-1))\end{array}}_{3,1}$ | $\boxed{\begin{array}{c}(.45,.55)\\((-1,0),(-1,-1))\end{array}}_{3,2}$ | $\boxed{\begin{array}{c}(.48,.52)\\((-1,0),(-1,0))\end{array}}_{3,3}$ |

Table 6: Political Situations (REVS & Position Profiles)

Applying the mapping $\Lambda_P(\cdot)$ to Table 6, after one iteration we have the following:

$$\Lambda_P(\text{Table 6}) = \begin{pmatrix} \boxed{\begin{array}{c}(.49,.51)\\((-1,+1),(0,-1))\end{array}}_{2,1} & \boxed{\begin{array}{c}(.51,.49)\\((-1,+1),(-1,0))\end{array}}_{2,3} \end{pmatrix}.$$

Again applying the mapping $\Lambda_P(\cdot)$, we obtain

$$\Lambda_P(\Lambda_P(\text{Table 6})) = \begin{pmatrix} \boxed{\begin{array}{c}(.49,.51)\\((-1,+1),(0,-1))\end{array}}_{2,1} \end{pmatrix}.$$

Thus, the farsightedly stable set consists of a single political situation,

$$F^* = \{((.49,.51),((-1,+1),(0,-1)))\},$$

in which candidate 2 is the expected winner.

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