Professor Robert Aumann

Autumn Quarter 1975-76

Economics 283

GAME THEORY

Final Examination

Instructions: Answer as many questions as you like. Correct answers will be awarded the indicated number of points. Incorrect answers will result in a penalty of 1/4 the indicated number of points. If you do not answer a question, you will be awarded neither positive nor negative credit for that question.

A grade of 95 to 100 entitles you to an A. A grade of over 100 entitles you to an A+. Except in the case of question 6, you are not expected to justify your answer. But please submit your scratch-work as well as your answers, and indicate which scratch-work corresponds to which question.

At the end of the exam, make a list indicating clearly on which questions you wish to be graded.

Please write your answers neatly and legibly; illegible answers may lead to a lower grade.

GOOD LUCK!

1. Find all equilibrium points and equilibrium payoffs (mixed and pure) of the following two-person games:

(a) (15 points)

1, 1	3, 0
4,0	2,1

(b) (5 points)

1, 1	0, 0
0, 0	0, 0

(c) (15 points)

1, 1	1, 1
1, 1	0, 0

2. Suppose that without changing the rules of play in chess, we define the payoff as follows: if one player wins, the loser must pay the winner $1; if the game is a draw, then both players must forfeit $1 to a neutral third party. Does the game so defined have an equilibrium point in pure strategy?

(10 points)

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3. Describe the unique symmetric N-M solution of the n-k game when

(a) $n = 7$, $k = 5$ (3 points)

(b) $n = 10$, $k = 6$ (5 points)

(c) $n = 5$, $k = 3$ (7 points)

4. Find a non-symmetric N-M solution of the n-k game for $n = 7$, $k = 5$. (40 points)

Remark: This isn't hard at all. The high number of points is given because it requires more originality than the other questions.

5. Find the core of the weighted majority game

$[8; 3,3,1,1,1,1]$ (15 points)

6. Let $N = \{1,2,3\}$. Define a pseudo-value on G^N to be a function from G^N to E^N that satisfies Shapley's conditions of symmetry, additivity, and efficiency, but not necessarily the null-player condition.

(a) Is there at least one pseudo-value on G^N? (5 points)

(b) Is there at most one pseudo-value on G^N? (20 points)

In each case, justify your answer.

7. Prove that the n-dimensional simplex is convex. (10 points)

8. Find a mixed strategy equilibrium payoff in the three-person game:

3, 3, 3	0, 0, 0
0, 0, 0	1, 1, 1

1, 1, 1	0, 0, 0
0, 0, 0	2, 2, 2

(15 points)